Searching for Galaxy Clusters in the Dark Energy Survey

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Abstract

This project aims to construct an algorithm capable of detecting galaxy clusters within data from the Dark Energy Survey. The algorithm is coded in MATLAB and tested with simulated data constructed for this project. After simulated data tests, real data is to be scanned. The method of detection is relatively simple, only looking at galactic density. No obvious galaxy clusters were detected in the survey data due to limits with the detection method. The project overall is still a success despite the lack of confirmed detection, since the algorithm performed exactly as intended and all other goals were met.

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1 Introduction

Galaxy clusters are the largest gravitationally bound objects in the known universe [1]. Clusters contain enough matter, therefore have sufficient gravity, to overcome the accelerating expansion of the universe which is driven by the elusive substance known as Dark Energy [2]. Certain cosmological parameters determine the ratio of 'normal' baryonic matter (the stuff we can see) to dark matter and dark energy. These density ratios are key to predicting how the Universe will evolve over time. By finding a count of the galaxy clusters present during a certain epoch of the Universe, the density parameter for dark energy (known as the cosmological constant 'lambda' Ω_Λ) can be estimated [3].

The aim of this project is to build an algorithm capable of detecting galaxy clusters within the data taken from the Dark Energy Survey (DES). This will be achieved by first creating our own simulated data containing clusters and then developing a way to detect these simulated clusters. The detection method will then be applied to the real data under certain filters to maximise the chance of positive cluster detection, and minimise false positives.

The DES is a photometric survey of the sky being performed in the Southern hemisphere, scanning a total area of 5000 square degrees. The main mission of the DES is to gain a better understanding of the accelerating expansion of the universe and what factors drive this expansion. One of the ways this survey is hoping to achieve this goal is to look specifically at the number density of galaxy clusters in the Universe at various epochs, in order to probe key cosmological parameters, including dark energy [4].

Once the DES has been completed there will be an enormous data catalogue published containing over 400 million total objects comprised of mainly stars and galaxies. For the purposes of this project, we will be using the Science Verification data (SV data) which was published shortly after the DES began its operation. The SV data contains a catalogue of over 25 million objects and exists as a proof of concept for the mission [5]. This much shorter survey proves the 8 year long mission has the ability to produce the useful results by collecting a sample of all the types of data possible from the full survey. The SV data is the environment in which will we test our algorithm. The full catalogue was not released at the time this project was carried out, however the SV catalogue is a viable substitute as it contains all the scientific elements of the full set of data covering a smaller portion of the sky. Using the SV data gives us a manageable amount of objects to process for the scale of this project.

Figure 1 below gives an idea of the scale of the DES. The objects plotted are only from the smaller scale SV survey. The full DES will consist of over 16 times more objects, covering a much larger portion of the total sky. All of the red areas containing the objects surveyed are situated in southern area of the sky. This is due to the DES being an Earth based experiment, the telescopes can only see the sky directly above them meaning the survey will not touch any northern areas of sky. The areas chosen by the survey are not completely random. Previous survey experiments have looked in these areas of sky before so, by looking again during a different search, the old and new data can be combined in order to ratify analysis making it a stronger scientific tool.

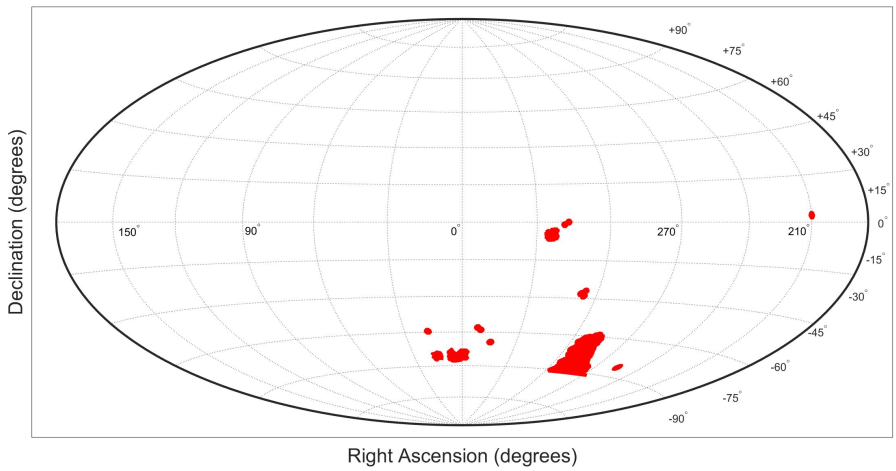

Figure 1: The red areas on the plot above show the area covered by the SV data survey. Every object in the SV catalogue is plotted as a red dot on this projection of the sky. At this resolution individual points cannot be seen but the general areas which have been surveyed are visible. There are over 25 million objects plotted on this map. The map itself is a projection of the sky in terms of the celestial coordinates, Right Ascension and Declination.

