3 pillars of category theory

1. Abstraction
Focus only on properties we care about

2. Composition
Things are composable as in we can take two things and make a new thing as a "closed" operation.

3. Identity
Things that may have been different in the details become similar when abstracted.
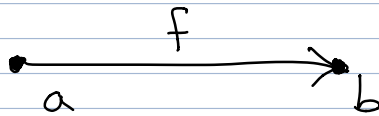
Concepts of Category Theory

A **category** contains objects.
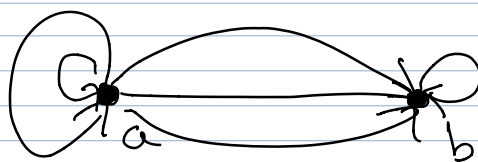(Important distinction that its <u>not</u> a SET of objects)

Objects have no properties.
They're the pinnacle of abstraction!
They're atomic in that they are
not decomposable.

• a

Morphisms are relationships between
objects. They are primitives, just
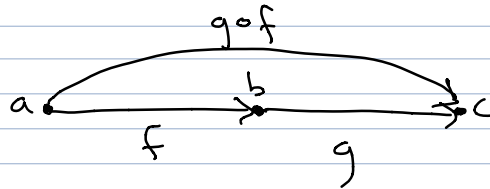like objects.

$$a \xrightarrow{f} b$$

There can be any number of
arrows between objects, even
infinite or uncountably infinite.

Notice here that the two arrows
from 'a' to itself are <u>not</u> identical.

Composition of Arrows

Arrows are composable. That is,

$$g \circ f$$

a •———————→• b ———→• c
     f   g

For two arrows f & g, a composition arrow $g \circ f$ from 'a' to 'c' __must__ exist that is __identical__ to 'f' then 'g'.

A category is defined by the compositions of its objects. Not the objects or arrows themselves.

2 Algebraic Properties

1. Identity

There exists an identity arrow id_a on a to or from any arrow f such that $id_a \circ f = f$ or $f \circ id_a = f$ respectively.

$$id_a \circ f = f \qquad\qquad f \circ id_a = f$$

2. Associativity

For arrows f, g, h w/ structure below,

$$h \circ (g \circ f) = (h \circ g) \circ f$$