

Foundation for Inference Part 1

DATA 606 - Statistics & Probability for Data Analytics

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One Minute Paper Results

What was the most important thing you learned during this class?

variables
using
z-scores
distributions
probability
scales
learned
used
area
use
can
also
data
distribution
normal
binomial
function
appreciate
getting
different

What important question remains unanswered for you?

know
good
question
model
plots
distributions
think
distribution
geometric
time
use
like
need
different
class
modeling
everything
normal
knowledge

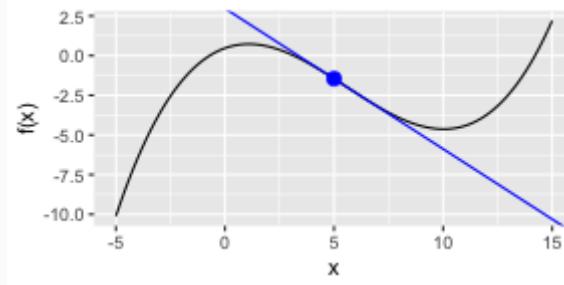
Crash Course in Calculus

Crash Course in Calculus

There are three major concepts in calculus that will be helpful to understand:

Limits - the value that a function (or sequence) approaches as the input (or index) approaches some value.

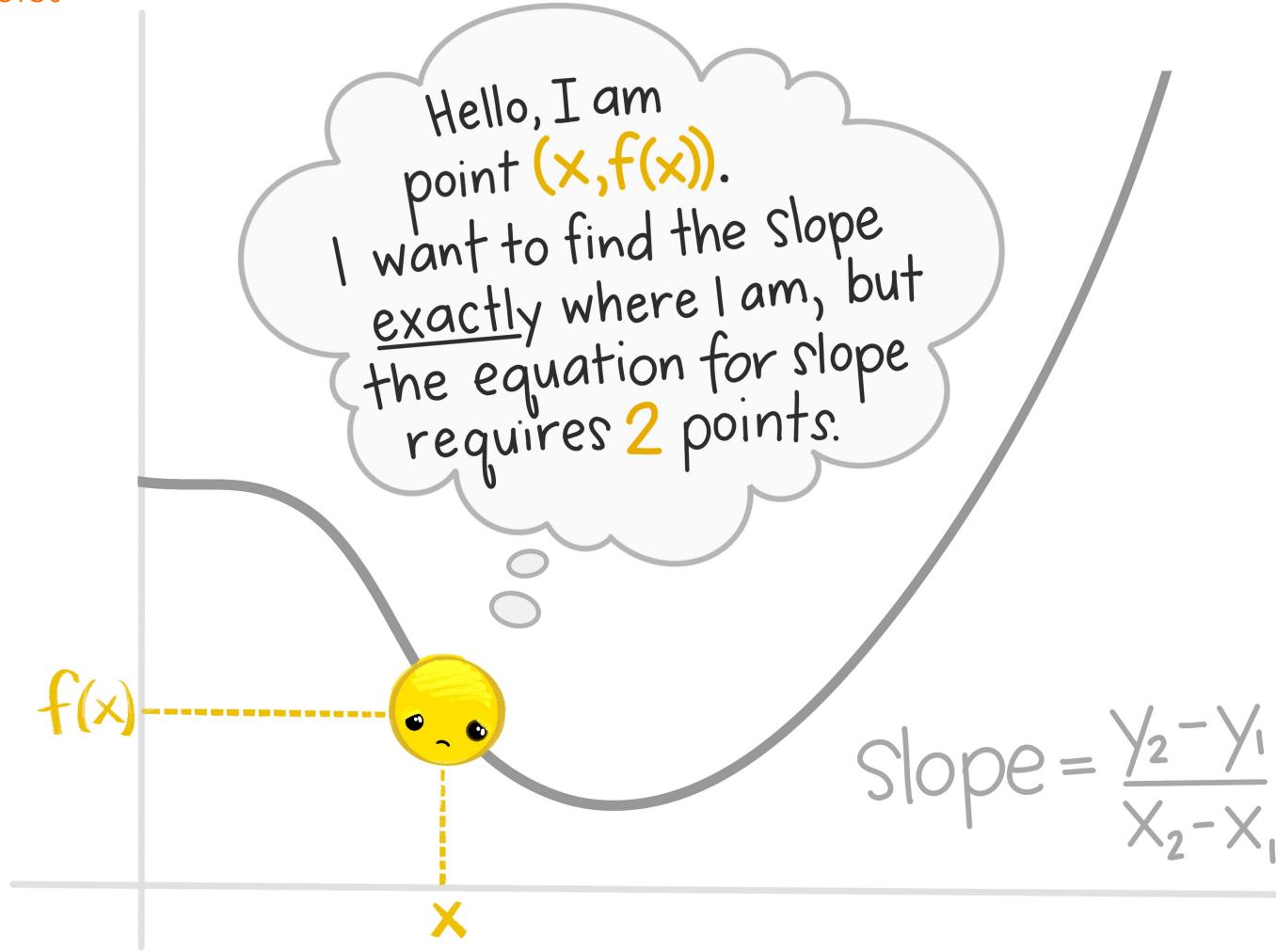
Derivatives - the slope of the line tangent at any given point on a function.



Integrals - the area under the curve.

