Introduction to Bayesian A/B testing

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Agenda

- Setting the scene
- Priors and probability distributions
- Running a Bayesian A/B test
- Posterior analysis
- When to stop a Bayesian A/B test?
- Summary and some final remarks
- Questions?

Setting the scene

Randomised Control Trials (RCTs)

A simplistic example:

- Users are assigned at **random** to two groups, A and B, with equal probability.
- Let A be our **control** group and B be our **treatment** group.

We want to know what effect our treatment has.

> **Tip**
>
> Early on during an experiment, differences between these groups could simply be due to the random allocation of participants. As the groups get larger, these random differences will diminish, bringing us closer to the difference caused by the treatment.
>
> Applying Bayesian inference effectively gives the experiment a guided head start by including more data in the form of **priors**.

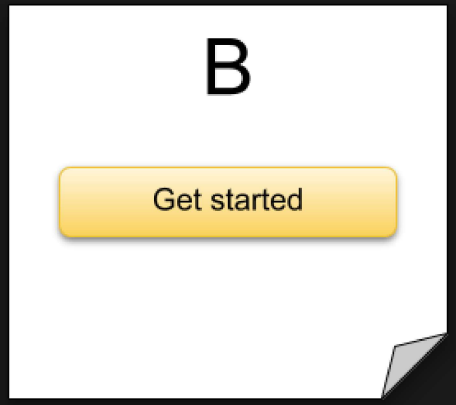
Hypothetical scenario

We have a button on a landing page that takes users to a sign up form.

At present, the button is labelled “Register your interest”.

We want to test whether changing it to “Get started” will result in an increased click-through rate (CTR).

The idea of “Get started” was suggested by an experienced and skilled UX design professional.



Priors and probability distributions

Prior knowledge and beliefs

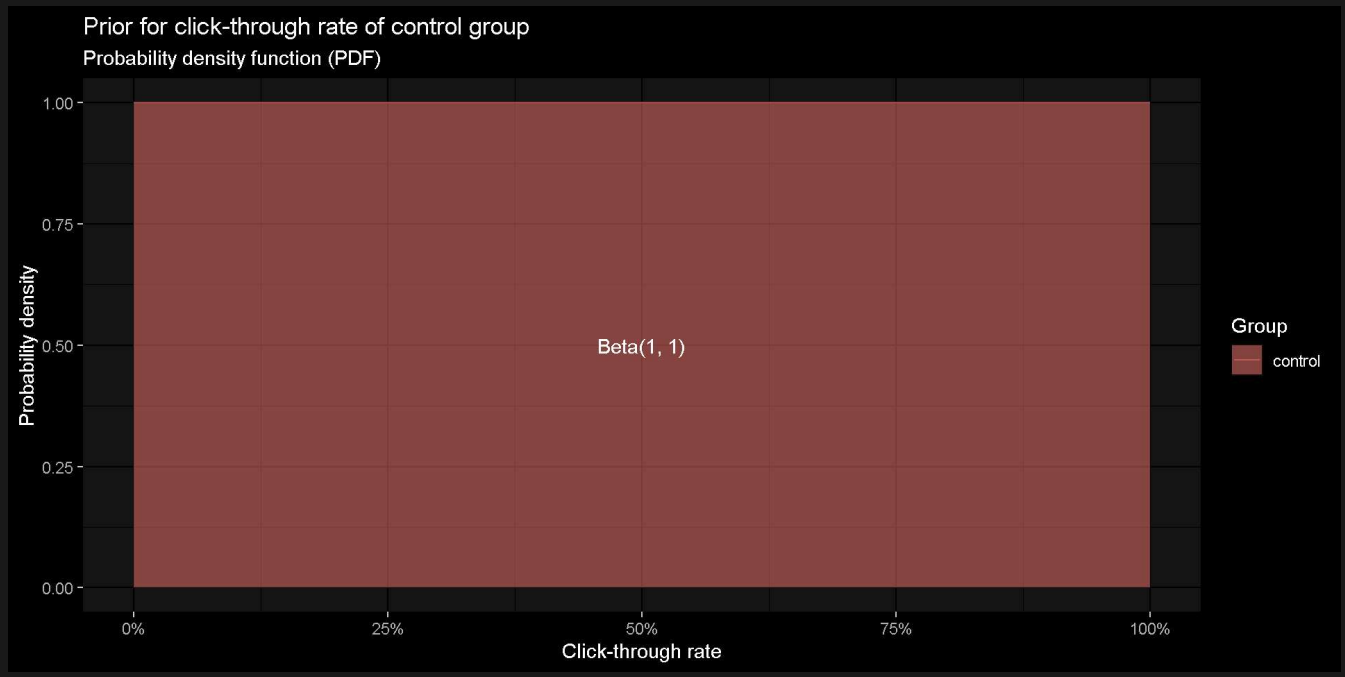
Before running an experiment, we form opinions about what we expect to see. We gather evidence such as:

- The baseline click-through rate of the button (with its current label) and knowledge of any outside variables that affects click-through rate, e.g. seasonality
- Effects we have seen from similar previous experiments
- Qualitative research, such as usability tests, focus groups, and surveys that are related to the test
- Opinions (including critical views) from interested parties, including experts

Priors are probability distributions

We express our prior beliefs about the control group using a **probability distribution**.

