ECON G6905
Topics in Trade

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Fall 2025, Week 1

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Outline of today

- Introduction + logistics
- Overview of the course
- Brief introduction to trade theory
- The CES Armington model of international trade

Logistics

This class

- Wednesdays, 8:10-10:00, IAB 1101
- Jonathan Dingel
  - Email: jid2106@columbia.edu
  - Office: IAB 1126B
  - Office hours: By appointment, please email
- Course materials: github.com/jdingel/econ6905 and courseworks2.columbia.edu

Broader context:

- This class is the economic-geography bridge between Weinstein's trade class and Davis's urban class
- I will emphasize computational aspects
- You should attend the Trade and Spatial Colloquium (Wednesdays, 12–1, IAB 1101)

Assessment

My goal is to introduce some concepts and tools in international trade and economic geography so you can tackle relevant research questions

- Grades based on assignments (70%) and a final exam (30%)
- Three types of assignments
  - Economics: Derive a theoretical result or survey an empirical literature.
  - Programming: Write a function that solves for equilibrium or estimates a parameter.
  - Referee report: Assess a recent working paper.
- Final exam at end of semester

Grab assignments from GitHub. Submit your work via Courseworks.

Coding

Submit transparent, self-contained code:

- Your code must reproduce your work in the “just press play” sense of the AEA Data Editor
- You may use Julia or Matlab. Use Julia.

See my recommended resources webpage for suggestions.

- Grant McDermott - Data science for economists
- Ivan Rudik - AEM 7130 Dynamic Optimization
- Paul Schrimpf and Jesse Perla - Computational Economics with Data Science Applications
- Jesus Fernandez-Villaverde - Computational Methods for Economists
- Perla, Sargent, Stachurski - Quantitative Economics

How many have used: Matlab? Julia? Git? Build automation?

Objectives

- My goal is to prepare students to tackle research questions in trade, spatial, and urban economics
- Writing papers is about matching skills with opportunities
- In my experience, spotting opportunities is a hard-to-teach combination of insight and luck
- This class will aim to equip you with skills so your technical quiver is full when you spot a target

Topics

1. The CES Armington model
2. Gains from trade and comparative advantage
3. Quantitative Ricardian trade models
4. Gravity regressions
5. Multiple factors of production
6. Increasing returns and home-market effects
7. Agglomeration economies
8. Quantitative spatial models
9. Quantitative urban models
10. Exact hat algebra and calibration
11. Spatial sorting of skills and sectors
12. Discrete choice estimation and simulations
13. Spatial environmental economics

See my comments on "Linkages between international trade and urban economics"

Why trade and spatial are interesting

- International trade has long intellectual history (Smith, Ricardo) and is hot policy topic today (Brexit, Trump)
- Healthy balance of theory and empirics (cf. theory-dominated from 1817 to 1990s) in which each informs the other
- Trade has tools and insights relevant for topics ranging from intracity commuting to national TFP growth
- I used to say trade economists sometimes have a data advantage because governments track cross-border transactions
- Spatial economics is a small but rapidly growing field (e.g., The rapid rise of spatial economics among JMCs, UEA history)

Why are you interested in trade/spatial/urban?

This week, we start with international trade

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Trade's interplay between theory and empirics

Descriptive facts motivate theoretical work

- Observed intra-industry trade motivated "new trade theory" (e.g., Krugman 1980)
- Observed firm-level heterogeneity motivated "new new trade theory" (e.g., Melitz 2003)

Empirical evidence comes from wide range of methods

- Descriptive statistics
- Estimated/calibrated quantitative models
- Applications employing sufficient statistics
- Quasi-natural experiments (rare, but see Japanese autarky, Suez Canal, the telegraph, etc)

Testing is tricky: See Harrigan (2001) and Adao et al (2023)

- Is it a "test"? Is there a clearly specified alternative hypothesis?
- How does the test isolate the distinctive GE prediction?
- Today, many have "abandon[ed] testing altogether"

International trade theory

- A dominant view is that international trade is an applied branch of general-equilibrium theory
- Any GE model has preferences + technology + equilibrium
- International trade theory focuses on locations, such that preferences (rarely) and technology (typically) are location-specific
- Trade theory traditionally has "international" goods markets and "domestic" factor markets
- Consumers have preferences over goods; factors are employed to produce goods
- Questions: How does international integration affect the goods market, the factor market, and welfare?
- One flavor of spatial economics is trade in goods plus mobile factors.

