Dynamics of Pipes Conveying Two-Phase Flows

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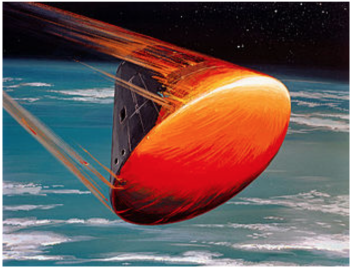

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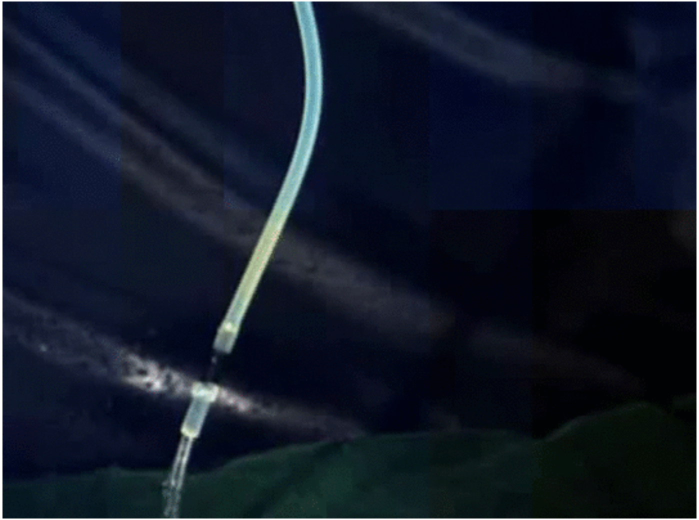
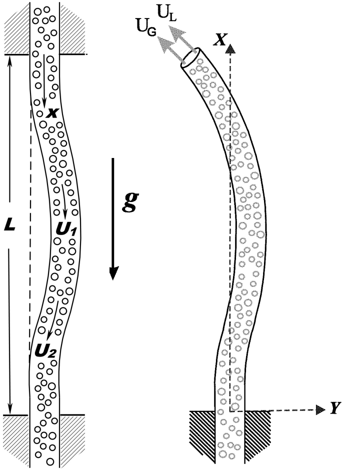
ME 506 Project — December 9, 2019

Flow Induced Instability



Introduction

- Dynamics of a pipe conveying 1ϕ flow is fairly understood
- Dynamics of a 2ϕ system does not translate well from 1ϕ system
- The heterogeneity of a 2ϕ system can impart oscillatory forces onto the pipe
- Extreme instabilities in 2ϕ system have been observed in experiments
- Theoretical studies have verified the presence of such instabilities in 2ϕ system
- Studying this problem is extremely important as the applications are endless



Methodology

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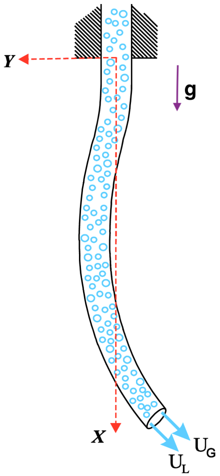
- Fluid-structure interaction is represented through an Equation of Motion (EOM)
- Dynamic response can be determined analytically or numerically
- Here, we used Galerkin method to study the frequency response of a system
- Complex frequencies of the motion will reveal the behavior of a system

Previous Work

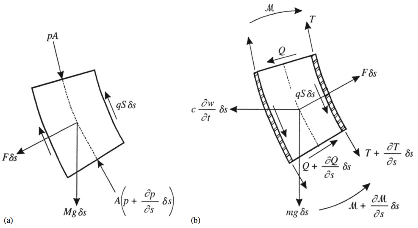
- Paidoussis (1970): Pipe conveying 1ϕ flow has been studied extensively
- Paidoussis and Issid (1974): 1ϕ system for cantilever pipes was studied numerically
- Hara (1974): First analytical work on 2ϕ fluid-structure interaction
- Monette and Pettigrew (2004): Experimental and analytical work on 2ϕ system of vertical cantilever pipes
- An & Su (2015): Analytical study of 2ϕ system for vertical clamped pipes
- Ebrahimi-Mamaghani et al. (2019): Recent extensive study of 2ϕ system of vertical cantilever pipes

