

BUSN 33946 & ECON 35101  
International Macroeconomics and Trade  
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# Open-economy growth

Open-economy mechanics may reverse closed-economy mechanics:

- ▶ Structural transformation: Matsuyama (1992, 2009)
- ▶ Neoclassical growth model: Ventura (1997), Acemoglu & Ventura (2002)

Economic integration as a source of economic growth:

- ▶ Rivera-Batiz & Romer (1991)
- ▶ Buera & Oberfield (2020)

See Chapter 19 of Acemoglu's *Introduction to Modern Economic Growth*

# Matsuyama (*JET* 1992) overview

“Agricultural productivity, comparative advantage, and economic growth”

- ▶ Model of endogenous growth driven by learning by doing in the manufacturing sector
- ▶ Income elasticity of demand for agricultural output  $< 1$
- ▶ Closed economy: Agricultural productivity raises growth
- ▶ Small open economy: Agricultural productivity lowers growth

# Matsuyama (1992) setup/mechanics

Learning by doing in manufacturing

$$X_t^M = M_t F(n_t)$$

$$X_t^A = AG(1 - n_t)$$

$$\dot{M}_t = \delta X_t^M \quad \delta > 0$$

$$AG'(1 - n_t) = p_t M_t F'(n_t) \quad (4)$$

Stone-Geary preferences with necessity  $\gamma$  where  $AG(1) > \gamma L > 0$ .

$$C_t^A = \gamma L + \beta p_t C_t^M \quad (7)$$

Closed-economy has  $C_t^M = X_t^M$  and  $C_t^A = X_t^A$ . Combining with equations (4) and (7), equilibrium  $n_t$  satisfies

$$\phi(n_t) \equiv G(1 - n_t) - \beta G'(1 - n_t) F(n_t) / F'(n_t) = \gamma L / A$$

Unique solution  $n_t = v(A)$  with  $v'() > 0$ .

Higher  $A$  raises *level* of manufacturing and thus economic growth *rate*

## Matsuyama (1992): small open economy

World economy has  $A^*$  and  $M_0^*$ . World relative price satisfies

$$A^* G'(1 - n^*) = p_t M_t^* F'(n^*)$$

SOE allocation must satisfy

$$AG'(1 - n_t) = p_t M_t F'(n_t)$$

Take ratio, set  $t = 0$ , find  $n_0 \leq n^*$ .

Growth rate result: When the Home initially has a comparative advantage in manufacturing, its manufacturing productivity will grow faster than the rest of the world and accelerate over time.

“a caution to the readers of the recent empirical work, which, in order to test implications of closed economy models of endogenous growth, uses cross-country data and treats all economies in the sample as if they were isolated from each other”

## Similar warning to empiricists (Matsuyama *JEEA* 2009)

“ Structural Change in an Interdependent World: A Global View of Manufacturing Decline”

*This paper presents a simple model of the world economy, in which productivity gains in manufacturing are responsible for the global trend of manufacturing decline, and yet, in a cross-section of countries, faster productivity gains in manufacturing do not necessarily imply faster declines in manufacturing. In doing so, it aims to draw attention to the common pitfall of using the cross-country evidence to test a closed economy model, and argues for a global perspective; in order to understand cross-country patterns of structural change, one needs a world economy model in which the interdependence across countries is explicitly spelled out.*

# Neoclassical Growth Model

- ▶ In a closed economy, neoclassical growth model predicts that:
  1. If there are diminishing marginal returns to capital, then different capital-labor ratios across countries lead to different growth rates along the transition path
  2. If there are constant marginal returns to capital (AK model), then different discount factors across countries lead to different growth rates in the steady state
- ▶ In an open economy, both predictions can be overturned

