ECON G6905

Topics in Trade

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Outline of today

1. Gains from trade (Samuelson 1939)
2. Comparative advantage (Deardorff 1980)
3. Ricardian model with two countries and continuum of goods (Dornbusch Fisher Samuelson 1977)

Neoclassical environment

"Neoclassical" trade models: perfect competition, constant returns to scale, and no distortions

- There are $n = 1, ..., N$ countries, each populated by $h = 1, ..., H_n$ households
- There are $g = 1, ..., G$ goods
  - Output vector in country n: $y^n \equiv (y_1^n, ..., y_G^n)$
  - Consumption vector of household h in country n: $c^{nh} \equiv \left(c_1^{nh}, ..., c_G^{nh}\right)$
  - Goods price vector in country n: $p^n \equiv (p_1^n, ..., p_G^n)$
- There are $f = 1, ..., F$ factors in fixed supply
  - The endowment vector in country n: $v^n \equiv (v_1^n, ..., v_F^n)$
  - The factor price vector in country n: $w^n \equiv (w_1^n, ..., w_F^n)$

Supply and the revenue function

Revenue function of country n is

$$r^n(p^n, v^n) = \max_{y^n} \{p^n \cdot y^n | (y^n, v^n) \text{ feasible}\}$$

Lots of handy properties in a neoclassical environment (see Dixit & Norman 1980 p.31-36)

- Revenue function summarizes all relevant properties of technology
- Under perfect competition, y^n maximizes r^n
- Derivatives w.r.t. goods prices give supply curves

$$\nabla_p r^n(p^n, v^n) = y^n(p^n, v^n)$$

- Derivatives w.r.t endowments give inverse factor demand curves

$$\nabla_v r^n(p^n, v^n) = w^n(p^n, v^n)$$

Demand and the expenditure function

Expenditure function for household h in country n with utility function u^{nh} is defined as

$$e^{nh}\left(p^n, u^{nh}\right) = \min_{c^{nh}} \left\{ p^n \cdot c^{nh} \middle| \, u^{nh}\left(c^{nh}\right) \geq u^{nh} \right\}$$

Familiar properties from consumer theory (see Dixit & Norman 1980 p.59-64)

- Optimization implies that $e^{nh}\left(p^n, u^{nh}\right) = p^n \cdot c^{nh}$ so

$$\nabla_p e^{nh}\left(p^n, u^{nh}\right) = c^{nh}\left(p^n, u^{nh}\right)$$

- $e^{nh}(p, u)$ is increasing in u

Gains from trade (representative household)

- The revealed-preference argument employs only the revenue and expenditure funtions
- Start with case of a single/representative household
- Drop the hn notation; use a to denote autarky vectors

Gains from trade (representative household)

In a neoclassical trade model with one representative household per country, all households are (weakly) better off under free trade than autarky.

Proof:

$$
\begin{aligned}
e\left(p, u^a\right) &\leq p \cdot c^a, && \text{by definition of the expenditure function} \\
&= p \cdot y^a, && \text{by market clearing under autarky} \\
&\leq r\left(p, v\right), && \text{by definition of the revenue function} \\
&= e\left(p, u\right), && \text{by budget and trade balance}
\end{aligned}
$$

Since expenditure is increasing in utility, we conclude that $u \geq u^a$.

- Weak inequalities to accommodate kinks in IC or PPF
- Gains from trade relative to autarky does not rank trading equilibria
- Draw the two-good case

Gains from trade (lump-sum transfers)

- With multiple households, trade is likely to generate winners and losers but we can show the winners win more than the losers lose
- Formally, there exist feasible domestic lump-sum transfers that make every household better off under free trade than autarky
- Reintroduce the household superscript notation:
  - c^{ah} and c^h denote the vector of consumptions of household h under autarky and free trade
  - v^h denotes the vector of endowments of household h under autarky and free trade
  - u^{ah} and u^h denote the utility levels of household h under autarky and free trade
  - τ^h denotes the lump-sum transfer (in trade equilibrium) from government to household h (lump-sum tax if negative)

Gains from trade (lump-sum transfers)

In a neoclassical trade model with multiple households per country, there exist domestic lump-sum transfers such that all households are (weakly) better off under free trade than autarky.

