Lecture 16 notes

Thursday, November 2, 2023 9:29 AM

$$VAR(u|x) = \sigma^2$$

$$VAR(y|x) = ?$$

$$y_i = \beta_0 + \beta_1 x + u_i$$

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{u}_i$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x$$

$$= VAR(\beta_0 + \beta_1 x + u | x)$$

$$= VAR(\beta_0 | x) + VAR(\beta_1 x | x) + VAR(u|x)$$

$$= 0 + 0 + \sigma^2$$

$$= \sigma^2$$

VARIANCE of OLS ESTIMATORS

$$SST_x = \sum (x_i - \bar{x})^2$$

$$VAR(\hat{\beta}_0) = \frac{\sigma^2 \frac{\sum x_i^2}{n}}{SST_x}$$

$$VAR(\hat{\beta}_1) = \frac{\sigma^2}{SST_x}$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$

$$= \beta_1 + \frac{\sum (x_i - \bar{x}) u_i}{SST_x}$$

$$VAR(aX) = a^2 VAR(X)$$

$$VAR(\hat{\beta}_1 | x) = VAR\left[\beta_1 + \frac{\sum (x_i - \bar{x}) u_i}{SST_x} \,\middle|\, X\right]$$

$$= VAR(\beta_1 | x) + VAR\left(\frac{\sum (x_i - \bar{x}) u_i}{SST_x} \,\middle|\, X\right)$$

$$= 0 + VAR\left(\frac{\sum (x_i - \bar{x}) u_i}{\sum (x_i - \bar{x})^2} \,\middle|\, x\right)$$

$$\left(\frac{}{\sum (x_i - \bar{x})^2} \mid X \right)$$

$$+ \text{VAR}\left(\frac{1}{SST_x} \sum (x_i - \bar{x}) u_i \mid X \right)$$

$$+ \left(\frac{1}{SST_x} \right)^2 \text{VAR}\left(\sum (x_i - \bar{x}) u_i \mid X \right)$$

$$+ \frac{1}{SST_x^2} \sum (x_i - \bar{x})^2 \, \text{VAR}(u_i \mid X)$$

$$+ \frac{\cancel{SST_x}}{SST_x^{\cancel{2}}} \sigma^2$$

$$\text{VAR}(\hat{\beta}_1) = \frac{\sigma^2}{SST_x}$$

$$SST_x = \sum (x_i - \bar{x})^2$$

$$\frac{SST_x}{n} = \boxed{\frac{\sum (x_i - \bar{x})^2}{n}} \longrightarrow \text{var}(x)$$

$$\frac{SST_x}{n} = \text{var}(x)$$

$$SST_x = n \cdot \text{var}(x)$$