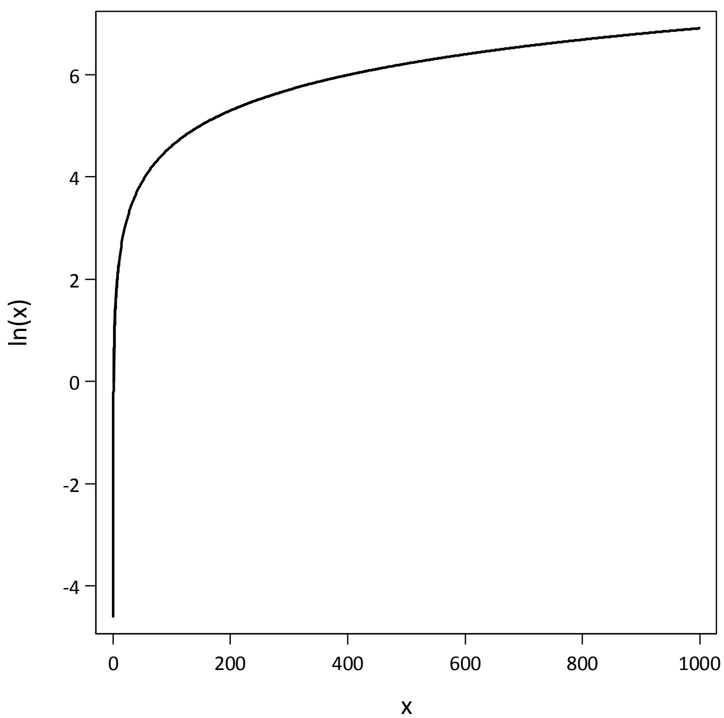
New York University
Wilf Family Department of Politics
Prof. Patrick Egan

QUANT I: Transforming X to Specify Diminishing Returns to X

```
set obs 1000
egen x = fill(.01,.02)
gen ln_x = ln(x)
egen y = fill (0,1)
gen ln_y = ln(y)
twoway (line ln_y y) (line ln_x x), legend(off) xtitle("x") ytitle (ln(x))
```



*A log transformation stretches out low values and compresses high values

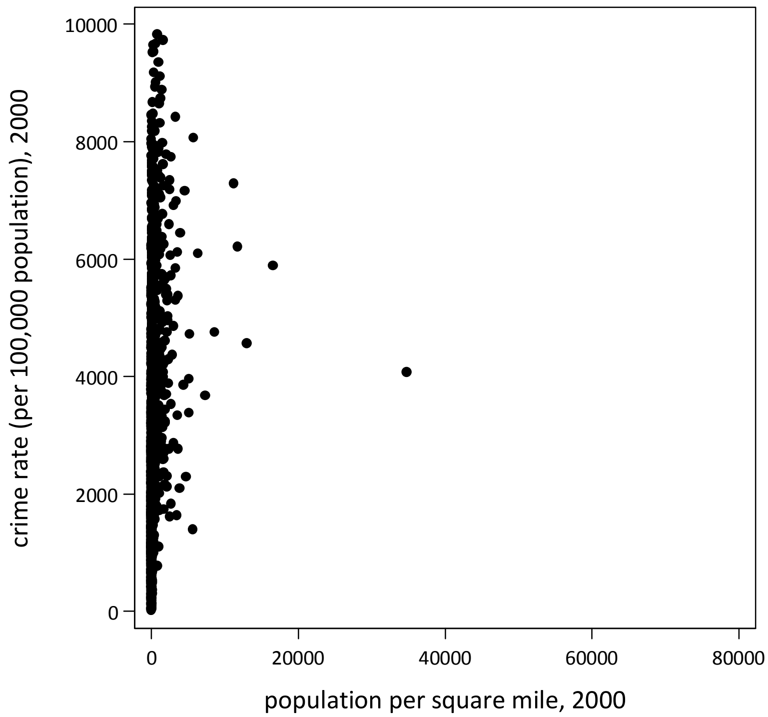
*thus when x’s marginal effect on y diminishes as x grows large (which is true of many social phenomena), the relationship between y and ln(x) is more likely to be linear than the relationship between y and x.

