

ECON G6905
Topics in Trade
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Fall 2025, Week 9



This week: Quantitative urban models

Subset of QSMs featuring commuting (choose residence-workplace pair)

- ▶ QUM's gravity equation features cross-hauling of homogeneous labor
 - ▶ Contrast with the canonical monocentric-city model
- Link to Davis (1995) on intra-industry trade

While GE machinery developed recently, gravity is a staple of urban economics:

- ▶ Gerald A. P. Carrothers (1956) “[An Historical Review of the Gravity and Potential Concepts of Human Interaction](#)”, *Journal of the American Institute of Planners*, 22:2, 94-102.
- ▶ Waldo Tobler (1975), “[Spatial Interaction Patterns](#)”. “A table of county-to-county interactions in the United States, for example, would yield nearly 10^7 numbers, an incomprehensible amount.”
- ▶ Colwell, P.F. (1982), “[Central Place Theory and The Simple Economic Foundations of the Gravity Model](#)” *Journal of Regional Science*, 22: 541-546.

Motivation: Why use a GE gravity model of commuting flows?

- ▶ Commuting frees us to separate choices of home and work locations
- ▶ A monocentric city with one employment area may be too stylized for many applications

[Redding \(2024\)](#): “Perhaps the most common empirical application is to transport infrastructure improvements (e.g., railroads, roads, public transit), but they also provide frameworks to evaluate zoning and land use regulations, other place-based policy interventions, and the implications of new communication technologies.”

- ▶ Freeway disamenities (Brinkman and Lin 2024)
- ▶ Los Angeles subway system (Chris Severen)
- ▶ Bus rapid transit in Bogota (Nick Tsivanidis)
- ▶ Commuting and formality in Mexico City (Roman David Zarate)
- ▶ Steam railway in 19th century London (Heblich Redding Sturm 2020)
- ▶ Revitalizing empty blocks in Detroit (Owens et al 2020)
- ▶ Transportation infrastructure with congestion (Allen Arkolakis 2022)

Baseline model: Economic environment

This is Dingel and Tintelnot (2025) with ARSW (2015) notation for β, z, d, ij

- ▶ Each location has productivity A and land endowment T
- ▶ Measure L individuals w/ one unit of labor and hired by competitive firms producing freely traded goods differentiated by location of production
- ▶ Individuals have Cobb-Douglas preferences over goods (β) and land ($1 - \beta$)
- ▶ Commuting costs: $d_{ij} = \underbrace{\bar{d}_{ij}}_{\text{time}} \times \underbrace{\lambda_{ij}}_{\text{disutility}}$
- ▶ Individuals have idiosyncratic tastes for pairs of residential and workplace locations, such that o 's utility from living in i and working in j is

$$U_{ij}^o = \epsilon \ln \left(\frac{w_j}{r_i^{1-\beta} P^\beta d_{ij}} \right) + z_{ij}^o \quad z_{ij}^o \stackrel{\text{iid}}{\sim} \text{T1EV}$$

- ▶ Labor-supply elasticity equals commuting elasticity ϵ
- ▶ $\epsilon \rightarrow \infty$ is the Rosen-Roback case

Baseline model: Equilibrium

Given economic primitives $(\beta, \epsilon, \sigma, L, \{A_j\}, \{T_i\}, \{d_{ij}\})$, an equilibrium is a set of wages $\{w_j\}$, rents $\{r_i\}$, and labor allocation $\{\ell_{ij}\}$ such that

labor allocation (gravity):

$$\frac{\ell_{ij}}{L} = \frac{w_j^\epsilon \left(r_i^{1-\beta} d_{ij}\right)^{-\epsilon}}{\sum_{i',j'} w_{j'}^\epsilon \left(r_{i'}^{1-\beta} d_{i'j'}\right)^{-\epsilon}} \quad \forall i, j \quad (1)$$

goods markets:

$$A_j \sum_i \frac{\ell_{ij}}{\bar{d}_{ij}} = \frac{(w_j/A_j)^{-\sigma}}{P^{1-\sigma}} Y \quad \forall j \quad (2)$$

land markets:

$$T_i = \frac{1-\beta}{r_i} \sum_j \underbrace{\frac{\ell_{ij}}{\bar{d}_{ij}} w_j}_{y_{ij}} \quad \forall i, j \quad (3)$$

Uniqueness: Sufficiently elastic labor demand or sufficiently small land share
 $\left(\frac{1+\epsilon}{\sigma+\epsilon}\right) \left(\frac{(1-\beta)\epsilon}{1+(1-\beta)\epsilon}\right) \leq \frac{1}{2} \implies$ unique equilibrium (Allen Arkolakis Li 2023)

Baseline model: Counterfactual outcomes by exact hat algebra

$$\hat{w}_j = \hat{A}_j \left(\sum_i \hat{y}_{ij} \frac{y_{ij}}{\sum_{i'} y_{i'j}} \right)^{\frac{1}{1-\sigma}} \left(\sum_{j'} \left(\frac{\hat{w}_{j'}}{\hat{A}_{j'}} \right)^{1-\sigma} \sum_i \frac{y_{ij'}}{Y} \right)^{\frac{1}{1-\sigma}} \hat{Y}^{\frac{1}{\sigma-1}} \quad (4)$$

$$\hat{r}_i = \hat{T}_i^{-1} \sum_j \hat{y}_{ij} \frac{y_{ij}}{\sum_{j'} y_{ij'}} \quad (5)$$

$$\hat{\ell}_{ij} = \begin{cases} 1, & \text{if } \ell_{ij} = 0 \\ \frac{\hat{w}_j^\varepsilon \left(\hat{r}_i^{1-\beta} \hat{\bar{d}}_{ij} \hat{\lambda}_{ij} \right)^{-\varepsilon}}{\sum_{i',j'} \hat{w}_{j'}^\varepsilon \left(\hat{r}_{i'}^{1-\beta} \hat{\bar{d}}_{i'j'} \hat{\lambda}_{i'j'} \right)^{-\varepsilon} \frac{\ell_{i'j'}}{L}} & \text{if } \ell_{ij} > 0 \end{cases} \quad (6)$$

Compute \hat{w}_j , \hat{r}_i , $\hat{\ell}_{ij}$, and $\hat{y}_{ij} = \hat{\ell}_{ij} \hat{w}_j / \hat{\bar{d}}_{ij}$ given elasticities β , σ and ϵ , baseline shares $\frac{\ell_{ij}}{L}$ and $\frac{y_{ij}}{Y}$, and relative exogenous parameters \hat{A}_j , \hat{T}_i , $\hat{\bar{d}}_{ij}$ and $\hat{\lambda}_{ij}$.

Commuting and the productivity elasticity of employment (1/3)

What is the productivity elasticity of employment? ($\hat{A}_{j^*} > 1$ is only shock)

To start, consider an economy of no commuting ($d_{ij} = \infty$ and $\ell_{ij} = 0$ if $i \neq j$) and perfectly elastic labor demand ($\sigma = \infty$)

$$\hat{w}_j = \hat{A}_j \quad \hat{r}_i = \hat{y}_{ii} = \hat{w}_{ii} \hat{\ell}_{ii} \quad \hat{\ell}_{ii} = \frac{\hat{w}_i^\varepsilon \hat{r}_i^{-\varepsilon(1-\beta)}}{\sum_{i'} \hat{w}_{i'}^\varepsilon \hat{r}_{i'}^{-\varepsilon(1-\beta)} \frac{\ell_{i'i'}}{L}}$$

Compare log changes to eliminate the denominator:

$$\frac{\hat{\ell}_{j^*j^*}}{\hat{\ell}_{jj}} = \frac{\hat{w}_{j^*}^\varepsilon (\hat{r}_{j^*})^{-\varepsilon(1-\beta)}}{\hat{r}_j^{-\varepsilon(1-\beta)}} = \frac{\hat{A}_{j^*}^{\beta\varepsilon} \left(\hat{\ell}_{j^*j^*}\right)^{-\varepsilon(1-\beta)}}{\hat{\ell}_{jj}^{-\varepsilon(1-\beta)}} = \hat{A}_{j^*}^{\frac{\beta\varepsilon}{1+\varepsilon(1-\beta)}}$$

With no commuting, *residential* land supply T_{j^*} is a brake on *employment* expansion in j^* . Lower land expenditure share (higher β) raises employment elasticity. ($\beta = 1$ makes T_{j^*} irrelevant)

