ECON G6905

Topics in Trade

Jonathan Dingel

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Columbia University
IN THE CITY OF NEW YORK

Empirical observations about regional economies

- Economic activity is geographically concentrated (see last week)
- Regions are connected via trade flows: most metropolitan areas spend more on shipments from other metros than on their own output (Allen Arkolakis 2025)
- The gravity equation for trade flows describes trade between regions well (Allen and Arkolakis 2025 graphs akin to those from Head and Mayer 2014)
- Trade costs seem essential to understanding the economic geography of some industries (e.g., Holmes and Stevens 2014 on sugar beet processing)

This week: Quantitative regional models

- Last week's models (i.e., Rosen-Roback) had no trade or spatial linkages
- Krugman (1991) introduces two-region model with trade costs and market-size consequences (core-periphery story is applied theory)
- Modern QRMs are multi-region models designed to be taken to data (quantitative counterfactual scenarios)

  Hallmarks (like QTMs): many locations, few elasticities, many shifters

Within broader class of "quantitative spatial models"

- A general-equilibrium approach: "locations are not independent observations in a cross-sectional regression but rather are systematically linked to one another through trade, commuting, and migration flows" (Redding & Rossi-Hansberg 2017)
- Distinguish quantitative regional economics (goods and labor move between cities-as-points) from quantitative urban economics (commuting flows within cities)
- What is a QSM? Proost and Thisse (Sec 5.2, 2019) almost answer

Krugman “Increasing Returns and Economic Geography” (1991)

- The second of Krugman’s pair of Nobel-winning papers
- Apply tools from ‘70s theoretical IO and ‘80s trade models to look at the geographic concentration of industry
- General story about IRS manufacturing vs CRS agriculture rather than industry localization
- Circular causation: “manufactures production will tend to concentrate where there is a large market, but the market will be large where manufactures production is concentrated”
- Formalize this unoriginal story
- Microfoundations with clear pecuniary externalities
- Comparative statics: Agglomeration depends on transport costs, economies of scale, and manufacturing share [exogenous parameters in Krugman’s account]
- Key restrictions: Immobile peasants and only two locations

Krugman (JPE 1991): "II. A Two Region Model"

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

$$C_M = \left[\sum_{i=1}^{N} c_i^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{2}$$

$$L_1 + L_2 = \mu \tag{3}$$

$$L_{Mi} = \alpha + \beta x_i \tag{4}$$

$$p_1 = \frac{\sigma}{\sigma - 1}\beta w_1 \tag{5}$$

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} \tag{6}$$

$$(p_1 - \beta w_1)x_1 = \alpha w_1 \tag{7}$$

$$x_1 = x_2 = \frac{\alpha}{\beta}(\sigma - 1) \tag{8}$$

$$\frac{n_1}{n_2} = \frac{L_1}{L_2} \tag{9}$$

- Immobile peasants, mobile workers, and "clever" choice of units in (3)
- Usual Dixit-Stiglitz monopolistic competition setup, but L_i endogenous
- Freely traded CRS good but still wages w_1 and w_2
- As in Krugman (1980), all action on extensive margin
- Iceberg trade costs: Only $\tau < 1$ arrives, send $1/\tau$ [inverse of modern notation]

III. Short-Run Equilibrium

c_{ij} is consumption in i of a variety from j; z_{1i} is relative expenditure in i on varieties from 1; that's awkward notation; short-run eqlbm is $w_i, z_{1i}|L_i$

$$\frac{c_{11}}{c_{12}} = \left(\frac{p_1 \tau}{p_2}\right)^{-\sigma} = \left(\frac{w_1 \tau}{w_2}\right)^{-\sigma} \tag{10}$$

$$z_{11} = \left(\frac{n_1}{n_2}\right)\left(\frac{p_1 \tau}{p_2}\right)\left(\frac{c_{11}}{c_{12}}\right) = \left(\frac{L_1}{L_2}\right)\left(\frac{w_1 \tau}{w_2}\right)^{-(\sigma-1)} \tag{11}$$

$$z_{12} = \left(\frac{L_1}{L_2}\right)\left(\frac{w_1}{w_2 \tau}\right)^{-(\sigma-1)} \tag{12}$$

$$w_1 L_1 = \mu\left[\left(\frac{z_{11}}{1+z_{11}}\right) Y_1 + \left(\frac{z_{12}}{1+z_{12}}\right) Y_2\right] \tag{13}$$

$$w_2 L_2 = \mu\left[\left(\frac{1}{1+z_{11}}\right) Y_1 + \left(\frac{1}{1+z_{12}}\right) Y_2\right] \tag{14}$$

$$Y_1 = (1-\mu)/2 + w_1 L_1 \tag{15}$$

$$Y_2 = (1-\mu)/2 + w_2 L_2 \tag{16}$$

III. Long-Run Equilibrium

- L_i is endogenous, not fixed. Let $f \equiv L_1/\mu$.
- Workers care about real wages ω_i, not nominal wages w_i.

$$P_1 = \left[f w_1^{-(\sigma-1)} + (1-f) \left(\frac{w_2}{\tau} \right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \tag{17}$$

$$P_2 = \left[f \left(\frac{w_1}{\tau} \right)^{-(\sigma-1)} + (1-f) \left(w_2 \right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \tag{18}$$

$$\omega_1 = w_1 P_1^{-\mu} \tag{19}$$

$$\omega_2 = w_2 P_2^{-\mu} \tag{20}$$

- Given w_i, greater f lowers P_1/P_2 and therefore raises ω_1/ω_2.
- A race between home market effect and price index effect (convergence) and competition for sales to peasants (divergence)
- See Figure 1 for numerical example varying τ

IV. Necessary Conditions for Mfg Concentration

Suppose there are n manufacturing firms and all are in region 1. Their value of sales is V_1 and a potential defector's value of sales is V_2.

