Vanderbilt University
Political Science Department
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Stats 1

(PSCI 8356)
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PRACTICE MIDTERM EXAMINATION:

These are meant to be examples of the types of questions that might appear on the exam. You will need either statistical tables like those appearing in the back of the textbook or access to a statistical software program (i.e., R) that can provide similar information. Consultation of Web resources is not permitted.

Be sure to show all your work and to use complete sentences to provide explanations.

1. (8.3 - 8.5) Suppose $\hat{\theta}$ is an estimator for θ and $E(\hat{\theta}) = a\theta + b$ for some nonzero constants a and b.

   (a) In terms of a, b, and θ, what is $B(\hat{\theta})$?

   (b) Find a function of $\hat{\theta}$ – say, $\hat{\theta}^*$ – that is an unbiased estimator for θ.

   (c) Express $MSE(\hat{\theta}^*)$ as a function of $VAR(\hat{\theta})$.

   (d) Give an example of a value for a for which $MSE(\hat{\theta}^*) < MSE(\hat{\theta})$.

   (e) Give an example of values for a and b for which $MSE(\hat{\theta}^*) > MSE(\hat{\theta})$

2. (8.17) If Y has a binomial distribution with parameters n and p, then $\hat{p}_1 = Y/n$ is an unbiased estimator of p. Another estimator of p is $\hat{p}_2 = (Y+1)/(n+2)$.

   (a) Derive the bias of $\hat{p}_2$.

   (b) Derive $MSE(\hat{p}_1)$ and $MSE(\hat{p}_2)$.

   (c) For what values of p is $MSE(\hat{p}_1) < MSE(\hat{p}_2)$?

3. (8.27) A random sample of 985 "likely voters" -— those who are judged to be likely to vote in an upcoming election —- were polled during a phone-athon conducted by the Republican Party, who is quite naturally interested in knowing whether they will win the election. Of those contacted, 592 indicated that they intended to vote for the Republican running in the election.

   (a) Conduct a hypothesis test, explicitly writing out each of the four components of the test.

   (b) Do you think the Republican candidate will be elected? Why? How confident are you in this conclusion?

   (c) Can you think of reasons that those polled might not be representative of those who actually vote in the election?

4. Please indicate whether these statements are True / False / Uncertain. Provide some explanation, either in words or in math, to receive full credit.

   (a) If two random variables have a covariance of 0 they must be independent. However, the reverse is not true that if they are independent must have a covariance of 0.

   (b) Type I and Type II error are positively related. The more likely you are to commit Type I error, the more likely you will also commit Type II error.

   (c) A one-tailed hypothesis test is easier to reject the null than a two-tailed hypothesis test for the same level of confidence.

   (d) The assumption that our data are drawn from an random sample is necessary to assume that the sampling distribution for an estimator converges on the Normal distribution.

   (e) We require both Slutzky's Theorem and consistency to prove that the standardized sampling distribution converges on the standard normal (i.e., $F\left(\frac{\bar{Y}-\mu}{S_u/\sqrt{n}}\right) \xrightarrow{p} \Phi$).