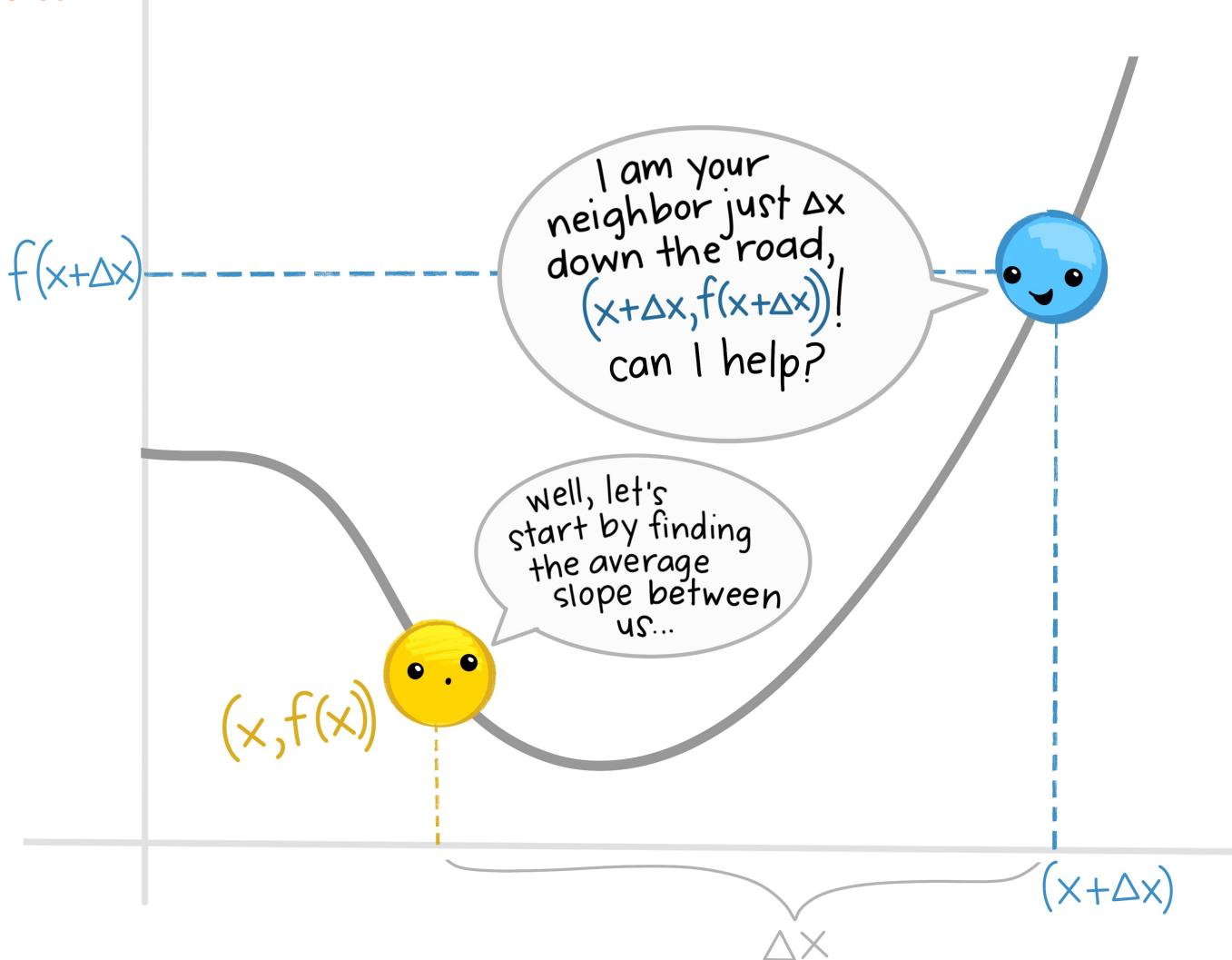
Derivatives

Source: @allison_horst



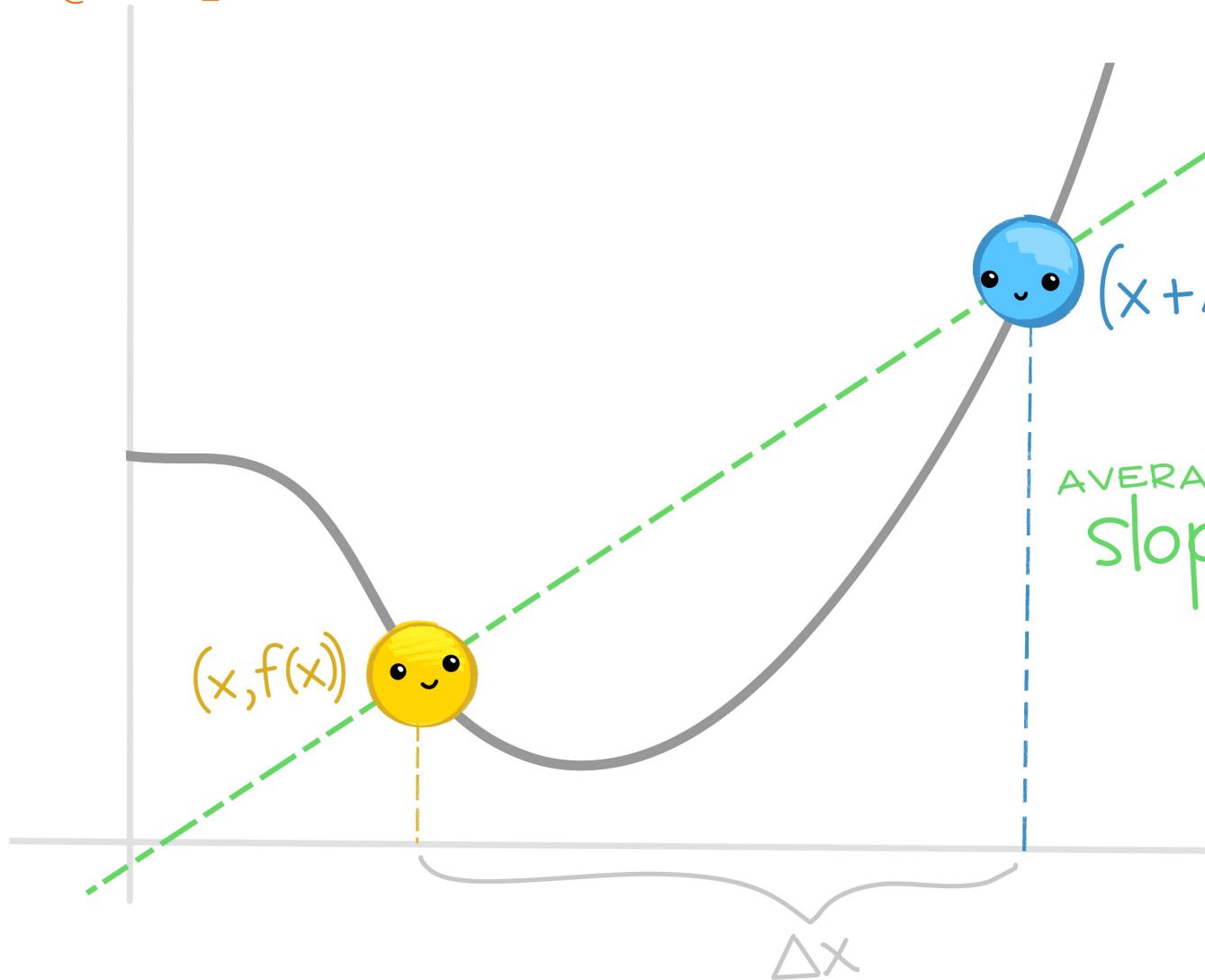
Derivatives

Source: @allison_horst



Derivatives

Source: @allison_horst



$(x + \Delta x, f(x + \Delta x))$

AVERAGE
slope

$$\begin{aligned} \text{AVERAGE slope} &= \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x} \\ &= \frac{f(x + \Delta x) - f(x)}{\Delta x} \end{aligned}$$

