

# StableMotifs Documentation

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## Module `StableMotifs`

### Sub-modules

- [StableMotifs.Attractor](#)
- [StableMotifs.AttractorRepertoire](#)
- [StableMotifs.DomainOfInfluence](#)
- [StableMotifs.Export](#)
- [StableMotifs.Format](#)
- [StableMotifs.RandomBooleanNetworks](#)
- [StableMotifs.Reduction](#)
- [StableMotifs.RestrictSpace](#)
- [StableMotifs.Succession](#)
- [StableMotifs.TimeReversal](#)

## Module `StableMotifs.Attractor`

### Classes

#### Class `Attractor`

```
class Attractor(
    reduction,
    reduction_attractor_id
)
```

Stores attractor data for a reduced network. Automatically initialized by the `AttractorRepertoire` class.

### Parameters

**reduction : `Reduction.MotifReduction`** Motif reduction to use as the representative (see attributes).  
**reduction\_attractor\_id : `int`** Reduction id to use for the representative (see attributes)

### Attributes

**logically\_fixed\_nodes : `partial state dictionary`** The nodes that are fixed by percolation on the expanded network (i.e., not by up-stream oscillations)  
**representative : `Reduction.MotifReduction, int tuple`** Entry 0 is a maximally reduced `Reduction.MotifReduction` object that contains the attractor. In general, other such objects contain the attractor, but they will correspond to equivalent reduced networks. Entry 1 is a unique identifier number (integer) for the attractor within the reduced network; this is necessary in cases when a fully reduced network contains multiple (complex) attractors.

**reductions : list of Reduction.MotifReduction** Maximally reduced MotifReductions that contain the attractor.

**attractor\_dict : dictionary** a dictionary describing the node states in the attractor according to the following key - 1 variable is "ON" 0 variable is "OFF" X variable is known to oscillate ? at least one such variable must oscillate ! the attractor may be false; if it is genuine, at least one such variable must oscillate

**stg : networkx.DiGraph** The state transition graph corresponding to the attractor (if computed)

**fixed\_nodes : partial state dictionary** All node states that are known to be fixed in the attractor.

**oscillation\_fixed\_nodes : partial state dictionary** Node states that are fixed in the attractor, but that are not fixed by percolation in the expanded network. These states are instead fixed by up-stream oscillation.

**reduced\_primes : PyBoolNet primes dictionary** Update rules for the maximally reduced network that contains the attractor.

**n\_unfixed : int** Number of nodes that are not logically fixed.

**size\_lower\_bound : int** Lower bound on number of states in attractor.

**size\_upper\_bound : int** Upper bound on number of states in attractor.

**explored : bool** True if all attractor states and transitions are explicitly computed.

**guaranteed : bool** True if and only if the attractor is known to be genuine. If False, the attractor may not actually be stable.

## Methods

### Method add\_reduction

```
def add_reduction(
    self,
    reduction
)
```

Add a reduction to the attractor. Does not check for compatibility.

Parameters

**reduction : Reduction.MotifReduction** Motif reduction that also contains the attractor.

## Module StableMotifs.AttractorRepertoire

### Classes

#### Class AttractorRepertoire

```
class AttractorRepertoire
```

The class that stores information about attractors. Initialize using either `from_primes` or `from_succession_diagram`.

### Attributes

**succession\_diagram : Succession.SuccessionDiagram** Succession diagram summarizing the stable motif structure of the model.

**attractors : list of Attractor.Attractor** List of (possible) attractors in the model.

**reduction\_attractors : dictionary** A dictionary with integer keys that correspond to the `succession_diagram.digraph` nodes. The dictionary values are lists of `Attractor.Attractor` objects that correspond to attractors that exist in the region of statespace corresponding to the reduced network represented by the key in the succession diagram.

**fewest\_attractors : int** A lower bound on the number of attractors in the model.

**most\_attractors** : **int** An upper bound on the number of attractors in the model.

**primes** : **PyBoolNet primes dictionary** The model rules.

**succession\_digraph** : **networkx digraph** Networkx digraph representation of the succession\_diagram object. If `AttractorRepertoire.simplify_diagram`, it is equivalent to `AttractorRepertoire.succession_digraph`. Otherwise, several of its nodes may be contracted (depending on input parameters).

**attractor\_equivalence\_classes** : **list** List of attractor equivalence classes. Each item is a dictionary with keys 'states', 'attractors', and 'reductions'. The 'states' value is a dictionary of variable values that all attractors in the class share. The 'attractors' value is a list of `Attractor` objects (i.e., a sublist of `self.attractors`); all attractors in this list have all relevant nodes equivalently characterized. The 'reductions' value is a list of `reduction_attractor` keys that collectively contain all the attractors in the class (and therefore cannot differ in any relevant node).

**relevant\_nodes** : **list** List of nodes that are "relevant", i.e., if trap spaces differ in the values of these variables, then the corresponding succession diagram nodes and attractors will not be merged.

## Static methods

Method `from_primes`

```
def from_primes(
    primes,
    max_simulate_size=20,
    max_stable_motifs=10000
)
```

Build the succession diagram and attractor repertoire from PyBoolNet formatted update rules rules.

Parameters

**primes** : **PyBoolNet primes dictionary** The model rules.

**max\_simulate\_size** : **int** Maximum number of variables for which to brute-force build a state transition graph (the default is 20).

**max\_stable\_motifs** : **int** Maximum number of output lines for PyBoolNet to process from the `AspSolver` (the default is 10000).

Returns

**AttractorRepertoire** `AttractorRepertoire` object for the input primes.

Method `from_succession_diagram`

```
def from_succession_diagram(
    succession_diagram
)
```

Build the succession diagram and attractor repertoire from a precomputed succession diagram.

Parameters

**succession\_diagram** : **Succession.SuccessionDiagram** Succession diagram summarizing the stable motif structure of the model.

Returns

**AttractorRepertoire** `AttractorRepertoire` object for the input succession diagram.

## Methods

### Method `analyze_system`

```
def analyze_system(  
    self,  
    primes,  
    max_simulate_size=20,  
    max_stable_motifs=10000  
)
```

Build and process the succession diagram for the model.

Parameters

**primes : PyBoolNet primes dictionary** The model rules.

**max\_simulate\_size : int** Maximum number of variables for which to brute-force build a state transition graph (the default is 20).

**max\_stable\_motifs : int** Maximum number of output lines for PyBoolNet to process from the AspSolver (the default is 10000).

### Method `simplify_diagram`

```
def simplify_diagram(  
    self,  
    projection_nodes,  
    merge_equivalent_reductions=True,  
    keep_only_projection_nodes=False,  
    condense_simple_paths=False  
)
```

Simplify the succession diagram for the model. This is done in two ways. First, variables can be designated ignorable using the `projection_nodes` parameter. If `keep_only_projection_nodes` is False, these variables are ignorable, otherwise, all other nodes are ignorable. When `merge_equivalent_reductions` is True, all nodes of the succession diagram that correspond to trap spaces whose fixed variables differ only in ignorable variables are contracted (in the graph theory sense). After this process, if `condense_simple_paths` is True, then all succession diagram nodes with in-degree equal to one are contracted with their parent node. This function constructs the `succession_digraph` and `attractor_equivalence_classes` attributes, which are described in the class documentation.

Parameters

**projection\_nodes : list of variable names** These nodes will be ignored if `keep_only_projection_nodes` is False (default); otherwise, all nodes except these will be ignored.

**merge\_equivalent\_reductions : bool** Whether to contract succession diagram nodes whose reductions differ only in ignorable nodes.

**keep\_only\_projection\_nodes : bool** Whether `projection_nodes` specifies non-ignorable nodes.

**condense\_simple\_paths : bool** Whether to contract nodes with in-degree one.

### Method `summary`

```
def summary(  
    self  
)
```

Prints a summary of the attractors to standard output.

