

## 2 Types of Semantics

Operational - Describing the operations necessary to execute code

Denotational - Mapping instructions to another model (like math)

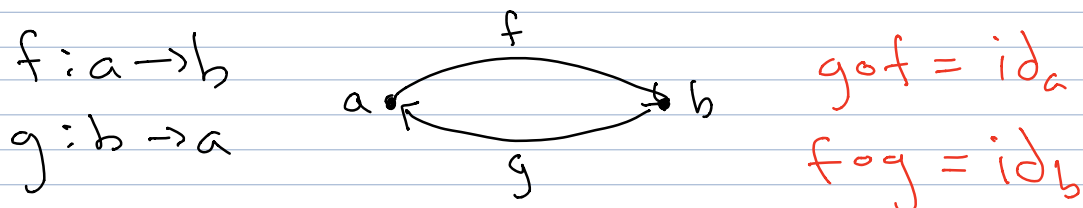
A function is a subset of the cartesian product of sets

Define  $f: A \rightarrow B$        $f \subset A \times B$

$(a, b) \in f$

In a function, many elements in the domain can map to a single element in the range, but one elem in domain cannot map to many in range

In Category Theory, an arrow  $f$ , that has an inverse,  $g$ , is an isomorphism



In Set Theory,  
A function has an inverse iff

1. It is **monic** (injective)
2. It is **epic** (surjective)

## Derivation of Epimorphism in Category Theory

Specifically, to show a function,  $f$ ,

$$f: A \rightarrow B$$

is an epimorphism (surjective)



compose  $f$  with any two functions

$$g_1: B \rightarrow C$$

$$g_2: B \rightarrow C$$

$f$  is an **epimorphism** iff

$$\forall g_1, g_2 \quad g_1 \circ f = g_2 \circ f \Rightarrow g_1 = g_2$$

If  $f$  were not epic, there would exist 2 f-ns  $g_1 \neq g_2$  that could still be equal under  $f$  composition, that is

$$g_1 \circ f = g_2 \circ f$$

(Video 2.1 40 mins for explanation)

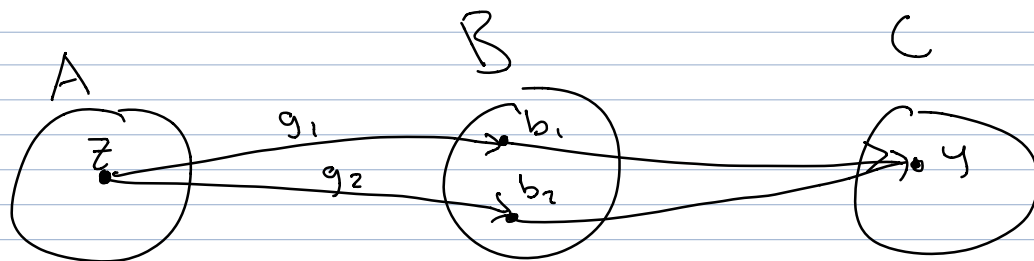
## Definition of Monomorphism

Similar to the proof above, function  $f$  is a **monomorphism** iff

$$\forall g_1, g_2 \quad f \circ g_1 = f \circ g_2 \Rightarrow g_1 = g_2$$

Where this time  $g_1: A \rightarrow B$   
 $g_2: A \rightarrow B$

An example where  $g_1 \neq g_2$  but  $f \circ g_1 = f \circ g_2$  and therefore  $f$  is not monic.



Notice that both definitions use the entire universe ( $\forall g_1, g_2$ ) to prove. This is characteristic of Category Theory.

Also important to note that the definition of an invertible function (epic & monic) does not translate to morphisms in Category Theory.