Defecting firm must pay workers wage premium to compensate for cost of living.

$$\frac{Y_2}{Y_1} = \frac{1-\mu}{1+\mu} \tag{21}$$

$$V_1 = \left(\frac{\mu}{n}\right)(Y_1 + Y_2) \tag{22}$$

$$\frac{w_2}{w_1} = \left(\frac{1}{\tau}\right)^{\mu} \tag{23}$$

$$V_2 = \left(\frac{\mu}{n}\right)\left[\left(\frac{w_2}{w_1\tau}\right)^{-(\sigma-1)} Y_1 + \left(\frac{w_2\tau}{w_1}\right)^{-(\sigma-1)} Y_2\right] \tag{24}$$

$$\frac{V_2}{V_1} = \frac{1}{2}\tau^{\mu(\sigma-1)}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}\right] \tag{25}$$

Defection profitable if $V_2/V_1 > w_2/w_1 = \tau^{-\mu}$.

IV. Necessary Conditions for Mfg Concentration

Defection profitable if $V_2/V_1 > \tau^{-\mu} \iff \nu < 1$.

$$
\begin{aligned}
\nu &= \frac{1}{2}\tau^{\mu\sigma}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}\right] && (26)\\
\frac{\partial\nu}{\partial\mu} &= \nu\sigma(\ln\tau) + \frac{1}{2}\tau^{\sigma\mu}\left[\tau^{\sigma-1} - \tau^{-(\sigma-1)}\right] < 0 && (27)\\
\frac{\partial\nu}{\partial\tau} &= \frac{\mu\sigma\nu}{\tau} + \frac{\tau^{\sigma\mu}(\sigma-1)\left[(1+\mu)\,\tau^{\sigma-1} - (1-\mu)\tau^{-(\sigma-1)}\right]}{2\tau} && (28)\\
\frac{\partial\nu}{\partial\sigma} &= \ln(\tau)\left\{\mu\sigma + \frac{1}{2}\tau^{\mu\sigma}\left[(1+\mu)\,\tau^{\sigma-1} - (1-\mu)\tau^{-(\sigma-1)}\right]\right\}\\
&= \ln(\tau)\left(\frac{\tau}{\sigma}\right)\left(\frac{\partial\nu}{\partial\tau}\right) && (29)
\end{aligned}
$$

Note $\tau = 1 \implies \nu = 1$.

The bifurcated "tomahawk" diagram

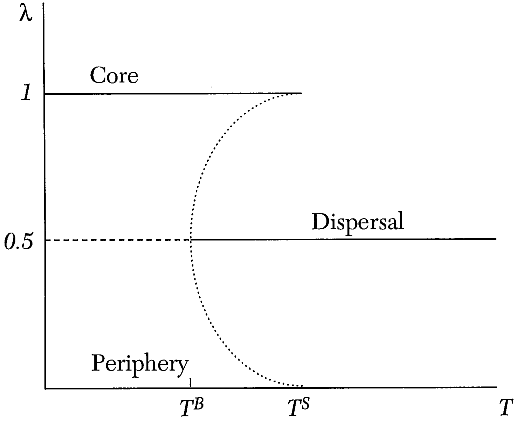


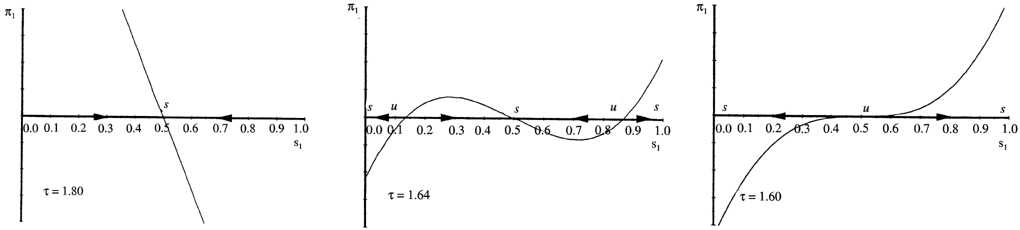
Figure 2. Agglomerated and Dispersed Equilibria as a Function of Trade Costs

- Notation: $1/\tau \to T$, $f \to \lambda$
- Krugman (1991) established a "sustain point" ν necessary for concentration
- There is also a "break point" that is sufficient for concentration to be the (stable) equilibrium outcome (Puga 1999)
- Higher trade costs can prevent regional divergence; sufficiently low trade costs rule out symmetric outcomes

Figure from Neary (2001)

More on that diagram and stability

- If workers "move faster" than firms, same conclusions about break and sustain points (Puga 1999; see Figure 1 below; $\tau \geq 1$)
- There are at most two interior asymmetric steady states. If they exist, they're unstable. (Robert-Nicoud 2005)



“Of Hype and Hyperbolas”

Neary (*JEL* 2001) on Fujita, Krugman, Venables monograph:

- “the model used throughout the book has a number of special features that make it less suitable for addressing some issues”
- “As the authors disarmingly admit, the book ‘sometimes looks as if it should be entitled *Games You Can Play with CES Functions*!”’
- “Though σ starts as a taste parameter, it ends up as an index of returns to scale
- “the Dixit-Stiglitz model has almost nothing to say about individual firms”
- “while costs may be fixed they are never sunk, so firms, industries, and even cities are always free to move”
- The “iceberg” assumption: “Of all industries, it seems to be characterized by very high ratios of fixed to variable costs”
- Davis (1998): This doesn’t work with comparable agricultural trade costs
- “The book deals in turn with regions, cities and countries, but there is nothing intrinsic to the models which conclusively identifies these units.”