# Preferences and technology

- ▶ For simplicity, assume:
  - ▶ No population growth  $l(t) = 1$  for all  $t$
  - ▶ No depreciation of capital
- ▶ Representative household at  $t = 0$  has log preferences,

$$U = \int_0^{\infty} \exp(-\rho t) \ln c(t) dt \quad (1)$$

- ▶ Final consumption good is produced according to,

$$y(t) = aF(k(t), l(t)) = af(k(t))$$

where output (per capita)  $f$  satisfies,

$$f' > 0 \quad \text{and} \quad f'' \leq 0$$



# Perfect competition, law of motion for capital, and no-Ponzi condition

- Firms maximize profits, taking factor prices  $w(t)$  and  $r(t)$  as given,

$$r(t) = af'(k(t)) \quad (2)$$

$$w(t) = af(k(t)) - k(t)af'(k(t)) \quad (3)$$

- Law of motion for capital is given by,

$$\dot{k}(t) = r(t)k(t) + w(t) - c(t) \quad (4)$$

- No-Ponzi condition:

$$\lim_{t \rightarrow \infty} \left[ k(t) \exp \left( - \int_0^t r(s) ds \right) \right] \geq 0 \quad (5)$$

# Competitive equilibrium

- **Definition** Competitive equilibrium of the neoclassical growth model consists in  $(c, k, r, w)$  such that the representative household maximizes (1) subject to (4) and (5) and factor prices satisfy (2) and (3).
- **Proposition 1** In any competitive equilibrium, consumption and capital follow the laws of motion given by,

$$\frac{\dot{c}(t)}{c(t)} = af'(k(t)) - \rho$$
$$\dot{k}(t) = af(k(t)) - c(t)$$

# Diminishing versus constant MPK

Suppose  $f'' < 0$ . In this case, Proposition 1 implies that:

1. The growth rate of consumption declines with  $k$
2. No long-run growth without exogenous technological progress
3. Starting from  $k(0) > 0$ , there exists a unique equilibrium converging monotonically to  $(c^*, k^*)$  such that,

$$af'(k^*) = \rho$$

$$c^* = af(k^*)$$

Now suppose  $af(k) = ak$  so that  $f'' = 0$ . In this case, Proposition 1 implies a unique equilibrium path with same growth for  $c$  and  $k$ :

$$g^* = a - \rho$$

Trade integration, through its effects on factor prices, may transform a model with diminishing marginal returns into an *AK* model and vice versa.

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$$\begin{aligned}af'(k^*) &= \rho \\c^* &= af(k^*)\end{aligned}$$

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# Ventura (1997): Assumptions

## “Growth and Interdependence”

- ▶ Neoclassical growth model with multiple countries indexed by  $j$ 
  - ▶ No differences in population size:  $I_j(t) = 1$  for all  $j$
  - ▶ No differences in discount rates:  $\rho_j = \rho$  for all  $j$
  - ▶ *Diminishing marginal returns*:  $f'' < 0$
- ▶ Capital and labor *services* are freely traded across countries
  - ▶ No trade in assets, so trade is balanced period-by-period.
- ▶ **Notation:**
  - ▶  $x_j^l(t), x_j^k(t) \equiv$  labor and capital services used in production of final good in country  $j$

$$y_j(t) = aF(x_j^k(t), x_j^l(t)) = ax_j^l(t)f(x_j^k(t)/x_j^l(t))$$

- ▶  $I_j(t) - x_j^l(t)$  and  $k_j(t) - x_j^k(t) \equiv$  net exports of factor services

## Ventura (1997): Free-trade equilibrium (1/2)

- ▶ Free trade equilibrium reproduces the integrated equilibrium.
- ▶ In each period:
  1. Free trade in factor services implies FPE:

$$r_j(t) = r(t)$$

$$w_j(t) = w(t)$$

2. FPE further implies identical capital labor ratios,

$$\frac{x_j^k(t)}{x_j^l(t)} = \frac{x^k(t)}{x^l(t)} = \frac{\sum_j k_j(t)}{\sum_j l_j(t)} = \frac{k^w(t)}{l^w(t)}$$

- ▶ Like static HO, countries with  $k_j(t)/l_j(t) > k^w(t)/l^w(t)$  export capital and import labor services.

