SOCIETY OF ACTUARIES

EXAM P PROBABILITY

EXAM P SAMPLE SOLUTIONS

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Some of the questions in this study note are taken from past examinations.

Some of the questions have been reformatted from previous versions of this note.

Questions 154-55 were added in October 2014.
Questions 156-206 were added January 2015.
Questions 207-237 were added April 2015.
Questions 238-240 were added May 2015.
Questions 241-242 were added November 2015.
Questions 243-326 were added September 2016.
Question 327 was added January 2018.
Questions 328-329 were added October 2018.
Question 330 was question 328 added May 2018.
Questions 331-332 were added August 2019

1. Solution: D

Let G = viewer watched gymnastics, B = viewer watched baseball, S = viewer watched soccer. Then we want to find

$$\Pr\left[\left(G \cup B \cup S\right)^c\right] = 1 - \Pr\left(G \cup B \cup S\right)$$
$$= 1 - \left[\Pr\left(G\right) + \Pr\left(B\right) + \Pr\left(S\right) - \Pr\left(G \cap B\right) - \Pr\left(G \cap S\right) - \Pr\left(B \cap S\right) + \Pr\left(G \cap B \cap S\right)\right]$$
$$= 1 - \left(0.28 + 0.29 + 0.19 - 0.14 - 0.10 - 0.12 + 0.08\right) = 1 - 0.48 = 0.52$$

2. Solution: A

Let R = referral to a specialist and L = lab work. Then

$$P[R \cap L] = P[R] + P[L] - P[R \cup L] = P[R] + P[L] - 1 + P[(R \cup L)^c]$$
$$= p[R] + P[L] - 1 + P[R^c \cap L'] = 0.30 + 0.40 - 1 + 0.35 = 0.05.$$

3. Solution: D

First note

$$P\left[A \cup B\right] = P\left[A\right] + P\left[B\right] - P\left[A \cap B\right]$$
$$P\left[A \cup B^c\right] = P\left[A\right] + P\left[B^c\right] - P\left[A \cap B^c\right]$$

Then add these two equations to get

$$P\left[A \cup B\right] + P\left[A \cup B^c\right] = 2P\left[A\right] + \left(P\left[B\right] + P\left[B^c\right]\right) - \left(P\left[A \cap B\right] + P\left[A \cap B^c\right]\right)$$
$$0.7 + 0.9 = 2P\left[A\right] + 1 - P\left[\left(A \cap B\right) \cup \left(A \cap B^c\right)\right]$$
$$1.6 = 2P\left[A\right] + 1 - P\left[A\right]$$
$$P\left[A\right] = 0.6$$

4. Solution: A

For i = 1,2, let R_i = event that a red ball is drawn from urn i and let B_i = event that a blue ball is drawn from urn i. Then, if x is the number of blue balls in urn 2,

$$\begin{aligned} 0.44 &= \Pr[(R_1 \cap R_2) \cup (B_1 \cap B_2)] = \Pr[R_1 \cap R_2] + \Pr[B_1 \cap B_2] \\ &= \Pr[R_1]\Pr[R_2] + \Pr[B_1]\Pr[B_2] \\ &= \frac{4}{10}\left(\frac{16}{x+16}\right) + \frac{6}{10}\left(\frac{x}{x+16}\right) \end{aligned}$$

Therefore,

$$\begin{aligned} &2.2 = \frac{32}{x+16} + \frac{3x}{x+16} = \frac{3x+32}{x+16} \\ &2.2x + 35.2 = 3x + 32 \\ &0.8x = 3.2 \\ &x = 4 \end{aligned}$$

5. Solution: D

Let $N(C)$ denote the number of policyholders in classification C. Then
N(Young and Female and Single)
= N(Young and Female) – N(Young and Female and Married)
= N(Young) – N(Young and Male) – [N(Young and Married) – N(Young and Married and Male)]
= 3000 – 1320 – (1400 – 600) = 880.

6. Solution: B

Let
H = event that a death is due to heart disease
F = event that at least one parent suffered from heart disease
Then based on the medical records,

$$P\left[H \cap F^c\right] = \frac{210-102}{937} = \frac{108}{937}$$

$$P\left[F^c\right] = \frac{937-312}{937} = \frac{625}{937}$$

and $P\left[H \mid F^c\right] = \dfrac{P\left[H \cap F^c\right]}{P\left[F^c\right]} = \dfrac{108}{937}\Big/\dfrac{625}{937} = \dfrac{108}{625} = 0.173$

7. Solution: D

Let A = event that a policyholder has an auto policy and H = event that a policyholder has a homeowners policy. Then,

$$\Pr(A \cap H) = 0.15$$
$$\Pr(A \cap H^c) = \Pr(A) - \Pr(A \cap H) = 0.65 - 0.15 = 0.50$$
$$\Pr(A^c \cap H) = \Pr(H) - \Pr(A \cap H) = 0.50 - 0.15 = 0.35$$

and the portion of policyholders that will renew at least one policy is given by

$$0.4 \Pr(A \cap H^c) + 0.6 \Pr(A^c \cap H) + 0.8 \Pr(A \cap H)$$
$$= (0.4)(0.5) + (0.6)(0.35) + (0.8)(0.15) = 0.53 \qquad (= 53\%)$$

8. Solution: D

Let C = event that patient visits a chiropractor and T = event that patient visits a physical therapist. We are given that

$$\Pr[C] = \Pr[T] + 0.14$$
$$\Pr(C \cap T) = 0.22$$
$$\Pr(C^c \cap T^c) = 0.12$$

Therefore,

$$0.88 = 1 - \Pr[C^c \cap T^c] = \Pr[C \cup T] = \Pr[C] + \Pr[T] - \Pr[C \cap T]$$
$$= \Pr[T] + 0.14 + \Pr[T] - 0.22$$
$$= 2\Pr[T] - 0.08$$

or

$$P[T] = (0.88 + 0.08)/2 = 0.48$$

9. Solution: B

Let M = event that customer insures more than one car and S = event that customer insurers a sports car. Then applying DeMorgan's Law, compute the desired probability as:

$$\Pr(M^c \cap S^c) = \Pr\left[(M \cup S)^c\right] = 1 - \Pr(M \cup S) = 1 - \left[\Pr(M) + \Pr(S) - \Pr(M \cap S)\right]$$
$$= 1 - \Pr(M) - \Pr(S) + \Pr(S|M)\Pr(M) = 1 - 0.70 - 0.20 + (0.15)(0.70) = 0.205$$

10. This question duplicates Question 9 and has been deleted

11. Solution: B

Let C = Event that a policyholder buys collision coverage and D = Event that a policyholder buys disability coverage. Then we are given that $P[C] = 2P[D]$ and $P[C \cap D] = 0.15$.
By the independence of C and D,

$$0.15 = [C \cap D] = P[C]P[D] = 2P[D]^2$$

$$P[D]^2 = 0.15/2 = 0.075$$

$$P[D] = \sqrt{0.075},\ P[C] = 2\sqrt{0.075}.$$

Independence of C and D implies independence of C^c and D^c. Then

$$P[C^c \cap D^c] = P[C^c]P[D^c] = (1 - 2\sqrt{0.075})(1 - \sqrt{0.075}) = 0.33.$$

12. Solution: E

“Boxed” numbers in the table below were computed.

	High BP	Low BP	Norm BP	Total
Regular heartbeat	0.09	0.20	0.56	0.85
Irregular heartbeat	0.05	0.02	0.08	0.15
Total	0.14	0.22	0.64	1.00

From the table, 20% of patients have a regular heartbeat and low blood pressure.

13. Solution: C

Let x be the probability of having all three risk factors.

$$\frac{1}{3} = P\left[A \cap B \cap C \mid A \cap B\right] = \frac{P\left[A \cap B \cap C\right]}{P\left[A \cap B\right]} = \frac{x}{x + 0.12}$$

It follows that

$$x = \frac{1}{3}\left(x + 0.12\right) = \frac{1}{3}x + 0.04$$

$$\frac{2}{3}x = 0.04$$

$$x = 0.06$$

Now we want to find

$$\begin{aligned} P\left[\left(A \cup B \cup C\right)^c \mid A^c\right] &= \frac{P\left[\left(A \cup B \cup C\right)^c\right]}{P\left[A^c\right]} \\ &= \frac{1 - P\left[A \cup B \cup C\right]}{1 - P\left[A\right]} \\ &= \frac{1 - 3\left(0.10\right) - 3\left(0.12\right) - 0.06}{1 - 0.10 - 2\left(0.12\right) - 0.06} \\ &= \frac{0.28}{0.60} = 0.467 \end{aligned}$$

14. Solution: A

$$p_k = \frac{1}{5} p_{k-1} = \frac{1}{5}\frac{1}{5} p_{k-2} = \frac{1}{5}\frac{1}{5}\frac{1}{5} p_{k-3} = \cdots = \left(\frac{1}{5}\right)^k p_0 \quad k \geq 0$$

$$1 = 1 = \sum_{k=0}^{\infty} p_k = \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k p_0 = \frac{p_0}{1-\frac{1}{5}} = \frac{5}{4} p_0,\ p_0 = 4/5$$

Therefore, P[N > 1] = 1 – P[N £1] = 1 – (4/5 + 4/5 x 1/5) = 1 – 24/25 = 1/25 = 0.04 .

15. Solution: C

Let x be the probability of choosing A and B, but not C, y the probability of choosing A and C, but not B, z the probability of choosing B and C, but not A.

We want to find $w = 1 - (x + y + z)$.
We have $x + y = 1/4$, $x + z = 1/3$, $y + z = 5/12$.
Adding these three equations gives

$$(x+y)+(x+z)+(y+z) = \frac{1}{4}+\frac{1}{3}+\frac{5}{12}$$

$$2(x+y+z) = 1$$

$$x+y+z = \frac{1}{2}$$

$$w = 1-(x+y+z) = 1-\frac{1}{2} = \frac{1}{2}.$$

Alternatively the three equations can be solved to give $x = 1/12$, $y = 1/6$, $z = 1/4$ again leading to

$$w = 1-\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right) = \frac{1}{2}$$

16. Solution: D

Let N_1 and N_2 denote the number of claims during weeks one and two, respectively. Then since they are independent,

$$\begin{aligned}
\mathrm{P}[N_1 + N_2 = 7] &= \sum_{n=0}^{7} \mathrm{P}[N_1 = n]\,\mathrm{Pr}[N_2 = 7-n] \\
&= \sum_{n=0}^{7} \left(\frac{1}{2^{n+1}}\right)\left(\frac{1}{2^{8-n}}\right) \\
&= \sum_{n=0}^{7} \frac{1}{2^9} \\
&= \frac{8}{2^9} = \frac{1}{2^6} = \frac{1}{64}
\end{aligned}$$

17. Solution: D

Let O = event of operating room charges and E = event of emergency room charges. Then

$$0.85 = P(O \cup E) = P(O) + P(E) - P(O \cap E)$$

$$= P(O) + P(E) - P(O)\mathrm{P}(E) \quad (\text{Independence})$$

Because $P\left(E^c\right) = 0.25 = 1 - P(E)$, $P(E) = 0.75$,

$$0.85 = P(O) + 0.75 - P(O)(0.75)$$

$$P(O)(1 - 0.75) = 0.85 - 0.75 = 0.10$$

$$P(O) = 0.10 / 0.25 = 0.40.$$

18. Solution: D

Let X_1 and X_2 denote the measurement errors of the less and more accurate instruments, respectively. If $N(\mu, \sigma)$ denotes a normal random variable then

$X_1 \sim N(0, 0.0056h)$, $X_2 \sim N(0, 0044h)$ and they are independent. It follows that

$Y = \dfrac{X_1 + X_2}{2} \sim N(0, \sqrt{\dfrac{0.0056^2 h^2 + 0.0044^2 h^2}{4}} = 0.00356h)$. Therefore,

$$P(-0.005h \le Y \le 0.005h) = P\left(-\frac{0.005h - 0}{0.00356h} \le Z \le \frac{0.005h - 0}{0.00356h}\right)$$

$$= P(-1.4 \le Z \le 1.4) = P(Z \le 1.4) - [1 - P(Z \le 1.4)] = 2(0.9192) - 1 = 0.84.$$

19. Solution: B

Apply Bayes' Formula. Let

$A =$ Event of an accident

$B_1 =$ Event the driver's age is in the range 16-20

$B_2 =$ Event the driver's age is in the range 21-30

$B_3 =$ Event the driver's age is in the range 30-65

$B_4 =$ Event the driver's age is in the range 66-99

Then

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) + P(A|B_4)P(B_4)}$$

$$= \frac{(0.06)(0.08)}{(0.06)(0.08) + (0.03)(0.15) + (0.02)(0.49) + (0.04)(0.28)} = 0.1584$$

20. Solution: D

Let
S = Event of a standard policy
F = Event of a preferred policy
U = Event of an ultra-preferred policy
D = Event that a policyholder dies
Then

$$P[U \mid D] = \frac{P[D \mid U]P[U]}{P[D \mid S]P[S] + P[D \mid F]P[F] + P[D \mid U]P[U]}$$

$$= \frac{(0.001)(0.10)}{(0.01)(0.50) + (0.005)(0.40) + (0.001)(0.10)}$$

$$= 0.0141$$

21. Solution: B

$$P[\text{Seri.} \mid \text{Surv.}]$$

$$= \frac{P[\text{Surv.} \mid \text{Seri.}]P[\text{Seri.}]}{P[\text{Surv.} \mid \text{Crit.}]P[\text{Crit.}] + P[\text{Surv.} \mid \text{Seri.}]P[\text{Seri.}] + P[\text{Surv.} \mid \text{Stab.}]P[\text{Stab.}]}$$

$$= \frac{(0.9)(0.3)}{(0.6)(0.1) + (0.9)(0.3) + (0.99)(0.6)} = 0.29$$

22. Solution: D

Let H = heavy smoker, L = light smoker, N = non-smoker, D = death within five-year period.

We are given that $P[D \mid L] = 2P[D \mid N]$ and $P[D \mid L] = \frac{1}{2}P[D \mid H]$

Therefore,

$$P[H \mid D] = \frac{P[D \mid H]P[H]}{P[D \mid N]P[N] + P[D \mid L]P[L] + P[D \mid H]P[H]}$$

$$= \frac{2P[D \mid L](0.2)}{\frac{1}{2}P[D \mid L](0.5) + P[D \mid L](0.3) + 2P[D \mid L](0.2)} = \frac{0.4}{0.25 + 0.3 + 0.4} = 0.42$$

23. Solution: D

Let
C = Event of a collision
T = Event of a teen driver
Y = Event of a young adult driver
M = Event of a midlife driver
S = Event of a senior driver
Then,

$$P[Y \mid C] = \frac{P[C \mid Y]P[Y]}{P[C \mid T]P[T] + P[C \mid Y]P[Y] + P[C \mid M]P[M] + P[C \mid S]P[S]}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08) + (0.08)(0.16) + (0.04)(0.45) + (0.05)(0.31)} = 0.22.$$

24. Solution: B

$$P\left[N \geq 1 \middle| N \leq 4\right] = \frac{P\left[1 \leq N \leq 4\right]}{P\left[N \leq 4\right]} = \left[\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right] \bigg/ \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right]$$

$$= \frac{10 + 5 + 3 + 2}{30 + 10 + 5 + 3 + 2} = \frac{20}{50} = \frac{2}{5}$$

25. Solution: B

Let Y = positive test result
D = disease is present
Then,

$$P[D \mid Y] = \frac{P[Y \mid D]P[D]}{P[Y \mid D]P[D] + P[Y \mid D^c]P[D^c]} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.005)(0.99)} = 0.657.$$

26. Solution: C

Let:
S = Event of a smoker
C = Event of a circulation problem
Then we are given that P[C] = 0.25 and $P[S|C] = 2\, P[S|C^c]$
Then,,

$$P[C \mid S] = \frac{P[S \mid C]P[C]}{P[S \mid C]P[C] + P[S \mid C^c]P[C^c]}$$

$$= \frac{2P[S \mid C^c]P[C]}{2P[S \mid C^c]P[C] + P[S \mid C^c](1 - P[C])} = \frac{2(0.25)}{2(0.25) + 0.75} = \frac{2}{2+3} = \frac{2}{5}$$

27. Solution: D

Let *B*, *C*, and *D* be the events of an accident occurring in 2014, 2013, and 2012, respectively. Let $A = B \cup C \cup D$.

