MATLAB

Class 8: Regressions and SVM

What is one of the ultimate goals of a model?

PREDICT

Computational models make predictions based on hypotheses.

They predict the state of an outcome variable (y) given new information x or at a new time point (xt)

How do computers learn?

In machine learning and computational modelling, the way you teach your algorithm to predict can take one of two forms: **supervised** learning and **Unsupervised** learning

How do computers learn?

supervised learning takes place when algorithms are fed examples of data to learn from like in regressions and SVM

Much like when you were a little kid, you were taught the alphabet with examples of words that start with each letter

How do computers learn?

Unsupervised learning takes place when algorithms

learn without examples, but through trial and error

without the need of labels

Much like when you learned how to ride a bike

Examples include k-means (see class on clustering)

What is a regression ?

To put it simply a regression is a set of statistical tools used to describe and quantify the relationship between dependent and independent variables

Dependent variables (y)
are the outcomes we wish to predict based on independent variables

Independent variables (x)
are the covariates/ predictors / features we
think change the value of the outcome variable
in a quantifiable way

Types of regression models

Linear Model:

Used when the outcome variables are continuous

Logistic Model:

Used when the outcome variables are binary

Hierarchical Model:

Used when your data has a defined structure (i.e., variables are nested)

*** not mutually exclusive

Linear regression models

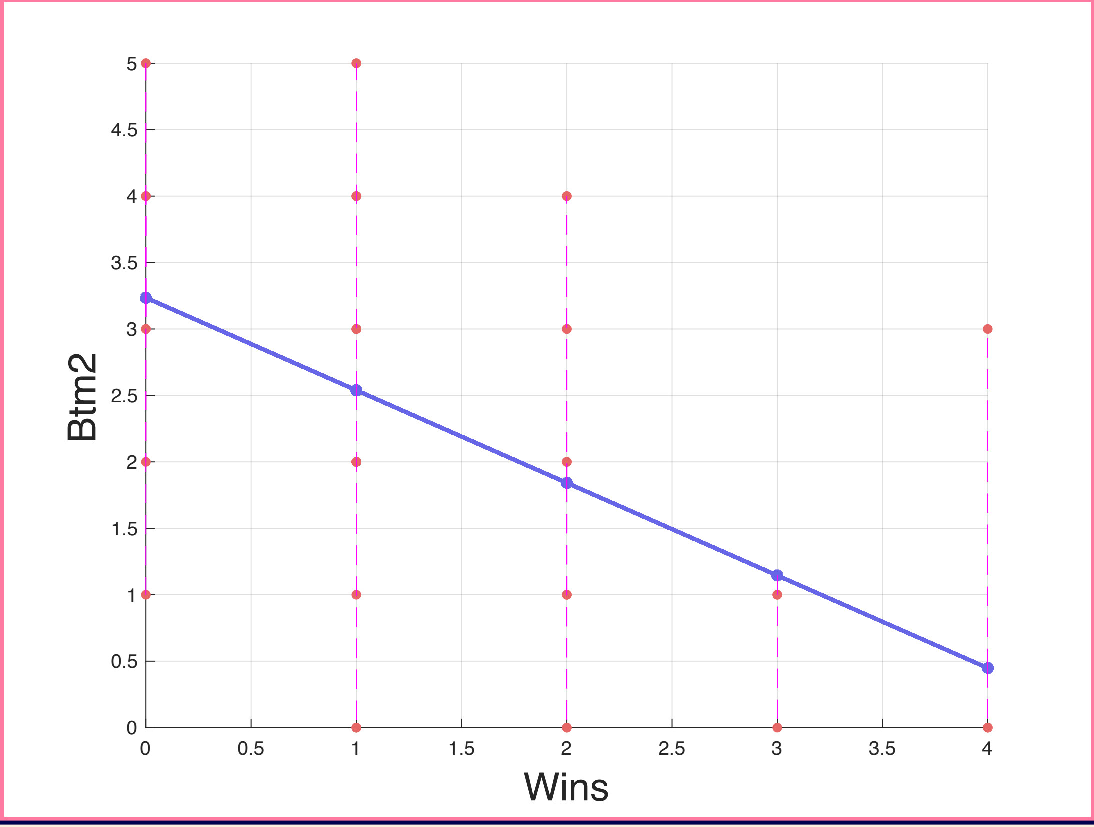
Intercept → the y-value of the regression line for an x value of 0

Slope → how steep the line is (for every 1 unit increase in x, how much do you expect y to increase)

These are also called beta coefficients

$$y = \beta_0 + \beta_1 * Age + \text{Residual}$$

Linear Regression Models



Linear Regression Models

Coding of your ordinal predictors is important for the interpretation of your betas/ coefficients:

Linear coding: 1 2 3 assumes that there is a linear relationship between the orders (i.e., x=1 is half of x=2, etc.)

Linear Regression Models

Coding of your categorial predictors is very important for the interpretation of your betas:

Dummy coding

Effect Coding

Dummy Coding

Attention (X1)	Difficulty (X2)	Var Attention	Var Difficulty
High	easy	1	0
Low	easy	0	0
High	easy	1	0
High	hard	0	1
Low	hard	1	1
High	hard	0	1

- Let's say we have two categorical variables:
  - Attention (high vs low)
  - Difficulty (easy vs hard)
- Dummy code them as 0 and 1

Dummy Coding

Attention (X1)	Difficulty (X2)	Var Attention	Var Difficulty
High	easy	1	0
Low	easy	0	0
High	easy	1	0
High	hard	0	1
Low	hard	1	1
High	hard	0	1

$$y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X1 * X2$$

- The intercept β_0 is the mean of y for all conditions when they are 0 (i.e., Low--easy)
- The slope β_1 is the difference between High and Low Attention for the easy task
- The slope β_2 is the difference between the easy and difficult task for low attention
- The slope β_3 is the interaction

Dummy Coding

Our regression model: $\boldsymbol{y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X1 * X2}$

$$y = \beta_0 + \beta_1 \mathbf{0} + \beta_2 \mathbf{0} + \beta_3 \mathbf{0} * \mathbf{0}$$
$$y = \beta_0$$

$$y = \beta_0 + \beta_1 \mathbf{1} + \beta_2 \mathbf{0} + \beta_3 \mathbf{1} * \mathbf{0}$$
$$y = \beta_0 + \beta_1$$

$$y = \beta_0 + \beta_1 \mathbf{0} + \beta_2 \mathbf{1} + \beta_3 \mathbf{0} * \mathbf{1}$$
$$y = \beta_0 + \beta_2$$

$$y = \beta_0 + \beta_1 \mathbf{1} + \beta_2 \mathbf{1} + \beta_3 \mathbf{1} * \mathbf{1}$$
$$y = \beta_0 + \beta_1 + \beta_2 + \beta_3$$

Dummy Coding

Our regression model: $\boldsymbol{y} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 X1} + \boldsymbol{\beta_2 X2} + \boldsymbol{\beta_3 X1 * X2}$

$$X(1,0) - \mathrm{X}(0,0) = \beta_0 + \beta_1 1 - \beta_0$$
$$X(1,0) - \mathrm{X}(0,0) = \beta_1$$

$$X(0,1) - \mathrm{X}(0,0) = \beta_0 + \beta_2 1 - \beta_0$$
$$X(0,1) - \mathrm{X}(0,0) = \beta_2$$

$$(X(1,1) - X(0,1)) - (X(1,0) - X(0,0))$$
$$= (\beta_0 + \beta_1 1 + \beta_2 1 + \beta_3 1 - \beta_0 - \beta_2 1) - (\beta_0 + \beta_1 1 - \beta_0)$$
$$= (\beta_1 + \beta_3) - (\beta_1)$$
$$= \beta_3$$

Dummy Coding

Our regression model: $\boldsymbol{y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X1 * X2}$

$$X(1,0) - \mathrm{X}(0,0) = \beta_0 + \beta_1 1 - \beta_0$$
$$X(1,0) - \mathrm{X}(0,0) = \beta_1$$

$$X(0,1) - \mathrm{X}(0,0) = \beta_0 + \beta_2 1 - \beta_0$$
$$X(0,1) - \mathrm{X}(0,0) = \beta_2$$

$$(X(1,1) - X(0,1)) - (X(1,0) - X(0,0))$$
$$= (\beta_0 + \beta_1 1 + \beta_2 1 + \beta_3 1 - \beta_0 - \beta_2 1) - (\beta_0 + \beta_1 1 - \beta_0)$$
$$= (\beta_1 + \beta_3) - (\beta_1)$$
$$= \beta_3$$

