# Lecture 17

## Quantitative Political Science

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# Agenda

1. Estimating Error Variance
2. Hypothesis Testing
3. Controls

# Estimating Error Variance

- Ended last lecture talking about $VAR(\hat{\beta}_1) = \frac{\sigma^2}{SST_x}$
  - This is a **conceptual** quantity
- How do we actually calculate it?
  - Recall from the univariate case where we wrote $VAR(\bar{Y}) = \frac{\sigma^2}{n}$
  - We said it is rare that we actually know $\sigma^2$, but we still estimate it with $S_u^2 = \frac{\sum(y_i - \bar{y})^2}{n-1}$
- Here, we do something very similar: $\hat{\sigma}^2 = \frac{\sum(u_i)^2}{n-2} = \frac{SSR}{n-2}$
  - Why $n-2$?

# Estimating Error Variance

- Thus we plug this into our two formulas for $VAR(\hat{\beta}_0)$ and $VAR(\hat{\beta}_1)$

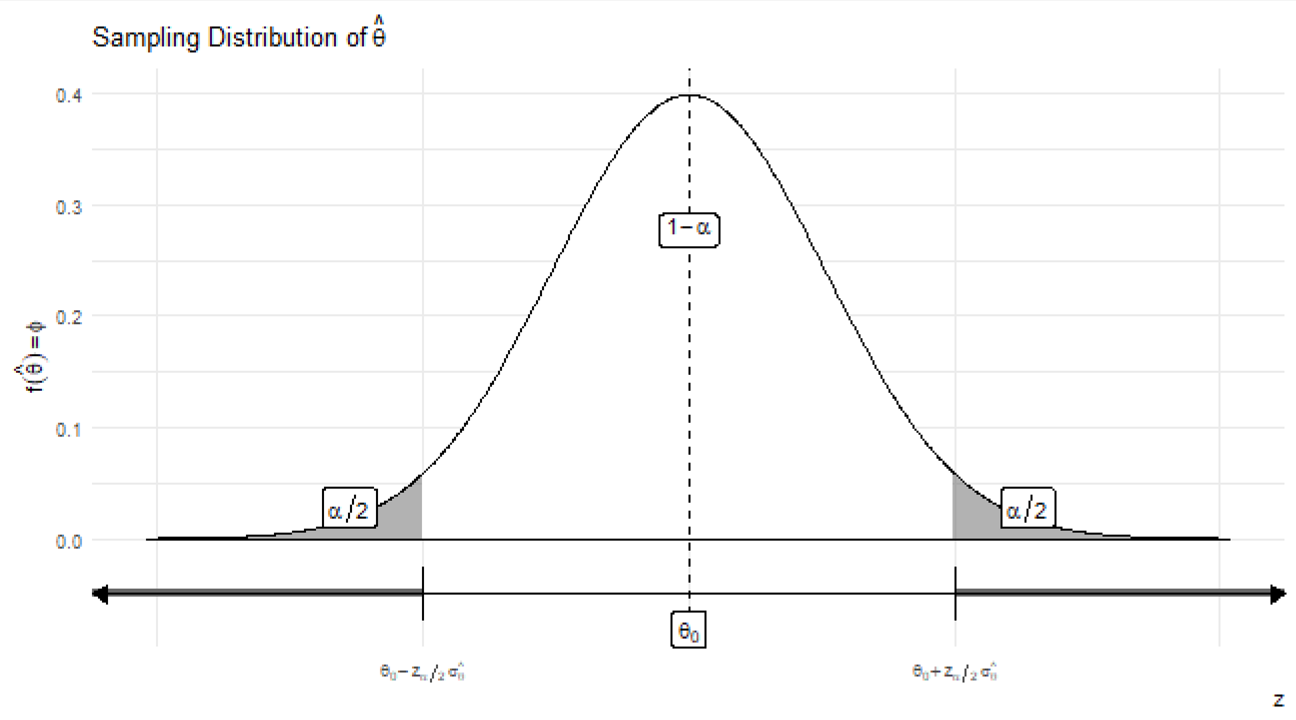
$$
\begin{aligned}
\widehat{VAR(\hat{\beta}_0)} &= \frac{\hat{\sigma}^2 \frac{\sum x_i^2}{n}}{SST_x} \\
&= \frac{\frac{SSR}{n-2} \frac{\sum x_i^2}{n}}{SST_x} \\
\widehat{VAR(\hat{\beta}_1)} &= \frac{\hat{\sigma}^2}{SST_x} \\
&= \frac{\frac{SSR}{n-2}}{SST_x}
\end{aligned}
$$

# Estimating Error Variance

- As we discussed last week in the theoretical case, $\hat{\sigma}^2$ is a very interesting quantity, because $\sqrt{\hat{\sigma}^2} = \hat{\sigma} \xrightarrow{p} \sigma$
  - $\hat{\sigma}$ is expressed in units of $y$
  - Tell us how far the typical fitted value of $y$ is from the observed value
  - Theoretically, the extent to which unexplained factors are affecting the value of $y$
  - **VERY INFORMATIVE STATISTIC THAT NO ONE REALLY PAYS ATTENTION TO**
- Terms for $\hat{\sigma}$:
  - Wooldridge: "standard error of regression" (SER)
  - Root MSE or RMSE
  - Standard error of the estimate (SEE)
  - `R`: Residual standard error

# Hypothesis Testing

- Remember all these fun times we had?