## Module StableMotifs.DomainOfInfluence

### Functions

#### Function GRASP

```
def GRASP(
    target,
    primes,
    GRASP_iterations,
    forbidden=None,
    GRASP_scores=<function GRASP_default_scores>
)
```

Search for drivers of target in primes using the method of Yang et al. 2018.

#### Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

**GRASP\_iterations : int** The number of times to run the GRASP method.

**forbidden : set of str variable names** Variables to be considered uncontrollable (the default is None).

**GRASP\_scores : function** Function to score candidates (the default is GRASP\_default\_scores; see that function for required inputs and outputs of the scoring function).

#### Returns

**solutions : list of partial state dictionaries** Each partial set dictionary represents a driver set whose LDOI contains the target.

#### Function GRASP\_default\_scores

```
def GRASP_default_scores(
    target,
    primes,
    candidates
)
```

Helper function for GRASP driver search. Scores candidate driver nodes.

#### Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

**candidates : list of partial state dictionaries** List of variable states that can potentially lead to the target.

#### Returns

**scores : list of ints** Logical domain of influence sizes for individual node states. If the node leads to a contradiction, the score will become the negative of the largest LDOI size. Scores are ordered in the same order as the candidates list.

#### Function all\_drivers\_of\_size

```
def all_drivers_of_size(
    driver_set_size,
    target,
    primes,
    external_search_vars=None,
```

```

        internal_search_vars=None
    )

```

Short summary.

Parameters

**driver\_set\_size : int** The number of driver nodes to try to find.

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

**external\_search\_vars : set of str variable names** Node set not in target to consider as potential drivers. If None, then all nodes not fixed in target (the default is None).

**internal\_search\_vars : set of str variable names** Node set in target to consider as potential drivers. If None, all nodes in partial state (the default is None).

Returns

**driver\_sets : list of partial state dictionaries** Each state dictionary in the list drives target.

**Function** `construct_GRASP_solution`

```

def construct_GRASP_solution(
    target,
    primes,
    candidates,
    scores
)

```

Helper function for GRASP driver search. Constructs individual driver set using the GRASP search method.

Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

**candidates : list of partial state dictionaries** List of variable states that can potentially lead to the target.

**scores : list of ints** Logical domain of influence sizes for individual node states. If the node leads to a contradiction, the score will become the negative of the largest LDOI size. Scores are ordered in the same order as the candidates list.

Returns

**partial state dictionary** A partial state that contains the target in its LDOI. If no such partial state is found, returns an empty dictionary instead.

**Function** `fixed_excludes_implicant`

```

def fixed_excludes_implicant(
    fixed,
    implicant
)

```

Returns True if and only if the (possibly partial) state “fixed” contradicts the implicant.

Parameters

**fixed : partial state dictionary** State (or partial state) representing fixed variable states.

**implicant : partial state dictionary** State (or partial state) representing the target implicant.

Returns



**bool** True if and only if the implicant contradicts the logical domain of influence of the fixed (partial) state.

**Function** `fixed_implies_implicant`

```
def fixed_implies_implicant(  
    fixed,  
    implicant  
)
```

Returns True if and only if the (possibly partial) state “fixed” implies the implicant.

Parameters

**fixed : partial state dictionary** State (or partial state) representing fixed variable states.  
**implicant : partial state dictionary** State (or partial state) representing the target implicant.

Returns

**bool** True if and only if the implicant is in the logical domain of influence of the fixed (partial) state.

**Function** `initial_GRASP_candidates`

```
def initial_GRASP_candidates(  
    target,  
    primes,  
    forbidden  
)
```

Helper function for GRASP driver search. Constructs initial candidates for driver nodes.

Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.  
**primes : PyBoolNet primes dictionary** Update rules.  
**forbidden : set of str variable names** Variables to be considered uncontrollable (the default is None).

Returns

**candidates : list of partial state dictionaries** List of variable states that can potentially lead to the target.

**Function** `internal_drivers`

```
def internal_drivers(  
    target,  
    primes,  
    max_drivers=None  
)
```

Find internal (logical) driver nodes of target through brute-force.

Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.  
**primes : PyBoolNet primes dictionary** Update rules.  
**max\_drivers : int** Maximum size of driver set to consider. If None, is set to the size of the partial state (as this is always sufficient to achieve the target) (the default is None).

Returns

**driver\_sets : list of partial state dictionaries** Each state dictionary in the list drives target. These are sorted by length (smallest first).

#### **Function** knock\_to\_partial\_state

```
def knock_to_partial_state(
    target,
    primes,
    min_drivers=1,
    max_drivers=None,
    forbidden=None
)
```

Find all partial states in primes that drive the target. Do not consider nodes in the forbidden list.

#### Parameters

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

**min\_drivers : int** Minimum size of driver set to consider. (the default is 1).

**max\_drivers : int** Maximum size of driver set to consider. If None, is set to the size of the partial state (as this is always sufficient to achieve the target) (the default is None).

**forbidden : set of str variable names** Variables to be considered uncontrollable (the default is None).

#### Returns

**knocked\_nodes : list of partial state dictionaries** Each state dictionary in the list drives target. Supersets of previously considered dictionaries are excluded.

#### **Function** local\_GRASP\_reduction

```
def local_GRASP_reduction(
    solution,
    target,
    primes
)
```

A helper function for GRASP driver search. Reduces valid solutions to attempt to remove redundancies.

#### Parameters

**solution : partial state dictionary** Solution to be reduced; must contain the target in its LDOI.

**target : partial state dictionary** PyBoolNet implicant that defines target fixed node states.

**primes : PyBoolNet primes dictionary** Update rules.

#### Returns

**partial state dictionary** Reduced solution that also contains target in its LDOI.

#### **Function** logical\_domain\_of\_influence

```
def logical_domain_of_influence(
    partial_state,
    primes,
    implied_hint=None,
    contradicted_hint=None
)
```

Computes the logical domain of influence (LDOI) (see Yang et al. 2018)

Parameters

**partial\_state** : **partial state dictionary** PyBoolNet implicant that defines fixed nodes.  
**primes** : **PyBoolNet primes dictionary** Update rules.  
**implied\_hint** : **partial state dictionary** Known subset of the LDOI; used during optimization.  
**contradicted\_hint** : **partial state dictionary** Known subset of the contradiction boundary; used during optimization.

Returns

**implied** : **partial state dictionary** The logical domain of influence.  
**contradicted** : **partial state dictionary** The contradiction boundary.

**Function** `minimal_drivers`

```
def minimal_drivers(  
    target,  
    primes,  
    max_drivers=None  
)
```

Finds smallest set(s) of (logical) driver nodes of target through brute-force. Unlike `minimal_drivers`, we are not limited to internal drivers nodes.

Parameters

**target** : **partial state dictionary** PyBoolNet implicant that defines target fixed node states.  
**primes** : **PyBoolNet primes dictionary** Update rules.  
**max\_drivers** : **int** Maximum size of driver set to consider. If None, is set to the size of the partial state (as this is always sufficient to achieve the target) (the default is None).

Returns

**driver\_sets** : **list of partial state dictionaries** Each state dictionary in the list drives target. These are sorted by length (smallest first).

**Function** `single_drivers`

```
def single_drivers(  
    target,  
    primes  
)
```

Finds all 1-node (logical) drivers of target under the rules given by primes.

Parameters

**target** : **partial state dictionary** PyBoolNet implicant that defines target fixed node states.  
**primes** : **PyBoolNet primes dictionary** Update rules.

Returns

**list of length-1 dictionaries** Each dictionary describes a single node state that contains the target in its logical domain of influence.

## Module `StableMotifs.Export`

### Functions

#### Function `attractor_dataframe`

```
def attractor_dataframe(
    ar
)
```

Summarize the input attractor repertoire in a pandas DataFrame (requires pandas).

Parameters

**ar : AttractorRepertoire** Attractor repertoire to summarize.

Returns

**pandas.DataFrame** Summary of the attractors.

#### Function `expanded_network`

```
def expanded_network(
    primes,
    single_parent_composites=False
)
```

Produce the expanded network for given input update rules.

