Name and school:

Problem 1: Mental Math (no calculators allowed)

Example:

Question 1.1:

Question 1.2:

Question 1.3:

Question 1.4:

Question 1.5:

Question 1.6:

Question 1.7:

Question 1.8:

Question 1.9:

Question 1.10:

Name and school:

Problem 2: Bouncing Numbers (Individual problem)

15 minutes, no calculators, 23 points possible

Sometimes it is easier to use a "simpler" number instead of the original number if this simpler number is close to the original. To find this simpler number we can **round** the original number. We can round an integer to the nearest ten, nearest hundred, nearest thousand, etc.

Part 1: How to Round Positive Integers

There are standard rules to rounding:

• Decide which place value you want to round to. For example,

- If you round to the nearest **ten**, then the **2nd** digit from the end is considered.
- If you round to the nearest **hundred**, then the **3rd** digit from the end is considered.
- If you round to the nearest **thousand**, then the **4th** digit from the end is considered.

• Leave this digit the same if the next digit is strictly less than 5 ("rounding down").

• Increase this digit by 1 if the next digit is 5 or more ("rounding up").

• Change each digit after the last one you want to keep to 0.

Examples

- Round 1234 to the nearest ten.
  The last digit you want to keep is the 2nd digit from the end. The next digit is 4, which is less than 5, so you keep the '3' and change the '4' to '0'.

  The result is: **1230**.
- Round 9876 to the nearest ten.
  The last digit you want to keep is the 2nd digit from the end. The next digit is 6, which is at least 5, so you increase the '7' to '8' and change the '6' to '0'.

  The result is: **9880**.
- Round 4359 to the nearest hundred.
  The last digit you want to keep is the 3rd digit from the end. The next digit is 5, which is at least 5, so you increase '3' to '4' and change '59' to '00'.

  The result is: **4400**.
- Round 4182 to the nearest thousand.
  The last digit you want to keep is the 4th digit from the end. The the next digit is 1, which is less than 5, so you keep the '4' and change '182' to '000'.

  The result is: **4000**.

Question 2.1: (1 points) What number do we get if we round 1234 to the nearest hundred?

Question 2.2: (1 points) What number do we get if we round 6700 to the nearest ten?

Question 2.3: (1 points) What number do we get if we round 7500 to the nearest thousand?

The **distance** between two numbers is a *non-negative* number. If the numbers are equal, then their distance is 0. Otherwise, you need to subtract the smaller number from the greater to get their distance.

Examples

- The distance of 2 and 7 is 5, because $7 - 2 = 5$.
  (Be careful, the distance is not -5.)
- The distance of 9 and 6 is 3, because $9 - 6 = 3$.

Question 2.4: (3 points) How many 4-digit numbers are there, so that if you round them to the nearest thousand, then the distance of the result and the original number is

part a) 23;

part b) 450;

part c) 793?

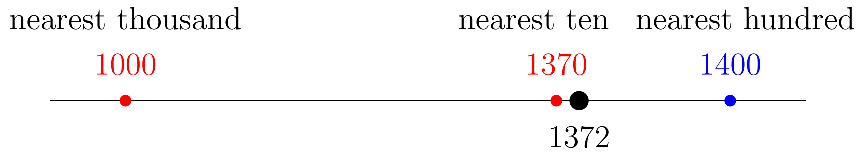
Part 2: Bouncing Numbers

When we round a number, the new number could be larger, smaller, or the same as the original number.

We say that a positive integer is **bouncing** if when we round it to the nearest ten, the result is smaller (and not equal); if we round it to the nearest hundred, the result is greater (and not equal); and if we round it to the nearest thousand, the result is smaller (and not equal) again.

Examples

- 1372 is **bouncing**, because
  - if we round it to the nearest ten, the result is 1370, smaller than 1372;
  - if we round it to the nearest hundred, the result is 1400, greater than 1372;
  - if we round it to the nearest thousand, the result is 1000, smaller than 1372;



- 6277 is **not bouncing**, because
  - if we round it to the nearest ten, the result is 6280, greater than 6277;
- 3871 is **not bouncing**, because
  - if we round it to the nearest thousand, the result is 4000, greater than 3871;

Question 2.5: (3 points) Are the following bouncing numbers? Answer Yes or No.

part a) 5693,

part b) 3464,

part c) 7290.

Question 2.6: (1 points) Give an example for a bouncing number that is equal to 9300 when rounded to the nearest hundred, and equal to 9000 when rounded to the nearest thousand.

Question 2.7: (2 points) What is the smallest 4-digit bouncing number?

Question 2.8: (2 points) What is the greatest 4-digit bouncing number?

Question 2.9: (4 points) How many 4-digit bouncing numbers are there?

Question 2.10: (2 points) How many 4-digit bouncing numbers are there that are multiples of 5?

Question 2.11: (3 points) How many 4-digit bouncing numbers are there that are multiples of 4?

Name and school:

Problem 3: Fast Car (Individual problem)

15 minutes, no calculators, 22 points possible

The Car Racing Team for the East Cupcake Elementary School has been hard at work studying what makes a car win races. However, they've found something strange happening with their win percentages. They've come to you for help to see if you can resolve the seeming paradoxes they've found.