Effect Coding

Attention (X1)	Difficulty (X2)	Var Attention	Var Difficulty
High	easy	1	-1
Low	easy	-1	-1
High	easy	1	-1
High	hard	-1	1
Low	hard	1	1
High	hard	-1	1

- In effect coding all columns of variables sum to 0
- To do this let's try replacing the 0's with -1

Effect Coding

Attention (X1)	Difficulty (X2)	Var Attention	Var Difficulty
High	easy	1	-1
Low	easy	-1	-1
High	easy	1	-1
High	hard	-1	1
Low	hard	1	1
High	hard	-1	1

$$y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X1 * X2$$

- The intercept β_0 is now the GRAND Mean
- The slope β_1 is the difference between High and Low Attention across both difficulties
- The slope β_2 is the difference between difficult and easy across Attention
- The slope β_3 is the interaction

Effect Coding

Our regression model: $\boldsymbol{y} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 X1} + \boldsymbol{\beta_2 X2} + \boldsymbol{\beta_3 X1} * \boldsymbol{X2}$

$$y = \beta_0 + \beta_1(\mathbf{-1}) + \beta_2(\mathbf{-1}) + \beta_3(\mathbf{-1}) * (\mathbf{-1})$$
$$y = \beta_0 - \beta_1 - \beta_2 + \beta_3$$

$$y = \beta_0 + \beta_1(\mathbf{1}) + \beta_2(\mathbf{-1}) + \beta_3(\mathbf{1}) * (\mathbf{-1})$$
$$y = \beta_0 + \beta_1 - \beta_2 - \beta_3$$

$$y = \beta_0 + \beta_1(\mathbf{-1}) + \beta_2(\mathbf{1}) + \beta_3(\mathbf{-1}) * (\mathbf{-1})$$
$$y = \beta_0 - \beta_1 + \beta_2 - \beta_3$$

$$y = \beta_0 + \beta_1(\mathbf{1}) + \beta_2(\mathbf{1}) + \beta_3(\mathbf{1}) * (\mathbf{1})$$
$$y = \beta_0 + \beta_1 + \beta_2 + \beta_3$$

Effect Coding

Our regression model: $\boldsymbol{y} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 X1} + \boldsymbol{\beta_2 X2} + \boldsymbol{\beta_3 X1} * \boldsymbol{X2}$

$$
\begin{aligned}
\text{X(1,0)} - \text{X(0,0)} &= \beta_0 + \beta_1 - \beta_2 - \beta_3 - (\beta_0 - \beta_1 - \beta_2 + \beta_3) \\
&= \beta_0 + \beta_1 - \beta_2 - \beta_3 - \beta_0 + \beta_1 + \beta_2 - \beta_3 \\
&= +\beta_1 + \beta_1 \\
&= 2\beta_1
\end{aligned}
$$

$$\frac{1}{2} * (\text{X(1,0)} - \text{X(0,0)}) = \beta_1$$

Thus, your main effect is twice as big, and your beta is half of the estimated effect

Effect Coding

Attention (X1)	Difficulty (X2)	Var Attention	Var Difficulty
High	easy	0.5	-0.5
Low	easy	-0.5	-0.5
High	easy	0.5	-0.5
High	hard	-0.5	0.5
Low	hard	0.5	0.5
High	hard	-0.5	0.5

- What about replacing 0's with -0.5 and 1's with 0.5

Effect Coding

Our regression model: $\boldsymbol{y} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 X1} + \boldsymbol{\beta_2 X2} + \boldsymbol{\beta_3 X1 * X2}$

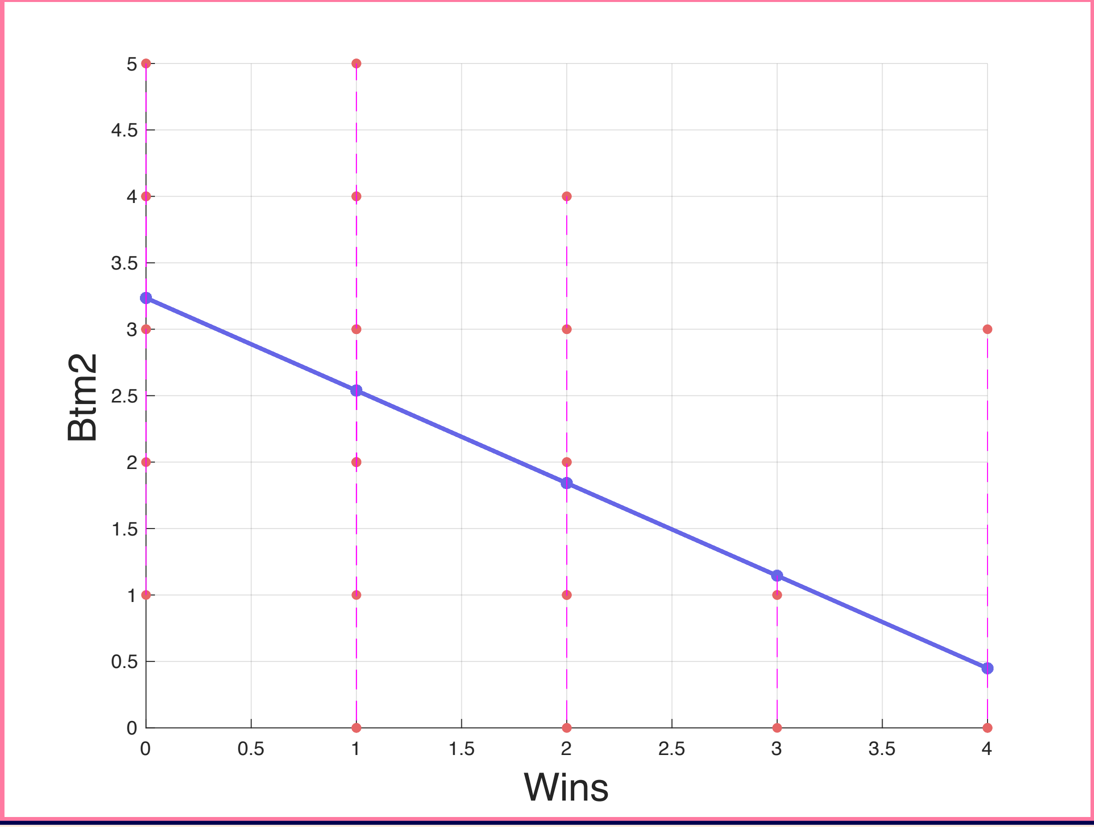
$$X(1,0) - X(0,0) = \beta_0 + \frac{1}{2}\beta_1 - \frac{1}{2}\beta_2 - \frac{1}{2}\beta_3 - \left(\beta_0 - \frac{1}{2}\beta_1 - \frac{1}{2}\beta_2 + \frac{1}{2}\beta_3\right)$$

$$= \beta_0 + \frac{1}{2}\beta_1 - \frac{1}{2}\beta_2 - \frac{1}{2}\beta_3 - \beta_0 + \frac{1}{2}\beta_1 + \frac{1}{2}\beta_2 - \frac{1}{2}\beta_3$$