On the largest scales the Universe forms a filament like structure with bright over-densities containing matter, and dark empty under-dense voids. Clusters sit at the centre of these bright webs where matter has clumped together as the Universe has evolved from initial tiny fluctuations. The simplest definition of a galaxy cluster is a collection of galaxies bound together by gravity. A cluster can contain a minimum of hundreds of galaxies to upwards of thousands of individual galaxies [6]. Using this simple idea of what makes a cluster, this project will focus on searching for areas containing high galactic density to locate potential galaxy clusters in the DES data.

The search for galaxy clusters is not only the main aim of this particular project, but also one of the targets of the survey itself. Locating and mapping out galaxy clusters throughout the Universe is one of four interconnected ways the survey plans to uncover the secrets of dark energy as well as constrain important cosmological parameters that determine the evolution of the Universe. For this much smaller scale project however, we will be only focusing on a simple method in order to detect galaxy clusters, purely by looking for visible light emitting galaxies in densely packed areas.

2 Theory

Since this project is not testing a phenomena easily seen in an Earth based laboratory, the main theoretical challenges take the form of modelling the algorithm to best fit the reality of the Universe in which the real data applies to. This means maximising the effectiveness of the model using our current understanding of the physics at play in regards to galaxy clusters, how big they tend to be, how far away they appear, as well as variables like the geometry of the Universe which affects how light from the galaxies travels to us. Key concepts for this project include the relationship between the redshift of an object and its angular distance from us, the geometry of the Universe, and how positions on the sky are calculated in order to locate the clusters we are searching for.

Cosmological redshift is due to the expansion of space over relatively large distances stretching the wavelengths of light emitted from objects deep in the Universe. The redshift, z, of a distant cosmological object as seen on Earth is intrinsically related to its distance from us. This relation is dependent on which type of distance you are interested in, as well as the Universe's assumed geometry. For the purposes streamlining the theory, we have assumed a flat Universe as current measurements indicate this is a good match to the Universe we currently live in [7]. This assumption also makes the calculations simpler going forward.

The simplest argument that relates the redshift of an object to its distance from us is via Hubble's law. Hubble's law states that objects which are further away from us are also receding faster from us at our point of view due to the expansion of the Universe [8]. The speed at which a star or galaxy travels away from us is directly related to its apparent redshift via the formula;

$$z = \frac{v}{c} \tag{1}$$

where z is the redshift of the object, v is the speed of its recession, and c is the speed of light. Therefore redshift is proportional to distance via Hubble's law, and the constants involved vary depending on which distance is being used.

Measuring distances to objects in the Universe is no simple task. There are multiple different ways to calculate the distance to a point in the Universe by using the light and information emitted by objects in an area of sky. However, compared to measuring distances between local objects here on Earth where we expect one value for distance intuitively, depending on the method used to measure the distance to a cosmological object the value can vary by very large amounts. Although this seems counter-intuitive since we are used to dealing with Euclidean geometry on much smaller scales, all of these different distances to the same point are valid and have different uses in the fields of Cosmology. It is worth noting all of these distances (luminosity, co-moving, angular diameter, light-travel) converge to the same Euclidean value for nearby objects [9].

For the purpose of this project we are going to utilise the Angular Diameter Distance. This measure of distance can be used to relate the redshift of an object to the size of an object as well as the angle it subtends on the sky. By making an assumption of the average size of a galaxy cluster, the objects we are interested in, as well as fixing the redshift (hence distance) we will look out to, we can fix the angular size of the objects. The method of detection we will use requires a grid of cells of particular size, used to divide up the area of sky we are scanning. By fixing the size of the cells to the angular size of the objects we are hoping to detect, we are maximising the chance of finding galaxy clusters amongst all of the background data noise.

By a geometric argument, where the distance from the object is much greater than the size of the object, the simplest definition of angular diameter distance to an object is the following;

$$D_A = \frac{l}{\theta} \tag{2}$$

where D_A is the angular distance, l is the fixed physical extent of the object and θ is the angle the object subtends on the sky. A typical galaxy cluster has (visible) physical extent of around one mega-parsec, and this value is unaffected by its relative distance from us. However the angle a cluster subtends on the sky can change with redshift (and hence distance as in equation 2). It is this relation between θ and z which we need to uncover. Fixing the redshift we look out to gives us a fixed angular size of galaxy clusters to look for.