Part 1: Things the elementary students know

Speed: The students know that in the United States speed is often determined by the number of miles an object would travel in one hour (miles per hour).

Example: If a car traveled 100 miles in 4 hours, then its average speed was $100/4 = 25$ miles per hour.

Data tables: The students have also learned how to collect data into useful tables. For example, when they asked the teachers and students in their school whether they preferred salty or sugary snacks, they found the following data:

	Salty	Sugary	Total
Teachers	20	10	30
Students	40	100	140
Total	60	110	170

For example, the table above shows that out of the 140 total students, 40 prefer salty snacks, whereas of the 30 teachers 10 prefer sugary snacks.

Can there be paradoxes in data? Sometimes the results of working with data are strange and you can get different conclusions depending on how you "split" the data.

For example, suppose we ask people if they like chocolate and Blue Moon ice cream both with and without sprinkles and get the following data

	with sprinkles	without sprinkles	Total
Chocolate	67/100 (.67)	150/300 (.5)	217/400 (.5425)
Blue Moon	180/300 (.6)	50/110 (.4545)	230/410 (.561)

where, for example, 67/100 means that of the 100 people in our survey who tried chocolate ice cream with sprinkles, 67 liked it. The above data is paradoxical, because it implies that more people like Blue Moon overall (56% compared to 54.25%), while more people seem to prefer chocolate with sprinkles over Blue Moon with sprinkles (67% to 60%) *and* prefer chocolate without sprinkles over Blue Moon without sprinkles (50% to 45.45%)!

Part 2: Speeds

First the students needed to better understand *speeds*. In the last race, car #1 went 51 miles in 30 minutes. Car #2 went 80 kilometers in 30 minutes.

Question 3.1: (1 points) What was the speed of car #1 in miles per hour?

Question 3.2: (1 points) If there is 1.6 kilometers in 1 mile, what was the speed of car #2 in miles per hour?

Question 3.3: (1 points) How many miles per hour faster was car #1 than car #2?

Part 3: Total Wins

The team decided that comparing each pair of the many cars they have was going to take too long, so they gathered up all the data they could find about how many races the different types of car on the team won over the last few years. Help them fill in the numbers they didn't have data for. (4 points total– 1 point each)

	Hybrid	Gas powered	Total wins
Rear wheel drive	17		41
Front wheel drive	35		
Total wins	52	113	

Question 3.4: (1 points) Who won more races, hybrid cars or gas powered cars?

Question 3.5: (1 points) Who won more races, rear wheel drive or front wheel drive?

Part 4: Win Percentages

Gas powered cars have been around longer than hybrid cars, so hybrid cars have competed in fewer races. Instead of comparing the number of wins, they should compare the percentage of wins. For example, if a car has won 20 races and competed in 50, it has won $\frac{20}{50} \cdot 100 = 40\%$ of its races. The person in charge of keeping records only kept track of wins, not of all races competed in. However, someone took pictures at every race, so they switched to considering paint color and whether or not the car had cow stickers on the hood.

From the pictures they determined the following table of wins:

Total wins	Cow stickers	No cow stickers	Total wins
Red	44	120	164
White	160	60	220
Total wins	204	180	384

Question 3.6: (1 points) If the cow sticker cars have been in 350 races, what is their win percentage?

Question 3.7: (1 points) If the cars without cow stickers have been in 250 races, what is their win percentage?

Instead of considering stickers and paint color separately, maybe they should look at each combination of stickers and paint. Here is a table giving the number of races each type of car has participated in.

Total races	Cow stickers	No cow stickers	Total races
Red	50	150	200
White	250	100	400
Total races	300	250	600

Question 3.8: (1 points) If we only consider the red cars with cow stickers, what is their win percentage?

Question 3.9: (1 points) If we only consider the red cars without cow stickers, what is their win percentage?

Question 3.10: (1 points) If we only consider the white cars with cow stickers, what is their win percentage?

Question 3.11: (1 points) If we only consider the white cars without cow stickers, what is their win percentage?

Question 3.12: (2 points) Which type of car (red with cow stickers, red without cow stickers, white with cow stickers, or white without cow stickers) has the highest win percentage?

Part 5: Simpson's paradox

Simpson's paradox is a phenomenon where if we look at subgroups of data, we reach one conclusion but if we look at all the data together we reach a different conclusion. For example, consider Harley and Logan's batting averages (the ratio of times they hit the ball to how many times they were at bat) for the last two years:

Player	2023	2024	Combined
Harley	12/48 (.25)	72/144 (.5)	84/192 (.4375)
Logan	104/400 (.26)	60/100 (.6)	164/500 (.328)

In both years, Logan has a higher (but not equal) batting average than Harley; in 2023 Logan's batting average was .26 while Harley's was .25, and in 2024 Logan's batting average was .6 while Harley's was .5. However, if we combine the two years, Harley (.4375) has a higher (but not equal) batting average than Logan (.328)!

Question 3.13: (1 points) Is the car racing data an example of Simpson's paradox? Use the following table to organize the win percentages for each type.

Win percentage	Red paint	White paint	Total win percentage
Cow stickers			
No cow stickers			

Yes or no?