Use Bayes' Theorem

$$P[B \mid A] = \frac{P[A|B]P[B]}{P[A|B][P[B] + P[A|C]P[C] + P[A|D]P[D]}$$

$$= \frac{(0.05)(0.16)}{(0.05)(0.16) + (0.02)(0.18) + (0.03)(0.20)} = 0.45.$$

28. Solution: A

Let

C = Event that shipment came from Company *X*

I = Event that one of the vaccine vials tested is ineffective

Then, $P[C \mid I] = \dfrac{P[I \mid C]P[C]}{P[I \mid C]P[C] + P\left[I \mid C^c\right]P\left[C^c\right]}$.

Now

$P[C] = \dfrac{1}{5}$

$P\left[C^c\right] = 1 - P[C] = 1 - \dfrac{1}{5} = \dfrac{4}{5}$

$P[I \mid C] = \binom{30}{1}(0.10)(0.90)^{29} = 0.141$

$P\left[I \mid C^c\right] = \binom{30}{1}(0.02)(0.98)^{29} = 0.334$

Therefore,

$$P[C \mid I] = \frac{(0.141)(1/5)}{(0.141)(1/5) + (0.334)(4/5)} = 0.096.$$

29. Solution: C

Let *T* denote the number of days that elapse before a high-risk driver is involved in an accident. Then *T* is exponentially distributed with unknown parameter l . We are given that

$$0.3 = P[T \le 50] = \int_0^{50} \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_0^{50} = 1 - e^{-50\lambda}.$$

Therefore, $e^{=50\lambda} = 0.7$ and $\lambda = -(1/50)\ln(0.7)$.

Then,

$$P[T \le 80] = \int_0^{80} \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_0^{80} = 1 - e^{-80\lambda} \quad = 1 - e^{-80l}$$

$$= 1 - e^{(80/50)\ln(0.7)} = 1 - 0.7^{8/5} = 0.435.$$

30. Solution: D

Let N be the number of claims filed. We are given $P[N=2]=\frac{e^{-\lambda}\lambda^2}{2!}=3P[N=4]=3\frac{e^{-\lambda}\lambda^4}{4!}$.

Then,

$\frac{1}{2}\lambda^2=\frac{3}{24}\lambda^4$ or $\lambda^2=4$ or $\lambda=2$, which is the variance of N.

31. Solution: D

Let X denote the number of employees who achieve the high performance level. Then X follows a binomial distribution with parameters $n = 20$ and $p = 0.02$. Now we want to determine x such that $P[X > x] < 0.01$ or equivalently $0.99 \le P\left[X \le x\right]=\sum_{k=0}^{x}\binom{20}{k}(0.02)^k(0.98)^{20-k}$

The first three probabilities (at 0, 1, and 2) are 0.668, 0.272, and 0.053. The total is 0.993 and so the smallest x that has the probability exceed 0.99 is 2. Thus $C = 120/2 = 60$.

32. Solution: D

Let

X = number of low-risk drivers insured

Y = number of moderate-risk drivers insured

Z = number of high-risk drivers insured

$f(x, y, z)$ = probability function of X, Y, and Z

Then f is a trinomial probability function, so

$$\begin{aligned}P\left[z \ge x+2\right] &= f(0,0,4)+f(1,0,3)+f(0,1,3)+f(0,2,2)\\ &=(0.20)^4+4(0.50)(0.20)^3+4(0.30)(0.20)^3+\frac{4!}{2!2!}(0.30)^2(0.20)^2\\ &=0.0488\end{aligned}$$

33. Solution: B

$$P\left[X>x\right]=\int_x^{20}0.005(20-t)\,dt=0.005\left(20t-\frac{1}{2}t^2\right)\Big|_x^{20}$$

$$=0.005\left(400-200-20x+\frac{1}{2}x^2\right)=0.005\left(200-20x+\frac{1}{2}x^2\right)$$

where $0 < x < 20$. Therefore,

$$P\left[X>16 \middle| X>8\right]=\frac{P\left[X>16\right]}{P\left[X>8\right]}=\frac{200-20(16)+\frac{1}{2}(16)^2}{200-20(8)+\frac{1}{2}(8)^2}=\frac{8}{72}=\frac{1}{9}.$$

34. Solution: C

We know the density has the form $C(10+x)^{-2}$ for 0 < *x* < 40 (equals zero otherwise). First, determine the proportionality constant *C*.

$$1=\int_0^{40} C(10+x)^{-2}\,dx=-C(10+x)^{-1}\Big|_0^{40}=\frac{C}{10}-\frac{C}{50}=\frac{2}{25}C$$

So *C* = 25/2 or 12.5. Then, calculate the probability over the interval (0, 6):

$$12.5\int_0^{6}(10+x)^{-2}\,dx=-12.5(10+x)^{-1}\Big|_0^{6}=12.5\left(\frac{1}{10}-\frac{1}{16}\right)=0.47\,.$$

35. This question duplicates Question 34 and has been deleted

36. Solution: B

To determine *k*,

$$1\ =\ 1=\int_0^1 k(1-y)^4\,dy=-\frac{k}{5}(1-y)^5\Big|_0^1\ =\frac{k}{5},\text{ so } k=5$$

We next need to find P[*V* > 10,000] = P[100,000 *Y* > 10,000] = P[*Y* > 0.1], which is

$\int_{0.1}^{1} 5(1-y)^4dy=-(1-y)^5\Big|_{0.1}^{1}\ =0.9^5=0.59$ and P[*V* > 40,000] which is

$\int_{0.4}^{1} 5(1-y)^4dy=-(1-y)^5\Big|_{0.4}^{1}\ =0.6^5=0.078$. Then,

$$P[V>40,000\,|\,V>10,000]=\frac{P[V>40,000\cap V>10,000]}{P[V>10,000]}=\frac{P[V>40,000]}{P[V>10,000]}=\frac{0.078}{0.590}=0.132.$$

37. Solution: D

Let T denote printer lifetime. The distribution function is $F(t)=1-e^{-t/2}$. The probability of failure in the first year is *F*(1) = 0.3935 and the probability of failure in the second year is *F*(2) – *F*(1) = 0.6321 – 0.3935 = 0.2386. Of 100 printers, the expected number of failures is 39.35 and 23.86 for the two periods. The total expected cost is 200(39.35) + 100(23.86) = 10,256.

38. Solution: A

The distribution function is $F(x)=P[X\le x]=\int_1^x 3t^{-4}dt=-t^{-3}\Big|_1^x=1-x^{-3}$. Then,

$$P[X<2\,|\,X\ge 1.5]=\frac{P\big[(X<2)\text{ and }(X\ge 1.5)\big]}{P\big[X\ge 1.5\big]}=\frac{P[X<2]-\Pr[\mathrm{X}<1.5]}{\Pr\big[X\ge 1.5\big]}$$

$$=\frac{F(2)-F(1.5)}{1-F(1.5)}=\frac{(1-2^{-3})-(1-1.5^{-3})}{1-(1-1.5^{-3})}\ =\frac{-1/8+8/27}{8/27}\ =\frac{37}{64}=0.578$$

39. Solution: E

The number of hurricanes has a binomial distribution with n = 20 and p = 0.05. Then
$P[X < 3] = 0.95^{20} + 20(0.95)^{19}(0.05) + 190(0.95)^{18}(0.05)^2 = 0.9245.$

40. Solution: B

Denote the insurance payment by the random variable Y. Then

$$Y = \begin{cases} 0 & \text{if} \quad 0 < X \le C \\ X - C & \text{if} \quad \text{C} < X < 1 \end{cases}$$

We are given that

$$0.64 = P[Y < 0.5] = P[0 < X < 0.5 + C] = \int_0^{0.5+C} 2x\,dx = x^2 \Big|_0^{0.5+C} = (0.5 + C)^2 .$$

The quadratic equation has roots at C = 0.3 and –1.3. Because C must be between 0 and 1, the solution is C = 0.3.

41. Solution: E

The number completing the study in a single group is binomial (10,0.8). For a single group the probability that at least nine complete the study is $\binom{10}{9}(0.8)^9(0.2) + \binom{10}{10}(0.8)^{10} = 0.376$
The probability that this happens for one group but not the other is 0.376(0.624) + 0.624(0.376) = 0.469.

42. Solution: D

There are two situations where Company B's total exceeds Company A's. First, Company B has at least one claim and Company A has no claims. This probability is 0.3(0.6) = 0.18. Second, both have claims. This probability is 0.3(0.4) = 0.12. Given that both have claims, the distribution of B's claims minus A's claims is normal with mean 9,000 – 10,000 = –1,000 and standard deviation $\sqrt{2,000^2 + 2,000^2} = 2,828.43.$ The probability that the difference exceeds zero is the probability that a standard normal variable exceeds $\dfrac{0-(-1,000)}{2,828.43} = 0.354.$ The probability is 1 – 0.638 = 0.362. The probability of the desired event is 0.18 + 0.12(0.362) = 0.223.

43. Solution: D

One way to view this event is that in the first seven months there must be at least four with no accidents. These are binomial probabilities:

$$\binom{7}{4}0.4^4 0.6^3 + \binom{7}{5}0.4^5 0.6^2 + \binom{7}{6}0.4^6 0.6 + \binom{7}{7}0.4^7$$

$$= 0.1935 + 0.0774 + 0.0172 + 0.0016 = 0.2897.$$

Alternatively, consider a negative binomial distribution where K is the number of failures before the fourth success (no accidents). Then

$$P[K < 4] = 0.4^4 + \binom{4}{1}0.4^4 0.6 + \binom{5}{2}0.4^4 0.6^2 + \binom{6}{3}0.4^4 0.6^3 = 0.2898$$

44. Solution: C

The probabilities of 1, 2, 3, 4, and 5 days of hospitalization are 5/15, 4/15, 3/15, 2/15, and 1/15 respectively. The expected payments are 100, 200, 300, 350, and 400 respectively. The expected value is [100(5) + 200(4) + 300(3) + 350(2) + 400(1)]/15 = 220.

45. Solution: D

$$E(X) = \int_{-2}^{0} x\frac{-x}{10}dx + \int_{0}^{4} x\frac{x}{10}dx = -\left.\frac{x^3}{30}\right|_{-2}^{0} + \left.\frac{x^3}{30}\right|_{0}^{4} = -\frac{8}{30} + \frac{64}{30} = \frac{56}{30} = \frac{28}{15}$$

46. Solution: D

The density function of T is

$$f(t) = \frac{1}{3}e^{-t/3}, \quad 0 < t < \infty$$

Therefore,

$$E[X] = E[\max(T,2)] = \int_0^2 \frac{2}{3}e^{-t/3}dt + \int_2^\infty \frac{t}{3}e^{-t/3}dt$$

$$= -2e^{-t/3}\Big|_0^2 - te^{-t/3}\Big|_2^\infty + \int_2^\infty e^{-t/3}dt = -2e^{-2/3} + 2 + 2e^{-2/3} - 3e^{-t/3}\Big|_2^\infty$$

$$= 2 + 3e^{-2/3}$$

Alternatively, with probability $1 - e^{-2/3}$ the device fails in the first two years and contributes 2 to the expected value. With the remaining probability the expected value is 2 + 3 = 5 (employing the memoryless property). The unconditional expected value is $(1-e^{-2/3})2 + (e^{-2/3})5 = 2 + 3e^{-2/3}$.

47. Solution: D

We want to find *x* such that

$$1000 = \text{E[P]} = \int_0^1 \frac{x}{10}\ \mathrm{e}^{-t/10}\ \mathrm{dt} + \int_1^3 \frac{x}{2}\frac{1}{10}\mathrm{e}^{-t/10}\ \mathrm{dt} =$$

$$1000 = \int_0^1 x(0.1)\,\mathrm{e}^{-t/10}\ dt + \int_1^3 0.5x(0.1)e^{-t/10}dt$$

$$= -xe^{-t/10}\Big|_0^1 - 0.5xe^{-t/10}\Big|_1^3$$

$$= -xe^{-1/10} + x - 0.5xe^{-3/10} + 0.5xe^{-1/10} = 0.1772x.$$

Thus $x = 5644$.

48. Solution: E

$$E[Y] = 4000(0.4) + 3000(0.6)(0.4) + 2000(0.6)^2(0.4) + 1000(0.6)^3(0.4)$$
$$= 2694$$

49. This question duplicates Question 44 and has been deleted

50. Solution: C

The expected payment is

$$\sum_{n=1}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!} = \left[\sum_{n=0}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}\right] - 10,000(-1)e^{-3/2}$$

$$= 10,000(1.5-1) + 10,000e^{-3/2} = 7,231.$$

51. Solution: C

The expected payment is

$$\int_{0.6}^{2} x\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx + \int_{2}^{\infty} 2\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx = 2.5(0.6)^{2.5}\left(\frac{-x^{-1.5}}{1.5}\Big|_{0.6}^{2} + \frac{-x^{-2.5}}{2.5}\Big|_{2}^{\infty}\right)$$

$$= 2.5(0.6)^{2.5}\left(\frac{-2^{-1.5}}{1.5} + \frac{0.6^{-1.5}}{1.5} + 2\frac{2^{-2.5}}{2.5}\right) = 0.9343.$$

52. Solution: A

First, determine K.

$$1 = K\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = K\left(\frac{60+30+20+15+12}{60}\right) = K\left(\frac{137}{60}\right)$$

$$K = \frac{60}{137}$$

Then, after applying the deductible, the expected payment is

$$0.05[(3-2)P(N=3)+(4-2)P(N=4)+(5-2)P(N=5)]$$

$$= 0.05(60/137)[1(1/3)+2(1/4)+3(1/5)] = 0.0314$$

53. Solution: D

The expected payment is

$$\int_1^{10} y\frac{2}{y^3}\,dy + \int_{10}^{\infty} 10\frac{2}{y^3}\,dy = -\frac{2}{y}\Bigg|_1^{10} - \frac{20}{2y^2}\Bigg|_{10}^{\infty} = -\frac{2}{10}+\frac{2}{1}-0+\frac{20}{200} = 1.9.$$

54. Solution: B

The expected payment is (in thousands)

$$(0.94)(0)+(0.02)(15-1)+0.04\int_1^{15}(x-1)0.5003e^{-x/2}dx$$

$$= 0.28+\left(0.020012\right)\left[-2e^{-x/2}(x-1)\Big|_1^{15} + \int_1^{15} 2\,\mathrm{e}^{-x/2}\,dx\right]$$

$$= 0.28+\left(0.020012\right)\left[-2e^{-7.5}(14)+\left(-4e^{-x/2}\Big|_1^{15}\right)\right]$$

$$= 0.28+\left(0.020012\right)\left[-2e^{-7.5}(14)-4e^{-7.5}+4e^{-0.5}\right]$$

$$= 0.28+\left(0.020012\right)\left(2.408\right)$$

$$= 0.328.$$

55. Solution: C

$1 = \displaystyle\int_0^{\infty} \frac{k}{(1+x)^4}\,dx = -\frac{k}{3}\frac{1}{(1+x)^3}\Bigg|_0^{\infty} = \frac{k}{3}$ and so k = 3.

The expected value is (where the substitution $u = 1 + x$ is used.

$$\int_0^{\infty} x\frac{3}{(1+x)^4}\,dx = \int_1^{\infty} 3(u-1)u^{-4}du = 3u^{-2}/(-2) - 3u^{-3}/(-3)\Big|_1^{\infty} = 3/2-1 = 1/2.$$

Integration by parts may also be used.

56. Solution: C

With no deductible, the expected payment is 500. With the deductible it is to be 125. Let *d* be the deductible. Then,

$$125 = \int_d^{1000} (x-d)(0.001)dx = \frac{(x-d)^2}{2}(0.001)\Big|_d^{1000} = 0.0005\left[(1000-d)^2 - 0\right]$$

$$250,000 = (1000-d)^2$$

$$500 = 1000 - d$$

$$d = 500.$$

57. Solution: B

This is the moment generating function of a gamma distribution with parameters 4 and 2,500. The standard deviation is the square root of the shape parameter times the scale parameter, or 2(2,500) = 5,000. But such recognition is not necessary.

$$M'(t) = 4(2500)(1-2500t)^{-5}$$

$$M''(t) = 20(2500)^2(1-2500t)^{-6}$$

$$E(X) = M'(0) = 10,000$$

$$E(X^2) = M''(0) = 125,000,000$$

$$Var(X) = 125,000,000 - 10,000^2 = 25,000,000$$

$$SD(X) = 5,000$$

58. Solution: E

Because the losses are independent, the mgf of their sum is the product of the individual mgfs, which is $(1 - 2t)^{-10}$. The third moment can be determined by evaluating the third derivative at zero. This is $(10)(2)(11)(2)(12)(2)(1- 2(0))^{-13} = 10,560$.

