ECON G6905

Topics in Trade

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This week: Identification, calibration, and exact hat algebra

An introduction to econometric issues in applied general equilibrium models using the CES Armington model of trade as our running example:

- Identification is a model property and precedes estimation
- Comparative statics are counterfactual scenarios
- Calibrating parameters to match observed shares is estimation
- Specification checks still apply to saturated models
- Finite-sample performance is poor with one parameter per observation

Identification, in general: A refresher

- "Econometric identification really means just one thing: model parameters or features being uniquely determined from the observable population that generates the data." (Lewbel 2019)
- For a parameter to be identified, alternative values of that parameter must imply different distributions of the observable data.
- Identification presumes structure: Identification of a structural feature (i.e., a parameter) is only defined in the context of a class defined by a maintained hypothesis (a "model").
- "identification logically precedes estimation, inference, and testing."
- A model is or is not point identified. (cf. "better identification")

Asides: Name your errors terms (are those supply shocks or demand shocks in the residual?). "Consideration of underlying structure is needed to convincingly argue that covariates included in the model as controls will actually function as they are intended."

Are CES Armington model parameters identified by trade flows?

A statement equivalent to "one can calibrate the model to observed shares without estimating the full structure of the model" is "the model's parameters are not identified by observed trade shares"

- If $\{T_i, L_i\}$, $\{\tau_{ij}\}$, ϵ, and $\{w_i\}$ satisfy

$$w_i L_i = \sum_j \frac{T_i \left(w_i \tau_{ij}\right)^{-\epsilon}}{\sum_\ell T_\ell \left(w_\ell \tau_{\ell j}\right)^{-\epsilon}} w_j L_j$$

  then $\{\delta_i^\epsilon T_i, L_i\}$, $\{\delta_i \tau_{ij}\}$, ϵ, and $\{w_i\}$ also satisfy this system with the same $\{X_{ij}\}$ $\forall \delta_i > 0$. Iceberg trade costs: productivity and export-cost shifter are indistinguishable
- If $\{T_i, L_i\}$, $\{\tau_{ij}\}$, ϵ, and $\{w_i\}$ satisfy this system, then $\{T_i, L_i\}$, $\{\xi_j \tau_{ij}\}$, ϵ, and $\{w_i\}$ also satisfy this system for any $\xi_j > 0$. Expenditure shares reflect relative prices.
- Similarly, $\{\mu_i^\epsilon T_i, \mu_i^{-1} L_i\}$, $\{\tau_{ij}\}$, ϵ, and $\{\mu_i w_i\}$ also satisfy this system for any $\mu_i > 0$. (Isomorphic endowment economy has $Q_i = T_i^{1/\epsilon} L_i$)

Are CES Armington model parameters identified by trade flows?

Suppose you want to identify *relative* trade costs, τ_{ij}/τ_{jj}. Normalize $\tau_{jj} = 1 \ \forall j$.

Using two prior results, if $\{T_i, L_i\}$, $\{\tau_{ij}\}$, ϵ, and $\{w_i\}$ satisfy this system, then $\{\delta_i^{\epsilon} T_i, L_i\}$, $\{\delta_i \xi_j \tau_{ij}\}$, ϵ, and $\{w_i\}$ also satisfy this system for any $\delta_i > 0$ and $\xi_j > 0$.

$$Y_i = \sum_j \frac{T_i \left(w_i \tau_{ij}\right)^{-\epsilon}}{\sum_\ell T_\ell \left(w_\ell \tau_{\ell j}\right)^{-\epsilon}} Y_j = \sum_j \frac{X_{ij}}{X_{jj}} X_{jj} = \sum_j \frac{T_i \left(w_i \tau_{ij}\right)^{-\epsilon}}{T_j \left(w_j \tau_{jj}\right)^{-\epsilon}} X_{jj}$$

$$= \sum_j \frac{\delta_i^{\epsilon} T_i \left(w_i \delta_i \xi_j \tau_{ij}\right)^{-\epsilon}}{\delta_j^{\epsilon} T_j \left(w_j \delta_j \xi_j \tau_{jj}\right)^{-\epsilon}} X_{jj}$$

If one chooses $\xi_j = \delta_j^{-1}$, then this preserves $\tau_{jj} = 1$.