Equation of Motion (EOM)



Cantilever Pipe Conveying Two-Phase Flow

- **Consider:** Control volume analysis of a small fluid/pipe element



FBD of Fluid/Pipe Element

- The EoM for a **pipe conveying two-phase flow** is given as [Monette & Pettigrew, 2004]:

$$\underbrace{EI(1+\mu i)\frac{\partial^4 y}{\partial x^4}}_{\text{Elastic force}}+\underbrace{\left(m_l U_l^2+m_g U_g^2\right)\frac{\partial^2 y}{\partial x^2}}_{\text{Centrifugal force}}+\underbrace{2\left(m_l U_l+m_g U_g\right)\frac{\partial^2 y}{\partial x \partial t}}_{\text{Coriolis force}}$$

$$+\underbrace{\left(m_p+m_l+m_g\right)\frac{\partial^2 y}{\partial t^2}}_{\text{Inertia force}}+\underbrace{\left(m_p+m_l+m_g\right)\boldsymbol{g}\left((x-L)\frac{\partial^2 y}{\partial x^2}+\frac{\partial y}{\partial x}\right)}_{\text{Gravity}}=0 \quad (1)$$

Nondimensional Equation of Motion

- Introducing the **nondimensional parameters**:

$$\xi = \frac{x}{L} \qquad \eta = \frac{y}{L} \qquad \tau = \frac{t}{T} \qquad T = L^2\sqrt{\frac{m_p + m_l + m_g}{EI}} \tag{2}$$

$$\beta_k = \frac{m_k}{m_p + m_l + m_g} \qquad u_k = U_k L\sqrt{\frac{m_k}{EI}} \qquad \gamma = \boldsymbol{g}L^3\sqrt{\frac{m_p + m_l + m_g}{EI}}$$

  where the subscript $k = l, g$ denotes different phases.

- Recalling the **dimensional EOM**:

$$EI(1+\mu i)\frac{\partial^4 y}{\partial x^4} + \left(m_l U_l^2 + m_g U_g^2\right)\frac{\partial^2 y}{\partial x^2} + 2\left(m_l U_l + m_g U_g\right)\frac{\partial^2 y}{\partial x \partial t} + \left(m_p + m_l + m_g\right)\frac{\partial^2 y}{\partial t^2} + \left(m_p + m_l + m_g\right)\boldsymbol{g}\left((x-L)\frac{\partial^2 y}{\partial x^2} + \frac{\partial y}{\partial x}\right) = 0 \tag{3}$$

- The resulting **nondimensional EOM** is given as:

$$(1+\mu i)\,\eta'''' + \left(u_l^2 + u_g^2\right)\eta'' + 2\left(\sqrt{\beta_l}u_l + \sqrt{\beta_g}u_g\right)\dot{\eta}' + \ddot{\eta} + \gamma\left[(\xi - L)\eta'' + \eta'\right] = 0 \tag{4}$$

- The **principle parameters** for defining a 2ϕ flow are:
  - Volumetric quality: $\boldsymbol{\epsilon} = \frac{Q_g}{Q}$
  - Void fraction: $\boldsymbol{\alpha} = \frac{A_g}{A}$
  - Slip ratio: $\boldsymbol{K_s} = \frac{U_g}{U_l}$
- These parameters are **related by** the following:

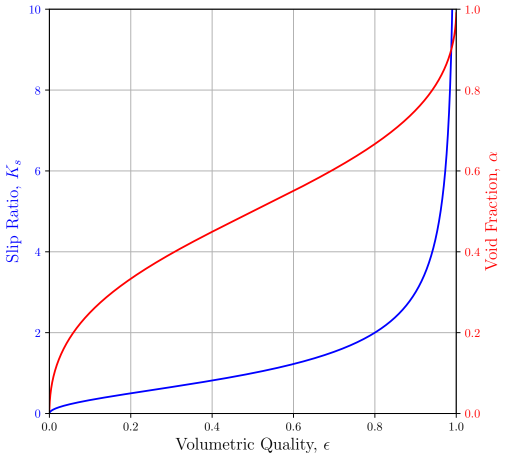
$$\frac{1-\epsilon}{\epsilon} = \left(\frac{1-\alpha}{\alpha}\right)\frac{1}{K_s} \tag{5}$$