Variants of trade models

One view: "positive trade theory uses a variety of models, each one suited to a limited but still important range of questions" (Jones and Neary 1980)

	Demand	Supply	Market structure
Goods markets	General; CES preferences; Translog, NHCES, etc	Constant returns to scale; Increasing returns	Perfect competition; Monopolistic competition; Oligopoly
Factor markets	Demand derived from supply of goods	Often perfectly inelastic	Almost always competitive

If you stop at the goods market, it's partial-equilibrium.

Neoclassical trade models

- "Neoclassical trade models" are characterized by three key assumptions:
  - perfect competition
  - constant returns to scale
  - no distortions
- Can accommodate decreasing returns to scale (DRS) using "hidden" factors in fixed supply; IRS is "new trade theory"
- Given the generality of these assumptions, there is not a wealth of results, but one can obtain two canonical insights:
  - gains from trade (Samuelson 1939)
  - law of comparative advantage (Deardorff 1980)
- By contrast, we are going to dive deeply into one very specific neoclassical model

The CES Armington model

Features:

- Concise: A one-elasticity model
- Relevant: Same macro-level predictions as other, important gravity-based models

Shortcomings:

- Supply side (endowment economy) is wholly uninteresting
- Preferences (national differentiation with IIA) are ad hoc

We will discuss

- Primitives
- Existence and uniqueness of equilibrium
- Solving for equilibrium
- Computing counterfactual outcomes

Armington model with CES prefences

- Each country has its own "signature" good (others have zero productivity in this good; maximal absolute advantage)
- Consumers in each country have identical CES preferences over the N goods with elasticity σ (see Dingel 2009 for CES refresher)
- Bilateral trade costs of the iceberg form τ_{ij}
- Demand: Consumer in j with total expenditure X_j spends X_{ij} on good from i

$$X_{ij} = \frac{(p_i \tau_{ij})^{1-\sigma}}{\sum_{\ell} (p_\ell \tau_{\ell j})^{1-\sigma}} X_j = \frac{(p_i \tau_{ij})^{1-\sigma}}{P_j^{1-\sigma}} X_j$$

- Economy i endowed with Q_i units so GDP is $Y_i = p_i Q_i$

$$X_{ij} = \frac{Y_i^{1-\sigma}}{Q_i^{1-\sigma}} \frac{X_j}{P_j^{1-\sigma}} \tau_{ij}^{1-\sigma}$$

- Balanced-trade equilibrium is $\{Y_i\}_{i=1}^{N}$ such that

$$X_i = Y_i = \sum_j X_{ij}$$

Equilibrium system of equations

Combine the last two equations to get N equations in N unknowns:

$$\begin{aligned} Y_i &= \sum_j X_{ij} \\ &= \sum_j \frac{Y_i^{1-\sigma}}{Q_i^{1-\sigma}} \frac{Y_j}{P_j^{1-\sigma}} \tau_{ij}^{1-\sigma} \\ &= \sum_j \frac{Y_i^{1-\sigma}}{Q_i^{1-\sigma}} \frac{Y_j}{\sum_\ell \left(\tau_{\ell j} Y_\ell / Q_\ell\right)^{1-\sigma}} \tau_{ij}^{1-\sigma} \end{aligned}$$

The N unknowns can be $\{Y_i\}_{i=1}^N$ or $\{p_i\}_{i=1}^N$:

$$p_i Q_i = \sum_j p_i^{1-\sigma} \frac{p_j Q_j}{\sum_\ell \left(p_\ell \tau_{\ell j}\right)^{1-\sigma}} \tau_{ij}^{1-\sigma}$$

Denote "trade elasticity" by $\epsilon \equiv \sigma - 1$ and expenditure shares by λ_{ij}

$$p_i Q_i = \sum_j \underbrace{\frac{p_i^{-\epsilon} \tau_{ij}^{-\epsilon}}{\sum_\ell \left(p_\ell \tau_{\ell j}\right)^{-\epsilon}}}_{\equiv \lambda_{ij}} p_j Q_j$$

Existence and uniqueness

- We want an equilibrium to exist: a model without an equilibrium leaves us little to analyze
- Should we want the equilibrium to be unique?
  - Certainly relevant for computing outcomes
  - May be relevant to identification (Lewbel 2019), but point identification concerns uniqueness of parameters given observable outcomes, not uniqueness of outcomes
  - May be relevant for counterfactual scenarios, but we can report sets of counterfactual equilibria (multiplicity seems more a threat to forecasting than counterfactual scenarios)

Allen, Arkolakis, Takahashi (2020) show

- $\sigma \neq 0$: an interior equilibrium exists
- $\sigma \geq 0$: all equilibria are interior
- $\sigma \geq 1$: interior eqlbm is unique (aggregate demand slopes down)

Recent related lit: Ouazad (2024) and Garg (2025) on enumerating all equilibria via polynomial roots; maybe check out HomotopyContinuation.jl

Solving for equilibrium numerically (1/2)

You want to find a fixed point $\{Y_i\}_{i=1}^N$ that satisfies

$$Y_i = \sum_j \frac{(Y_i/Q_i)^{-\epsilon}\,\tau_{ij}^{-\epsilon}}{\sum_\ell (Y_\ell \tau_{\ell j}/Q_\ell)^{-\epsilon}} Y_j.$$

Choose a numeraire to pin this down.

