Quant I – Fall 2013

Pr(Type I error) = Pr(reject H_0| H_0 is true) = α

If H_0 is true, then the parameter θ equals θ_0. Therefore, under the null, the sampling distribution of our estimator $\hat{\theta}$ is Normal with mean θ_0 and standard deviation $\sigma_{\hat{\theta}}$.

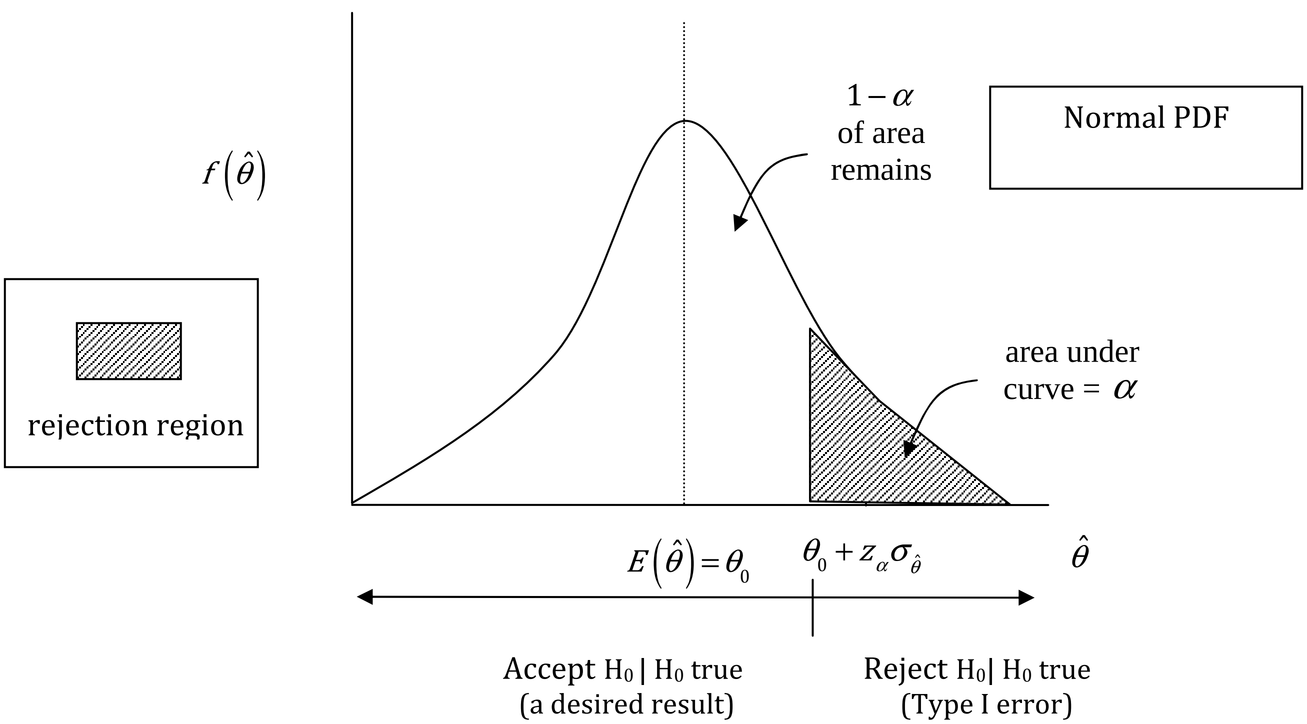
In a one-tail test, our procedure is to specify an alternative hypothesis (for example, H_A: $\theta > \theta_0$), decide upon a confidence coefficient $1 - \alpha$, and from there specify a rejection region (the shaded area in the diagram below). We reject the null if we observe an estimate greater than $\theta_0 + z_\alpha \sigma_{\hat{\theta}}$.