Helpman (1998): Overview

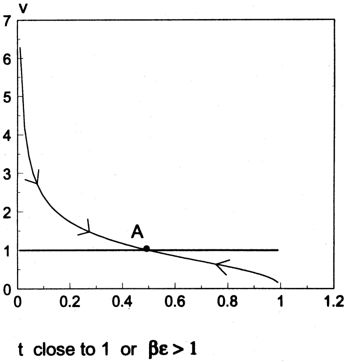
- Vocab: centripetal = agglomeration, centrifugal = dispersion
- Replace freely traded agriculture with non-traded fixed factor (housing) $\Rightarrow$ Cobb-Douglas preferences over housing and varieties
- Replace location-bound peasant income with assumption that housing is owned equally by all individuals (regardless of location) who spend income where they live
- These two deviations flip the comparative statics for trade costs in Krugman (1991)! "While in Krugman's model low transport costs lead to agglomeration and high transport costs lead to dispersion, in my model, low transport costs lead to dispersion and high transport costs lead to agglomeration."
- The first deviation (freely traded ag) is the key
- Question: What's the difference in equilibria with freely traded manufactures?
- Also evaluates welfare efficiency of market equilibrium

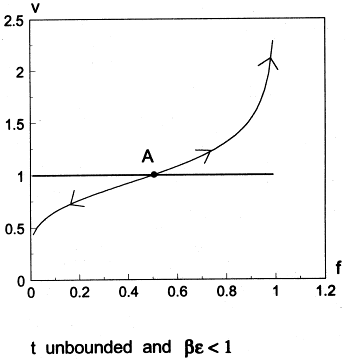
Helpman (1998): Setup

- β is Cobb-Douglas expenditure share on housing
- ϵ is elasticity of substitution across differentiated varieties
- $t > 1$ is the iceberg trade cost
- $f = N_1/N$ is the population share of region 1
- $v = u_1/u_2$ is the relative utility level of 1

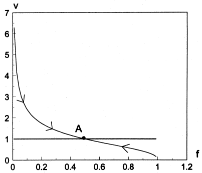
Helpman (1998)

- When $t \to 1$, only housing prices matter: Go to less populous place
- When $\beta\epsilon > 1$, housing prices relatively more important than traded prices
- When $\beta\epsilon < 1$, differentiated products are poor substitutes and demand for housing is low. In this case, trade costs matter.

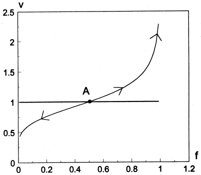




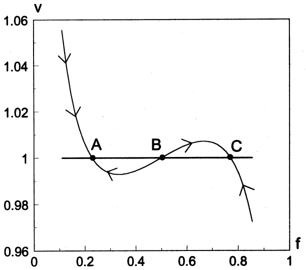
Helpman (1998): The “tomahawk” reverses



t close to 1 or $\beta\varepsilon > 1$

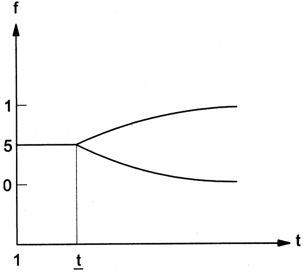


t unbounded and $\beta\varepsilon < 1$



ε=2 , β=0.4 , $\beta\varepsilon < 1$

t=6



$\beta\varepsilon < 1$

Redding and Sturm (2008)

- "We exploit the division of Germany after the Second World War and the reunification of East and West Germany in 1990 as a source of exogenous variation to provide evidence for the causal importance of market access for economic development."
- "The key idea behind our empirical approach is that West German cities close to the new border experienced a disproportionate loss of market access relative to other West German cities."
- Extend Helpman (1998) to a multi-region version (assume that $\beta\epsilon > 1 \Rightarrow$ unique equilibrium)
- Difference-in-differences design compares proximate vs distant West German cities before and after division
- Triple difference: "the greater dependence of small cities on markets in other cities implies that this effect will be particularly pronounced for small cities."