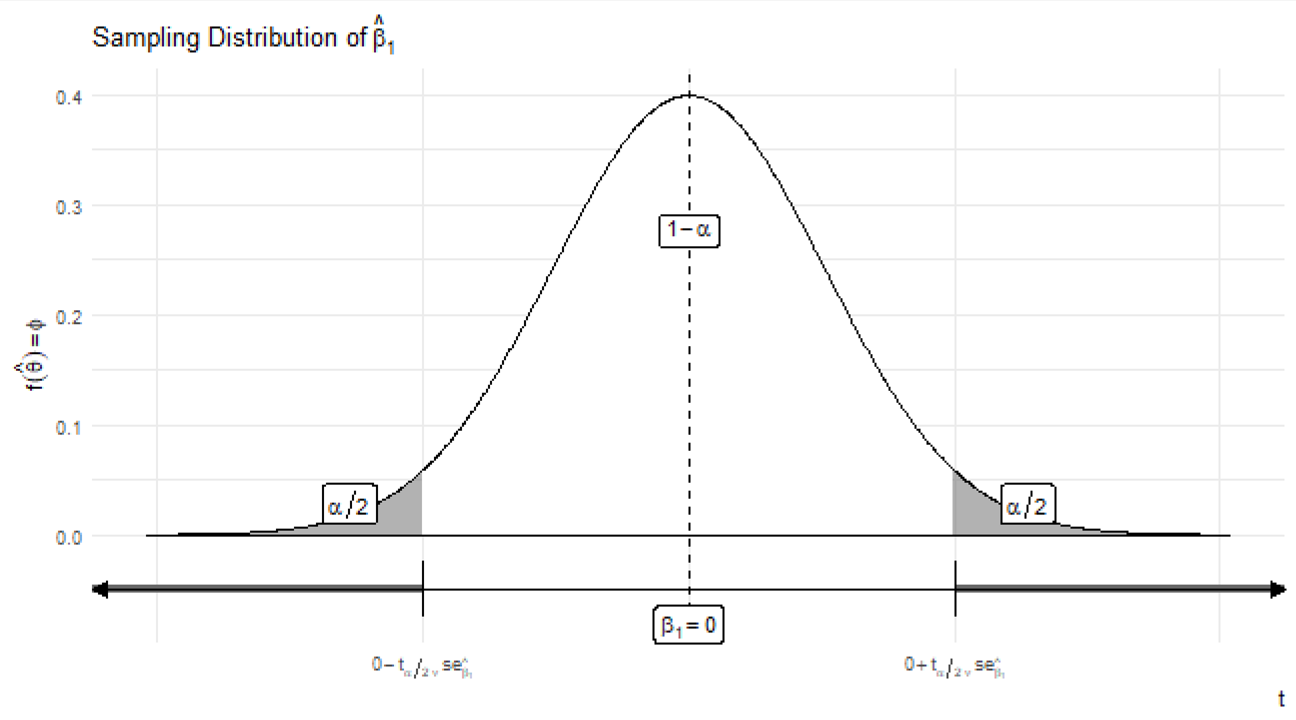
# Hypothesis Testing

- We now have the tools to do this with $\hat{\beta}_1$! (And $\hat{\beta}_0$, although that is rarely the quantity of interest.)
- Note that we typically are interested in whether $\hat{\beta}_1$ is zero:
  - Null $H_0$: $\beta_1 = 0$
  - Alternative $H_A$: $\beta_1 \neq 0$
  - Test statistic: Critical $t$ value for Student's T-test for our estimator $\hat{\beta}_1$
  - Rejection Region: $\hat{\beta}_1 < 0 - t_{\alpha/2,\nu} * \sqrt{V\widehat{AR}(\hat{\beta}_1)}$ or $\hat{\beta}_1 > 0 + t_{\alpha/2,\nu} * \sqrt{V\widehat{AR}(\hat{\beta}_1)}$

# Hypothesis Testing

- What is $\sqrt{\widehat{VAR(\hat{\beta}_1)}}$?
    - The **standard error** of the estimator $\hat{\beta}_1$, or $\widehat{se(\hat{\beta}_1)}$, (or often just $se_{\hat{\beta}_1}$)
- What is $\nu$?
    - The **degrees of freedom**: This will be $n-k-1$. $n$ observations minus $k$ parameters (in this case just one: $\hat{\beta}_1$) - 1 (for the intercept $\hat{\beta}_0$)

# Hypothesis Testing



```r
require(tidyverse)
set.seed(123)
n <- 100
X <- rnorm(n)
Y <- rnorm(n,mean = X)

summary(lm(Y~X))
```

```
##
## Call:
## lm(formula = Y ~ X)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9073 -0.6835 -0.0875  0.5806  3.2904
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.10280    0.09755  -1.054    0.295
## X            0.94753    0.10688   8.865  3.5e-14 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9707 on 98 degrees of freedom
## Multiple R-squared:  0.4451,    Adjusted R-squared:  0.4394
## F-statistic:  78.6 on 1 and 98 DF,  p-value: 3.497e-14
```