Commuting and the productivity elasticity of employment (2/3)

Now consider an economy with costless commuting ($d_{ij} = 1 \forall ij$) and perfectly elastic labor demand ($\sigma = \infty$)

$$\hat{w}_j = \hat{A}_j \quad \hat{r}_i = \sum_j \hat{y}_{ij} \frac{y_{ij}}{\sum_{j'} y_{ij'}} \quad \hat{\ell}_{ij} = \frac{\hat{w}_j^\varepsilon \hat{r}_i^{-\varepsilon(1-\beta)}}{\sum_{i',j'} \hat{w}_{j'}^\varepsilon \hat{r}_{i'}^{-\varepsilon(1-\beta)} \frac{\ell_{i'j'}}{L}}$$

Because commuting is costless, the residence i share of workplace j employees

$$\frac{\ell_{ij}}{\sum_{i'} \ell_{i'j}} = \frac{r_i^{-\varepsilon(1-\beta)}}{\sum_{i'} r_{i'}^{-\varepsilon(1-\beta)}}$$
 is independent of j . What happens to employment?

$$\frac{\hat{L}_{j^*}}{\hat{L}_j} = \frac{\sum_i \hat{\ell}_{ij^*} \frac{\ell_{ij^*}}{L_j^*}}{\sum_i \hat{\ell}_{ij} \frac{\ell_{ij}}{L_j}} = \frac{\hat{w}_{j^*}^\varepsilon \sum_i \hat{r}_i^{-\varepsilon(1-\beta)} \frac{\ell_{ij^*}}{L_j^*}}{\hat{w}_j^\varepsilon \sum_i \hat{r}_i^{-\varepsilon(1-\beta)} \frac{\ell_{ij}}{L_j}} = \frac{\hat{A}_{j^*}^\varepsilon}{\hat{A}_j^\varepsilon} = \hat{A}_{j^*}^\varepsilon$$

With costless commuting, the productivity elasticity of employment is independent of the land expenditure share β . Choices of workplace and residences are wholly separated (everyone is a teleworker).

Commuting and the productivity elasticity of employment (3/3)

Monte, Redding, Rossi-Hansberg (2018) study how the local employment elasticity varies with commuting ties between US counties

- ▶ Commuting complicates studies of “local labor markets” (Also, counties vary greatly in size)
- ▶ In a model of counties with commuting, productivity elasticity of county employment varies
- ▶ Share of residents who work where they live predicts this variation in elasticities
- ▶ Million-dollar plants elicit larger employment increases in more open (lower $\frac{\ell_{ii}}{\sum_j \ell_{ij}}$) counties
- ▶ Reduced commuting costs could substitute for relaxing housing-supply restrictions

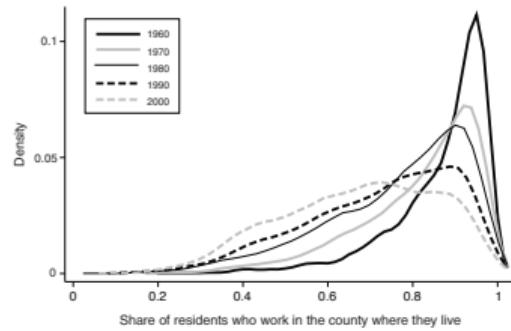


FIGURE 1. KERNEL DENSITIES OF THE SHARE OF RESIDENTS WHO WORK IN THE COUNTY WHERE THEY LIVE

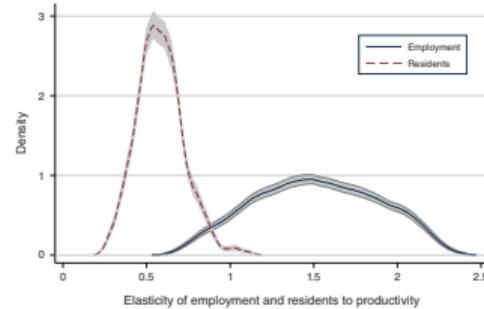


FIGURE 2. KERNEL DENSITY FOR THE DISTRIBUTION OF EMPLOYMENT AND RESIDENTS ELASTICITIES IN RESPONSE TO A PRODUCTIVITY SHOCK ACROSS COUNTIES

Ahlfeldt, Redding, Sturm, Wolf (Ecma, 2015)

ARSW is often referred to as “the Berlin Wall paper”:

- ▶ Develop a quantitative model of the city to identify intra-city agglomeration and dispersion forces
- ▶ Estimate using 1936, 1986 and 2006 data for thousands of city blocks on land prices, workplace employment, and employment by residence
- ▶ Use the division of Berlin after WWII and its reunification in 1989 as sources of exogenous variation in the surrounding concentration of economic activity

Won the [2018 Frisch Medal](#) for best applied paper in *Econometrica*: “The paper provides an outstanding example of how to credibly and transparently use a quasi-experimental approach to structurally estimate model parameters that can serve as critical inputs for counterfactual policy analyses.” ([cf. Haile](#))

Dividing Berlin

- ▶ A protocol signed during WWII organized Germany into American, British, French, and Soviet occupation zones
- ▶ Although 200km inside the Soviet zone, Berlin was to be jointly occupied and organized into four sectors (initially three roughly equal-sized, then British sector was split between French and British)
- ▶ Protocol envisioned a joint city administration (“Kommandatura”), but Cold War:
 - ▶ East and West Germany founded as separate states and separate city governments created in East and West Berlin in 1949
 - ▶ The adoption of Soviet-style policies of command and control in East Berlin limited economic interactions with West Berlin
 - ▶ To stop civilians leaving for West Germany, the East German authorities constructed the Berlin Wall in 1961
- ▶ Germany reunified in 1989

ARSW model: Overview

- ▶ The city consists of a set of discrete blocks indexed by i
- ▶ Single, freely traded (numeraire) final good ($\sigma = \infty$)
- ▶ Floor space can be used for residential or commercial use
- ▶ Firms choose a block of production and inputs of labor and floor space
- ▶ Workers choose block of residence, block of employment, and consumption of the final good
- ▶ Reservation level of utility (\bar{U}) for living outside the city
Individuals who choose Berlin and realize utility below the city-wide average cannot leave.

ARSW model: Preferences

- ▶ Utility for worker o residing in block i and working in block j :

$$C_{ijo} = \frac{B_i z_{ijo}}{d_{ij}} \left(\frac{c_{ij}}{\beta} \right)^\beta \left(\frac{\ell_{ij}}{1 - \beta} \right)^{1-\beta}, \quad 0 < \beta < 1$$

- ▶ Consumption of numeraire final good c_{ij}
- ▶ Residential amenity B_i
- ▶ Residential floor space ℓ_{ij} at price Q_i
- ▶ Commuting costs d_{ij}
- ▶ (sorry)
- ▶ Idiosyncratic shock z_{ijo}
- ▶ Indirect utility given wage w_j at workplace j

$$U_{ijo} = z_{ijo} B_i w_j Q_i^{\beta-1} / d_{ij},$$

- ▶ Idiosyncratic part of worker productivity is Fréchet distributed:

$$F(z_{ijo}) = e^{-T_i E_j z_{ijo}^{-\epsilon}}, \quad T_i, E_j > 0, \epsilon > 1$$

If x has a Fréchet distribution, then $\ln(x)$ has a Gumbel distribution

ARSW model: Location decisions within Berlin

- ▶ Probability worker chooses to live in i and work in j is

$$\pi_{ij} = \frac{T_i E_j \left(d_{ij} Q_i^{1-\beta} \right)^{-\epsilon} (B_i w_j)^\epsilon}{\sum_{r=1}^S \sum_{s=1}^S T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^\epsilon} \equiv \frac{\Phi_{ij}}{\Phi}.$$

- ▶ People choose Berlin (or not) before idiosyncratic shocks z_{ijo} are realized. Indifference equates expected utility in Berlin to reservation utility:

$$\mathbb{E} \left[\max_{ij} U_{ij}^o \right] = \gamma \left[\sum_{r=1}^S \sum_{s=1}^S T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^\epsilon \right]^{1/\epsilon} = \bar{U},$$

- ▶ Residential and workplace choice probabilities

$$\pi_{Ri} = \sum_{j=1}^S \pi_{ij} = \frac{1}{\Phi} \sum_{j=1}^S \Phi_{ij}, \quad \pi_{Mj} = \sum_{i=1}^S \pi_{ij} = \frac{1}{\Phi} \sum_{i=1}^S \Phi_{ij}.$$