Parameters

**primes : PyBoolNet primes dictionary** The update rules for which to construct the expanded network.

**single\_parent\_composites : bool** Whether to insert composite nodes between virtual nodes when one is a prime implicant of the other. If False, the number of nodes is decreased; if True, then the expanded network is bipartite (the default is False).

Returns

**networkx.DiGraph** Digraph representing the expanded network. Nodes have a 'type' attribute that can be either 'virtual' or 'composite'.

#### Function `format_reduction_label`

```
def format_reduction_label(
    s
)
```

Helper function to make graph labels more readable. Removes single quotes and square brackets from the input string.

Parameters

**s : str** Input label.

Returns

**str** Label with characters ', [, and ] removed.

#### Function `networkx_succession_diagram`

```
def networkx_succession_diagram(
    ar,
    include_attractors_in_diagram=True,
    use_compressed_diagram=True
)
```

Label the succession diagram and (optionally) attractors of the input attractor repertoire according to the conventions of Rozum et al. (2021). This is an alias for the function `Export.networkx_succession_diagram_reduced_network_based`.

Parameters

**ar : AttractorRepertoire** Attractor repertoire object for which to build the diagram.  
**include\_attractors\_in\_diagram : bool** Whether attractors should be represented as nodes in the diagram (the default is True).  
**use\_compressed\_diagram : bool** Whether to use the (potentially compressed) succession diagram stored in `ar.succession_digraph` instead of the complete one `ar.succession_diagram.digraph`. These are equivalent unless `ar.simplify_diagram` is called. See `AttractorRepertoire.py` for additional details. The default is True.

Returns

**networkx.DiGraph** A labeled digraph that represents the succession diagram.

**Function** `networkx_succession_diagram_motif_based`

```
def networkx_succession_diagram_motif_based(
    ar,
    include_attractors_in_diagram=True
)
```

Label the succession diagram and (optionally) attractors of the input attractor repertoire according to the conventions of Zanudo and Albert (2015). If attractors are not included, this is the line graph of the succession diagram defined in Rozum et al. (2021). Does not support compression.

Parameters

**ar : AttractorRepertoire** Attractor repertoire object for which to build the diagram.  
**include\_attractors\_in\_diagram : bool** Whether attractors should be represented as nodes in the diagram (the default is True).

Returns

**networkx.DiGraph** A labeled digraph that represents the succession diagram.

**Function** `networkx_succession_diagram_reduced_network_based`

```
def networkx_succession_diagram_reduced_network_based(
    ar,
    include_attractors_in_diagram=True,
    use_compressed_diagram=True
)
```

Label the succession diagram and (optionally) attractors of the input attractor repertoire according to the conventions of Rozum et al. (2021).

Parameters

**ar : AttractorRepertoire** Attractor repertoire object for which to build the diagram.  
**include\_attractors\_in\_diagram : bool** Whether attractors should be represented as nodes in the diagram (the default is True).  
**use\_compressed\_diagram : bool** Whether to use the (potentially compressed) succession diagram stored in `ar.succession_digraph` instead of the complete one `ar.succession_diagram.digraph`. These are equivalent unless `ar.simplify_diagram` is called. See `AttractorRepertoire.py` for additional details. The default is True.

Returns

**networkx.DiGraph** A labeled digraph that represents the succession diagram.

### Function `plot_nx_succession_diagram`

```
def plot_nx_succession_diagram(
    G,
    pos=None,
    fig_dimensions=(None, None),
    nx_node_kwargs=None,
    nx_edge_kwargs=None,
    draw_node_labels=True,
    labeling_convention='label',
    draw_edge_labels=False,
    nx_node_label_kwargs=None,
    nx_edge_label_kwargs=None
)
```

Plot the input succession diagram. Requires matplotlib. For finer control over plot appearance, it is recommended to plot `g` directly.

#### Parameters

**G : `networkx.DiGraph`** Labeled succession diagram, e.g., as is output from `Export.networkx_succession_diagram_reduced_network_based()`.

**fig\_dimensions : (`int,int`)** Dimensions of the output figure. If (`None, None`), then the dimensions are calculated based on the number of nodes in `g` (the default is (`None, None`)).

**pos : `str` or `graphviz_layout`** Layout for the nodes; A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2. If none, we attempt to use `pydot/graphviz` to construct a layout, otherwise we fall back to the `networkx` `planar_layout` function (succession diagrams are always planar).

**draw\_node\_labels : `bool`** Whether node labels should be drawn (`True`) or left as metadata (`False`) (the default is `True`).

**draw\_edge\_labels : `bool`** Whether edge labels should be drawn (`True`) or left as metadata (`False`); only affects reduced-network-based (default) succession diagrams, not motif-based succession diagrams. (The default value is `False`.)

**labeling\_convention : `str`** Whether edge labels should be just the stable motifs (`'label'`) or all stabilized states (`'states'`) (the default is `'label'`).

**nx\_node\_kwargs : `dictionary`** Keyword arguments passed to `nx.draw_networkx_nodes` (in addition to `G` and `pos`). If `None`, we pass `{'node_size':50*G.number_of_nodes()}` by default.

**nx\_edge\_kwargs : `dictionary`** Keyword arguments passed to `nx.draw_networkx_edges` (in addition to `G` and `pos`). If `None`, we pass `{'arrowstyle':'->', 'width':2, 'arrowsize':30}` by default.

**nx\_node\_label\_kwargs : `dictionary`** Keyword arguments passed to `nx.draw_networkx_labels` (in addition to `G` and `pos`). If `None`, we pass `{'font_size':16}` by default.

**nx\_edge\_label\_kwargs : `dictionary`** Keyword arguments passed to `nx.draw_networkx_edge_labels` (in addition to `G` and `pos`). If `None`, we pass `{'font_size':16}` by default.

### Function `save_to_graphml`

```
def save_to_graphml(
    G,
    model_name
)
```

Export a labeled succession diagram to graphml format.

#### Parameters

**G : `networkx.DiGraph`** Labeled succession diagram to export.

**model\_name : `str`** Name of file to save to (.graphml extension will be appended).

## Module StableMotifs.Format

### Functions

#### Function bnet2sympy

```
def bnet2sympy(  
    rule  
)
```

Converts a BNet string expression to a sympy string expression.

Parameters

**rule** : **str** Boolean expression in BNET format.

Returns

**str** Expression in sympy format.

#### Function bnetDNF2list

```
def bnetDNF2list(  
    bnet  
)
```

Converts a BNet string expression to a list of prime implicant dictionaries. Requires that the input be in disjunctive normal form, but this is not checked explicitly.

Parameters

**bnet** : **str** BNET formatted expression in disjunctive normal form.

Returns

**list of partial state dictionaries** Variable states specified by each dictionary are to be thought of as “AND”- connected, and the dictionaries as “OR”-connected.

#### Function booleannet2bnet

```
def booleannet2bnet(  
    rules  
)
```

Converts BooleanNet rules to BNet format. e.g., an input of “A\*=B or C and not D” returns A, B | C & !D

Also replaces ~ with !

Parameters

**rules** : **str** BooleanNet formatted rules.

Returns

**str** BNET formatted rules.

#### Function build\_rule\_using\_bnetDNFs

```
def build_rule_using_bnetDNFs(  
    expr0,  
    expr1  
)
```

Converts a BNet string expression (expr1) and its negation (expr0) to a PyBoolNet rule list. Note that this function does not test for consistency between expr0 and expr1.

Parameters

expr0 : **str** Rule, in BNET format, for the “OFF” state of a variable.

expr1 : **str** Rule, in BNET format, for the “ON” state of a variable.

Returns

**PyBoolNet rule list** The complementary expressions as they would appear in a PyBoolNet primes dictionary for a variable whose update rule is given by expr1.