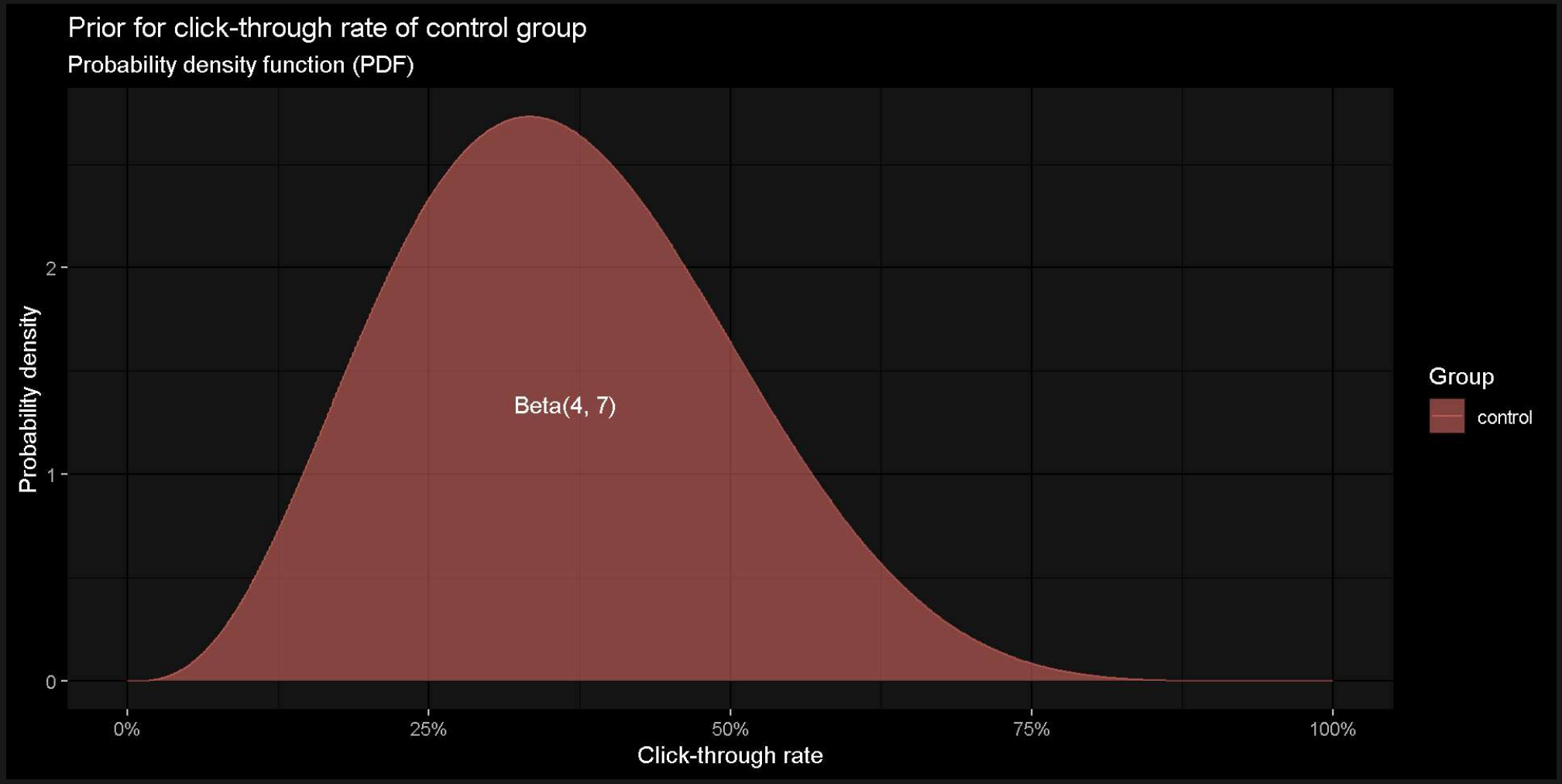
This plot shows an example of an extremely uninformative prior – a uniform prior that says every outcome is equally likely, i.e. naive.



Tip

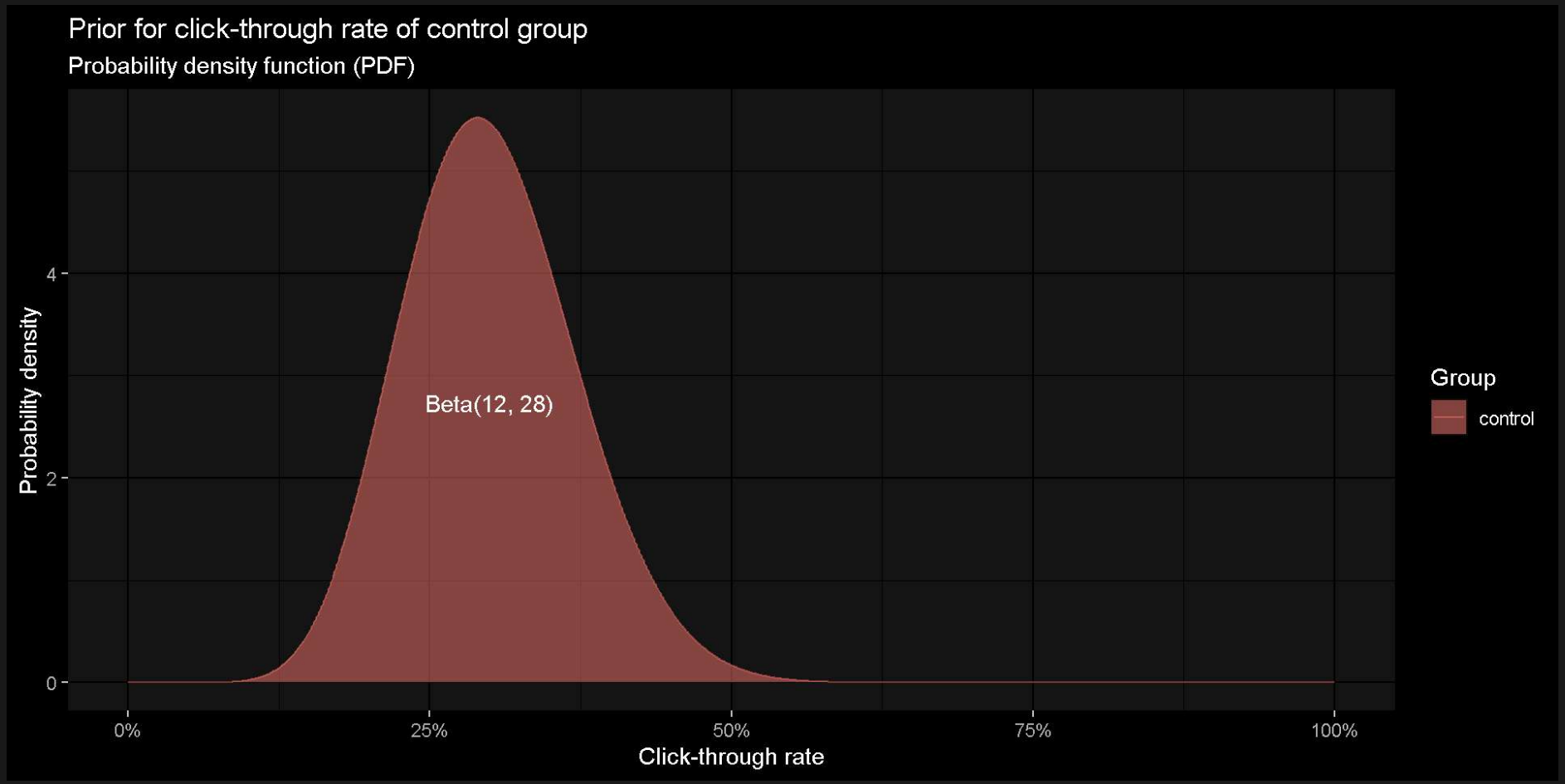
The **Beta distribution** is a **probability density function (PDF)** with two **shape parameters**: $B(shape1, shape2)$. It's used to describe proportions, such as click-through rate.