59. Solution: B

The distribution function of X is

$$F(x)=\int_{200}^{x}\frac{2.5(200)^{2.5}}{t^{3.5}}dt=\left.\frac{-(200)^{2.5}}{t^{2.5}}\right|_{200}^{x}=1-\frac{(200)^{2.5}}{x^{2.5}},\quad x>200$$

The pth percentile x_p of X is given by

$$\frac{p}{100}=F(x_p)=1-\frac{(200)^{2.5}}{x_p^{2.5}}$$

$$1-0.01p=\frac{(200)^{2.5}}{x_p^{2.5}}$$

$$(1-0.01p)^{0.4}=\frac{200}{x_p}$$

$$x_p=\frac{200}{(1-0.01p)^{0.4}}$$

It follows that $x_{70}-x_{30}=\frac{200}{(0.30)^{0.4}}-\frac{200}{(0.70)^{0.4}}=93.06$.

60. Solution: E

Let X and Y denote the annual cost of maintaining and repairing a car before and after the 20% tax, respectively. Then $Y = 1.2X$ and $Var(Y)=Var(1.2X)=1.44Var(X)=1.44(260)=374.4$.

61. This question duplicates Question 59 and has been deleted

62. Solution: C

First note that the distribution function jumps ½ at $x = 1$, so there is discrete probability at that point. From 1 to 2, the density function is the derivative of the distribution function, $x – 1$. Then,

$$E(X)=\frac{1}{2}(1)+\int_{1}^{2}x(x-1)dx=\frac{1}{2}+\left.\left(\frac{x^3}{3}-\frac{x^2}{2}\right)\right|_{1}^{2}=\frac{1}{2}+\frac{8}{3}-\frac{4}{2}-\frac{1}{3}+\frac{1}{2}=\frac{4}{3}$$

$$E(X^2)=\frac{1}{2}(1)^2+\int_{1}^{2}x^2(x-1)dx=\frac{1}{2}+\left.\left(\frac{x^4}{4}-\frac{x^3}{3}\right)\right|_{1}^{2}=\frac{1}{2}+\frac{16}{4}-\frac{8}{3}-\frac{1}{4}+\frac{1}{3}=\frac{23}{12}$$

$$Var\left(X\right)=E(X^2)-\left[E(X)\right]^2=\frac{23}{12}-\left(\frac{4}{3}\right)^2=\frac{23}{12}-\frac{16}{9}=\frac{5}{36}.$$

63. Solution: C

$$E[Y] = \int_0^4 x(0.2)dx + \int_4^5 4(0.2)dx = 0.1x^2\Big|_0^4 + 0.8 = 2.4$$
$$E[Y^2] = \int_0^4 x^2(0.2)dx + \int_4^5 4^2(0.2)dx = (0.2/3)x^3\Big|_0^4 + 3.2 = 7.46667$$
$$\text{Var[Y]} = E[Y^2] - \left(E[Y]\right)^2 = 7.46667 - 2.4^2 = 1.707.$$

64. Solution: A

The mean is 20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30) = 55. The second moment is 400(0.15) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30) = 3500. The variance is $3500 – 55^2 = 475$. The standard deviation is 21.79. The range within one standard deviation of the mean is 33.21 to 76.79, which includes the values 40, 50, 60, and 70. The sum of the probabilities for those values is 0.05 + 0.20 + 0.10 + 0.10 = 0.45.

65. Solution: B

Let *Y* be the amount of the insurance payment.

$$E[Y] = \int_{250}^{1500} \frac{1}{1500}(x-250)dx = \frac{1}{3000}(x-250)^2\Big|_{250}^{1500} = \frac{1250^2}{3000} = 521$$
$$E[Y^2] = \int_{250}^{1500} \frac{1}{1500}(x-250)^2 dx = \frac{1}{4500}(x-250)^3\Big|_{250}^{1500} = \frac{1250^3}{4500} = 434,028$$
$$\text{Var[Y]} = 434,028 - (521)^2 = 162,587$$
$$SD[Y] = 403.$$

66. DELETED

67. Solution: B

The expected amount paid is (where *N* is the number of consecutive days of rain)

$$1000P[N=1] + 2000P[N>1] = 1000\frac{e^{-0.6}0.6}{1!} + 2000\left(1 - e^{-0.6} - e^{-0.6}0.6\right) = 573.$$

The second moment is

$$1000^2 P[N=1] + 2000^2 P[N>1] = 1000^2 \frac{e^{-0.6}0.6}{1!} + 2000^2\left(1 - e^{-0.6} - e^{-0.6}0.6\right) = 816,893.$$

The variance is $816,893 – 573^2 = 488,564$ and the standard deviation is 699.

68. Solution: C

X has an exponential distribution. Therefore, c = 0.004 and the distribution function is $F(x) = 1 - e^{-0.004x}$. For the moment, ignore the maximum benefit. The median is the solution to $0.5 = F(m) = 1 - e^{-0.004m}$, which is $m = -250\ln(0.5) = 173.29$. Because this is below the maximum benefit, it is the median regardless of the existence of the maximum. Note that had the question asked for a percentile such that the solution without the maximum exceeds 250, then the answer is 250.

69. Solution: D

The distribution function of an exponential random variable, T, is $F(t) = 1 - e^{-t/\theta}$, $t > 0$. With a median of four hours, $0.5 = F(4) = 1 - e^{-4/\theta}$ and so $\theta = -4/\ln(0.5)$. The probability the component works for at least five hours is $P[T \geq 5] = 1 - F(5) = 1 - 1 + e^{5\ln(0.5)/4} = 0.5^{5/4} = 0.42$.

70. Solution: E

This is a conditional probability. The solution is

$$\begin{aligned}
0.95 &= P[X \leq p \mid X > 100] = \frac{P[100 \leq X \leq p]}{P[X > 100]} = \frac{F(p) - F(100)}{1 - F(100)} \\
&= \frac{1 - e^{-p/300} - 1 + e^{-100/300}}{1 - 1 + e^{-100/300}} = \frac{e^{-100/300} - e^{-p/300}}{e^{-100/300}} = 1 - e^{-(p-100)/300} \\
0.05 &= e^{-(p-100)/300} \\
-2.9957 &= -(p-100)/300 \\
p &= 999
\end{aligned}$$

71. Solution: A

The distribution function of Y is given by $G(y) = P(T^2 \leq y) = P(T \leq \sqrt{y}) = F(\sqrt{y}) = 1 - 4/y$ for $y > 4$. Differentiate to obtain the density function $g(y) = 4y^{-2}$.
Alternatively, the density function of T $f(t) = F'(t) = 8t^{-3}$. We have $t = y^{0.5}$ and $dt = 0.5y^{-0.5}dy$.
Then $g(y) = f(y^{0.5})\left|dt/dy\right| = 8(y^{0.5})^{-3}(0.5y^{-0.5}) = 4y^{-2}$.

72. Solution: E

The distribution function of V is given by

$$F(v) = P[V \le v] = P\left[10{,}000e^R \le v\right] = P\left[R \le \ln(v) - \ln(10{,}000)\right]$$

$$= \int_{0.04}^{\ln(v)-\ln(10{,}000)} \frac{1}{0.04}\,dr = \left.\frac{r}{0.04}\right|_{0.04}^{\ln(v)-\ln(10{,}000)} = 25\ln(v) - 25\ln(10{,}000) - 1$$

$$= 25\left[\ln\left(\frac{v}{10{,}000}\right) - 0.04\right].$$

73. Solution: E

$$F(y) = P[Y \le y] = P[10X^{0.8} \le y] = \Pr\left[X \le (0.1y)^{1.25}\right] = 1 - e^{-(0.1y)^{1.25}}.$$

Therefore, $f(y) = F'(y) = 0.125(0.1y)^{0.25}e^{-(0.1y)^{1.25}}$.

74. Solution: E

First note $R = 10/T$. Then,

$F(r) = P[R \le r] = P[10/T \le r] = P[T \ge 10/r] = \int_{10/r}^{12} 0.25dt = 0.25(12 - 10/r)$. The density function is $f(r) = F'(r) = 2.5/r^2$.

Alternatively, $t = 10/r$ and $dt = -10/r^2 dr$. Then $f_R(r) = f_T(10/r)\,|dt/dr| = 0.25(10/\mathrm{r}^2) = 2.5/r^2$

.

75. Solution: A

Let Y be the profit for Company II, so $Y = 2X$.

$$G(y) = P[Y \le y] = P[2X \le y] = P[X \le y/2] = F(y/2)$$

$$g(y) = G'(y) = F'(y/2) = (1/2)f(y/2).$$

Alternatively, $X = Y/2$ and $dx = dy/2$. Then,

$$g(y) = f(y/2)\,|dx/dy| = f(y/2)(1/2).$$

76. Solution: A

The distribution function of X is given by

$$F(x) = \int_1^x \frac{3}{t^4}\,dt = -t^{-3}\Big|_1^x = 1 - x^{-3}, \quad x > 1$$

Next, let X_1, X_2, and X_3 denote the three claims made that have this distribution. Then if Y denotes the largest of these three claims, it follows that the distribution function of Y is given by

$$G(y) = P[X_1 \le y]P[X_2 \le y]P[X_3 \le y] = (1 - y^{-3})^3.$$

The density function of Y is given by

$$g(y) = G'(y) = 3(1 - y^{-3})^2(3y^{-4}).$$

Therefore,

$$E[Y] = \int_1^\infty y3(1 - y^{-3})^2 3y^{-4}dy = 9\int_1^\infty y^{-3} - 2y^{-6} + y^{-9}dy$$

$$= 9\left[-y^{-2}/2 + 2y^{-5}/5 - y^{-8}/8\Big|_1^\infty\right] = 9\left[1/2 - 2/5 + 1/8\right]$$

$$= 2.025 \text{ (in thousands).}$$

77. Solution: D

The probability it works for at least one hour is the probability that both components work for more than one hour. This probability is

$$\int_1^2\int_1^2 \frac{x+y}{8}dxdy = \int_1^2 \frac{0.5x^2 + xy}{8}\Bigg|_1^2 dy = \int_1^2 \frac{1.5+y}{8}dy = \frac{1.5y + 0.5y^2}{8}\Bigg|_1^2 = 0.375.$$

The probability of failing within one hour is the complement, 0.625.

78. This question duplicates Question 77 and has been deleted

79. Solution: E

Let s be on the horizontal axis and t be on the vertical axis. The event in question covers all but the upper right quarter of the unit square. The probability is the integral over the other three quarters. Answer (A) is the lower left quarter. Answer B is the left half. Answer (C) is the upper right quarter. Answer (D) is the lower half plus the left half, so the lower left quarter is counted twice. Answer (E) is the lower right corner plus the left half, which is the correct region.

For this question, the regions don’t actually need to be identified. The area is 0.75 while the five answer choices integrate over regions of area 0.25, 0.5, 0.25, 1, and 0.75 respectively. So only (E) can be correct.

80. Solution: C

The mean and standard deviation for the 2025 contributions are 2025(3125) = 6,328,125 and 45(250) = 11,250. By the central limit theorem, the total contributions are approximately normally distributed. The 90th percentile is the mean plus 1.282 standard deviations or 6,328,125 + 1.282(11,250) = 6,342,548.

81. Solution: C

The average has the same mean as a single claim, 19,400. The standard deviation is that for a single claim divided by the square root of the sample size, 5,000/5 = 1,000. The probability of exceeding 20,000 is the probability that a standard normal variable exceeds (20,000 – 19,400)/1,000 = 0.6. From the tables, this is 1 – 0.7257 = 0.2743.

82. Solution: B

A single policy has a mean and variance of 2 claims. For 1250 polices the mean and variance of the total are both 2500. The standard deviation is the square root, or 50.
The approximate probability of being between 2450 and 2600 is the same as a standard normal random variable being between (2450 – 2500)/50 = –1 and (2600 – 2500)/50 = 2. From the tables, the probability is 0.9772 – (1 – 0.8413) = 0.8185.

83. Solution: B

Let n be the number of bulbs purchased. The mean lifetime is $3n$ and the variance is n. From the normal tables, a probability of 0.9772 is 2 standard deviations below the mean. Hence $40 = 3n - 2\text{sqrt}(n)$. Let m be the square root of n. The quadratic equation is $3m^2 - 2m - 40$. The roots are 4 and –10/3. So n is either 16 or 100/9. At 16 the mean is 48 and the standard deviation is 4, which works. At 100/9 the mean is 100/3 and the standard deviation is 10/3. In this case 40 is two standard deviations above the mean, and so is not appropriate. Thus 16 is the correct choice.

84. Solution: B

Observe that (where Z is total hours for a randomly selected person)

$$E[Z] = E[X+Y] = E[X] + E[Y] = 50 + 20 = 70$$

$$Var[Z] = Var[X+Y] = Var[X] + Var[Y] + 2Cov[X,Y] = 50 + 30 + 20 = 100.$$

It then follows from the Central Limit Theorem that T is approximately normal with mean 100(70) = 7000 and variance 100(100) = 10,000 and standard deviation 100. The probability of being less than 7100 is the probability that a standard normal variable is less than (7100 – 7000)/100 = 1. From the tables, this is 0.8413.

85. Solution: B

A single policy has an exponential distribution with mean and standard deviation 1000. The premium is then 1000 + 100 = 1100. For 100 policies, the total claims have mean 100(1000) = 100,000 and standard deviation 10(1000) = 10,000. Total premiums are 100(1100) = 110,000. The probability of exceeding this number is the probability that a standard normal variable exceeds (110,000 – 100,000)/10,000 = 1. From the tables this probability is 1 – 0.8413 = 0.1587.

86. Solution: E

For a single recruit, the probability of 0 pensions is 0.6, of 1 pension is 0.4(0.25) = 0.1, and of 2 pensions is 0.4(0.75) = 0.3. The expected number of pensions is 0(0.6) + 1(0.1) + 2(0.3) = 0.7. The second moment is 0(0.6) + 1(0.1) + 4(0.3) = 1.3. The variance is 1.3 – 0.49 = 0.81. For 100 recruits the mean is 70 and the variance is 81. The probability of providing at most 90 pensions is (with a continuity correction) the probability of being below 90.5. This is (90.5 – 70)/9 = 2.28 standard deviations above the mean. From the tables, this probability is 0.9887.

87. Solution: D

For one observation, the mean is 0 and the variance is 25/12 (for a uniform distribution the variance is the square of the range divided by 12). For 48 observations, the average has a mean of 0 and a variance of (25/12)/48 = 0.0434. The standard deviation is 0.2083. 0.25 years is 0.25/0.2083 = 1.2 standard deviations from the mean. From the normal tables the probability of being within 1.2 standard deviations is 0.8849 – (1 – 0.8849) = 0.7698.

88. Solution: C

For a good driver, the probability is $1-e^{-3/6}$ and for a bad driver, the probability is $1-e^{-2/3}$. The probability of both is the product, $(1-e^{-3/6})(1-e^{-2/3}) = 1-e^{-1/2}-e^{-2/3}+e^{-7/6}$.

89. Solution: B

The probability both variables exceed 20 is represented by the triangle with vertices (20,20), (20,30), and (30,20). All the answer choices have *x* as the outer integral and *x* ranges from 20 to 30, eliminating answers (A), (D), and (E). For a given value of *x*, the triangle runs from the base at *y* = 20 to the diagonal line at *y* = 50 – *x*. This is answer (B).