Question 3.14: (1 points) Suppose you are given the following table where, for example, 2/10 means that out of 10 trials, the Type 1 objects of Group A were succesful 2 times.

	Type 1	Type 2
Group A	2/10	?/20
Group B	5/20	7/10

What is the smallest number that can replace the ? and make this an example of Simpson's paradox? Remember, the percentages cannot be equal for the paradox to hold!

Question 3.15: (1 points) What is the largest number that can replace the ? and make this an example of Simpson's paradox? Once again, remember that the percentages can not be equal for the paradox to hold.

Name and school:

Problem 4: Roadtrip! (Individual problem)

15 minutes, no calculators, X points possible

Meriadoc and Pippin are planning a road trip. They'll start in Madison and they want to visit Chicago, Nashville, Seattle, and Minneapolis, each exactly one time, and ending in one of those four cities.

Here's a table of the driving distances (in miles, rounded to the nearest 10) between their destinations (for example, the distance between Nashville and Chicago is 470 miles):

	Madison	Chicago	Nashville	Seattle	Minneapolis
Madison	0	150	620	1920	270
Chicago	150	0	470	2060	410
Nashville	620	470	0	2390	890
Seattle	1920	2060	2390	0	1660
Minneapolis	270	410	890	1660	0

And here's a map:

Part 1: Distance

Meriadoc believes they should follow the route

Madison → Chicago → Seattle → Nashville → Minneapolis

While Pippin thinks

Madison → Seattle → Nashville → Minneapolis → Chicago

would be faster.

Question 4.1: (1 points) Whose route is shorter?

Question 4.2: (2 points) Given that they have to start in Madison, how many possible routes are there if they visit each city exactly one time?

Pippen and Meriadoc don't want to check that many routes, so they try to find a way to eliminate some of the possibilities.

Question 4.3: (1 points) Which city should they end in as they seek the shortest path?

Question 4.4: (1 points) Knowing where they should start, and where they should end, how many paths remain between the cities in which each is visited exactly one time?

Now that they know were to start and end, they want to find the shortest path. Below (and continuing on the next page) are 8 boxes where you can organize information. For each different route starting in Madison and ending in the proper ending city, put the distance of that route. You will NOT need all these boxes.

Question 4.5: (1 points) How many miles is the shortest route?

Part 2: Cost

Meriadoc and Pippen were happy to find the shortest route, but they realized they needed to think about their budget as well. Gas is \$3.00 per gallon, and their car can drive 30 miles per gallon.

Question 4.6: (1 points) How much does it cost for them to drive 10 miles?

Also, although they drive very fast, they know they will have to stop to rest at a hotel in the second-to-last city they visit. So, for example, on the route:

$$\text{Madison} \rightarrow \text{Seattle} \rightarrow \text{Nashville} \rightarrow \text{Chicago} \rightarrow \text{Minneapolis}$$

they stop overnight in Chicago, but only Chicago.

The hotels in Chicago, Nashville, and Seattle are all \$100 per night, but due to a J.R.R. Tolkien convention in Minneapolis that week, the price is \$176 for a hotel in Minneapolis.

Question 4.7: (1 points) How much would the cheapest trip cost?

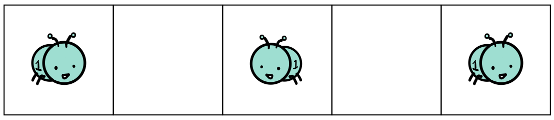
Question 4.8: (2 points) What's the lowest price of gas that makes the shortest possible route also the cheapest (with ties allowed)?

Name:

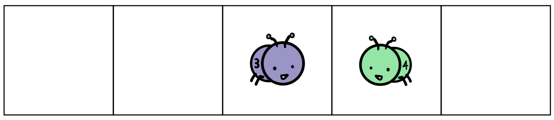
Problem 5: Ants on a log (Group problem)

30 minutes, no calculators, 14 points possible

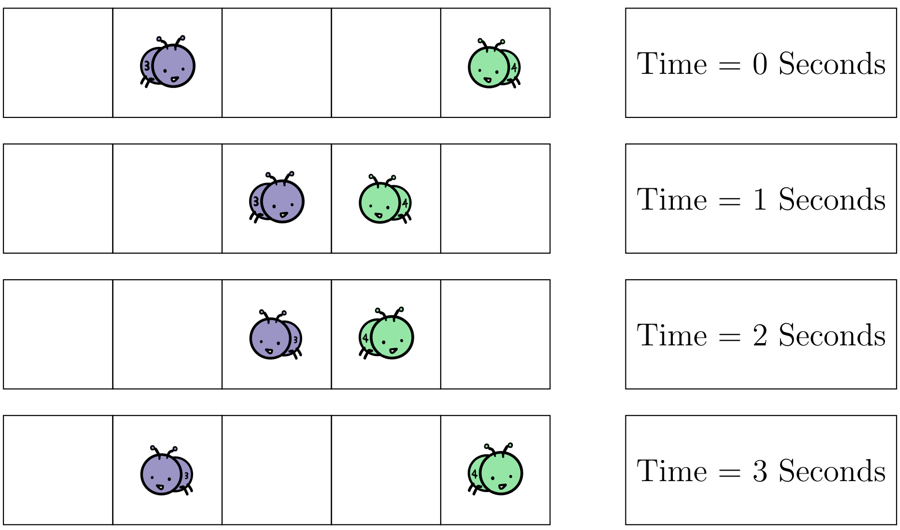
A colony of ants are walking on a thin log. Each second, ants occupy a 1cm segment of the stick, facing either left or right, and move forward into the segment directly in front of them in the following second. Here is a configuration of three ants on a 5 centimeter stick, with two ants facing to the right of the stick, and one ant facing to the left:



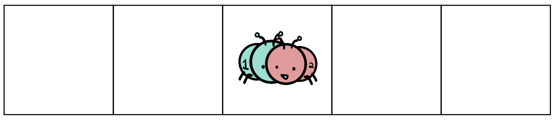
Ants continue moving in the direction they are facing, changing the direction they face only when they bump into other ants. This can happen in 2 different ways. In the first case, ants meet when they directly face one another, like so:



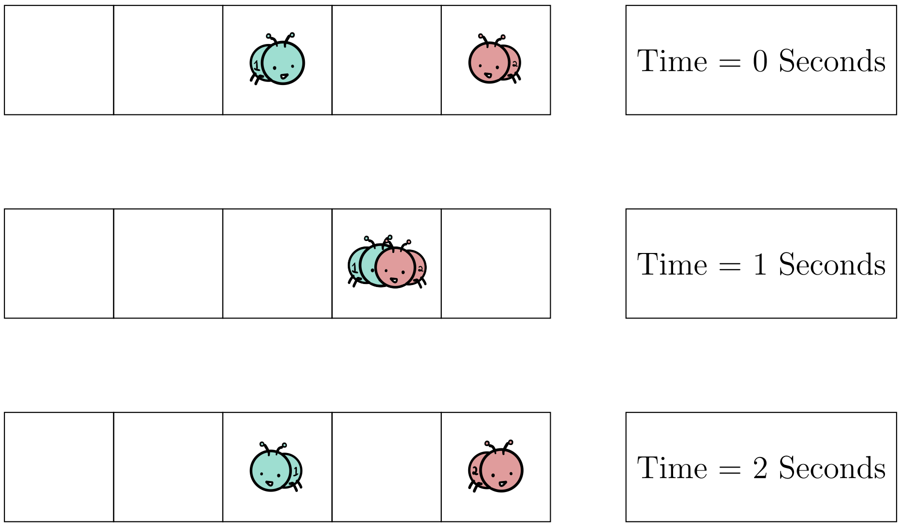
These ants bounce off each other, taking one second to turn around, and then travel in the opposite direction. Here is an example of two ants bumping off one another in this way, with diagrams drawn for each second following the initial configuration:



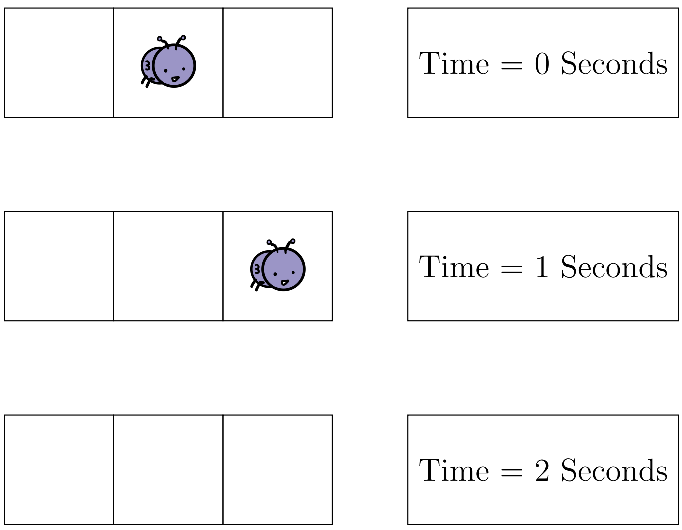
The second case occur when two ants meet in the same square, like so:



These ants also bounce off one another, but more powerfully, instantly turning around and returning to the squares they came from in the following second. Here's an example:



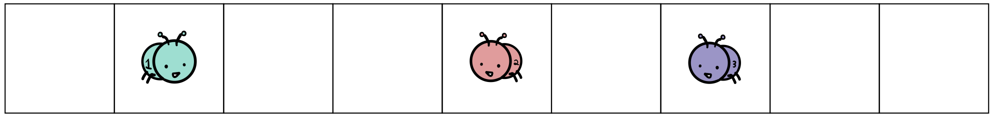
When an ant reaches the end of the stick, it falls off the log. Like this:



Here is a simple example of the behaviour of three ants travelling on a 5 centimeter log:

Part 1: Where Do the Ants Go?

Question 5.1: (3 points) Suppose that there are three ants on a 9 centimeter stick in the following configuration:



Time = 0 Seconds

Let us number the ants from left to right with the numbers 1, 2, and 3. Draw the *positions* of each ant after the specified amount of time passes. Write your answers like in the following example: After 1 second, the configuration is:

		1	2		3			

Time = 1 Seconds

(A) (1 point) What position are the ants in 2 seconds after starting?