The relationship between the angular diameter distance D_A and redshift z of an object is found to take the form;

$$D_A = \frac{D_M}{(1+z)} \tag{3}$$

where D_M is the 'proper' distance to the object. This proper distance can be calculated analytically by inputting cosmological parameters which have been assumed when taking into account the flat geometry of the Universe. D_M takes a complex form but this is not the distance we are interested for this project. A script can be written with fixed cosmological parameters in order to plot the relation between redshift and angular diameter distance, which is what we are interested in.

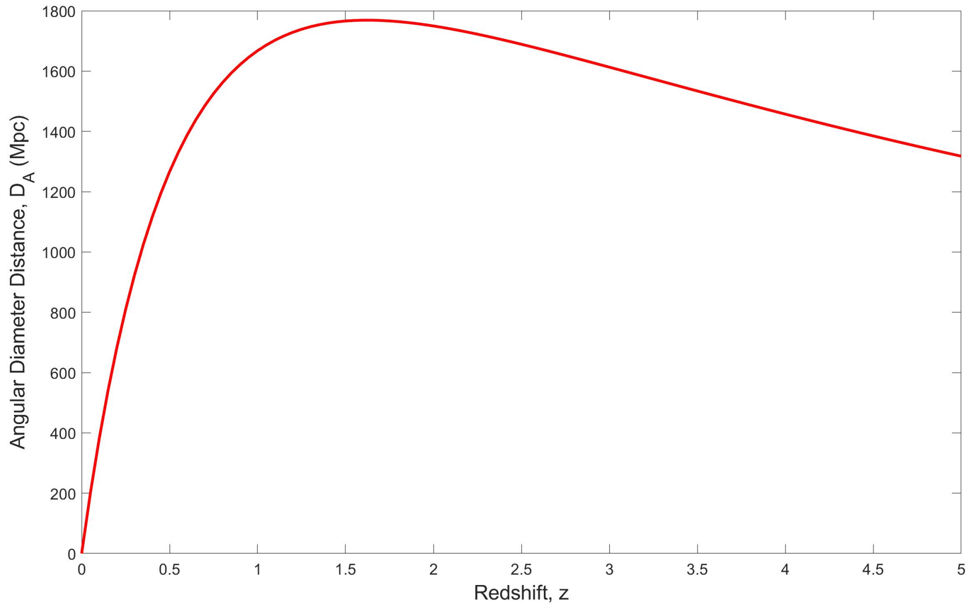

Figure 2: The red line shows the shape of the analytical function joining angular distance and redshift. Redshift is a dimensionless quantity, whereas the distance on this plot is measured in mega parsecs. An oddity of this relation is the turnaround seen at about z=1.5. This indicates, after a certain amount of redshifting, an object 'behind' another will appear larger on the sky and therefore has a smaller angular distance to Earth. This strange phenomenon is not important to our project, since we will be working at a redshift value below 1 as this works best for the optical data captured by the DES.

The end result of analytically calculating the relation in equation 3 gives the plot shown in figure 2. This function can then be used to read off corresponding z and D_A values. The redshift we are interest in is at z = 0.5 which is equal to a D_A value of 1272.5 Mpc. The culmination of the theory for this project is using the redshift and angular distance together in order to calculate the angular size of a galaxy cluster at a set distance away. Using equations 2 and 3, as well as the function plotted on figure 2, θ can be found for a set value of z and an estimate of l for an average galaxy cluster. This angular size of a cluster as it appears on the sky is key because it fixes the resolution of our detection method. With this angular size, we have a much needed component used to figure out what the algorithm should be looking for.

Locating galaxy clusters is the end goal for this project and in order to do that we need to understand the celestial coordinate system used to locate objects in space. Just as longitude and latitude are used as an angular positioning system on Earth, Right Ascension (RA) and Declination (DEC) are used to fix the positions of objects in space relative to us on a projected sphere extending to all of space. RA can take any value between 0 and 360 degrees, forming a circle around this projected sphere in a similar fashion to the equator around the centre of the Earth. The DEC coordinate, which is also measured in degrees, is the angular distance north or south of the celestial equator formed by the RA coordinate and can take any value between 90 and -90 degrees. The important point here is both of these numbers can be used together in order to locate potential galaxy cluster candidates in a particular area of the sky.

3 Method

At its core the main challenge of this project is to create an algorithm capable of counting the number of galaxies in a patch of sky and, by filtering out background noise, determine if the local galactic density is high enough that a cluster could potentially reside in that specific area. The technique will we use is purely photometric, only using optical data looking at the presence of stars and galaxies. This is because the data itself is from a purely optical survey. The object catalogue we will be using for this experiment contains various types of information associated with each object aside from their positions that we can utilise for filtering in order to improve our algorithms accuracy.

