2 Types of Semantics

Operational - Describing the operations necessary to execute code

Denotational - Mapping instructions to another model (like math)

A **function** is a subset of the cartesian product of sets

Define $f: A \to B$ $\quad f \subset A \times B$

$(a,b) \in f$

In a function, many elements in the domain can map to a single element in the range, but one elem in domain cannot map to many in range

In Category Theory, a function, f, that has an inverse, g, is an **isomorphism**

$f : a \to b$ $\quad\quad$ $g \circ f = id_a$

$g : b \to a$ $\quad\quad$ $f \circ g = id_b$

In Set Theory,
A function has an inverse iff

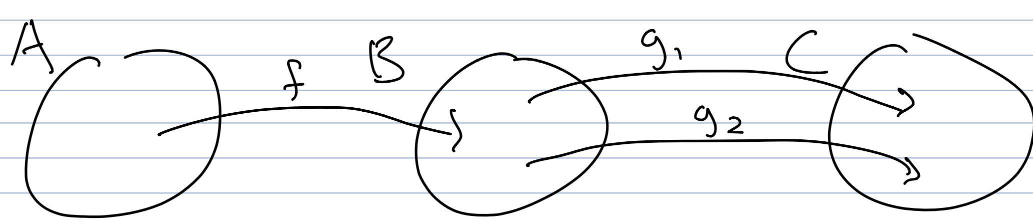
1. It is **monic** (injective)
2. It is **epic** (surjective)

Derivation of Epimorphism in Category Theory

Specifically, to show a function, f,

$f : A \to B$

is an epimorphism (surjective)



compose f with any two functions

$$g_1 : B \to C$$
$$g_2 : B \to C$$

f is an **epimorphism** iff

$$\forall g_1, g_2 \quad g_1 \circ f = g_2 \circ f \Rightarrow g_1 = g_2$$

If f were not epic, there would exist 2 f-ns $g_1 \neq g_2$ that could still be equal under f composition, that is

$$g_1 \circ f = g_2 \circ f$$

(Video 2.1 40 mins for explanation)

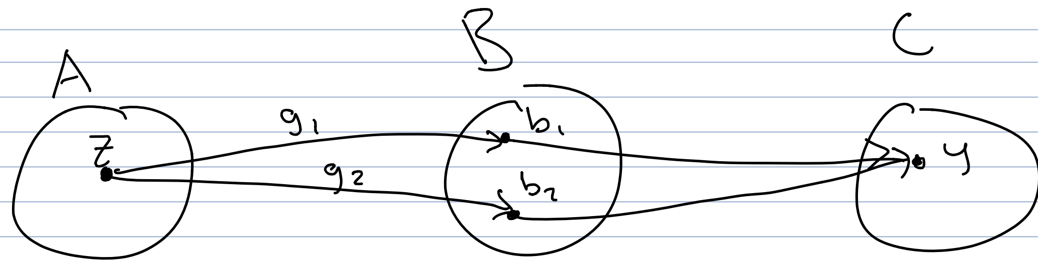
Definition of Monomorphism

Similar to the proof above, function f is a **monomorphism** iff

$$\forall g_1, g_2 \quad f \circ g_1 = f \circ g_2 \Rightarrow g_1 = g_2$$

Where this time $g_1 : A \to B$
$g_2 : A \to B$

An example where $g_1 \neq g_2$ but $f \circ g_1 = f \circ g_2$ and therefore f is <u>not</u> monic.



Notice that both definitions use the entire universe ($\forall g_1, g_2$) to prove. This is characteristic of Category Theory.

Also important to note that the definition of an invertible function (epic + monic) does <u>not</u> translate to morphisms in Category Theory.