We cannot recover productivities and *directed* trade costs from directed trade flows.

We can normalize $T_i = L_i = 1 \ \forall i$ and $\tau_{ii} = 1 \ \forall i$

Using three prior results: If $\{T_i, L_i\}$, $\{\tau_{ij}\}$, ϵ, and $\{w_i\}$ satisfy this system, then $\{\mu_i^\epsilon \delta_i^\epsilon T_i, \mu_i^{-1} L_i\}$, $\{\delta_i \xi_j \tau_{ij}\}$, ϵ, and $\{\mu_i w_i\}$ also satisfy this system for any $\mu_i > 0$, $\delta_i > 0$, and $\xi_j > 0$.

Choose $\mu_i = L_i$, $\delta_i = \mu_i^{-1} T_i^{-1/\epsilon}$, and $\xi_j = \delta_j^{-1}$. This defines a trade-cost matrix τ and wage vector w satisfying the system of equations given $T_i = L_i = 1 \ \forall i$ and ϵ.

By $L_i = 1$, $w_i = Y_i = \sum_j X_{ij}$. That's observed. We can thus identify relative trade costs from relative expenditures and relative producer prices:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{Y_i}{Y_j}\right)^{-\epsilon} \left(\frac{\tau_{ij}}{\tau_{jj}}\right)^{-\epsilon} \implies \frac{\tau_{ij}}{\tau_{jj}} = \left(\frac{X_{ij}}{X_{jj}}\right)^{-1/\epsilon} \left(\frac{Y_i}{Y_j}\right)^{-1}$$

In sum, imposing $\tau_{ii} = T_i = L_i = 1 \ \forall i$ determines a unique solution for τ_{ij} given ϵ:

$$\tau_{ij} = (X_{ij}/X_{jj})^{-1/\epsilon} Y_j/Y_i$$

Trade flows alone are insufficient

- Even after normalizing $\{T_i, L_i, \tau_{ii}\}_{i=1}^{N}$, the union of trade costs $\{\tau_{ij}\}$ and the trade elasticity ϵ are not identified from trade flows $\{X_{ij}\}$ alone.
- Recall from week 2 that trade-cost proxies like distance are insufficient to identify ϵ: one needs trade costs to estimate trade elasticity (versus distance elasticity)
- Researchers often use import tariffs and freight charges, which provide needed ad valorem trade costs assuming that they are appropriately orthogonal to other trade costs in the error term

Given the trade elasticity, trade flows are sufficient

- Given trade flows X_{ij}, the trade elasticity ϵ, and the assumption $T_i = L_i = \tau_{ii} = 1 \; \forall i$, there exists a (unique) solution for τ_{ij}
- Therefore, there exist parameter values consistent with the observed sufficient statistics for *any* (non-negative) trade matrix

  $\tau_{ii} = 1$ normalization does presume $X_{ii} > 0$

  obviously the parameter vector is not unique
- In other words, this is a "saturated" model: $N(N-1)$ degrees of freedom in τ_{ij} accommodate $N(N-1)$ observable outcomes without any restrictions

  Trade matrix has N^2 elements. Why is one row redundant?
- Bottom line: Any trade matrix $\{X_{ij}\}_{ij}$ can be rationalized by bilateral trade costs

Factory-gate prices do not discipline the Armington model much

- So far, we use only trade volumes (values), not distinguishing prices and quantities
- Recall that bilateral trade costs τ_{ij} are isomorphic to bilateral preference shifters β_{ij} in the Armington model (week 1).
- Conditional on $Y_i = p_i Q_i$, any observed p_i can be rationalized by introducing preference shifters.
- Similarly, in Melitz (2003), firm-level productivity φ might shift TFPR by lowering costs (TFPQ) or raising demand (quality). Given this isomorphism, firm-level prices can do little to discipline the model.
- If you've committed to the Eaton and Kortum (2002) model with symmetric preferences, then the price index may be more informative. But an isomorphic "quality" interpretation of T_i seems straightforward.