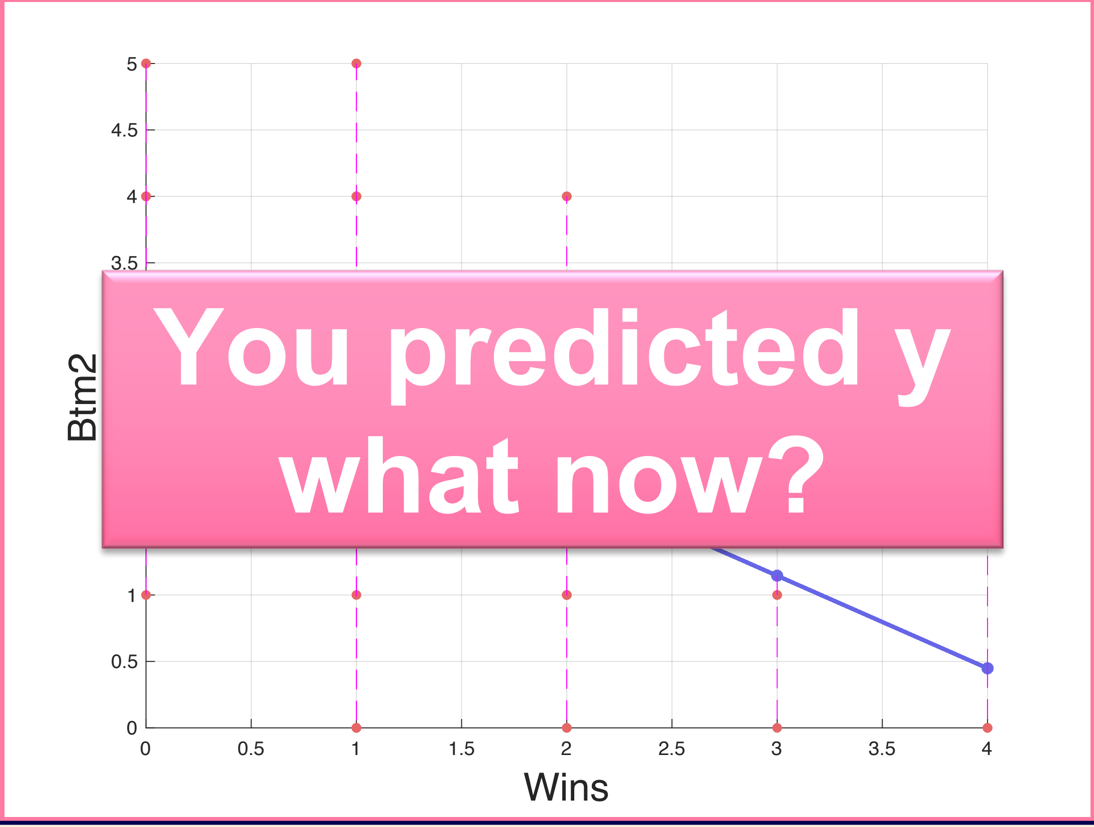
$$= +\frac{1}{2}\beta_1 + \frac{1}{2}\beta_1$$

$$= \beta_1$$

Linear Regression Models



Linear Regression Models



Linear Regression Models

- Regressions can be used in different fashions: testing statistical relationships, predicting future data, removing effects from your data
- After fitting regressions, you can compute and utilizes the residuals (i.e., the unexplained variance or y) for further analysis… we want the residuals of a regression to be normal

Linear Regression Models

Because of how linear regressions compute effects (holding all other effects at zero), you can add nuisance variables into your regression to compute your desired effect while controlling for the effect of another uninteresting variable

$$\text{Alpha power} = \beta_0 + \beta_1 Age + \beta_2 Attenion + \beta_3\ Sleep\ Quality$$

Linear Regression Models

Because of how linear regressions compute effects (holding all other effects at zero), you can add nuisance variables into your regression to compute your desired effect while controlling for the effect of another uninteresting variable

$$Alpha\ power = \beta_0 + \beta_1 Age + \beta_2 Attenion + \beta_3\ Sleep\ Quality$$

controlling for age effects / confounds

Multicollinearity

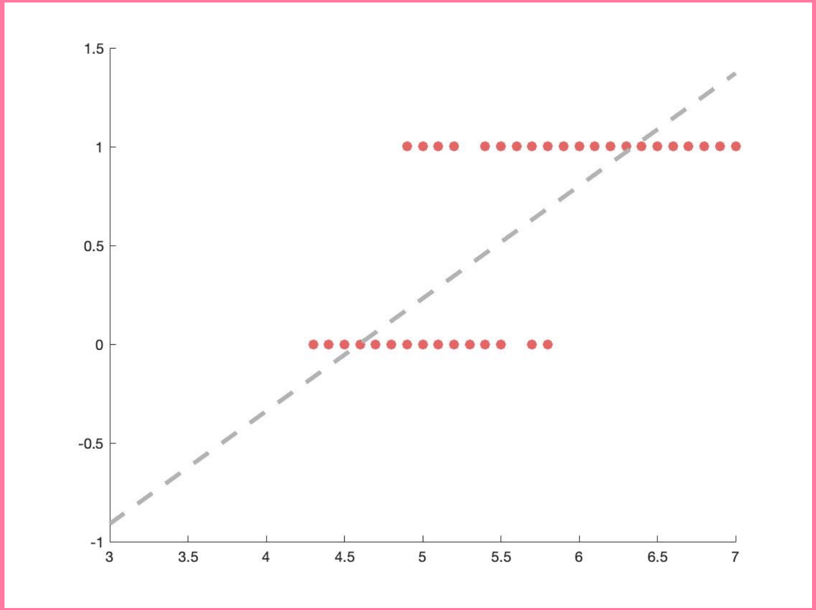
A problem when two or more of your predictors / regressors/ measures are related to one another (i.e., correlated)

For example, participant height and weight

One assumption of regressions are that the measures are independent, and if this violated the beta coefficients estimated become unstable and sensitive to small changes

Logistic Regression

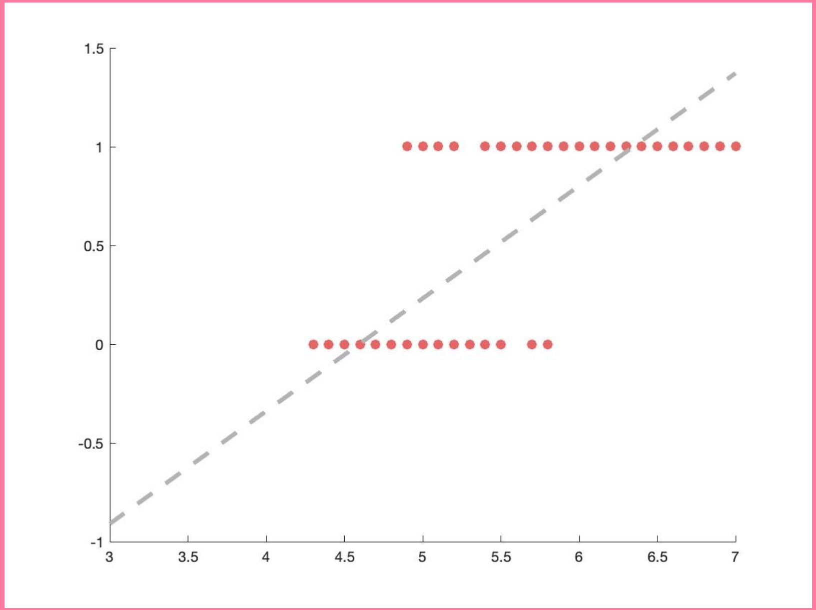
Linear regression work very well on continuous outcomes, but what happens when your outcome is categorical or binary? Fitting a line just doesn't cut it



Logistic Regression

But if we fit a different function to the data, it might fit it much better...

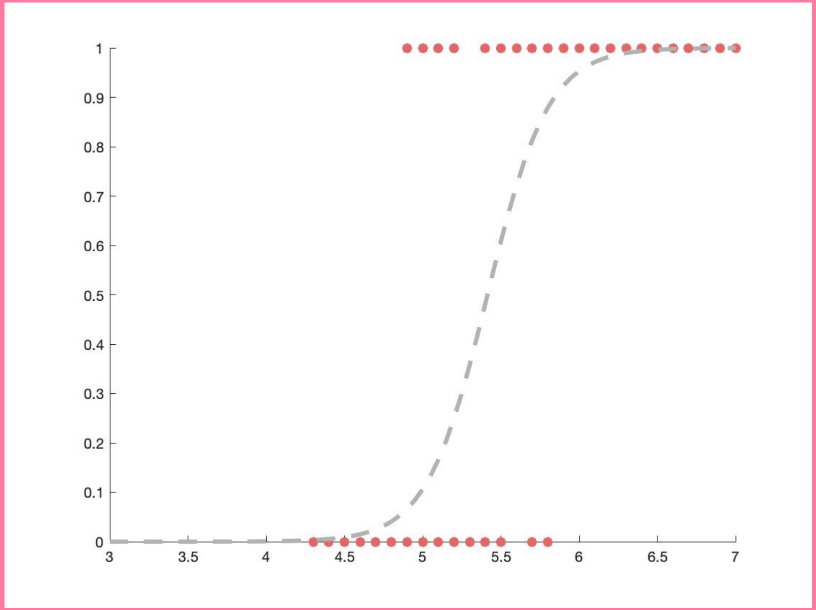
Here comes in Logistic Functions



Logistic Regression

But if we fit a different function to the data, it might fit it much better...