Time = 2 Seconds

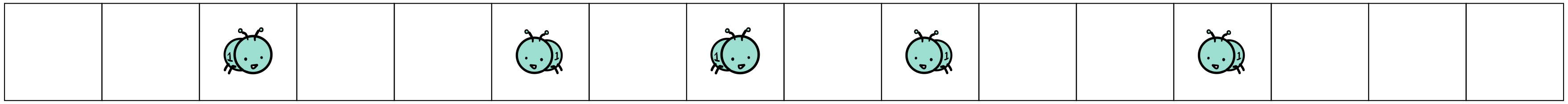
(B) (1 point) What position are the ants in 4 seconds after starting?

Time = 4 Seconds

(C) (1 point) What position are the ants in 6 seconds after starting?

Time = 6 Seconds

Question 5.2: (3 points) Consider 5 ants on a 16 centimeter stick in the following configuration:



Draw the *directions* the ants are facing after a certain period of time. Write your answers like in the following example: After 1 second, the configuration is:

			→	←				→←			←				

(A) (1 point) What position are the ants in 2 seconds after starting?

(B) (1 point) What position are the ants in 4 seconds after starting?

(C) (1 point) What position are the ants in 12 seconds after starting?

Part 2: How Much Time Till The Ants All Fall?

Question 5.3: (6 points) The following page lists 5 different initial configurations of ants, labelled (A), (B), (C), (D), and (E). In each configuration, how long does it take for all of the ants to fall from the log? As an example, in the final example on the introduction, it takes 5 seconds for all ants to fall from the log.

Note: Configurations D and E are intended to be read as a single stick that does not fit on the page, top to bottom. Configuration D is a single stick of length 19 with ants at positions 3, 5, 7, 11, 13, 15, 17, 19, and configuration E is a single stick of length 39.

(A) (1 Point).

(B) (1 Point).

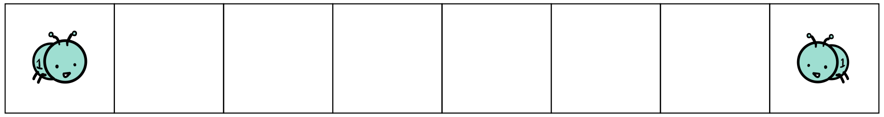
(C) (1 Point).

(D) (1 Points).

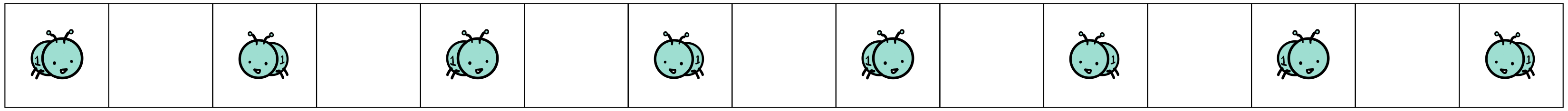
(E) (2 Points).

Question 5.4: (3 points) Suppose there are 40 ants on a 100 centimeter log. Like in all of the previous initial positions of ants above, all ants must start with at least one empty box between them and any other ant. What is the smallest amount of time you need to wait in order to guarantee that all the ants fall, for any possible arrangement of the 40 ants?

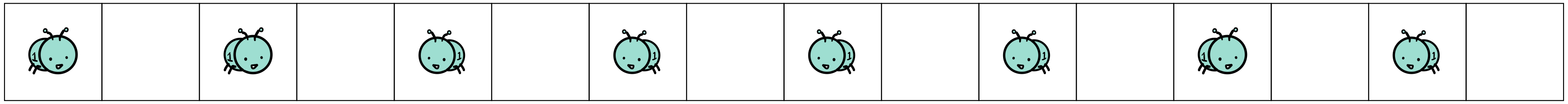
Configuration (A)



Configuration (B)

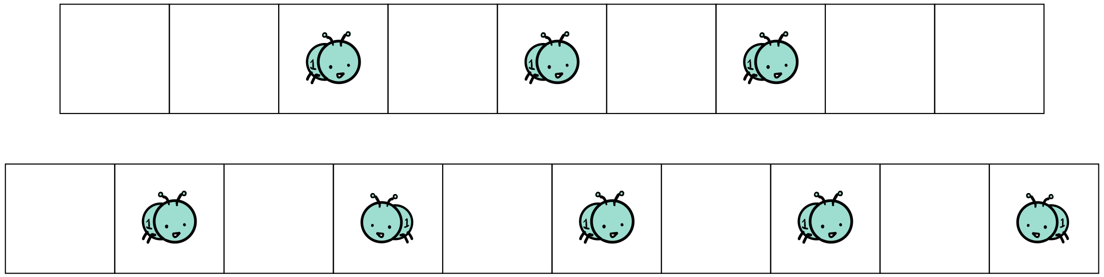


Configuration (C)