# Manual Calculation!

```
b1_hat <- cov(X,Y)/var(X)
b0_hat <- mean(Y) - (cov(X,Y)/var(X))*mean(X)

preds <- b0_hat + b1_hat*X
resids <- Y - preds
mean(resids)
```

```
## [1] -1.621641e-17
```

# Manual Calculation!

```
SSR <- sum(resids^2)
sigma2_hat <- SSR/(n-2)
sigma_hat <- sqrt(sigma2_hat)

SST_x <- sum((X - mean(X))^2)
S_xx <- sum(X^2) - n*mean(X)^2 # Equivalent ways

VAR0_hat <- (sigma2_hat*(sum(X^2)/n))/SST_x
se0_hat <- sqrt(VAR0_hat)

VAR1_hat <- sigma2_hat/SST_x
se1_hat <- sqrt(VAR1_hat)
```

# Manual Calculation!

```
cat(c(b0_hat,se0_hat),'\n',c(b1_hat,se1_hat))
```

```
## -0.1028031 0.09755118
##  0.9475284 0.1068786
```

```
summary(lm(Y~X))
```

```
## 
## Call:
## lm(formula = Y ~ X)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -1.9073 -0.6835 -0.0875  0.5806  3.2904 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -0.10280    0.09755  -1.054    0.295    
## X            0.94753    0.10688   8.865  3.5e-14 ***
## ---
## Signif. codes:
```

# Other output

- Regression output includes two additional columns
  - `t value` and `Pr(>|t|)`
- `t value` is just $\frac{\hat{\beta}_1}{\widehat{se(\hat{\beta}_1)}}$ (or $\frac{\hat{\beta}_0}{\widehat{se(\hat{\beta}_0)}}$)
  - `b1_hat / se1_hat =` 8.8654627
- `Pr(>|t|)` is literally the probably of observing a value as large as the absolute value of the t-value
  - I.e., the $p$**-value**!: "attained significance level" or "smallest level of $\alpha$ for which we would **reject** $H_0$

# Controlling for a variable

- We talked last week about OVB (**O**mitted **V**ariable **B**ias)
  - Subset of broader conversation about bias
- We might want to "control" for $z$ to remove OVB (preventing the $\beta_2 z_i + \nu_i$ from being "buried in the $u$)
  - But remember that failing to control for $z$ is only a problem **if** either (1) $\beta_2 \neq 0$ or (2) $cov(z, x) \neq 0$
- *Ceteris paribus*: "all things being equal"
  - Want to estimate a *ceteris paribus* relationship between $X$ and $Y$
  - What would the relationship look like if all other aspects of our units were the same?
  - Commonly invoked for causal claims, but more on that next semester

# Controlling for a variable

- Let's get precise with our terminology:
  - $Z$ is a potential **confound**
  - If $Z$ "confounds" the relationship between $X$ and $Y$, it **renders the relationship spurious**
- How about some examples?

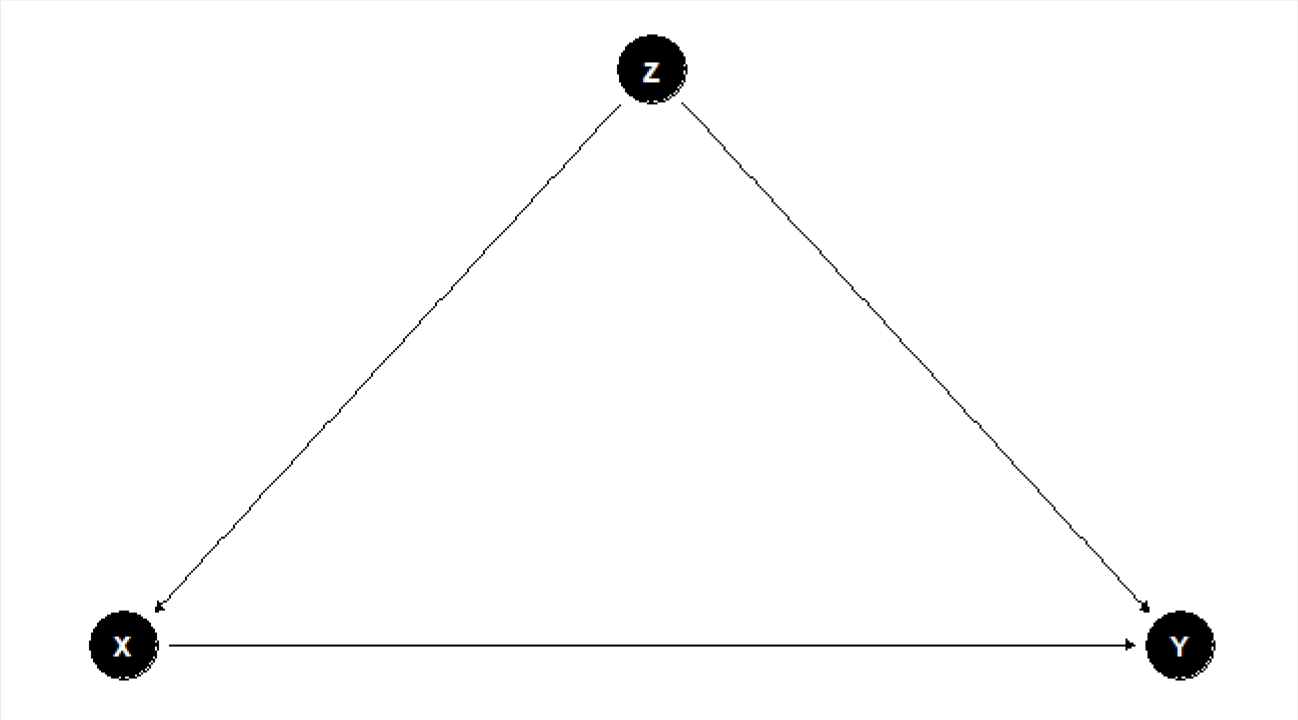
| $X$ | $Y$ | $Z$ |
|---|---|---|
| College degree | Salary at age 25 | Ability |
| Female | Pro-Choice | Democrat |
| First-born | IQ Score | Parental involvement |
| Asian-American | Trump Support | Vietnamese |
| Own a home | Participated in Women's March | Year of Birth |
| ... | ... | ... |