Here comes in Logistic Functions



Logistic Regression

Logistic regression run on the odds $\frac{p}{1-p}$

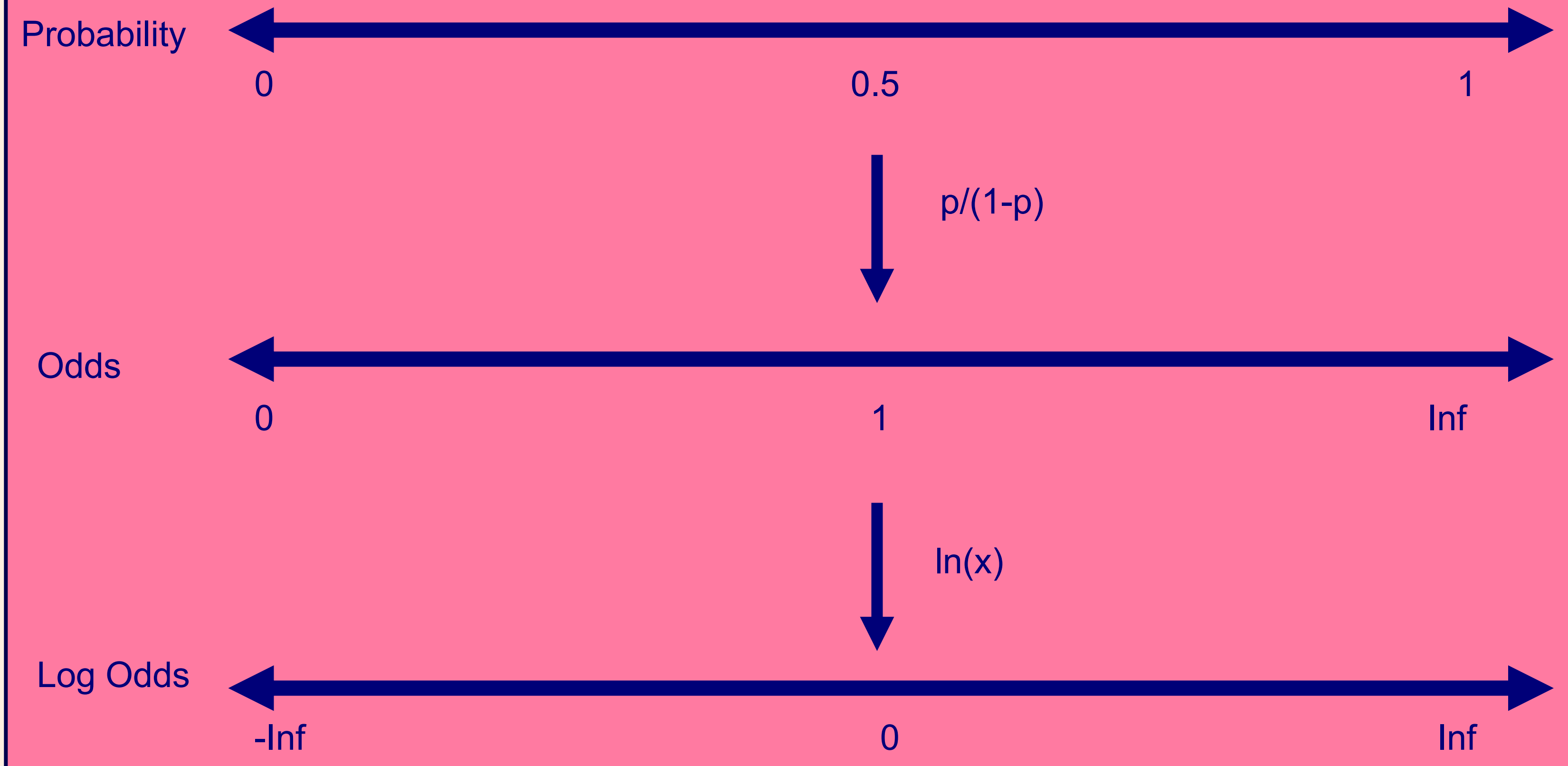
(specifically the log odds)

an Odds of 1 means equally likely to happen or not happen

An Odds < 1 means less likely to happen

An Odds >1 means more likely to happen

Odds



Logistic Regression

Logistic regression are harder to converge and require more data to fit

$$log_e \frac{p}{1-p} = \beta_{0j} + \beta_{1j} * Age$$

The Intercept is the log Odds of your event when x is 0
If the intercept is ZERO → you have a 50/50 chance
If the intercept is negative → you're less likely
If the intercept is positive → you're more likely

Logistic Regression

Logistic regression are harder to converge and require more data to fit

$$log_e \frac{p}{1-p} = \beta_{0j} + \beta_{1j} * Age$$

The Slope controls how steep the relationship is
If the Slope is small → the relationship between x & y is noisy
If the Slope is negative → small values of X are more likely
If the Slope is positive → large values of X are more likely

Nested data

What is the best song to dance to?

Nested data

What is the best song to dance to?

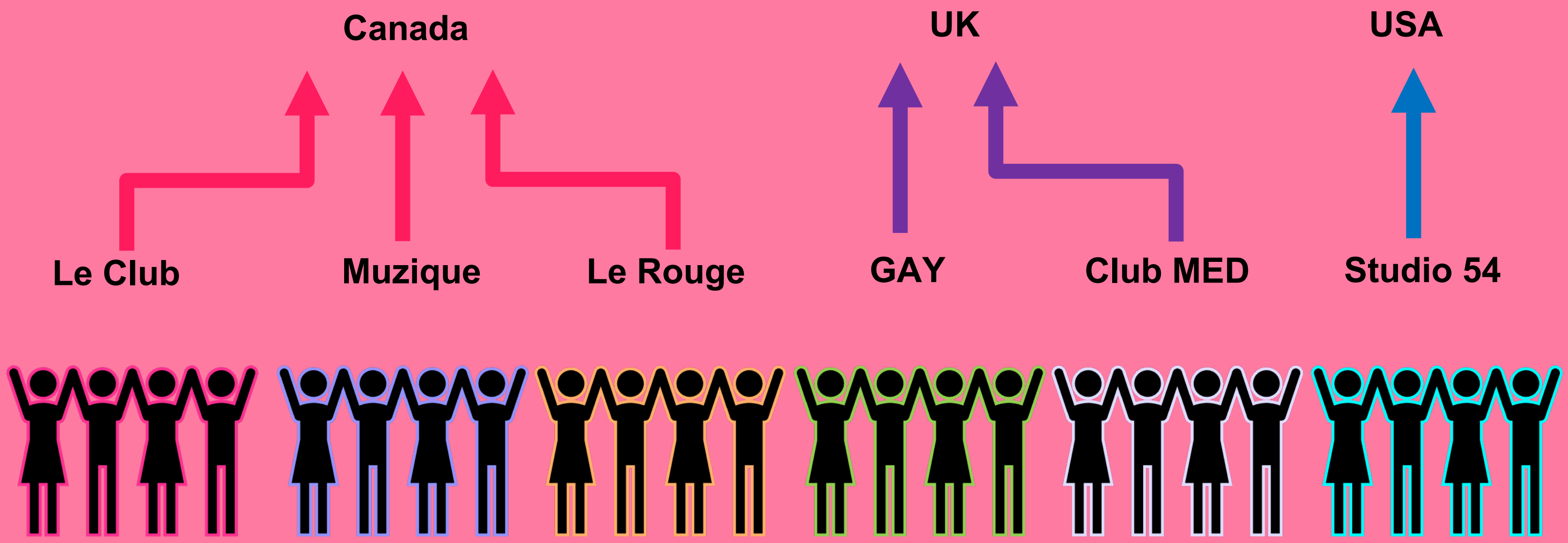
Nested data

What is the best song to dance to?

Le Club | Muzique | Le Rouge | GAY | Club MED | Studio 54

Nested data

What is the best song to dance to?