90. Solution: C

Let B be the time until the next Basic Policy claim, and let D be the time until the next Deluxe policy claim. Then the joint pdf of B and D is

$$f(b,d)=\left(\frac{1}{2}e^{-b/2}\right)\left(\frac{1}{3}e^{-d/3}\right)=\frac{1}{6}e^{-b/2}e^{-d/3}.$$

The desired probability is

$$P[B>D]=\int_0^\infty\int_0^b \frac{1}{6}e^{-b/2}e^{-d/3}\,dddb=\int_0^\infty -\frac{1}{2}e^{-b/2}e^{-d/3}\Big|_0^b\,db$$

$$=\int_0^\infty -\frac{1}{2}e^{-b/2}e^{-b/3}+\frac{1}{2}e^{-b/2}db=\frac{3}{5}e^{-5b/6}-e^{-b/2}\Big|_0^\infty=\frac{2}{5}=0.4.$$

91. Solution: D

$$P[X+Y\ge 1]=\int_0^1\int_{1-x}^2 \frac{2x+2-y}{4}dydx=\int_0^1 \frac{2xy+2y-0.5y^2}{4}\Big|_{1-x}^2\,dx$$

$$=\int_0^1 \frac{4x+4-2}{4}-\frac{2x(1-x)+2(1-x)-0.5(1-x)^2}{4}dx$$

$$=\int_0^1 \frac{2.5x^2+3x+0.5}{4}dx=\frac{2.5/3+3/2+0.5}{4}=0.708.$$

92. Solution: B

Because the distribution is uniform, the probability is the area of the event in question divided by the overall area of 200(200) = 40,000. It is easier to get the complement as it comprises two triangles. One triangle has vertices at (2000,2020), (2000,2200), and (2180,2200). The area is 180(180)/2 = 16,200. The other triangle has the same area for a total of 32,400. The area in question is 7,600 for a probability of 0.19.

93. Solution: C

Because losses are uniformly distribution, the desired probability is the area of the event in question divided by the total area, in this case 10(10) = 100. Let X be the loss on the policy with a deductible of 1 and Y the loss on the policy with a deductible of 2. The area where total payment is less than 5 can be broken down as follows:

For $0 < X < 1$, there is no payment on that policy. Then Y must be less than 7. Area equals 7.

For $1 < X < 6$, the payment on that policy is $X - 1$. The other policy must have a loss of less than $2 + 5 - (X - 1) = 8 - X$. This is a trapezoid with a base of width 5 and a height that starts at 7 and decreases linearly to 2. The area is 5(7 + 2)/2 = 22.5.

The total area is 29.5 and so the probability is 0.295.

94. Solution: C

First note that due to symmetry the two random variables have the same mean. Second note that probability is on the square from 0 to 6 but the upper right corner is cut off by the diagonal line where the sum is 10. The integral must be split into two parts. The first runs horizontally from 0 to 4 and vertically from 0 to 6. The second runs horizontally from 4 to 6 and vertically from 0 to 10 – t_1. The area of this total region is 4(6) + 2(6 + 4)/2 = 34. So the constant density is 1/34. The mean is

$$E[T_1] = \int_0^4\int_0^6 t_1 \frac{1}{34} dt_2 dt_1 + \int_4^6\int_0^{10-t_1} t_1 \frac{1}{34} dt_2 dt_1 = \int_0^4 t_1 \left.\frac{t_2}{34}\right|_0^6 dt_1 + \int_4^6 t_1 \left.\frac{t_2}{34}\right|_0^{10-t_1} dt_1$$

$$= \int_0^4 \frac{6t_1}{34} dt_1 + \int_4^6 \frac{1}{34}\left(10t_1 - t_1^2\right) dt_1 = \left.\frac{3t_1^2}{34}\right|_0^4 + \left.\frac{5t_1^2 - t_1^3/3}{34}\right|_4^6$$

$$= \frac{48}{34} + \frac{1}{34}\left(180 - 72 - 80 + \frac{64}{3}\right) = 2.86.$$

The expected sum is 2(2.86) = 5.72.

95. Solution: E

$$M(t_1, t_2) = E\left[e^{t_1 W + t_2 Z}\right] = E\left[e^{t_1(X+Y) + t_2(Y-X)}\right] = E\left[e^{(t_1-t_2)X} e^{(t_1+t_2)Y}\right]$$

$$= E\left[e^{(t_1-t_2)X}\right] E\left[e^{(t_1+t_2)Y}\right] = e^{\frac{1}{2}(t_1-t_2)^2} e^{\frac{1}{2}(t_1+t_2)^2} = e^{\frac{1}{2}\left(t_1^2 - 2t_1t_2 + t_2^2\right)} e^{\frac{1}{2}\left(t_1^2 + 2t_1t_2 + t_2^2\right)} = e^{t_1^2 + t_2^2}.$$

96. Solution: E

The tour operator collects 21x50 = 1050 for the 21 tickets sold. The probability that all 21 passengers will show up is $(1 - 0.02)^{21} = (0.98)^{21} = 0.65$. Therefore, the tour operator’s expected revenue is 1050 – 100(065) = 985.

97. Solution: C

The domain has area $L^2/2$ and so the density function is $2/L^2$. Then,

$$E[T_1^2 + T_2^2] = \int_0^L\int_0^{t_2} (t_1^2 + t_2^2)\frac{2}{L^2} dt_1 dt_2 = \int_0^L \left.\left(t_1^3/3 + t_2^2 t_1\right)\frac{2}{L^2}\right|_0^{t_2} dt_2$$

$$= \int_0^L \left(t_2^3/3 + t_2^3\right)\frac{2}{L^2} dt_2 = \left.(t_2^4/3)\frac{2}{L^2}\right|_0^L = \frac{2L^2}{3}.$$

98. Solution: A

The product of the three variables is 1 only if all three are 1, so $P[Y = 1] = 8/27$. The remaining probability of 19/27 is on the value 0. The mgf is

$$M(t) = E[e^{tY}] = e^{t(0)}(19/27) + e^{t(1)}(8/27) = \frac{19}{27} + \frac{8}{27}e^{t}.$$

99. Solution: C

First obtain the covariance of the two variables as (17,000 – 5,000 – 10,000)/2 = 1,000.
The requested variance is

$$\begin{aligned} Var(X + 100 + 1.1Y) &= Var(X) + Var(1.1Y) + 2Cov(X, 1.1Y) \\ &= Var(X) + 1.21Var(Y) + 2(1.1)Cov(X, Y) \\ &= 5,000 + 1.21(10,000) + 2.2(1,000) = 19,300. \end{aligned}$$

100. Solution: B

$P(X = 0) = 1/6$
$P(X = 1) = 1/12 + 1/6 = 3/12$
$P(X = 2) = 1/12 + 1/3 + 1/6 = 7/12$.
$E[X] = (0)(1/6) + (1)(3/12) + (2)(7/12) = 17/12$
$E[X^2] = (0)^2(1/6) + (1)^2(3/12) + (2)^2(7/12) = 31/12$
$\text{Var}[X] = 31/12 - (17/12)^2 = 0.58$.

101. Solution: D

Due to the independence of X and Y

$$Var(Z) = Var(3X - Y - 5) = 3^2 Var(X) + (-1)^2 Var(Y) = 9(1) + 2 = 11.$$

102. Solution: E

Let X and Y denote the times that the generators can operate. Now the variance of an exponential random variable is the square of them mean, so each generator has a variance of 100. Because they are independent, the variance of the sum is 200.

103. Solution: E

Let S, F, and T be the losses due to storm, fire, and theft respectively. Let $Y = \max(S,F,T)$. Then,

$$\begin{aligned} P[Y > 3] &= 1 - P[Y \le 3] = 1 - P[\max(S, F, T) \le 3] = 1 - P[S \le 3]P[F \le 3]P[T \le 3] \\ &= 1 - (1 - \mathrm{e}^{-3/1})(1 - e^{-3/1.5})(1 - e^{-3/2.4}) = 0.414. \end{aligned}$$

104. Solution: B

First determine k:

$$1 = \int_0^1 \int_0^1 kxdxdy = \int_0^1 0.5kx^2\Big|_0^1 dy = \int_0^1 0.5kdy = 0.5k$$

$k = 2.$

Then

$$E[X] = \int_0^1\int_0^1 2x^2 dydx = \int_0^1 2x^2 dx = \frac{2}{3}$$

$$E[Y] = \int_0^1\int_0^1 y2x\ dxdy = \int_0^1 ydy = \frac{1}{2}$$

$$E[XY] = \int_0^1\int_0^1 2x^2 ydxdy = \int_0^1 (2/3)ydy = \frac{1}{3}$$

$$\text{Cov}(X,Y) = E[XY] - E[X]E[Y] = \frac{1}{3} - \frac{2}{3}\frac{1}{2} = 0.$$

Alternatively, note that the density function factors as the product of one term that depends only on x and one that depends only on y. Therefore, the two variables are independent and the covariance must be 0.

105. Solution: A

Note that although the density function factors into expressions involving only x and y, the variables are not independent. An additional requirement is that the domain be a rectangle.

$$E[X] = \int_0^1\int_x^{2x} \frac{8}{3}x^2 y\, dy\, dx = \int_0^1 \frac{4}{3}x^2 y^2\Big|_x^{2x} dx = \int_0^1 \frac{4}{3}x^2\left(4x^2 - x^2\right)dx = \int_0^1 4x^4 dx = \frac{4}{5}x^5\Big|_0^1 = \frac{4}{5}$$

$$E[Y] = \int_0^1\int_x^{2x} \frac{8}{3}xy^2 dy\, dx = \int_0^1 \frac{8}{9}xy^3\Big|_x^{2x} dx = \int_0^1 \frac{8}{9}x\left(8x^3 - x^3\right)dx = \int_0^1 \frac{56}{9}x^4 dx = \frac{56}{45}x^5\Big|_0^1 = \frac{56}{45}$$

$$E[XY] = \int_0^1\int_x^{2x} \frac{8}{3}x^2 y^2 dy\, dx = \int_0^1 \frac{8}{9}x^2 y^3\Big|_x^{2x} dx = \int_0^1 \frac{8}{9}x^2\left(8x^3 - x^3\right)dx = \int_0^1 \frac{56}{9}x^5 dx = \frac{56}{54} = \frac{28}{27}$$

$$\text{Cov}(X,Y) = E[XY] - E[X]E[Y] = \frac{28}{27} - \left(\frac{56}{45}\right)\left(\frac{4}{5}\right) = 0.04.$$

106. Solution: C

The joint density function of *X* and *Y* is

$$f(x,y) = f(x)f(y\,|\,x) = \frac{1}{12}\frac{1}{x} = \frac{1}{12x},\ 0 < x < 12,\ 0 < y < x.$$

$$E[X] = \int_0^{12}\int_0^x \frac{x}{12x}dydx = \int_0^{12} \frac{y}{12}\Big|_0^x dx = \int_0^{12} \frac{x}{12}dx = \frac{x^2}{24}\Big|_0^{12} = \frac{144}{24}$$

$$E[Y] = \int_0^{12}\int_0^x \frac{y}{12x}dydx = \int_0^{12} \frac{y^2}{24x}\Big|_0^x dx = \int_0^{12} \frac{x}{24}dx = \frac{x^2}{48}\Big|_0^{12} = \frac{144}{48}$$

$$E[XY] = \int_0^{12}\int_0^x \frac{xy}{12x}dydx = \int_0^{12} \frac{y^2}{24}\Big|_0^x dx = \int_0^{12} \frac{x^2}{24}dx = \frac{x^3}{72}\Big|_0^{12} = \frac{1728}{72}$$

$$Cov(X,Y) = \frac{1728}{72} - \frac{144}{24}\frac{144}{48} = 6.$$

107. Solution: A

First obtain Var(*X*) = 27.4 – 25 = 2.4, Var(*Y*) = 51.4 – 49 = 2.4, Cov(*X*,*Y*) = (8 – 2.4 – 2.4)/2 = 1.6. Then,

$$\begin{aligned} Cov(C_1,C_2) &= Cov(X+Y, X+1.2Y) = Cov(X,X) + 1.2Cov(X,Y) + Cov(Y,X) + 1.2Cov(Y,Y) \\ &= Var(X) + 1.2Var(Y) + 2.2Cov(X,Y) \\ &= 2.4 + 1.2(2.4) + 2.2(1.6) = 8.8. \end{aligned}$$

108. Solution: A

The joint density of T_1 and T_2 is given by $f(t_1,t_2) = e^{-t_1}e^{-t_2},\ \ t_1 > 0,\ \ t_2 > 0$

Therefore,

$$\begin{aligned} P[X \le x] &= P[2T_1 + T_2 \le x] \\ &= \int_0^x \int_0^{(x-t_2)/2} e^{-t_1}e^{-t_2}dt_1dt_2 = \int_0^x -e^{-t_1}e^{-t_2}\Big|_0^{(x-t_2)/2} dt_2 = \int_0^x e^{-t_2}\left(1 - e^{-(x-t_2)/2}\right)dt_2 \\ &= \int_0^x \left(e^{-t_2} - e^{-x/2}e^{-t_2/2}\right)dt_2 = -e^{-t_2} + 2e^{-x/2}e^{-t_2/2}\Big|_0^x = -e^{-x} + 2e^{-x} + 1 - 2e^{-x/2} \\ &= 1 - 2e^{-x/2} + e^{-x}. \end{aligned}$$

The density function is the derivative,

$$\frac{d}{dx}\left[1 - 2e^{-x/2} + e^{-x}\right] = e^{-x/2} - e^{-x}.$$

109. Solution: B

Let
U be annual claims,
V be annual premiums,
$g(u, v)$ be the joint density function of U and V,
$f(x)$ be the density function of $X = U/V$, and
$F(x)$ be the distribution function of X.
Then because U and V are independent,
$g(u,v) = e^{-u}(0.5e^{-v/2}) = 0.5e^{-u}e^{-v/2}, \quad 0 < u, v < \infty.$
Then,

$$
\begin{aligned}
F(x) &= P[X \le x] = P[U / V \le x] = P[U \le Vx] \\
&= \int_0^\infty \int_0^{vx} g(u,v)dudv = \int_0^\infty \int_0^{vx} 0.5e^{-u}e^{-v/2}dudv \\
&= \int_0^\infty -0.5e^{-u}e^{-v/2}\Big|_0^{vx} dv = \int_0^\infty -0.5e^{-vx-v/2} + 0.5e^{-v/2}dv \\
&= \frac{0.5}{x+0.5}e^{-vx-v/2} - e^{-v/2}\Big|_0^\infty = -\frac{0.5}{x+0.5} + 1 = -\frac{1}{2x+1} + 1.
\end{aligned}
$$

Then, $f(x) = F'(x) = \dfrac{2}{(2x+1)^2}$.

110. Solution: C

The calculations are:

$$f(y \mid x = 1/3) = \frac{f(1/3, y)}{f_X(1/3)}, \quad 0 < y < \frac{2}{3}$$

$$f_X(1/3) = \int_0^{2/3} 24(1/3)ydy = 8(2/3)^2/2 = 16/9$$

$$f(y \mid x = 1/3) = \frac{8y}{16/9} = 4.5y, \quad 0 < y < \frac{2}{3}$$

$$P[Y < X \mid X = 1/3] = P[Y < 1/3 \mid X = 1/3] = \int_0^{1/3} 4.5ydy = 4.5(1/3)^2/2 = 1/4.$$

111. Solution: E

$$P[1 < Y < 3 \mid X = 2] = \int_1^3 \frac{f(2,y)}{f_X(2)}dy$$

$$f(2,y) = \frac{2}{2^2(2-1)} y^{-(4-1)/(2-1)} = 0.5y^{-3}$$

$$f_X(2) = \int_1^\infty f(2,y)dy = \int_1^\infty 0.5y^{-3}dy = -0.25y^{-2}\Big|_1^\infty = 0.25$$

$$P[1 < Y < 3 \mid X = 2] = \int_1^3 \frac{0.5y^{-3}}{0.25}dy = -y^{-2}\Big|_1^3 = -1/9 + 1 = 8/9.$$

112. Solution: D

Because only those with the basic policy can purchase the supplemental policy, $0 < y < x < 1$. Then,

$$P[Y < 0.05 \mid X = 0.10] = \int_0^{0.05} \frac{f(0.10,y)}{f_X(0.10)}dy$$

$$f(0.10,y) = 2(0.10 + y),\ 0 < y < 0.10$$

$$f_X(0.10) = \int_0^{0.10} f(0.10,y)dy = \int_0^{0.10} 2(0.10+y)dy = 0.2y + y^2\Big|_0^{0.10} = 0.03$$

$$P[Y < 0.05 \mid X = 0.10] = \int_0^{0.05} \frac{2(0.10+y)}{0.03}dy = \frac{0.2y+y^2}{0.03}\Bigg|_0^{0.05} = \frac{0.01+0.0025}{0.03} = 0.417.$$

113. Solution: E

Because the husband has survived, the only possible claim payment is to the wife. So we need the probability that the wife dies within ten years given that the husband survives. The numerator of the conditional probability is the unique event that only the husband survives, with probability 0.01. The denominator is the sum of two events, both survive (0.96) and only the husband survives (0.01). The conditional probability is 0.01/(0.96 + 0.01) = 1/97. The expected claim payment is 10,000/97 = 103 and the expected excess is 1,000 – 103 = 897.