Something a little more informative



As the curve narrows, notice that the shape parameters of the Beta distribution increase.

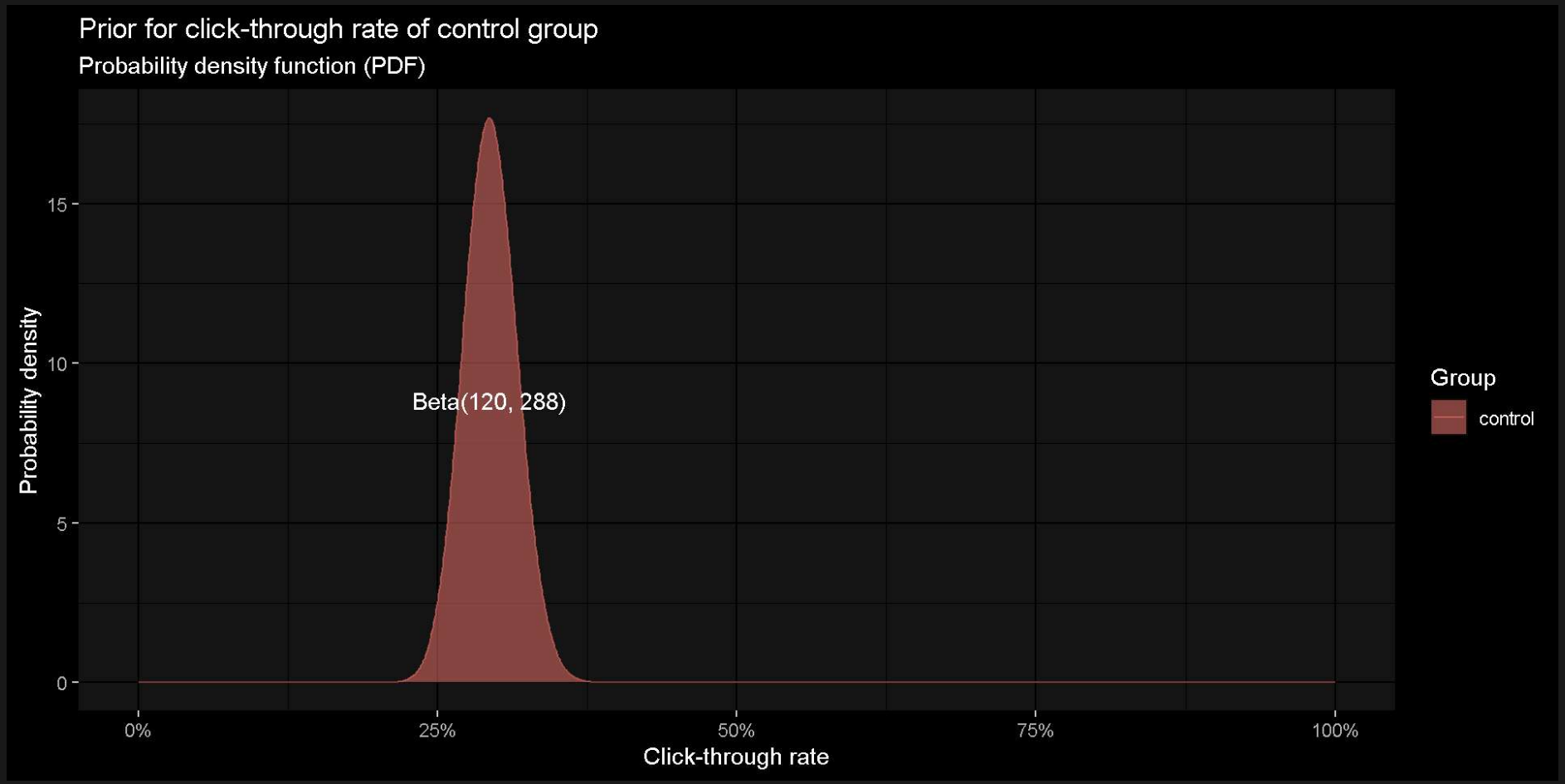
Something even more informative



The shape parameters (`shape1` and `shape2`) of the Beta distribution can be thought of as counts of **successes** and **failures**, respectively. Therefore, the mean probability of success (i.e. average click-through rate) is simply calculated with this formula:

$$\frac{shape1}{shape1 + shape2}$$

Let’s say we’ve settled on this:

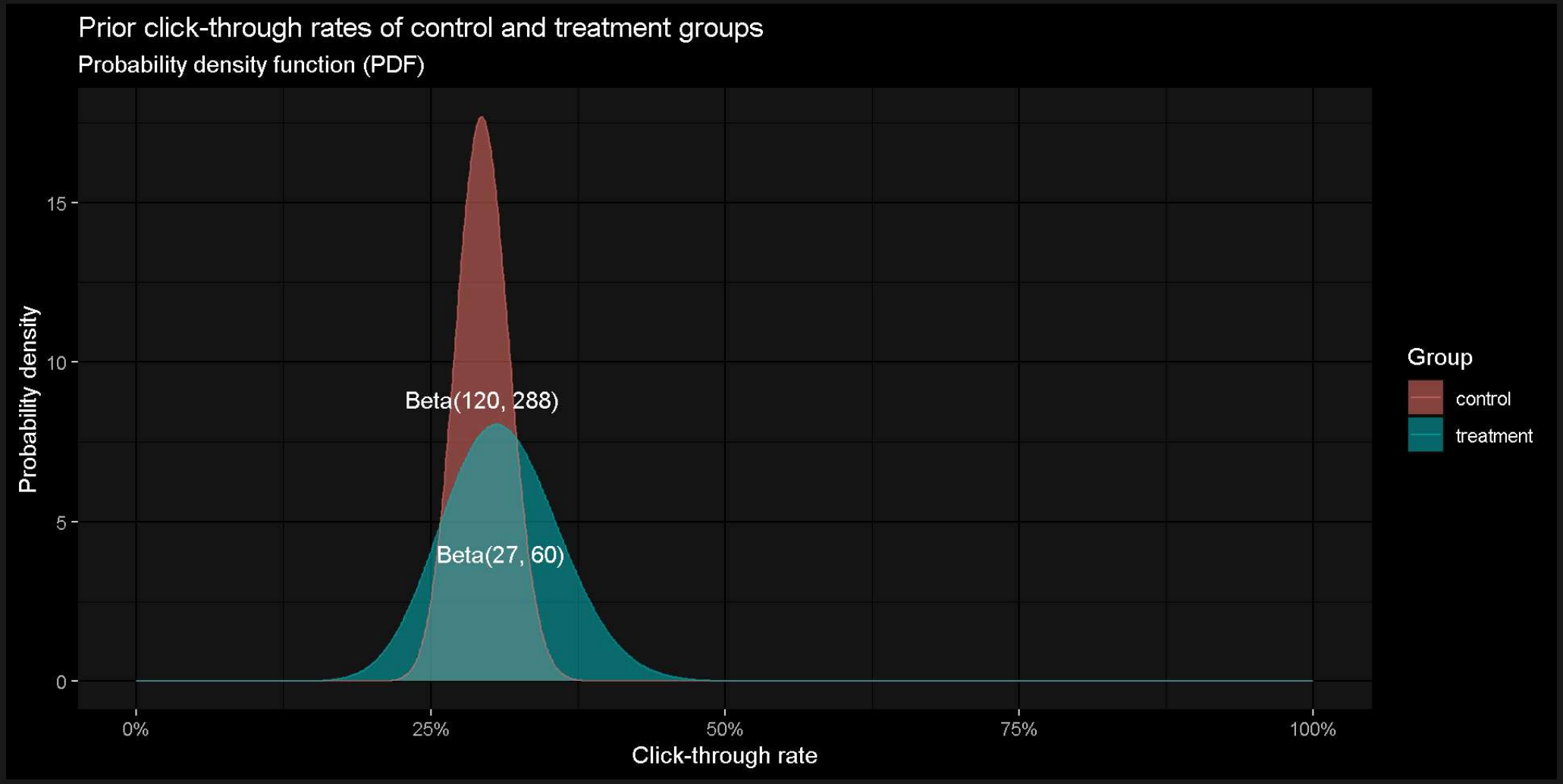


The more confident we are about our beliefs, the narrower the curve.

What about the treatment group?

- We expect that the click-through rates of the treatment and control groups will be correlated.
- We don’t know how correlated they will be, but we’re not expecting a dramatic difference between them.
- We hope that the click-through rate of the treatment group will be an improvement, but we’re open to other possibilities.
- We’re more confident than not that the click-through rate of the treatment group will be an improvement over control.
- We don’t want to bias the results of the experiment in favour of treatment or control, or towards a conclusion of there being a difference or no difference.

We've settled on these priors:



Running a Bayesian A/B test

Prior agreement

Agreement must be reached on the priors before collecting and analysing data from the experiment.

Once we've agreed on the priors and have locked them in, we can start the experiment.

Here's a summary of the priors we have chosen:

```
priors %>%
    mutate(
        mean = (shape1 / (shape1 + shape2)) %>%
            scales::percent(),
        sd = sqrt(
            shape1 * shape2 /
                ((shape1 + shape2) ^ 2 * (shape1 + shape2 + 1))
        ) %>%
            scales::percent(suffix = " p.p.")
    )
```

group	shape1	shape2	mean	sd
control	120	288	29.4%	2.3 p.p.
treatment	27	60	31.0%	4.9 p.p.

Let’s run a simulated experiment

Let’s pretend that there’s some true theoretical click-through rate for the control and treatment groups, 32% and 35% respectively. That equates to a relative uplift of 9.4%.

> **Note**
>
> Remember that this is just a hypothetical simulation. We wouldn’t know these in a real experiment ¯_(ツ)_/¯.

If we’re successful at applying Bayesian inference then we should hope to see (but can’t guarantee due to randomness) results that somewhat match with these theoretical CTRs.

1 day since the experiment started…

Let’s pretend that on average 150 users enter our experiment each day, and we’ve received the following data from day 1:

group	total_users	clicked	not_clicked	CTR
control	67	25	42	37.31%
treatment	84	32	52	38.10%