Nested data

- Observations within the same country, for example, are more similar to themselves than to a different country
- When data is nested, it is important that your model takes into account the correlational structure of your data

Nested data

Regressions assume that all your predictors (i.e., measures) are **independent**. But this may not always be the case, specifically for **repeated measure** designs where participants see multiple experimental conditions

Nested data

The independence of each observation is often violated in real life—data is nested in a big wonderful world

People are more similar to themselves, their family, their friends, people from their city, country etc

Nested data

- Regression models need to consider the correlated structure of your observations to better deal with noise
- Adding this info to your regression will allow you to better predict out of sample

Hierarchical regressions

Hierarchical regressions allow you to take into account the structure of your data to help better estimate your effects and deal with noise that can be accounted for by your nested data

Hierarchical regressions

This is achieved by allowing Intercepts or slopes to vary as a function of the levels/ nests within your data

For example, allowing every person in your sample to get a different intercept accounts for the fact that people have differences in response times, but fixing the slope of the regression lets you test the effect across your conditions

Hierarchical regressions

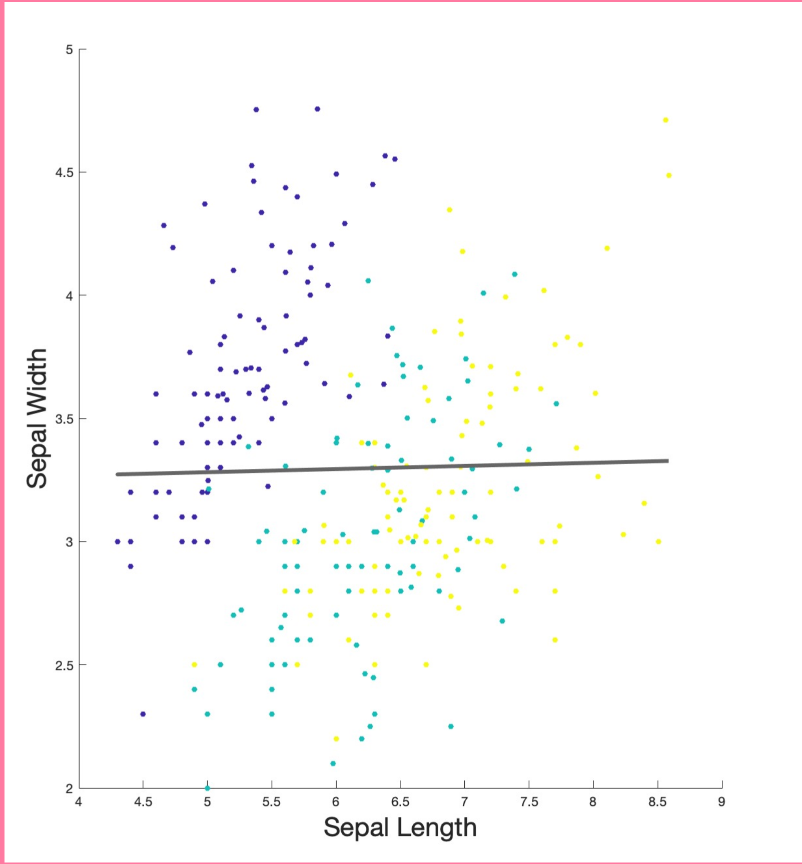
Fitting a line to our data

sometimes does not

capture the

complicated effect

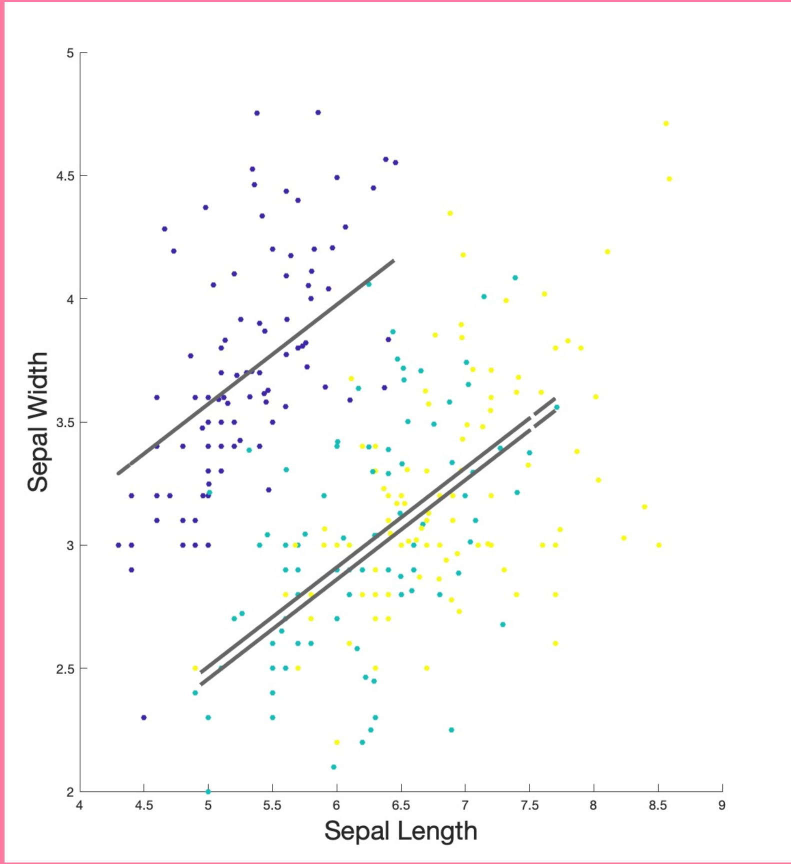
we may have



Hierarchical regressions

Random Intercept

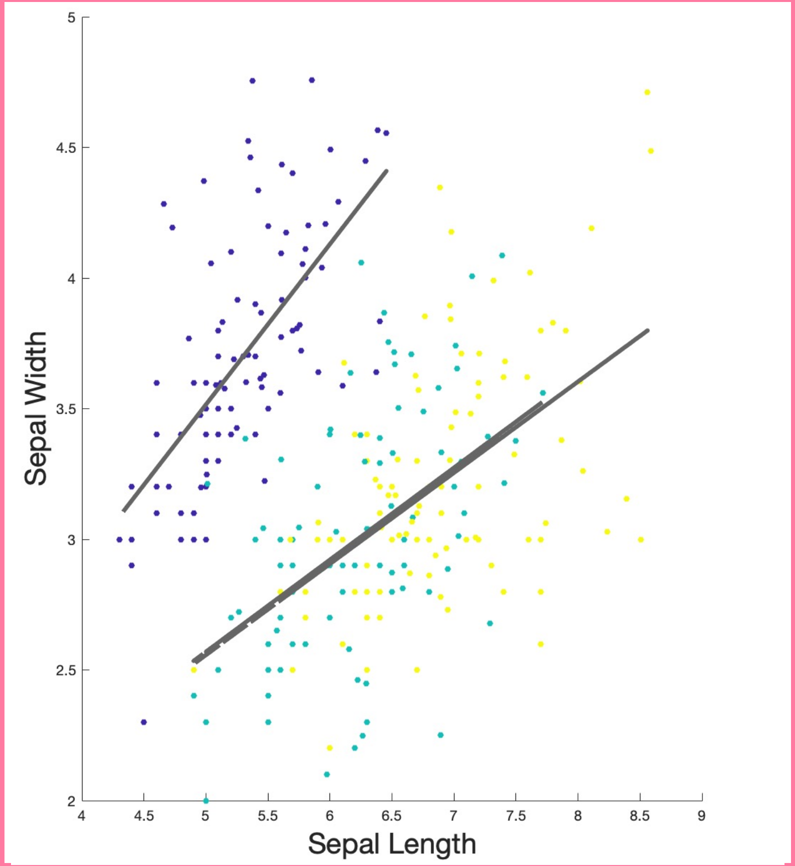
Sometime our effect is constant across individuals, but they may vary in their baseline ability