There are many methods that make counting objects within a 2D surface efficient for large quantities of data. For this project, the 'Count-in-Cells' method will be utilised [10]. This system is based on the most basic idea of dividing a patch of sky into a grid of even squares and then counting up all of the visible galaxies in each cell. From this, a mean background count of all the galaxies present can be found. Areas with a galactic density above a certain threshold for this background are likely locations for a galaxy cluster to be present. The theory fixes the redshift we will be searching in, but we also need a way to determine which areas of the sky are most likely to contain a cluster. The formula below is used to calculate a contrast between the mean background noise and over-dense areas;

$$\sigma_{cl} = \frac{N_{cluster} - N_{field}}{\sigma_{field}} \tag{4}$$

where σ_{cl} is the contrast or enhancement above the background, $N_{cluster}$ is the total number of objects in the current cell, N_{field} is the total number of objects in the entire area being scanned and σ_{field} is the variance across the entire area. This contrast is central to the function of the algorithm. By comparing each cell locally to the average background distribution of galaxies, the density of objects across the area of sky can be found. Our algorithm defines a cluster to be an area with a density over a certain threshold above the mean background, meaning a large collection of galaxies reside in that one particular cell.

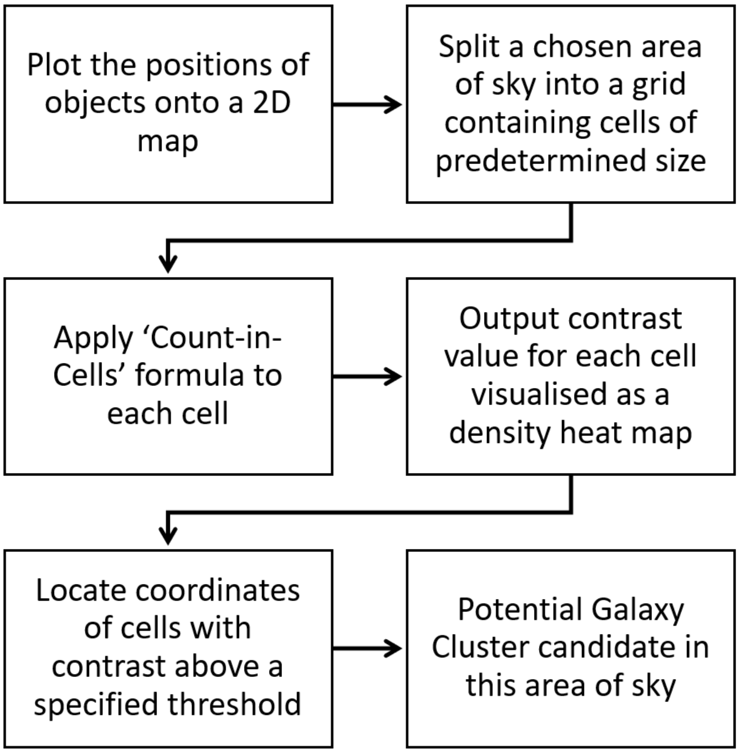

Figure 3: A flowchart to demonstrate the basic outline of the algorithm used to detect galaxy clusters. By mapping object positions on the sky, they can be grouped into cells to find the local object density. By assuming a galaxy cluster will be located in an area of relatively high density, the count-in-cells formula can be used to filter for cells with a high contrast in order to detect clusters.

Figure 3 gives a simple guide to the idea behind the algorithm constructed for this project. The detection method is simple in design, looking at spatial distribution of galaxies from our point of view of Earth. The algorithm will be designed around this idea in order to sift through the DES data in search of galaxy clusters.

To gain an understanding of how our detection algorithm functioned, we needed to test it worked correctly on data in which we already knew what the outcome should look like. This was achieved by creating a random distribution of points on a 2D grid to simulate lone background galaxies, and overlaying dense regions of points designed to replicate the kinds of galaxies clusters we hope to see in the real data. Running the algorithm over this 'fake' area of sky with clusters in known positions allowed us to check our code worked as intended, as well as give us an idea of how the real data results should appear to us.

Each object detected by the survey has a brightness as seen from Earth known as its apparent magnitude. Since we are dealing with apparent magnitudes, this value does not tell us anything about the distance to the object. A fainter object could be far away and extremely bright, or very close and produce low optical emission. This magnitude value only tells us about the amount of light that we are receiving. The first way in which we filtered the data was to decide on a suitable 'cutoff' of which magnitudes to include, any objects with magnitudes outside of this cutoff would then be discarded. To find a reasonable value for this cutoff, all of the object magnitudes were binned together in a histogram to easily visualise the data.

