3 pillars of category theory

1. Abstraction

Focus only on properties we care about

2. Composition

Things are composable as in we can take two things and make a new thing as a "closed" operation.

3. Identity

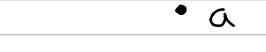
Things that may have been different in the details become similar when abstracted.

Concepts of Category Theory

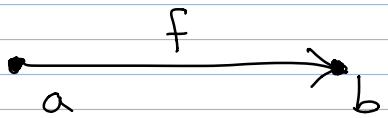
A **category** contains objects.

(Important distinction that its <u>not</u> a SET of objects)

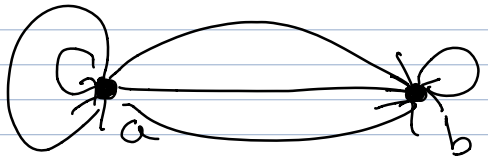
Objects have no properties. They're the pinnacle of 'abstraction'. They're atomic in that they are not decomposable.



Morphisms are relationships between objects. They are primitives, just like objects.



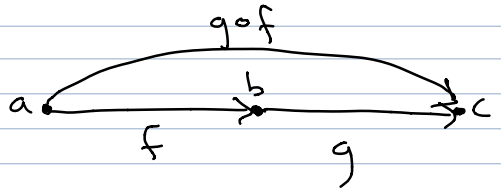
There can be any number of arrows between objects, even infinite or uncountably infinite.



Notice here that the two arrows from 'a' to itself are <u>not</u> identical.

Composition of Arrows

Arrows are composable. That is,



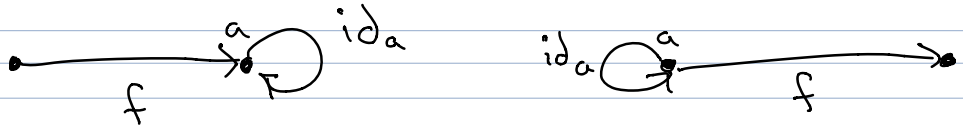
For two arrows f & g, a composition arrow g∘f from 'a' to 'c' must exist that is identical to 'f' then 'g'.

A category is defined by the compositions of its objects. Not the objects or arrows themselves.

2 Algebraic Properties of Composition

1. Identity

There exists an identity arrow id_a on a to or from any arrow f such that $id_a \circ f = f$ or $f \circ id_a = f$ respectively.



$id_a \circ f = f$ $\qquad$ $f \circ id_a = f$

2. Associativity

For arrows f, g, h w/ structure below,

$$h \circ (g \circ f) = (h \circ g) \circ f$$