- Set transfers such that each household can still afford its autarky consumption bundle under free trade
$$\tau^h = (p - p^a) \cdot c^{ah} - (w - w^a) \cdot v^h$$
- These are feasible (government revenue is non-negative)

$$\begin{aligned}
-\sum_h \tau^h = & \quad (p^a - p) \cdot \sum_h c^{ah} - (w^a - w) \cdot \sum_h v^h, \text{ by definition of } \tau^h \\
= & \quad (p^a - p) \cdot y^a - (w^a - w) \cdot v, \text{ market clearing under autarky} \\
= & \quad -p \cdot y^a + w \cdot v, \text{ income equals expenditure under autarky} \\
\geq & \quad -r(p, v) + w \cdot v, \text{ from definition of revenue function} \\
= & \quad 0, \text{ income equals expenditure under free trade}
\end{aligned}$$

Gains from trade (commodity and factor taxation)

- Domestic lump-sum transfers are not typically feasible
- Let government set specific taxes on goods and factors so that, e.g., the price of good g is $p_g^{\text{consumer}} = p_g + \tau_g$
- Set $\tau_g = p_g^a - p_g$ and $\tau_f = w - w_f^a$ so household is indifferent
- Government revenue is positive (similar to above):

$$
\begin{aligned}
T &= \sum\nolimits_g \tau_g \sum\nolimits_h c_g^{ah} + \sum\nolimits_f \tau_f \sum\nolimits_h v_f^h \\
&= (p^a - p) \cdot \sum_h c^{ah} - (w^a - w) \cdot \sum_h v^h \geq 0
\end{aligned}
$$

- Remember that you cannot just rebate the revenue, you need to change a consumer price to achieve the strict improvement (Kemp & Wan *JIE* 1986)
- There's probably a Pareto-improving direction of change in consumer prices in the neighborhood of the autarky price vector (Dixit & Norman *JIE* 1986)

Introducing comparative advantage

- "Comparative advantage" – differences in autarkic relative marginal costs – is the basis for trade
- If autarkic relative prices are identical, then "zero trade" is a free-trade equilibrium allocation at those prices
- Theory of comparative advantage (2x2 case): If two countries engage in trade, each will export the good in which it has lower relative marginal cost prior to trade

Law of comparative advantage for free-trade equilibria

In a neoclassical trade model with representative households with autarkic and free-trade prices p^{na} and p, $(p - p^{na}) \cdot t^n \geq 0$, where $t^n = y^n - c^n$ is the vector of country n's net exports. Proof (Deardorff 1980, 1994):

$$
\begin{aligned}
p^{na} \cdot y^n &\leq r\left(p^{na}, v^n\right), \quad \text{by def of revenue function} \\
p^{na} \cdot c^n &\geq e\left(p^{na}, u^n\right), \quad \text{by def of expenditure function} \\
p^{na} \cdot t^n &\leq r\left(p^{na}, v^n\right) - e\left(p^{na}, u^n\right), \quad \text{by previous two inequalities} \\
e\left(p^{na}, u^n\right) &\geq e\left(p^{na}, u^{na}\right), \text{ since } u^n \geq u^{na} \text{ and } \frac{\partial e\left(p, u\right)}{\partial u} \geq 0 \\
p^{na} \cdot t^n &\leq r\left(p^{na}, v^n\right) - e\left(p^{na}, u^{na}\right), \text{ by previous two inequalities} \\
p^{na} \cdot t^n &\leq 0, \text{ since autarkic income equals autarkic expenditure} \\
p \cdot t^n &= 0, \text{ by balanced trade} \\
(p - p^{na}) \cdot t^n &\geq 0, \text{ by combining previous two expressions}
\end{aligned}
$$

Comments on general validity of law of CA

- $(p - p^{na}) \cdot t^n$ is a correlation result because covariance of two vectors is simply their inner product if one of the vectors (i.e., normalized $p - p^{na}$) sums to zero
- $(p - p^{na}) \cdot t^n$ depends on both autarky and free-trade prices
- Corollaries 3 and 4 of Deardorff (1980) state result in terms of only autarkic prices (requires world market-clearing assumption)
- Deardorff (1980) covers the case of costly trade, distinguishing consumer price p^q, producer price p^t, and world price p^w
- Core of the proof is that $p^{na} \cdot t^n \leq 0$: gains from trade mean that consumption is at most barely attainable under autarky ($p^a y^n \leq p^a c^n$)

