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Quantitative Research in Political Science I
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Some Helpful Probability Concepts

1 Univariate Analysis

1.1 For a *discrete* random variable, Y

probability function $p(y) \equiv P(Y = y)$

1.2 For a *continuous* random variable, Y

cumulative distribution function, or CDF $F(y) \equiv P(Y \leq y) = \int\limits_{-\infty}^{y} f(t)dt$

probability density function, or PDF $f(y) \equiv \frac{dF(y)}{dy} = F'(y)$

2 Multivariate Analysis

2.1 For two *discrete* random variables Y_1 and Y_2

joint probability function	$p(y_1, y_2) \equiv P(Y_1 = y_1, Y_2 = y_2)$
joint distribution function, or joint CDF	$F(y_1, y_2) \equiv P(Y_1 \le y_1, Y_2 \le y_2) = \sum_{t_1 \le y_1} \sum_{t_2 \le y_2} p(t_1, t_2)$
marginal probability function of Y_1	$p_1(y_1) \equiv P(Y_1 = y_1) = \sum_{all\ y_2} p(y_1, y_2)$
conditional probability function of Y_1 given Y_2	$p(y_1\|y_2) \equiv P(Y_1 = y_1\|Y_2 = y_2) = \frac{p(y_1,y_2)}{p_2(y_2)},\ p_2(y_2) > 0.$

2.2 For two *jointly continuous* random variables Y_1 and Y_2

joint distribution function, or joint CDF	$F(y_1, y_2) \equiv P(Y_1 \le y_1, Y_2 \le y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1$
joint density function, or joint PDF	$f(y_1, y_2) \equiv \frac{\partial^2 F(y_1,y_2)}{\partial y_1 \partial y_2}$
marginal density function of Y_1	$f_1(y_1) \equiv \int_{-\infty}^{\infty} f(y_1, y_2) dy_2$
conditional distribution function of Y_1 given Y_2	$F(y_1\|y_2) \equiv P(Y_1 \le y_1\|Y_2 = y_2) = \int_{-\infty}^{y_1} \frac{f(t_1,y_2)}{f_2(y_2)} dt_1$
conditional density function of Y_1 given $Y_2 = y_2$	$f(y_1\|y_2) \equiv \frac{f(y_1,y_2)}{f_2(y_2)},\ f_2(y_2) > 0.$