  Note the shift from qualitative applied theory question to quantitative empirical application

Redding and Sturm (2008): Difference in differences

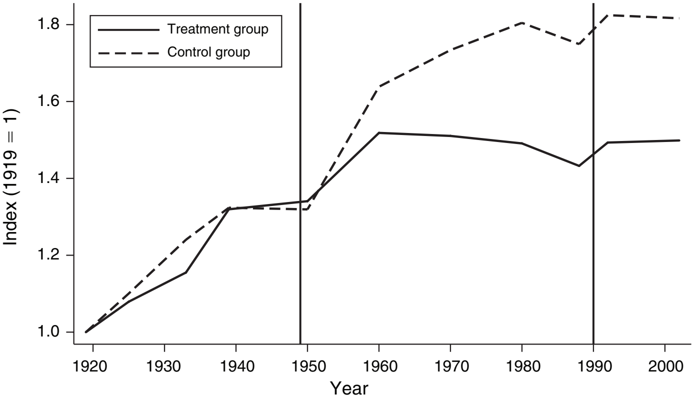


FIGURE 3. INDICES OF TREATMENT AND CONTROL CITY POPULATION

Table 2 reports estimate of -0.75 (se 0.18) for the coefficient on the interaction of "within 75km of border" dummy and 1950–1988 dummy

Redding and Sturm (2008): Quantitative model fit

Model can explain "the quantitative magnitude of the relative decline of small and large cities along the East-West German border relative to other West German cities"

- Choose parameter values to minimize the distance between small-large third difference in the simulation and data
- Finding "plausible parameter values" is "further evidence that their relative decline is indeed due to a loss of market access"
- Contrast this fit check with non-falsifiable calibrations in later literature

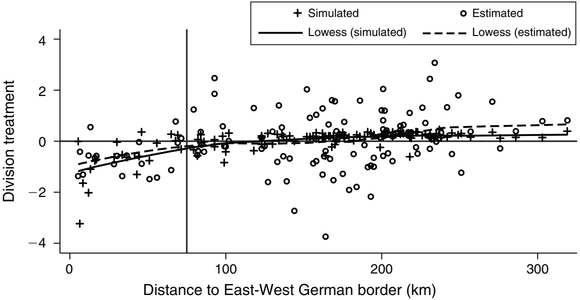


Figure 6. Simulated and Estimated Division Treatments

Notes: Simulated division treatments based on the parameter configuration that minimizes the sum of squared deviations between the simulated and estimated division treatments for small and large cities. Lowess is a locally weighted linear least squares regression of the division treatment against distance to the East-West German border (bandwidth 0.8).

Allen and Arkolakis: “Trade and Topography of the Spatial Economy”

- Stylized geographies (e.g., line or circle) are awkward for empirical application
- Authors develop a quantitative framework (more general than Redding-Sturm) that extends new economic geography to much broader class
- Derive sufficient conditions for existence and uniqueness of equilibrium in spatial geography models (with a continuum of locations)
- RR17: “a major contribution of this quantitative economic geography literature has been to preserve sufficient analytical tractability to provide conditions under which there exists a unique spatial equilibrium distribution of economic activity and to permit some analytical comparative statics”
- Illustrative quantitative exercise: What was the welfare effect of the interstate highway system?

AA '14: Model – Geography

- Continuum of locations, $i \in S$ (S is a closed and bounded set of a finite dimensional Euclidean space)
- Location $i \in S$ with population $L(i)$:
  - Endowed with differentiated variety (Armington assumption)
  - Productivity: $A(i) = \bar{A}(i)L(i)^{\alpha}$ [$\bar{A}(i)$ is exogenous]
  - Amenity: $u(i) = \bar{u}\,(i)\,L(i)^{\beta}$ [$\bar{u}(i)$ is exogenous part]
- Spillovers are local (only affect i). Presumably $\alpha \geq 0$, $\beta \leq 0$.
- For all $i, j \in S$, symmetric iceberg bilateral trade cost $T\,(i, j)$
- Together, $\bar{A}$, $\bar{u}$, and T comprise the *geography* of S
- A geography is *regular* if $\bar{A}$, $\bar{u}$, and T are continuous and bounded above and below by strictly positive numbers.

AA '14: Model – Workers

- Can choose to live/work in any location (static model with spatial equilibrium)
- Receive wage $w(i)$ for their inelastically supplied unit of labor
- CES preferences over locations' varieties with elasticity of substitution $\sigma > 1$
- Welfare in location i is

$$W(i) = \left(\int_{s \in S} q(s,i)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} u(i) = \frac{w(i)}{P(i)} u(i)$$

where $q(s,i)$ is the per capita quantity consumed in location i of the good produced in location s and $u(i)$ is the local amenity.

AA '14: Model – Production

- Labor is the only factor of production, $L(i)$ is the density of workers.
- Productivity of worker in location i is $A(i)$
- Price of good from i is $\frac{w(i)}{A(i)} T\left(i, j\right)$ in location j
- Trade flows: $X(i,j) = \left(\frac{T(i,j)w(i)}{A(i)P(j)}\right)^{1-\sigma} w(j)L(j)$
- Price index: $P(j)^{1-\sigma} = \int_S T(s,j)^{1-\sigma} A(s)^{\sigma-1} w(s)^{1-\sigma} ds$

AA '14: Model – Equilibrium

- A spatial equilibrium is a distribution of economic activity (functions w and L) such that:
  - Markets clear, i.e. $w(i)\,L(i) = \int_S X(i,s)\,ds$,
  - Welfare is equalized, i.e. $W \in \mathbb{R}_{++}$ such that for all $i \in S$, $W(i) \leq W$, with equality if $L(i) > 0$,
  - The aggregate labor market clears, i.e. $\int_S L(s)\,ds = \bar{L}$.
- A spatial equilibrium is *regular* if L and w are continuous and strictly positive (i.e. every location is inhabited).
- A spatial equilibrium is point-wise locally stable if $\frac{dW(i)}{dL(i)} < 0$ for all $i \in S$ (i.e. no small number of workers can increase welfare by moving to another location; similar to Henderson 1974).