Deardorff (1980) environment

Notation differs: country i, "natural trade" n, quantity Q

$$(Q^i, T^i) \in F^i \Rightarrow (Q^i + T^i, 0) \in F^i \quad \text{non-negative trade costs} \tag{1}$$

$$\text{local non-satiation} \tag{2}$$

$$(Q^{ai}, 0) \in F^i \quad \text{autarky eqlbm feasible} \tag{3}$$

$$p^{ai} Q^{ai} \geq p^{ai} Q \ \forall (Q, 0) \in F^i \quad \text{profit maximizing} \tag{4}$$

$$U^i(Q^{ai}) \geq U^i(Q) \ \forall Q : p^{ai} Q \leq p^{ai} Q^{ai} \quad \text{utility maximizing} \tag{5}$$

$$(Q^{ni}, T^{ni}) \in F^i \quad \text{trade eqlbm feasible} \tag{6}$$

$$p^{qi} Q^{ni} + p^{ti} T^{ni} \geq p^{qi} Q + p^{ti} T \ \forall (Q, T) \in F^i \quad \text{profit maximizing} \tag{7}$$

$$U^i(Q^{ni}) \geq U^i(Q) \ \forall Q : p^{qi} Q \leq p^{qi} Q^{ni} \quad \text{utility maximizing} \tag{8}$$

$$p^w T^{ni} = 0 \quad \text{balanced trade} \tag{9}$$

$$(p^w_g - p^{ti}_g) T^{ni}_g \geq 0 \ \forall g \quad \text{"natural trade"} \tag{10}$$

$$\sum_i T^{ni} = 0 \quad \text{world market clears} \tag{11}$$

How should we take comparative advantage to data?

Canonical 2×2 insight we teach in principles classes isn't amenable to empirical investigation. Now we have a general formulation.

- Good news: "while the classical theory predicts only the direction and not the magnitude of trade, it nonetheless permits one to infer a negative infer a negative correlation between relative costs and net exports" (Deardorff 1980)
- Bad news: "relative antarky prices are not observable. Almost all countries have engaged in trade throughout history, so that there is no experience with autarky from which to draw data." (Deardorff 1984)
- Long-standing approach: Use model with observable primitives (technology and factor endowments) to infer autarkic prices. Joint test of CA and model.

Bernhofen and Brown: Sometimes we observe autarky

- Japan had “sudden and complete opening up to international trade in the 1860s” due to US military
- Bernhofen and Brown use this a natural experiment to test law of comparative advantage
- Key prediction is $p^{na} \cdot t^n \leq 0$, but we never simultaneously observe autarky prices p^a and trade-equilibrium net exports t^n
- “the comparison of autarky with free trade should be understood as a comparison between two alternative histories, not as a change that takes place over time” (Helpman and Krugman 1985)
- If preferences and technology in 1868-1875 (observed trade years) are same as those in 1858, hope that p^a from 1858 is valid measure of p^a for 1868-1875
- Test $p^{na} \cdot t^n \leq 0$ by computing $p^{a,1858} \cdot t^{n,1868}$ (roughly speaking)

Assumptions

- Read Section III of BB (2004) on the assumptions that this is a relevant and valid natural experiment
  1. Competitive economy in autarky
  2. Japanese are price takers in international markets
  3. Exports not subsidized
  4. PPF shifts from 1859 to 1868 not biased toward importables (if $\mathbf{p_2^a} = \mathbf{p_1^a} + \epsilon$, then $\epsilon\mathbf{T} \leq 0$ is sufficient for $\mathbf{p}_1^a\mathbf{T} \leq 0 \Rightarrow \mathbf{p}_2^a\mathbf{T} \leq 0$)
- What is the test of $p^{na} \cdot t^n \leq 0$?
  - Alternative hypothesis H_1: $p^{na} \cdot t^n > 0$
  - Alternative hypothesis H_2: $\Pr\left(p^{na} \cdot t^n \leq 0\right) = \frac{1}{2}$