Let’s now incorporate our priors

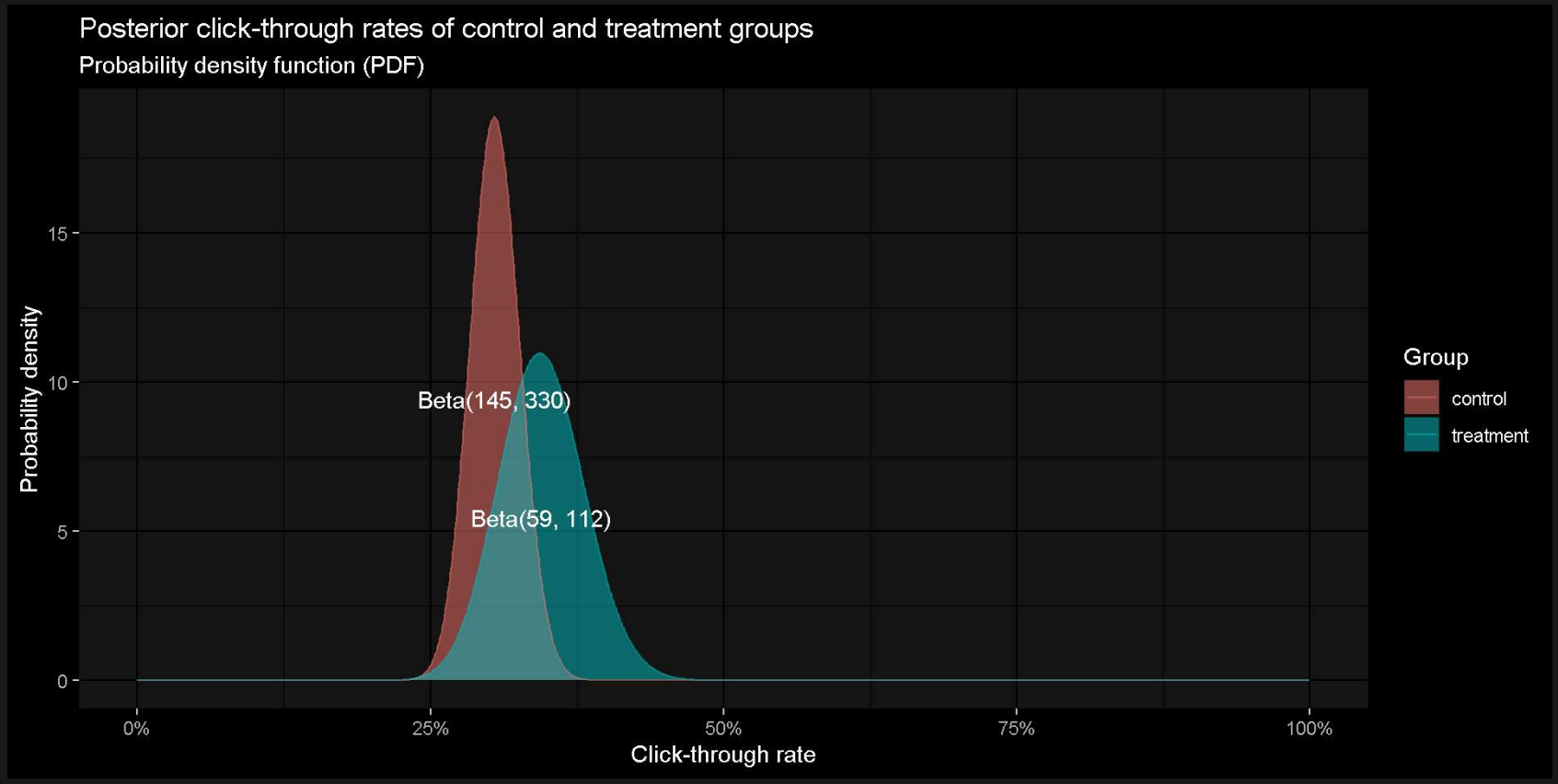
For each experiment group, we derive our **posterior** shape parameters through simple arithmetic addition:

- Increment the first shape parameter by the count users who had **clicked**
- Increment the second shape parameter by the count users who **didn’t click**

count	control	treatment
prior_shape1	120	27
clicked	25	32
posterior_shape1	145	59

count	control	treatment
prior_shape2	288	60
not_clicked	42	52
posterior_shape2	330	112

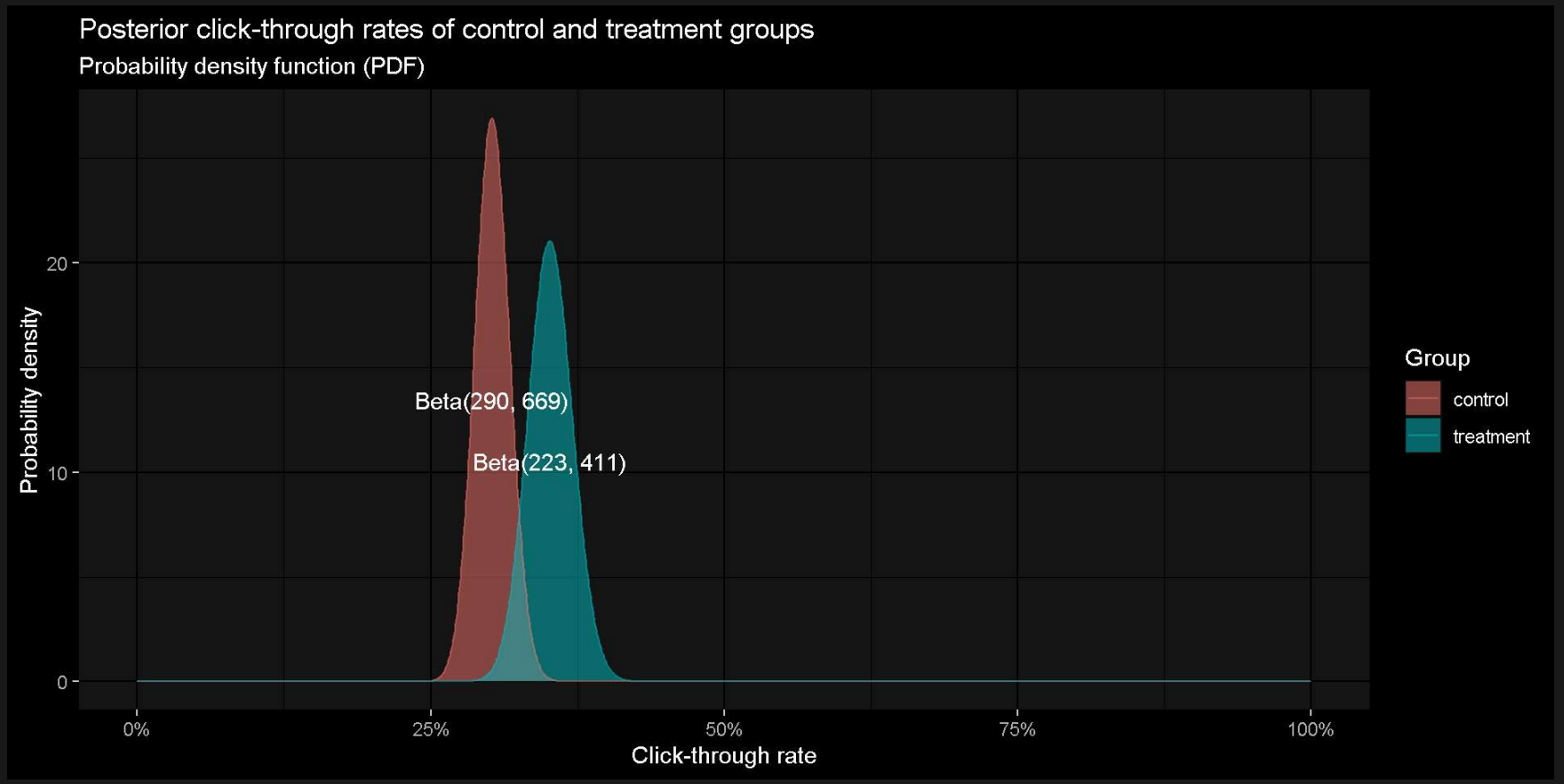
Posterior distribution of each group



We have now updated our beliefs. These posteriors can now be thought of as our updated priors.