*another interpretation: change in y is predicted less well by the <u>absolute</u> change in x than the <u>percentage</u> change in x.

*Why does this matter? OLS Assumption 1: we assume we have correctly specified a population model that is linear in its parameters. Violation of this assumption can lead to bias and imprecision.

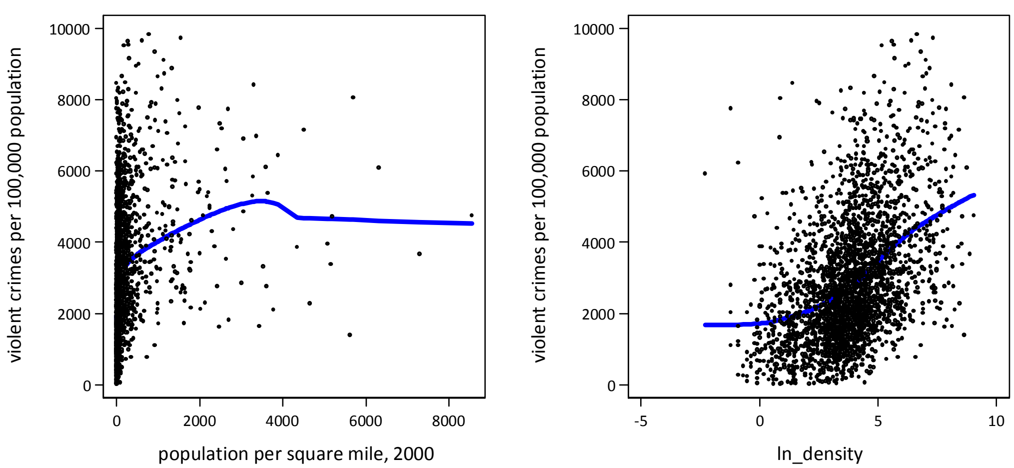
* An example where we look at the relationships among crime, poverty and population density:

```
use "counties.dta", clear
twoway (scatter   crimerate density, msize(small))
```




```
*looks like there are some outliers.  Let’s see what they are:

. list county state density if density>=10000

      +--------------------------------------------+
      |                    county   state   density |
      |--------------------------------------------|
2245. |             Yukon-Koyukuk      AK         . |
2295. |  Yellowstone National Park      MT         . |
2296. |              South Boston      VA         . |
2315. |                   Suffolk      MA     11692 |
2806. |                    Queens      NY     20453 |
      |--------------------------------------------|
2808. |                    Hudson      NJ     12957 |
2810. |                  New York      NY     66835 |
2811. |                     Bronx      NY     31730 |
2812. |                     Kings      NY     34723 |
2867. |              Philadelphia      PA     11241 |
      |--------------------------------------------|
2990. |             San Francisco      CA     16526 |
      +--------------------------------------------+

*note this includes all with value “.” which Stata treats as a very large number.

*we don’t want these observations to play too strong a role in our analysis; leave
them out for now.
```

```
twoway (scatter   crimerate density, msize(tiny)) (lowess crimerate density,
lw(thick))  if density<10000, legend(off)

graph save Graph "density.gph", replace

*we have reason to think that while density is associated with crime, this
association might be diminishing in density.  To see if this is the case, let's
create a log transformation of density:

gen ln_density = ln(density)
twoway (scatter   crimerate ln_density, msize(tiny)) (lowess crimerate ln_density,
lw(thick))  if density<10000, legend(off)

graph save Graph "ln_density.gph", replace

graph combine "density.gph" "ln_density.gph", ycommon
```




```
* ln_density appears to be a better predictor of crime than density.
* Bivariate regressions confirm this:
```

```
. reg  crimerate density if density<10000

      Source |       SS       df       MS              Number of obs =    2662
-------------+------------------------------           F(  1,  2660) =  223.48
       Model |   618563579     1   618563579           Prob > F      =  0.0000
    Residual |  7.3627e+09  2660  2767930.03           R-squared     =  0.0775
-------------+------------------------------           Adj R-squared =  0.0772
       Total |  7.9813e+09  2661  2999345.15           Root MSE      =  1663.7

------------------------------------------------------------------------------
   crimerate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     density |   .8876232   .0593764    14.95   0.000     .7711945    1.004052
       _cons |   2686.712   34.18831    78.59   0.000     2619.674     2753.75
------------------------------------------------------------------------------

. reg  crimerate ln_density if density<10000

      Source |       SS       df       MS              Number of obs =    2662
-------------+------------------------------           F(  1,  2660) =  660.40
       Model |  1.5874e+09     1  1.5874e+09           Prob > F      =  0.0000
    Residual |  6.3939e+09  2660  2403706.28           R-squared     =  0.1989
-------------+------------------------------           Adj R-squared =  0.1986
       Total |  7.9813e+09  2661  2999345.15           Root MSE      =  1550.4

------------------------------------------------------------------------------
   crimerate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  ln_density |    459.948   17.89808    25.70   0.000     424.8525    495.0436
       _cons |   1096.143   74.80349    14.65   0.000     949.4638    1242.822
------------------------------------------------------------------------------
```

