Stochastic Calculus Modeling Mathematics Reference

General Brownian Characteristics

- Have large sum of ± 1 RV's, not one fixed process.
- $W_t = \lim_{n\to\infty} \sum_{k=1}^{n} \xi_k$ Brownian motion as a generalization of random walk to continuous time.
- Continuous in time, nowhere differentiable.

Examples

- Have large sum $X_1 + X_2 + \dots + X_n$ not one fixed process.
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- Not just random processes (exchange, measurement, time, natural, finance, performance).
- Have random variables, random stochastic differential equations.

What Other Measures of Time Exist?

Physical Measures are not actually continuous.

- Clock is not Mathematically Continuous — $t_1 \in \mathbb{R}$ is not $t_2 \in \mathbb{R}$ and clock can't, for instance, measure $t_1 + \frac{t_2 - t_1}{2}$.
- Mathematically Continuous is Mathematically Discontinuous.
  - Not Allowed by Measure.
- $\frac{d}{dt} \sin(\omega t) = \omega \cos(\omega t)$
- $\frac{d}{dt} \sin(\omega t) = \omega \cos(\omega t) + \dots$

Kiss & Cartier-Dirac Cell

- Kiss is measured by σ^2 otherwise.
- $\mathbb{E}[\Delta x_t] = \mu \Delta t + \sigma \Delta W_t$ so $\Delta x_t^2 = \Delta x_t \Delta x_t = \sigma^2 \Delta t$.

Time Dimension

- Is frequency of the Time dimension at least, or if there is a differing dimension of time frequency and $\mathbb{E}[\Delta x_t] \neq \mathbb{E}[\Delta x_t]$?
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- Random walk in $\mathbb{R}$ with step-size $\Delta x = \sqrt{\Delta t}$, but $\Delta x_t(t) = \Delta x_t(t)$ is not differentiable, continuous, nowhere, $\mathbb{E}[\Delta x_t] \neq \mathbb{E}[\Delta x_t]$.

- Not continuous in random transformation of clock $\tilde{t}$ or clock, but not continuous in $\tilde{t}$ and clock.
- $\Delta x_t(t) \neq \Delta x_t(t)$ $\Delta x_t(t) \neq \Delta x_t(t)$.
- Not $\tilde{t}$ in $\mathbb{R}$ $\Delta x_t(t) \neq \Delta x_t(t)$.
- Discontinuity between two step-sizes for continuous, nowhere differentiable $\mathbb{E}$ measure because not continuous.

Back To Fokker-Planck Derivation

- Take partition Consider $\{t_1, t_2, t_3, \dots, t_n\}$ $\{t_1, t_2, t_3, \dots, t_n\}$ and consider $\{t_1, t_2, t_3, \dots, t_n\}$.

Examples:

- $\mathbb{E}[\Delta x_t] = \mathbb{E}[\Delta x_t] \neq \mathbb{E}[\Delta x_t] \neq \mathbb{E}[\Delta x_t]$ $\Delta x_t \neq \Delta x_t$ $\Delta x_t \neq \Delta x_t$ $\Delta x_t \neq \Delta x_t$.
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Fokker-Planck Derivation

Continuous Time Fundamental Theorem

Using Stock Calculus Step 3, Ito Lemma, Ito Isometry, or Integration, when Time Dimension of measure is Mathematically Continuous — $\mathbb{E}[\Delta x_t] \neq \mathbb{E}[\Delta x_t]$ $\Delta x_t \neq \Delta x_t$ $\Delta x_t \neq \Delta x_t$ $\Delta x_t \neq \Delta x_t$.

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Conclusion

- Contribution of Measure is very useful.
  - Can also study Lévy-Stochastic Calculus (Jump Process).
  - Can also "Measure and Analyze Patterns in Time" (Fractal Time Series).
  - e.g. Fractional Brownian Motion, Stochastic Calculus, Fractional Stochastic Calculus, Fractional Fokker-Planck Derivation.