Lecture 11

Quantitative Political Science

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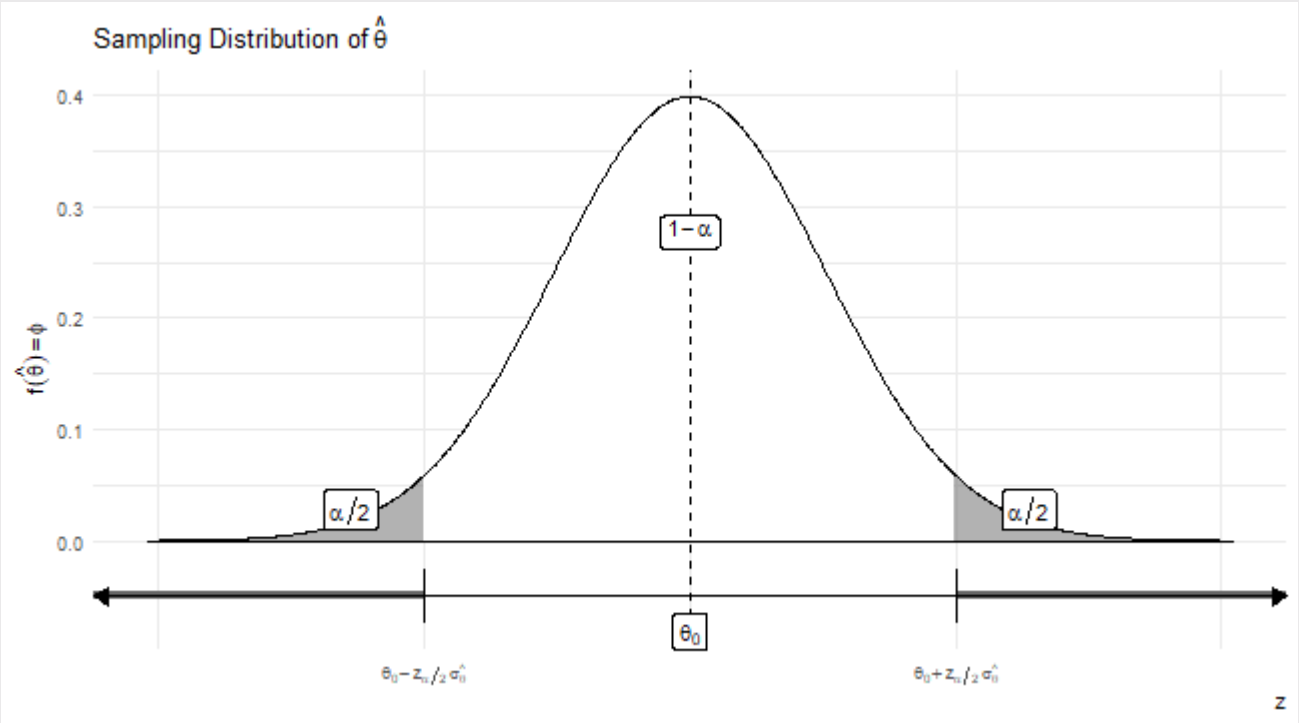
Agenda