- ▶ Conditional on living in block i , the probability that a worker commutes to block j

$$\pi_{ij|i} = \frac{E_j (w_j/d_{ij})^\epsilon}{\sum_{s=1}^S E_s (w_s/d_{is})^\epsilon},$$

ARSW model: Commuting

- ▶ Employment in block j equals the sum across all blocks i of people living in residence times the probability of commuting from i to j :

$$H_{Mj} = \sum_{i=1}^S \frac{E_j (w_j/d_{ij})^\epsilon}{\sum_{s=1}^S E_s (w_s/d_{is})^\epsilon} H_{Ri}, \quad d_{ij} = e^{\kappa \tau_{ij}}.$$

- ▶ This is the labor-supply curve for workplace j

ARSW model: Production

- ▶ A single final good (numeraire) is produced by competitive firms with constant returns to scale (and no trade costs):

$$y_j = A_j (H_{Mj})^\alpha (L_{Mj})^{1-\alpha}, \quad 0 < \alpha < 1$$

- ▶ H_{Mj} is workplace employment
- ▶ L_{Mj} is measure of floor space used commercially
- ▶ Isocost curve in commercial floor space q_j and labor w_j with shifter A_j :

$$q_j = (1 - \alpha) \left(\frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}$$

ARSW model: Floor space prices

- The share of floor space used commercially:

$$\theta_i = 1 \quad \text{if} \quad q_i > \xi_i Q_i,$$

$$\theta_i \in [0, 1] \quad \text{if} \quad q_i = \xi_i Q_i,$$

$$\theta_i = 0 \quad \text{if} \quad q_i < \xi_i Q_i.$$

- $\xi_i \geq 1$ represents the tax equivalent of regulations restricting commercial land use
- Assume observed land price is maximum of commercial and residential price:
$$Q_i = \max\{q_i, Q_i\}$$

ARSW model: Floor-space- and land-market clearing

- ▶ Floor space L is supplied competitively using Cobb-Douglas combination of local land K_i at price \mathbb{R}_i and freely traded capital input M

$$L_i = M_i^\mu K_i^{1-\mu} \implies Q_i = \chi \mathbb{R}_i^{1-\mu}$$

- ▶ Cobb-Douglas demand for residential floor space:

$$(1 - \theta_i)L_i = \frac{(1 - \beta)\mathbb{E}(w | i)}{Q_i} H_{Ri}$$

- ▶ Cobb-Douglas demand for commercial floor space:

$$\theta_i L_i = \left(\frac{(1 - \alpha)A_i}{q_i} \right)^{\frac{1}{\alpha}} H_{Mi}$$

- ▶ Density of development (M_i^μ) from land-market clearing:

$$M_i^\mu = \frac{L_i}{K_i^{1-\mu}} = \frac{(1 - \theta_i)L_i + \theta_i L_i}{K_i^{1-\mu}}$$

Equilibrium with exogenous locational characteristics

- ▶ **Proposition 1:** Given the model's parameters $[\alpha, \beta, \mu, \epsilon, \kappa]$, the reservation utility \bar{U} , and vectors of exogenous location characteristics $[T, E, A, B, \phi, K, \xi, \tau]$, there exists a unique general equilibrium vector $[\pi_M, \pi_R, H, Q, q, w, \theta]$, where H denotes total city population.
- ▶ These seven components are determined by the system of seven equations: commercial land market clearing, residential land market clearing, zero profits, no arbitrage between alternative uses of land, residential choice probability π_{Ri} , workplace choice probability π_{Mi} , and indifference with reservation utility.
- ▶ Notice that B_i and A_j are exogenous in this proposition

Amenity and productivity spillovers

- ▶ Productivity (A_j) depends on fundamentals (a_j) and spillovers (Υ_j):

$$A_j = a_j \Upsilon_j^\lambda, \quad \Upsilon_j \equiv \left[\sum_{s=1}^S e^{-\delta \tau_{js}} \left(\frac{H_{Ms}}{K_s} \right) \right],$$

- ▶ δ is the rate of decay of spillovers
- ▶ λ captures the relative importance of spillovers (Pardon the clash with λ_{ij})
- ▶ Residential amenities are influenced by both fundamentals (b_i) and spillovers (Ω_i)

$$B_i = b_i \Omega_i^\eta, \quad \Omega_i \equiv \left[\sum_{s=1}^S e^{-\rho \tau_{is}} \left(\frac{H_{Rs}}{K_s} \right) \right].$$

- ▶ These local externalities are the heart of the paper

Subsequent research allows explicit trips to enjoy amenities (Hausman, Samuels, Cohen, Sasson 2025) or consume non-traded services (Miyauchi, Nakajima, Redding 2025)

Overview of remainder of paper

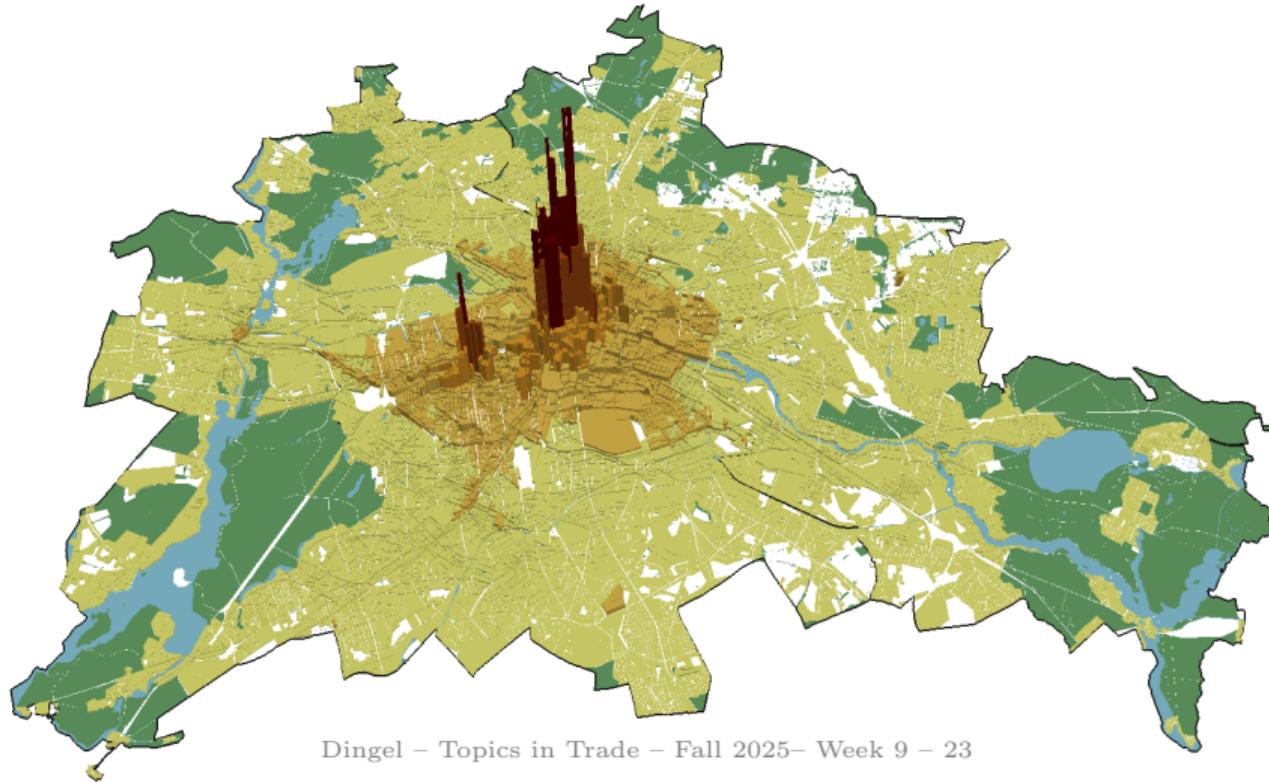
- ▶ Relate land prices to distance to pre-war CBD
- ▶ Estimate the model without any agglomeration effects. In counterfactuals, this model “is unable to account quantitatively for the observed impact of division and reunification on the pattern of economic activity within West Berlin.”
- ▶ Estimate model with local production and residential amenity externalities. These are interesting in their own right and improve model fit (they also create possibility of multiple equilibria, which must be handled carefully)