AA '14: Solving for equilibrium

Plugging the expression for trade flows and indirect utility function into the goods market clearing condition yields:

$$L(i)w(i)^{\sigma} = \int_S W(s)^{1-\sigma} T(i,s)^{1-\sigma} A(i)^{\sigma-1} u(s)^{\sigma-1} L(s) w(s)^{\sigma} ds$$

Combining the price index with the indirect utility function yields:

$$w(i)^{1-\sigma} = \int_S W(s)^{1-\sigma} T(s,i)^{1-\sigma} u\,(i)^{\sigma-1}\, A(s)^{\sigma-1} w(s)^{1-\sigma} ds$$

We are looking for functions $w(i)$ and $L(i)$ that solve these equations and will look for equilibria in which every location is inhabited (regular equilibrium); hence we consider $W(s) = W \; \forall s$

AA '14: Solving for equilibrium without spillovers

With no productivity nor amenity spillovers ($\alpha = \beta = 0$) and welfare equalized across space, the (discrete analogues of the) previous equations can be written as:

$$g = \lambda K g$$

$$f = \lambda K' f$$

with eigenfunctions $g(i) = L(i)w(i)^{\sigma}$ and $f(i) = w(i)^{1-\sigma}$ and eigenvalues $\lambda = W^{1-\sigma}$. Let $K(i,j) = T(i,j)^{1-\sigma}\bar{A}(i)^{\sigma-1}\bar{u}(j)^{\sigma-1} > 0 \ \forall i,j$. The two kernels K and K' are transposes of each other.

"extensions of standard results in linear algebra guarantee the existence and uniqueness of a positive solution"

Solution by function iteration:

$$f_{n+1}(i) = \frac{\int_S K(i,s) f_n(s) \mathrm{d}s}{\int_S \int_S K(i,s) f_n(s) \mathrm{d}s \mathrm{d}i}$$

Connecting no-spillovers case to Roback (1982)

With free trade ($T(i,s) = 1 \forall i,s$) and equal welfare:

$$L(i)w(i)^{\sigma} = W^{1-\sigma} \int_S \bar{A}(i)^{\sigma-1} \bar{u}(s)^{\sigma-1} L(s) w(s)^{\sigma} \mathrm{d}s$$
$$w(i)^{1-\sigma} = W^{1-\sigma} \int_S \bar{u}(i)^{\sigma-1} \bar{A}(s)^{\sigma-1} w(s)^{1-\sigma} \mathrm{d}s$$

Labor demand is $L(s) = A(s)^{\sigma-1} w(s)^{-\sigma} Y/P$ and labor supply is $W = u(i)w(i)/P$.
If $\bar{A}(i)^{\sigma-1} w(i)^{1-\sigma} = \phi L(i) w(i)^{\sigma} \bar{u}(i)^{\sigma-1}$ with $\phi > 0$ then two equations reduce to

$$L(i)^{\tilde{\sigma}} = \bar{u}(i)^{(1-\tilde{\sigma})(\sigma-1)} \bar{A}(i)^{\tilde{\sigma}(\sigma-1)} W^{1-\sigma} \int_S \bar{A}(s)^{(1-\tilde{\sigma})(\sigma-1)} \bar{u}(s)^{\tilde{\sigma}(\sigma-1)} L(s)^{\tilde{\sigma}} \mathrm{d}s$$

where $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1} < 1$. Solve for $L(i)$ and W.
(Draw $\ln w(i)$ vs $\ln L(i)$ diagram)

AA '14: Solving for equilibrium with spillovers

When there are productivity or amenity spillovers and welfare is equalized across space, the previous equations yield:

$$L(i)^{1-\alpha(\sigma-1)}w(i)^{\sigma} = W^{1-\sigma}\int_S T(i,s)^{1-\sigma}\bar{A}(i)^{\sigma-1}\bar{u}(s)^{\sigma-1}L(s)^{1+\beta(\sigma-1)}w(s)^{\sigma}ds$$

$$w(i)^{1-\sigma}L(i)^{\beta(1-\sigma)} = W^{1-\sigma}\int_S T(s,i)^{1-\sigma}\bar{A}(s)^{\sigma-1}\bar{u}(i)^{\sigma-1}w(s)^{1-\sigma}L(s)^{\alpha(\sigma-1)}ds$$

If trade costs are symmetric, it turns out the system can be written as

$$A(i)^{\sigma-1}w(i)^{1-\sigma} = \phi L(i)w(i)^{\sigma}u(i)^{\sigma-1} \tag{1}$$

$$L(i)^{\tilde{\sigma}\gamma_1} = K_1(i)W^{1-\sigma}\int_S T\left(s,i\right)^{1-\sigma}K_2(s)\left(L(s)^{\tilde{\sigma}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}}ds, \tag{2}$$

where $K_1(i)$ and $K_2(i)$ are functions of $\bar{A}(i)$ and $\bar{u}(i)$, γ_1, γ_2, and $\tilde{\sigma}$ are functions of α, β, and σ.

The last equation is a Hammerstein non-linear integral equation

AA '14: Existence and uniqueness (with spillovers)

Theorem 2: Consider any regular geography with endogenous productivity and amenities with T symmetric. Define $\gamma_1 = 1 - \alpha(\sigma - 1) - \beta\sigma$, and $\gamma_2 = 1 + \alpha\sigma + (\sigma - 1)\beta$. If $\gamma_1 \neq 0$, then:

1. There exists a regular equilibrium.
2. If $\gamma_1 < 0$, no regular equilibria are point-wise locally stable.
3. If $\gamma_1 > 0$, all equilibria are regular and point-wise locally stable.
4. If $\frac{\gamma_2}{\gamma_1} \in (-1, 1]$, the equilibrium is unique and can be computed iteratively.