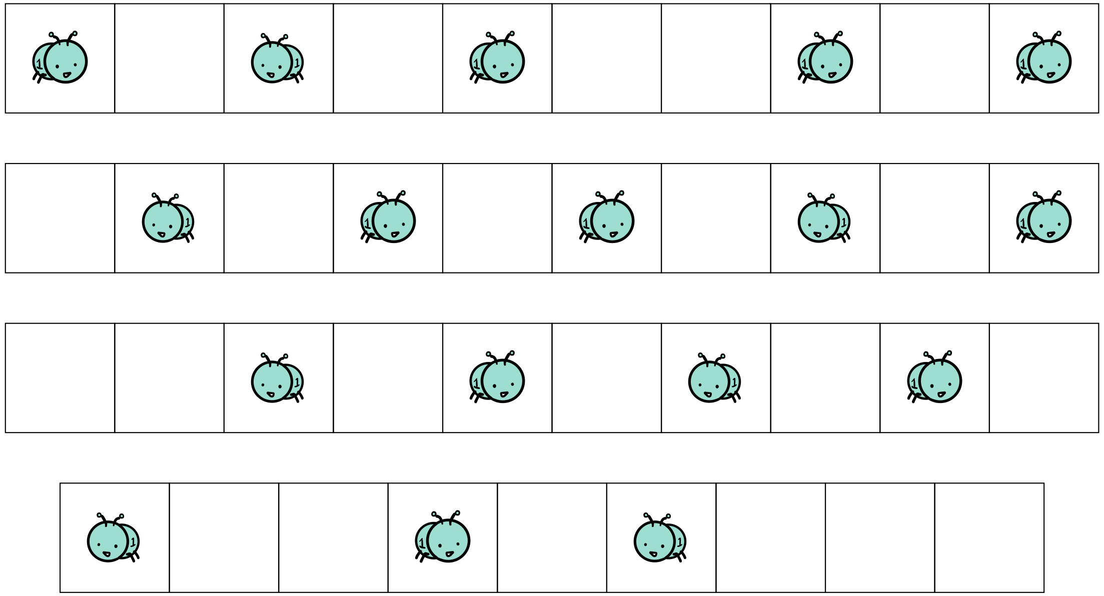
Configuration (D)

NOTE: this is meant to be one stick, but couldn't fit on a page-line. Read from left to right, top to bottom (as discussed on note on previous page)



Configuration (E)

NOTE: this is meant to be one stick, but couldn't fit on a page-line. Read from left to right, top to bottom (as discussed in note on previous page)



Part 3: Collision counting

Question 5.5: (6 points) Please refer to the five configurations (A), (B), (C), (D), and (E) from the previous two pages. For each configuration, determine the *total number* of collisions that occur between ants until all the ants have jumped off the stick (For example, in the last example of the introduction, there are two collisions – you can go back and check!).

(A) (1 Point)

(B) (1 Point)

(C) (1 Point)

(D) (1 Point)

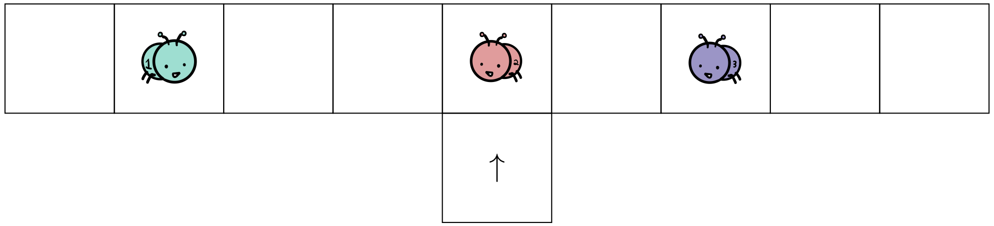
(E) (2 Points)

Question 5.6: (1 points) Suppose a 20 centimeter stick has 5 ants on it, with 3 ants facing right, and 2 ants facing left, but we do not know where these ants are located on the stick. From this information, is it possible to determine the total number of collisions that occur between ants? Answer: Yes or No.

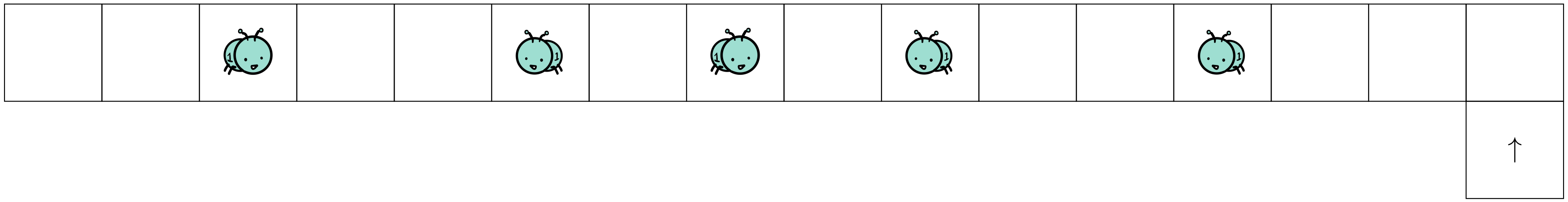
Question 5.7: (1 points) If the answer above is yes: give the total number of collisions. If the answer above is no: what is the minimum number of collisions that could occur with a configuration of a 20 centimeter stick with 5 ants on it, with 3 ants facing right, and 2 ants facing left?

Part 4: It's not always fun and games

Question 5.8: (X points) Suppose there is an anteater below the exact middle segment in the ant arrangement in problem 1, part 1. We have reproduced the configuration below. The anteater sticks its tongue out every five seconds (starting at time 0), and will eliminate any ant that is on that segment at that time. We have indicated the middle segment with an arrow below it. How many ants will eventually jump off the stick without falling victim to the anteater?