Derivatives

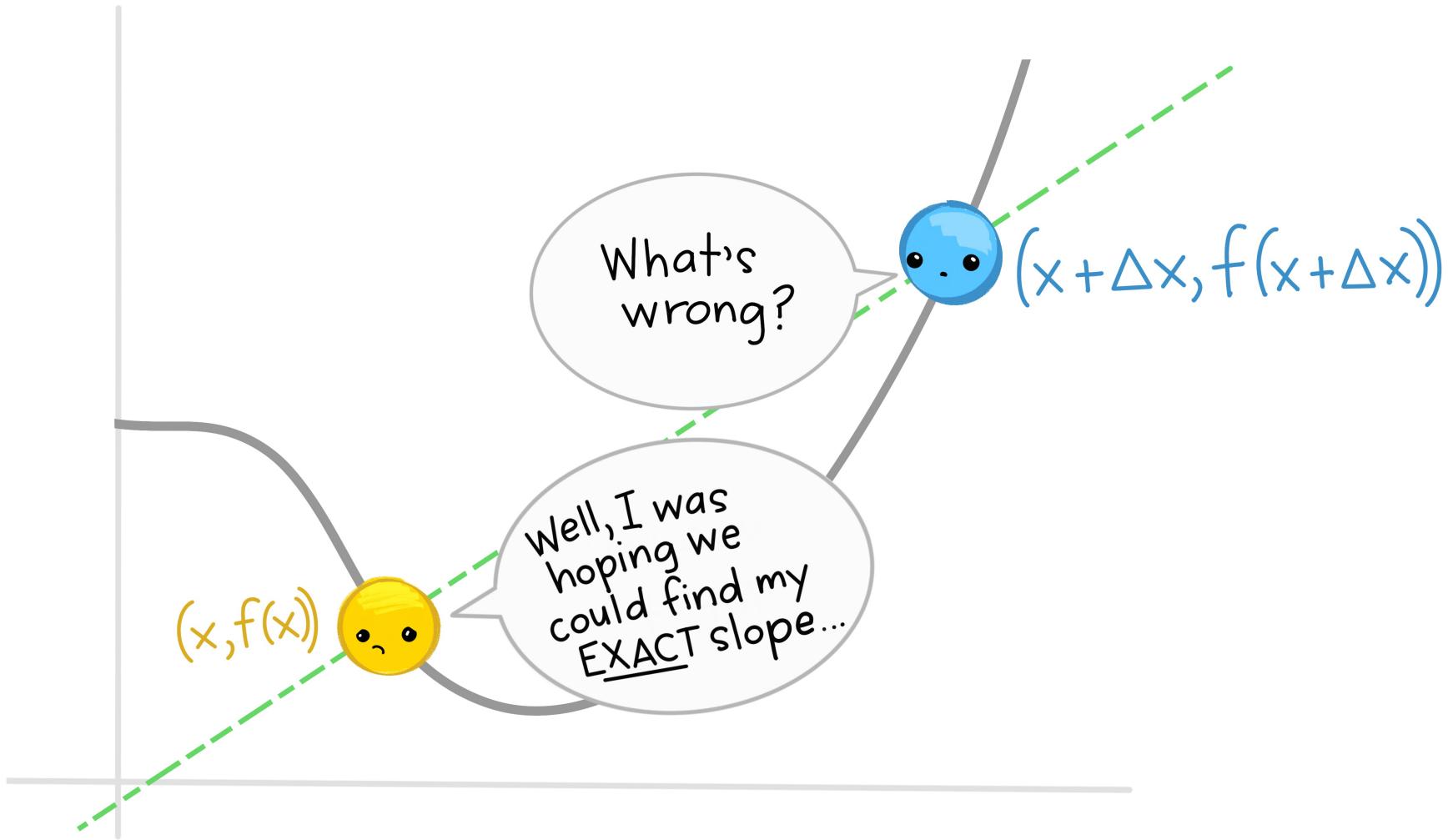
Source: @allison_horst

So: the average slope between ANY 2 POINTS on function $f(x)$ separated by Δx is

$$m = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Derivatives

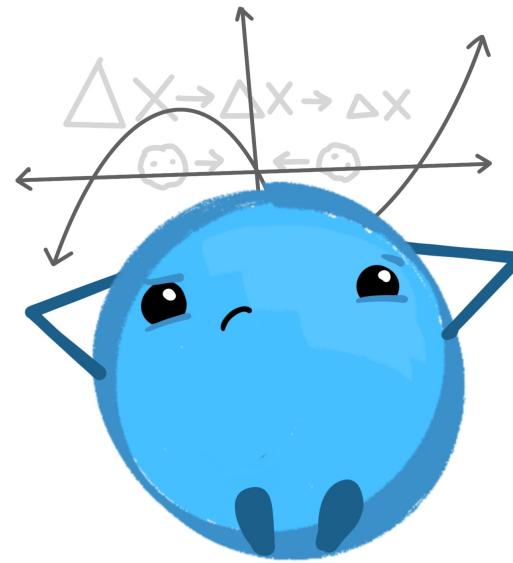
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Derivatives

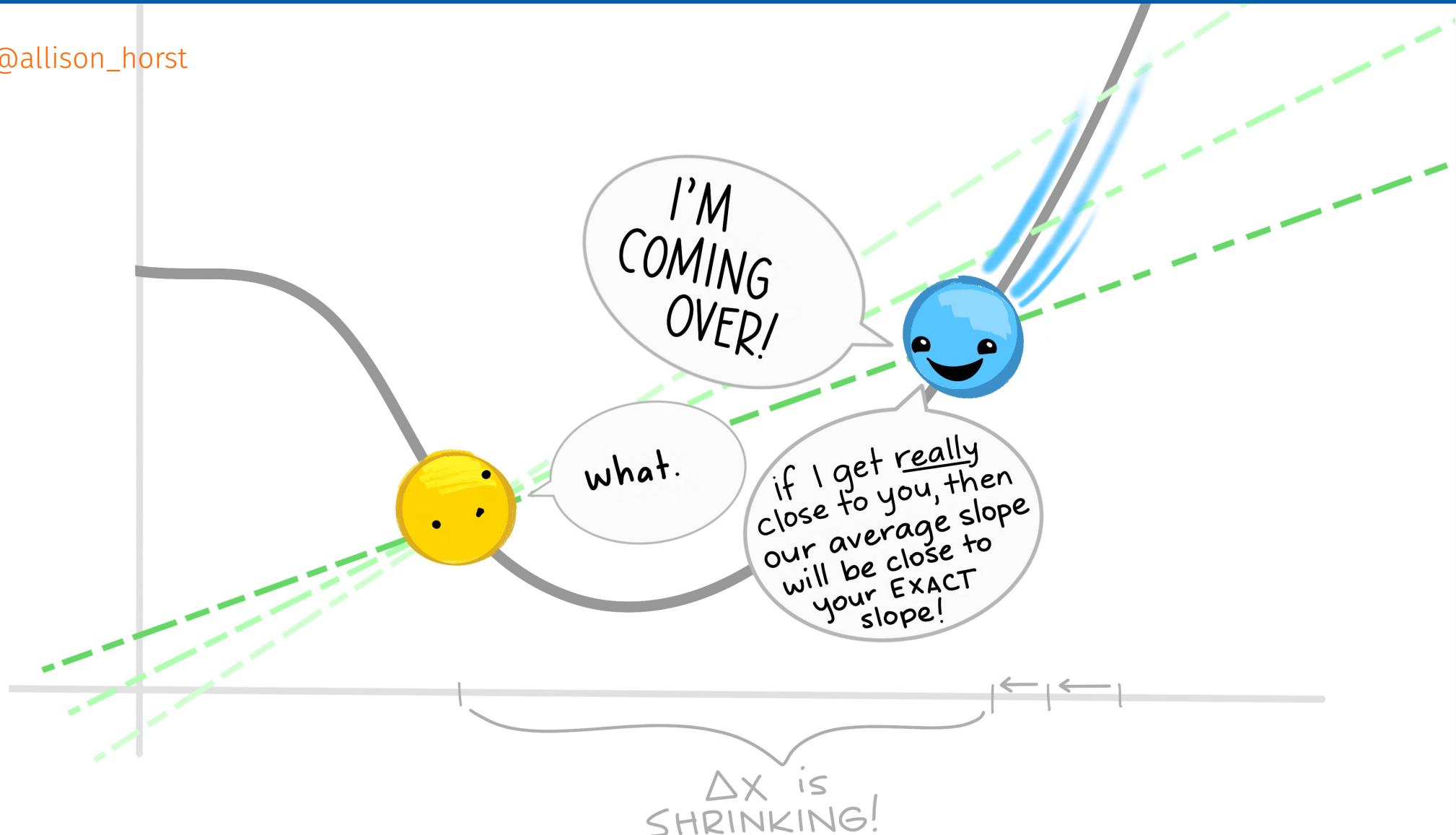
Source: @allison_horst

BRAINSTORM MONTAGE!



