The Infinite Actuary's
Exam FM
Sample Exam 1

by James Washer, FSA, MAAA

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Take this sample exam under strict exam conditions. Start a timer for 3 hours and stop immediately when the timer is done. Do not stop the clock when you go to the bathroom. Do not look at your notes. Do not look at the answer key.

This exam contains 35 questions. Do not spend too much time on any one question. Choose the best available answer for each question.

1. Paul deposits X into a bank account. Paul's account is credited interest at a nominal rate of interest of 12% convertible monthly.

   At the same time, Mary deposits X into a different bank account. Mary's account is credited interest at a force of interest of δ.

   After 5 years, the value of each account is the same.

   Calculate δ.

   A. 0.1192 B. 0.1194 C. 0.1196 D. 0.1198 E. 0.1200

2. You are given the following term structure of interest rates:

Term (in years)	Spot Rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

Calculate $1000\,(Da)_{\overline{4}|}$, rounded to the nearest 100.

A. 8700 B. 8800 C. 8900 D. 9000 E. 9100

3. A borrower repays a loan with 20 annual payments of 500 made at the end of each year. The annual effective rate of interest is 6%. Calculate the total amount of principal repaid in the odd-numbered payments (i.e., payment 1, payment 3, and so on).

A. 2744 B. 2784 C. 2867 D. 2921 E. 2978

4. In retail lending, which of the following would not be a factor that might be present with a loan and that works in favor of the lender?

   A. Being able to offer different rates to borrowers based on their creditworthiness
   B. The existence of reserve funds on deposit with the Federal Reserve
   C. Collateral for the loan
   D. A federal guarantee
   E. None of the above

5. Fred has a mortgage with monthly payments of \$850. The payments are made at the end of each month. There are twelve years remaining on the loan, which accrues interest at a nominal annual rate of 6% convertible monthly.

   Fred chooses to refinance his mortgage. The refinanced mortgage would have the same remaining term and level payments, but the interest rate would be reduced to 5%, also convertible monthly. The refinancing process carries a flat fee of \$2000.

   Fred has a bank account that earns 3% nominal annual interest, convertible monthly. He deposits the difference between his old payment and new payment in this account each month. We assume that the refinancing fee was also withdrawn from this account.

   Counting the refinancing fee and the interest it would have earned for Fred, how much does Fred gain by refinancing, evaluated at the death of the loan?

   A. \$4420 B. \$4820 C. \$5220 D. \$5620 E. \$6020

6. An n-year 1000 par value bond is purchased for 843.20 to yield 7% annually. The bond pays 5% annual coupons and has a redemption value of C.

   Find the book value of the bond immediately after the third coupon is paid.

   A. 862 B. 872 C. 882 D. 892 E. 902

7. You are the CFO of Infinite Life. Infinite Life only has one liability of $5000 due in 8 years. Infinite Life uses a nominal rate of 6% convertible semiannual to discount all liability cash flows.

   You call up your favorite bond broker and ask him what bonds he has for sale today. Your broker says he has 5-year and 10-year bonds. Both bonds are priced to yield 6% convertible semiannually. The 5-year bond pays 6% coupons semiannually and the 10-year bond is a zero-coupon bond. The bonds can be bought in any face amount.

   What face amount of the 5-year bond should you buy in order to meet the first two conditions of immunization?

   A. 777 B. 888 C. 999 D. 1111 E. 2222

8. A 20-year 1000 par value bond with 6% annual coupons is callable on coupon dates, beginning with the 11th coupon. If the bond is called during the years 11–15, the redemption amount would be 1200. If the bond is called during the years 16–20, the redemption amount would be 1100.

   Find the minimum yield an investor might earn if the investor pays 950 for this bond.

   A. 6.3% B. 6.7% C. 7.1% D. 7.5% E. 7.9%

9. Today the LIBOR spot rates are:

Years to Maturity	Spot Rate
1	4.50%
2	5.00%
3	5.75%
4	6.75%
5	8.00%

Today you enter into a 5-year interest rate swap (with a notional amount of 100,000) to pay a fixed rate and to receive a floating rate based on future 1-year LIBOR rates. If the swap has annual payments, what is the fixed rate you should pay?

A. 7.68% B. 7.78% C. 7.88% D. 7.98% E. 8.08%

10. A 10-year 1000 par value bond was purchased to yield 6% convertible semiannually. The bond has a coupon rate of 4% and pays semiannual coupons.

    The bond investor reinvests the coupons into an account paying a nominal annual rate of 8% convertible semiannually.

    Find the overall yield earned by the bond investor over the life of the bond, expressed as an effective annual rate.