If we do this, we will (purely by chance):

- observe an estimate $\hat{\theta}$ in the rejection region 100 x α % of the time
  (and thus falsely reject the null even though it's true = commit Type I error)

and

- observe an estimate $\hat{\theta}$ in the rejection region 100 x (1-α) % of the time
  (and thus fail to reject the null given that it's true = a desired result)



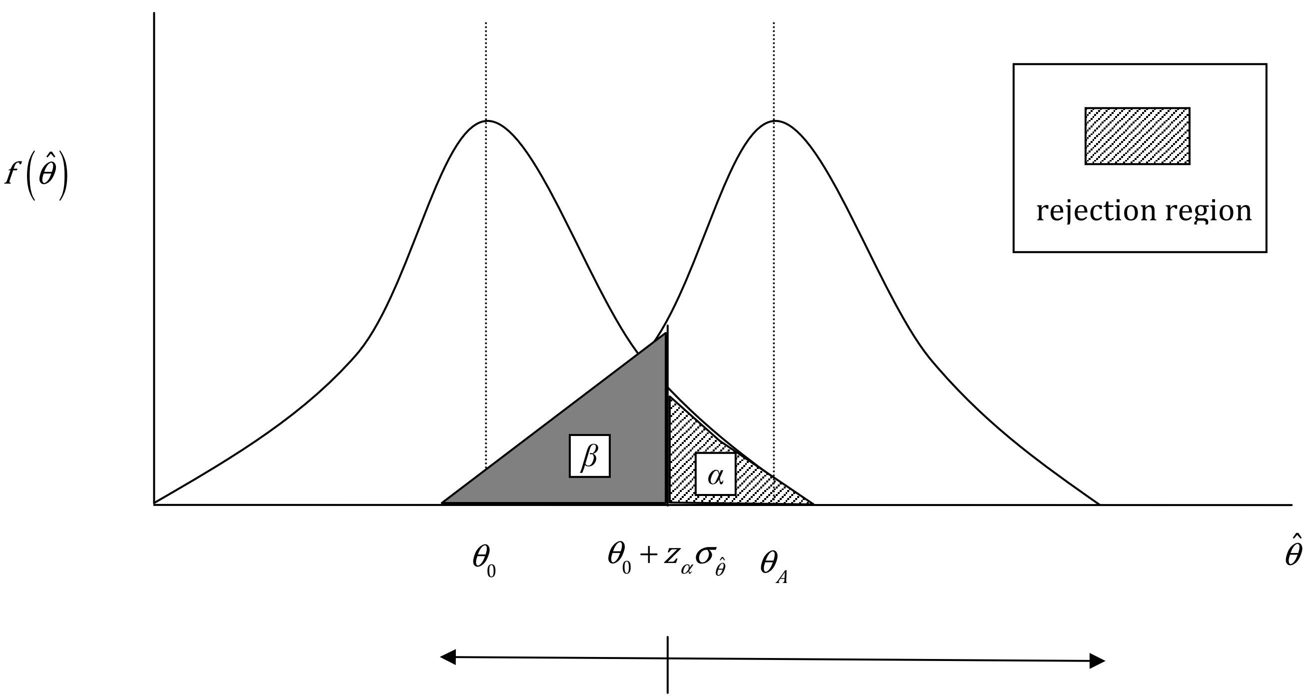
Thus the probability of committing type I error is $\boldsymbol{\alpha}$.

Pr(Type II error) = Pr(accept H_0| H_A is true) = β

Now consider the following thought experiment: suppose that instead of H_0 being true, it is the case that some exact alternative hypothesis, H_A: $\theta = \theta_A$, is true.

But as is always the case, we conduct our hypothesis test under the assumption that the null hypothesis, H_0 is true, i.e. H_0: $\theta = \theta_0$. As before, we proceed with the knowledge that under the null, the sampling distribution of our estimator $\hat{\theta}$ is Normal with mean θ_0 and standard deviation $\sigma_{\hat{\theta}}$. As before, we reject the null if we observe an estimate that is greater than $\theta_0 + z_\alpha \sigma_{\hat{\theta}}$.

But what is <u>actually</u> the case is that the sampling distribution of our estimator $\hat{\theta}$ is Normal with <u>mean θ_A</u> and standard deviation $\sigma_{\hat{\theta}}$.



Accept H_0| H_A true (Type II error)

Reject H_0| H_A true (a desired result)

β = Pr(Accept H_0| H_A true)

α = Pr(Reject H_0| H_0 true)

1 - β = **Power**
= 1- Pr(Reject H_0| H_A true)
= $1 - \Pr\left(\hat{\theta} < \theta_0 + z_\alpha \sigma_{\hat{\theta}} \mid \theta = \theta_A\right)$