# Controlling for a variable

- To determine whether $Z$ renders the relationship between $X$ and $Y$ spurious, we...
  - "control for $Z$"
  - "condition on $Z$"
  - "hold $Z$ constant"
- These all mean the same thing conceptually, but there are several different ways to do this
- Ideally, we would do exactly what "holding $Z$ constant" suggests: divide our units by categories of $Z$ and examine the relationship between $X$ and $Y$ within each category of $Z$
  - I.e., if women are more pro-choice, we want to see this among *both* Democrats and Republicans
  - If the relationship persists after holding $Z$ constant, we say it is not spurious
  - If it no longer holds, we say that $Z$ is a confound rendering the relationship between $X$ and $Y$ spurious
- In practice, we usually do something much less careful

# Controlling for a variable

- We often will make our assumptions explicit with a **D**irected **A**cyclic **G**raph (DAG)
    - This encodes our intuition about what the population parameters of interest might be



# Controlling for a variable

- Let's tackle a classic: education and income
  - $Y$: income
  - $X$: education
- What is $Z$?
  - Parent's education?

# Controlling for a variable

- Start by looking at all three relationships separately

```r
require(tidyverse)
require(haven) # To open .dta files

dat <- read_dta('../Materials/gss7222_r1.dta')

dat <- dat %>%
  select(realinc,educ,paeduc) %>%
  sample_n(size = 10000,replace = F) %>%
  drop_na()

gc()
```

```
##           used (Mb) gc trigger   (Mb)  max used   (Mb)
## Ncells 1558977 83.3    2867194  153.2   2867194  153.2
## Vcells 2631858 20.1  568045588 4333.9 484482476 3696.4
```

- (`gc()` helps save memory after I dropped thousands of rows and columns)

# Controlling for a variable

- Start by looking at all three relationships separately

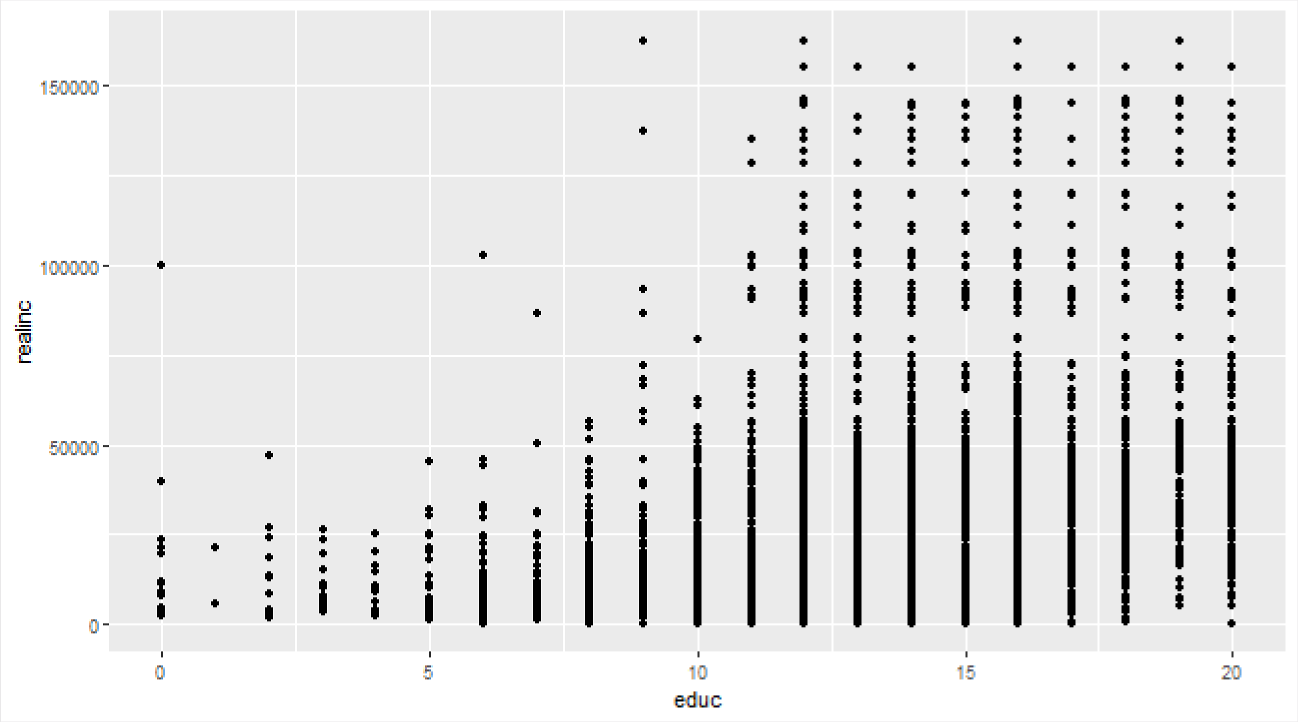
```
dat %>%
  group_by(educ) %>%
  summarise(income = mean(realinc,na.rm=T))
```

```
## # A tibble: 21 × 2
##    educ                        income
##    <dbl+lbl>                    <dbl>
##  1 0 [no formal schooling]   16684.
##  2 1                         13372.
##  3 2                         14913.
##  4 3                         10643.
##  5 4                          9529.
##  6 5                         12811.
##  7 6                         15210.
##  8 7                         13519.
##  9 8                         15339.
## 10 9                         19075.
## # … with 11 more rows
```