Data

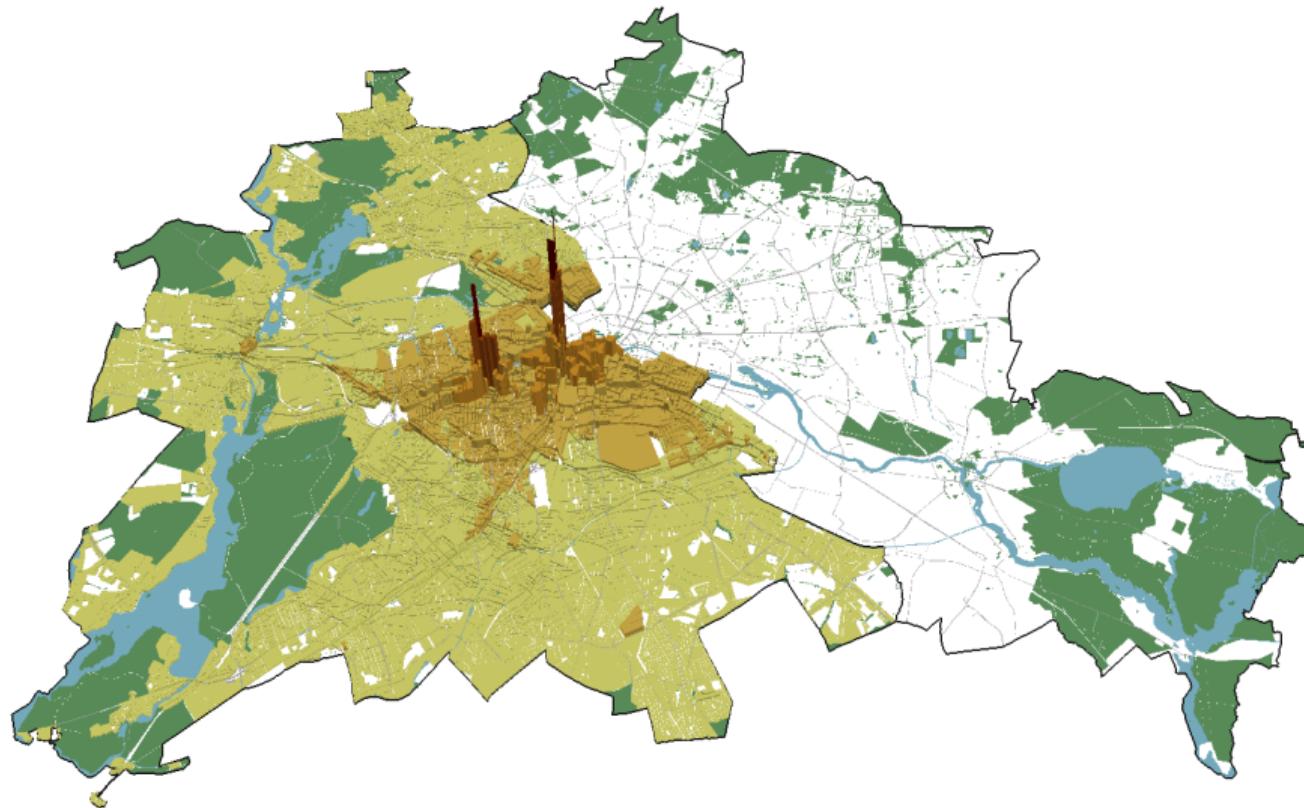
- ▶ Data on land prices, workplace employment, residence employment and bilateral travel times
- ▶ Data for Greater Berlin in 1936 and 2006 and data for West Berlin in 1986 by
 - ▶ Pre-war districts (“Bezirke”), 20 in Greater Berlin, 12 in West Berlin
 - ▶ Statistical areas (“Gebiete”), around 90 in West Berlin
 - ▶ Statistical blocks, around 9,000 in West Berlin
- ▶ Land prices: official assessed land value of a representative undeveloped property or the fair market value of a developed property if not developed
- ▶ Geographical Information Systems (GIS) data on land area, land use, building density, proximity to U-Bahn (underground) and S-Bahn (suburban) stations, schools, parks, lakes, canals and rivers, Second World War destruction, location of government buildings and urban regeneration programs

Land prices in Berlin in 1936

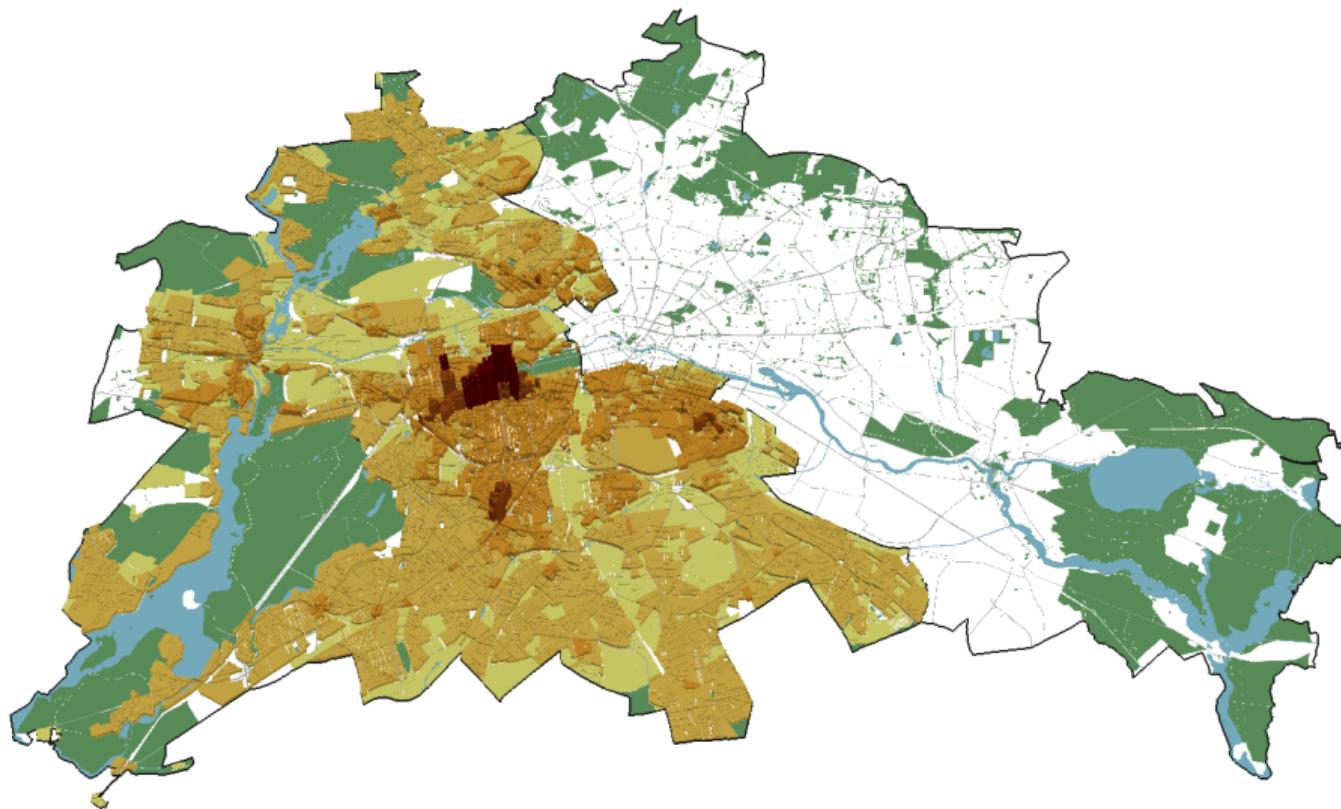
Land prices are normalized to have a mean of 1 in each year.



