Sections: 3.1 and 3.2 **Math 340 Worksheet 8** Date: 9.7.2022

It may be useful for you to have this worksheet for future discussion sections. It may be in your interest to solve the questions not in the order listed, but according to which questions you need practice with. Your TA may or may not give you specific advice or directions on which questions to try first.

Exercise 1.
Compute the following determinants

1. $\begin{vmatrix} 4 & -3 & 5 \\ 5 & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix}$.

2. $\begin{vmatrix} 4 & 1 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 2 \end{vmatrix}$.

Exercise 2.
Compute the determinant

$$\begin{vmatrix} 1 & 3 & 7 & 9 & 5 \\ 2 & 0 & 3 & 0 & 1 \\ 4 & 0 & 6 & 0 & 2 \\ 0 & 1 & 2 & 8 & 3 \\ 2 & 0 & 2 & 2 & 0 \end{vmatrix}.$$

Exercise 3.
For which values of t is the following determinant nonzero?

$$\begin{vmatrix} t+1 & 4 \\ 2 & t-3 \end{vmatrix}$$

Exercise 4.
Let's compute a formula for the determinant of the 3×3 matrix

$$A = \begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix}$$

where x_1, x_2, x_3 are arbitrary numbers. A is called the 3×3 *Vandermonde matrix.* To do this, let's use the definition of the determinant given by sums over permutations, i.e. for the sum

$$\det(A) = \sum_j \operatorname{sgn}(j) a_{1j_1} a_{2j_2} a_{3j_3}.$$

where j ranges over all permutations of $\{1, 2, 3\}$.

1. Start by listing out all permutations of $\{1, 2, 3\}$. There should be $3! = 6$ of them. Write them in the first column of the grid below.

2. For each permutation j, compute the *sign* $\operatorname{sgn}(j)$ of the permutation. Write these values in the second column of the grid below.

3. Finally, for each permutation j, compute $a_{1j_1} a_{2j_2} a_{3j_3}$. Write these values in the third column of the grid below.

4. One can now calculate the determinant by summing up the values in the third column multiplied by the signs in the second column. By doing some algebra, show that
$$\det(A) = (x - y)(y - z)(z - x).$$
Challenge: Can one find a 4×4 matrix A for which a similar result holds?

Permutations	Signs	Products