# Controlling for a variable

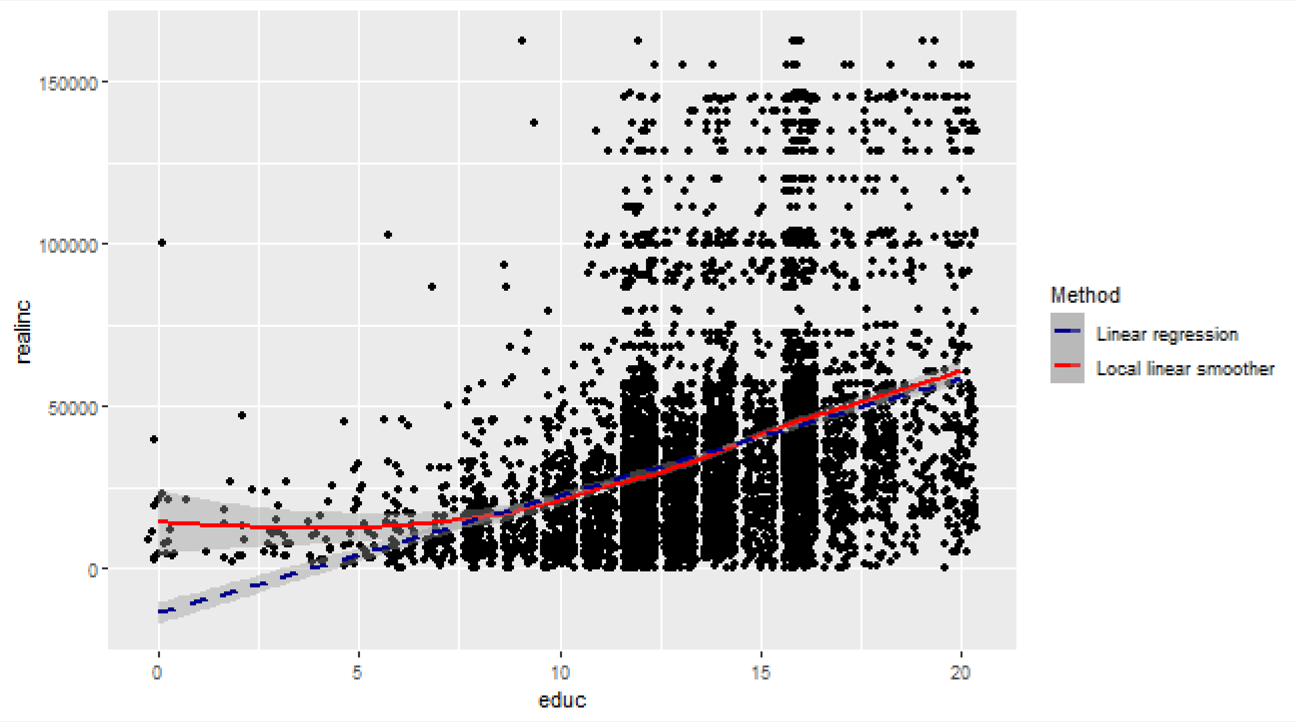
- Start by looking at all three relationships separately

```
dat %>%
  ggplot(aes(x = educ,y = realinc)) +
  geom_point()
```