- Relation between **gas** (g) and **liquid** (l) phases:

$$\beta_g = \left(\frac{\rho_g \epsilon}{\rho_l (1-\epsilon) K_s}\right)\beta_l \qquad u_g = u_l\sqrt{K_s\left(\frac{\rho_g \epsilon}{\rho_l (1-\epsilon)}\right)} \tag{6}$$

- The **slip ratio** in a **vertical hanging pipes** is given as [Monette & Pettigrew, 2004]:

$$K_s = \sqrt{\frac{\epsilon}{1-\epsilon}} \tag{7}$$



Slip Ratio and Void Fraction **vs** Volumetric Quality

Relation between 1ϕ and 2ϕ Flow

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- 1ϕ liquid only (l_0):

$$\beta_{l_0} = \frac{m_l}{m_p + m_l} \tag{8}$$

- **Constant** across 1ϕ and 2ϕ:

$$\frac{m_p}{A} = \rho_l \left(\frac{1-\beta_{l_0}}{\beta_{l_0}} \right) \tag{9}$$

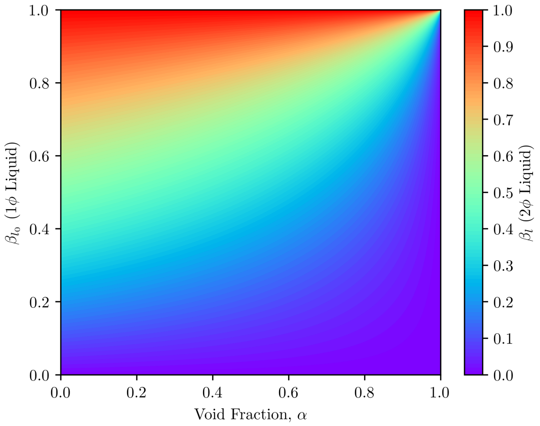
- For 2ϕ liquid:

$$\beta_l = \frac{m_l}{m_p + m_l + m_g} \tag{10}$$

$$= \frac{\rho_l (1-\alpha)}{\frac{m_p}{A} + \rho_l (1-\alpha) + \rho_g \alpha} \tag{11}$$

- **Relation** between 1ϕ and 2ϕ flow:

$$\beta_l = \frac{\rho_l (1-\alpha)}{\rho_l \left(\frac{1-\beta_{l_0}}{\beta_{l_0}} \right) + \rho_l (1-\alpha) + \rho_g \alpha} \tag{12}$$



Variation of 2ϕ Liquid Mass Ratio **with** Void Fraction and 1ϕ Liquid Mass Ratio

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Galerkin Method

- **General solution** form:

$$\eta(\xi,\tau)=\sum_{r=1}^{n}\phi_r(\xi)q_r(\tau) \tag{13}$$

- $\phi_r(\xi)$ are the orthogonal **eigen-functions** of the system:

$$\phi_r(\xi)=\cosh(\lambda_r\xi)-\cos(\lambda_r\xi)-\sigma_r\left\{\sinh(\lambda_r\xi)-\sin(\lambda_r\xi)\right\}$$
$$\sigma_r=\frac{\sinh(\lambda_r)-\sin(\lambda_r)}{\cosh(\lambda_r)+\cos(\lambda_r)}$$

- λ_r are the **eigenvalues** of r-th mode of the system, obtained from the system's **frequency equation**:

$$\cos(\lambda_r)\cosh(\lambda_r)+1=0$$

- Writing the EOM in **weak formulation** yields a **system of ODEs**:

$$[M]\,\ddot{\boldsymbol{q}}(\tau)+[C]\,\dot{\boldsymbol{q}}(\tau)+[K]\,\boldsymbol{q}(\tau)=\mathbf{0} \tag{14}$$