Counterfactual outcomes by exact hat algebra in trade model

Recall exact hat algebra for Armington CES model from week 1:

$$\hat{w}_i \hat{L}_i = \hat{Y}_i = \sum_{j=1}^{N} \gamma_{ij} \hat{\lambda}_{ij} \hat{Y}_j = \sum_{j=1}^{N} \frac{\gamma_{ij} \hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon}}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon} \hat{\tau}_{lj}^{-\epsilon}} \hat{w}_j \hat{L}_j$$

- The name refers to Jones (1965) "hat algebra" for comparative statics
- It's "exact" because it's global given whole demand and supply system

A system of equations characterizing counterfactual endogenous variables $\hat{w}_i$ in terms of initial equilibrium shares $\lambda_{ij} \equiv \frac{X_{ij}}{X_j}$ and $\gamma_{ij} \equiv \frac{X_{ij}}{Y_i}$, the trade elasticity ϵ, and counterfactual exogenous changes $\hat{L}_i, \hat{T}_i, \hat{\tau}_{ij}$

Exact hat algebra: Sufficient statistics for comparative statics

$$\hat{w}_i \hat{L}_i = \sum_{j=1}^{N} \frac{\gamma_{ij} \hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon}}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon} \hat{\tau}_{lj}^{-\epsilon}} \hat{w}_j \hat{L}_j$$

Exact hat algebra concerns comparative statics, not calibration (blog post)

- The system of equations defines counterfactual outcomes for any model of the baseline shares λ_{ij} and γ_{ij} and the trade elasticity ϵ
- Model parameters $\{T_i, L_i\}$ and $\{\tau_{ij}\}$ matter (only) because they determine the baseline shares

Identifying comparative statics may not require identifying all model parameters

- $\{T_i, L_i\}$ and $\{\tau_{ij}\}$ matter only because they determine the baseline shares
- Even if you did distinguish between T_i and L_i, it would be of no consequence for counterfactual changes if they delivered the same λ_{ij} and γ_{ij}
- Though one might find it strange to know $\hat{T}_i \equiv T_i'/T_i$ when you do not know T_i

Counterfactual scenarios and required parameter values

Some counterfactual scenarios require (knowledge of) more parameters than just the information encoded in initial equilibrium shares λ_{ij} and γ_{ij} and the trade elasticity ϵ.

Eaton and Kortum (2002) conduct two thought experiments:

- Autarky: $\tau_{ij} \to \infty$ for $i \neq j$. $\hat{\tau}_{ij} = \infty$.
- "Zero gravity": $\tau_{ij} \to \tau_{jj}$ for $i \neq j$. $\hat{\tau}_{ij} = \frac{\tau_{jj}}{\tau_{ij}}$.

Knowing model parameters up to λ_{ij}, γ_{ij}, ϵ is *insufficient* to compute the zero-gravity scenario because that requires knowing $\hat{\tau}_{ij} = \frac{\tau_{jj}}{\tau_{ij}}$.