**Function** `implicant2bnet`

```
def implicant2bnet(
    partial_state
)
```

Converts a partial state dictionary to a BNet string e.g., {'A':1,'B':0} returns 'A & !B'

Parameters

partial\_state : **partial state dictionary** Partial state to convert.

Returns

**str** BNET representation of the partial state.

**Function** `import_primes`

```
def import_primes(
    fname,
    format='BooleanNet',
    remove_constants=False
)
```

Import boolean rules from file and return PyBoolNet formatted primes list.

Parameters

fname : **str** Path to (plaintext) file containing Boolean rules in format specified by the 'format' option.

format : **str** Boolean rule format; options are 'BooleanNet' or 'BNet' (the default is 'BooleanNet').

remove\_constants : **bool** If True, variables that are constant are removed and their influence is percolated. Otherwise, they remain and we consider initial conditions in opposition to their values (the default is False).

Returns

**PyBoolNet primes dictionary** Update rules in PyBoolNet format.

**Function** `longbnet2primes`

```
def longbnet2primes(
    BNET,
    remove_constants=False
)
```

A modified version of PyBoolNet's `bnet2primes` that does not do path-checking, as this can cause errors if the bnet rules are very long. Assumes BNET is a bnet string, not a file.

Parameters

BNET : **str** BNET formatted rules to convert.



**remove\_constants : bool** Whether or not to remove and percolate constant input values (the default is False).

Returns

**primes : PyBoolNet primes dictionary** Update rules in PyBoolNet format.

**Function** `pretty_print_prime_rules`

```
def pretty_print_prime_rules(  
    primes  
)
```

Prints PyBoolNet a prime dictionary as Boolean rules The output format is of the form:  $A^* = B \ \& \ C \mid !D$ , for example.

Parameters

**primes : PyBoolNet primes dictionary** Update rules to print.

**Function** `pretty_print_primes`

```
def pretty_print_primes(  
    primes  
)
```

Prints PyBoolNet a prime dictionary in a more readable format. Prints both state updates (1 and 0).

Parameters

**primes : PyBoolNet primes dictionary** Update rules to print.

**Function** `pretty_print_rspace`

```
def pretty_print_rspace(  
    L,  
    simplify=True,  
    silent=True  
)
```

Produces string representation of the Boolean rule describing the input rspace L (see `RestrictSpace.rspace`).

Parameters

**L : rspace list** Restrict space list (see `RestrictSpace.rspace` for details).

**simplify : bool** Whether to simplify the rule (the default is True).

**silent : bool** Whether to suppress output of the rule (the default is True).

Returns

**str** BNET expression that is true in and only in the rspace specified by L.

**Function** `primes2bnet`

```
def primes2bnet(  
    primes  
)
```

A simpler version of PyBoolNet's `FileExchange.primes2bnet` function with fewer options and less organized output. Should handle prime rules with tautologies better than the PyBoolNet version though.

Parameters

**primes : PyBoolNet primes dictionary** Update rules to convert.

Returns

**str** BNET representation of update rules.

#### Function primes2booleannet

```
def primes2booleannet(  
    primes,  
    header=''  
)
```

Convert a PyBoolNet primes dictionary to a BooleanNet string representation.

Parameters

**primes : PyBoolNet primes dictionary** Update rules to convert.

**header : str** Text to include at the beginning of the file, e.g., comment lines. For example, the legacy Java version of StableMotifs requires rules files to begin with the line “#BOOLEAN RULES”.

Returns

**str** BooleanNet representation of update rules.

#### Function remove\_comment\_lines

```
def remove_comment_lines(  
    stream,  
    comment_char='#'  
)
```

Removes commented out lines from stream, e.g., those starting with ‘#’.

Parameters

**stream : iterable of str** Lines from which comments should be excluded.

**comment\_char : str** Lines beginning with this character will be excluded.

Returns

**list of str** Lines that do not begin with comment\_char.

#### Function rule2bnet

```
def rule2bnet(  
    rule  
)
```

Converts a PyBoolNet prime rule into a BNet string. e.g., [{‘A’:1,‘B’:0},{‘C’:0}] returns ‘A & !B | !C’

Parameters

**rule : list of PyBoolNet partial states** Update rule to convert.

Returns

**str** BNET representation of Boolean expression.

#### Function statedict2str

```
def statedict2str(  
    statedict  
)
```

Converts a state dictionary to a statestring using alphabetical sorting.

Parameters

**statedict** : **partial state dictionary** State to convert to a binary string representation.

Returns

**str** A binary string, with each position corresponding to the variable name at the same position in the alphabetized keys in statedict.

**Function** statelist2dict

```
def statelist2dict(
    names,
    statestrings
)
```

Converts a collection of statestrings to a dictionary.

Parameters

**names** : **list of str** An ordered list of variable names; (alphabetical order is PyBoolNet's default, e.g. sorted(primes)).

**c** : **iterable of str** Each element should be a binary string, with each position corresponding to the variable name at the same position in names.

Returns

**dictionary** Dictionary summarizing c. If a node takes the same value in every state, the corresponding dictionary value matches its fixed value; otherwise, the dictionary value is 'X'.

**Function** statestring2dict

```
def statestring2dict(
    statestring,
    names
)
```

Converts a state string, which specifies a node in an STG, to the corresponding dictionary representation.

Parameters

**statestring** : **str** A binary string, e.g., '01101'.

**names** : **list of str** An ordered list of variable names; (alphabetical order is PyBoolNet's default, e.g. sorted(primes)).

Returns

**dictionary** The keys are the elements of names and the values are the corresponding value in statestring.

**Function** sympy2bnet

```
def sympy2bnet(
    rule
)
```

Converts a sympy string expression to a BNET string expression.

Parameters

**rule** : **str** Boolean expression in sympy format.

Returns

**str** Expression in BNET format.

## Module `StableMotifs.RandomBooleanNetworks`

### Functions

#### Function `Binary_Rule_From_Decimal`

```
def Binary_Rule_From_Decimal(  
    node_rule_decimal,  
    node_input_list  
)
```

Convert single decimal rule to its binary form.

Parameters

**node\_rule\_decimal : int** Decimal form of a truth table's output column.

**node\_input\_list : list of str** Variable names that correspond to each column of the truth table.

Returns

**list of int** Binary rule list corresponding to an output column of a truth table.

#### Function `Binary_Rules_From_Decimal`

```
def Binary_Rules_From_Decimal(  
    node_rules_decimal_dictionary  
)
```

Construct Binary format rules from decimal format rules.

Parameters

**node\_rules\_decimal\_dictionary : dictionary** Rules in decimal format to convert.

Returns

**dictionary** Binary rules dictionary.

#### Function `Random_Boolean_Network_Ensemble_Kauffman`

```
def Random_Boolean_Network_Ensemble_Kauffman(  
    N,  
    K,  
    p,  
    N_ensemble,  
    seed=1000,  
    write_Boolean_network=False  
)
```

Generate a sample from the Kauffman NK RBN ensemble.

Parameters

**N : int** Number of nodes of RBN.

**K : int** Number of inputs of each node in the RBN.

**p : float** Probability that each entry in each truth table output column is equal to 1.

**N\_ensemble : int** Number of networks to generate.

**seed : int** Random seed for generating the RBN ensemble (the default is 1000).

**write\_Boolean\_network : bool** Whether to write each network in the ensemble as a CSV file in a new directory (the default is False).

Returns

**RBN\_ensemble\_rules : list of str** Each string are the Boolea rules of an ensemble in boolean-net format. Each element in **RBN\_ensemble\_rules** can be used as an input for the **Format.booleannet2bnet** function.

#### **Function** String\_Rule\_From\_Binary

```
def String_Rule_From_Binary(  
    node_rule_binary,  
    node_input_list  
)
```

Convert binary rule to BooleanNet format.