114. Solution: C

$$P[Y = 0 \mid X = 1] = \frac{P(X=1, Y=0)}{P(X=1)} = \frac{P(X=1,Y=0)}{P(X=1,Y=0)+P(X=1,Y=1)} = \frac{0.05}{0.05+0.125} = 0.286$$

$$P[Y = 1 \mid X = 1] = 1 - 0.286 = 0.714.$$

The conditional variable is Bernoulli with $p = 0.714$. The variance is (0.714)(0.286) = 0.204.

115. Solution: A

$$f(y \mid x) = \frac{f(x,y)}{f_X(x)}$$
$$f_X(x) = \int_x^{x+1} 2x dy = 2x$$
$$f(y \mid x) = \frac{2x}{2x} = 1,\ x < y < x+1.$$

The conditional variance is uniform on the interval (x, x + 1). A uniform random variable on a unit interval has variance 1/12. Alternatively the integrals can be done to obtain the mean of x + 0.5 and second moment of $x^2 + x + 1/3$. The second moment minus the square of them mean gives the variance of 1/12.

116. Solution: D

With no tornadoes in County P the probabilities of 0, 1, 2, and 3 tornadoes in County Q are 12/25, 6/25, 5/25, and 2/25 respectively.
The mean is (0 + 6 + 10 + 6)/25 = 22/25.
The second moment is (0 + 6 + 20 + 18)/25 = 44/25.
The variance is $44/25 - (22/25)^2 = 0.9856$.

117. Solution: C

The marginal density of X is

$$f_X(x) = \int_0^{1-x} 6(1-x-y)dy = 6(y - xy - 0.5y^2)\Big|_0^{1-x} = 6[1-x-x(1-x)-0.5(1-x)^2]$$
$$= 6[1-x-x+x^2-0.5+x-0.5x^2] = 6(0.5x^2 - x + 0.5),\ 0 < x < 1.$$

The requested probability is

$$P[X < 0.2] = \int_0^{0.2} 6(0.5x^2 - x + 0.5)dx = x^3 - 3x^2 + 3x\Big|_0^{0.2} = 0.008 - 0.12 + 0.6 = 0.488.$$

118. Solution: E

$$g(y) = \int_y^{\sqrt{y}} 15y\,dx = 15yx\Big|_y^{\sqrt{y}} = 15y(\sqrt{y} - y) = 15y^{3/2}(1 - y^{1/2}),\ 0 < y < 1$$

The limits are found by noting that x must be less than the square root of y and also must be greater than y. While not directly stated, the only values of x for which the square is smaller are $0 < x < 1$. This implies y is constrained to the same range and thus its square root must be larger, ensuring that the integral has the correct sign.

119. Solution: D

The joint density is $f(x,y) = f_X(x) f(y \mid x) = 1(1) = 1,\ 0 < x < 1,\ x < y < x+1.$

The marginal density of Y is $f_Y(y) = \int_{\max(y-1,0)}^{\min(y,1)} 1dx = \min(y,1) - \max(y-1,0),\ 0 < y < 2.$ Thus from 0 to 1 the density is y and from 1 to 2 it is $2 - y$. The requested probability is

$$\int_{0.5}^{1} ydy + \int_{1}^{2} 2 - ydy = 0.5y^2\Big|_{0.5}^{1} + -0.5(2-y)^2\Big|_{1}^{2} = 0.5 - 0.125 - 0 + 0.5 = 0.875 = 7/8.$$

Because the joint density is uniform, an alternative is to find the area represented by the event $Y > 0.5$.

120. Solution: A

Let Y be the processing time. The joint density is

$$f(x,y) = f_X(x) f(y \mid x) = \frac{3x^2}{8}\frac{1}{x},\ 0 \le x \le 2,\ x \le y \le 2x.$$

The marginal density of Y is $f_Y(y) = \int_{y/2}^{\min(y,2)} \frac{3x}{8} dx = \frac{3}{16}[\min(y,2)^2 - y^2/4],\ 0 \le y \le 4.$

$$P[Y \ge 3] = \int_3^4 f_Y(y)dy = \int_3^4 \frac{3}{16}[4 - y^2/4]dy = \frac{3}{16}[4y - y^3/12]\Big|_3^4$$

$$= \frac{3}{16}[16 - 64/12 - 12 + 27/12] = 0.172.$$

121. Solution: C

The marginal density of X is given by

$$f_X(x) = \int_0^1 \frac{1}{64}\left(10 - xy^2\right)dy = \frac{1}{64}\left(10y - \frac{xy^3}{3}\right)\Bigg|_0^1 = \frac{1}{64}\left(10 - \frac{x}{3}\right)$$

Then,

$$E[X] = \int_2^{10} x f_X(x)dx = \int_2^{10} x\frac{1}{64}\left(10 - \frac{x}{3}\right)dx = \frac{1}{64}\left(\frac{10x^2}{2} - \frac{x^3}{9}\right)\Bigg|_2^{10}$$

$$= \frac{1}{64}\left(\frac{1000}{2} - \frac{1000}{9} - \frac{40}{2} + \frac{8}{9}\right) = 5.78.$$

122. Solution: D

The marginal distribution of Y is $f_Y(y) = \int_0^y 6e^{-x}e^{-2y}dx = -6e^{-x}e^{-2y}\Big|_0^y = -6e^{-3y} + 6e^{-2y}$.

The expected value of Y can be found via integration by parts, or recognition, as illustrated here.

$$E[Y] = \int_0^\infty y(-6e^{-3y} + 6e^{-2y})dy = -2\int_0^\infty y3e^{-3y}dt + 3\int_0^\infty y2e^{-2y}dt = -2(1/3) + 3(1/2) = 5/6 = 0.83.$$

With the constants factored out, the integrals are the expected value of exponential distributions with means 1/3 and 1/2, respectively.

123. Solution: C

From the Law of Total Probability:

$$\begin{aligned} P[4 < S < 8] &= P[4 < S < 8 \mid N = 1]P[N = 1] + P[4 < S < 8 \mid N > 1]P[N > 1] \\ &= (e^{-4/5} - e^{-8/5})(1/3) + (e^{-4/8} - e^{-8/8})(1/6) = 0.122. \end{aligned}$$

124. Solution: A

Because the domain is a rectangle and the density function factors, X and Y are independent. Also, Y has an exponential distribution with mean 1/2. From the memoryless property, Y given $Y > 3$ has the same exponential distribution with 3 added. Adding a constant has no effect on the variance, so the answer is the square of the mean, 0.25.

More formally, the conditional density is

$$f(y \mid Y > 3) = \frac{f(y)}{P(Y > 3)} = \frac{2e^{-2y}}{e^{-6}} = 2e^{6-2y},\ y > 3$$

$$E[Y \mid Y > 3] = \int_3^\infty y2e^{6-2y}dy = \int_0^\infty (u+3)2e^{-2u}du = 0.5 + 3 = 3.5$$

$$E[Y^2 \mid Y > 3] = \int_3^\infty y^2 2e^{6-2y}dy = \int_0^\infty (u+3)^2 2e^{-2u}du = 0.5 + 6(0.5) + 9 = 12.5$$

$$Var[Y \mid Y > 3] = 12.5 - 3.5^2 = 0.25.$$

The second integrals use the transformation $u = y - 3$ and then the integrals are moments of an exponential distribution.

125. Solution: E

The joint density is $f(x, y) = f_X(x)f(y \mid x) = 2x(1/x) = 2,\ 0 < y < x < 1$.

The marginal density of Y is $f_Y(y) = \int_y^1 2dx = 2(1 - y)$.

The conditional density is $f(x \mid y) = \dfrac{f(x, y)}{f_Y(y)} = \dfrac{2}{2(1-y)} = \dfrac{1}{1-y}$.

126. Solution: C

Due to the equal spacing of probabilities, $p_n = p_0 - nc$ for c = 1, 2, 3, 4, 5. Also, $0.4 = p_0 + p_1 = p_0 + p_0 - c = 2p_0 - c.$ Because the probabilities must sum to 1, $1 = p_0 + p_0 - c + p_0 - 2c + p_0 - 3c + p_0 - 4c + p_0 - 5c = 6p_0 - 15c.$ This provides two equations in two unknowns. Multiplying the first equation by 15 gives $6 = 30p_0 - 15c.$ Subtracting the second equation gives $5 = 24p_0 \Rightarrow p_0 = 5/24$. Inserting this in the first equation gives c = 1/60. The requested probability is $p_4 + p_5 = 5/24 - 4/60 + 5/24 - 5/60 = 32/120 = 0.267.$

127. Solution: D

Because the number of payouts (including payouts of zero when the loss is below the deductible) is large, apply the central limit theorem and assume the total payout S is normal. For one loss, the mean, second moment, and variance of the payout are

$$\int_0^{5,000} 0\frac{1}{20,000}dx + \int_{5,000}^{20,000}(x-5,000)\frac{1}{20,000} = 0 + \frac{(x-5,000)^2}{40,000}\bigg|_{5,000}^{20,000} = 5,625$$

$$\int_0^{5,000} 0^2\frac{1}{20,000}dx + \int_{5,000}^{20,000}(x-5,000)^2\frac{1}{20,000} = 0 + \frac{(x-5,000)^3}{60,000}\bigg|_{5,000}^{20,000} = 56,250,000$$

$56,250,000 - 5,625^2 = 24,609,375.$

For 200 losses the mean, variance, and standard deviation are 1,125,000, 4,921,875,000, and 70,156. 1,000,000 is 125,000/70,156 = 1.7817 standard deviations below the mean and 1,200,000 is 75,000/70,156 = 1.0690 standard deviations above the mean. From the standard normal tables, the probability of being between these values is 0.8575 – (1 – 0.9626) = 0.8201.

128. Solution: B

Let H be the percentage of clients with homeowners insurance and R be the percentage of clients with renters insurance.

Because 36% of clients do not have auto insurance and none have both homeowners and renters insurance, we calculate that 8% (36% – 17% – 11%) must have renters insurance, but not auto insurance.

(H – 11)% have both homeowners and auto insurance, (R – 8)% have both renters and auto insurance, and none have both homeowners and renters insurance, so (H + R – 19)% must equal 35%. Because H = 2R, R must be 18%, which implies that 10% have both renters and auto insurance.

129. Solution: B

Let Y be the reimbursement. Then, $G(115) = P[Y < 115 \mid X > 20]$. For Y to be 115, the costs must be above 120 (up to 120 accounts for a reimbursement of 100). The extra 15 requires 30 in additional costs. Therefore, we need

$$P[X \le 150 \mid X > 20] = \frac{P[X \le 150] - P[X \le 20]}{P[X > 20]} = \frac{1 - e^{-150/100} - 1 + e^{-20/100}}{1 - 1 + e^{-20/100}}$$

$$= \frac{-e^{-1.5} + ^{-0.2}}{e^{-0,2}} = 1 - e^{-1.3} = 0.727.$$

130. Solution: C

$$E[100(0.5)^X] = 100E[e^{X \ln 0.5}] = 100M_X(\ln 0.5) = 100\frac{1}{1 - 2\ln 0.5} = 41.9.$$

131. Solution: E

The conditional probability function given 2 claims in April is

$$p_{N_1}(2) = \sum_{n_2=1}^{\infty} \frac{3}{4}\frac{1}{4}e^{-2}(1-e^{-2})^{n_2-1} = \frac{3e^{-2}}{16}\frac{1}{1-(1-e^{-2})} = \frac{3}{16}$$

$$p(n_2 \mid N_1 = 2) = \frac{p(2, n_2)}{p_{N_1}(2)} = \frac{3}{4}\frac{1}{4}e^{-2}(1-e^{-2})^{n_2-1}\frac{16}{3} = e^{-2}(1-e^{-2})^{n_2-1}.$$

This can be recognized as a geometric probability function and so the mean is $1/e^{-2} = e^2$.

132. Solution: C

The number of defective modems is 20% x 30 + 8% x 50 = 10.

The probability that exactly two of a random sample of five are defective is $\dfrac{\dbinom{10}{2}\dbinom{70}{3}}{\dbinom{80}{5}} = 0.102$.

133. Solution: B

P(40 year old man dies before age 50) = P(T < 50 | T > 40)

$$= \frac{\Pr(40 < T < 50)}{\Pr(T > 40)} = \frac{F(50) - F(40)}{1 - F(40)}$$

$$= \frac{1 - \exp\left(\frac{1-1.1^{50}}{1000}\right) - 1 + \exp\left(\frac{1-1.1^{40}}{1000}\right)}{1 - 1 + \exp\left(\frac{1-1.1^{40}}{1000}\right)} = \frac{\exp\left(\frac{1-1.1^{40}}{1000}\right) - \exp\left(\frac{1-1.1^{50}}{1000}\right)}{\exp\left(\frac{1-1.1^{40}}{1000}\right)}$$

$$= \frac{0.9567 - 0.8901}{0.9567} = 0.0696$$

Expected Benefit = 5000(0.0696) = 348.

134. Solution: C

Letting t denote the relative frequency with which twin-sized mattresses are sold, we have that the relative frequency with which king-sized mattresses are sold is 3t and the relative frequency with which queen-sized mattresses are sold is (3t+t)/4, or t. Thus, t = 0.2 since t + 3t + t = 1. The probability we seek is 3t + t = 0.80.

135. Solution: E

$Var(N) = E[Var(N \mid \lambda)] + Var[E(N \mid \lambda)] = E[\lambda] + Var(\lambda) = 1.5 + 0.75 = 2.25.$ The variance of a uniform random variable is the square of the range divided by 12, in this case $3^2/12 = 0.75$.

136. Solution: D

X follows a geometric distribution with p = 1/6 and Y = 2 implies the first roll is not a 6 and the second roll is a 6. This means a 5 is obtained for the first time on the first roll (probability = 0.2) or a 5 is obtained for the first time on the third or later roll (probability = 0.8).

$E[X \mid X \geq 3] = \frac{1}{p} + 2 = 6 + 2 = 8.$, The expected value is 0.2(1) + 0.8(8) = 6.6.

137. Solution: E

Because X and Y are independent and identically distributed, the moment generating function of $X + Y$ equals $K^2(t)$, where $K(t)$ is the moment generating function common to X and Y. Thus, $K(t) = 0.3e^{-t} + 0.4 + 0.3e^{t}$. This is the moment generating function of a discrete random variable that assumes the values –1, 0, and 1 with respective probabilities 0.3, 0.4, and 0.3. The value we seek is thus 0.3 + 0.4 = 0.7.

138. Solution: D

Suppose the component represented by the random variable X fails last. This is represented by the triangle with vertices at (0, 0), (10, 0) and (5, 5). Because the density is uniform over this region, the mean value of X and thus the expected operational time of the machine is 5. By symmetry, if the component represented by the random variable Y fails last, the expected operational time of the machine is also 5. Thus, the unconditional expected operational time of the machine must be 5 as well.

139. Solution: B

The unconditional probabilities for the number of people in the car who are hospitalized are 0.49, 0.42 and 0.09 for 0, 1 and 2, respectively. If the number of people hospitalized is 0 or 1, then the total loss will be less than 1. However, if two people are hospitalized, the probability that the total loss will be less than 1 is 0.5. Thus, the expected number of people in the car who are hospitalized, given that the total loss due to hospitalizations from the accident is less than 1 is

$$\frac{0.49}{0.49+0.42+0.09\cdot 0.5}(0)+\frac{0.42}{0.49+0.42+0.09\cdot 0.5}(1)+\frac{0.09\cdot 0.5}{0.49+0.42+0.09\cdot 0.5}(2)=0.534$$

140. Solution: B

Let X equal the number of hurricanes it takes for two losses to occur. Then X is negative binomial with "success" probability $p = 0.4$ and $r = 2$ "successes" needed.

$$P[X=n]=\binom{n-1}{r-1}p^r(1-p)^{n-r}=\binom{n-1}{2-1}(0.4)^2(1-0.4)^{n-2}=(n-1)(0.4)^2(0.6)^{n-2}, \text{ for } n \geq 2.$$

We need to maximize $P[X = n]$. Note that the ratio

$$\frac{P[X=n+1]}{P[X=n]}=\frac{n(0.4)^2(0.6)^{n-1}}{(n-1)(0.4)^2(0.6)^{n-2}}=\frac{n}{n-1}(0.6).$$

This ratio of "consecutive" probabilities is greater than 1 when $n = 2$ and less than 1 when $n \geq 3$. Thus, $P[X = n]$ is maximized at $n = 3$; the mode is 3. Alternatively, the first few probabilities could be calculated.