    A. 6.1% B. 6.2% C. 6.3% D. 6.4% E. 6.5%

11. For 10,000, Kelly purchases an annuity-immediate that pays 400 quarterly for the next 10 years. Calculate the annual effective interest rate earned by Kelly's investment.

A. 10.0% B. 10.3% C. 10.5% D. 10.7% E. 11.0%

12. Consider a $2n$-year par value bond with annual coupons of 100. The bond has a redemption value of C, and the bond is priced at P to yield an annual effective rate of i.

    Find the price of an n-year annuity-immediate with payments of 100 if it also has an annual effective yield rate of i.

    A. $P + Cv_i^n$ B. $P - Cv_i^n$ C. $\frac{P - Cv_i^{2n}}{1 - v_i^n}$ D. $\frac{P - Cv_i^{2n}}{1 + v_i^n}$ E. $\frac{P - Cv_i^{2n}}{2}$

13. Which of the following statements are false?

I. If a set of cashflows has Redington immunization, then they also have full immunization.

II. A set of cashflows with full immunization requires rebalancing over time to maintain immunization.

III. Full immunization assumes a flat yield curve and parallel shifts in the yield curve.

A. I only
B. II only
C. III only
D. I and II only
E. None of the above

14. Deposits are made at the beginning of every month into a fund earning a nominal annual rate of 6% convertible monthly. The first deposit is 100 and deposits increase 2% every year. In other words, deposits 1–12 are 100, deposits 13–24 are $100 \times 1.02 = 102$, deposits 25–36 are $100 \times 1.02^2 = 104.04$, and so on.

    Calculate the fund balance at the end of 10 years.

    A. 16,569 B. 16,893 C. 17,257 D. 17,770 E. 17,859

15. Jane takes out a $100,000 loan at a nominal annual rate of 12% convertible monthly. Jane makes monthly interest-only payments for the first 3 years, and then she pays $1500 per month until the loan is paid off. How many total payments does Jane make, including the final smaller payment?

A. 110 B. 111 C. 142 D. 146 E. 147

16. Jack borrows \$90 at an annual effective rate of discount d. He repays the loan 6 months later in full for \$100.

Jill borrows \$90 at a simple rate of discount d and repays her loan in full six months later for X. Determine X.

A. 81.45 B. 94.75 C. 99.45 D. 100.00 E. 101.35

17. A company has a single liability at time 6. The present value of this liability is 4000.

The company wants to achieve Redington immunization for this liability. It can purchase the following assets:

Asset	Present value	Macaulay duration	Macaulay convexity
I	3000	4	40
II	1000	9	24
III	3000	5	44
IV	1000	12	10

Which combination of assets could be used to achieve Redington immunization?

A. I and II B. I and IV C. II and III D. III and IV E. None of the answers are correct

18. At an annual effective interest rate i:

(i) 1 grows to 5 in x years

(ii) 2 grows to 8 in y years

(iii) 3 grows to 20 in z years

Calculate $12(1+i)^{3x-2y+z}$.

A. 625 B. 650 C. 675 D. 700 E. 725

19. A 4-year 100 par value bond paying 5% annual coupons has a redemption value of 110. If the adjustment to the book value in the third year is a write-down of \$2.25, find the original price of the bond.

A. \$117.90 B. \$118.40 C. \$118.90 D. \$119.40 E. \$119.90

20. Deposits of \$100 are made at the end of each calendar year from 2014 through 2019 inclusive. The resulting fund is to be used to buy a perpetuity with annual payments following the pattern

$$X, 4X, 7X, 10X, \ldots$$

with the first payment on December 31, 2021.

If the annual effective rate of interest is 5%, find X.

A. 0.558 B. 0.567 C. 0.576 D. 0.585 E. 0.594

21. For a three-year annuity-immediate, you are given:

(i) Annual payments are \$200.

(ii) $i = 0.06$.

(iii) The convexity is X.

(iv) The Macaulay convexity is Y.

Determine X/Y.

A. 0.783 B. 0.794 C. 1.000 D. 1.259 E. 1.277

22. An investor holds two securities in his portfolio:

Security	Market value	Modified duration	Convexity
A	\$5000	3	5
B	\$8000	7	12

Suppose the risk-free interest rate drops by 1%. Estimate the new market value of the investor's portfolio.

A. \$12,284 B. \$12,296 C. \$13,710 D. \$13,716 E. \$13,722

23. An annuity-immediate has the following payments:

$$10,\ 20,\ 30,\ 40,\ 50,\ 40,\ 30,\ 20,\ 10$$

Which of the following expressions represents the present value of the annuity?