Correlation of p^a and t in 1869

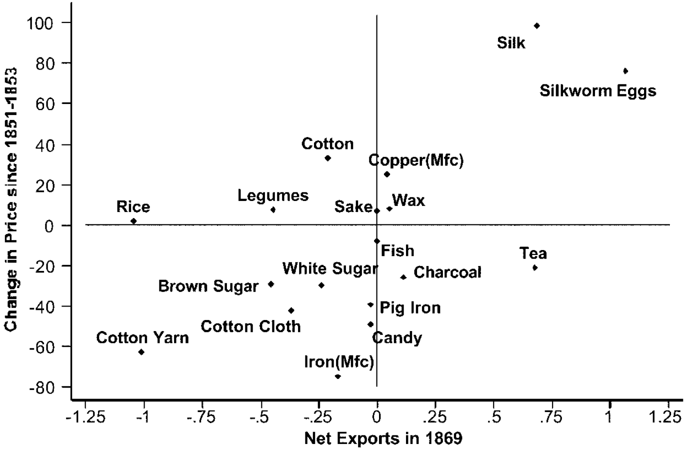


FIG. 4.—Net exports and price changes for 1869. Source: Japan Bureau of Revenue (1893) for trade data and Kinyu Kenkyukai (1937), Miyamoto (1963), Ono (1979), Yamazaki (1983), and Mitsui Bunko (1989) for price data.

Inner product of p^a and t, year by year

TABLE 2

APPROXIMATE INNER PRODUCT IN VARIOUS TEST YEARS (Millions of Ryō)

	YEAR OF NET EXPORT VECTOR							
COMPONENTS	1868	1869	1870	1871	1872	1873	1874	1875
1. Imports with observed autarky prices	−2.24	−4.12	−8.44	−7.00	−5.75	−5.88	−7.15	−7.98
2. Imports of woolen goods	−.98	−.82	−1.29	−1.56	−2.16	−2.50	−1.56	−2.33
3. Imports with approximated autarky prices (Shinbo index)	−1.10	−.95	−.70	−.85	−1.51	−2.08	−1.60	−2.65
4. Exports with observed autarky prices	4.07	3.40	4.04	5.16	4.99	4.08	5.08	4.80
5. Exports with approximated autarky prices (Shinbo index)	.09	.03	.07	.07	.15	.07	.11	.10
Total inner product (sum of rows 1–5)	**−.18**	**−2.47**	**−6.31**	**−4.17**	**−4.28**	**−6.31**	**−5.11**	**−8.06**

SOURCE.—For sources of price data, see Sec. IV*B* and n. 17. For rows 3 and 5, current silver yen values are converted to values of 1851–53 by deflating them with the price indices for exports and imports found in Shinbo (1978, table 5–10).

NOTE.—All values are expressed in terms of millions of ryō. The ryō equaled about \$1.00 in 1873 and was equivalent to the yen when it was introduced in 1871. The estimates are of the approximation of the inner product ($\tilde{\mathbf{p}}_1^a\tilde{\mathbf{T}}$) valued at autarky prices prevailing in 1851–53. An explanation of the assumptions underlying the approximation is contained in the text.

"The p-value is exactly 1/256, where 1/256 is the probability of obtaining eight heads in eight tosses with a balanced coin."

Comments on Bernhofen and Brown (2004)

- What is the autarky price of a good not produced in autarky?
- Plot of prices changes $p - p^a$ in Figure 4 okay if $p \cdot t = 0$ by balanced trade (check Figure 3)
- What is the power of this test in the absence of a competing theory?
- Computation of p-value assumes independence of observations
- Does $p^a \cdot t$ exhibit a trend?