Hierarchical regressions

Random Slope

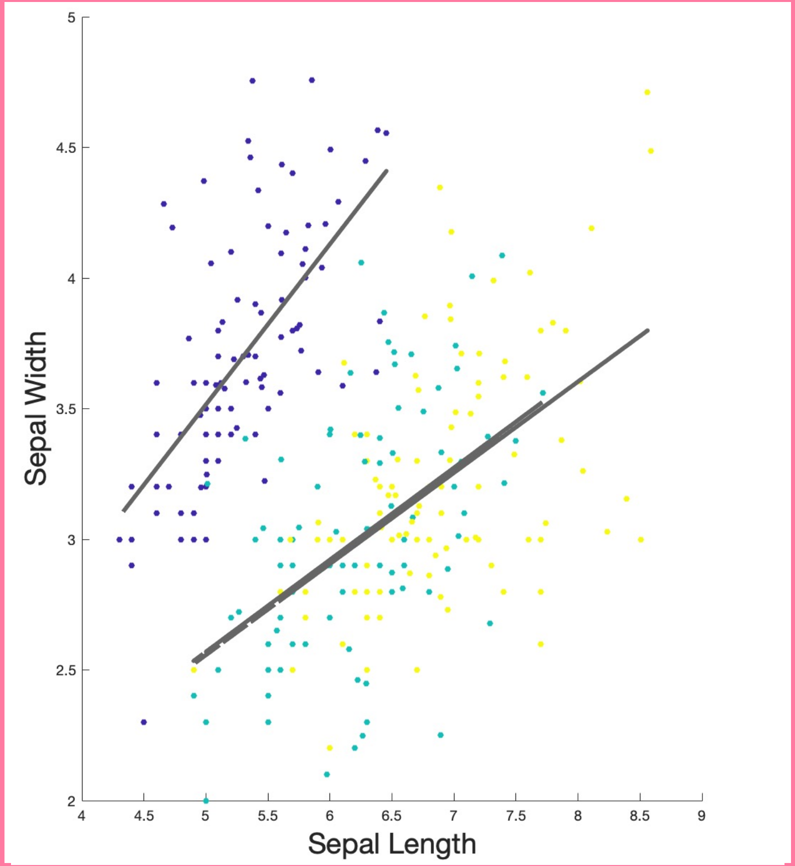
Sometime our effect
changes across
groups too



Hierarchical regressions

See

http://mfviz.com/hierarchical-models/ for interactive example demonstrating fixed vs random slopes and intercepts



Fixed vs Random effects

Note that hierarchical models have both ***fixed*** and ***random*** effects

Fixed effects—assume you have every desired level of variable (e.g., experimental condition)

Random effects—assume you do not have every possible level you wish to test (e.g., participant)

Fixed vs Random effects

Fixed effects—sometimes are also referred to as variables you MANIPULATE

Random effects—sometimes referred to as variables that are sources of variation in your data that you do not care about

Model selection revisited

We can try and quantify a measure that balances both the **fit** of the model

as well as **parsimony** (i.e., the number of parameters)

Akaike information criterion (AIC).

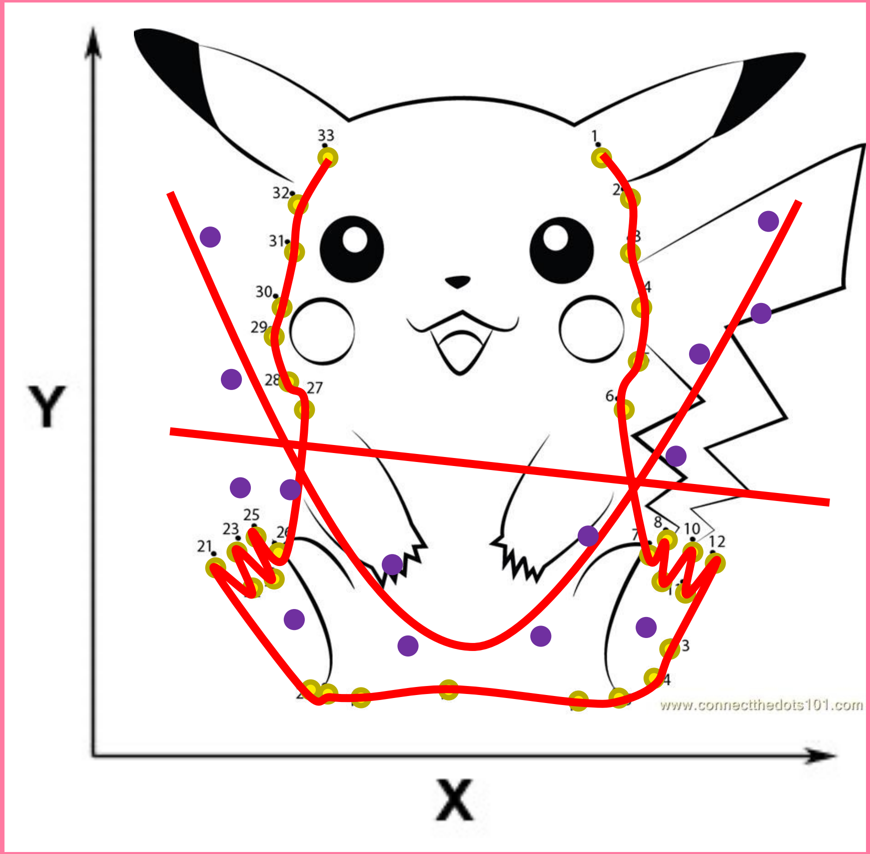
$-2\log L(\theta) + 2k$

Bayesian information criterion (BIC)

$-2\log L(\theta) + k*\log(T)$

Where k is the number of parameters fit to T observations

Overfitting



$$Y = \beta X + \alpha$$

$$Y = \beta X^2 + \gamma X + \alpha$$

$$Y = \beta X^2 + \gamma X + \frac{\xi \sin\left(\tau\left(\sqrt{X} - \varphi\right)\right)}{\sqrt[3]{e^{-\omega X}}} + \alpha$$

Support Vector Machines

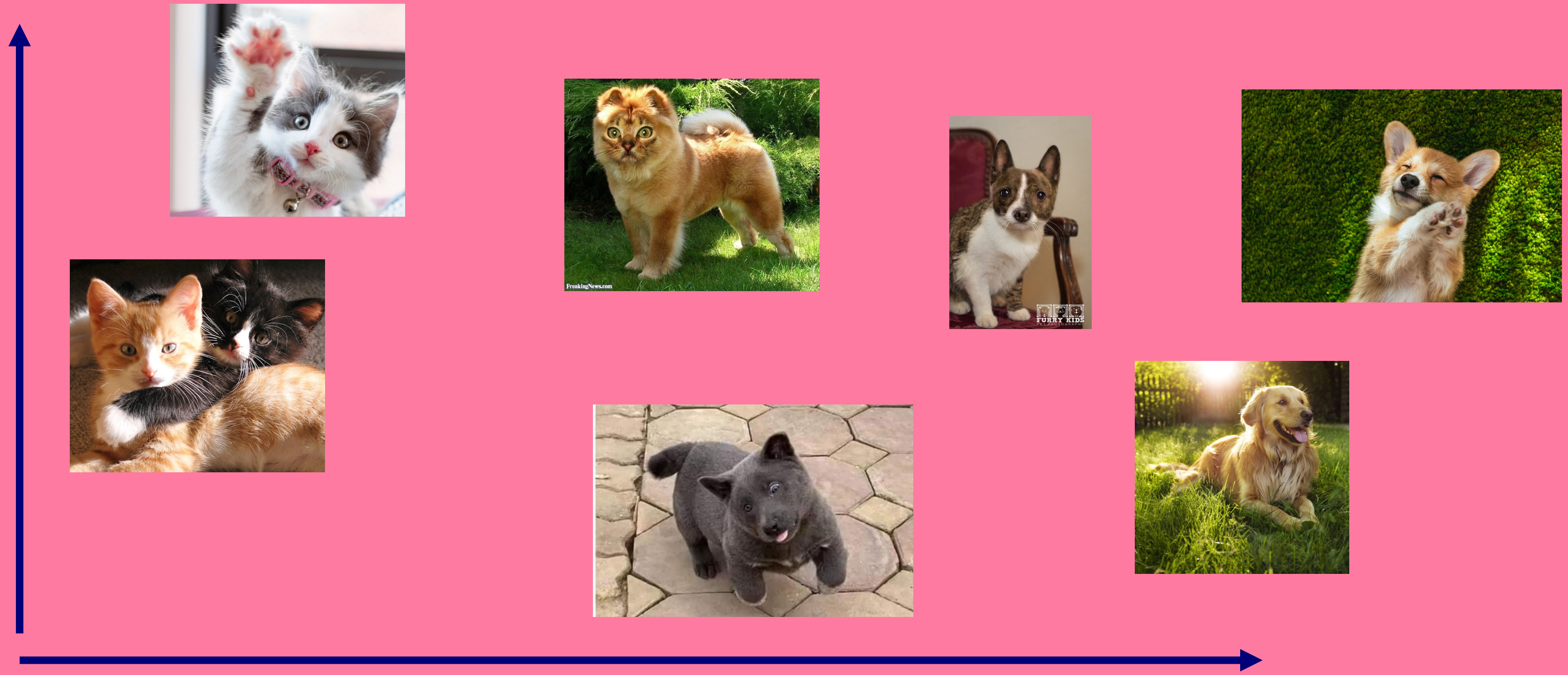
Supervised machine learning algorithm that learns to classify two classes of data (can be generalized to multi class problems) given a bunch of examples of each class