You might define a differentiable objective function and find its minimum (the fixed point where it is zero). This can be slow.

$$\min_{\{Y_i\}_{i=1}^N} \left(Y_i - \sum_j \frac{(Y_i/Q_i)^{-\epsilon}\,\tau_{ij}^{-\epsilon}}{\sum_\ell (Y_\ell \tau_{\ell j}/Q_\ell)^{-\epsilon}} Y_j \right)^2$$

An iterative approach can be quite fast. Function iteration means

- Guess $\{Y_i^s\}_{i=1}^N$ starting with $s = 0$.
- Compute implied LHS when using $\{Y_i^s\}_{i=1}^N$ in RHS
- Update $\mathbf{Y}^{s+1}$ based on convex combination of $\mathbf{Y}^s$ and implied $\mathbf{Y}$
- Iterate until $\mathbf{Y}^{s+1} = \mathbf{Y}^s$

Solving for equilibrium numerically (2/2)

As in Alvarez and Lucas (2007), define the excess demand function

$$f_i(\mathbf{p}) = \frac{1}{p_i}\sum_j \lambda_{ij} p_j Q_j - Q_i = \frac{1}{p_i}\sum_j \frac{\left(p_i\tau_{ij}\right)^{-\epsilon}}{\sum_\ell \left(p_\ell \tau_{\ell j}\right)^{-\epsilon}} p_j Q_j - Q_i$$

Compute equilibrium by defining mapping with damper $\kappa \in (0,1]$:

$$M_i(\mathbf{p}) = p_i\left[1 + \kappa f_i(\mathbf{p})/Q_i\right]$$

If we start with prices such that $\sum_{i=1}^N p_i Q_i = 1$, then

$$\begin{aligned}\sum_i M_i(\mathbf{p}) Q_i &= \sum_i p_i Q_i + \sum_i p_i \kappa f_i(\mathbf{p}) = 1 + \kappa \sum_i p_i \left[\frac{1}{p_i}\sum_j \lambda_{ij} p_j Q_j - Q_i\right] \\ &= 1 + \kappa \sum_i \sum_j \lambda_{ij} p_j Q_j - \kappa \sum_i p_i Q_i = 1\end{aligned}$$

This maps the set $\left\{\mathbf{p} \in \mathbb{R}_+^N : \sum_i p_i Q_i = 1\right\}$ to itself. Iteration converges to $M_i(\mathbf{p}) = p_i$ (see Alvarez and Lucas 2007).

Introducing a production function

Switch from an endowment economy to a simple production function

- One factor of production in fixed supply: L_i
- Constant returns to scale: $Q_i = A_i L_i$
- Perfect competition: $p_i = w_i / A_i$ and $Y_i = w_i L_i$
- (Choose units to define $T_i \equiv A_i^{\epsilon}$)

Our equilibrium system of equations is now

$$w_i L_i = \sum_j \frac{T_i \left(w_i \tau_{ij}\right)^{-\epsilon}}{\sum_\ell T_\ell \left(w_\ell \tau_{\ell j}\right)^{-\epsilon}} w_j L_j$$

Introducing asymmetric preferences

Consider an Armington model with asymmetric preferences:

$$U_j = \left(\sum_i \beta_{ij} q_{ij}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

$$\Rightarrow \frac{X_{ij}}{X_j} = \beta_{ij} \left(\frac{p_{ij}}{P_j} \right)^{1-\sigma} = \frac{w_i^{1-\sigma}}{P_j^{1-\sigma}} \beta_{ij} \tau_{ij}^{1-\sigma}$$

Bilateral trade costs and bilateral preferences are observationally equivalent.

(The CES price index P_j on this slide differs from previous P_j.)