Derivatives

Source: @allison_horst



Derivatives

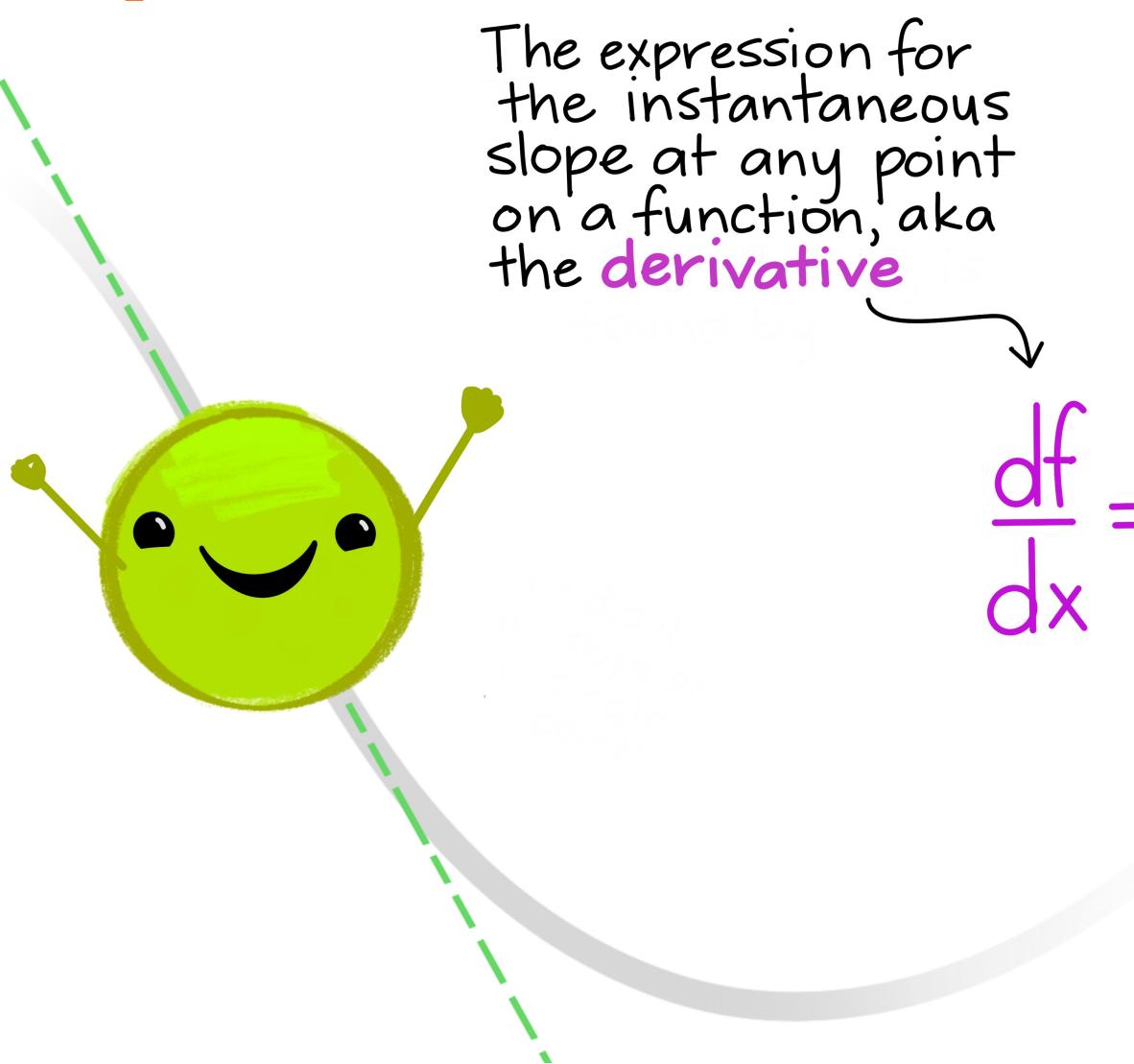
Source: [@allison_horst](#)

$$\rightarrow \Delta x \leftarrow$$

And if the distance between us gets infinitely small ($\Delta x \rightarrow 0$), our AVERAGE SLOPE becomes the INSTANTANEOUS SLOPE at a single point!

Derivatives

Source: [@allison_horst](#)



The expression for
the instantaneous
slope at any point
on a function, aka
the **derivative**

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

IS FOUND BY:

- ① Finding an expression for the **slope** between 2 points separated by Δx ...

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- ② Evaluating that slope as the points get infinitely close together.

Function for Normal Distribution

$$f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

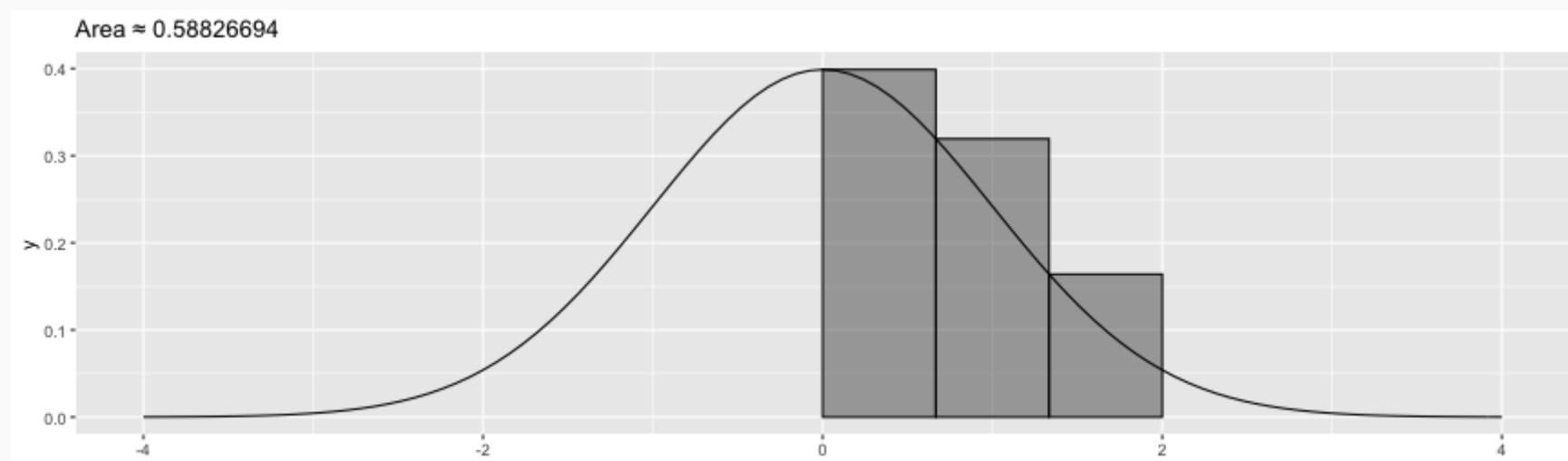
```
f <- function(x, mean = 0, sigma = 1) {  
  1 / (sigma * sqrt(2 * pi)) * exp(1)^(-1/2 * ( (x - mean) / sigma )^2)  
}
```

```
min <- 0; max <- 2  
ggplot() + stat_function(fun = f) + xlim(c(-4, 4)) +  
  geom_vline(xintercept = c(min, max), color = 'blue', linetype = 2) + xlab('x')
```