141. Solution: C

There are 10 (5 choose 3) ways to select the three columns in which the three items will appear. The row of the rightmost selected item can be chosen in any of six ways, the row of the leftmost selected item can then be chosen in any of five ways, and the row of the middle selected item can then be chosen in any of four ways. The answer is thus (10)(6)(5)(4) = 1200. Alternatively, there are 30 ways to select the first item. Because there are 10 squares in the row or column of the first selected item, there are 30 − 10 = 20 ways to select the second item. Because there are 18 squares in the rows or columns of the first and second selected items, there are 30 − 18 = 12 ways to select the third item. The number of permutations of three qualifying items is (30)(20)(12). The number of combinations is thus (30)(20)(12)/3! = 1200.

142. Solution: B

The expected bonus for a high-risk driver is 0.8(12)(5) = 48.
The expected bonus for a low-risk driver is 0.9(12)(5) = 54.
The expected bonus payment from the insurer is 600(48) + 400(54) = 50,400.

143. Solution: E

Liability but not property = 0.01 (given)
Liability and property = 0.04 – 0.01 = 0.03.
Property but not liability = 0.10 – 0.03 = 0.07
Probability of neither = 1 – 0.01 – 0.03 – 0.07 = 0.89

144. Solution: E

The total time is to e less than 60 minutes, so if x minutes are spent in the waiting room (in the range 0 to 60), from 0 to 60 − x minutes are spent in the meeting itself.

145. Solution: C

$$f(y \mid x = 0.75) = \frac{f(0.75, y)}{\int_0^1 f(0.75, y)dy} = \frac{f(0.75, y)}{\int_0^{0.5} 1.5dy + \int_{0.5}^1 0.75dy} = \frac{f(0.75, y)}{1.125}..$$

Thus,

$$f(y \mid x = 0.75) = \begin{cases} 1.50/1.125 = 4/3 & \text{for } 0 < y < 0.5 \\ 0.75/1.125 = 2/3 & \text{for } 0.5 < y < 1 \end{cases},$$

Then,

$$E(Y \mid X = 0.75) = \int_0^{0.5} y(4/3)\,\mathrm{dy} + \int_{0.5}^1 y(2/3)\,\mathrm{dy} = (1/8)(4/3) + (3/8)(2/3) = 5/12$$

$$E(Y^2 \mid X = 0.75) = \int_0^{0.5} y^2(4/3)\,\mathrm{dy} + \int_{0.5}^1 y^2(2/3)\,\mathrm{dy} = (1/24)(4/3) + (7/24)(2/3) = 18/72$$

$$Var(Y \mid X = 0.75) = 18/72 - (5/12)^2 = 11/144 = 0.076.$$

146. Solution: B

C = the set of TV watchers who watched CBS over the last year
N = the set of TV watchers who watched NBC over the last year
A = the set of TV watchers who watched ABC over the last year
H = the set of TV watchers who watched HGTV over the last year

The number of TV watchers in the set $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A}$ is 34 + 15 + 10 – 7 – 6 – 5 + 4 = 45.

Because $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A}$ and H are mutually exclusive, the number of TV watchers in the set $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A} \cup \mathrm{H}$ is 45 + 18 = 63.

The number of TV watchers in the complement of $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A} \cup \mathrm{H}$ is thus 100 – 63 = 37.

147. Solution: A

Let X denote the amount of a claim before application of the deductible. Let Y denote the amount of a claim payment after application of the deductible. Let λ be the mean of X, which because X is exponential, implies that λ^2 is the variance of X and $E(X^2) = 2\lambda^2$.

By the memoryless property of the exponential distribution, the conditional distribution of the portion of a claim above the deductible given that the claim exceeds the deductible is an exponential distribution with mean λ. Given that $E(Y) = 0.9\lambda$, this implies that the probability of a claim exceeding the deductible is 0.9 and thus $E(Y^2) = 0.9(2\lambda^2) = 1.8\lambda^2$. Then, $\mathrm{Var}(Y) = 1.8\lambda^2 - (0.9\lambda)^2 = 0.99\lambda^2$. The percentage reduction is 1%.

148. Solution: C

Let N denote the number of hurricanes, which is Poisson distributed with mean and variance 4.

Let X_i denote the loss due to the i^{th} hurricane, which is exponentially distributed with mean 1,000 and therefore variance $(1{,}000)^2 = 1{,}000{,}000$.

Let X denote the total loss due to the N hurricanes.

This problem can be solved using the conditional variance formula. Note that independence is used to write the variance of a sum as the sum of the variances.

$$\begin{aligned}
\mathrm{Var}(X) &= Var[E(X \mid N)] + E[Var(X \mid N)] \\
&= Var[E(X_1 + \cdots + X_N)] + E[Var(X_1 + \cdots + X_N)] \\
&= Var[NE(X_1)] + E[NVar(X_1)] \\
&= Var(1{,}000N) + E(1{,}000{,}000N) \\
&= 1{,}000^2 Var(N) + 1{,}000{,}000E(N) \\
&= 1{,}000{,}000(4) + 1{,}000{,}000(4) = 8{,}000{,}000.
\end{aligned}$$

149. Solution: B

Let N denote the number of accidents, which is binomial with parameters 3 and 0.25 and thus has mean 3(0.25) = 0.75 and variance 3(0.25)(0.75) = 0.5625.

Let X_i denote the unreimbursed loss due to the i^{th} accident, which is 0.3 times an exponentially distributed random variable with mean 0.8 and therefore variance $(0.8)^2 = 0.64$. Thus, X_i has mean 0.8(0.3) = 0.24 and variance $0.64(0.3)^2 = 0.0576$.

Let X denote the total unreimbursed loss due to the N accidents.

This problem can be solved using the conditional variance formula. Note that independence is used to write the variance of a sum as the sum of the variances.

$$\begin{aligned}
\mathrm{Var}(X) &= Var[E(X \mid N)] + E[Var(X \mid N)] \\
&= Var[E(X_1 + \cdots + X_N)] + E[Var(X_1 + \cdots + X_N)] \\
&= Var[NE(X_1)] + E[NVar(X_1)] \\
&= Var(0.24N) + E(0.0576N) \\
&= 0.24^2 Var(N) + 0.0576E(N) \\
&= 0.0576(0.5625) + 0.0576(0.75) = 0.0756.
\end{aligned}$$

150. Solution: B

The 95th percentile is in the range when an accident occurs. It is the 75th percentile of the payout, given that an accident occurs, because (0.95 – 0.80)/(1 – 0.80) = 0.75. Letting x be the 75th percentile of the given exponential distribution, $F(x) = 1 - e^{-\frac{x}{3000}} = 0.75$, so x = 4159. Subtracting the deductible of 500 gives 3659 as the (unconditional) 95th percentile of the insurance company payout.

151. Solution: C

The ratio of the probability that one of the damaged pieces is insured to the probability that none of the damaged pieces are insured is

$$\frac{\dfrac{\binom{r}{1}\binom{27-r}{3}}{\binom{27}{4}}}{\dfrac{\binom{r}{0}\binom{27-r}{4}}{\binom{27}{4}}} = \frac{4r}{24-r},$$

where *r* is the total number of pieces insured. Setting this ratio equal to 2 and solving yields *r* = 8.

The probability that two of the damaged pieces are insured is

$$\frac{\binom{r}{2}\binom{27-r}{2}}{\binom{27}{4}} = \frac{\binom{8}{2}\binom{19}{2}}{\binom{27}{4}} = \frac{(8)(7)(19)(18)(4)(3)(2)(1)}{(27)(26)(25)(24)(2)(1)(2)(1)} = \frac{266}{975} = 0.27.$$

152. Solution: A

The probability that Rahul examines exactly *n* policies is $0.1(0.9)^{n-1}$. The probability that Toby examines more than *n* policies is 0.8^n. The required probability is thus

$$\sum_{n=1}^{\infty} 0.1(0.9)^{n-1}(0.8)^n = \frac{1}{9}\sum_{n=1}^{\infty} 0.72^n = \frac{0.72}{9(1-0.72)} = 0.2857.$$

An alternative solution begins by imagining Rahul and Toby examine policies simultaneously until at least one of the finds a claim. At each examination there are four possible outcomes:

1. Both find a claim. The probability is 0.02.
2. Rahul finds a claim and Toby does not. The probability is 0.08.
3. Toby finds a claim and Rahul does not. The probability is 0.18
4. Neither finds a claim. The probability is 0.72.

Conditioning on the examination at which the process ends, the probability that it ends with Rahul being the first to find a claim (and hence needing to examine fewer policies) is 0.08/(0.02 + 0.08 + 0.18) = 8/28 = 0.2857.

153. Solution: E

Let a be the mean and variance of X and b be the mean and variance of Y. The two facts are $a = b - 8$ and $a + a^2 = 0.6(b + b^2)$. Substituting the first equation into the second gives

$$b-8+(b-8)^2 = 0.6b+0.6b^2$$

$$b-8+b^2-16b+64 = 0.6b+0.6b^2$$

$$0.4b^2-15.6b+56 = 0$$

$$b = \frac{15.6 \pm \sqrt{15.6^2-4(0.4)(56)}}{2(0.4)} = \frac{15.6 \pm 12.4}{0.8} = 4 \text{ or } 35.$$

At $b = 4$, a is negative, so the answer is 35.

154. Solution: C

Suppose there are N red sectors. Let w be the probability of a player winning the game. Then, w = the probability of a player missing all the red sectors and

$$w = 1-\left[\frac{9}{20}+\left(\frac{9}{20}\right)^2+\cdots+\left(\frac{9}{20}\right)^N\right]$$

Using the geometric series formula,

$$w = 1-\frac{\frac{9}{20}-\left(\frac{9}{20}\right)^{N+1}}{1-\frac{9}{20}} = 1-\frac{9}{20}\,\frac{1-\left(\frac{9}{20}\right)^N}{1-\frac{9}{20}} = \frac{2}{11}+\frac{9}{11}\left(\frac{9}{20}\right)^N$$

Thus we need

$$0.2 > w = \frac{2}{11}+\frac{9}{11}\left(\frac{9}{20}\right)^N$$

$$2.2 > 2+9\left(\frac{9}{20}\right)^N$$

$$0.2 > 9\left(\frac{9}{20}\right)^N$$

$$\frac{2}{90} > \left(\frac{9}{20}\right)^N$$

$$\left(\frac{20}{9}\right)^N > 45$$

$$N > \frac{\ln(45)}{\ln(20/9)} \approx 4.767$$

Thus N must be the first integer greater than 4.767, or 5.

155. Solution: B

The fourth moment of X is

$$\int_0^{10} \frac{x^4}{10} dx = \left.\frac{x^5}{50}\right|_0^{10} = 2000.$$

The Y probabilities are 1/20 for Y = 0 and 10, and 1/10 for Y = 1,2,…,9.

$$E[Y^4] = (1^4 + 2^4 + \cdots + 9^4)/10 + 10^4/20 = 2033.3.$$

The absolute value of the difference is 33.3.

156. Solution: E

$$P(x=1, y=1) = P(y=1 \mid x=1)P(x=1) = 0.3(0.5)^2 = 0.075$$
$$P(x=2, y=0) = P(y=0 \mid x=2)P(x=2) = 0.25(0.5)^3 = 0.03125$$
$$P(x=0, y=2) = P(y=2 \mid x=0)P(x=0) = 0.05(0.5)^1 = 0.025$$

The total is 0.13125.

157. Solution: C

$$E(X) = \int_1^{\infty} x \frac{p-1}{x^p} dx = (p-1)\int_1^{\infty} x^{1-p} dx$$
$$(p-1)\left.\frac{x^{2-p}}{2-p}\right|_1^{\infty} = \frac{p-1}{p-2} = 2$$
$$p-1 = 2(p-2) = 2p-4$$
$$p = 3$$

158. Solution: D

The distribution function plot shows that X has a point mass at 0 with probability 0.5. From 2 to 3 it has a continuous distribution. The density function is the derivative, which is the constant (1 – 0.5)/(3 – 2) = 0.5. The expected value is 0(0.5) plus the integral from 2 to 3 of 0.5x. The integral evaluates to 1.25, which is the answer. Alternatively, this is a 50-50 mixture of a point mass at 0 and a uniform(2,3) distribution. The mean is 0.5(0) + 0.5(2.5) = 1.25.

159. Solution: E

The dice rolls that satisfy this event are (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (4,6), (5,3), (5,4), (5,5), (5,6), (6,4), (6,5), and (6,6). They represent 24 of the 36 equally likely outcomes for a probability of 2/3.

160. Solution: D

$$0.64 = \rho = \frac{Cov(M,N)}{\sqrt{Var(M)Var(N)}}$$
$$Cov(M,N) = 0.64\sqrt{1600(900)} = 768$$
$$Var(M+N) = Var(M) + Var(N) + 2Cov(M,N) = 1600 + 900 + 2(768) = 4036$$

161. Solution: C

$$\int_1^6 (x-1)0.5e^{-0,5x}dx + \int_6^\infty 5(0.5)e^{-0.5x}dx = -(x-1)e^{-0.5x}\Big|_1^6 + \int_1^6 e^{-0.5x}dx - 5e^{-0.5x}\Big|_6^\infty$$
$$= -5e^{-3} + 0 - 2e^{-0.5x}\Big|_1^6 + 5e^{-3} = -2e^{-3} + 2e^{-1/2}$$

162. Solution: A

First, observe that
$Var[(X+Y)/2] = (0.5)^2 Var(X+Y) = 0.25[Var(X) + Var(Y) + 2Cov(X,Y)]$.
Then,
$$E(X) = \int_0^2\int_0^2 x\frac{x+y}{8}dxdy = \frac{1}{8}\int_0^2 \frac{8}{3} + 2ydy = \frac{1}{8}\left(\frac{16}{3}+4\right) = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$
$$E(X^2) = \int_0^2\int_0^2 x^2\frac{x+y}{8}dxdy = \frac{1}{8}\int_0^2 4 + \frac{8}{3}ydy = \frac{1}{8}\left(8+\frac{16}{3}\right) = 1 + \frac{4}{6} = \frac{10}{6}$$
$Var(X) = 10/6 - (7/6)^2 = 11/36.$
By symmetry, the mean and the variance of Y are the same. Next,
$$E(XY) = \int_0^2\int_0^2 xy\frac{x+y}{8}dxdy = \frac{1}{8}\int_0^2 \frac{8}{3}y + 2y^2dy = \frac{1}{8}\left(\frac{16}{3}+\frac{16}{3}\right) = \frac{8}{6},$$
$Cov(X,Y) = 8/6 - (7/6)(7/6) = -1/36.$
Finally,
$Var(X+Y) = 0.25[11/36 + 11/36 + 2(-1/36)] = 5/36 = 10/72.$

163. Solution: A

Let C be the number correct. C has a binomial distribution with n = 40 and p = 0.5. Then the mean is 40(0.5) = 20 and the variance is 40(0.5)(0.5) = 10. With the exact probability we have
$$0.1 = P(C > N) = \Pr\left(Z > \frac{N + 0.5 - 20}{\sqrt{10}}\right)$$
$$1.282 = \frac{N + 0.5 - 20}{\sqrt{10}}, \quad N = 1.282\sqrt{10} + 19.5 = 23.55.$$
At N = 23 the approximate probability will exceed 0.1 (Z = 1.107).