Parameters

**node\_rule\_binary : list of int** Binary rule list corresponding to an output column of a truth table.

**node\_input\_list : list of str** Variable names that correspond to each column of the truth table.

Returns

**str** BooleanNet representation of rule.

#### **Function** String\_Rules\_From\_Binary

```
def String_Rules_From_Binary(  
    node_rules_binary_dictionary  
)
```

Convert from binary dictionary rule format to BooleanNet format.

Parameters

**node\_rules\_binary\_dictionary : dictionary** Binary dictionary representation of rules.

Returns

**str** BooleanNet representation of rules.

#### **Function** get\_criticality\_K\_Kauffman

```
def get_criticality_K_Kauffman(  
    p  
)
```

The Kauffman RBN is at criticality when  $K = 2/(p(1-p))$ .

Parameters

**p : float** Probability that each entry in each truth table output column is equal to 1.

Returns

**K\_criticality : int** Number of inputs of each node in the RBN.

#### **Function** get\_criticality\_p\_Kauffman

```
def get_criticality_p_Kauffman(  
    K  
)
```

The Kauffman RBN is at criticality when  $K = 2/(p(1-p))$ .

Parameters

**K : int** Number of inputs of each node in the RBN.

Returns

**p\_criticality : float** Probability that each entry in each truth table output column is equal to 1.

### **Function** read\_Boolean\_network\_decimal

```
def read_Boolean_network_decimal(
    filename
)
```

Imports rules from csv in decimal format.

Parameters

**filename : str** Path to csv from which to import decimal-formatted rules.

Returns

**str** Rules in BooleanNet format.

### **Function** write\_Boolean\_network\_decimal

```
def write_Boolean_network_decimal(
    node_rules_decimal_dictionary,
    filename
)
```

Write the decimal format of the Boolean rules to file.

Parameters

**node\_rules\_decimal\_dictionary : dictionary** Update rule truth table in decimal format.

**filename : str** Path to file for csv output of the truth table.

## **Classes**

### **Class** RandomBooleanNetworks

```
class RandomBooleanNetworks
```

Generator of random Boolean networks (RBN) and ensembles of RBN. The RandomBooleanNetworks class object is a Boolean model and stores information of how the Boolean model was generated. It has functions that generate ensembles of RBN by generating multiple RandomBooleanNetworks objects.

### **Attributes**

**node\_names : list of str** List of variable names.

**node\_inputs\_dictionary : dictionary** Each value is a (fixed order) list of the names of the nodes whose values are inputs into the key variable's update function.

**node\_rules\_binary\_dictionary : dictionary** Each value is a list of outputs for the key variable's update function, stored as a list in ascending order of the numerical representation of the input row.

**node\_rules\_decimal\_dictionary : dictionary** Decimal conversion of node\_rules\_binary\_dictionary.

**node\_rules\_string\_dictionary : dictionary** BooleanNet (str) conversion of node\_rules\_binary\_dictionary values.

**node\_rules\_string : str** BooleanNet representation of update rules.

**random\_Boolean\_type : str** Description of generative process. Currently only "Kauffman NK" is implemented.

**N : int** Number of nodes in the Boolean network.

**random\_Boolean\_Network\_parameters** : **list** For Kauffman NK generation -  $[K,p]$ , where  $K$  is the in-degree and  $p$  is the bias.  $K$  is a positive integer less than or equal to  $N$ , and  $p$  is a float between 0 and 1 (inclusive).  
**random\_seed** : **int** Seed for random functions.  
**filename** : **str** Path to file where network data are stored. If None, no files are written.

## Methods

### Method Random\_Boolean\_Network

```
def Random_Boolean_Network(
    self,
    random_Boolean_type,
    N,
    rbn_parameters,
    seed=None,
    filename=None
)
```

Construct network using specified generative process.

Parameters

**random\_Boolean\_type** : **str** Description of generative process. Currently only “Kauffman NK” is implemented.  
**N** : **int** Number of nodes in the Boolean network.  
**random\_Boolean\_Network\_parameters** : **list** For Kauffman NK generation -  $[K,p]$ , where  $K$  is the in-degree and  $p$  is the bias.  $K$  is a positive integer less than or equal to  $N$ , and  $p$  is a float between 0 and 1 (inclusive).  
**random\_seed** : **int** Seed for random functions.  
**filename** : **str** Path to file where network data are stored. If None, no files are written.

### Method Random\_Boolean\_Network\_Rules

```
def Random_Boolean_Network_Rules(
    self
)
```

Generate various conversions of the node\_rules\_binary\_dictionary attribute.

## Module StableMotifs.Reduction

### Functions

#### Function delete\_node

```
def delete_node(
    primes,
    node
)
```

Reduces Boolean rules given by primes by deleting the variable specified by node. The deleted node may not appear in its own update function. Any update rules depending on the deleted node will have that dependence replaced by the update function of the deleted node. The rules are simplified after node deletion.

Parameters

**primes** : **PyBoolNet primes dictionary** Update rules.  
**node** : **str** Name of the node to delete.

Returns

**new\_primes : PyBoolNet primes dictionary** The reduced primes.  
**constants : partial state dictionary** Node states that became logically fixed during simplification.

#### Function deletion\_reduction

```
def deletion_reduction(  
    primes,  
    max_in_degree=inf  
)
```

Implements the reduction method of Veliz-Cuba (2011). Deletion order is such that nodes with low in-degree are prioritized for removal. Deletion proceeds until all remaining nodes have self-loops.

Parameters

**primes : PyBoolNet primes dictionary** Update rules.  
**max\_in\_degree : int or float** Will not try to delete nodes with in-degree larger than this. Deleting nodes with large in-degree can be computationally expensive (the default is float('inf')).

Returns

**reduced : PyBoolNet primes dictionary** The reduced primes.  
**constants : partial state dictionary** Node states that became logically fixed during reduction.

#### Function mediator\_reduction

```
def mediator_reduction(  
    primes  
)
```

Network reduction method of Saadatpour, Albert, Reluga (2013) Preserves fixed points. Number of complex attractors is often, but not always conserved (despite initial claims). Can be viewed as a more restrictive version of the deletion reduction method of Veliz-Cuba (2011).

Parameters

**primes : PyBoolNet primes dictionary** Update rules.

Returns

**reduced : PyBoolNet primes dictionary** The reduced primes.  
**constants : partial state dictionary** Node states that became logically fixed during reduction.

#### Function reduce\_primes

```
def reduce_primes(  
    fixed,  
    primes  
)
```

Simplifies boolean rules when some nodes are held fixed

Parameters

**fixed : partial state dictionary** Node states to be held fixed.  
**primes : PyBoolNet primes dictionary** Update rules.



Returns

**reduced\_primes : PyBoolNet primes dictionary** Simplified update rules  
**percolated\_states : partial state dictionary** Fixed node states (including inputs) that were simplified and removed.

**Function** `remove_outdag`

```
def remove_outdag(  
    primes  
)
```

Removes the terminal directed acyclic part of the regulatory network. This part of the network does not influence the attractor repertoire.

Parameters

**primes : PyBoolNet primes dictionary** Update rules.

Returns

**reduced : PyBoolNet primes dictionary** The reduced primes.  
**constants : partial state dictionary** Node states that became logically fixed during reduction.

**Function** `simplify_primes`

```
def simplify_primes(  
    primes  
)
```

Simplifies PyBoolNet primes (e.g.,  $A \mid A \ \& \ B$  becomes  $A$ )

Parameters

**primes : PyBoolNet primes dictionary** Rules to simplify.

Returns

**PyBoolNet primes dictionary** Simplified rules.

**Function** `simplify_using_expression_and_negation`

```
def simplify_using_expression_and_negation(  
    node,  
    expr0,  
    expr1,  
    bnet  
)
```

Simplify the expression `bnet` by substituting the value for `node` given by `node = expr1 = !expr0` (does not check that `expr1 != expr0`)

Parameters

**node : str** Name of node to substitute  
**expr0 : str** Expression to substitute for `!node`  
**expr1 : str** Expression to substitute for `node`  
**bnet : str** BNET expression in which to perform the substitutions.