Question 5.9: (X points) Suppose there is an anteater below the far right segment in the ant arrangement in problem 1, part 2 (reproduced below) and that this anteater is faster! The anteater sticks its tongue out every three seconds (starting at time 0), and will eliminate any ant that is on that segment at that time. (We have again marked the dangerous square with an arrow.) How many ants will jump off the stick without falling victim to the anteater?



Name:

Problem 6: Geometry of paper and scissors (Group problem)

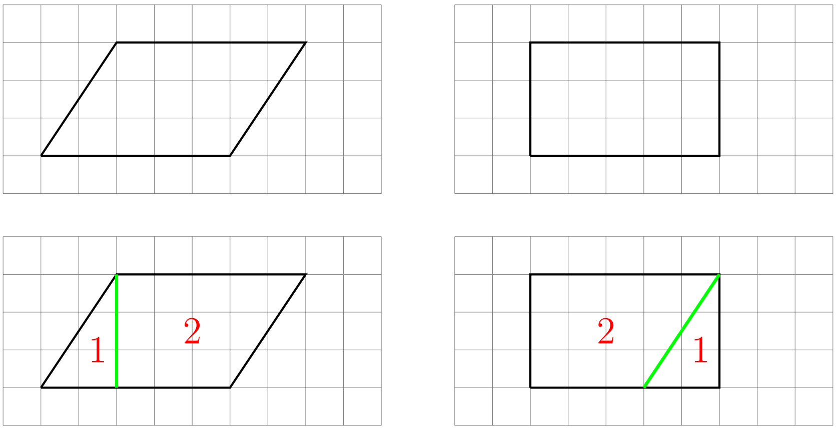
30 minutes, no calculators, ?? points possible

Part 1: Introduction

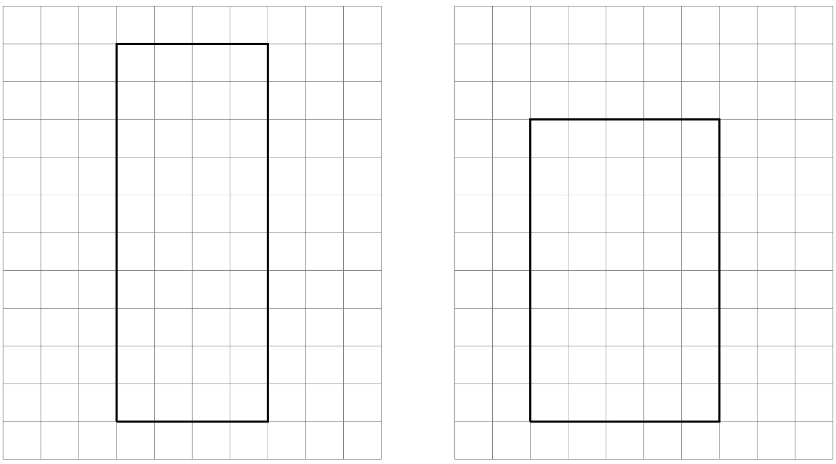
A polygon is a shape that is made from straight lines. Suppose we have 2 polygons that are made from paper. Suppose that we want to take scissors, cut one of the polygons into pieces, put those pieces back together in a different manner, and obtain the polygon. When is this possible?

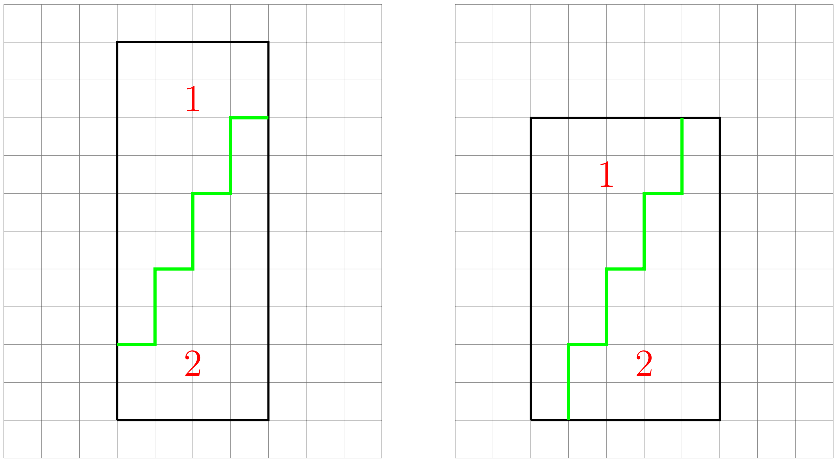
Let's begin with some examples. To make problems to be more fun (and easier to grade for us), we will restrict how many pieces allowed.

Example: Cut the polygon on the left using one straight line cut, which separates the shape into two pieces.