*interpretation: a one-percent increase in population density is associated with an increase in the crime rate of approximately 460 crimes per 100,000 people.

*This improved specification has implications for our estimates of other variables. Let's say we want to look at the association between poverty and crime, controlling for density. First look at correlation matrix:

```
. pwcorr  povrate crimerate density ln_density if density<10000

             |  povrate crimer~e  density ln_den~y
-------------+------------------------------------
     povrate |   1.0000
   crimerate |   0.0643   1.0000
     density |  -0.1174   0.2784   1.0000
  ln_density |  -0.2511   0.4460   0.5579   1.0000
```

*ln_density is much more highly correlated with poverty and crime than density. This will have implications for our estimates.

```
. reg  crimerate  povrate density if density<10000

      Source |       SS       df       MS              Number of obs =    2662
-------------+------------------------------           F(  2,  2659) =  128.38
       Model |   702809198     2   351404599           Prob > F      =  0.0000
    Residual |  7.2784e+09  2659   2737287.8           R-squared     =  0.0881
-------------+------------------------------           Adj R-squared =  0.0874
       Total |  7.9813e+09  2661  2999345.15           Root MSE      =  1654.5

------------------------------------------------------------------------------
   crimerate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     povrate |   28.90484   5.210233     5.55   0.000     18.68832    39.12136
     density |   .9321349   .0595895    15.64   0.000     .8152884    1.048981
       _cons |   2246.886   86.26309    26.05   0.000     2077.737    2416.036
------------------------------------------------------------------------------
```

povrate estimated more precisely because effect of density is now more properly specified.

```
. reg  crimerate  povrate ln_density if density<10000

      Source |       SS       df       MS              Number of obs =    2662
-------------+------------------------------           F(  2,  2659) =  414.21
       Model |  1.8959e+09     2   947949932           Prob > F      =  0.0000
    Residual |  6.0854e+09  2659  2288588.79           R-squared     =  0.2375
-------------+------------------------------           Adj R-squared =  0.2370
       Total |  7.9813e+09  2661  2999345.15           Root MSE      =  1512.8

------------------------------------------------------------------------------
   crimerate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     povrate |   57.07723   4.916074    11.61   0.000     47.43751    66.71694
  ln_density |   518.8822   18.18696    28.53   0.000     483.2202    554.5442
       _cons |   18.89011   118.0527     0.16   0.873    -212.5944    250.3746
------------------------------------------------------------------------------
```

Association between povrate and crimerate now found to be much larger. Looking back at the correlation matrix, can you see why?