1. Type 1 and Type II Error
2. Calculating Power
3. p-values

Type I Error

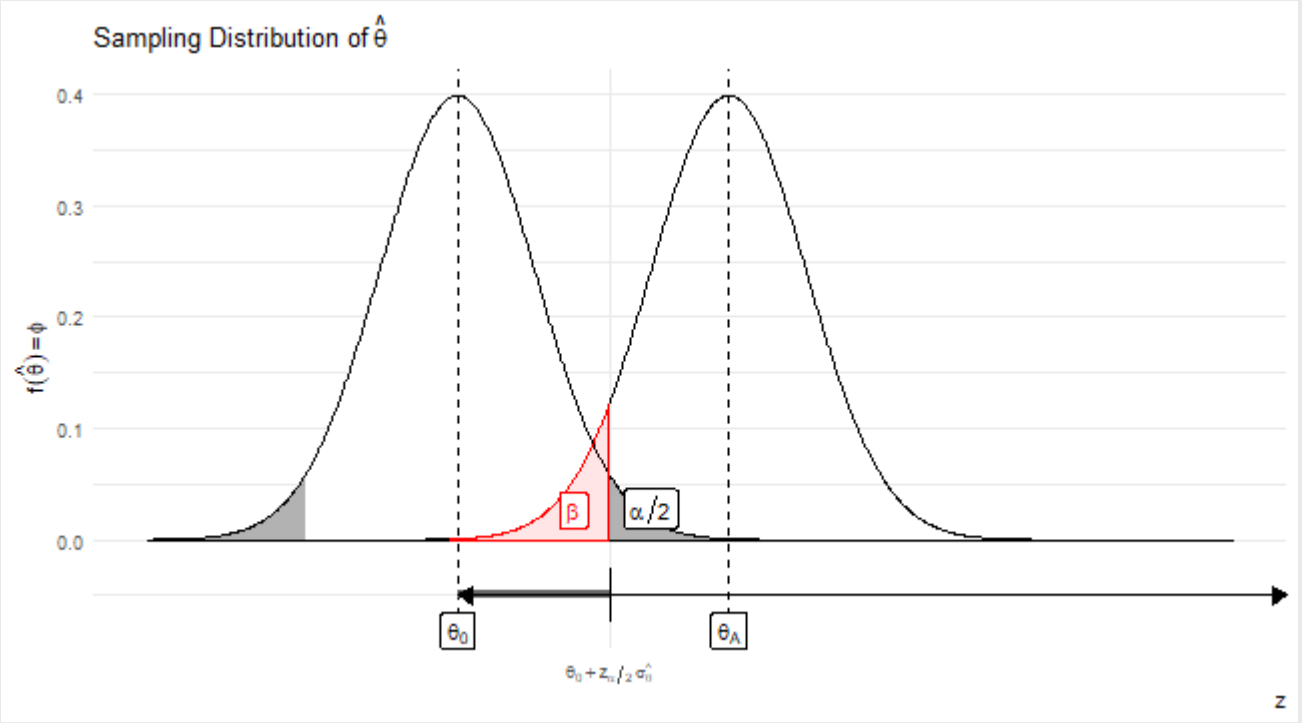
- **Type I error**: reject H_0 when it is actually true
  - What does this look like?



Type I error

- We will (purely by chance):
  - Observe an estimated $\hat{\theta}$ in the *RR* $100 * \alpha\%$ of the time
  - Thus falsely reject the null even though it's true
- This is Type I error!

Type II error



Type II error

- Suppose that the alternative hypothesis is true
- But we always conduct our hypothesis test **under the assumption that the null is true**
- If the sampling distribution of our estimator $\hat{\theta} \sim N(\theta_A, \sigma_{\hat{\theta}})$, we will mistakenly accept the null $100 * \beta\%$ of the time
- Define **power** as $1 - \beta$

$$\begin{aligned} \text{Power} &= 1 - \beta \\ &= 1 - Pr(\text{Accept } H_0 | H_A \text{ true}) \\ &= 1 - Pr(\hat{\theta} < \theta_0 + z_{\alpha/2}\sigma_{\hat{\theta}} | \theta = \theta_A) \end{aligned}$$

Power

- We can do this!

$$
\begin{aligned}
\beta &= \Pr(\hat{\theta} < \theta_0 + z_\alpha \sigma_{\hat{\theta}} | \theta = \theta_A) \\
&= \Pr\left(\frac{\hat{\theta} - \theta_A}{\sigma_{\hat{\theta}}} < \frac{\theta_0 + z_\alpha \sigma_{\hat{\theta}}}{\sigma_{\hat{\theta}}} | \theta = \theta_A\right) \\
&= \Phi\left(\frac{\theta_0 + z_\alpha \sigma_{\hat{\theta}} - \theta_A}{\sigma_{\hat{\theta}}}\right) \\
&= \Phi\left(\frac{\theta_0 - \theta_A}{\sigma_{\hat{\theta}}} + z_\alpha\right)
\end{aligned}
$$

Power

- We know θ_0 and θ_A (or we can specify them)
- We have also specified α and therefore z_α

$$\text{Power} = 1 - \Phi\left(\frac{\theta_0 - \theta_A}{\frac{\sigma}{\sqrt{n}}} + z_\alpha\right)$$

- Stare at this for a second: can you figure out the following signs?

$$\frac{\frac{\partial \text{Power}}{\partial \alpha}}{\frac{\partial \text{Power}}{\partial \sigma}} \frac{\frac{\partial \text{Power}}{\partial n}}{\frac{\partial \text{Power}}{\partial (|\theta_0 - \theta_A|)}}$$