## Ventura (1997): Free-trade equilibrium (2/2)

- ▶ Let  $c(t) \equiv \sum_j c_j(t)/l^w(t)$  and  $k(t) \equiv \sum_j k_j(t)/l^w(t)$ .
- ▶ World consumption and capital per capita satisfy

$$\frac{\dot{c}(t)}{c(t)} = af'(k(t)) - \rho$$
$$\dot{k}(t) = f(k(t)) - c(t)$$

- ▶ For each country, however, we have,

$$\frac{\dot{c}_j(t)}{c_j(t)} = af'(k(t)) - \rho \tag{6}$$

$$\dot{k}_j(t) = af'(k(t))k_j(t) + w(t) - c_j(t) \tag{7}$$

- ▶ If  $k(t)$  is fixed, equations (6) and (7) imply that it is *as if* countries were facing an  $AK$  technology.



## Ventura (1997): Summary and implications

- ▶ Ventura (1997) hence shows that trade may help countries avoid the curse of diminishing marginal returns:
  - ▶ As long as country  $j$  is “small” relative to the rest of the world,  $k_j(t) \ll k(t)$ , the return to capital is independent of  $k_j(t)$ .
  - ▶ This is the ‘factor price insensitivity’ result from the small open (or partial-equilibrium large) economy HO model.
  - ▶ “International trade converts an excess production of capital-intensive goods into exports, instead of falling prices.”
- ▶ This insight may help explain growth miracles in East Asia:
  - ▶ Asian economies, which were more open than many developing countries, accumulated capital more rapidly but without rising interest rates or diminishing returns.
  - ▶ These economies were also heavily industrializing along their development path. HO mechanism requires this.
- ▶ Growth miracles cannot go on forever in this model

# Acemoglu and Ventura (2002): Assumptions

## “The World Income Distribution”

- ▶ Now we go in the opposite direction.
- ▶  $AK$  model with multiple countries indexed by  $j$ .
  - ▶ No differences in population size,  $l_j(t) = 1$  for all  $j$ .
  - ▶ Constant marginal returns  $f'' = 0$ .
- ▶ Like in an “Armington” model, capital services are differentiated by country of origin.
- ▶ Capital services are freely traded and combined into a unique final good (for consumption or investment) per

$$c_j(t) = \left[ \sum_{j'} \mu_{j'}^{1/\sigma} x_{jj'}(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$
$$i_j(t) = \left[ \sum_{j'} \mu_{j'}^{1/\sigma} x_{jj'}^i(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

# Acemoglu and Ventura (2002): Free trade equilibrium

► **Lemma** *In each period,  $c_j(t) = \rho_j k_j(t)$*

► **Proof:**

1. Euler equation implies

$$\frac{\dot{c}_j(t)}{c_j(t)} = r_j(t) - \rho_j$$

2. Budget constraint at time  $t$  requires:

$$\dot{k}_j(t) = r_j(t)k_j(t) - c_j(t)$$

3. Combining these two expressions, we obtain,

$$[k_j(t)/c_j(t)] = \rho_j[k_j(t)/c_j(t)] - 1$$

4. That plus the no-Ponzi condition implies

$$k_j(t)/c_j(t) = 1/\rho_j$$

# Acemoglu and Ventura (2002): Free trade equilibrium

- **Proposition 2** *In steady-state equilibrium, we must have:*

$$\frac{\dot{k}_j(t)}{k_j(t)} = \frac{\dot{c}_j(t)}{c_j(t)} = g^*$$

- **Proof:**

1. In steady state, by definition, we have  $r_j(t) = r_j^*$ .
2. Lemma + Euler equation  $\implies \frac{\dot{k}_j(t)}{k_j(t)} = r_j(t) - \rho_j$ .
3.  $1 + 2 \implies \frac{\dot{k}_j(t)}{k_j(t)} = g_j^*$ .
4. Market clearing implies,

$$r_j^* k_j(t) = \mu_j (r_j^*)^{1-\sigma} \sum_{j'} r_{j'}^* k_{j'}(t), \quad \text{for all } j.$$

5. From 4, all countries must grow at the same rate,  $g_j^* = g^*$ .
6.  $5 + \text{Lemma} \implies \frac{\dot{c}_j(t)}{c_j(t)} = g^*$ .