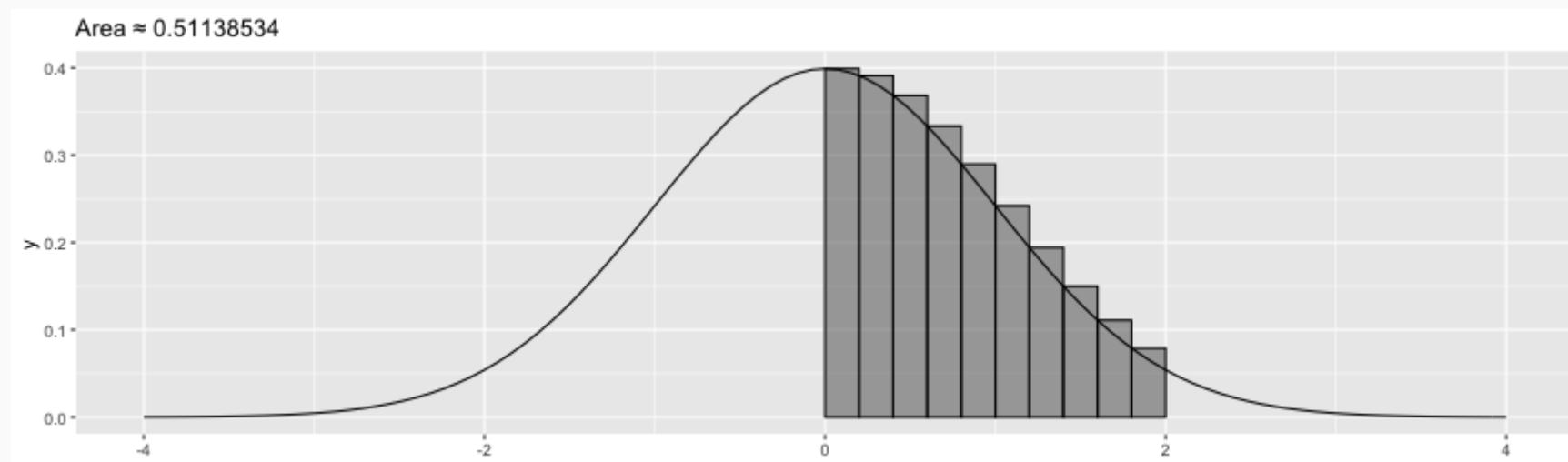


Reimann Sums

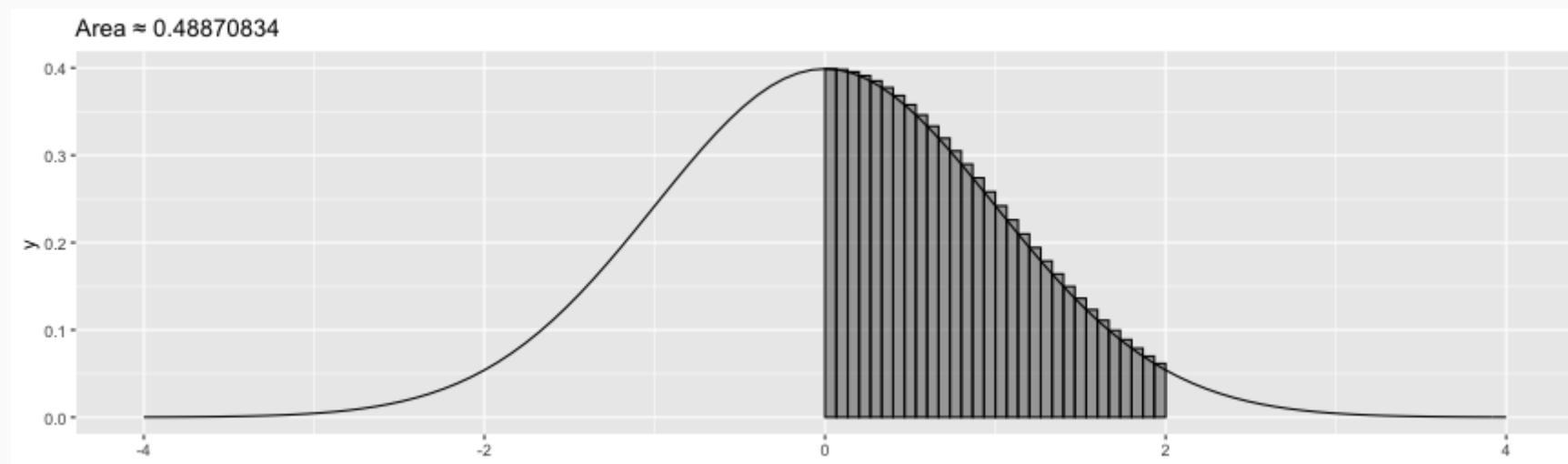
One strategy to find the area between two values is to draw a series of rectangles. Given n rectangles, we know that the width of each is $\frac{2-0}{n}$ and the height is $f(x)$. Here is an example with 3 rectangles.



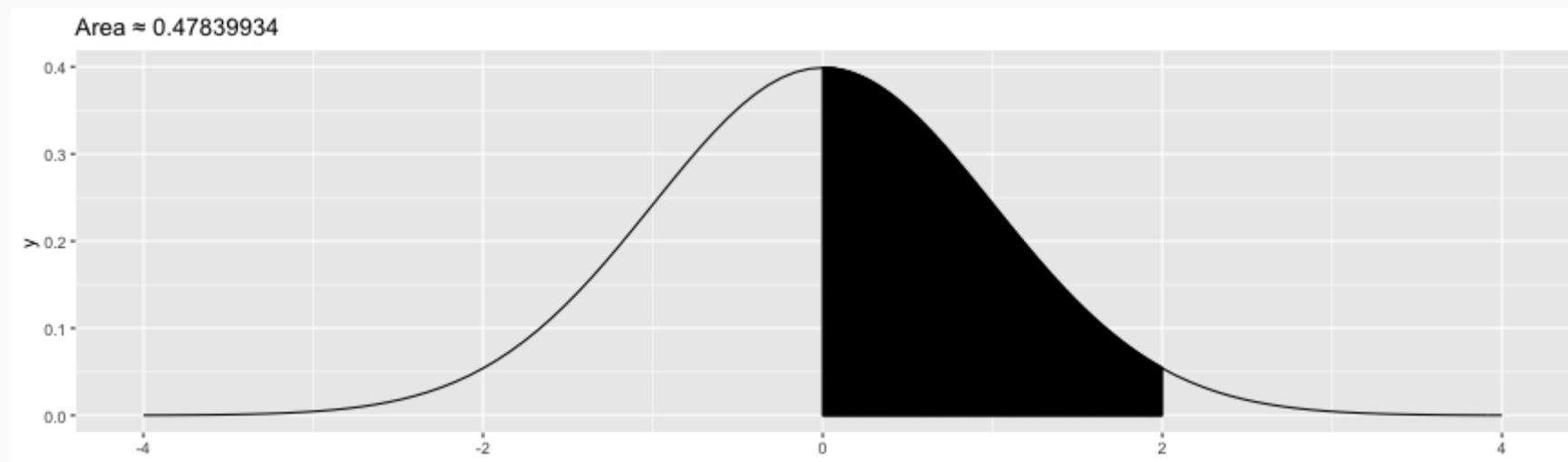
Reimann Sums (10 rectangles)



Reimann Sums (30 rectangles)



Reimann Sums (300 rectangles)



$n \rightarrow \infty$

As n approaches infinity we are going to get the *exact* value for the area under the curve. This notion of letting a value get increasingly close to infinity, zero, or any other value, is called the **limit**.

The area under a function is called the integral.

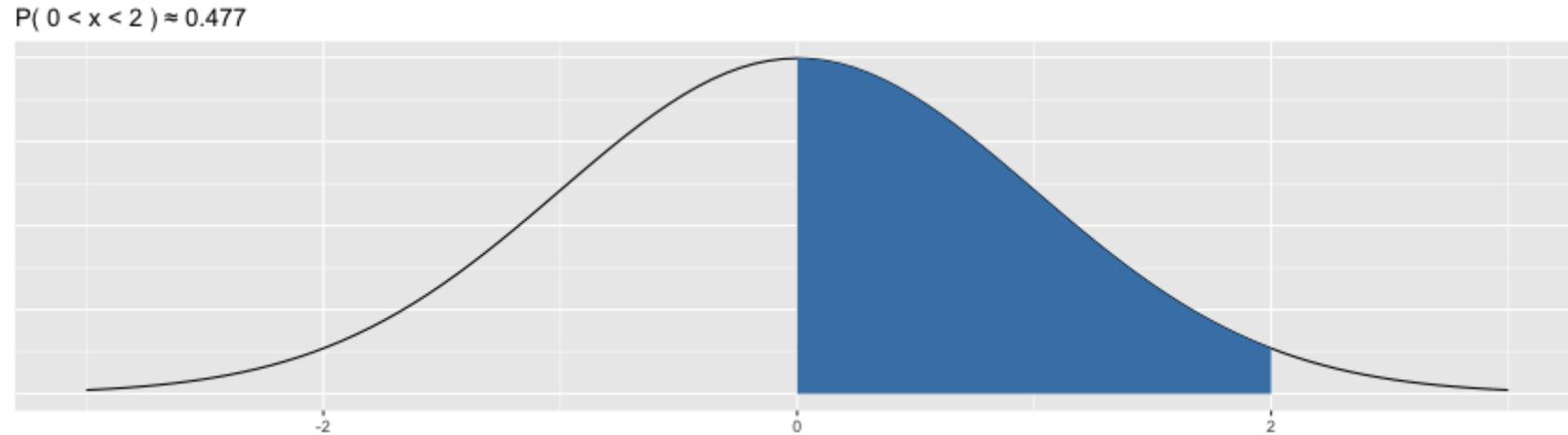
```
integrate(f, 0, 2)
```

```
## 0.4772499 with absolute error < 5.3e-15
```

```
DATA606::shiny_demo('calculus')
```