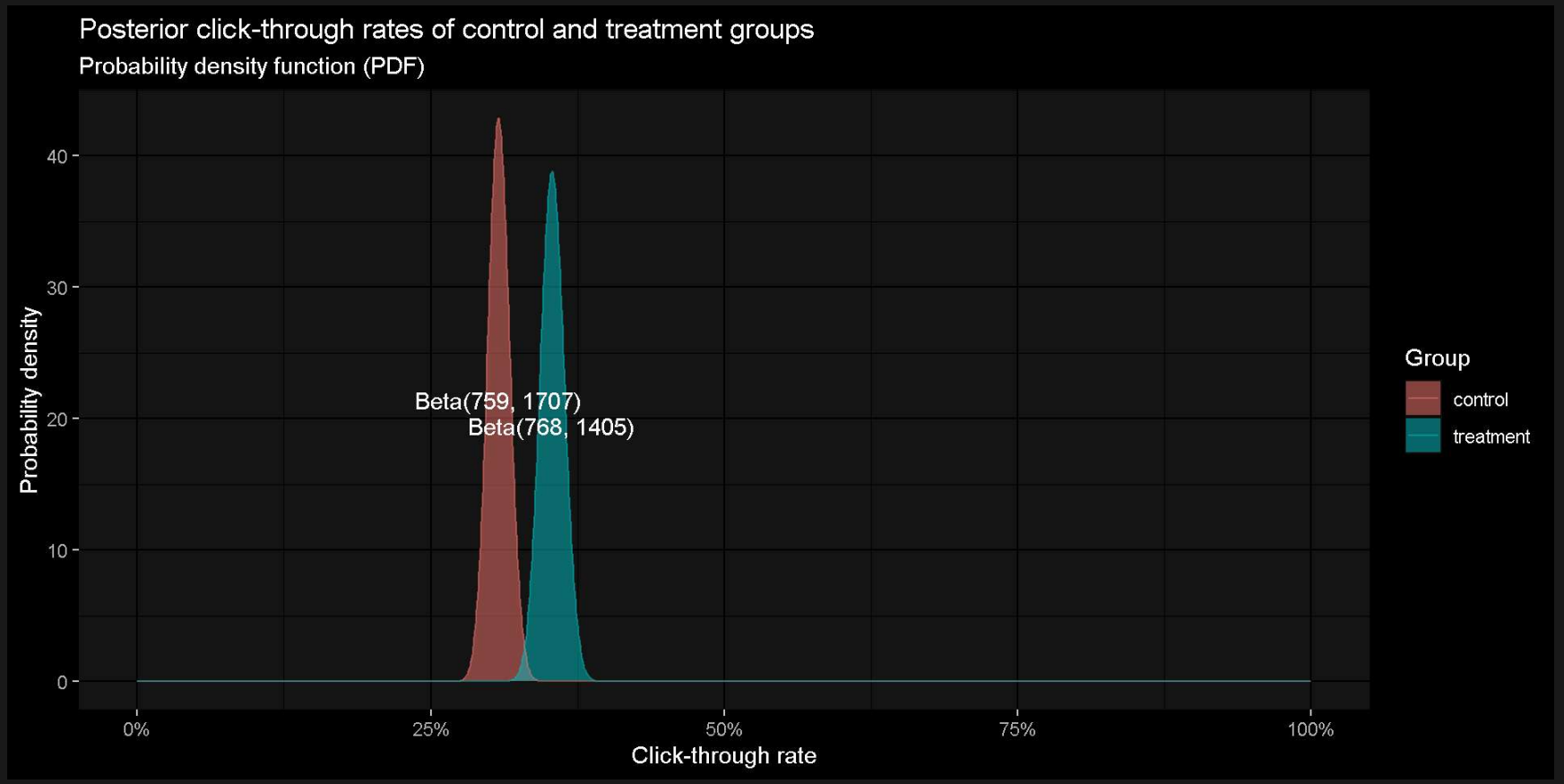
Another six days later…

We've now collected more data, so let's update our priors to form new posteriors.



Another three weeks later…

We've now collected even more data, so let's again update our priors to form new posteriors.



Posterior analysis

Statistical inferences using the posterior distributions

Monte Carlo simulation

We can draw a very very large number of random samples from our posterior distributions to make inferences about the experiment.

This is called Monte Carlo simulation – named after a well known casino.

Credits: Sam Garza from Los Angeles, USA, CC BY 2.0, via Wikimedia Commons

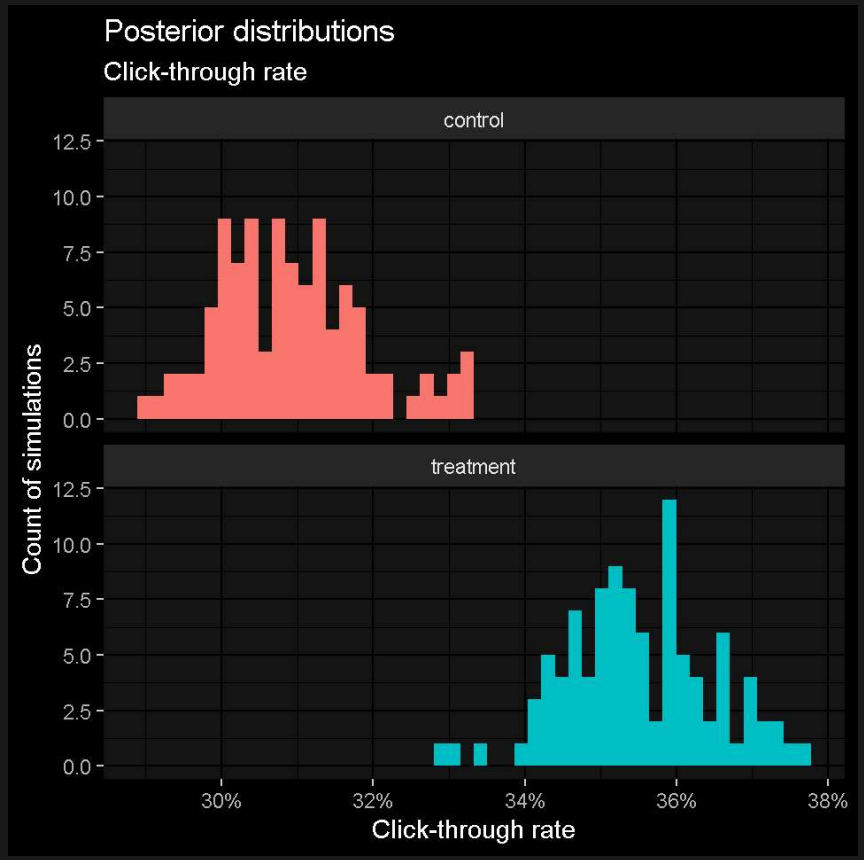
> **Note**
>
> The more samples drawn, the greater the reliability and precision of the inferences you make, but this comes at the cost of computational time and memory. Nowadays, computer processing speed and memory are more than adequate for what we need.