Returns

**str** Simplified BNET expression after substitutions are performed.

## Classes

### Class MotifReduction

```
class MotifReduction(  
    motif_history,  
    fixed,  
    reduced_primes,  
    max_simulate_size=20,  
    prioritize_source_motifs=True,  
    max_stable_motifs=10000  
)
```

Class to generate and store data about a network reduction that arises during the stable motif succession diagram construction algorithm.

### Parameters

**motif\_history** : **list of partial state dictionaries** Stable motifs that can lock in to give the reduced network (in order).  
**fixed** : **partial state dictionary** Nodes values that have been fixed and reduced by stable motifs and their logical domain of influence.  
**reduced\_primes** : **PyBoolNet primes dictionary** Update rules for the reduced network.  
**max\_simulate\_size** : **int** Maximum number of variables for which to brute-force build a state transition graph (the default is 20).  
**prioritize\_source\_motifs** : **bool** Whether source nodes should be considered first (the default is True).  
**max\_stable\_motifs** : **int** Maximum number of output lines for PyBoolNet to process from the AspSolver (the default is 10000).

### Attributes

**merged\_history\_permutations** : **list of lists of int** Permutations of motif\_history (by index) that also yield this reduction.  
**logically\_fixed\_nodes** : **partial state dictionary** Nodes values that have been fixed and reduced by stable motifs and their logical domain of influence.  
**time\_reverse\_primes** : **PyBoolNet primes dictionary** Update rules of the time reversed reduced system.  
**stable\_motifs** : **list of partial state dictionaries** Stable motifs of the reduced system.  
**time\_reverse\_stable\_motifs** : **list of partial state dictionaries** Stable motifs of the time reversed system.  
**merged\_source\_motifs** : **list of partial state dictionaries** List of source-like stable motifs that have been merged into a single motif to avoid redundancy.  
**source\_independent\_motifs** : **list of partial state dictionaries** Stable motifs that exist independent of the values of the source nodes  
**merge\_source\_motifs** : **list of partial state dictionaries** Stable motifs generated by merging the stable motifs corresponding to source nodes.  
**rspace** : **rspace list** The rspace, or “restrict space” of the reduced network, describing a necessary condition for the system to avoid activating additional stable motifs (see RestrictSpace.py for further details).  
**motif\_history** : **list of partial state dictionaries** Stable motifs that can lock in to give the reduced network (in order)  
**reduced\_primes** : **PyBoolNet primes dictionary** Update rules for the reduced network.  
**fixed\_rspace\_nodes** : **partial state dictionary** Nodes values that are fixed in the rspace.  
**rspace\_constraint** : **str** BNET expression that is true in and only in the rspace.  
**reduced\_rspace\_constraint** : **str** S simplification of the rspace\_constraint given the fixed\_rspace\_nodes states are satisfied  
**rspace\_update\_primes** : **PyBoolNet primes dictionary** The update rules obtained from simplifying under the assumption that the fixed\_rspace\_nodes are fixed

**conserved\_functions** : **list of PyBoolNet expressions** Boolean functions that are constant within every attractor, in PyBoolNet update rule format

**rspace\_attractor\_candidates** : **list of str** Attractors (lists of statestrings) in the rspace\_update\_primes that satisfy the reduced\_rspace\_constraint

**partial\_STG** : **networkx.DiGraph** Subgraph of the state transition graph of the reduced network that contains any and all attractors that do not lie in any of the reduced network's stable motifs.

**no\_motif\_attractors** : **list of str** Complex attractors that do not "lock in" any additional stable motifs, stored as collections of state strings.

**attractor\_dict\_list** : **list of dictionaries** Dictionaries corresponding to attractors that are in this reductions, but not in any of its subreductions (if it has any). Each describes the node states in the attractors according to the following 1 variable is "ON" 0 variable is "OFF" X variable is known to oscillate ? at least one such variable must oscillate ! the attractor may be false; if it is genuine, at least one such variable must oscillate

**terminal** : **str** One of "yes", "no", or "possible", indicating whether the reduction contains attractors that are not in any of its subreductions.

**delprimes** : **PyBoolNet prime dictionary** Update rules for the system's deletion projection. Steady states and stable motif activation are preserved. These rules may yield additional, spurious complex attractors.

**deletion\_STG** : **networkx.DiGraph** Portion of the deletion projection's STG that contains all motif-avoidant attractors.

**deletion\_no\_motif\_attractors** : **list of str** Motif avoidant attractors of the deletion projection. The number of these is an upper bound on the number of motif avoidant attractors in the reduction.

## Methods

### Method build\_K0

```
def build_K0(
    self
)
```

Helper function for smart STG building. Builds initial set of nodes that are not part of any motif-avoidant attractor.

Returns

**set of str** Statestrings that do not belong to any motif-avoidant attractor.

### Method build\_deletion\_STG

```
def build_deletion_STG(
    self,
    max_stable_motifs=10000
)
```

Build a piece of the STG that is guaranteed to contain all motif-avoidant attractors of the deletion projection. Complex attractors found here may be spurious.

Parameters

**max\_stable\_motifs** : **int** Maximum number of output lines for PyBoolNet to process from the AspSolver (the default is 10000).

### Method build\_inspace

```
def build_inspace(
    self,
    ss,
```

```

        names,
        tr_stable_motifs=None
    )

```

Helper function for smart STG building. List all time reversal stable motifs to which (partial) state *ss* belongs.

Parameters

**ss** : **str** Statestring (possibly on a subspace).  
**names** : **list of str** Variable names ordered to correspond to the positions of *ss*.  
**tr\_stable\_motifs** : **list of partial state dictionaries** Time reverse stable motifs. If None, use all time reverse stable motifs in the reduced system (the default is None).

Returns

**list of partial state dictionaries** Time reverse stable motifs that are active in the state *ss*.

**Method** `build_partial_STG`

```

def build_partial_STG(
    self
)

```

Build a piece of the STG that is guaranteed to contain all motif-avoidant attractors of the reduction.

**Method** `find_constants_in_complex_attractor`

```

def find_constants_in_complex_attractor(
    self,
    c
)

```

Given a set of strings representing the states of a complex attractor the function finds the nodes that are constant in the full complex attractor.

Parameters

**c** : **a set of binary strings** Set of statestrings, e.g. `set(['000', '010', '100'])`.

Returns

**list of str** An array consisting of 0s, 1s, and Xs. X represents an oscillating node, and the 0s and 1s represent nodes stabilized to those states.

**Method** `find_deletion_no_motif_attractors`

```

def find_deletion_no_motif_attractors(
    self,
    max_stable_motifs=10000
)

```

Identify motif-avoidant attractors in the deletion projection.

Parameters

**max\_stable\_motifs** : **int** Maximum number of output lines for PyBoolNet to process from the AspSolver (the default is 10000).

**Method** find\_no\_motif\_attractors

```
def find_no_motif_attractors(
    self
)
```

Find attractors of the reduction that are not present in any of its subreductions.

**Method** generate\_attr\_dict

```
def generate_attr_dict(
    self
)
```

Generate a list of attractors that are present in the reduction, but not in any of its subreductions.

Returns

**list of dictionaries** Dictionaries corresponding to attractors that are in this reductions, but not in any of its subreductions (if it has any). Each describes the node states in the attractors according to the following 1 variable is “ON” 0 variable is “OFF” X variable is known to oscillate ? at least one such variable must oscillate ! the attractor may be false; if it is genuine, at least one such variable must oscillate

**Method** in\_motif

```
def in_motif(
    self,
    ss,
    names
)
```

Tests whether the (partial) state ss is in any stable motifs

Parameters

**ss : str** Statestring (possibly on a subspace).

**names : list of str** Variable names ordered to correspond to the positions of ss.

Returns

**bool** Whether ss is in any stable motif of the reduced system.

**Method** merge\_source\_motifs

```
def merge_source_motifs(
    self
)
```

Merges stable motifs (and time-reversal stable motifs) that correspond to source nodes, e.g.  $A=A$ , into combined motifs to avoid combinatorial explosion. For example,  $A=A$ ,  $B=B$ ,  $C=C$  produces six motifs that can stabilize in 8 ways; without merging, these 8 combinations lead to  $8 \cdot 3! = 48$  successions because they can be considered in any order. This is silly because source nodes all stabilize simultaneously.