Normal Distribution

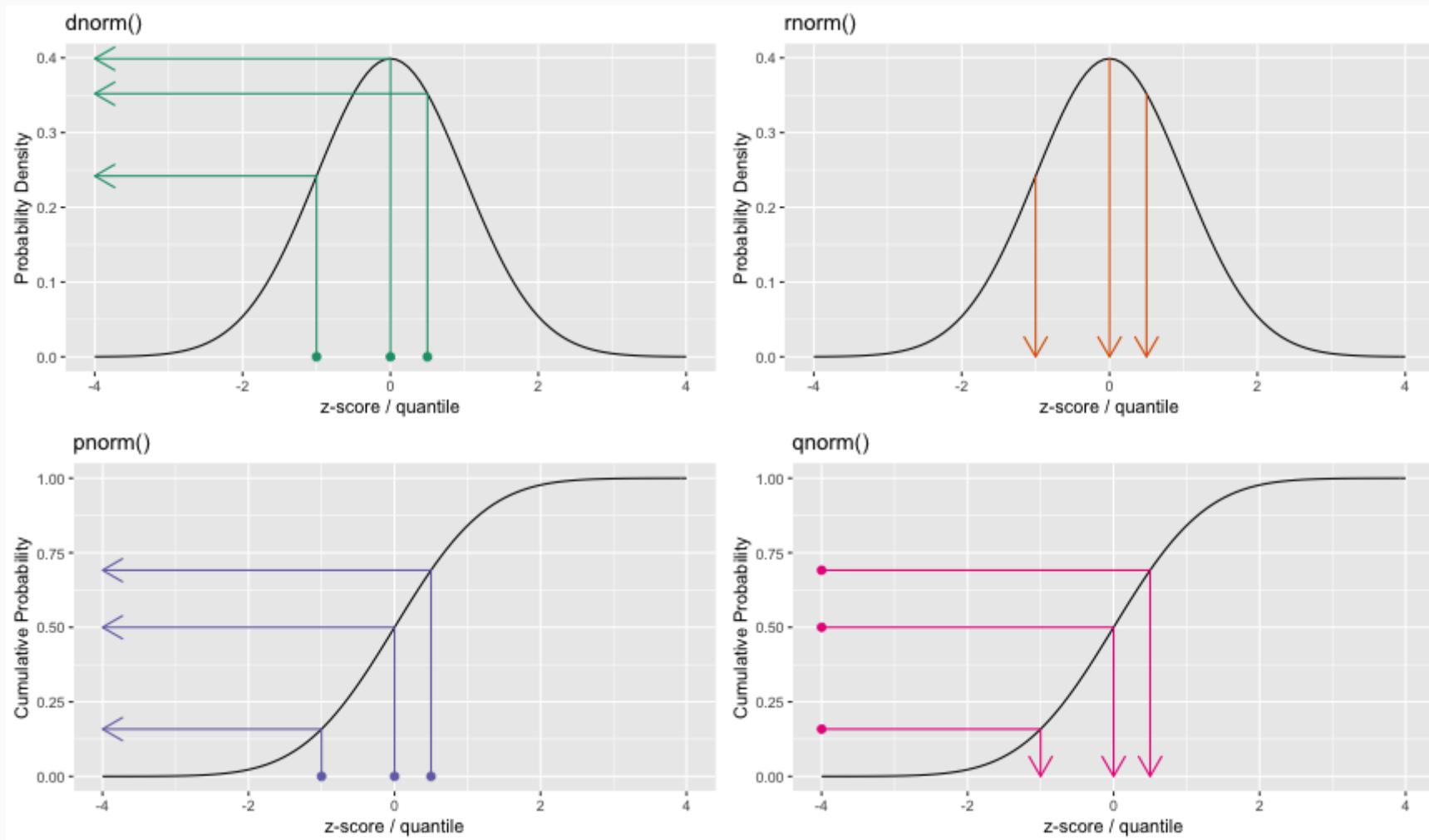
```
normal_plot(cv = c(0, 2))
```



```
pnorm(2) - pnorm(0)
```

```
## [1] 0.4772499
```

R's built in functions for working with distributions



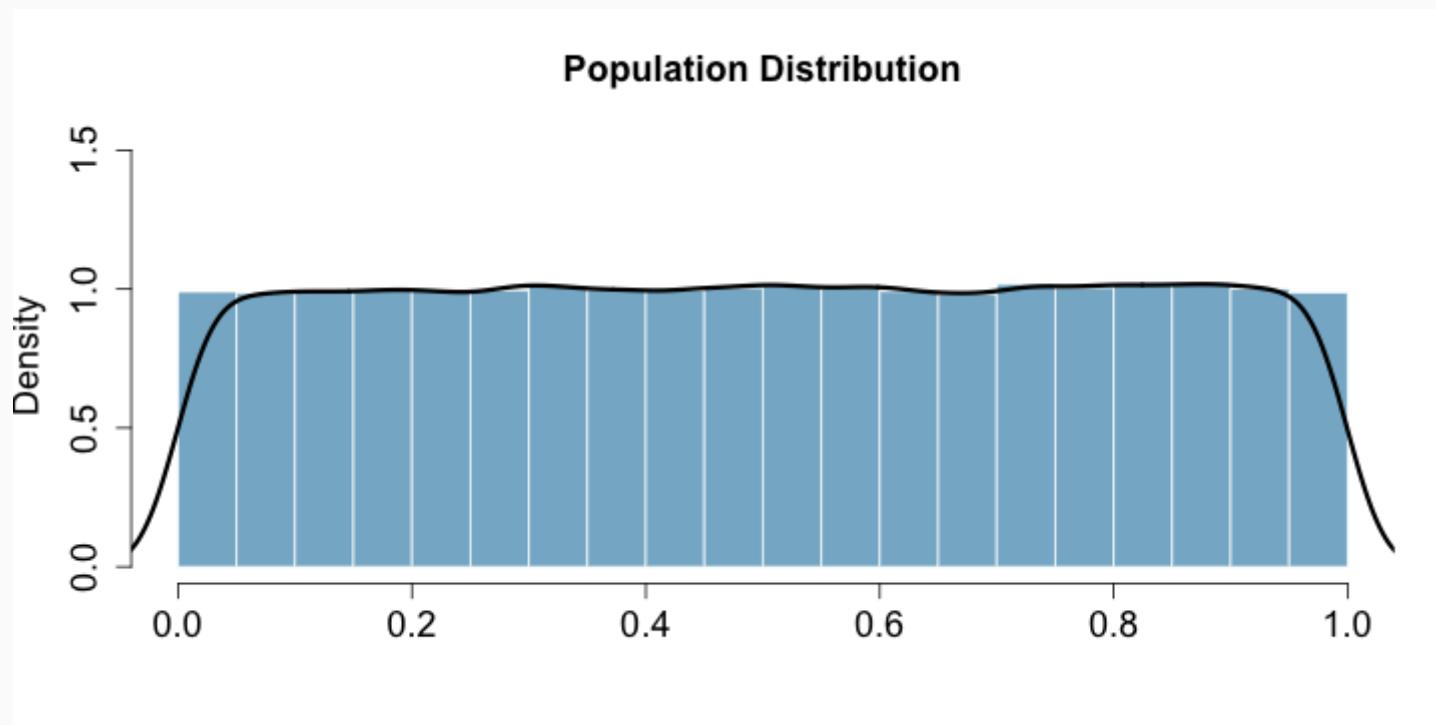
See <https://github.com/jbryer/DATA606Fall2021/blob/master/R/distributions.R>

Foundation for Inference

Population Distribution (Uniform)

```
n <- 1e5  
pop <- runif(n, 0, 1)  
mean(pop)
```

```
## [1] 0.5012861
```