100 simulations

Let's start slowly by drawing 100 random samples from our distributions and plotting them using histograms…

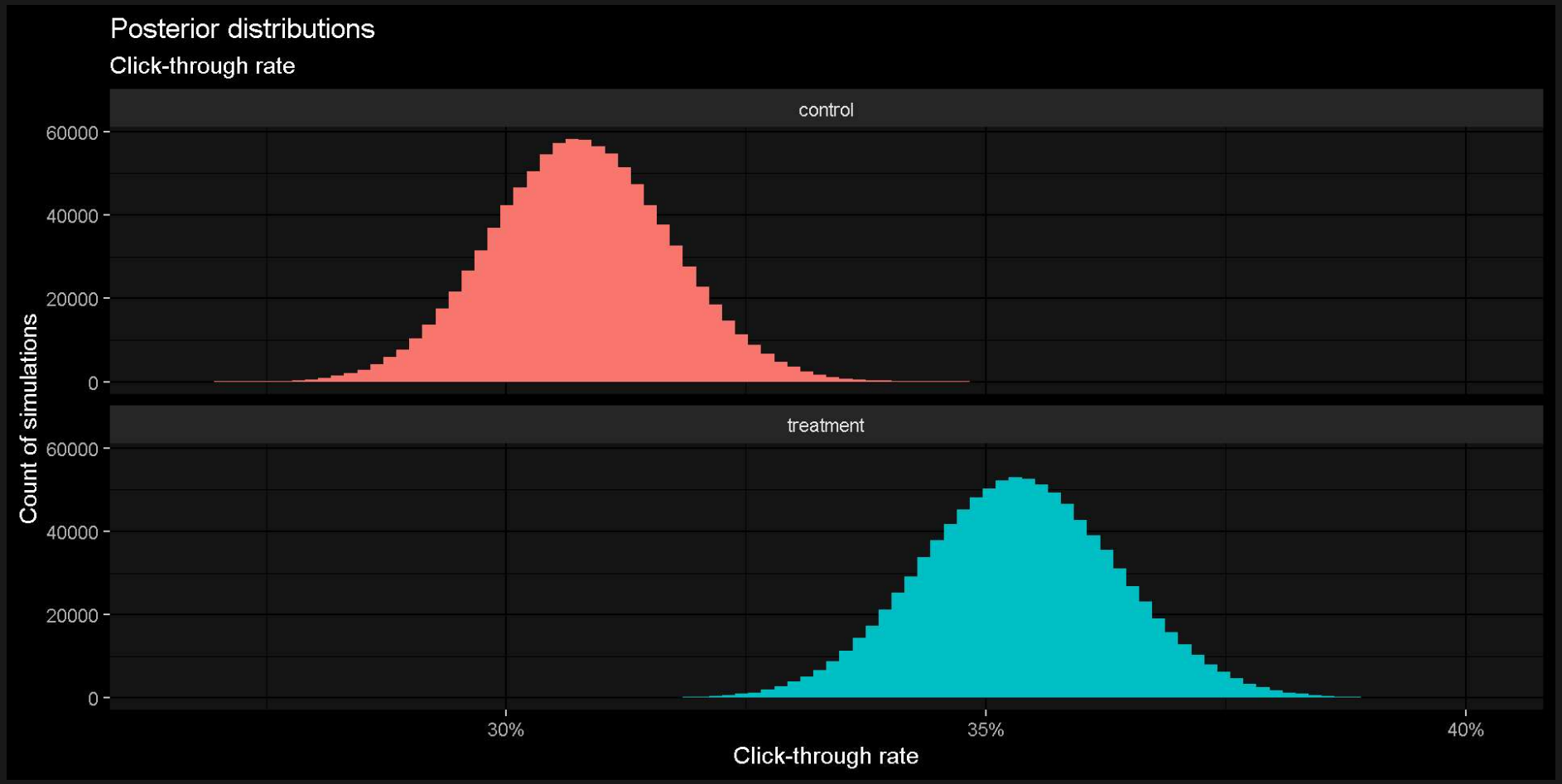
Here's some of our Monte Carlo samples:

control	treatment	uplift	beats_control
0.315	0.353	0.121	TRUE
0.300	0.352	0.172	TRUE
0.303	0.373	0.231	TRUE
0.304	0.355	0.167	TRUE
0.308	0.364	0.181	TRUE
0.309	0.350	0.131	TRUE
0.311	0.375	0.207	TRUE



Let’s now beef it up a bit…

We’ll now draw 1 000 000 samples…



We can now make some inferences

Here's a summary of our posterior predictive distributions as a result of the 1 000 000 simulations:

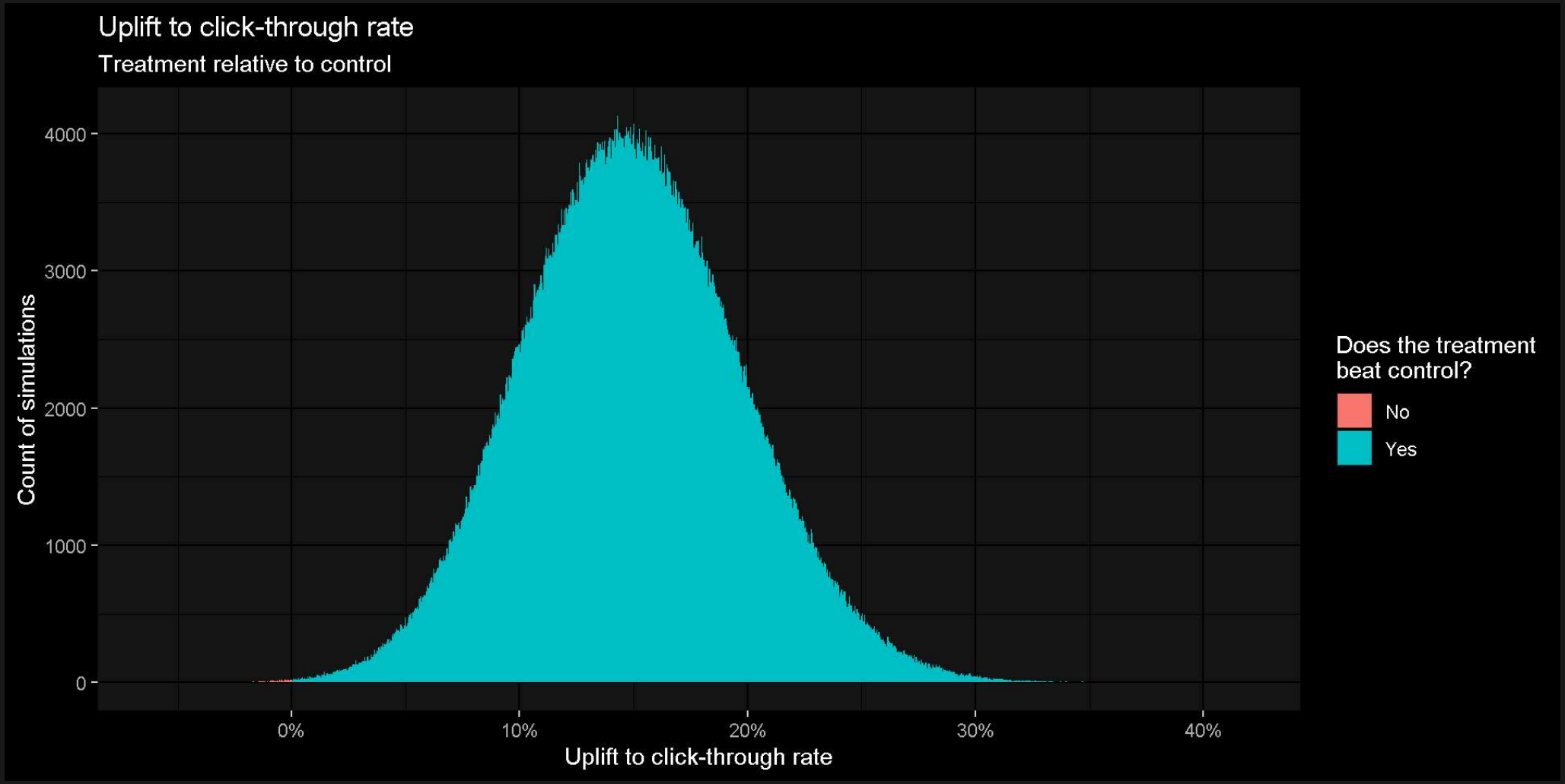
```
    control          treatment           uplift          beats_control
 Min.   :0.2661   Min.   :0.3044   Min.   :-0.06095   Mode :logical
 1st Qu.:0.3015   1st Qu.:0.3465   1st Qu.: 0.11630   FALSE:468
 Median :0.3077   Median :0.3534   Median : 0.14833   TRUE :999532
 Mean   :0.3078   Mean   :0.3534   Mean   : 0.14937
 3rd Qu.:0.3140   3rd Qu.:0.3603   3rd Qu.: 0.18127
 Max.   :0.3552   Max.   :0.4005   Max.   : 0.41846
```

What is the probability that the CTR of the treatment is greater than that of control?

```
1 with(
2     posterior_comparison,
3     mean(beats_control)
4 ) %>% scales::percent(0.01)
```

```
[1] "99.95%"
```

Posterior distribution of the CTR uplift



When to stop a Bayesian A/B test?

If using uninformative priors…

If your original priors are uninformative or too weak, then you face the same risks as with frequentist experiments.

Perform **power analysis** ahead of running the experiment. This is to determine the required sample size before any inferences are made.

Prior to the experiment commencing, decide on:

- The **minimum detectable effect** size
- The accepted **false positive rate**
- The accepted **false negative rate**

If using informative priors…

If your priors are relatively informative and chosen carefully, then this can reduce the chances of false positives and negatives. But:

- Be careful to not bias the results of the experiment.
- Power analysis is still recommended in order to gauge the worse case scenario for how long the experiment might run.

Bayesian inference, with informative priors, can make it possible to end an experiment early.

If deciding to end early…

Ask yourself:

- Has the experiment run for at least a couple of cycles? (e.g. at least two full weeks)
- Have the results stabilised? Is there a clear winner?
- Could it be worth running longer to learn more?
- What are the risks of continuing or ending now? What if the results you see are just a fluke and are therefore misguiding you? What is the impact of making the wrong choice? What are the chances?

Summary and some final remarks

Before starting an experiment

Gather prior knowledge and articulate beliefs:

- Establish a baseline - what do you know about the control group?
- What do you expect the effect of the treatment to be? How sure are you?

Express those beliefs and knowledge as distributions - these are your priors for your control and treatment groups.

Important

Ensure that the priors encapsulate the collective knowledge and beliefs of all interested parties so that there is agreement. This helps to avoid the results from being challenged later. This is because everyone would have already had an opportunity to provide their opinions.

Running the experiment

- Start the experiment, gather data, and update your priors to form posteriors
- Draw inferences by running a large number of Monte Carlo simulations using the posteriors
- Know when to end the experiment – try to plan for this ahead of running the experiment

Final remarks

Null-hypothesis significance testing (NHST) is not what Bayesian is for:

- Bayesian tells you the probability of some effect being within some range, given the data. I.e. Given everything we know so far, what are the risks associated with the choices we have?
- NHST tells you the probability of data at least as extreme as what has been observed, given there is no real effect. I.e. How ridiculous would this outcome be if it were due to chance alone?

NHST is often referred to as the frequentist approach, where decisions are made using p-values and some arbitrary threshold α (i.e. false positive rate).

Unlike NHST, Bayesian A/B testing doesn't give you a yes/no answer – it instead informs you about the probabilities and risks associated with the choices you have.

Questions?

Tip

These presentation slides and simulations have been produced in RStudio using Quarto. You can download the source code and slides from Github at: https://github.com/jdeboer/measurecamp2022