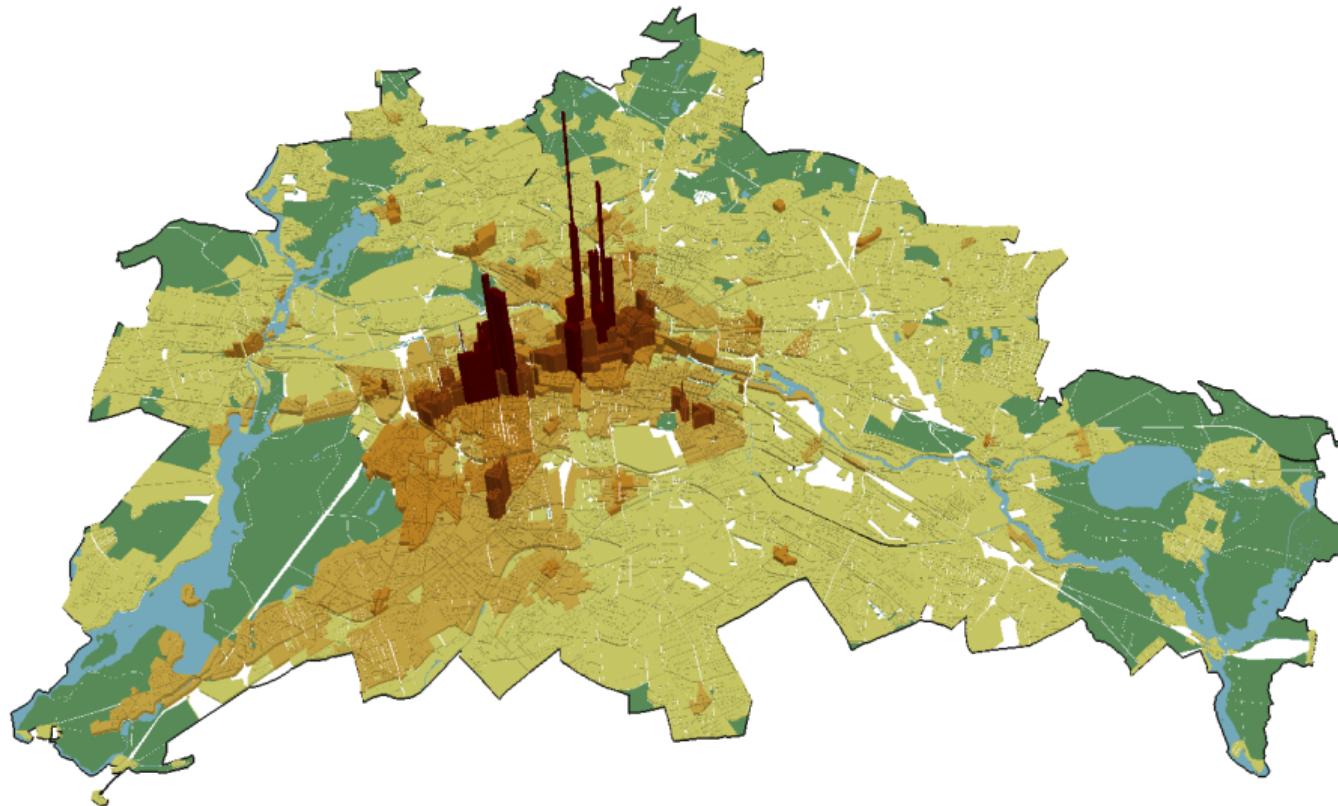
Land prices in West Berlin in 1936



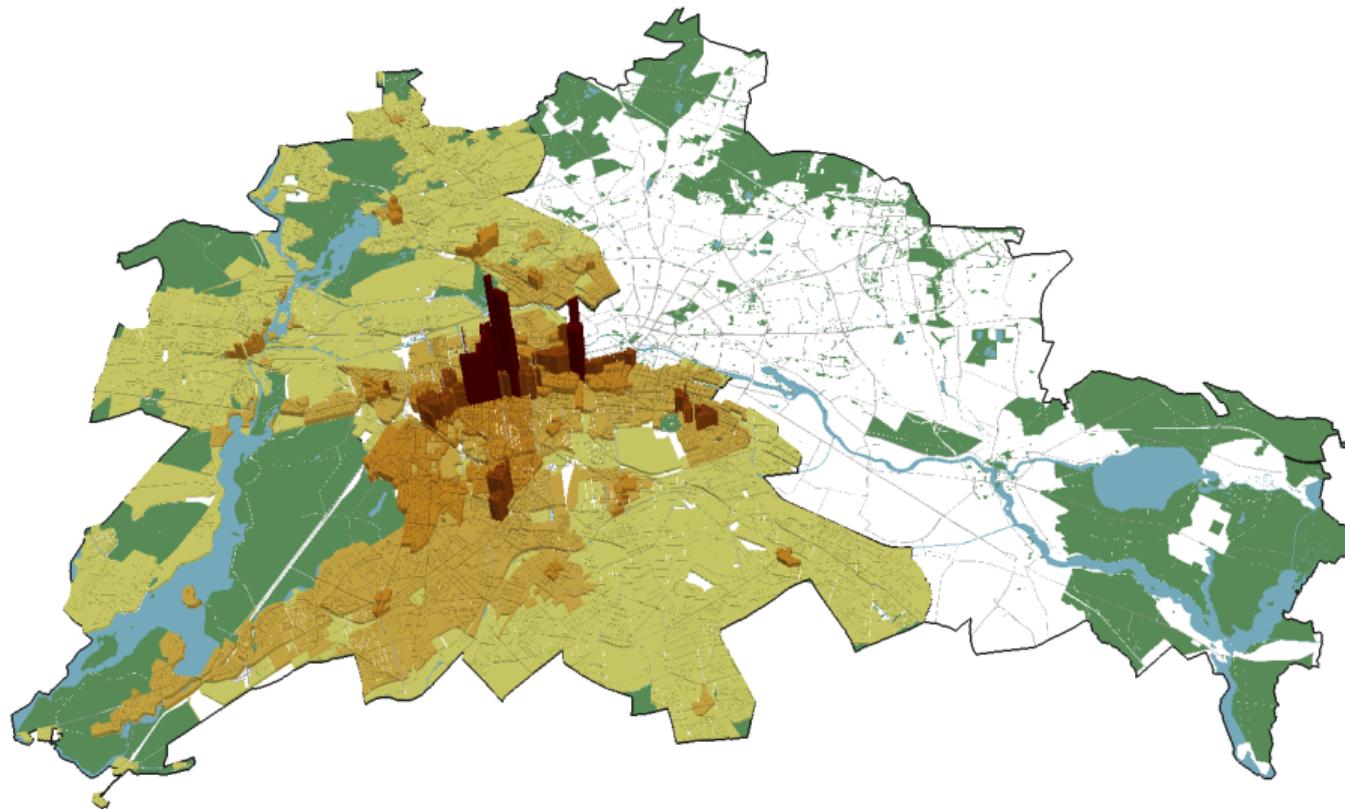
Land prices in West Berlin in 1986



Land prices in Berlin in 2006



Land prices in West Berlin in 2006



Diff-in-diffs specification

- ▶ Estimate difference-in-differences specification for division and reunification separately (for areas in West Berlin):

$$\Delta \ln Q_i = \psi + \sum_{k=1}^K \mathbf{1}_{ik} \beta_k + \ln X_i \zeta + \chi_i,$$

- ▶ $\mathbf{1}_{ik}$ is a $(0, 1)$ dummy which equals one if block i lies within distance grid cell k from the pre-war CBD and zero otherwise
- ▶ Observable block characteristics (X_i): Land area, land use, distance to nearest U-Bahn station, S-Bahn station, school, lake, river or canal, and park, war destruction, government buildings and urban regeneration programs