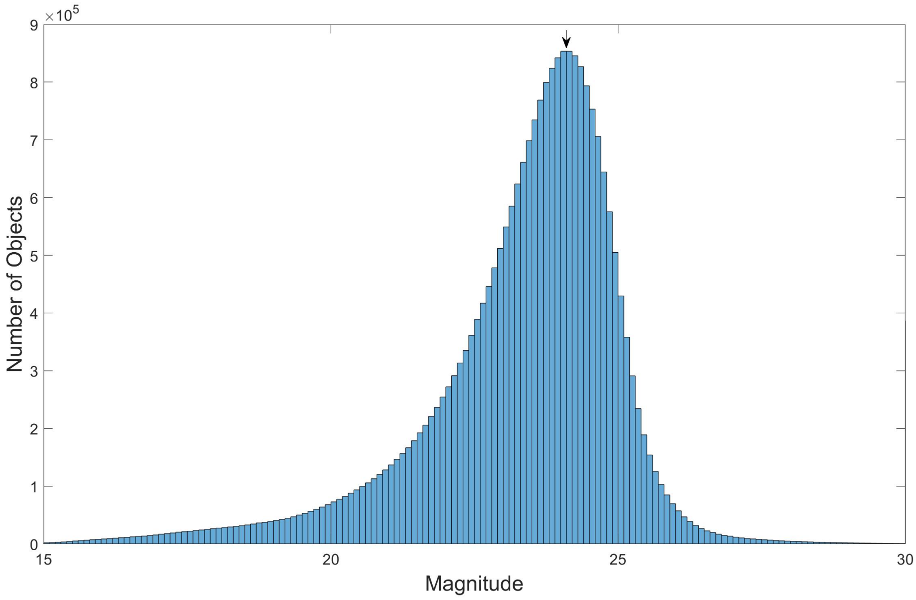

Figure 4: This histogram contains every object - over 25 million - from the SV data catalogue. The black arrow at the top of the figure indicates where the turnover point occurs. Beyond this point, higher magnitudes (corresponding to fainter objects) are missing objects since they become increasingly harder to detect. We are able to remove bias in the results by only including objects below this point

This cutoff is introduced in order to ensure there is no bias in the type of objects being looked at. As figure 4 demonstrates, after a certain magnitude the number of fainter objects begins to decrease. This is due to the limitations of the equipment performing the survey not being able to detect all objects beyond a certain apparent magnitude for the simple reason of them being too faint or far away. By only including objects with a magnitude lower than the turnover point we avoid the problem of 'missing' objects which would be present if experimental limits were not present. This also allows us the reasonable assumption we have enough information from the object to determine if it is a galaxy or not since it is above a certain brightness threshold.

Since the main idea behind our search for galaxy clusters is to hunt for areas with a high galaxy density, it makes sense to only include objects that are most likely to be galaxies. This survey is searching the optical wavebands and so will pick up anything which emits visible light. Besides galaxies, the other main visible light emitting objects in the data are likely to be nearby and/or bright stars. For this reason the catalogue was also filtered, using parameters already present in the SV data, to remove any objects deemed more likely to be a star rather than a galaxy.

4 Results

The key theoretical result central to the function of our algorithm is the cell size used to divide up the area of the sky. This cell size corresponds to the predicted angular size a galaxy cluster should appear on the sky for a specified redshift and estimated cluster size. An average galaxy cluster should span approximately one mega parsec perpendicular to the line of sight. The value of redshift chosen was 0.5 as this is relatively close by in terms of distance and works well for optical wavelength data which is what we are going to be working with. With these parameters, the cell size θ can then be calculated by using the function relating D_A and z to find the distance and then inputting this into equation 2 to solve for the angle. θ was found to be equal to 0.457 degrees. This is the angular size of our cells which will be used throughout testing.

Initially testing our algorithm on simulated data allowed us to understand exactly how the code would perform and output the results. By creating our own noisy background of lone galaxies and then overlaying pre-made simulated clusters, we also were able to test the function of algorithm and alter tolerances on cluster sizes and densities in order to maximise correct detection in the DES data. The size of the area of sky chosen for testing is completely arbitrary at this point since it is all simulated random distributions, however this area size was used for testing a realistic amount of data at once as we will go on to do for the surveyed data.