Note that

$$W(i) = \frac{\left(\int_S T(i,s)^{1-\sigma} P(s)^{\sigma-1} w(s) L(s) ds\right)^{\frac{1}{\sigma}}}{P(i)} \bar{A}(i)^{\frac{\sigma-1}{\sigma}} \bar{u}(i) L(i)^{-\frac{\gamma_1}{\sigma}}$$

hence parts 2 and 3 follow from $\text{sign}\left(\frac{dW(i)}{dL(i)}\right) = -\text{sign}(\gamma_1)$

Sufficient conditions for uniqueness satisfied only if no net spillovers, i.e. $\alpha + \beta \leq 0$

AA '14: Existence and uniqueness (with spillovers)

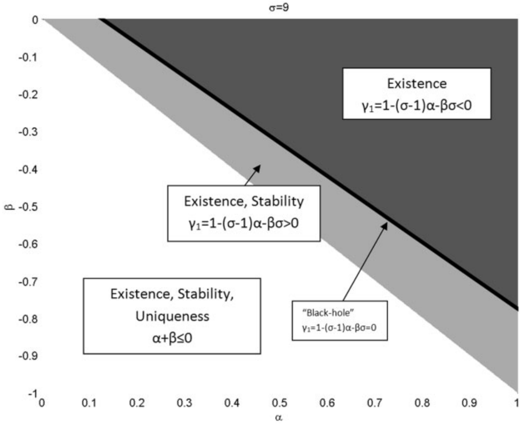


FIGURE I

Equilibria with Amenity and Productivity Spillovers

This figure shows the regions of values for the productivity spillover α and the amenity spillover β for which there exists an equilibrium, for which there exists a point-wise locally stable equilibrium, and whether that equilibrium is unique. The elasticity of substitution σ is chosen to equal 9.

AA '14: Geographic component T

- Apart from everything above, Allen and Arkolakis (2014) introduce the "fast marching method" into spatial economics
- Suppose S is a compact surface (e.g., line, plane, cow)
- Let $\tau : S \to \mathbb{R}_+$ be a continuous function where $\tau(i)$ is the instantaneous cost of traveling over location i
- Trade cost $T(i,j) = f(t(i,j)), f' > 0, f(0) = 1$ is the total iceberg trade cost incurred along least-cost route from i to j

$$t(i,j) = \min_{g \in \Gamma(i,j)} \int_0^1 \tau(g(t)) \left\| \frac{dg(t)}{dt} \right\| dt$$

where $g : [0,1] \to S$ is a path and Γ is set of possible continuous once-differentiable paths

AA '14: Eikonal equations and FMM

- Previous equation is oft-studied in physics (wave propagation)
- A necessary condition for its solution is the following eikonal partial differential equation
$$\|\nabla t(i,j)\| = \tau(j)$$
where the gradient is taken with respect to the destination j.
- One solution algorithm for this is the fast marching method
- "FMM can be interpreted as a generalization of Dijkstra to continuous spaces"
- Treb's website has an example of implementing FMM in Matlab
- In R: fastmaRching
- Julia is popular with people who solve PDEs: DifferentialEquations.jl
- I have not yet tried EikonalInv.jl, but I hope to soon

Application to the US economy

- Estimate bilateral trade costs
- Given trade costs, identify (composite) productivities and amenities
- Quantify the importance of geographic location
- Perform counterfactual exercise: remove the Interstate Highway System.
- Note: Cannot identify α, β, σ; they do analysis for a large variety of (α, β) while assuming $\sigma = 9$.

AA '14: Estimating trade costs

Goal: Find trade costs that best rationalize the bilateral trade flows observed in 2007 Commodity Flow Survey (CFS).

Three-step process:

1. Using fast marching method and observed transportation network, calculate the (normalized) distance between every CFS area for each major mode of travel (road, rail, air, and water).
2. Using a discrete-choice framework and observed mode-specific bilateral trade shares, estimate the relative cost of each mode of travel. ("the discrete choice framework is entirely distinct from the economic geography model developed in Section II and is used only as a tool to estimate trade costs based on mode-specific trade shares")
3. Using a gravity model and observed total bilateral trade flows, pin down normalization (and incorporate non-geographic trade costs).

Practicalities

- The Commodity Flow Survey only covers agriculture, manufacturing, mining, and wholesaling. Where do tradable services show up in this model?
- The market-clearing condition $w(i)\,L(i) = \int_S X(i,s)\,ds$ is a balanced-trade condition.
- CFS areas are not counties
- Gravity does not aggregate simply: sum of log-linear equations is not a log-linear equation
- "each CFS area (in the estimation of trade costs) and each county (in the estimation of overall productivities and amenities) are distinct locations"

AA '14: Estimating trade costs

- For any $i, j \in S$, assume $\exists$ traders t choosing mode $m \in \{1, ..., M\}$ of transit where cost is: $\exp\left(\tau_m d_m(i,j) + f_m + \nu_{tm}\right)$
- Then mode-specific bilateral trade shares are:

$$\pi_m(i,j) = \frac{\exp\left(-a_m d_m(i,j) - b_m\right)}{\sum_k \left(\exp\left(-a_k d_k(i,j) - b_k\right)\right)}, \tag{3}$$

  where $a_m = \theta \tau_m$ and $b_m = \theta f_m$.
- Combined with model, yields gravity equation:

$$\ln X_{ij} = \frac{\sigma - 1}{\theta} \ln \sum_m \left(\exp\left(-a_m d_{mij} - b_m\right)\right) + (1 - \sigma)\,\beta' \ln \mathbf{C}_{ij} + \delta_i + \delta_j \tag{4}$$

- Estimate a_m and b_m using bilateral trade share eq (3), θ using gravity eq (4). Assume $\sigma = 9$.