A. $10a_{\overline{5}|}\,a_{\overline{5}|}$
B. $10a_{\overline{5}|}\,\ddot{a}_{\overline{5}|}$
C. $10a_{\overline{5}|}\,a_{\overline{4}|}$
D. $10a_{\overline{5}|}\,\ddot{a}_{\overline{4}|}$
E. $10\,(Ia)_{\overline{5}|} + 10\,(Da)_{\overline{4}|}$

24. For a 20-year bond, you are given:

(i) The par value is \$100.

(ii) The redemption value is \$105.

(iii) The coupon rate is $r\%$ payable quarterly.

(iv) The bond is priced at \$85.37 to yield 12% convertible quarterly.

Calculate $r\%$.

A. 10.0% B. 10.1% C. 10.2% D. 10.3% E. 10.4%

25. Which of the following statements are true?

I. At the time a swap is arranged, the market value of the swap is zero.

II. An amortizing swap is a swap whose market value is decreasing for the swap counterparty who pays the fixed rate.

III. The notional amount of a swap is the monetary amount to which the interest rates are applied in order to determine the monetary swap payments.

A. I and II only
B. I and III only
C. II and III only
D. I, II and III
E. The correct answer not given by (A), (B), (C) or (D)

26. John buys a 20 year annuity-immediate with quarterly payments:

(i) year 1 payments are 1 per quarter,

(ii) year 2 payments are 2 per quarter,

(iii) year 3 payments are 3 per quarter,

(iv) and so on.

Calculate the purchase price if $i = 4\%$.

A. 508 B. 513 C. 518 D. 523 E. 528

27. A loan is repaid with 10 annual payments. The first payment occurs one year after the loan. The first payment is 100 and each subsequent payment increases by 10. The annual effective rate of interest is 5%. The amount of principal repaid in the 4th payment is X.

Determine X.

A. 71 B. 76 C. 80 D. 84 E. 91

28. Which economic theory regarding yield curves says that interest rates for longer terms tend to be higher because lenders attach additional risk and potential opportunity cost to having their money tied up in longer term investments?

A. Market Segmentation Theory
B. Expectations Theory
C. Liquidity Preference Theory
D. Preferred Habitat Theory
E. None of the above

29. An insurance company has expected liabilities of 5,000 and 10,000 at the end of year 1 and year 2 respectively. The following annual coupon bonds are available:

Maturity (years)	Annual Coupon Rate	Annual Effective Yield	Par
1	0%	10%	1000
2	12%	12%	1000

Determine the cost to the company today to match its expected liabilities exactly.

A. 11,000 B. 11,500 C. 12,000 D. 12,500 E. 13,000

30. A fairly smart actuary (also known as an FSA) is offered the following rates on a loan:

I. $X\%$ nominal annual rate of interest compounded monthly

II. $X\%$ nominal annual rate of discount compounded monthly

III. $X\%$ annual effective rate of interest

IV. $X\%$ annual effective rate of discount

V. $X\%$ constant force of interest

Which rate does the FSA take?

A. I B. II C. III D. IV E. V

31. An investor wishes to accumulate a total of 10,000 at the end of 10 years by immediately making a single deposit into a fund earning an annual effective rate of 10%. The interest is paid at the end of each year and deposited into a separate fund earning an annual effective rate of 9%. Interest on the separate fund is also paid at the end of each year and immediately reinvested at an annual effective rate of 8%.

Calculate the single deposit.

A. 1206 B. 2412 C. 3267 D. 3992 E. 4440

32. A loan of amount X has level payments of R for n years payable at the end of each year. The annual effective interest rate is i. You are given:

(i) The interest portion of the first payment is 644.17.

(ii) The principal portion of the $(n-4)$-th payment is 816.70.

(iii) The outstanding balance after $n-1$ payments is 1111.11.

Calculate X.

A. 7850 B. 7900 C. 7950 D. 8000 E. 8050

33. You are given the following information about 100 par value notes in the Treasury market:

Term	Coupon	Price
1	0%	96.62
2	0%	X
3	0%	88.90

It is known that the 1-year forward rate starting in two years is 4.5%.

Calculate X.

A. 87.65 B. 89.70 C. 92.90 D. 93.45 E. 95.50

34. The stock of Company ABC sells for 75 per share assuming an annual effective interest rate of 11%. Annual dividends will be paid at the end of each year forever.

The first dividend is X, with each subsequent dividend 3% greater than the previous year's dividend.

Calculate X.

A. 4 B. 5 C. 6 D. 7 E. 8

35. A loan of 50,000 is being repaid using monthly payments of 600 for 20 years. The first payment is due one month after the loan is made.

The loan has an annual effective interest rate of i throughout the life of the loan.

What is the outstanding balance after the 60^{th} payment, rounded to the nearest hundred?

A. 37,500 B. 40,500 C. 43,500 D. 46,500 E. 49,500