Support Vector Machines

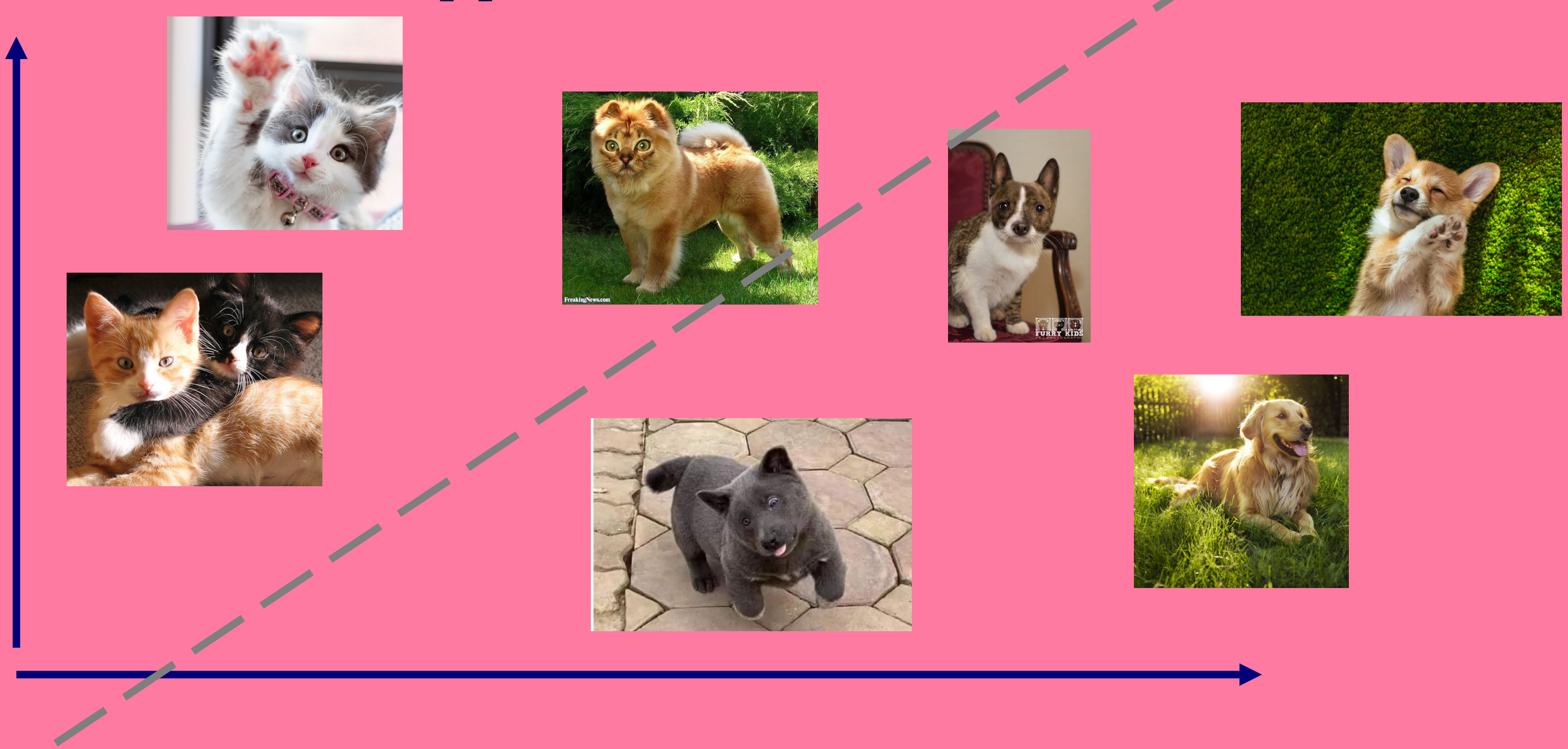
- It works using **support vectors** that help create a decision boundary (i.e., a hyperplane) along your multidimensional feature space to classify your inputs.
- Support vectors are cases that are very close to the decision boundary (i.e., ambiguous cases)