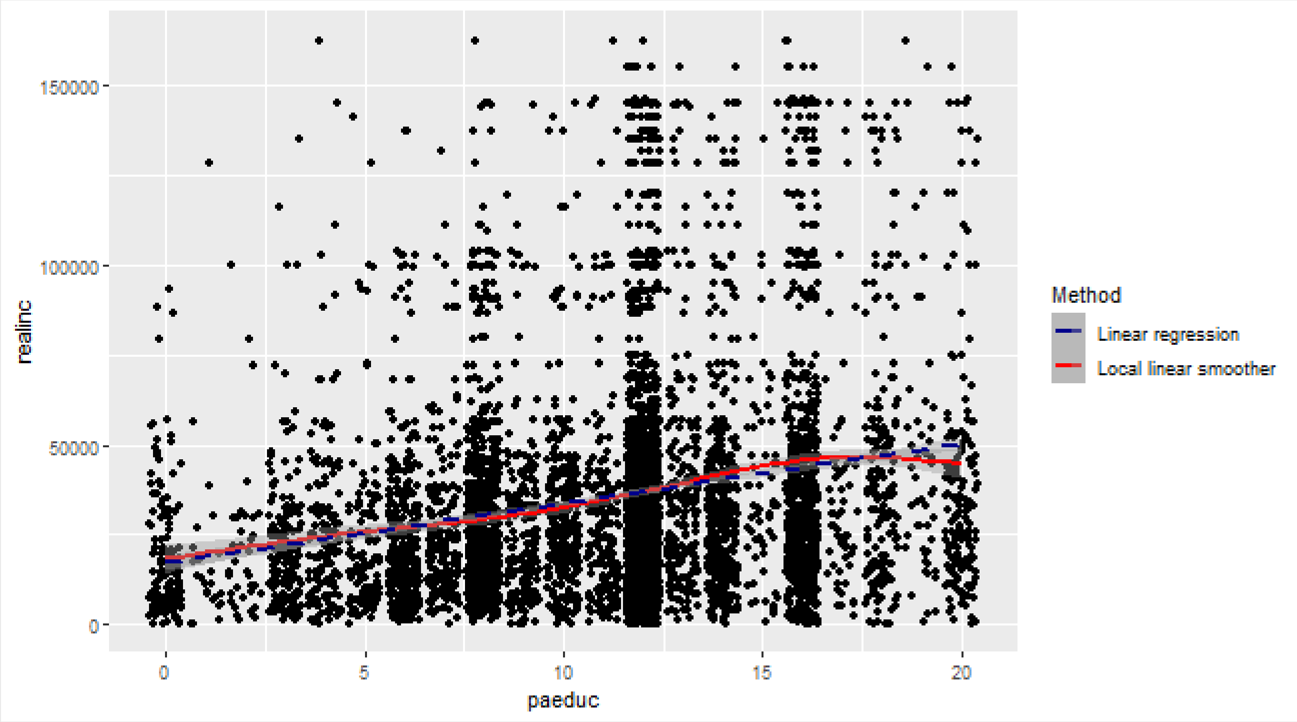
# Controlling for a variable

- Start by looking at all three relationships separately



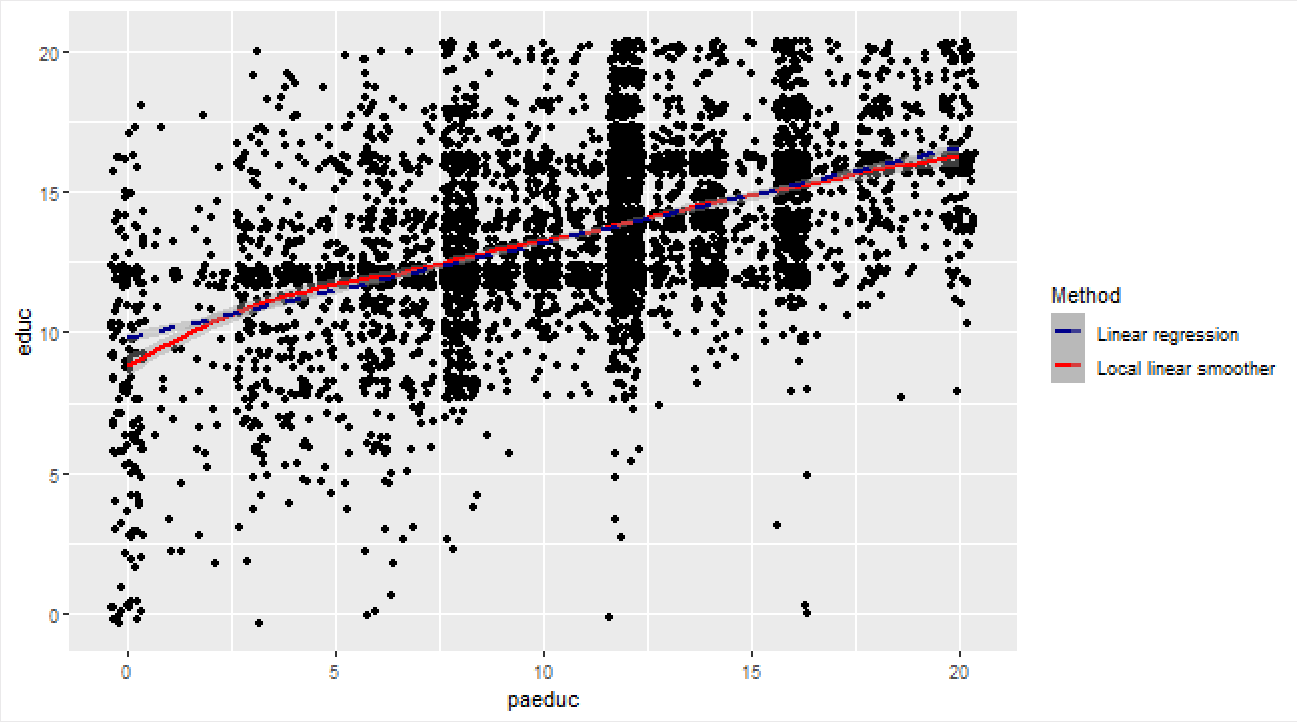
# Controlling for a variable

- Start by looking at all three relationships separately



# Controlling for a variable

- Start by looking at all three relationships separately



# Controlling for a variable

- Clearly evidence of:
    - $\beta_1 \neq 0$
    - $\beta_2 \neq 0$
    - $cov(educ, paeduc) \neq 0$
- I.e., OVB!
- Think through what this will mean for the following regression: $realinc_i = \beta_0 + \beta_1 educ_i + u_i$