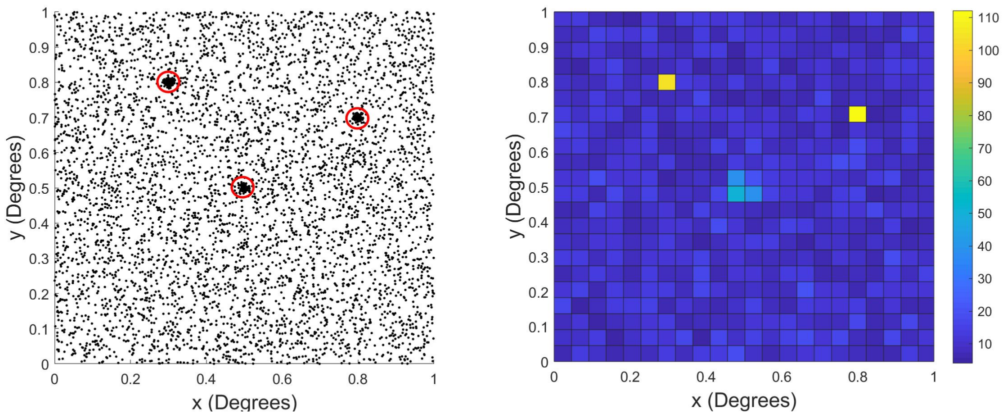

Figure 5: The plot on the left displays the simulated data as points in 2D space. The background is a random distribution of 5000 galaxies as single points with 3 galaxy clusters (circled in red) randomly overlaid. Each galaxy cluster contains 100 'galaxies' tightly packed together. On the right is a heat map of the galaxy density. The colour of each cell corresponds to the amount of objects contained within it. As the colour-bar shows, high density is orange/yellow and low density is blue. Both plots represent one square degree of the sky, although in this case the x and y directions are purely arbitrary.

The first important result for our algorithm is shown in figure 5. This tests if the algorithm can correctly spilt an area into a grid of cells of predetermined size then correctly count all objects within each cell. The simulated distribution is randomised, as are the positions of the dense regions. This initial test also checks the algorithm can accurately locate cells with high densities using parameters designed to mirror the real survey data.

The next major step taken before tackling the real data was to develop the contrast output section of the algorithm. This is the main ingredient in our design to determine which areas of sky may contain a cluster. In order to find the contrast for a given area using equation 4 we had to use the count density found above and visualised in figure 5. Using the count values for each cell the mean for the entire area was calculated as well as the variance. From these numbers the contrast above the mean background can be calculated for each cell. Visualised as a heat map the contrast appears almost identical to the right plot in figure 5 since they both represent how the density varies across the area of sky, however the key difference is the contrast is not dependent on the actual number of objects present. This is important because the contrast needs to be able to work no matter the area of sky and amount of galaxies seen.

The contrast values are dimensionless numbers and represent the intensity of deviation away from the average value of objects per cell. A contrast value of zero for a cell indicates the density of objects it contains is exactly the average value for that area. For this reason the contrast can also be called the 'enhancement above the mean background' as a more accurate name. The enhancement value is quantified as the amount of standard deviation from the mean since the distribution of objects is modelled as a standard Gaussian distribution. Generally, a deviation of three 'sigma' (or 3σ) or higher from the mean is considered a solid detection of something which is very unlikely to be just part of the background. What this means in terms of our project is, any cell found with a contrast value of three or above is considered to contain a cluster.

Figure 6 displays the process of hunting galaxy clusters in the real DES SV data. The scatter plot of the real data is much less evenly distributed and there are no obvious densely pack areas alluding to a cluster being present, in comparison to our simulated data where the clusters were easily discernible. For this random section of sky taken from within the limits of the survey, our algorithm has highlighted one potential cluster. The final step in this sequence is to check if our code has correctly located a cluster using this method of detection. In order to do this, the cells location in terms of real world right ascension and declination is output and checked against a catalogue of known cosmological objects. The accuracy of the coordinates is up to four decimal places in the units of degrees which gives a semi-precise location to look in for an already known object.