Taxonomy of neoclassical trade models

- In a neoclassical model, comparative advantage (lower relative autarkic marginal cost) is the basis for trade
- Autarky costs might reflect demand or supply differences
- Demand differences typically neglected by assumption
- Supply-side explanations for autarkic cost differences:
  - Technological differences (Ricardian theory)
  - Factor-endowment differences (Ricardo-Viner and Heckscher-Ohlin)
  - Increasing returns to scale (beyond neoclassical scope)
- In theoretical models, the roles of factor proportions and technological differences are typically kept separate:
  - Ricardian model assumes one factor of production
  - Factor-proportions theory typically assumes common production function

Technology vs factors

Different models for different questions?[1]

- What is the effect of rising Chinese productivity on US real wages? (DFS 1977, its interpretation, Hsieh and Ossa 2016)
- What are distributional consequences of trade? Need multiple factors

Interaction of technology and factors might matter

- Does fact of intra-industry trade necessitate increasing returns in theory? No, says Davis (1995).
- Chor (JIE 2010) and Morrow (JIE 2010)
- Factor-biased technical change

[1] Jones & Neary (1980): "positive trade theory uses a variety of models, each one suited to a limited but still important range of questions"

Canonical Ricardian model of DFS 1977

- Two countries, Home and Foreign; asterisk denotes latter
- One factor of production, call it labor, endowed in amounts L and L^* and paid wages w and w^*

  [efficiency units, and see "Hicksian composite"]
- Unit labor costs for good z are $a(z)$ and $a^*(z)$
- WLOG, order goods such that $A(z) \equiv \frac{a^*(z)}{a(z)}$ is decreasing
- Home has comparative advantage in low-z goods

Recall the concepts and insights of two-good case

Consider two goods, z and z'

- Home has *absolute advantage* in z when $a(z) < a^*(z)$
- Home has *comparative advantage* in z when its relative autarkic marginal cost is lower: $\frac{a(z)}{a(z')} < \frac{a^*(z)}{a^*(z')}$

What is equilibrium pattern of specialization?

- For factor markets to clear, Home cannot be least-cost provider of both goods. It is not possible that

$$wa(z) < w^*a^*(z) \text{ and } wa(z') < w^*a^*(z')$$

- If Home has comparative advantage in z, it must be that

$$\frac{a(z)}{a^*(z)} \leq \frac{w^*}{w} \leq \frac{a(z')}{a^*(z')}$$

- Absolute advantage determines wages; comparative advantage determines specialization

Pattern of specialization in DFS 1977

- Let $p(z)$ denote the price of good z under free trade
- Profit maximization and factor-market clearing require

$$p(z) \leq wa(z) \text{ and } p(z) \leq w^* a^*(z)$$

  with equality if produced in Home or Foreign, respectively
- There exists $\tilde{z}$ such that Home produces all of $z < \tilde{z}$ and Foreign produces all of $z > \tilde{z}$ (proof by contradiction)
- Countries specialize according to comparative advantage
- Define relative wage $\omega \equiv \frac{w}{w^*}$
- Given relative wages, cost-minimizing specialization is $[0, \tilde{z}]$ at Home and $[\tilde{z}, 1]$ in Foreign such that $A(\tilde{z}) = \omega$
- Continuum of z and $A'(\tilde{z}) < 0$ makes $\tilde{z} = A^{-1}(\omega)$
- Second curve in z-ω space requires demand

Cobb-Douglas preferences

Identical Cobb-Douglas preferences with expenditure shares $b(z)$

$$b\left(z\right)=\frac{p\left(z\right)c\left(z\right)}{wL}=b^{*}\left(z\right)=\frac{p^{*}\left(z\right)c^{*}\left(z\right)}{w^{*}L^{*}}$$
$$\int_{0}^{1}b\left(z\right)dz=\int_{0}^{1}b^{*}\left(z\right)dz=1$$

Denote the share of expenditure on Home goods by $\theta\left(\tilde{z}\right)$

$$\theta\left(\tilde{z}\right)=\int_{0}^{\tilde{z}}b\left(z\right)dz\quad\text{and}\quad 1-\theta\left(\tilde{z}\right)=\int_{\tilde{z}}^{1}b\left(z\right)dz$$

Trade balance then requires $\theta\left(\tilde{z}\right)w^{*}L^{*}=\left[1-\theta\left(\tilde{z}\right)\right]wL$, which implies

$$\omega=\frac{\theta\left(\tilde{z}\right)}{1-\theta\left(\tilde{z}\right)}\frac{L^{*}}{L}\equiv B\left(\tilde{z}\right)$$