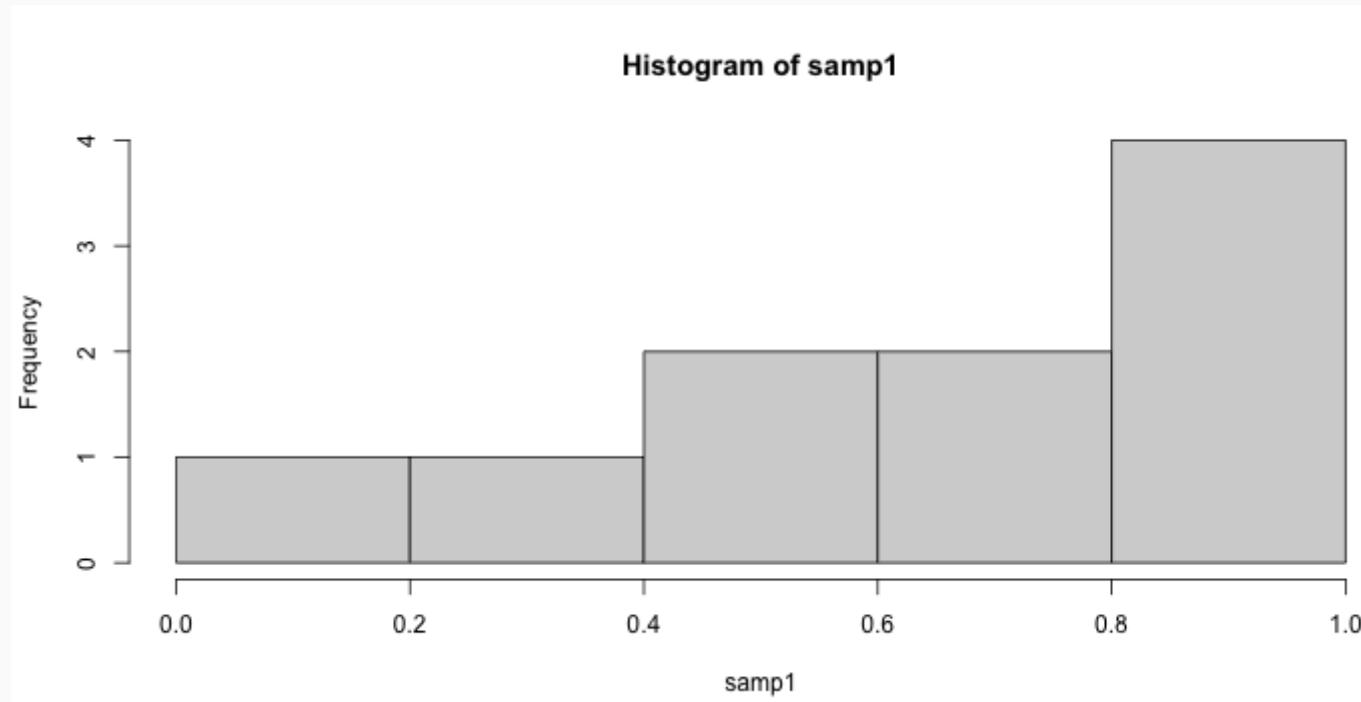


Random Sample (n=10)

```
samp1 <- sample(pop, size=10)  
mean(samp1)
```

```
## [1] 0.6548842
```

```
hist(samp1)
```

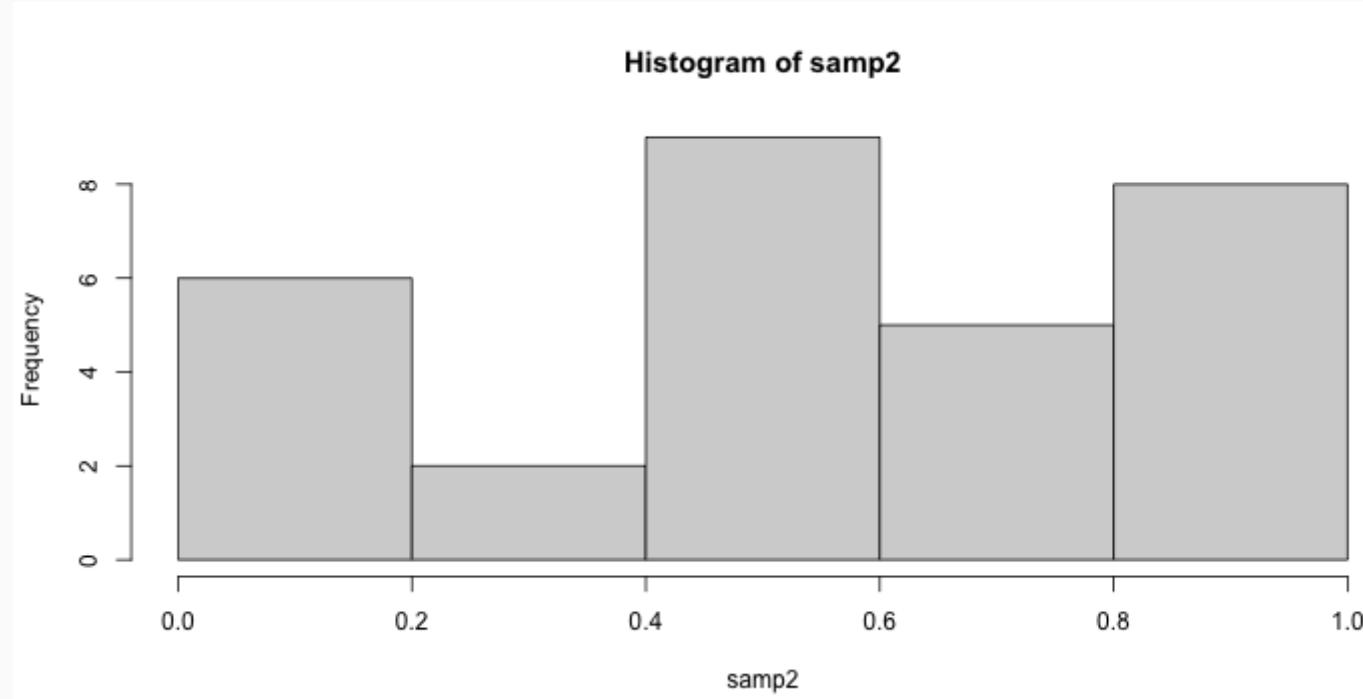


Random Sample (n=30)

```
samp2 <- sample(pop, size=30)  
mean(samp2)
```

```
## [1] 0.53561
```

```
hist(samp2)
```



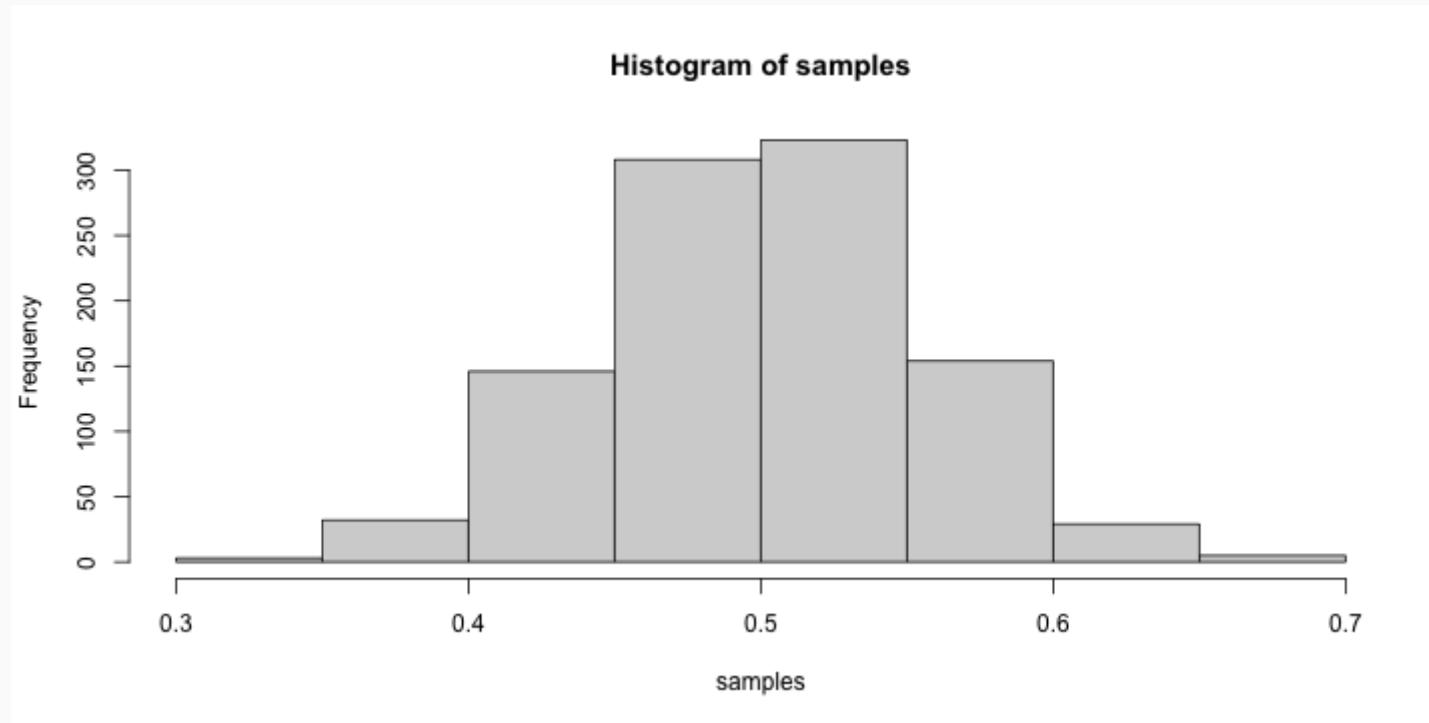
Lots of Random Samples

```
M <- 1000
samples <- numeric(length=M)
for(i in seq_len(M)) {
  samples[i] <- mean(sample(pop, size=30))
}
head(samples, n=8)
```

```
## [1] 0.4767379 0.4492937 0.4860282 0.5802728 0.4947736 0.4974327 0.4978299
## [8] 0.4391328
```