# Controlling for a variable

- Let's "control" for parent's education in three ways

1. Fewest Assumptions: Just visualize it with a local linear smoother, and subset to different values of parent's education
    - No linearity assumption between $X$ and $Y$
    - Different relationship between $X$ and $Y$ for different values of $Z$
2. More Assumptions: Run multiple linear regressions, subsetting to different values of parent's education
    - Linearity assumption between $X$ and $Y$
    - Different (linear) relationships between $X$ and $Y$ for different values of $Z$
3. Most Assumptions: Run single linear regression, adding $Z$ as an additional predictor
    - Linearity assumption between $X$ and $Y$
    - Same linear relationship between $X$ and $Y$ for all values of $Z$

# Controlling for a variable

- First, let's take `paeduc` and transform it into a categorical measure

```
dat <- dat %>%
  mutate(paeduc_cat = factor(ifelse(paeduc < 9,'8th gr or less',
                             ifelse(paeduc < 12,'HS dropout',
                                    ifelse(paeduc == 12,'HS degree',
                                           ifelse(paeduc < 16,'Some college',
                                                  ifelse(paeduc == 16,'College degree','Post grad'))))),
                             levels = c('8th gr or less','HS dropout','HS degree','Some
college','College degree','Post grad')))
dat %>% count(paeduc_cat)
```

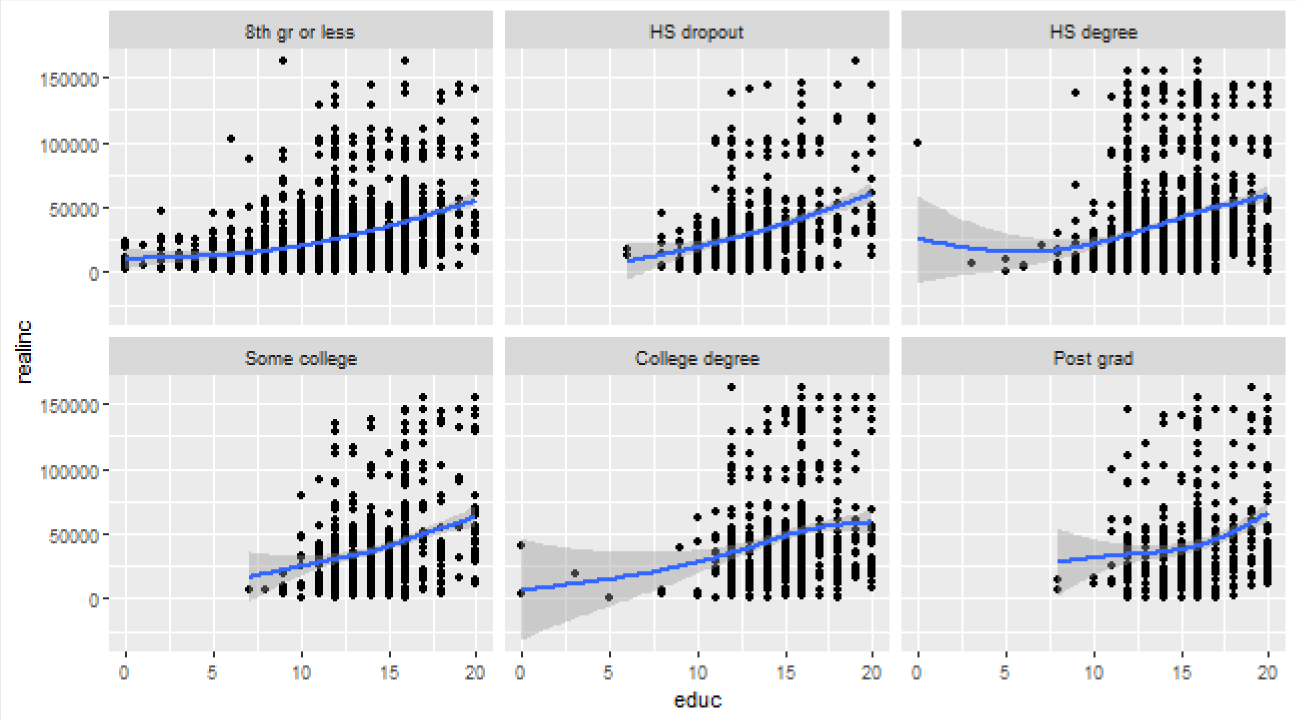
```
## # A tibble: 6 × 2
##   paeduc_cat         n
##   <fct>          <int>
## 1 8th gr or less  1970
## 2 HS dropout       659
## 3 HS degree       1790
## 4 Some college     633
## 5 College degree   656
## 6 Post grad        440
```

# Controlling for a variable: Fewest assumptions

```
p <- dat %>%
  ggplot(aes(x = educ,y = realinc)) +
  geom_point() +
  geom_smooth() +
  facet_wrap(~paeduc_cat)
```