# Acemoglu and Ventura (2002): Summary

- ▶ Under autarky,  $AK$  model predicts that countries with different discount rates  $\rho_j$  should grow at different rates.
- ▶ Under free trade, Proposition 2 shows that all countries grow at the same rate.
- ▶ Because of terms of trade effects, everything is *as if* we were back to a model of diminishing marginal returns.
- ▶ From a theoretical standpoint, Acemoglu and Ventura (2002) is the mirror image of Ventura (1997).

# Economic integration as a source of economic growth

- ▶ Is “economic integration” trade in goods or flows of ideas?
- ▶ Openness vs trade: Recall Ed Prescott quote from week 1
- ▶ There is no general trade-growth relationship from economic theory. In some models, trade restrictions slow global growth rate. In others, they speed it up.

# Rivera-Batiz & Romer (1991)

Economic integration:

- ▶ Scale effects of integration (similar countries)
- ▶ Two models: goods flows vs ideas flows

Primitives:

- ▶ Like Romer (1990), production function (for consumption and capital goods) uses human capital  $H$ , labor  $L$ , and machines (varieties indexed by  $i$ )

$$Y(H, L, x(\cdot)) = H^\alpha L^\beta \int_0^A x(i)^{1-\alpha-\beta} di$$

$$Y = C + \int_0^A x(i) di = C + K$$

- ▶ Patent holder for machine  $j$  rents it out to all manufacturing firms.
- ▶ Patent value is  $P_A$ , NPV of monopoly rent minus cost of the machines.

# Rivera-Batiz & Romer: Two possible R&D functions

Knowledge-driven R&D:

$$\dot{A} = \delta H_A A$$

Prior knowledge stock  $A$  is freely available, so price of innovation is hiring human capital, with  $H = H_A + H_Y$

Lab-equipment R&D:

$$\dot{A} = B H^\alpha L^\beta \int_0^A x(i)^{1-\alpha-\beta} di$$

In lab-equipment model,  $P_A = 1/B$  because opportunity cost is manufactured output

Collapse to one output equation

$$C + \dot{K} + \dot{A}/B = H^\alpha L^\beta A (K/A)^{1-\alpha-\beta} = H^\alpha L^\beta K^{1-\alpha-\beta} A^{\alpha+\beta}$$



# Rivera-Batiz & Romer (1991): Balanced Growth

Euler equation from CRAA prefs with elasticity  $\sigma$  yields

$$r = \rho + \sigma \frac{\dot{C}}{C} = \rho + \sigma g$$

Knowledge-driven R&D:

$$r = (\delta H - g) \frac{(\alpha + \beta)(1 - \alpha - \beta)}{\alpha} = (\delta H - g) / \Lambda$$

Equilibrium  $r$  is therefore

$$g = (\delta H - \Lambda \rho) / (\Lambda \sigma + 1)$$

Lab-equipment R&D:

$$r = \Gamma H^\alpha L^\beta$$

Equilibrium  $r$  is therefore

$$g = (\Gamma H^\alpha L^\beta - \rho) / \sigma$$

In both models, the scale of  $H$  (and  $L$ ) affects the growth rate.

Integration of two countries ( $2H$  and  $2L$ ) raises growth (but, welfare on transition path?)

# Rivera-Batiz & Romer (1991): Flows of goods and ideas

- ▶ Knowledge-driven R&D: Trade in goods doesn't affect growth
  - ▶ Integration doubles marginal product of  $H$  in manufacturing
  - ▶ Integration doubles value of patent holding ( $MPH$  in R&D)
  - ▶ Level effect courtesy of doubling machine varieties
  - ▶  $H_A/H_Y$  unaffected, so no growth effect because  $\dot{A} = \delta H_A A$
- ▶ Knowledge-driven R&D: Ideas flows do affect growth
  - ▶ Rise in research productivity:  $\dot{A} = \delta H_A (A + A^*)$
  - ▶ Level effect is negative because  $H$  drawn away from manufacturing
  - ▶ Trade in goods ensures non-overlapping varieties invented
- ▶ Lab-equipment R&D: Trade in goods akin to complete integration
  - ▶ Don't think goods = level effect and ideas = growth effect
  - ▶ Larger market for patent usage and  $P_A = 1/B \rightarrow$  equilibrium requires higher interest rate
  - ▶ Higher  $r$  yields same  $g$  as  $2H, 2L$  integration
- ▶ Growth is about increasing returns in both models