Galerkin Method of Solution

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Galerkin Method

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- $\phi_r(\xi)$ are the orthogonal **eigen-functions** of the system:

$$\phi_r(\xi) = \cosh(\lambda_r \xi) - \cos(\lambda_r \xi) - \sigma_r \{\sinh(\lambda_r \xi) - \sin(\lambda_r \xi)\}$$

$$\sigma_r = \frac{\sinh(\lambda_r) - \sin(\lambda_r)}{\cosh(\lambda_r) + \cos(\lambda_r)}$$

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$$[M]\,\ddot{\boldsymbol{q}}(\tau) + [C]\,\dot{\boldsymbol{q}}(\tau) + [K]\,\boldsymbol{q}(\tau) = \boldsymbol{0} \tag{14}$$

Method of Solution

- 2^{nd} order ODE $\Longrightarrow$ 1^{st} order ODE

$$\boldsymbol{Z}(\tau) = \begin{bmatrix} \boldsymbol{q}(\tau) \\ \dot{\boldsymbol{q}}(\tau) \end{bmatrix}$$

$$[B]\,\dot{\boldsymbol{Z}}(\tau) + [E]\,\boldsymbol{Z}(\tau) = \boldsymbol{0}$$

where:

$$[B] = \begin{bmatrix} 0 & [M] \\ [M] & [C] \end{bmatrix} \qquad [E] = \begin{bmatrix} -[M] & 0 \\ 0 & [K] \end{bmatrix}$$

- Assuming the form: $\boldsymbol{q}(\tau) = \boldsymbol{a} e^{i\omega\tau}$
- Yields the **eigenvalue problem**:

$$([Y] - i\omega\,[I])\,\boldsymbol{a} = \boldsymbol{0} \tag{15}$$

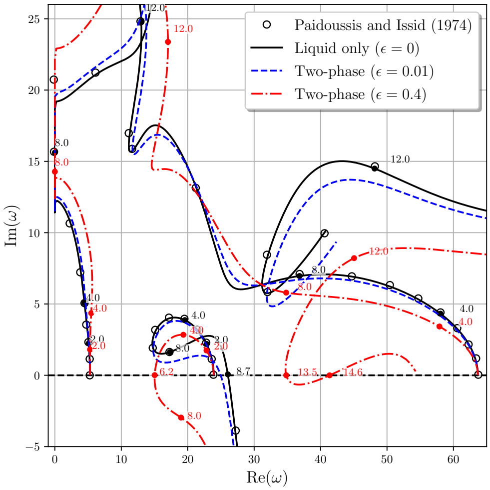
where $[Y] = -[B]^{-1}[E]$.

Overview of Study

- **Application:** Cantilever pipe conveying 2ϕ flow with variable flow velocity
- **Flow Components:** Water & Air
- **Verification of Galerkin Solver:** Compared results vs. published literature
- **Analysis:** Stability via dimensional complex frequency response
- **Investigated:**
  - Instability modes
  - Critical flow velocity
  - Effect of 2ϕ flow parameters
- **Case I: Vertical Hanging Cantilever Pipe (Fixed-Free)**
  - $\rho_g = 1.2$ kg/m^3, $\rho_l = 1000$ kg/m^3
  - $\mu = 0,\ \beta_{l_0} = \{0.3,\ 0.2,\ 0.65\},\ \gamma = \{10,\ 100\}$
- **Case II: Vertically Clamped Pipe (Fixed-Fixed)**
  - $\rho_g = 1.2$ kg/m^3, $\rho_l = 1000$ kg/m^3
  - $\mu = 0,\ \beta_{l_0} = 0.645,\ \gamma = 0$