Support Vector Machines

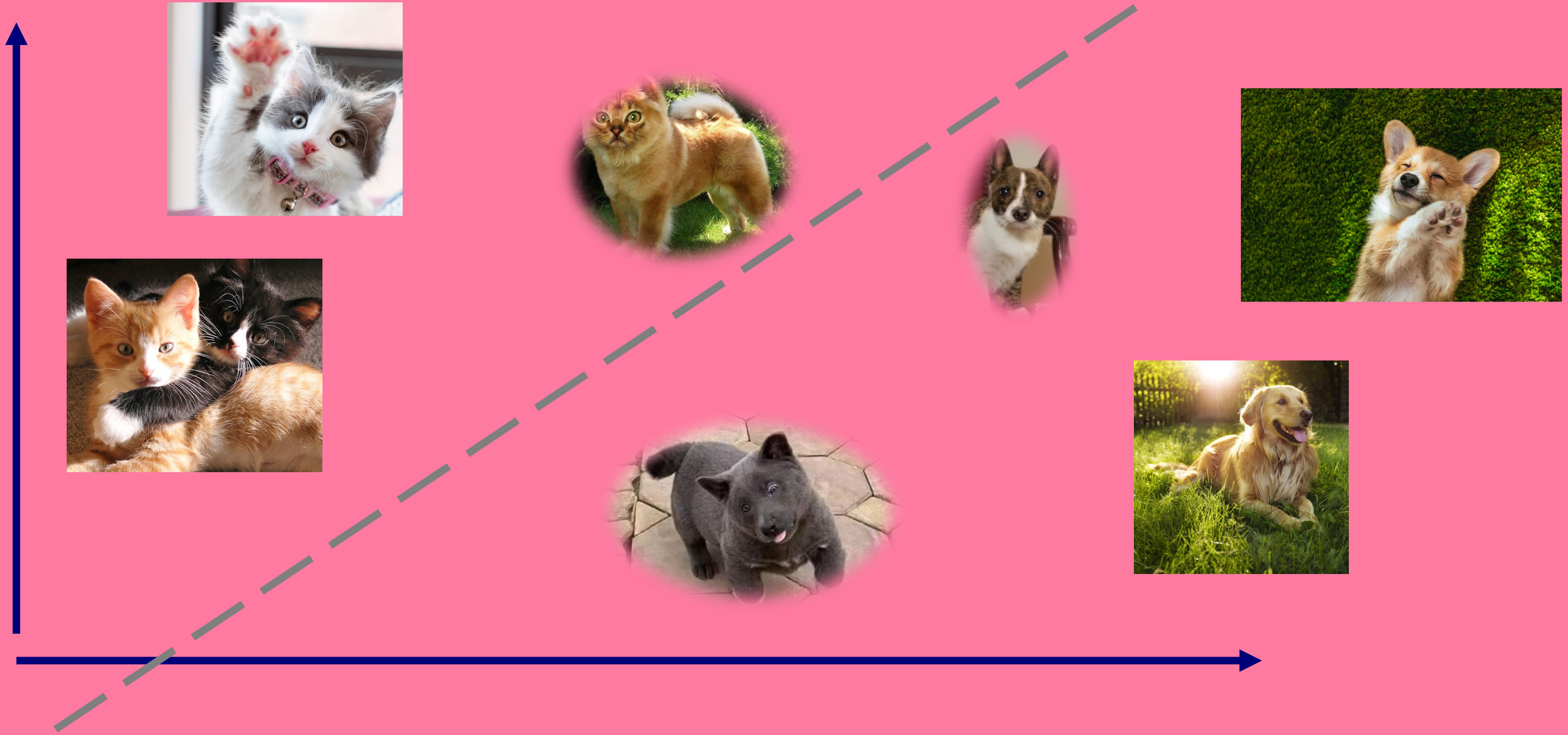
Support Vector Machines



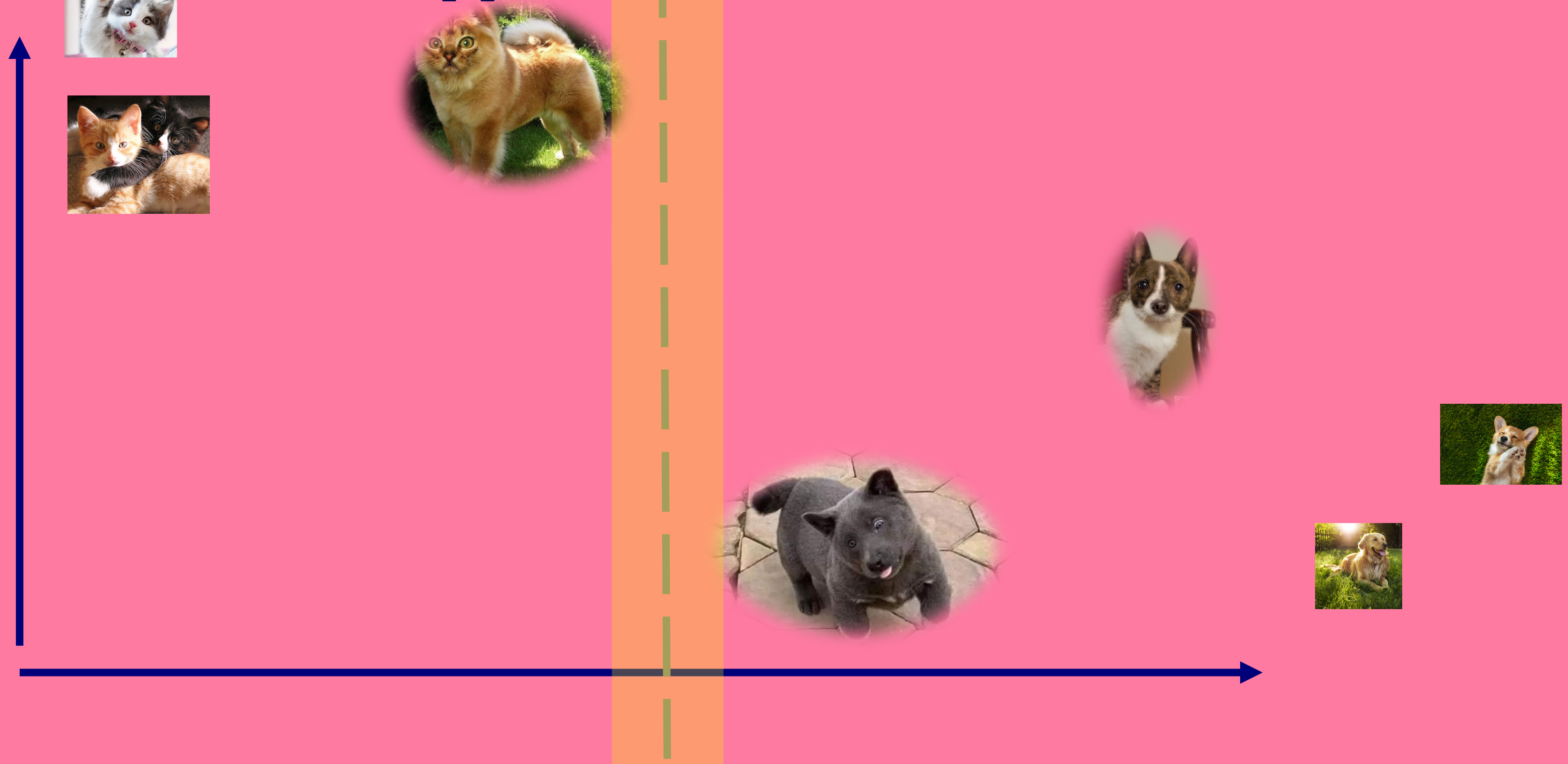
Support Vector Machines



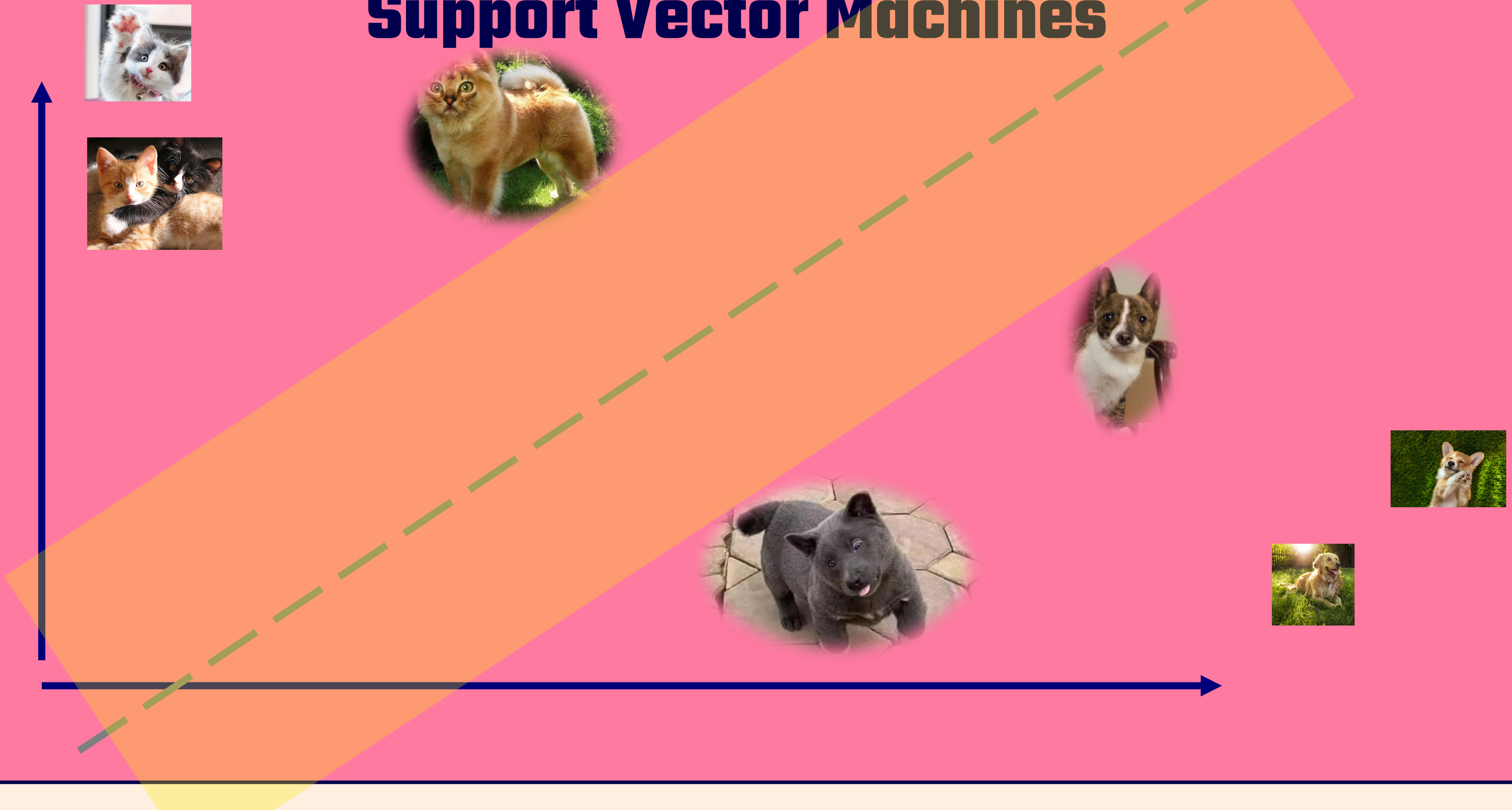
Support Vector Machines



Support Vector Machines



Support Vector Machines



Support Vector Machines

SVM tries to find a hyperplane such that it leaves the widest lane between the edge examples