# Buera and Oberfield (2020): Overview

- ▶ Openness vs trade: Recall Ed Prescott quote from week 1
- ▶ Knowledge diffusion/technology imitation models (c.f. Lucas and Moll, Perla and Tonetti 2014) neglect spatial dimension
- ▶ Model with geography of buyers and sellers lends itself to geography of knowledge diffusion
- ▶ Learning from foreign sellers (trade  $\rightarrow$  imitating imports)
- ▶ Learning from domestic producers (trade  $\rightarrow$  pool of peers)
- ▶ Productivity is Fréchet so the static mechanics follow Eaton & Kortum (2002)

# Closed economy that motivates Fréchet distribution

Fixed interval of goods,  $s \in [0, 1]$ . Linear production:

$$y(s) = ql(s) \quad (1)$$

New ideas via combination of exogenous ingenuity  $z$  with arrival rate  $A_t(z)$  and random draw from existing  $q'$

$$q = zq'^\beta$$

$\beta \in [0, 1)$ . Two observations about this function:

- ▶  $\frac{zq_1^\beta}{zq_2^\beta} = \left(\frac{q_1}{q_2}\right)^\beta$  :  $\beta$  governs  $q$ 's sensitivity to quality of prior  $q'$
- ▶  $\frac{q}{q'} = zq'^{\beta-1}$  declines with  $q'$ : better  $q'$  are harder to improve

# The closed economy's knowledge frontier $\rightarrow$ Fréchet

$F_t(q)$  denotes the fraction of goods for which no producer's productivity exceeds  $q$ .

$$\frac{d}{dt} \ln F_t(q) = \lim_{\Delta \rightarrow 0} \frac{F_{t+\Delta}(q) - F_t(q)}{\Delta F_t(q)} = - \int_0^\infty A_t(q/q'^\beta) dG_t(q')$$

- ▶ Assumption 1:  $A_t(z) = \alpha_t z^{-\theta}$  (later,  $\alpha_t = \alpha_0 \exp(\gamma t)$ )
- ▶ Proposition 1: the appropriately-scaled frontier of knowledge converges asymptotically to Fréchet with shape  $\theta$
- ▶ Assumption 2: Initial distribution  $F_0(q)$  is Fréchet
- ▶ Proposition 1 and Assumption 2 imply knowledge growth is

$$\frac{d}{dt} \ln F_t(q) = -\alpha_t q^{-\theta} \int_0^\infty x^{\beta\theta} dG_t(x) \quad (4)$$

- ▶ If source distribution is domestic distribution,  $G_t(q) = F_t(q)$ ,

$$\dot{\lambda}_t = \alpha_t \Gamma(1 - \beta) \lambda_t^\beta$$

Let's discuss exogenous vs semi-endogenous vs endogenous growth

# Trade model

- ▶ The static trade model is Bernard, Eaton, Jensen, Kortum (2003) with EK's subscript order

$$\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}}$$

- ▶ Learning from sellers:  $G_i(q) = G_i^S(q) \equiv \sum_j H_{ij}(q)$

$$\dot{\lambda}_{it} = \alpha_{it} \int_0^\infty x^{\beta\theta} dG_i^S(q) = \Gamma(1 - \beta) \alpha_{it} \sum_j \pi_{ij} \left( \frac{\lambda_j}{\pi_{ij}} \right)^\beta$$

Trade costs impede goods transfers, not idea transfers

- ▶ Learning from domestic producers (w/ triangle inequality):  
 $G_i(q) = G_i^P(q) \equiv \frac{H_{ii}(q)}{H_{ii}(\infty)}$

$$\dot{\lambda}_{it} = \alpha_{it} \int_0^\infty x^{\beta\theta} dG_i^P(q) = \Gamma(1 - \beta) \alpha_{it} \left( \frac{\lambda_i}{\pi_{ii}} \right)^\beta$$