AA '14: Trade cost estimates

Table II: Estimated mode-specific relative cost of travel

	All CFS Areas				Only MSAs			
Geographic trade costs	Road	Rail	Water	Air	Road	Rail	Water	Air
Variable cost	0.5636***	0.1434***	0.0779***	0.0026	0.4542***	0.1156***	0.0628***	0.0021
	(0.0120)	(0.0063)	(0.0199)	(0.0085)	(0.0233)	(0.0210)	(0.0265)	(0.0176)
Fixed cost	0	0.4219***	0.5407***	0.5734***	0	0.34***	0.4358***	0.4621***
	N/A	(0.0097)	(0.0236)	(0.0129)	N/A	(0.0235)	(0.0375)	(0.0264)
Estimated shape parameter (θ)	14.225***				17.6509***			
	(0.3375)				(1.4194)			
Non-geographic trade costs								
Similar ethnicity	-0.0888***				-0.0803***			
	(0.0153)				(0.0275)			
Similar language	0.063***				0.0286			
	(0.0223)				(0.0359)			
Similar migrants	-0.0191				-0.0135			
	(0.0119)				(0.0203)			
Same state	-0.2984***				-0.3104***			
	(0.0101)				(0.0176)			
R-squared (within)	0.4487				0.4113			
R-squared (overall)	0.6456				0.5995			
Observations with positive bilateral flows	9601	9601	9601	9601	3266	3266	3266	3266
Observations with positive mode-specific bilateral flows	9311	1499	78	1016	3144	340	26	471

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AA '14: Recovering productivities and amenities

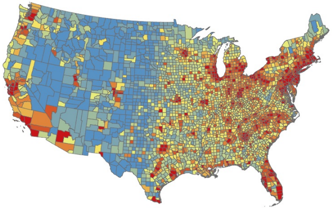
- Given data on wages and population across space, productivities A and amenities u can be recovered.
- To see this, plug (1) into the indirect utility function (after substituing the price index into it). This yields:

$$u(i)^{1-\sigma} = \frac{W^{1-\sigma}}{\phi} \int_S T(s,i)^{1-\sigma} w(i)^{\sigma-1} w(s)^{\sigma} L(s) u(s)^{\sigma-1} ds$$

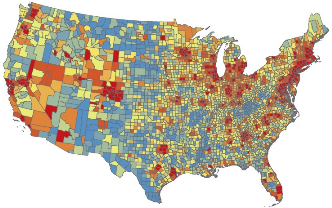
- Again, this is a Hammerstein non-linear integral equation, which can be uniquely solved for $u(i)$.
- Then A(i) can be recovered from (1).
- Note: $\bar{A}$ and $\bar{u}$ cannot be identified without knowledge of α and β.

AA '14: Population and wages – data

Figure 12: United States population density and wages in 2000



Population density

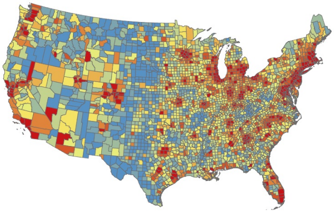


Wages

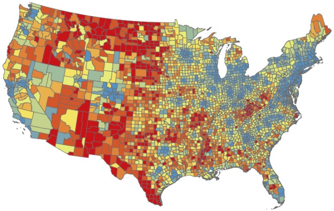
Notes: This figure shows the relative population density (top) and wages (bottom) within the United States in the year 2000 by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles. (Source: MPC (2011a)).

AA '14: Productivities and amenities

Figure 13: Estimated composite productivity and amenity



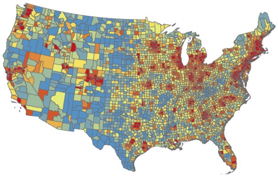
Composite productivity



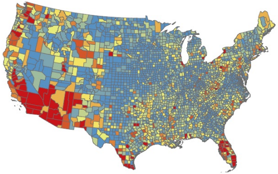
Composite amenity

Notes: This figure shows the estimated composite productivity (top) and amenity (bottom) by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

Figure 14: Estimated exogenous productivity and amenity



Exogenous productivity

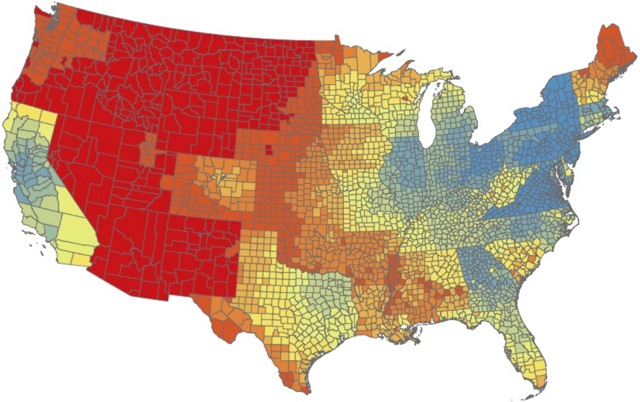


Exogenous amenity

Notes: This figure shows the estimated exogenous productivity $\bar{A}$ (top) and amenity $\bar{u}$ (bottom) by decile assuming $\alpha = 0.1$ and $\beta = -0.3$. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA '14: Price index