Division and Pre-War CBD



Diff-in-diffs West Berlin 1936-86

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln \text{EmpR}$	(7) $\Delta \ln \text{EmpR}$	(8) $\Delta \ln \text{EmpW}$	(9) $\Delta \ln \text{EmpW}$
CBD 1	-0.800*** (0.071)	-0.567*** (0.071)	-0.524*** (0.071)	-0.503*** (0.071)	-0.565*** (0.077)	-1.332*** (0.383)	-0.975*** (0.311)	-0.691* (0.408)	-0.639* (0.338)
CBD 2	-0.655*** (0.042)	-0.422*** (0.047)	-0.392*** (0.046)	-0.360*** (0.043)	-0.400*** (0.050)	-0.715** (0.299)	-0.361 (0.280)	-1.253*** (0.293)	-1.367*** (0.243)
CBD 3	-0.543*** (0.034)	-0.306*** (0.039)	-0.294*** (0.037)	-0.258*** (0.032)	-0.247*** (0.034)	-0.911*** (0.239)	-0.460** (0.206)	-0.341 (0.241)	-0.471** (0.190)
CBD 4	-0.436*** (0.022)	-0.207*** (0.033)	-0.193*** (0.033)	-0.166*** (0.030)	-0.176*** (0.026)	-0.356** (0.145)	-0.259 (0.159)	-0.512*** (0.199)	-0.521*** (0.169)
CBD 5	-0.353*** (0.016)	-0.139*** (0.024)	-0.123*** (0.024)	-0.098*** (0.023)	-0.100*** (0.020)	-0.301*** (0.110)	-0.143 (0.113)	-0.436*** (0.151)	-0.340*** (0.124)
CBD 6	-0.291*** (0.018)	-0.125*** (0.019)	-0.094*** (0.017)	-0.077*** (0.016)	-0.090*** (0.016)	-0.360*** (0.100)	-0.135 (0.089)	-0.280** (0.130)	-0.142 (0.116)
Inner Boundary 1-6		Yes	Yes	Yes		Yes			Yes
Outer Boundary 1-6		Yes	Yes	Yes		Yes			Yes
Kudamm 1-6			Yes	Yes		Yes			Yes
Block Characteristics				Yes		Yes			Yes
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6260	6260	6260	6260	6260	5978	5978	2844	2844
R-squared	0.26	0.51	0.63	0.65	0.71	0.19	0.43	0.12	0.33

Note: Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1-CBD6 are six 500m distance grid cells for distance from the pre-war CBD. Inner Boundary 1-6 are six 500m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1-6 are six 500m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1-6 are six 500m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A2 of the web appendix. Block characteristics include the logarithm of distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999). * significant at 10%; ** significant at 5%; *** significant at 1%.

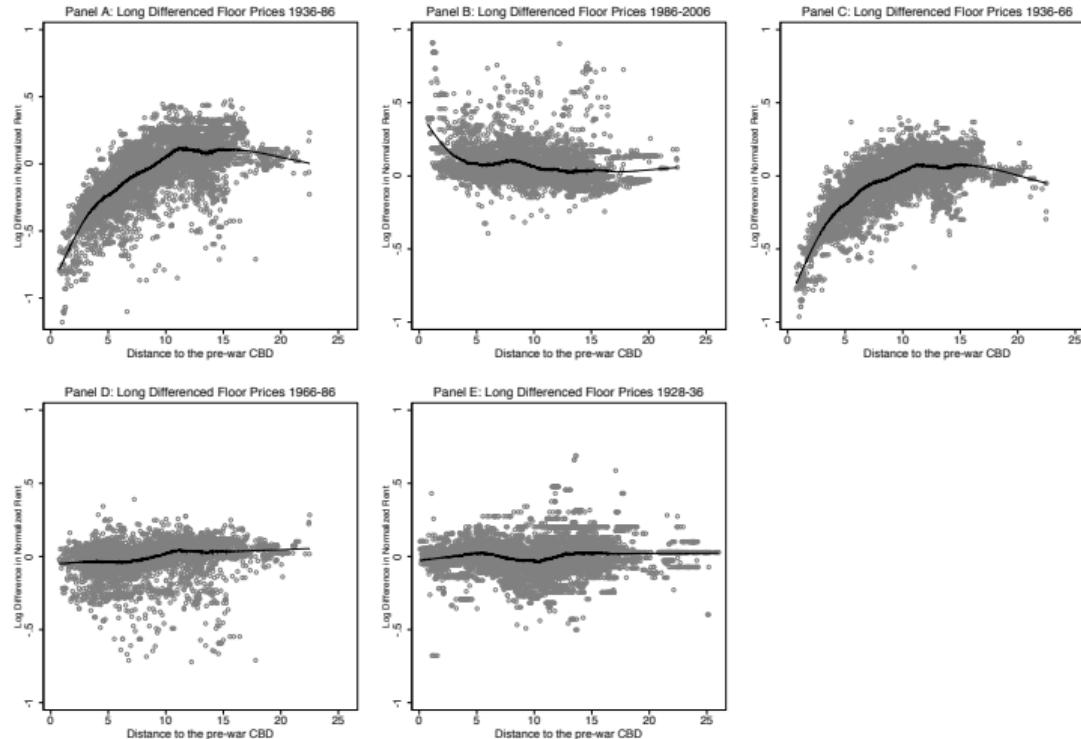
Diff-in-diffs West Berlin 1986-2006

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln \text{EmpR}$	(7) $\Delta \ln \text{EmpR}$	(8) $\Delta \ln \text{EmpW}$	(9) $\Delta \ln \text{EmpW}$
CBD 1	0.398*** (0.105)	0.408*** (0.090)	0.368*** (0.083)	0.369*** (0.081)	0.281*** (0.088)	1.079*** (0.307)	1.025*** (0.297)	1.574*** (0.479)	1.249** (0.517)
CBD 2	0.290*** (0.111)	0.289*** (0.096)	0.257*** (0.090)	0.258*** (0.088)	0.191** (0.087)	0.589* (0.315)	0.538* (0.299)	0.684** (0.326)	0.457 (0.334)
CBD 3	0.122*** (0.037)	0.120*** (0.033)	0.110*** (0.032)	0.115*** (0.032)	0.063** (0.028)	0.340* (0.180)	0.305* (0.158)	0.326 (0.216)	0.158 (0.239)
CBD 4	0.033*** (0.013)	0.031 (0.023)	0.030 (0.022)	0.034 (0.021)	0.017 (0.020)	0.110 (0.068)	0.034 (0.066)	0.336** (0.161)	0.261 (0.185)
CBD 5	0.025*** (0.010)	0.018 (0.015)	0.020 (0.014)	0.020 (0.014)	0.015 (0.013)	-0.012 (0.056)	-0.056 (0.057)	0.114 (0.118)	0.066 (0.131)
CBD 6	0.019** (0.009)	-0.000 (0.009)	-0.000 (0.012)	-0.003 (0.012)	0.005 (0.011)	0.060 (0.039)	0.053 (0.041)	0.049 (0.095)	0.110 (0.098)
Inner Boundary 1-6		Yes	Yes	Yes			Yes		Yes
Outer Boundary 1-6		Yes	Yes	Yes			Yes		Yes
Kudamm 1-6			Yes	Yes			Yes		Yes
Block Characteristics				Yes			Yes		Yes
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	7050	7050	7050	7050	7050	6718	6718	5602	5602
R-squared	0.08	0.32	0.34	0.35	0.43	0.04	0.07	0.03	0.06

Note: Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1-CBD6 are six 500m distance grid cells for distance from the pre-war CBD. Inner Boundary 1-6 are six 500m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1-6 are six 500m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1-6 are six 500m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A4 of the web appendix. Block characteristics include the logarithm of distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999). * significant at 10%; ** significant at 5%; *** significant at 1%.

Land prices over time

1928–1936 is placebo test. Most of 1936–1986 change occurs by 1966.



Note: Log floor prices are normalized to have a mean of zero in each year before taking the one difference. Solid lines are fitted values from locally-weighted linear least squares regressions.

Gravity regression for commuting between districts

Log commuters from residence I to workplace J (Jensen's inequality):

$$\ln \pi_{IJ} = -\nu \tau_{IJ} + \vartheta_I + \varsigma_J + e_{IJ},$$

where τ_{IJ} is transit minutes, $\nu = \epsilon \kappa$, and ϑ_I and ς_J are fixed effects

	(1)	(2)	(3)	(4)
	ln Bilateral Commuting Probability 2008	ln Bilateral Commuting Probability 2008	ln Bilateral Commuting Probability 2008	ln Bilateral Commuting Probability 2008
Travel Time ($-\kappa \epsilon$)	-0.0697*** (0.0056)	-0.0702*** (0.0034)	-0.0771*** (0.0025)	-0.0706*** (0.0026)
Estimation	OLS	OLS	Poisson PML	Gamma PML
More than 10 Commuters		Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Observations	144	122	122	122
R-squared	0.8261	0.9059	-	-

Note: Gravity equation estimates based on representative micro survey data on commuting for Greater Berlin for 2008.

Observations are bilateral pairs of 12 workplace and residence districts (post 2001 Bezirk boundaries). Travel time is measured in minutes. Fixed effects are workplace district fixed effects and residence district fixed effects. The specifications labelled more than 10 commuters restrict attention to bilateral pairs with 10 or more commuters. Poisson PML is Poisson Pseudo Maximum Likelihood estimator. Gamma PML is Gamma Pseudo Maximum Likelihood Estimator. Standard errors in parentheses are heteroscedasticity robust. *significant at 1%, **significant at 5%, *** significant at 1%.