We will assume that stable motifs and time reverse stable motifs have already been computed.

Note that a source node in the forward time system is a source node in the time reverse system as well. This follows from  $A^* = A \Rightarrow A^- = \sim(A^*(A=\sim A)) = \neg A = A$ .

If  $A^* = A$  or  $X$  (i.e.,  $A=1$  is a stable motif), then  $A^- = \neg(A \mid X) = A \& \sim X$ , so  $A=0$  is a time-reverse stable motif. A similar argument applies for the  $A=0$  stable motif. Thus, a motif is only a source motif if it is also a time-reverse motif.

### Method summary

```
def summary(  
    self,  
    show_original_rules=True,  
    hide_rules=False,  
    show_explicit_permutations=False  
)
```

Print a summary of the reduction.

Parameters

**show\_original\_rules : bool** Show rules of the unreduced system (the default is True)?  
**hide\_rules : bool** Hide rules of the reduced system (the default is False)?  
**show\_explicit\_permutations : bool** Show motif permutations explicitly, instead of by index (the default is False)?

## Module StableMotifs.RestrictSpace

### Functions

#### Function attractor\_space\_candidates

```
def attractor_space_candidates(  
    maxts,  
    trmaxts  
)
```

Merge the maximum trap spaces maxts and time-reverse maximum trap spaces to obtain a list of attractor-conserved quantities. Note that any Boolean function of these is also conserved in attractors.

Parameters

**maxts : list of partial state dictionaries** Stable motifs, i.e., maximum trap spaces for the system.  
**trmaxts : list of partial state dictionaries** Stable motifs, i.e., maximum trap spaces for the time-reversed system.

Returns

**rspace() list** Restrict space list (see RestrictSpace.rspace for details).

#### Function fixed\_rspace\_nodes

```
def fixed_rspace_nodes(  
    L,  
    primes  
)
```

Finds the nodes that must have a fixed value in order for the rspace constraint L to be satisfied in the system given by primes.

Parameters

**L : rspace() list** Restrict space list (see RestrictSpace.rspace for details).  
**primes : PyBoolNet primes dictionary** Update rule for the system.

Returns

**dictionary** Nodes that are fixed everywhere in the rspace L. Returns {'0':1} if L is a self-contradictory.

**Function** `partial_state_contradicts_rspace`

```
def partial_state_contradicts_rspace(
    state,
    L
)
```

Tests to see if state lies entirely outside the rspace L.

Parameters

**state** : **partial state dictionary** State, or partial state to test.

**L** : **rspace()** **list** Restrict space list (see `RestrictSpace.rspace` for details).

Returns

**bool** True if and only if state is not in L.

**Function** `reduce_rspace`

```
def reduce_rspace(
    L,
    primes
)
```

Reduce the rspace L for the system given by primes so that trivially fixed nodes are factored out. The first element of the returned rspace (L2) will specify these trivially fixed nodes (i.e., they are factored on the left).

Parameters

**L** : **rspace()** **list** Restrict space list (see `RestrictSpace.rspace` for details).

**primes** : **PyBoolNet primes dictionary** Update rule for the system.

Returns

**L2** : **rspace()** **list** Reduced restrict space list (see `RestrictSpace.rspace` for details).

**Function** `reduce_rspace_string`

```
def reduce_rspace_string(
    s,
    fd,
    simplify=True
)
```

Replaces variables in the string s with the fixed values given by the dictionary fd.

Parameters

**s** : **str** Boolean expression in BNET format.

**fd** : **partial state dictionary** Node values that are to be considered fixed.

**simplify** : **bool** Whether to simplify the expression using espresso (the default is True).

Returns

**str** String with substitutions made according to fd.

**Function** `rspace`

```
def rspace(
    maxts,
    trmaxts,
    primes
)
```

In order for none of the trap spaces to “lock in”, we would require that their single-node drivers are all sustained in a negated state. We can use this idea to hone the exclusion space. `rspace` will return the region that

- 1) has the negations of 1-node drivers of each `maxts` active and . . .
- 2) has the update rules of these 1-node drivers taking the appropriate value

In addition, a time-reverse trap space (`trmaxts`) describes a region that, once exited, cannot be reentered. Thus, if the LDOL of the region contains any contradiction, the region cannot contain any attractor. Therefore, we include a third criterion for the `rspace`:

- 3) is not in a state belonging to an attractor-free time-reversed trap space

The return value is a list `L` of lists of prime implicants. Each element of `L` is to be interpreted as a list of OR-separated prime implicants; `L` is to be interpreted as AND-separated. e.g., `L=[[{'A':0,'B':1},{C:0}],[{B':0,'D':1},{A':1}]]` should be read as `L = (!A&B | !C) & (!B & D | A)`

Parameters

**maxts : list of partial state dictionaries** Stable motifs, i.e., maximum trap spaces for the system.

**trmaxts : list of partial state dictionaries** Stable motifs, i.e., maximum trap spaces for the time-reversed system.

**primes : PyBoolNet primes dictionary** Update rule for the system.

Returns

**L : `rspace()` list** Description of `rspace` in list form (see summary above for details).

**Function** `state_in_rspace`

```
def state_in_rspace(
    state,
    L
)
```

Tests to see if state is in the `rspace` `L`.

Parameters

**state : partial state dictionary** State, or partial state to test.

**L : `rspace()` list** Restrict space list (see `RestrictSpace.rspace` for details).

Returns

**bool** True if and only if state is in `L`.

## Module `StableMotifs.Succession`

### Functions

**Function** `build_succession_diagram`

```
def build_succession_diagram(
    primes,
    fixed=None,
    motif_history=None,
    diagram=None,
    merge_equivalent_motifs=True,
    max_simulate_size=20,
    prioritize_source_motifs=True,
    max_stable_motifs=10000
)
```



Recursively construct a succession diagram from the input update rules. Generally, it is preferable to construct this from within the `AttractorRepertoire` class (using, e.g., `AttractorRepertoire.from_primes`).

Parameters

**primes** : **PyBoolNet primes dictionary** Update rules.  
**fixed** : **partial state dictionary** Used only for recursion. Specifies nodes to be fixed in the next reduced network to be added to the diagram.  
**motif\_history** : **list of partial state dictionaries** Used only for recursion. Specifies stable motif history for the next reduced network to be added to the diagram.  
**diagram** : **SuccessionDiagram** Used only for recursion. The `SuccessionDiagram` object that is under construction.  
**merge\_equivalent\_motifs** : **bool** If False, equivalent reduced networks have their data re-computed and copied. Making this False is only recommended if the succession diagram must be represented in a form that has no feedforward loops; making this True provides large computational advantages, both in terms of speed and memory usage (the default is True).  
**max\_simulate\_size** : **int** Maximum number of variables for which to brute-force build a state transition graph (the default is 20).  
**prioritize\_source\_motifs** : **bool** Whether source nodes should be considered first (the default is True).  
**max\_stable\_motifs** : **int** Maximum number of output lines for PyBoolNet to process from the `AspSolver` (the default is 10000).

Returns

**SuccessionDiagram** The succession diagram for the input update rules.

## Classes

**Class** `SuccessionDiagram`

```
class SuccessionDiagram
```

Class describing the succession diagram of a Boolean system. See, e.g., Zanudo and Albert (2015) or Rozum et al. (2021).

## Attributes

**motif\_reduction\_dict** : **dictionary** MotifReduction-valued dictionary with integer (index) keys (see `Reduction.py`).  
**digraph** : **networkx.DiGraph** Topological structure of the succession diagram. Nodes are integers that align with the entries of `motif_reduction_dict`.