Welfare

- There is only one factor of production and it is inelastically supplied
- If we know the CES price index, we can study the real wage w_i/P_i, real income w_iL_i/P_i, and so forth for each country
- Real wage in country j with symmetric preferences:

$$\frac{w_j}{P_j} = \frac{w_j}{\left(\sum_{i=1}^{N} \left(p_i \tau_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} = \frac{w_j}{\left(\sum_{i=1}^{N} \left(w_i \tau_{ij}/A_i\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}$$

Counterfactual outcomes

Counterfactual scenarios:

- If our model has parameters $\{T_i, L_i, \tau_{ij}, \epsilon\}$, a counterfactual scenario is an alternative parameter vector $\{T_i', L_i', \tau_{ij}', \epsilon'\}$.
- The model's baseline equilibrium outcomes are $\{w_i\}$ and the counterfactual outcomes by primes are $\{w_i'\}$

  (Be careful with $\epsilon \to \epsilon'$ exercises)

We can address many counterfactuals even in this simple model. Examples:

- How large are the gains from trade relative to autarky?
- How much would countries gain from frictionless trade?
- Which countries gain from Chinese productivity growth?
- When is productivity growth immiserizing?

Counterfactual outcomes by exact hat algebra

One way of stating counterfactual outcomes is "exact hat algebra" (Costinot and Rodriguez-Clare 2014)

- A counterfactual equilibrium can be expressed in terms of counterfactual endogenous outcomes relative to baseline endogenous outcomes, counterfactual exogenous parameters relative to baseline exogenous parameters, elasticities, and baseline equilibrium shares.
- The name refers to the "hat algebra" of Jones (1965): obtaining comparative statics by totally differentiating a model in logarithms
- It's "exact" because it's global (not only small changes) thanks to knowing the whole demand and supply system
- We will discuss the use (and misuse) of this technique (and its name) more later in the course

Counterfactual Armington outcomes by EHA (1/2)

Start from the market-clearing condition and the gravity equation:

$$w_i L_i = \sum_{j=1}^{N} \lambda_{ij} w_j L_j \qquad \lambda_{ij} = \frac{T_i \left(\tau_{ij} w_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}$$

We consider a shock to $\hat{T}_i \equiv \frac{T_i'}{T_i}$. By assumption, $\hat{\tau} = 1$ and $\hat{L} = 1$. We want to solve for the endogenous variables $\hat{\lambda}_{ij}, \hat{X}_{ij}$ and $\hat{w}_i$. In the following derivation, define "sales shares" by $\gamma_{ij} \equiv \frac{X_{ij}}{Y_i}$.

$$w_i L_i = \sum_{j=1}^{N} \lambda_{ij} w_j L_j, \quad w_i' L_i' = \sum_{j=1}^{N} \lambda_{ij}' w_j' L_j = \sum_{j=1}^{N} X_{ij}'$$

$$\hat{w}_i \hat{L}_i = \sum_{j=1}^{N} \frac{X_{ij}'}{w_i L_i} = \sum_{j=1}^{N} \frac{X_{ij}}{w_i L_i} \hat{X}_{ij} \equiv \sum_{j=1}^{N} \gamma_{ij} \hat{X}_{ij} \tag{1}$$

Counterfactual Armington outcomes by EHA (2/2)

$$\lambda_{ij} = \frac{T_i \left(\tau_{ij} w_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}, \quad \lambda'_{ij} = \frac{T'_i \left(\tau_{ij} w'_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T'_l \left(\tau_{lj} w'_l\right)^{-\epsilon}}$$

$$\hat{\lambda}_{ij} \equiv \frac{\lambda'_{ij}}{\lambda_{ij}} = \hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon} \frac{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}{\sum_{l=1}^{N} T'_l \left(\tau_{lj} w'_l\right)^{-\epsilon}} = \frac{\hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon}}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon} \hat{\tau}_{lj}^{-\epsilon}} \tag{2}$$

Combining equations (1) and (2) under the assumptions that $\hat{Y}_i = \hat{X}_i$ and $\hat{\tau} = \hat{L} = 1$, we obtain a system of equation characterizing an equilibrium $\hat{w}_i$ as a function of shocks $\hat{T}_i$, initial equilibrium shares λ_{ij} and γ_{ij}, and the trade elasticity ϵ:

$$\hat{w}_i \hat{L}_i = \sum_{j=1}^{N} \gamma_{ij} \hat{X}_{ij} = \sum_{j=1}^{N} \gamma_{ij} \hat{\lambda}_{ij} \hat{w}_j \Rightarrow \hat{w}_i = \sum_{j=1}^{N} \frac{\gamma_{ij} \hat{T}_i \hat{w}_i^{-\epsilon} \hat{w}_j}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon}}$$

Given a model parameterization that defines ϵ, λ_{ij}, and γ_{ij}, we can choose arbitrary productivity shocks $\{\hat{T}_i\}_{i=1}^{N}$ and solve for $\{\hat{w}\}_{i=1}^{N}$. (This generalizes to arbitrary $\hat{\tau}, \hat{L}$.)

