New York University
Wilf Family Department of Politics
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Quantitative Research in Political Science I

Professor Patrick Egan

PROBLEM SET 6: Due Tuesday, November 6 at 10 a.m.

A reminder: you may work with others in the class on this problem set, and you are in fact encouraged to do so. However, the work you hand in must be your own. Handwritten work is acceptable, but word-processed work (e.g., using LaTeX) is preferred.

1. You are studying the i.i.d. random variable, Y. You are conducting a hypothesis test with null and alternative $H_0 : \mu_Y = 24$, $H_A : \mu_Y > 24$. You draw a sample of $N = 25$ and find that $\overline{Y} = 25$ and $S_U = 12$.

   (a) If $\alpha = .10$, for what value of $\overline{Y}$ or greater do we reject H_0 if we conduct a z-test?

   (b) If $\alpha = .10$, for what value of $\overline{Y}$ or greater do we reject H_0 if we conduct a t-test?

   (c) Now answer (a) and (b) again, but this time stipulate $\alpha = .001$.

   (d) What do the results in (a) through (c) suggest about how the difference between z-tests and t-tests changes with α? Given what we know about the shape of the t and the Normal distributions, why does this make sense? (If you're stumped, it may help to look at Figure 7.3, page 360 of your text.) Provide a diagram (you can draw it by hand) explaining your answer.

   (e) What is the *attained level of significance* (also known as the p-value) associated with the finding $N = 25, \overline{Y} = 25, S_U = 12$ if we conduct a t-test?

   (f) What is the *power* of this test if we stipulate $H_A : \mu_Y = 27$ and $\alpha = .10$?

   (g) Given these parameters, what *sample size* would we need for our hypothesis test's power to equal or exceed .80?

2. Consider two 95% CIs drawn around some parameter θ that has standard deviation σ_θ. The first CI is drawn with the assumption that the sampling distribution of the standardized version of estimator $\widehat{\theta}$ of the parameter $\left(\frac{\widehat{\theta}-\theta}{\sigma_{\widehat{\theta}}}\right)$ follows the t-distribution. The second CI is drawn with the assumption that the sampling distribution follows the standard Normal.

   (a) Show that the magnitude of the difference between the lower bounds of these two confidence intervals is always equal to

   $$\left(t_{\frac{\alpha}{2},n-1} - z_{\frac{\alpha}{2}}\right)\frac{\sigma_\theta}{\sqrt{n}}.$$

   (b) Express the magnitude of the difference in the lower bounds of these two confidence intervals in terms of σ_θ for $n = 1000$; $n = 100$; and $n = 25$. Do these differences seem very large to you? In a few sentences, say what you learn from this analysis.

3. When we conduct a t-test, we are assuming that we are working with a sample drawn from a population whose distribution follows the Normal. But how well does the t-test work in small samples that are not Normal and are in fact quite skewed? Using techniques learned in lab, do the following:

   (a) We'll be simulating a random variable that follows the *gamma distribution* with parameters $\alpha = 1$ and $\beta = 1$. To get a sense of what this distribution looks like, first draw a sample of $n = 1,000$ from the distribution using the Stata function **rgamma(1,1)** and graph the distribution of these observations. Pretty skewed, right?

   (b) A random variable that follows the gamma distribution has mean $\alpha\beta$ and variance $\alpha\beta^2$. So in our case, $\mu = 1$ and $\sigma^2 = 1$. Show that therefore a hypothesis test with a .05 level of significance should inaccurately reject the null $H_0 : \mu = 1$ in favor of the alternative $H_A : \mu \neq 1$ only 5% of the time.

   (c) Now let's see if this is the case when we conduct a t-test on small samples drawn from this distribution. Using Stata, do the following:

      - Generate 1,000 samples of $n = 15$ each from a gamma distribution with $\alpha = 1$ and $\beta = 1$.
      - In each of the samples, conduct a t-test pitting the null $H_0 : \mu = 1$ against the alternative $H_0 : \mu \neq 1$.
      - Record the percentage of times the test (inaccurately) rejects the null.
      - How close is this to 5%? Does the t-test give us more or less confidence that we are avoiding Type I error than it should? By how much?

   (d) Now run the same experiment with samples of $n = 29$. How do the results change from what you found in (b)?

   (e) Finally, run the experiment again with samples of $n = 29$, but now use a z-test. How do the results change from what you found in (c)?