## Methods

**Method** `add_motif_permutation`

```
def add_motif_permutation(
    self,
    reduction_index,
    permutation
)
```

Adds a permutation of a preexisting stable motif history to a precomputed `MotifReduction` object.

Parameters

**reduction\_index** : **int** Index of the preexisting reduced network.  
**permutation** : **list of int** Permutation that maps the preexisting history to the input history.

**Method** `add_motif_reduction`

```
def add_motif_reduction(
    self,
    motif_reduction
)
```

Inserts a given MotifReduction into the succession diagram. Does not check for consistency, but will insert a properly constructed MotifReduction into the correct place in the diagram.

Parameters

**motif\_reduction : MotifReduction** Reduced network to be appended to the succession diagram.

**Method** `find_equivalent_reduction`

```
def find_equivalent_reduction(
    self,
    fixed
)
```

Extracts the MotifReduction object that has the frozen node values specified by fixed, if such an object exists (returns None otherwise).

Parameters

**fixed : partial state dictionary** Nodes values that have been fixed and reduced by stable motifs and their logical domain of influence.

Returns

**MotifReduction** Reduced network that has the frozen node values specified by fixed, if such an object exists (returns None otherwise).

**Method** `find_motif_permutation`

```
def find_motif_permutation(
    self,
    motif_history
)
```

Check whether some permutation of the input motif\_history is already represented in the succession diagram. If so, return the preexisting reduction's index and the permutation that maps between the two histories.

Parameters

**motif\_history : list of partial state dictionaries** Stable motifs that can lock in to give a given reduced network (in order).

Returns

**reduction\_index : int** Index of the preexisting reduced network. This value is None if no such reduced network exists.

**permutation : list of int** Permutation that maps the preexisting history to the input history. This value is None if no such history exists.

**Method** `get_motifs`

```
def get_motifs(
    self
)
```

Extract the stable motifs of a system and its reduced networks from its attractor repertoire. Notably, these include both the system's primary stable motifs and conditionally stable motifs (see, e.g., Deritei et al. 2019).

Returns

**list of dictionaries** Stable motifs that appear in the system or during reduction (in no particular order).

#### **Method** `reduction_drivers`

```
def reduction_drivers(
    self,
    target_index,
    method='internal',
    max_drivers=None,
    GRASP_iterations=None
)
```

Find control strategies that lead to the reduced network specified by the target index. Several control strategies are implemented. See `Succession.SuccessionDiagram.reprogram_to_trap_spaces` for a detailed description of control methods available. Generally, this method should not be used directly. Instead, use `reprogram_to_trap_spaces`.

Parameters

**target\_index : int** Index of the target reduced network.

**method : str** One of 'internal', 'minimal', or 'GRASP'. See `Succession.SuccessionDiagram.reprogram_to_trap_spaces` for details.

**max\_drivers : int** Maximum number of driver nodes to consider (not used in GRASP methods). If none, the upper limit is given by the number of free variables (the default is None).

**GRASP\_iterations : int** Number of times to construct GRASP driver sets; only used in GRASP methods. If none, the number of iterations is chosen based on the network size (the default is None).

Returns

**list** Control strategies found; interpretation depends on method selected See `Succession.SuccessionDiagram.reprogram_to_trap_spaces` for details.

#### **Method** `reductions_indices_with_states`

```
def reductions_indices_with_states(
    self,
    logically_fixed,
    optimize=True
)
```

Find all reductions (by index) that have the nodes states specified logically fixed.

Parameters

**logically\_fixed : partial state dictionary** Nodes states that should be fixed in all returned network reductions.

**optimize : bool** Whether to remove reduced networks that are subnetworks of valid reductions. This is generally recommended so as to obtain the most parsimonious control strategies (the default is True).

Returns

**list of int** Indices of reduced networks that have the appropriate fixed states.

## Method reprogram\_to\_trap\_spaces

```
def reprogram_to_trap_spaces(
    self,
    logically_fixed,
    target_method='history',
    driver_method='internal',
    max_drivers=None,
    GRASP_iterations=None,
    GRASP_score_override=None
)
```

Find driver sets that lead to fixing the node states specified.

### Parameters

**logically\_fixed** : **partial state dictionary** Targeted fixed nodes.

**target\_method** : **str** Either 'history' or 'merge'; see Notes below for details.

**driver\_method** : **str** Either 'internal', 'minimal', or 'GRASP' see Notes below for details.

**max\_drivers** : **int** Maximum number of driver nodes to consider (not used in GRASP methods). If none, the upper limit is given by the number of free variables (the default is None).

**GRASP\_iterations** : **int** Number of times to construct GRASP driver sets; only used in GRASP methods. If none, the number of iterations is chosen based on the network size (the default is None).

**GRASP\_score\_override** : **function** Optional heuristic score function override (see Domain-Influence.GRASP for details). Only used in GRASP methods (the default is None).

### Returns

**list** Control strategies found; interpretation depends on method selected See Notes below for details.

### Notes

The various combinations of **target\_method** and **driver\_method** options result in different control strategies, which are outlined below.

**target\_method = history, driver\_method = internal**: Finds all shortest stable motif histories that result in the target node states being logically fixed. Each stable motif is searched for internal driver nodes. The resulting internal drivers are combined into a single control set. The return value consists of all such control sets for all stable motif histories. Each control set eventually becomes self-sustaining.

**target\_method = history, driver\_method = minimal**: Similar to the history method, except the search for stable motif drivers includes external driver nodes for the motif and does not extend to driver sets of larger size once one driver set has been found for a motif. Because the search includes external driver nodes, special care must be taken in interpreting the effect of the drivers, as their influence may impact the effect of motifs stabilizing. Thus, the control is only guaranteed to work if the interventions are temporary and implemented in the order specified by the motif history.

For this reason, the output consists of lists of ordered interventions. Each element of the return value is a list of lists of dictionaries. Each element of the return value represents a control strategy. To implement such a strategy, select a dictionary from the first element of the strategy and fix the node states it specifies until their influence has propagated through the system. Then repeat this process iteratively for each element of the strategy list, in order. For example, if `nonredundant_drivers = [ [{ 'xD':1, 'xE':1 } ], [ { 'xA':1 }, { 'xB':1 }, { 'xC':1 } ] ]` then there are two control strategies available: 1) fix `xD=xE=1` temporarily and 2) first fix either `xA=1` or `xB=1` temporarily, then fix `xC=1` temporarily.

**target\_method = history, driver\_method = GRASP**: The same as history, minimal, except external driver nodes are searched for using the GRASP algorithm using **GRASP\_iterations**

iterations.

`target_method = merge, driver_method = internal`: Finds all shortest stable motif histories that result in the target node states being logically fixed. All node states in the motifs in the history are merged into a stable module dictionary. This is then searched for internal driver nodes. Each element of the return value is a dictionary corresponding to a control set. Each control set eventually becomes self-sustaining.

`target_method = merge, driver_method = minimal`: Similar to the merge method, except the search for drivers is conducted over all nodes, not just those internal to the merged stable module. Furthermore, the search is truncated when a control set is found such that the search does not proceed to driver sets larger than the smallest found. Each element of the return value is a dictionary corresponding to a control set. The control sets are only guaranteed to result in activation of the target if they are temporary interventions.

`target_method = merge, driver_method = GRASP`: The same as merge, minimal, except external driver nodes are searched for using the GRASP algorithm using `GRASP_iterations` iterations.

## Module `StableMotifs.TimeReversal`

### Functions

**Function** `time_reverse_primes`

```
def time_reverse_primes(  
    primes  
)
```

Computes the time reversal of the input system (under general asynchronous update). The time reverse system has the same STG as the input system, but with each edge reversed.

Parameters

**primes** : **PyBoolNet prime dictionary** System update rules.

Returns

**trprimes** : **PyBoolNet prime dictionary** Time-reversed system update rules.

---

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