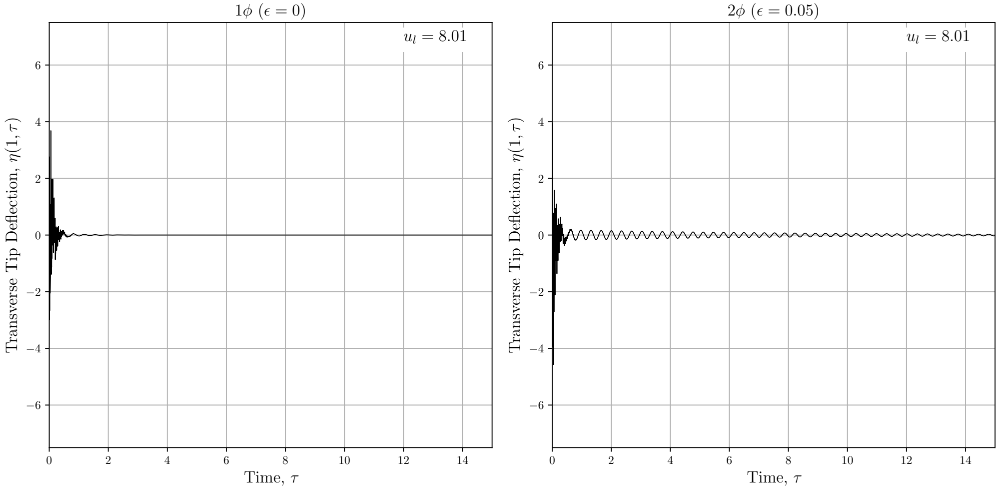
Verification

- 1ϕ response matches well with Paidoussis and Issid (1974)
- The response deviates from 1ϕ system at higher u_l with increasing ϵ
- 2nd Mode **Instability**
  - 1ϕ: $u_l^c = 10.5$
  - 2ϕ: $u_l^c = 6.5$ for $\epsilon = 0.4$ $(\alpha = 0.45)$
  - Hopf bifurcation single mode flutter
- 4th Mode **Restabilization**
  - Unstable at $u_l^c = 13.5$
  - Restabilizes at $u_l^c = 14.1$
  - Unstable ultimately at $u_l^c = 15.0$



Argand Diagram for $\beta_{l_0} = 0.3$, $\gamma = 10$, $\mu = 0$

Response Comparison: 1ϕ vs 2ϕ



Results: Case I (Cantilever Pipe)

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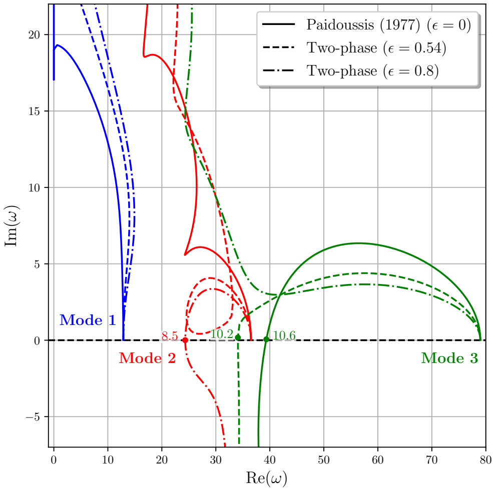
Importance of ϵ Parameter

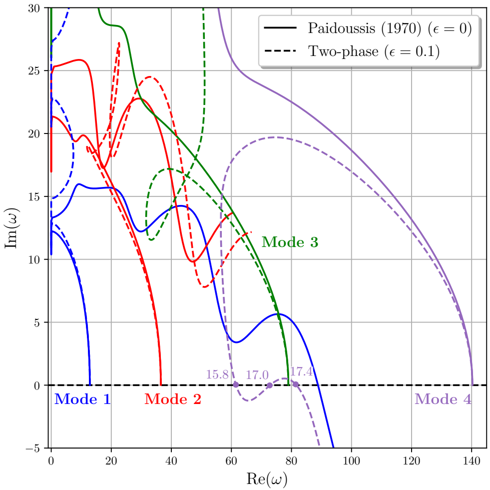
- $\epsilon = \mathbf{0.00}$**:** 3rd Mode => Unstable (1ϕ)
- $\epsilon = \mathbf{0.54}$**:** 2nd Mode => Moves closer to stability line
- $\epsilon = \mathbf{0.80}$**:** 2nd Mode => Unstable, 3rd Mode => Stable
- Volumetric quality ϵ is an important parameter to asses 2ϕ pipe flow instability