Given the trade elasticity, trade shares are sufficient

- Given trade flows data (including the diagonal), you observe the sufficient statistics $\lambda_{ij} \equiv \frac{X_{ij}}{X_j}$ and $\gamma_{ij} \equiv \frac{X_{ij}}{Y_i}$
- Given trade flows X_{ij}, the trade elasticity ϵ, and the assumption $T_i = L_i = \tau_{ii} = 1 \; \forall i$, there exists a (unique) solution for τ_{ij}
- Therefore, there exist parameter values consistent with the observed sufficient statistics for *any* (non-negative) trade matrix
  $\tau_{ii} = 1$ normalization does presume $X_{ii} > 0$
  obviously the parameter vector is not unique
- Ergo, feel free to use the observed baseline shares λ_{ij} and γ_{ij} without estimating T_i, L_i, and τ_{ij}

Prior descriptions of exact hat algebra and calibration

Costinot and Rodriguez-Clare (*Handbook* 2014):
"We have described how to use gravity models to perform welfare and counterfactual analysis. An appealing feature of this approach, which we have referred to as the exact hat algebra, is that the impact of various counterfactual scenarios can be computed without estimating the full structure of the model. All the relevant information about preferences, technology, and trade costs can be inferred directly from the cross-section of bilateral trade flows and estimates of the trade elasticity."

Caliendo and Parro (2015):
"we can solve for the general equilibrium of the model without needing to estimate parameters which are difficult to identify in the data, [such] as productivities and iceberg trade costs." (c.f. Lewbel 2019)

Using observed shares is calibrating (combinations of) parameters

Finding (products of) parameters to match observed shares is calibrating them.

For purposes of computing counterfactual outcomes, using observed shares typically leverages two properties of the model and the counterfactual scenario:

- The model has enough degrees of freedom that there exist parameters that can rationalize any observed pattern of trade flows. ("Saturated" model)
- The baseline shares are sufficient to calibrate the combinations of parameters needed to compute counterfactual outcomes.

Corollaries to these two features when using observed shares:

- No pattern of baseline shares can reject the model specification. (Models with fewer degrees of freedom admit tests.)
- The set of admissible counterfactual scenarios is restricted. (e.g., zero gravity)

Admissible counterfactuals and model specification

Heckman & Vytlacil (2007): "for many decisions (policy problems), only combinations of explicit economic parameters are required—no single economic parameter need be identified"

> *Following Marschak's Maxim, we postulate specific economic questions that are interesting to address and ask what combinations of underlying economic parameters or functionals are required to answer them. Answering one question well usually requires fewer assumptions, and places less demands on the data, than answering a wide array of questions – the original goal of structural econometrics.*

Gene Grossman (2022 transcript):

> *When you force a simple model to fit a complicated world and you don't get standard errors, you don't know what damage you're doing. And then you immediately run to counterfactuals, and what am I to make of those numbers that it generates?*

(Using sufficient statistics does not require you to use an unfalsifiable model)

Sufficient statistics and model validation

Chetty (2009): "A second potential weakness of sufficient-statistic formulas is that they are more easily misapplied than structural methods. This is because one can draw policy conclusions from a sufficient-statistic formula without assessing the validity of the model upon which it is based. . . In contrast, because structural methods require full estimation of the model, policy conclusions can be drawn only from models that fit the data."

A saturated model may still be falsifiable

- We cannot use baseline shares (nor prices) to evaluate the model
- The CES model of trade flows is a demand system and we know how to test demand systems (at least, the one-elasticity straightjacket of CES/IIA)
- One approach: Estimate an encompassing specification and test the parameter restrictions imposed by CES (e.g., Hausman and McFadden (1984) test IIA logit by estimating a nested-logit specification and testing whether $\lambda = 1$)
- Other tests specifically leverage IIA: Estimate on subsets of countries and see if elasticities are common (the other Hausman and McFadden (1984) test)
- One can also test the complete passthrough from tariffs to prices (Head, Mayer 2023)
- One could test the triangle inequality on trade costs (cf. Foellmi, Hepenstrick, Torun 2024)

The CES/IIA model with one elasticity does not fit the data

Adao, Costinot, and Donaldson (2017) study reduced exchange economies:

- "the mixed CES demand system allows data to speak to whether this independence of irrelevant alternatives embodied in CES holds empirically"
- "the deviations from IIA…are a systematic feature of the data…related to the similarity of competitors in terms of per capita GDP"