164. Solution: B

The months in question have 1, 1, 0.5, 0.5, and 0.5 respectively for their means. The sum of independent Poisson random variables is also Poisson, with the parameters added. So the total number of accidents is Poisson with mean 3.5. The probability of two accidents is

$$\frac{e^{-3.5}3.5^2}{2!} = 0.185.$$

165. Solution: B

For either distribution the moments can be found from

$$M(t) = (1-1.5t)^{-2}$$

$$M'(t) = 2(1.5)(1-1.5t)^{-3} = 3(1-1.5t)^{-3}$$

$$M''(t) = 3(3)(1.5)(1-1.5t)^{-4} = 13.5(1-1.5t)^{-4}$$

$$E(X) = E(Y) = M'(0) = 3$$

$$E(X^2) = E(Y^2) = M''(0) = 13.5$$

$$Var(X) = Var(Y) = 13.5 - 3^2 = 4.5$$

$$Var(X+Y) = Var(X) + Var(Y) = 4.5 + 4.5 = 9.$$

The standard deviation is the square root, 3.

If it is recognized that this is the moment generating function of the gamma distribution, then the parameters (1.5 and 2) and the moments can be obtained without calculations as 1.5(2) = 3 and 1.5(1.5)(2) = 4.5.

166. Solution: B

The payments are 0 with probability 0.72 (snowfall up to 50 inches), 300 with probability 0.14, 600 with probability 0.06, and 700 with probability 0.08. The mean is 0.72(0) + 0.14(300) + 0.06(600) + 0.08(700) = 134 and the second moment is 0.72(0^2) + 0.14(300^2) + 0.06(600^2) + 0.08(700^2) = 73,400. The variance is 73,400 – 134^2 = 55,444. The standard deviation is the square root, 235.

167. Solution: D

$$1 = \int_0^{20} c(x^2 - 60x + 800)dx = c(x^3/3 - 30x^2 + 800x)\Big|_0^{20} = c20,000/3 \Rightarrow c = 3/20,000$$

$$P(X > d) = \int_d^{20} c(x^2 - 60x + 800)dx = c(x^3/3 - 30x^2 + 800x)\Big|_d^{20} = 1 - \frac{3}{20,000}(d^3/3 - 30d^2 + 800d)$$

$$P(X > 10 \mid X > 2) = \frac{P(X > 10)}{P(X > 2)} = \frac{0.2}{0.7776} = 0.2572$$

168. Solution: A

Each event has probability 0.5. Each of the three possible intersections of two events has probability 0.25 = (0.5)(0.5), so each pair is independent. The intersection of all three events has probability 0, which does not equal (0.5)(0.5)(0.5) and so the three events are not mutually independent.

169. Solution: C

Let event A be the selection of the die with faces (1,2,3,4,5,6), event B be the selection of the die with faces (2,2,4,4,6,6) and event C be the selection of the die with all 6's. The desired probability is, using the law of total probability,

$$P(6,6) = P(6,6 \mid A)P(A) + P(6,6 \mid B)P(B) + P(6,6 \mid C)P(C)$$

$$= (1/36)(1/2) + (1/9)(1/4) + 1(1/4) = 1/72 + 2/72 + 18/72 = 21/72 = 0.292.$$

170. Solution: D

$$\frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{\binom{12}{6}} = \frac{15(6)(1)}{924} = 0.097$$

171. Solution: A

The random variable has a uniform distribution on a diamond with vertices at (0,-1), (-1,0), (0,1), and (1,0). The marginal density of X is the joint density (1/2) times the length of the segment from the bottom to the top of the triangle for the given value of x. For positive x the bottom is at $x - 1$ and the top is at $1 - x$. The length is $1 - x - (x - 1) = 2 - 2x$. Multiplying by (1/2) gives the density as $1 - x$. By symmetry, for negative x, the density is $1 + x$. Also by symmetry, the mean is zero. The variance is then

$$\int_{-1}^{0} x^2(1+x)dx + \int_{0}^{1} x^2(1-x)dx = x^3/3 + x^4/4\Big|_{-1}^{0} + x^3/3 - x^4/4\Big|_{0}^{1} = 1/12 + 1/12 = 1/6.$$

172. Solution: D

Consider the three cases based on the number of claims.
If there are no claims, the probability of total benefits being 48 or less is 1.
If there is one claim, the probability is 48/60 = 0.8, from the uniform distribution.
If there are two claims, the density is uniform on a 60x60 square. The event where the total is 48 or less is represented by a triangle with base and height equal to 48. The triangle's area is 48x48/2 = 1152. Dividing by the area of the square, the probability is 1152/3600 = 0.32.

Using the law of total probability, the answer is 0.7(1) + 0.2(0.8) + 0.1(0.32) = 0.892.

173. Solution: B

The sum of independent Poisson variables is also Poisson, with the means added. Thus the number of tornadoes in a three week period is Poisson with a mean of 3x2 = 6. Then,

$$P(N<4)=p(0)+p(1)+p(2)+p(3)=e^{-6}\left(\frac{6^0}{0!}+\frac{6^1}{1!}+\frac{6^2}{2!}+\frac{6^3}{3!}\right)=0.1512.$$

174. Solution: A

The number of components that fail has a binomial(2, 0.05) distribution. Then,

$$P(N\ge 2)=p(2)+p(3)=\binom{3}{2}(0.05)^2(0.95)+\binom{3}{3}(0.05)^3=0.00725.$$

175. Solution: E

The profit variable *X* is normal with mean 100 and standard deviation 20. Then,

$$P(X\le 60\mid X>0)=\frac{P(0<X\le 60)}{P(X>0)}=\frac{P\left(\frac{0-100}{20}<Z\le\frac{60-100}{20}\right)}{P\left(Z>\frac{0-100}{20}\right)}=\frac{F(-2)-F(-5)}{1-F(-5)}.$$

For the normal distribution, $F(-x) = 1 - F(x)$ and so the answer can be rewritten as $[1 - F(2) - 1 + F(5)]/[1 - 1 + F(5)] = [F(5) - F(2)]/F(5)$.

176. Solution: A

Let *B* be the event that the policyholder has high blood pressure and *C* be the event that the policyholder has high cholesterol. We are given $P(B) = 0.2$, $P(C) = 0.3$, and $P(C \mid B) = 0.25$. Then,

$$P(B\mid C)=\frac{P(B\cap C)}{P(C)}=\frac{P(C\mid B)P(B)}{P(C)}=\frac{0.25(0.2)}{0.3}=1/6.$$

177. Solution: D

This is a hypergeometric probability,

$$\frac{\binom{20}{1}\binom{5}{1}}{\binom{25}{2}}=\frac{20(5)}{25(24)/2}=\frac{100}{300}=0.333\ ,$$

Alternatively, the probability of the first worker being high risk and the second low risk is (5/25)(20/24) = 100/600 and of the first being low risk and the second high risk is (20/25)(5/24) = 100/600 for a total probability of 200/600 = 0.333.

178. Solution: C

$$E\left(\frac{X}{1-X}\right)=60\int_0^1\frac{x}{1-x}x^3(1-x)^2\,dx=60\int_0^1 x^4(1-x)dx=60(x^5/5-x^6/6)\Big|_0^1=60(1/5-1/6)=2$$

$$E\left[\left(\frac{X}{1-X}\right)^2\right]=60\int_0^1\frac{x^2}{(1-x)^2}x^3(1-x)^2\,dx=60\int_0^1 x^5dx=60(x^6/6)\Big|_0^1=60(1/6)=10$$

$$Var\left(\frac{X}{1-X}\right)=10-2^2=6$$

179. Solution: B

P(at least one emergency room visit **or** at least one hospital stay) = 1 – 0.61 = 0.39 = *P*(at least one emergency room visit) + *P*(at least one hospital stay) – *P*(at least one emergency room visit **and** at least one hospital stay).

P(at least one emergency room visit **and** at least one hospital stay) = 1 – 0.70 + 1 – 0.85 – 0.39 = 0.060.

180. Solution: A

Let Y be the loss and X be the reimbursement. If the loss is less than 4, $P(X\le x)=P(Y\le x)=0.2x$ for $x<4$ because Y has a uniform distribution on [0, 5]. However, the probability of the reimbursement being less than or equal to 4 is 1 because 4 is the maximum reimbursement.

181. Solution: B

The number of males is 0.54(900) = 486 and of females is then 414.
The number of females over age 25 is 0.43(414) = 178.
The number over age 25 is 395. Therefore the number under age 25 is 505. The number of females under age 25 is 414 – 178 = 236. Therefore the number of males under 25 is 505 – 236 = 269 and the probability is 269/505 =0.533.

182. Solution: C

Let R be the event the car is red and G be the event the car is green. Let E be the event that the claim exceeds the deductible. Then,

$$P(R\,|\,E)=\frac{P(R)P(E\,|\,R)}{P(R)P(E\,|\,R)+P(\mathrm{G})P(E\,|\,\mathrm{G})}=\frac{0.3(0.09)}{0.3(0.09)+0.7(0.04)}=\frac{0.027}{0.055}=0.491.$$

Note that if A is the probability of an accident,
$P(E\,|\,R)=P(E\,|\,R\text{ and }A)P(A\,|\,R)=0.1(0.9)=0.09.$

183. Solution: A

$$F(t) = P(T \le t) = P(X^2 \le t) = P\left(-\sqrt{t} \le X \le \sqrt{t}\right)$$
$$= \int_{-\sqrt{t}}^{\sqrt{t}} f(x)dx = \int_{-\sqrt{t}}^{0} 2e^{4x}dx + \int_0^{\sqrt{t}} e^{-2x} = 0.5e^{4x}\Big|_{-\sqrt{t}}^{0} - 0.5e^{-2x}\Big|_0^{\sqrt{t}} = 0.5 - 0.5e^{-4\sqrt{t}} - 0.5e^{-2\sqrt{t}} + 0.5$$
$$= 1 - 0.5e^{-4\sqrt{t}} - 0.5e^{-2\sqrt{t}}$$
$$f(t) = F'(t) = -0.5e^{-4\sqrt{t}}[-4(0.5)/\sqrt{t}] - 0.5e^{-2\sqrt{t}}[-2(0.5)/\sqrt{t}] = e^{-4\sqrt{t}}/\sqrt{t} + 0.5e^{-2\sqrt{t}}/\sqrt{t}$$
$$= \frac{e^{-2\sqrt{t}}}{2\sqrt{t}} + \frac{e^{-4\sqrt{t}}}{\sqrt{t}}$$

184. Solution: B

Let X and Y be the selected numbers. The probability Paul wins is $P(|X-Y| \le 3)$. Of the 400 pairs, it is easiest to count the number of outcomes that satisfy this event:
If X = 1, then Y can be 1, 2, 3, or 4 (4 total)
If X = 2, then Y can be 1, 2, 3, 4, or 5 (5 total)
For X = 3 there are 6, and for X = 4 through 17 there are 7. For X = 18, 19, and 20 the counts are 6, 5, and 4 respectively. The total is then 4 + 5 + 6 + 14(7) + 6 + 5 + 4 = 128. The probability is 128/400 = 0.32.

185. Solution: C

Let C and K denote respectively the event that the student answers the question correctly and the event that he actually knows the answer. The known probabilities are
$P(C \mid K^c) = 0.5, \quad P(C \mid K) = 1, \quad P(K \mid C) = 0.824, \quad P(K) = N/20.$ Then,
$$0.824 = P(K \mid C) = \frac{P(C \mid K)P(K)}{P(C \mid K)P(K) + P(C \mid K^c)P(K^c)} = \frac{1(N/20)}{1(N/20) + 0.5(20-N)/20} = \frac{N}{N + 0.5(20-N)}$$
$$0.824(0.5N + 10) = N$$
$$8.24 = 0.588N$$
$$N = 14.$$

186. Solution: D

The probability that a randomly selected cable will not break under a force of 12,400 is $P(Y > 12,400) = P[Z > (12,400 - 12,432)/25 = -1.28] = 0.9.$ The number of cables, N, that will not break has the binomial distribution with n = 400 and p = 0.9. This can be approximated by a normal distribution with mean 360 and standard deviation 6. With the continuity correction, $P(N \ge 349) = P[Z \ge (348.5 - 360)/6 = -1.9167] = 0.97.$

187. Solution: D

Because the mode is 2 and 3, the parameter is 3 (when the parameter is a whole number the probabilities at the parameter and at one less than the parameter are always equal). Alternatively, the equation $p(2) = p(3)$ can be solved for the parameter. Then the probability of selling 4 or fewer policies is 0.815 and this is the first such probability that exceeds 0.75. Thus, 4 is the first number for which the probability of selling more than that number of policies is less than 0.25.

188. Solution: E

Of the 36 possible pairs, there are a total of 15 that have the red number larger than the green number. Note that a list is not needed. There are 6 that have equal numbers showing and of the remaining 30 one-half must have red larger than green. Of these 15, 9 have an odd sum for the answer, 9/15 = 3/5. This is best done by counting, with 3 combinations adding to 7, 2 combinations each totaling 5 and 9, and 1 combination each totaling 3 and 11.

189. Solution: B

From the table the 93rd percentile comes from a z-score between 1.47 and 1.56. 1.47 implies a test score of 503 + 1.47(98) = 647.1. Similarly, 1.56 implies a score of 655.9. The only multiple of 10 in this range is 650. Abby's z-score is then (650 – 521)/101 = 1.277. This is at the 90th percentile of the standard normal distribution.

190. Solution: C

Let X, Y, and Z be the three lifetimes. We want
$P(X+Y>1.9Z) = P(W = X+Y-1.9Z>0)$.
A linear combination of independent normal variables is also normal. In this case W has a mean of 10 + 10 – 1.9(10) = 1 and a variance of 9 + 9 + 1.9(1.9)(9) = 50.49 for a standard deviation of 7.106.
Then the desired probability is that a standard normal variable exceeds (0 – 1)/7.106 = –0.141. Interpolating in the normal tables gives 0.5557 + (0.5596 – 0.5557)(0.1) = 0.5561, which rounds to 0.556.

191. Solution: B

$$57 = Var(X \mid Y = 28.5) = (1-\rho^2)Var(X) = (1-\rho^2)76$$
$$1-\rho^2 = 57/76 = 0.75$$
$$Var(Y \mid X = 25) = (1-\rho^2)Var(Y) = 0.75(32) = 24$$

192. Solution: C

We have

$$0.3 = P\left[\text{insurer must pay at least } 1.2\right] = P\left[\text{loss} \geq 1.2 + d\right] = \frac{2 - 1.2 - d}{2 - 0} = \frac{0.8 - d}{2}$$

$d = 0.8 - 2(0.3) = 0.2.$

Then,

$$P\left[\text{insurer must pay at least } 1.44\right] = P\left[\text{loss} \geq 1.44 + d\right] = \frac{2 - 1.44 - 0.2}{2 - 0} = 0.18.$$

193. Solution: E

The cumulative distribution function for the exponential distribution of the lifespan is $F(x) = 1 - e^{-\lambda x}$, for positive *x*.

The probability that the lifespan exceeds 4 years is $0.3 = 1 - F(4) = e^{-4\lambda}$. Thus $\lambda = -(\ln 0.3)/4$.

For positive *x*, the probability density function is

$$f(x) = \lambda e^{-\lambda x} = -\frac{\ln 0.3}{4} \mathrm{e}^{(\ln 0.3)x/4} = -\frac{\ln 0.3}{4}(0.3)^{x/4}.$$

194. Solution: C

It is not necessary to determine the constant of proportionality. Let it be *c*. To determine the mode, set the derivative of the density function equal to zero and solve.

$$0 = f'(x) = \frac{d}{dx} cx^2(1+x^3)^{-1} = 2cx(1+x^3)^{-1} + cx^2[-(1+x^3)^{-2}]3x^2$$

$$= 2cx(1+x^3) - 3cx^4 \quad \text{(multiplying by } (1+x^3)^2\text{)}$$

$$= 2cx + 2cx^4 - 3cx^4 = 2cx - cx^4$$

$$= 2 - x^3 \Rightarrow x = 2^{1/3} = 1.26.$$

195. Solution: C

It is not necessary to determine the constant of proportionality. Let it be *c*. To determine the mode, set the derivative of the density function equal to zero and solve.

$$0 = f'(x) = \frac{d}{dx} cxe^{-x^2} = ce^{-x^2} - cx(2x)e^{-x^2} = ce^{-x^2}(1 - 2x^2)$$

$$= 1 - 2x^2 \quad \text{(multiplying by } \mathrm{c}e^{x^2}\text{)}$$

$$\Rightarrow x = (1/2)^{1/2} = 0.71.$$

196. Solution: E

A geometric probability distribution with mean 1.5 will have p = 2/3. So Pr(1 visit) = 2/3, P(two visits) = 2/9, etc. There are four disjoint scenarios in which total admissions will be two or less.