Gains from trade in DFS 1977

- It's a neoclassical model with a free-trade equilibrium, so last week's results apply: there are gains from trade
- Given functional forms, we can speak to magnitudes

$$\ln(U/L) = \ln w - \int_0^1 b(z) \ln p(z) dz$$

- Choose $w = 1$ in both autarky and trade equilibria

$$\int_0^1 b(z) \ln a(z) dz \text{ vs } \int_0^{\tilde{z}} b(z) \ln a(z) dz + \int_{\tilde{z}}^1 b(z) \ln \left[w^* a^*(z) \right] dz$$

- Connect to double-factoral terms of trade and dissimilarity as source of GFT

Comparative statics for population growth

An increase in L^*/L moves $B(\tilde{z})$ schedule. See DFS Figure 2:

- Equilibrium is a decrease in $\bar{z}$ and increase in $\bar{\omega}$
- At initial $\bar{\omega}$, larger L^*/L means trade surplus for Home, so its terms of trade must improve
- Goods produced at Home before and after shock have no change in price
- Goods produced in Foreign before and after shock become cheaper for Home consumers
- What about the goods that switch?
- Each good is produced using CRS, but akin to country-level DRS

Comparative statics for technical change

What happens with each of the following shocks?

- Uniform global technical progress: $\mathrm{d}\ln a(z) = \mathrm{d}\ln a^*(z) = x < 0$
- Uniform Foreign technical progress: $\mathrm{d}\ln a^*(z) = x < \mathrm{d}\ln a(z) = 0$
- Technical transfer: Convergence to $a(z) = a^*(z)$

Trade costs

"Iceberg" trade costs $g(z) = g < 1$ (in fact, shipping ice is IRS)

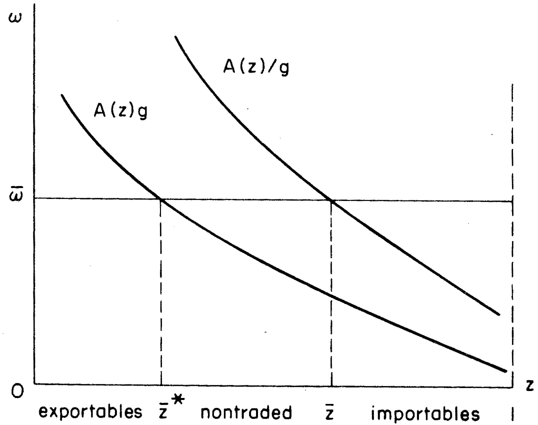


FIGURE 3

- Home produces if $wa(z) \leq (1/g)w^*a^*(z)$
- Foreign produces if $w^*a^*(z) \leq (1/g)wa(z)$
- Trade balance is $(1-\lambda)wL = (1-\lambda^*)w^*L^*$

DFS with non-homothetic preferences (Matsuyama 2000) in one slide

Switch to hierarchical demand of Murphy, Shleifer, Vishny (1989):

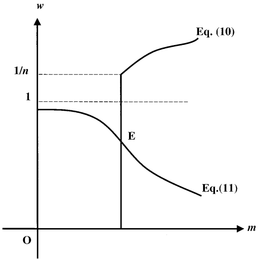
- $z \in [0, \infty)$ and cutoff good m given by $w = A(m)$ with $w^* = 1$
- $A(z)$ schedule in Figure 1 same as DFS Figure 1

$$V = \int_0^\infty b(z)x(z)dz \text{ where } x(z) \in \{0, 1\}$$

- $b(z)/a(z)$ and $b(z)/a^*(z)$ decreasing so households "prioritize" low-z goods
- Expenditure to consume up to z is $E(z) \equiv \int_0^z p(s)ds$ (monotone in utility).

Why might income elasticities of goods be interesting?

- Terms of trade might be shaped by global growth
- Product cycles in which rich buy innovations first
- "Neutral" productivity shifts aren't neutral
- Scope for normative implications



Wrapping up

Wilson (Ecma, 1980):

> *The DFS paper represents a significant contribution in demonstrating how one might modify the standard Ricardian model in order to make it more tractable for comparative statics analysis. Their assumptions are so restrictive, however, that the extent to which their approach can be generalized is not readily apparent. Besides the possibility of relaxing their assumptions on demand, it is not at all clear from their examples how the analysis would proceed if we wished to allow for more than two countries.*

Next week: Quantitative Ricardian models that handle many countries