Lind, Ramondo (2023) develop a Ricardian model with richer productivity patterns:

- "By relaxing the independence assumptions used in the literature, the model generates import demand systems spanning the entire generalized extreme value (GEV) class (McFadden 1978, 1981)."
- "significant sharing of technologies across countries and sectors…manifests in considerable heterogeneity in correlation in productivity, which, in turn, changes the answers to standard counterfactuals"
- "our [latent factor model] estimates capture quite accurately departures from IIA within and across sectors"

What's in those baseline shares?

Gene Grossman (2022 transcript):

> *So the models often fit the data by introducing things we can't measure like amenities and productivity. In my generation, we used to call that the error term, but now it's part of the model. Then when you do counterfactuals you don't know what the hell they are, but you have to hold them constant. So why would we think these things would remain constant once we start changing the environment?*

Overfitting

- One overfits a model by using a more flexible parameterization that improves in-sample fit but worsens out-of-sample performance.
- Adding more covariates always improves the R^2 of OLS, but "use as many variables as possible" is not the preferred specification of forecasters (Perhaps more relevant: Consider 2SLS with N observations and N instrumental variables. The first-stage R^2 of 1.0 is not good news: $\beta^{\text{OLS}} = \beta^{\text{2SLS}}$. Now imagine using N white-noise draws as IVs.)
- Hastie, Tibshirani, and Friedman (2009): "Unfortunately training error is not a good estimate of the test error, as seen in Figure 7.1. Training error consistently decreases with model complexity, typically dropping to zero if we increase the model complexity enough. However, a model with zero training error is overfit to the training data and will typically generalize poorly."

A saturated model is overfit if there's noise

Belloni, Chernozhukov, Hansen (2014) come close to describing the calibrated-shares procedure:

The key concept underlying the analysis of high-dimensional data is that dimension reduction or "regularization" is necessary to draw meaningful conclusions. The need for regularization can easily be seen when one considers an example where there are exactly as many variables (plus a constant) as there are observations. In this case, the ordinary least squares estimator will fit the data perfectly, returning an R^2 of one. However, using the estimated model is likely to result in very poor forecasting properties out-of-sample because the model estimated by least squares is overfit: the least-squares fit captures not only the signal about how predictor variables may be used to forecast the outcome, but also fits the noise that is present in the given sample, and is not useful for forming out-of-sample predictions. Producing a useful forecasting model in this simple case requires regularization; that is, the estimates must be constrained so that overfitting is avoided and useful out-of-sample forecasts can be obtained.

A more critical perspective on calibrated shares

Antras and Chor (Handbook, 2022):

“ practitioners of this approach often praise how parsimonious it is relative to CGE models, which involve the estimation of thousands of parameters. An often glossed-over fact, however, is that the hat-algebra approach requires the model to fit the data *exactly*, which amounts to calibrating all parameters of the model (or combinations of them) to values that ensure this exact fit …

although quantitative work often requires strong assumptions on functional forms, calibrating thousands of parameters to fit the data exactly can be problematic for the validity or reliability of the counterfactual predictions of those models. The problem is similar to overfitting in regression analysis leading to poor out-of-sample performance. As recently shown in Dingel and Tintelnot (2020), this is a particularly severe problem in spatial environments in which the data the model is fitted to contains a significant number of zeros. Note that even in the WIOD – a WIOT focusing on relatively rich countries – the share of zeroes is 13.7% in the matrix of input-use coefficients and 46.8% in the matrix of final-use column vectors.”

Dingel and Tintelnot (2025)

The calibrated-shares procedure can perform poorly in high-dimensional settings because matching (noisy) observed shares amount to overfitting the model parameters.

Go to Dingel and Tintelnot (2025) slidedeck.

First half of paper concerns this overfitting problem. Second half of paper introduces a finite model to quantify uncertainty.