

4. Answer each of the following questions. Show your work.

- (a) You wish to determine whether women are more Democratic than men.
- You draw a random sample of 50 men and 50 women, and find that 43 percent of men identify as Democrats, while 56 percent of women do. How sure does this make you that women are more Democratic than men in the general population?
  - Now you draw a random sample of  $n$  men and  $n$  women, and find that 43 percent of men identify as Democrats, while 56 percent of women do. Find the largest  $n$  at which you are **not** sure with 95 percent confidence that women are more Democratic than men in the general population.
- (b) You are the leader of an environmental group who hopes that major environmental legislation is passed by Congress and signed into law at some point during the 2013-2014 Congressional session. The number of major environmental laws passed per session can be modeled as the random variable  $Y$  with  $E(Y) = 1$ . What is the chance that at least one piece of major environmental legislation gets passed during the session? Be precise about how you are modeling this process.
- (c) You have discovered some observation  $Y_i$  of the random variable  $Y$  that is two standard deviations greater than  $Y$ 's mean,  $\mu_Y$ .
- Find  $P(Y \geq \mu_Y + 2\sigma_Y | Y \sim \text{Standard Normal})$ .
  - Explain why it is non-sensical to model  $Y$  as distributed Uniform. HINT: Show that  $P(Y \geq \mu_Y + 2\sigma_Y | Y \sim \text{Uniform}) = 0$ .

ANSWER:

- Consider a Uniform  $Y$  with range  $b$  and thus density  $\frac{1}{b}$ .
- Variance of  $Y$  is thus  $\frac{(b)^2}{12}$ .
- Standard deviation of  $Y$ ,  $\sigma_Y = \sqrt{\frac{(b)^2}{12}} = \frac{b}{2\sqrt{3}}$ .
- Thus  $\mu_Y + 2\sigma_Y = \mu_Y + \frac{b}{\sqrt{3}}$ .
- But max of a Uniform  $Y$  with mean  $\mu$  and range  $b$  is  $\mu + \frac{b}{2}$ .
- Because  $\mu + \frac{b}{2} < \mu + \frac{b}{\sqrt{3}}$ , it is impossible that  $Y \geq \mu_Y + 2\sigma_Y$ .
- Thus  $P(Y \geq \mu_Y + 2\sigma_Y | Y \sim \text{Uniform}) = 0$ .