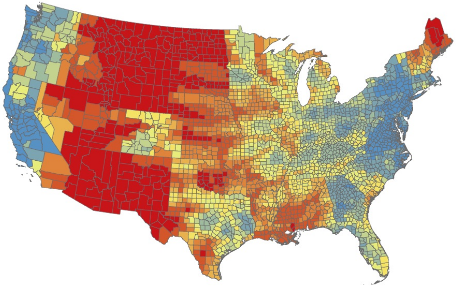
Figure 15: Estimated price index



Notes: This figure shows the estimated price index by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

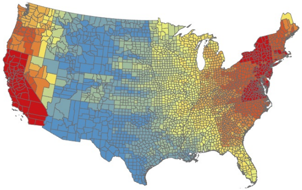
AA ‘14: Counterfactual scenario: Remove interstate highways

Figure 17: Estimated increase in the price index from removing the Interstate Highway System

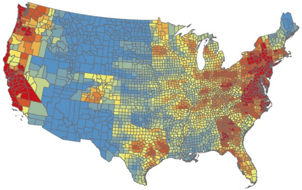


Notes: This figure depicts the estimated increase in the price index (by decile) across space from removing the Interstate Highway System (IHS), holding wages and productivities constant at the 2000 U.S. levels. Red (blue) indicate higher (lower) deciles (e.g. the removal of the IHS disproportionately increased the economic remoteness in red regions).

Figure 18: Estimated change in the population from removing the Interstate Highway System



$\alpha = 0,\ \beta = 0$



$\alpha = 0.1,\ \beta = -0.3$

Notes: This figure shows the estimated change in population (in deciles) from the removal of the Interstate Highway System (IHS). The top map reports the estimated population changes when there are no spillovers (i.e. $\alpha = \beta = 0$), while the bottom map reports the estimated population changes when spillovers are chosen to approximately match those from the literature (i.e. $\alpha = 0.1$ and $\beta = -0.3$). Red (blue) indicates higher (lower) deciles (e.g. the removal of the IHS increased the relative population in red areas).

Bartelme (2018): Market access slows the shift to the Sun Belt

- South's share of US population rose from 24% in 1950 to 30% in 2000 (Glaeser and Tobio 2008)
- Bartelme (2018) "assess[es] the quantitative contribution of trade costs to US economic geography"
- Preferences a la Helpman (1998) with Eaton-Kortum production structure and agglomeration elasticity
- Compact system of equations with two key elasticities: response of wages to market access and response of population to market access
- Proposes shift-share instrument for market access based on neighbor's industrial composition
- Counterfactual: "reducing trade costs would result in large population shifts from the Northeast towards the South and West, along with a flattening of the city size distribution"

Bartelme (2018): System of equations

$$w_n = \Phi_n^{\epsilon_w} \chi_n^w$$
$$L_n = \Phi_n^{\epsilon_l} \chi_n^l$$
$$\Phi_n = \sum_i \frac{w_i L_i}{\Phi_i} \tau_{in}^{-\theta}$$

How to get exogenous variation in market access Φ_n over time?

1. Exogenous shifts in trade costs τ_{in} (Redding and Sturm 2008; Donaldson and Hornbeck 2016)
2. Shocks to other regions' fundamentals that shift $w_i L_i$ in other regions

Bartelme (2018): Shift-share design

- Shift-share design requires a multi-industry model that still delivers aggregate gravity: "assume a common cost function w_i (up to a productivity shifter T_{nk}), common trade costs, and equality between the upper and lower tier elasticities, $\sigma - 1 = \theta$"
- Construct predicted change in market access by summing over predicted partner's growth using national industry growth times local employment share
- I expect that industrial employment shares are spatially correlated
- See Adao, Kolesar, Morales (2019) on concerns for shift-share designs
- I am not aware of an existence-and-uniqueness theorem for multi-sector models with varying parameters

Caliendo, Parro, Rossi-Hansberg, Sarte (2018): Multi-sector model

- Two factors: labor and equipped land
- Labor moves to equalize welfare across regions $U = I_n/P_n$
- Multi-sector EK/CP production with intermediates
- Imbalanced trade with regional transfers
- EK and equipped land are dispersion forces; no agglomeration mechanism; equilibrium is presumably unique
- Calibrated shares: "we do not need direct information on transport costs since all the relevant information is embedded in the observed trade flows"
- Computing counterfactuals: Function iteration on relative regional factor prices $\hat{\omega}$ (Appendix A.3)
- "the effects of disaggregated productivity changes depend in complex ways on the details of which sectors and regions are affected, and how these are linked through input–output and trade relationships to other sectors and regions"

What's missing/next?

Allen and Arkolakis (2025) handbook chapter on quantitative regional economics:

- Multiple sectors
- Central place theory
- Migration costs
- Inter-regional knowledge spillovers
- Heterogeneous people
- Granular firms
- Market power

Next week

Quantitative urban models featuring commuting