# Controlling for a variable: Fewest assumptions

```
p
```



# Controlling for a variable: More assumptions

```r
res <- list()
for(i in unique(dat$paeduc_cat)) {
  res[[i]] <- lm(realinc ~ educ,dat %>%
                               filter(paeduc_cat == i))
}

# 8th grade or less
summary(res$`8th gr or less`)$coefficients
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -3498.564  1848.5655 -1.892583 5.855994e-02
## educ         2559.554   151.1077 16.938610 3.478382e-60
```

```r
# High school dropout
summary(res$`HS dropout`)$coefficients
```

```
##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -22065.206  5442.1609 -4.054494 5.626777e-05
## educ          4028.299   401.9979 10.020697 4.332182e-22
```

# Controlling for a variable: More assumptions

```
summary(res$`HS degree`)$coefficients
```

```
##                Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -16113.26  4140.4210 -3.891697 1.031964e-04
## educ          3852.58   293.9332 13.106994 1.521904e-37
```

```
summary(res$`Some college`)$coefficients
```

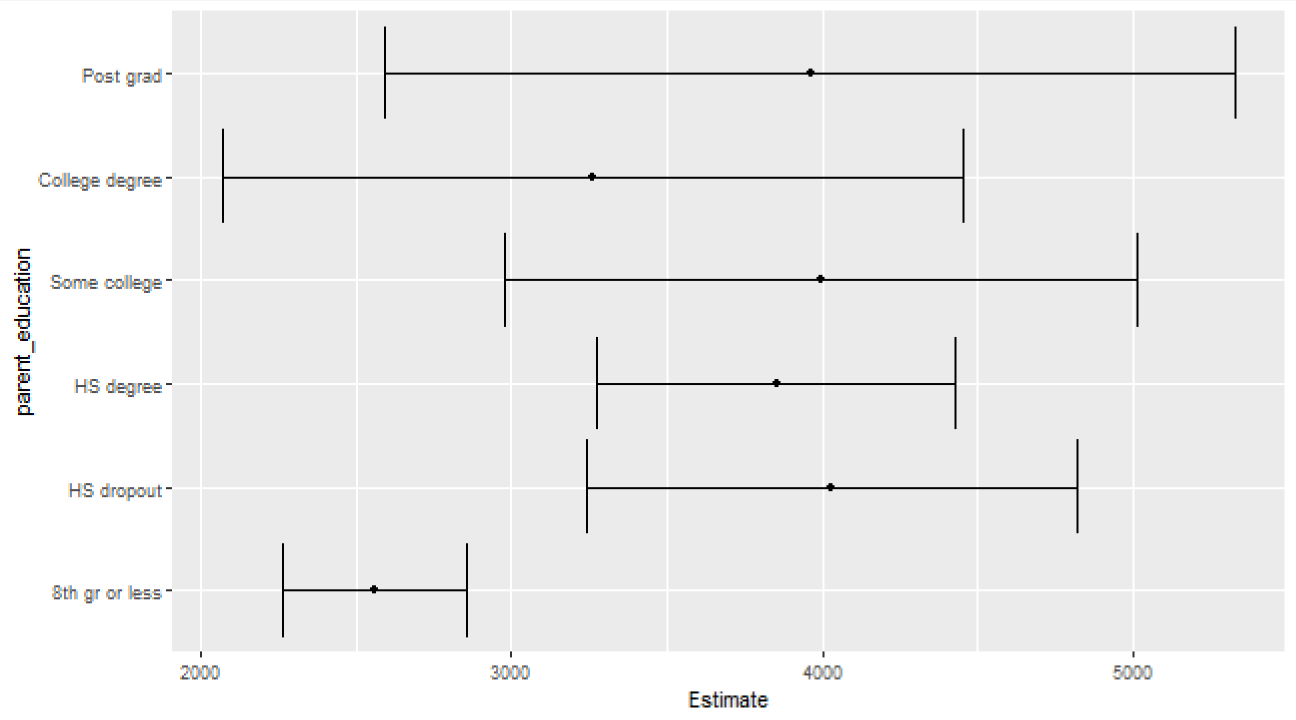
```
##                 Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -18203.119  7684.9483 -2.368672 1.815234e-02
## educ          3994.444   519.7771  7.684917 5.880018e-14
```

```
summary(res$`College degree`)$coefficients
```

```
##                Estimate Std. Error    t value     Pr(>|t|)
## (Intercept) -1461.172  9359.9190 -0.1561095 8.759949e-01
## educ         3262.281   607.6131  5.3690103 1.101091e-07
```

```
summary(res$`Post grad`)$coefficients
```

# Controlling for a variable: More assumptions



# Controlling for a variable: Most assumptions

```
summary(lm(realinc ~ educ + paeduc,dat))
```

```
##
## Call:
## lm(formula = realinc ~ educ + paeduc, data = dat)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -56540 -17957  -6298   7800 145804
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14498.45    1689.61  -8.581  < 2e-16 ***
## educ          3228.40     138.71  23.274  < 2e-16 ***
## paeduc         561.52      97.47   5.761 8.77e-09 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 29090 on 6145 degrees of freedom
## Multiple R-squared:  0.1301,    Adjusted R-squared:  0.1298
## F-statistic: 459.4 on 2 and 6145 DF,  p-value: < 2.2e-16
```