Ejemplo 2

Simetrica $\Rightarrow A = A^T$

Definida positiva $\Rightarrow$ Cada subdeterminante > 0

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 5 & -5 \\ 1 & -5 & 6 \end{pmatrix}$$

Recuerda $\Rightarrow$ $L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}$

Algoritmo:

$$l_{11} = \sqrt{a_{11}}$$

$$l_{i,1} = \frac{a_{i,1}}{l_{11}}, \quad i \geq 2$$

$$l_{k,k} = \sqrt{a_{kk} - \sum_{r=1}^{k-1} l_{kr}^2}, \quad k \geq 2$$

$$l_{ik} = \frac{a_{ik} - \sum_{r=1}^{k-1} l_{ir} l_{kr}}{l_{kk}}, \quad i > k$$

1) Simetrica ✓

2) Definida positiva $\Rightarrow$ i) $|1| = 1 > 0$

ii) $\begin{vmatrix} 1 & -1 \\ -1 & 5 \end{vmatrix} = 4 > 0$

Así A es definida positiva y podemos hacer $A = LL^T$.

Entonces

$$l_{11} = \sqrt{1} = 1$$

$$l_{21} = \frac{-1}{1} = -1$$

$$l_{31} = \frac{1}{1} = 1$$

$$l_{22} = \sqrt{5-(-1)^2} = 2$$

$$l_{32} = \frac{-5-1(-1)}{2} = -2$$

$$l_{33} = \sqrt{6-1-(-2)^2} = 1$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -2 & 1 \end{pmatrix}$$

así

$$LL^T = A \Rightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}}_{LL^T} = \underbrace{\begin{pmatrix} 1 & -1 & 1 \\ -1 & 5 & -5 \\ 1 & -5 & 6 \end{pmatrix}}_{A}$$