Backing out productivities

- ▶ Composites: $\tilde{A}_j \equiv A_j E_j^{\alpha/\epsilon}$, $\tilde{a}_j \equiv a_j E_j^{\alpha/\epsilon}$ $\omega_j \equiv \tilde{w}_j^\epsilon = E_j w_j^\epsilon$
- ▶ Using estimated ν , and data on residence and workplace employment, one can solve for transformed wages $E_j w_j^\epsilon$ from commuting equation (summed over origins)
- ▶ Recover overall productivity A_j from zero-profit equation:

$$\ln \left(\frac{\tilde{A}_{jt}}{\bar{\tilde{A}}_t} \right) = (1 - \alpha) \ln \left(\frac{\bar{Q}_{jt}}{\bar{Q}_t} \right) + \frac{\alpha}{\epsilon} \ln \left(\frac{\omega_{jt}}{\bar{\omega}_t} \right)$$

where $\bar{\tilde{A}}_t = \exp \left(1/S \sum_{s=1}^S \ln \tilde{A}_{st} \right)$ (geometric mean)

Zero-profit condition of Rosen-Roback model

- ▶ High floor prices and wages require high final good productivity for zero profits to be satisfied

Backing out amenities

- ▶ Composites: $\tilde{B}_i \equiv b_i T_i^{1/\epsilon} \zeta_{Ri}^{1-\beta}$, $\tilde{b}_i \equiv b_i T_i^{1/\epsilon} \zeta_{Ri}^{1-\beta}$, where $\zeta_{Ri} = 1$ or $\zeta_{Ri} = \xi_i$ based on land use
- ▶ Recover amenities B_i from residential choice probabilities:

$$\ln \left(\frac{\tilde{B}_{it}}{\bar{\tilde{B}}_t} \right) = \frac{1}{\epsilon} \ln \left(\frac{H_{Rit}}{\bar{H}_{Rt}} \right) + (1 - \beta) \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t} \right) - \frac{1}{\epsilon} \ln \left(\frac{W_{it}}{\bar{W}_t} \right)$$

where $W_{it} = \sum_{s=1}^S \omega_{st} / e^{\nu \tau_{ist}}$ and variables with an upper bar denote that variable's geometric mean.

If $\epsilon = \infty$, this is Rosen-Roback indifference condition

- ▶ High floor prices and high residence employment must be explained either by high wage commuting access or high amenities
- ▶ (So far not making assumptions about the relative importance of production and residential externalities versus fundamentals)

Backing out amenities and productivities

- ▶ Set α, β, μ “to central estimates from the existing empirical literature”
- ▶ Use estimate of $\nu = \epsilon\kappa = 0.07$ from gravity regression
- ▶ “To calibrate the value of the Fréchet shape parameter (ϵ), we use our data on the dispersion of log wages by workplace across the districts of West Berlin for 1986.”
- ▶ “This value of $\epsilon = 6.83$ for commuting decisions is broadly in line with the range of estimates for the Fréchet shape parameter for international trade flows.”
- ▶ Table IV columns 1-4 show changes in amenities and productivities over time.
- ▶ Table IV columns 5-6 show the impact of the division and reunification on West Berlin land-price gradient, holding productivity and amenities constant at their 1936 values. Poor fit ($-.408 \neq -.800$; $-.010 \neq +.398$)

Changes in fundamentals and counterfactuals with exogenous A , B

TABLE IV
PRODUCTIVITY, AMENITIES, AND COUNTERFACTUAL FLOOR PRICES^a

	(1) $\Delta \ln A$ 1936–1986	(2) $\Delta \ln B$ 1936–1986	(3) $\Delta \ln A$ 1986–2006	(4) $\Delta \ln B$ 1986–2006	(5) $\Delta \ln QC$ 1936–1986	(6) $\Delta \ln QC$ 1986–2006
CBD 1	−0.207*** (0.049)	−0.347*** (0.070)	0.261*** (0.073)	0.203*** (0.054)	−0.408*** (0.038)	−0.010 (0.020)
CBD 2	−0.260*** (0.032)	−0.242*** (0.053)	0.144** (0.056)	0.109* (0.058)	−0.348*** (0.017)	0.079** (0.036)
CBD 3	−0.138*** (0.021)	−0.262*** (0.037)	0.077*** (0.024)	0.059** (0.026)	−0.353*** (0.022)	0.036 (0.031)
CBD 4	−0.131*** (0.016)	−0.154*** (0.023)	0.057*** (0.015)	0.010 (0.008)	−0.378*** (0.021)	0.093*** (0.026)
CBD 5	−0.095*** (0.014)	−0.126*** (0.013)	0.028** (0.013)	−0.014* (0.007)	−0.380*** (0.022)	0.115*** (0.033)
CBD 6	−0.061*** (0.015)	−0.117*** (0.015)	0.023** (0.010)	0.001 (0.005)	−0.354*** (0.018)	0.066*** (0.023)
Counterfactuals					Yes	Yes
Agglomeration Effects					No	No
Observations	2,844	5,978	5,602	6,718	6,260	7,050
R^2	0.09	0.06	0.02	0.03	0.07	0.03

ARSW (2015) Section 7: Estimation of structural model

Assumed Parameter	Source	Value
Residential land	$1 - \beta$	Davis & Ortalo-Magne (2011)
Commercial land	$1 - \alpha$	Valentinyi-Herrendorf (2008)
Fréchet Scale	T	(normalization)
Expected Utility	\bar{u}	(normalization)

Estimated Parameter		
Production externalities elasticity	λ	
Production externalities decay	δ	
Residential externalities elasticity	η	
Residential externalities decay	ρ	
Commuting semi-elasticity	$\nu = \epsilon\kappa$	
Commuting heterogeneity	ϵ	

GMM estimation procedure

- ▶ Use exogenous variation from Berlin's division and reunification “to structurally estimate” the agglomeration parameters $\{\lambda, \delta, \eta, \rho\}$.

$$\Delta \ln \left(\frac{a_{it}}{\bar{a}_t} \right) = (1 - \alpha) \Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t} \right) + \frac{\alpha}{\epsilon} \Delta \ln \left(\frac{\omega_{it}}{\bar{\omega}_t} \right) - \lambda \Delta \ln \left(\frac{\Upsilon_{it}}{\bar{\Upsilon}_t} \right)$$

$$\begin{aligned} \Delta \ln \left(\frac{b_{it}}{\bar{b}_t} \right) &= \frac{1}{\epsilon} \Delta \ln \left(\frac{H_{Rit}}{\bar{H}_{Rt}} \right) + (1 - \beta) \Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t} \right) \\ &\quad + \frac{1}{\epsilon} \Delta \ln \left(\frac{W_{it}}{\bar{W}_t} \right) - \eta \Delta \ln \left(\frac{\Omega_{it}}{\bar{\Omega}_t} \right) \end{aligned}$$

- ▶ Production externalities Υ_{it} depend on travel-time weighted sum of observed workplace employment densities
- ▶ Residential externalities Ω_{it} depend on travel-time weighted sum of observed residence employment densities
- ▶ Adjusted fundamentals relative to geometric mean are “structural residuals”

Moment conditions

- ▶ Changes in adjusted fundamentals uncorrelated with exogenous change in surrounding economic activity from division/reunification

$$\mathbb{E} [\mathbb{I}_k \times \Delta \ln (a_{it}/\bar{a}_t)] = 0, \quad k \in \{1, \dots, K_{\mathbb{I}}\},$$

$$\mathbb{E} [\mathbb{I}_k \times \Delta \ln (b_{it}/\bar{b}_t)] = 0, \quad k \in \{1, \dots, K_{\mathbb{I}}\}.$$

where \mathbb{I}_k are indicators for distance grid cells from pre-war CBD

- ▶ Other moments are fraction of workers that commute less than 30 minutes and wage dispersion

$$\mathbb{E} \left[\psi H_{Mj} - \sum_{i \in \mathbb{N}_j}^S \frac{\omega_j / e^{\nu \tau_{ij}}}{\sum_{s=1}^S \omega_s / e^{\nu \tau_{is}}} H_{Ri} \right] = 0,$$

$$\mathbb{E} \left[(1/\epsilon)^2 \ln (\omega_j)^2 - \sigma_{\ln w_i}^2 \right] = 0,$$

Why estimate $\nu = \epsilon\kappa$ twice?