Argand Diagram for $\beta_{l_0} = 0.2$, $\gamma = 100$, $\mu = 0$

Restabilization (1/2)

- **"Z"** type frequency **response** indicates **unstable-stable-unstable** response
- $\epsilon = 0.1$: 4th Mode shows unstable-stable-unstable response
- The system can be stabilized by increasing the velocity up to a certain point



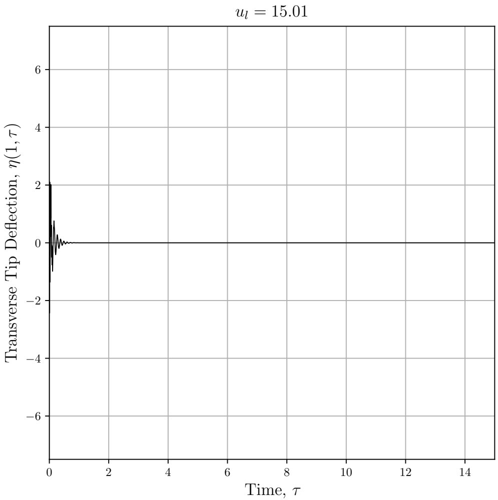
Argand Diagram for $\beta_{l_0} = 0.65$, $\gamma = 100$, $\mu = 0$

Restabilizing System Response

- Transverse tip deflection:

$$\eta(1,\tau) = \sum_{r=1}^{n} \phi_r(1) q_r(\tau)$$

- System response to $\uparrow u_l$
  1. **Unstable** at $\mathbf{u_l^c = 15.8}$
  2. **Restabilizes** at $\mathbf{u_l^c = 17.0}$
  3. **Unstable** ultimately at $\mathbf{u_l^c = 17.4}$

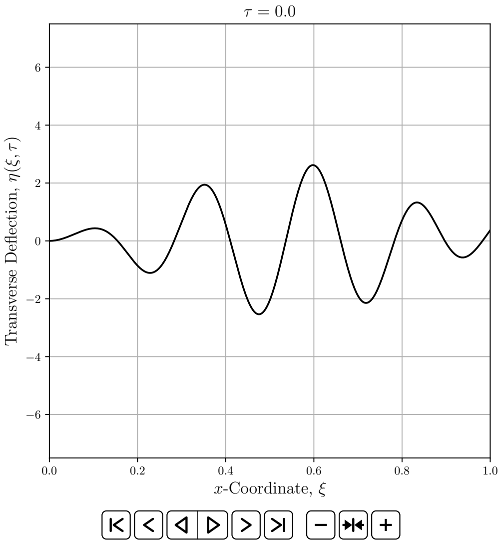


Unstable Transient Motion of Pipe

- Transverse deflection:

$$\eta(\xi,\tau) = \sum_{r=1}^{n} \phi_r(\xi) q_r(\tau)$$

- Fixed liquid velocity: $u_l = 15.85$ $(> u_l^c = 15.8)$
- **Flutter instability with growth**



Unstable Transient Motion of Pipe

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$$\eta(\xi,\tau) = \sum_{r=1}^{n} \phi_r(\xi) q_r(\tau)$$

