Lecture 18 notes

Tuesday, November 14, 2023   9:26 AM

Dot product:

$$\vec{u} = [u_1, u_2, \dots, u_n]$$

$$\vec{v} = [v_1, v_2, \dots v_n]$$

$$\vec{u} \cdot \vec{v} = [u_1 v_1 + u_2 v_2 + \dots + u_n v_n]$$

$$= \sum_{i}^{n} u_i v_i$$

Matrix Mult:

$$A = [\quad]_{n \times m}$$

$$B = [\quad]_{m \times p}$$

$$A = \begin{array}{c} \begin{bmatrix} 2 & 10 \\ 0 & 1 \\ -1 & 5 \end{bmatrix} \begin{array}{l} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \end{array} \end{array} \quad 3 \times 2$$

$$B = \begin{array}{c} \begin{bmatrix} 1 & 4 \\ -1 & 10 \end{bmatrix} \\ \vec{b}_1 \quad \vec{b}_2 \end{array} \quad 2 \times 2$$

$$= \begin{bmatrix} \vec{a}_1 \vec{b}_1 & \vec{a}_1, \vec{b}_2 \\ \vec{a}_2 \vec{b}_1 & \vec{a}_2 \vec{b}_2 \\ \vec{a}_3 \vec{b}_1 & \vec{a}_3 \vec{b}_2 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + 10 \cdot -1 & 2 \cdot 4 + 10 \cdot 10 \\ 0 \cdot 1 + 1 \cdot -1 & 0 \cdot 4 + 1 \cdot 10 \\ -1 \cdot 1 + 5 \cdot -1 & -1 \cdot 4 + 5 \cdot 10 \end{bmatrix} = \begin{bmatrix} -8 & 108 \\ -1 & 10 \\ -6 & 46 \end{bmatrix}$$

Properties of Matrix Mult:

1. Associative: $(AB)C = A(BC)$
2. Distributive: $A(B+C) = AB + AC$
3. Transpose Rule: $(AB)^T = B^T A^T$
4. NOT commutative: $AB \neq BA$

④ NOT commutative: $AB \neq BA$

$$B(\hat{\theta}) = E(\hat{\theta}) - \theta$$

$$E(X) = \begin{bmatrix} E(x_{11}) & E(x_{12}) \\ E(x_{21}) & E(x_{22}) \end{bmatrix}$$

$$\boxed{\vec{y} = A\vec{x}}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{n1} & a_{n2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{\partial \vec{y}}{\partial \vec{x}} = A^T$$

$$y = a_1 x_1^1 + a_2 x_2^1$$

$$\frac{\partial y}{\partial a_1} = a_1 x_1^0$$

$$\frac{\partial y}{\partial a_1} = a_1$$

$$y = a^T x$$

$$\frac{\partial y}{\partial x} = a$$

Special Matrixes

0: zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot 0 = 0$$

I: identity matrix: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$AI = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$I_{n \times n} : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix Inversion

$$\frac{a}{b} = a \cdot b^{-1}$$

$$b \cdot b^{-1} = \frac{b}{b} = 1$$

$$A = \begin{bmatrix} & \end{bmatrix}_{n \times m}$$

$$A^{-1} = AA^{-1} = I$$

$$(A^{-1})^{-1} = A$$

$$(A^{T})^{-1} = (A^{-1})^{T}$$

What is A^{-1}?

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

$$2x_1 + 1x_2 = 10$$

$$2x_1 - 1x_2 = -10$$

$$\vec{b} = \begin{bmatrix} 10 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$