Section 6 uses bilateral flows in 2008 between 12 districts:

- ▶ Table III: $\nu = \epsilon\kappa = 0.07$
- ▶ “minimize the squared difference between the variances across districts of log adjusted wages in the model and log wages in the data” $\implies \epsilon = 6.83$
- ▶ “While the model uses measures of bilateral travel times that we construct based on the transport network, the micro survey data include self-reported travel times for each commuter... assume that this measurement error is uncorrelated with self-reported travel times.”

Section 7 uses fraction of Berlin commutes under 30 minutes during division:

- ▶ ν : “none of the other parameters $\{\epsilon, \lambda, \delta, \eta, \rho\}$ affect the commuting moment condition” $\nu = \epsilon\kappa = 0.10$
- ▶ ϵ : “none of the other parameters $\{\lambda, \delta, \eta, \rho\}$ affect the wage moment condition” $\epsilon = 6.69$

Severen (2023) on workplace amenities

$$\mathbb{E} \left[(1/\epsilon)^2 \ln (\omega_j)^2 - \sigma_{\ln w_i}^2 \right] = 0 \quad \omega_j = E_j w_j^\epsilon$$

The urban economic geography literature often identifies ϵ from a combination of modeling assumptions and cross-sectional variation in travel time. Because I observe workplace wage, I can test a prominent assumption. Ahlfeldt et al. (2015) select ϵ to rescale the variance of model-implied wages.²⁴ This implicitly disallows positive correlation between w and e and requires either (i) $\mathbb{E}[w_j e_j]/\mathbb{V}[e_j] = -1/2\epsilon$ or (ii) $\mathbb{V}[e_j] = 0$.²⁵ Panel C of table 3 presents tests of (i), where estimates of ϵ from Panel A are enforced to calculate $e_j = \hat{\omega}_j - \hat{\epsilon} w_j$. Most specifications show a negative correlation between w_j and e_j , as required if $\mathbb{E}[w_j e_j]/\mathbb{V}[e_j] = -1/2\epsilon$. The first row of p -values treats ϵ as a constant, the second as a random variable (see appendix D for details). If ϵ is fixed, most specifications reject the null hypothesis that $\mathbb{E}[w_j e_j]/\mathbb{V}[e_j] = -1/2\epsilon$. However, if uncertainty in ϵ is taken into account, only columns 3 and 4 reject the null.

²⁴To see this, Ahlfeldt et al. (2015) require $\mathbb{V}(\hat{\omega}_j) = \mathbb{V}(w_j)$, where $\hat{\omega} = \hat{w}_j + \frac{1}{\epsilon} e_j$, \hat{w}_j are model-implied wages, and $\mathbb{V}(w_j)$ is the variance of average wage across twelve districts of Berlin. Rearranging, noting that e_j is mean zero, and substituting observed wage w_j for \hat{w}_j and w_j yields $2\epsilon \mathbb{E}[w_j e_j] + \mathbb{V}[e_j] = 0$. Consider three cases: (i) if w_j and e_j are negatively correlated, $\mathbb{E}[w_j e_j]/\mathbb{V}[e_j] = -1/2\epsilon$; (ii) $\mathbb{E}[w_j e_j] \Leftrightarrow \mathbb{V}[e_j] = 0$; and (iii) positive correlation w_j and e_j imply a negative variance of e .

“Supplementary results in appendix D indicate that wages explain only a relatively small amount of the variation in workplace fixed effects ω .”

ARSW: Estimated parameters

TABLE V
GENERALIZED METHOD OF MOMENTS (GMM) ESTIMATION RESULTS^a

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\varepsilon$)	0.0951*** (0.0016)	0.1011*** (0.0016)	0.0987*** (0.0016)
Commuting Heterogeneity (ε)	6.6190*** (0.0939)	6.7620*** (0.1005)	6.6941*** (0.0934)
Productivity Elasticity (λ)	0.0793*** (0.0064)	0.0496*** (0.0079)	0.0710*** (0.0054)
Productivity Decay (δ)	0.3585*** (0.1030)	0.9246*** (0.3525)	0.3617*** (0.0782)
Residential Elasticity (η)	0.1548*** (0.0092)	0.0757** (0.0313)	0.1553*** (0.0083)
Residential Decay (ρ)	0.9094*** (0.2968)	0.5531 (0.3979)	0.7595*** (0.1741)

Very localized externalities

TABLE VI
EXTERNALITIES AND COMMUTING COSTS^a

	(1) Production Externalities $(1 \times e^{-\delta\tau})$	(2) Residential Externalities $(1 \times e^{-\rho\tau})$	(3) Utility After Commuting $(1 \times e^{-\kappa\tau})$
0 minutes	1.000	1.000	1.000
1 minute	0.696	0.468	0.985
2 minutes	0.485	0.219	0.971
3 minutes	0.338	0.102	0.957
5 minutes	0.164	0.022	0.929
7 minutes	0.079	0.005	0.902
10 minutes	0.027	0.001	0.863
15 minutes	0.004	0.000	0.802
20 minutes	0.001	0.000	0.745
30 minutes	0.000	0.000	0.642

Counterfactuals

TABLE VII
COUNTERFACTUALS^a

	(1) $\Delta \ln QC$ 1936–1986	(2) $\Delta \ln QC$ 1936–1986	(3) $\Delta \ln QC$ 1936–1986	(4) $\Delta \ln QC$ 1936–1986	(5) $\Delta \ln QC$ 1986–2006	(6) $\Delta \ln QC$ 1986–2006	(7) $\Delta \ln QC$ 1986–2006
CBD 1	-0.836*** (0.052)	-0.613*** (0.032)	-0.467*** (0.060)	-0.821*** (0.051)	0.363*** (0.041)	1.160*** (0.052)	0.392*** (0.043)
CBD 2	-0.560*** (0.034)	-0.397*** (0.025)	-0.364*** (0.019)	-0.624*** (0.029)	0.239*** (0.028)	0.779*** (0.044)	0.244*** (0.027)
CBD 3	-0.455*** (0.036)	-0.312*** (0.030)	-0.336*** (0.030)	-0.530*** (0.036)	0.163*** (0.031)	0.594*** (0.045)	0.179*** (0.031)
CBD 4	-0.423*** (0.026)	-0.284*** (0.019)	-0.340*** (0.022)	-0.517*** (0.031)	0.140*** (0.021)	0.445*** (0.042)	0.143*** (0.021)
CBD 5	-0.418*** (0.032)	-0.265*** (0.022)	-0.351*** (0.027)	-0.512*** (0.039)	0.177*** (0.032)	0.403*** (0.038)	0.180*** (0.032)
CBD 6	-0.349*** (0.025)	-0.222*** (0.016)	-0.304*** (0.022)	-0.430*** (0.029)	0.100*** (0.024)	0.334*** (0.034)	0.103*** (0.023)
Counterfactuals	Yes						
Agglomeration Effects	Yes						
Observations	6,260	6,260	6,260	6,260	7,050	6,260	7,050
R ²	0.11	0.13	0.07	0.13	0.12	0.24	0.13

Two primary contributions of ARSW

- ▶ Estimates of very localized externalities (“The economics of density”)
e.g., Stuart S. Rosenthal and William C. Strange “[How Close Is Close? The Spatial Reach of Agglomeration Economies](#)” *JEP* 2020
- ▶ Canonical model of commuting within a city
i.e., the baseline model described in Dingel and Tintelnot (2025) at start of this class

How these models work: Normative properties

Davis, Gregory (2021) - “Place-Based Redistribution in Simple Location-Choice Models”

- ▶ Fajgelbaum and Gaubert (2020) and subsequent papers prescribe fiscal transfers (trade deficits) to high-MU-of-tradables places (cf. Glaeser and Gottlieb 2008)
- ▶ Idiosyncratic errors terms are uninsured and affect location choices, the level of utility, and marginal utility
- ▶ Frechet vs Weibull: marginal utilities can differ even when allocations and elasticities for marginal households coincide, thus making the planning problems differ even when positive predictions coincide
- ▶ I conjecture that if you use GEV errors the model is not identified
- ▶ I am skeptical that their proposed computational approach to the planning problem will scale to empirical applications

Next week

Next week: Identification, calibration, and exact hat algebra