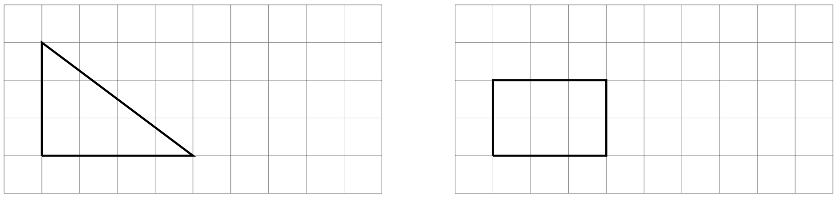
Example: Cut the polygon on the left using one non-straight line cut, which separates the shape into two pieces.



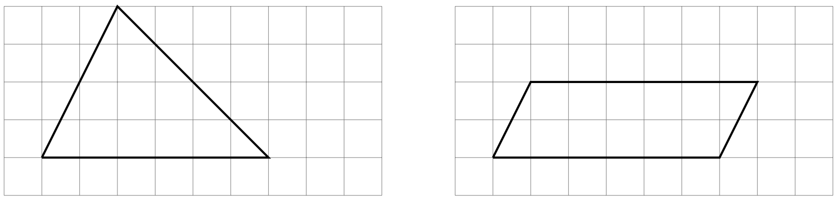


Part 2: Problems

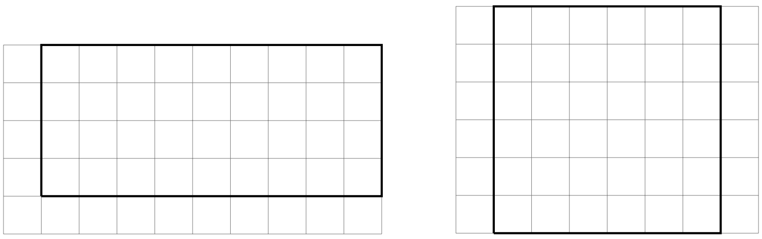
Question 6.1: (1 points) Cut the polygon on the left (the triangle) using 1 straight line cut, that separates the shape into two pieces. Label the pieces on the left and then also label the corresponding pieces on the right.



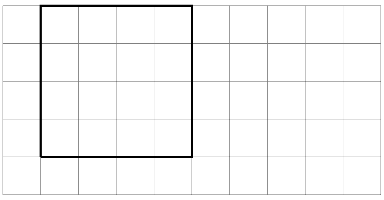
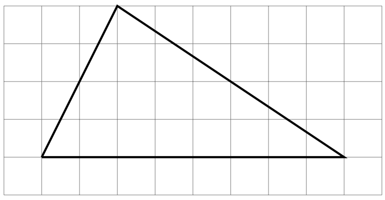
Question 6.2: (1 points) Cut the polygon on the left (the triangle) using 1 straight line cut, that separates the shape into two pieces. Label the pieces on the left and then also label the corresponding pieces on the right.



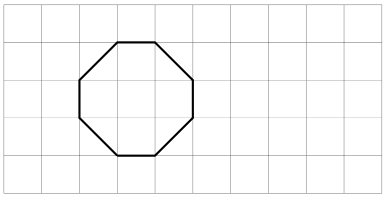
Question 6.3: (1 points) Cut the polygon on the left (the rectangle) using 1 non-straight line cut, that separates the shape into two pieces. Label the pieces on the left and then also label the corresponding pieces on the right.

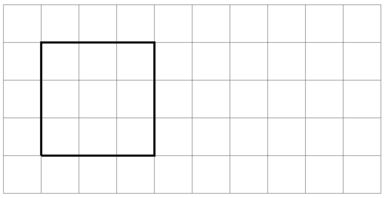
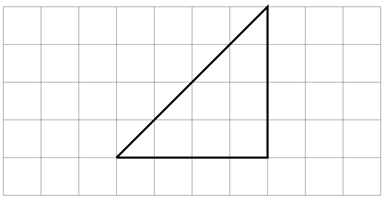


Question 6.4: (1 points) Cut the polygon on the left (the square) using exactly two straight line cuts, that separate the shape into 3 pieces. Label the pieces on the left and then also label the corresponding pieces on the right.

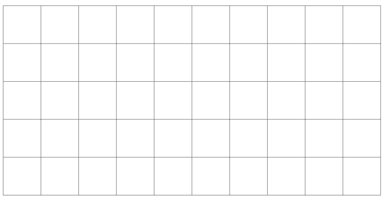
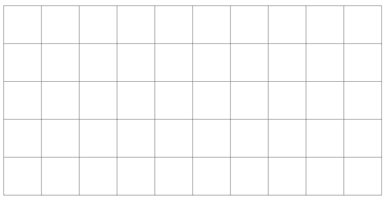
Question 6.5: (2 points) Which polygons can be constructed one from other? Consider the 4 polygons given below.

(A): 
(B): 

(C): 
(D): 

a) Which 2 polygons above can be constructed from each other in the manner of cuts we are considering? Your answer should consist of the two letters corresponding to the shapes. (**Note:** At this point you do not need to provide the actual cuts.)

b) Show how to cut the 2 polygons you chose in part a) so that one can be constructed from the other. Do this with precisely 2 cuts that divide the shapes into 3 parts. You will need to carefully reproduce the shapes below.

Question 6.6: (2 points) Cut the polygon on the left using exactly two straight line cuts, that separates the shape into three pieces. Label the pieces on the left and then also label the corresponding pieces on the right.