Counterfactual outcomes: Autarky and free trade

Autarky

- The autarky counterfactual scenario is the alternative parameter vector in which $\tau_{ij} = \infty \; i \neq j$ ($\{T_i, L_i, \{\tau_{ij}^{-1}\} = I_N, \epsilon\}$)
- Can compute by exact hat algebra: $\hat{\tau}_{ij} = \infty$ for $i \neq j$

Free trade

- Given $\{T_i, L_i, \tau_{ij}, \epsilon\}$ where $\tau_{ii} = 1 \; \forall i$, the free-trade counterfactual scenario is the alternative parameter vector in which $\tau_{ij} = 1 \; \forall ij$ ($\{T_i, L_i, \mathbf{1}_{N \times N}, \epsilon\}$)
- Cannot compute using only shares. Need level of τ_{ij}.

Special case: Symmetric trade costs

When $\tau_{ij} = \tau_{ji}\ \forall i, j$, we can rewrite the system in terms of market access $\Phi_i \equiv P_j^{1-\sigma}$ (see Appendix A.1.3 of Dingel, Meng, Hsiang):

$$Y_i = w_i L_i = \sum_j \left(\frac{w_i}{A_i}\right)^{-\epsilon} \tau_{ij}^{-\epsilon} \frac{w_j L_j}{\Phi_j} = \left(\frac{w_i}{A_i}\right)^{-\epsilon} \Omega_i$$

$$\Rightarrow \frac{w_i}{A_i} = \left(\frac{\Omega_i}{A_i L_i}\right)^{\frac{1}{\epsilon+1}}$$

$$\Rightarrow \Phi_i = \sum_j \tau_{ji}^{-\epsilon} \left(\frac{w_j}{A_j}\right)^{-\epsilon} = \sum_j \tau_{ji}^{-\epsilon} \left(A_j L_j / \Omega_j\right)^{\frac{\epsilon}{\epsilon+1}}$$

$$= \sum_j \tau_{ji}^{-\epsilon} \left(A_j L_j / \Phi_j\right)^{\frac{\epsilon}{\epsilon+1}}$$

The last equality exploits the fact that we can normalize incomes such that $\Phi_i = \Omega_i$ when trade is balanced and $\tau_{ij}^{-\epsilon}$ is symmetric (Anderson and van Wincoop 2003; Head and Mayer 2014).

Multi-sector Armington model

- **Preferences**. Cobb-Douglas over sectors $s = 1, \ldots, S$ and CES within:

$$P_i = \prod_{s=1}^{S} P_{is}^{\alpha_{is}} \text{ and } P_{is} = \left(\sum_{i=1}^{N} p_i(\omega_s)^{1-\sigma_s} \right)^{1/(1-\sigma_s)}$$

- **Production**. Sector-specific productivities A_{is} and trade costs τ_{ijs}.
- **Gravity equation**. Denote sales from i to j in sector s by X_{ijs} and j's total expenditure by $X_j \equiv \sum_{i=1}^{N} \sum_{s=1}^{S} X_{ijs}$.

$$\lambda_{ijs} = \frac{X_{ijs}}{X_{js}} = \frac{T_{is} \left(\tau_{ijs} w_i\right)^{-\epsilon_s}}{\sum_{l=1}^{N} T_{ls} \left(\tau_{ljs} w_l\right)^{-\epsilon_s}} = \frac{T_{is} \left(\tau_{ijs} w_i\right)^{-\epsilon_s}}{\Phi_{js}}.$$

- **Equilibrium**. Labor-market clearing, goods-market clearing, and budget constraints mean total income $Y_i = w_i L_i$ and sectoral income $Y_{is} = w_i L_{is}$ satisfy $Y_{is} = \sum_{j=1}^{N} X_{ijs}$, $Y_i = \sum_{s=1}^{S} Y_{is}$, and $X_{is} = \alpha_{is} Y_i$ for all countries.

$$Y_{is} = \sum_{j=1}^{N} \lambda_{ijs} \alpha_{js} \sum_{s'=1}^{S} Y_{js'}.$$

(Recall $T_{is} = A_{is}^{\epsilon_s} = A_{is}^{\sigma_1 - 1}$)

Wrapping up

Next week: Gains from trade and comparative advantage

Extra time? Discuss assignment 4