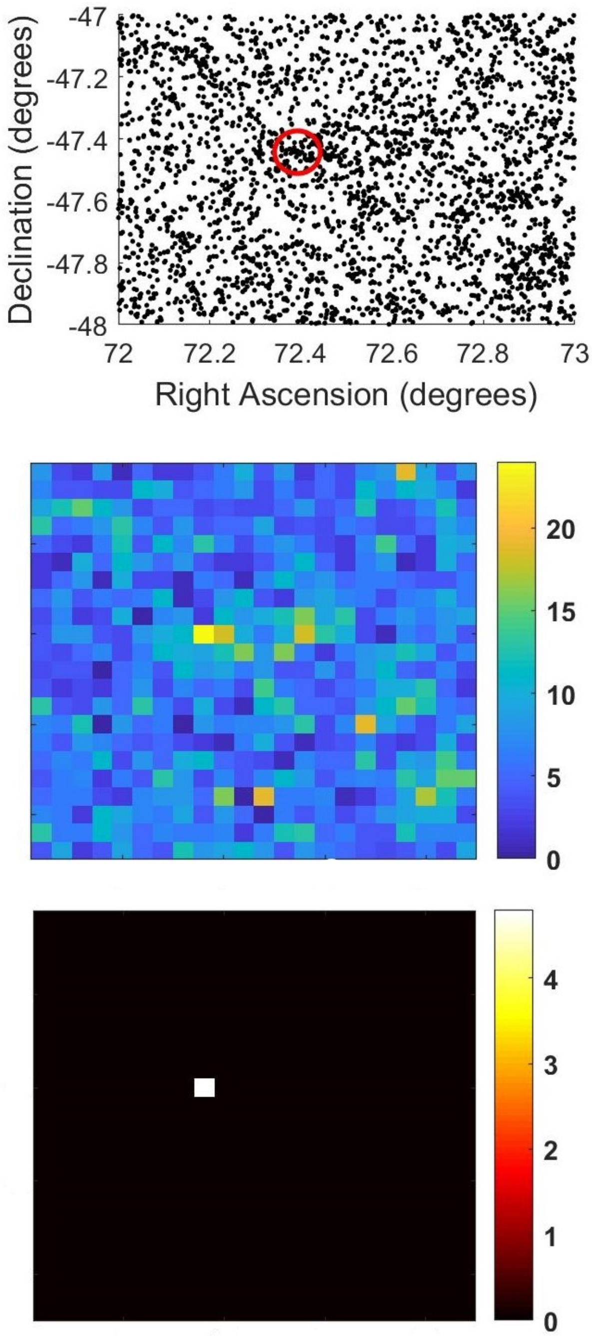

Figure 6: The three plots here represent real DES data. The top figure is a scatter plot of the positions of the galaxies on the sky with their real RA and DEC coordinates in degrees. The middle figure displays the density variation for the counts of each cell across the area of sky, the same as shown for the simulated data above. Finally the bottom figure is a visualisation of the contrast for this area. In order to highlight cluster cells, all values of contrast below the threshold have been flattened to zero. This particular area of sky (chosen at random) appears to potentially contain one galaxy cluster, according to our algorithm. The approximate area of this cluster candidate is circled in red on the top figure.

Initial hunts for solid galaxy cluster candidates in published image catalogues unfortunately proved unsuccessful. After retroactively fixing some minor coding glitches that incorrectly modified the potential cluster coordinates, as well as increasing the contrast threshold up to 4σ in order to combat false positives amongst the background noise, locating a real cluster still proved difficult. Although our search did not yield any obvious galaxy clusters, a few coordinates did indicate the presence of a large collection of galaxies. Below figure 7 shows an optical image from the DES database of one of the potential cluster locations highlighted by our algorithm. The top right corner of the image contains many bright galaxies in close vicinity to each other, indicating a positive detection by our algorithm.

