3. Indicate whether each of the following statements is true or false. Explain each of your answers, using mathematics where necessary.

   (a)

   (b)

   (c) All things being equal, Type I errors are more likely with small samples than with large samples.
   False. The probability of a Type I error is specified by the researcher *a priori* as the confidence coefficient α, which remains constant regardless of sample size.

   (d) All things being equal, Type II errors are more likely with large samples than with small samples.
   True, as shown by when conducting a z-test with a null θ_0 and alternative θ_A, where $\theta_0 < \theta_A$:

$$\begin{aligned} P\left(\text{Type II error}\right) &= \beta = P\left(\text{Reject } H_0 | H_A \text{ true}\right) \\ &= P\left(\widehat{\theta} < \theta_0 + z_\alpha \sigma_{\widehat{\theta}} | \theta = \theta_A\right) \\ &= \Pr\left(\frac{\widehat{\theta} - \theta_A}{\sigma_{\widehat{\theta}}} < \frac{\theta_0 + z_\alpha \sigma_{\widehat{\theta}} - \theta_A}{\sigma_{\widehat{\theta}}} | \theta = \theta_A\right) \\ &= \Phi\left(\frac{\theta_0 + z_\alpha \sigma_{\widehat{\theta}} - \theta_A}{\sigma_{\widehat{\theta}}}\right) \\ &= \Phi\left(\frac{\theta_0 - \theta_A}{\sigma_{\widehat{\theta}}} + z_\alpha\right), \text{ so} \\ \beta &= \Phi\left(\frac{\theta_0 - \theta_A}{\frac{\sigma}{\sqrt{n}}} + z_\alpha\right). \end{aligned}$$

   Noting that $\theta_0 - \theta_A < 0$,we see that:

$$\frac{\partial \frac{\theta_0 - \theta_A}{\frac{\sigma}{\sqrt{n}}}}{\partial n} < 0,$$

   and so

$$\frac{\partial \beta}{\partial n} < 0.$$

   Type II errors are thus *less* likely with large samples than small samples.

   (e)

$$\frac{\partial \sigma^2_{\overline{Y}}}{\partial \sigma^2_Y} < 0.$$

   FALSE.

$$\begin{aligned} \sigma^2_{\overline{Y}} &= \frac{\sigma^2_Y}{n}, \text{ so} \\ \frac{\partial \sigma^2_{\overline{Y}}}{\partial \sigma^2_Y} &= \frac{1}{n} > 0. \end{aligned}$$

   (f) In an i.i.d. random sample of size n drawn from the population Y, the observation Y_1 is an unbiased estimate of μ_Y.
   True. Identicality ensures that $E\left(Y_1\right) = E\left(Y\right) = \mu_Y$.