Sampling Distribution

```
hist(samples)
```



Central Limit Theorem (CLT)

Let X_1, X_2, \dots, X_n be independent, identically distributed random variables with mean μ and variance σ^2 , both finite. Then for any constant z ,

$$\lim_{n \rightarrow \infty} P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z\right) = \Phi(z)$$

where Φ is the cumulative distribution function (cdf) of the standard normal distribution.

In other words...

The distribution of the sample mean is well approximated by a normal model:

$$\bar{x} \sim N \left(\text{mean} = \mu, SE = \frac{\sigma}{\sqrt{n}} \right)$$

where SE represents the **standard error**, which is defined as the standard deviation of the sampling distribution. In most cases σ is not known, so use s .

CLT Shiny App

```
library(DATA606)
shiny_demo('sampdist')
shiny_demo('CLT_mean')
```



Standard Error

```
samp2 <- sample(pop, size=30)  
mean(samp2)
```

```
## [1] 0.6192882
```

```
(samp2.se <- sd(samp2) / sqrt(length(samp2)))
```

```
## [1] 0.05183611
```

Confidence Interval

The confidence interval is then $\mu \pm CV \times SE$ where CV is the critical value. For a 95% confidence interval, the critical value is ~1.96 since

$$\int_{-1.96}^{1.96} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \approx 0.95$$

```
qnorm(0.025) # Remember we need to consider the two tails, 2.5% to the left, 2.5% to the right.
```

```
## [1] -1.959964
```

```
(samp2.ci <- c(mean(samp2) - 1.96 * samp2.se, mean(samp2) + 1.96 * samp2.se))
```

```
## [1] 0.5176894 0.7208870
```

Confidence Intervals (cont.)

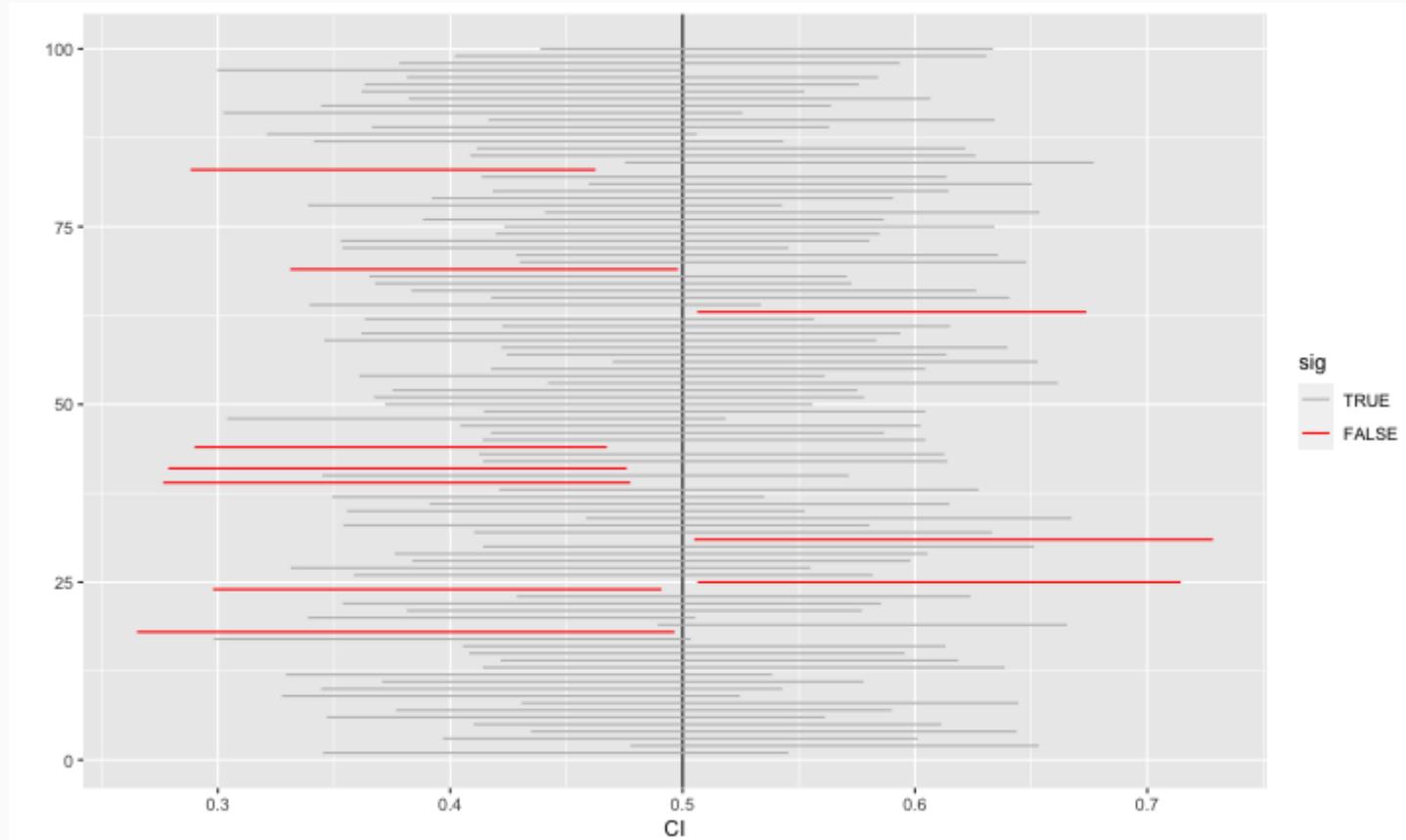
We are 95% confident that the true population mean is between 0.5176894, 0.720887.

That is, if we were to take 100 random samples, we would expect at least 95% of those samples to have a mean within 0.5176894, 0.720887.

```
ci <- data.frame(mean=numeric(), min=numeric(), max=numeric())
for(i in seq_len(100)) {
  samp <- sample(pop, size=30)
  se <- sd(samp) / sqrt(length(samp))
  ci[i,] <- c(mean(samp),
              mean(samp) - 1.96 * se,
              mean(samp) + 1.96 * se)
}
ci$sample <- 1:nrow(ci)
ci$sig <- ci$min < 0.5 & ci$max > 0.5
```

Confidence Intervals

```
ggplot(ci, aes(x=min, xend=max, y=sample, yend=sample, color=sig)) +  
  geom_vline(xintercept=0.5) +  
  geom_segment() + xlab('CI') + ylab('') +  
  scale_color_manual(values=c('TRUE'='grey', 'FALSE'='red'))
```



One Minute Paper

Complete the one minute paper:

<https://forms.gle/qxRnsCyydx1nf8sXA>

1. What was the most important thing you learned during this class?
2. What important question remains unanswered for you?