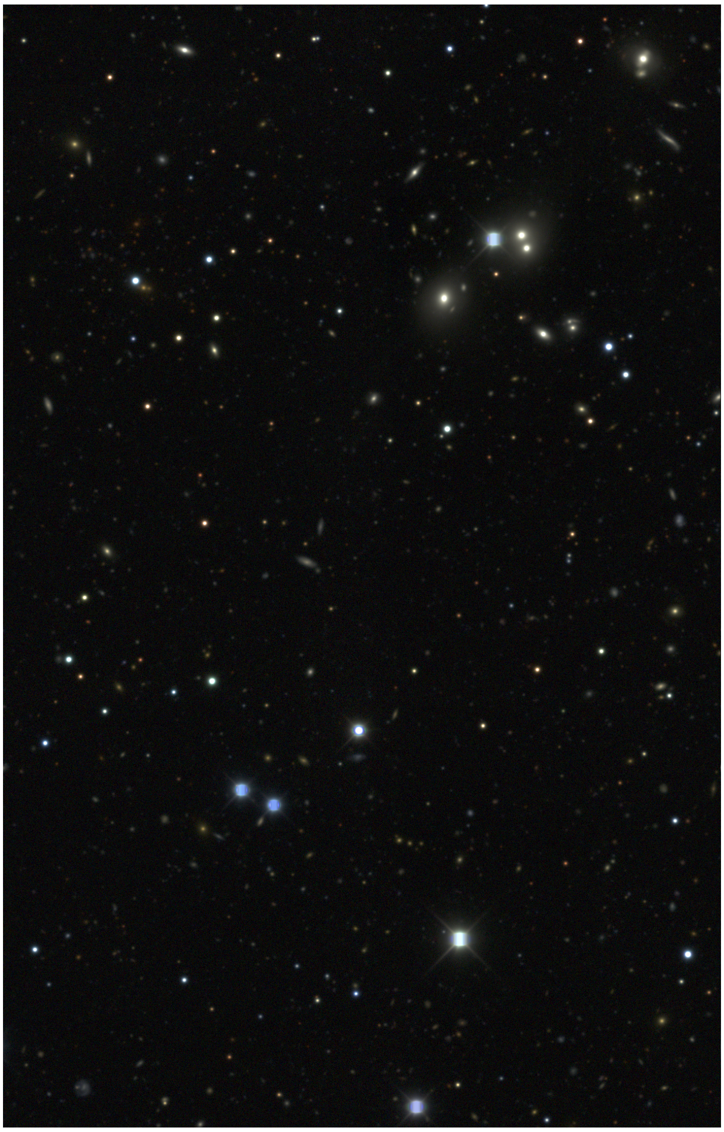

Figure 7: This optical image is taken directly from the DES database. There are many objects visible in this area of sky with a potential galaxy cluster being found in the top right of the image with multiple bright objects (assumed to be galaxies) bunched together. This is one of the few images that had any slight hint of a visible galaxy cluster.

Looking over many different (a total of approximately 20 square degrees was scanned) parts of the sky data from the survey proved to be more of the same. Each square degree of the sky that the algorithm looked over showed the potential to contain a few clusters at most on average. None of the coordinates of these cells showed an obvious cluster presence, except for a small number that were on the borderline and hinted at a cluster like the image shown in figure 7.

Although it is disappointing to conclude this project without concrete evidence this method of galaxy cluster detection was successful, the results are overall still positive. It is clear from the trials on the simulated data that the algorithm we constructed performed its basic purpose efficiently and as we had desired. It was able to repeatedly correctly locate simulated clusters wherever they would appear in the randomly generated data. Applying the real world DES data to our code most likely performed less successfully due to galaxy clusters being much harder to find with our relatively simple detection criteria.

5 Discussion

The biggest flaw with how our algorithm determines what constitutes a galaxy cluster is purely spatial in nature. We have assumed all the way through this experiment that all the galaxy positions seen on our two-dimensional projection of the SV catalogue are at the same distance away from Earth. Without further detailed analysis of each object, our simple assumptions have overlooked this factor since it is beyond the scope of this project. From our point of view, galaxies may appear in the same area at face value but the reality may be they all sit at different distances from us in the same patch of sky. An improvement to the accuracy of this project would be made by taking the problem of space being three dimensional into consideration in order to rule out any false positives generated by this phenomenon.

A high density of galaxy type objects in a confined area of the sky is the only marker our algorithm uses to check for clusters. Extending the algorithm to incorporate various other physics that indicates a galaxy cluster type object would be the best way to improve detection when it comes to real data. Although these extensions were beyond the scope of this project, many ideas were highlighted as the experiment was performed. The Red Sequence method provides a solution to this 'line-of-sight' problem. It has been found that all galaxy clusters contain early-type elliptical galaxies which form a very tight red sequence relationship between there colour and brightness [11]. The redder the elliptical galaxy, the brighter it shines. This is a general feature found in all clusters, and it is useful since galaxies which are lined up by chance on the sky appearing to be in a cluster will not have this very strong relation, so by checking for it the false positive detection can be widely reduced.

Red Sequence clustering is one of many methods which applies other known cluster physics, aside from galaxy concentration, in order to improve the search. Another phenomena of galaxy clusters that can be exploited is their ability to cause gravitational lensing of light sources such as other galaxies and stars. Clusters contain hundreds of galaxies which each contain trillions of stars, their relatively huge masses lead to an incredibly strong gravitational pull by the cluster. A large enough gravitational well, such as the one produced by a galaxy cluster, can act as a cosmological magnifying glass by 'bending' light originating from behind it in our line of sight [12]. Searching for areas with these distorted magnified images of galaxies sat behind clusters can aid in finding clusters that may be otherwise hidden or difficult to detect.

Overall, by the end of this project the algorithm we set out to build was successfully completed and performed as we had intended. The shortcomings of the results came when applying our code to the actual data from the DES. The main factor that affected any obvious positive galaxy cluster detection came from the simplicity of the method of detection. Simulated data ignored many real world physical limits and alterations which the real data contains. Extending the method to take at least one of the previously outlined corrections into account would alleviate this issue and improve the overall detection success.

6 Conclusion

Although our search for galaxy clusters in the Dark Energy Survey data did not prove as fruitful as we had hoped, overall the project was a success. The algorithm constructed for the purpose of looking for these clusters worked as intended. Calculations performed as a basis for the theory also came out as expected when compared to known results. The limitation of this project is the relative simplicity of the method of detection used. The methods utilised throughout this endeavour are a good foundation for a strong galaxy cluster detection tool. Many extensions can be made on the designs created here in order to improve the detection in the final results. However, within the boundaries set up at the beginning of this experiment, we were still able to achieve the goals set at the start.

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