## Gains from trade

- ▶ ACR (2012) formula applies:  $y_i \equiv \frac{w_i}{P_i} \propto \left( \frac{\lambda_i}{\pi_{ii}} \right)^{1/\theta}$
- ▶ Assuming exogenous growth in arrival rate  $A_t(z)$  at  $\gamma$  on a BGP, detrend variables  $\hat{\lambda}_{it} = \lambda_{it} \exp \left( \frac{\gamma}{1-\beta} t \right)$
- ▶ The detrended stocks of knowledge on a BGP solve:

$$\text{Sellers: } \hat{\lambda}_i = \frac{(1-\beta)\Gamma(1-\beta)}{\gamma} \hat{\alpha}_i \sum_{j=1}^n \pi_{ij}^{1-\beta} \hat{\lambda}_j^\beta \quad (10)$$

$$\text{Producers: } \hat{\lambda}_i = \frac{(1-\beta)\Gamma(1-\beta)}{\gamma} \hat{\alpha}_i \left( \frac{\hat{\lambda}_i}{\pi_{ii}} \right)^\beta \propto \hat{\alpha}_i^{\frac{1}{1-\beta}} \pi_{ii}^{-\frac{\beta}{1-\beta}} \quad (11)$$

- ▶ Gains relative to autarky along the BGP:

$$\text{Sellers: } \frac{\lambda_i}{\lambda_i^{\text{aut}}} = \left( \sum_{j=1}^n \pi_{ij}^{1-\beta} \left( \frac{\lambda_j}{\lambda_i} \right)^\beta \right)^{\frac{1}{(1-\beta)}} \quad (12)$$

$$\text{Producers: } \frac{\lambda_i}{\lambda_i^{\text{aut}}} = \pi_{ii}^{-\frac{\beta}{1-\beta}} \quad (13)$$

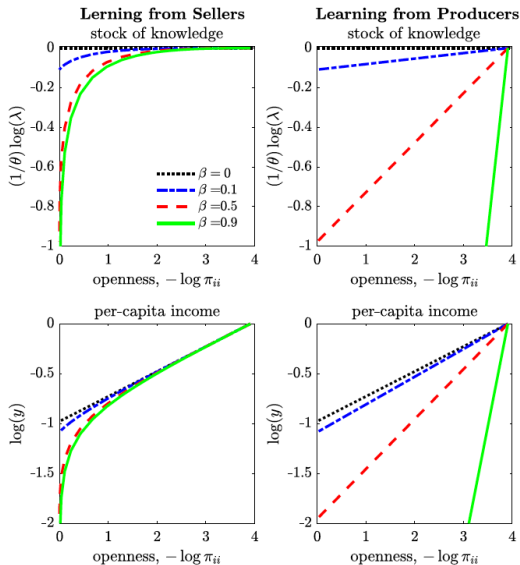


FIGURE 1.—Gain From Reducing Common Trade Barriers. Note: This figure shows each country's stock of knowledge and per-capita income relative to their values under costless trade. See footnote 27 for additional details of the calibration used in this figure.



# Quantitative exploration

Preferred calibration: “both the gains from trade and the fraction of variation of TFP growth accounted for by changes in trade more than double relative to a model without diffusion”

Discuss the following assumptions/choices

- ▶  $\gamma = 0.01$  (and  $\beta$ ) to match average US population growth 1962-2000
- ▶ the exogeneity of physical and human capital paths
- ▶ the rationalization of trade imbalances
- ▶ the exclusion of countries with large reexports
- ▶ backing out  $\kappa_{ijt}$  and the triangle inequality

# Conclusions

Thinking about growth with trade in mind

- ▶ Growth miracles occurred in export-oriented economies
- ▶ Emerging economies import ideas, not just goods
- ▶ Closed-economy conclusions can be reversed by trade