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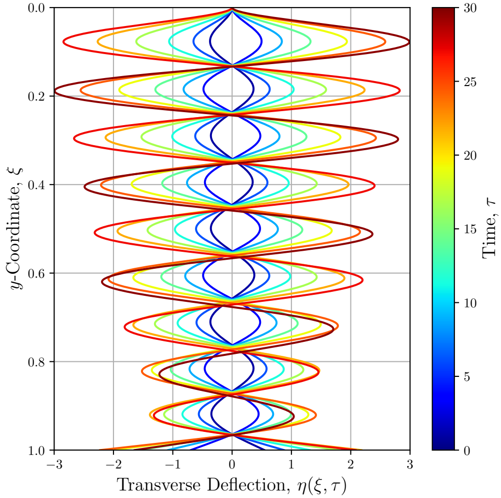
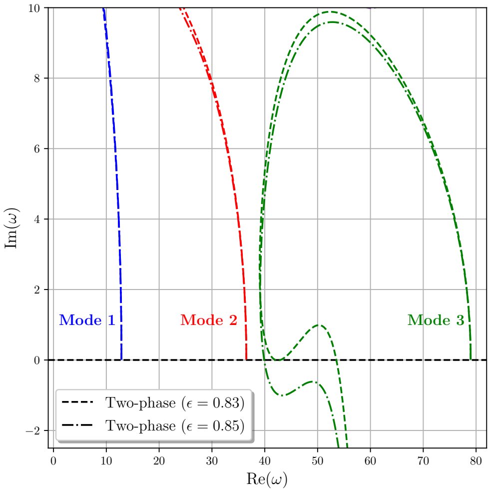


Transient of η vs ξ at $u_l = 15.85$ for $\epsilon = 0.1$, $\beta_{l_0} = 0.65$, $\gamma = 100$, $\mu = 0$

Results: Case I (Cantilever Pipe)

Restabilization (2/2)

- **"Z"** type response is extremely **sensitive** to change in ϵ
- $\epsilon = 0.83$: 3rd Mode response is almost **tangent** to the stability line
- The response changes drastically even with slight changes in ϵ ($\epsilon = 0.83$ to $\epsilon = 0.85$)



Argand Diagram for $\beta_{l_0} = 0.65$, $\gamma = 100$, $\mu = 0$

Results: Case II (Clamped Pipe)

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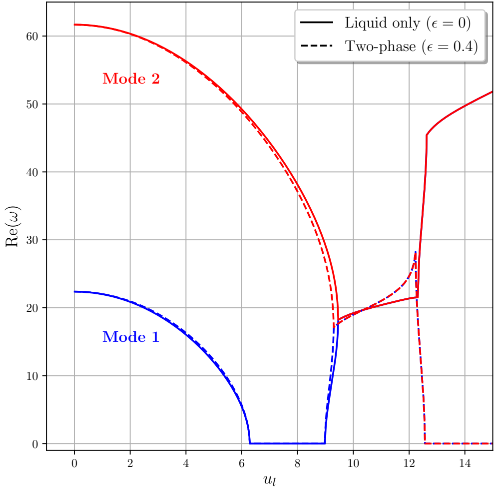
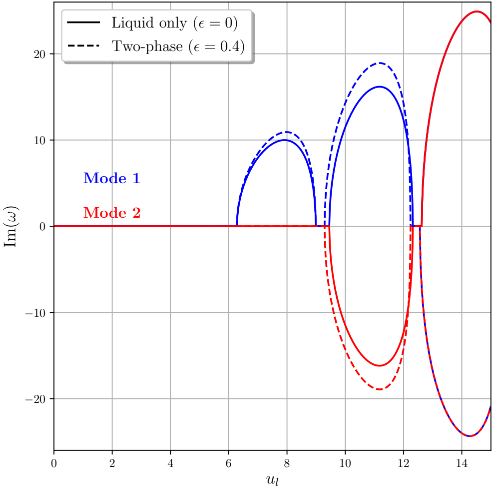
Couple-Mode Instability



Re(ω) **vs** u_l



Im(ω) **vs** u_l

Conclusions

- 2ϕ system response is drastically different to 1ϕ
- Volumetric quality ϵ is an important parameter to asses 2ϕ pipe flow instability
- The dynamic response of 2ϕ system is erratic
- Galerkin method is an elegant numerical technique to study frequency response of a system
- This work can be extended to model more realistic 2ϕ system with water-vapor flow
- The predictability of the system response can be improved with better 2ϕ flow modeling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