Scenario 1: No employees have hospital admissions. Probability = $0.8^5 = 0.32768$.

Scenario 2: One employee has one admission and the other employees have none. Probability =
$\binom{5}{1}(0.2)(0.8)^4(2/3) = 0.27307$.

Scenario 3: One employee has two admissions and the other employees have none. Probability =
$\binom{5}{1}(0.2)(0.8)^4(2/9) = 0.09102$.

Scenario 4: Two employees each have one admission and the other three employees have none.
Probability = $\binom{5}{2}(0.2)^2(0.8)^3(2/3)(2/3) = 0.09102$.

The total probability is 0.78279.

197. Solution: C

The intersection of the two events (third malfunction on the fifth day and not three malfunctions on first three days) is the same as the first of those events. So the numerator of the conditional probability is the negative binomial probability of the third success (malfunction) on the fifth day, which is
$\binom{4}{2}(0.4)^2(0.6)^2(0.4) = 0.13824$.
The denominator is the probability of not having three malfunctions in three days, which is
$1-(0.4)^3 = 0.936$.
The conditional probability is 0.13824/0.936 = 0.1477.

198. Solution: C

Let p_i represent the probability that the patient's cancer is in stage i, for i = 0, 1, 2, 3, or 4. The probabilities must sum to 1. That fact and the three facts given the question produce the following equations.

$$\begin{aligned}
&p_0 + p_1 + p_2 + p_3 + p_4 = 1 \\
&p_0 + p_1 + p_2 = 0.75 \\
&p_1 + p_2 + p_3 + p_4 = 0.8 \\
&p_0 + p_1 + p_3 + p_4 = 0.8
\end{aligned}.$$

Therefore, we have

$$\begin{aligned}
&p_0 = (p_0 + p_1 + p_2 + p_3 + p_4) - (p_1 + p_2 + p_3 + p_4) = 1 - 0.8 = 0.2 \\
&p_2 = (p_0 + p_1 + p_2 + p_3 + p_4) - (p_0 + p_1 + p_3 + p_4) = 1 - 0.8 = 0.2. \\
&p_1 = (p_0 + p_1 + p_2) - p_0 - p_2 = 0.75 - 0.2 - 0.2 = 0.35.
\end{aligned}$$

199. Solution: D

Using the law of total probability, the requested probability is

$$\sum_{k=0}^{\infty} P(k + 0.75 < X \le k + 1 \mid k < X \le k + 1) P(k < X \le k + 1).$$

The first probability is

$$P(k + 0.75 < X \le k + 1 \mid k < X \le k + 1) = \frac{P(k + 0.75 < X \le k + 1)}{P(k < X \le k + 1)}$$

$$= \frac{F(k+1) - F(k+0.75)}{F(k+1) - F(k)} = \frac{1 - e^{-(k+1)/2} - 1 + e^{-(k+0.75)/2}}{1 - e^{-(k+1)/2} - 1 + e^{-k/2}} = \frac{e^{-0.375} - e^{-0.5}}{1 - e^{-0.5}} = 0.205.$$

This probability factors out of the sum and the remaining probabilities sum to 1 so the requested probability is 0.205.

200. Solution: B

The requested probability can be determined as

$$P(\text{3 of first 11 damaged}) P(\text{12th is damaged} \mid \text{3 of first 11 damaged})$$

$$= \frac{\binom{7}{3}\binom{13}{8}}{\binom{20}{11}} \frac{4}{9} = \frac{35(1,287)}{167,960} \frac{4}{9} = 0.119.$$

201. Solution: E

Let M be the size of a family that visits the park and let N be the number of members of that family that ride the roller coaster. We want $P(M = 6 \mid N = 5)$. By Bayes theorem

$$
\begin{aligned}
&P(M=6 \mid N=5) \\
&= \frac{P(N=5 \mid M=6)P(M=6)}{\sum_{m=1}^{7} P(N=5 \mid M=m)P(M=m)} \\
&= \frac{\frac{1}{6}\frac{2}{28}}{0+0+0+0+\frac{1}{5}\frac{3}{28}+\frac{1}{6}\frac{2}{28}+\frac{1}{7}\frac{1}{28}} = \frac{\frac{1}{3}}{\frac{3}{5}+\frac{1}{3}+\frac{1}{7}} = \frac{35}{63+35+15} = \frac{35}{113} \approx 0.3097.
\end{aligned}
$$

202. Solution: C

Let S represent the event that the selected borrower defaulted on at least one student loan.
Let C represent the event that the selected borrower defaulted on at least one car loan.

We need to find $P(C \mid S) = \dfrac{P(C \cap S)}{P(S)}$.

We are given $P(S) = 0.3, \quad P(S \mid C) = \dfrac{P(C \cap S)}{P(C)} = 0.4, \quad P(C \mid S^c) = \dfrac{P(C \cap S^c)}{P(S^c)} = 0.28$.

Then,

$P(C \cap S^c) = 0.28P(S^c) = 0.28(1-0.3) = 0.196$.

Because

$P(C) = P(C \cap S) + P(C \cap S^c]$ and $P(C) = P(C \cap S)/0.4$ we have

$P(C \cap S)/0.4 = P(C \cap S) + 0.196 \Rightarrow P(C \cap S) = 0.196/1.5 = 0.13067.$

Therefore,

$$P(C \mid S) = \frac{P(C \cap S)}{P(S)} = \frac{0.13067}{0.3} = 0.4356,$$

203. Solution: C

The conditional density of Y given $X = 2$ is

$$f_{Y|X}(y\,|\,2) = \frac{f_{X,Y}(2,y)}{f_X(2)} = \frac{\frac{1}{18}e^{-(2+y)/6}}{\int_2^\infty \frac{1}{18}e^{-(2+y)/6}dy} = \frac{\frac{1}{18}e^{-(2+y)/6}}{-\frac{1}{3}e^{-(2+y)/6}\Big|_2^\infty} = \frac{\frac{1}{18}e^{-(2+y)/6}}{\frac{1}{3}e^{-2/3}} = \frac{1}{6}e^{-(y-2)/6}, \quad y > 2$$, and is

zero otherwise.

While the mean and then the variance can be obtained from the usual integrals, it is more efficient to recognize that this density function is 2 more than an exponential random variable with mean 6. The variance is then the same as that for an exponential random variable with mean 6, which is 6x6 = 36.

204. Solution: A

Let Y denote the time between report and payment. Then

$$f(t,y) = f(y\,|\,t)\,f(t) = \left(\frac{1}{8-t}\right)\left(\frac{8t-t^2}{72}\right) = \frac{t}{72}, \quad 0 < t < 6,\, 2+t < y < 10$$

$$\mathrm{P}(T+Y<4) = \int_0^1\int_{2+t}^{4-t}\frac{t}{72}dydt = \int_0^1 t\frac{(4-t)-(2+t)}{72}dt = \int_0^1 t\frac{2-2t}{72}dt = \frac{t^2-2t^3/3}{72}\Bigg|_0^1 = 1/216 = 0.005.$$

205. Solution: D

First, note that $W = 0$ if T is greater than 8 or less than 1.5. Therefore, $P(W=0) = \frac{2+1.5}{10} = 0.35$

.

For $W > 0$,

$$P(0 < W < 79) = P(100e^{-0.04T} < 79 \text{ and } 1.5 \le T < 8) = P(-0.04T < \ln 0.79 \text{ and } 1.5 \le T < 8)$$

$$= P(5.893 < T < 8) = \frac{8-5.893}{10} = 0.211.$$

Then,

$P(W < 79) = 0.35 + 0.211 = 0.561.$

206. Solution: E

Without the deductible, the standard deviation is, from the uniform distribution, $b/\sqrt{12} = 0.28868b$. Let Y be the random variable representing the payout with the deductible.

$$E(Y) = \int_{0.1b}^{b} (y-0.1b)\frac{1}{b}dy = \left.\frac{y^2}{2b} - 0.1y\right|_{0.1b}^{b} = 0.5b - 0.1b - 0.005b + 0.01b = 0.405b$$

$$E(Y^2) = \int_{0.1b}^{b} (y-0.1b)^2\frac{1}{b}dy = \left.\frac{y^3}{3b} - 0.1y^2 + 0.01by\right|_{0.1b}^{b}$$

$$= b^2/3 - 0.1b^2 + 0.01b^2 - 0.001b^2/3 + 0.001b^2 - 0.001b^2 = 0.243b^2$$

$$Var(Y) = 0.243b^2 - (0.405b)^2 = 0.078975b^2$$

$$SD(Y) = 0.28102b.$$

The ratio is 0.28102/0.28868 = 0.97347.

207. Solution: C

i) is false because G includes having one accident in year two.
ii) is false because there could be no accidents in year one.
iii) is true because it connects the descriptions of F and G (noting that "one or more" and "at least one" are identical events) with "and."
iv) is true because given one accident in year one (F), having a total of two or more in two years is the same as one or more in year two (G).
v) is false because it requires year two to have at least two accidents.

208. Solution: B

$$P[D] = P[H]P[D|H] + P[M]P[D|M] + P[L]P[D|L]$$

$$0.009 = P[H]P[D|H] + P[M]\left(\frac{1}{2}P[D|H]\right) + P[L]\left(\frac{1}{2}\frac{1}{3}P[D|H]\right)$$

$$0.009 = 0.20P[D|H] + 0.35\left(\frac{1}{2}P[D|H]\right) + 0.45\left(\frac{1}{6}P[D|H]\right) = 0.45P[D|H]$$

$$P[D|H] = 0.009/0.45 = 0.02$$

209. Solution: C

If the deductible is less than 60 the equation is,
$0.10(60-d) + 0.05(200-d) + 0.01(3000-d) = 30 \Rightarrow d = 100.$.
So this cannot be the answer. If the deductible is between 60 and 200, the equation is $0.05(200-d) + 0.01(3000-d) = 30 \Rightarrow d = 166.67$. This is consistent with the assumption and is the answer.

210. Solution: C

The probability that none of the damaged houses are insured is

$$\frac{1}{120}=\frac{\binom{10-k}{0}\binom{k}{3}}{\binom{10}{3}}=\frac{k(k-1)(k-2)}{720}.$$

$$k(k-1)(k-2)=6$$

This cubic equation could be solved by expanding, subtracting 6, and refactoring. However, because k must be an integer, the three factors must be integers and thus must be 3(2)(1) for k = 3.

The probability that at most one of the damaged houses is insured equals

$$\frac{1}{120}+\frac{\binom{10-3}{1}\binom{3}{2}}{\binom{10}{3}}=\frac{1}{120}+\frac{7(3)}{120}=\frac{22}{120}=\frac{11}{60}.$$

211. Solution: B

This question is equivalent to "What is the probability that 9 different chips randomly drawn from a box containing 4 red chips and 8 blues chips will contain the 4 red chips?" The hypergeometric probability is

$$\frac{\binom{4}{4}\binom{8}{5}}{\binom{12}{9}}=\frac{1(56)}{220}=0.2545.$$

212. Solution: D

Let N be the number of sick days for an employee in three months. The sum of independent Poisson variables is also Poisson and thus N is Poisson with a mean of 3.. Then,

$$\mathrm{P}[N\le 2]=e^{-3}\left(\frac{3^0}{0!}+\frac{3^1}{1!}+\frac{3^2}{2!}\right)=e^{-3}(1+3+4.5)=0.423.$$

The answer is the complement, 1 – 0.423 = 0.577.

213. Solution: B

$$A = P(N > 3) = 1 - \left[P(N = 0) + P(N = 1) + P(N = 2) + P(N = 3)\right]$$
$$= 1 - e^{-3}\left(1 + \frac{3}{1} + \frac{9}{2} + \frac{27}{6}\right) = 1 - 13e^{-3} = 0.3528$$
$$B = P(N > 1.5) = 1 - \left[P(N = 0) + P(N = 1)\right]$$
$$= 1 - e^{-1.5}\left(1 + \frac{1.5}{1}\right) = 1 - 2.5e^{-1.5} = 0.4422$$

$B - A = 0.4422 - 0.3528 = 0.0894$.

214. Solution: E

For Policy A, the relevant equation is

$$0.64 = P(L > 1.44) = e^{-1.44/\mu}$$
$$\ln(0.64) = -0.44629 = -1.44/\mu$$
$$\mu = 3.2266.$$

For Policy B, the relevant equation is

$$0.512 = P(L > d) = e^{-d/3.2266}$$
$$\ln(0.512) = -0.6694 = -d/3.2266$$
$$d = 2.1599.$$

215. Solution: B

Because the density function must integrate to 1, $1 = \int_0^5 cx^a dx = c\frac{5^{a+1}}{a+1} \Rightarrow c = \frac{a+1}{5^{a+1}}$.

From the given probability,

$$0.4871 = \int_0^{3.75} cx^a dx = c\frac{3.75^{a+1}}{a+1} = \frac{a+1}{5^{a+1}}\frac{3.75^{a+1}}{a+1} = \left(\frac{3.75}{5}\right)^{a+1}$$
$$\ln(0.4871) = -0.71929 = (a+1)\ln(3.75/5) = -0.28768(a+1)$$
$$a = (-0.71929)/(-0.28768) - 1 = 1.5.$$

The probability of a claim exceeding 4 is,

$$\int_4^5 cx^a dx = c\frac{5^{a+1} - 4^{a+1}}{a+1} = \frac{a+1}{5^{a+1}}\frac{5^{a+1} - 4^{a+1}}{a+1} = 1 - \left(\frac{4}{5}\right)^{1.5+1} = 0.42757.$$

216. Solution: A

Let N denote the number of warranty claims received. Then,
$0.6 = P(N=0) = e^{-c} \Rightarrow c = -\ln(0.6) = 0.5108.$

The expected yearly insurance payments are:

$$\begin{aligned}
&5000[P(N=2)+2P(N=3)+3P(N=4)+\cdots] \\
&= 5000[P(N=1)+2P(N=2)+3P(N=3)+\cdots]-5000[P(N=1)+P(N=2)+P(N=3)+\cdots] \\
&= 5000E(N)-5000[1-P(N=0)] = 5000(0.5108)-5000(1-0.6) = 554.
\end{aligned}$$

217. Solution: D

If L is the loss, the unreimbursed loss, X is

$$X = \begin{cases} L, & L \le 180 \\ 180, & L > 180. \end{cases}$$

The expected unreimbursed loss is

$$144 = E(X) = \int_0^{180} l[f(l)]dl + 180\,\Pr(L>180) = \int_0^{180} l\frac{1}{b}dl + 180\frac{b-180}{b}$$

$$= \left.\frac{l^2}{2b}\right|_0^{180} + 180 - \frac{180^2}{b} = \frac{180^2}{2b} + 180 - \frac{180^2}{b}$$

$$144b = 180^2/2 + 180b - 180^2$$

$$16,200 = 36b$$

$$b = 450.$$

218. Solution: B

Let X be normal with mean 10 and variance 4. Let Z have the standard normal distribution. Let p = 12th percentile. Then

$$0.12 = P(X \le p) = P\left(\frac{X-10}{2} \le \frac{p-10}{2}\right) = P\left(Z \le \frac{p-10}{2}\right).$$

From the tables, $P(Z \le -1.175) = 0.12$. Therefore,

$$\frac{p-10}{2} = -1.175;\ p-10 = -2.35;\ p = 7.65.$$

219. Solution: D

From the normal table, the 14th percentile is associated with a z-score of -1.08. Since the means are equal and the standard deviation of company B's profit is $\sqrt{2.25} = 1.5$ times the standard deviation of company A's profit, a profit that is 1.08 standard deviations below the mean for company A would be1.08/1.5 = 0.72 standard deviations below the mean for company B. From the normal table, a z-score of -0.72 is associated with the 23.6th percentile.

220. Solution: C

The conditional variance is

$$Var(X \mid X \geq 10) = E(X^2 \mid X \geq 10) - E(X \mid X \geq 10)^2$$

$$= \frac{\int_{10}^{\infty} x^2 (0.2) e^{-0.2(x-5)} dx}{\int_{10}^{\infty} 0.2 e^{-0.2(x-5)} dx} - \left[\frac{\int_{10}^{\infty} x (0.2) e^{-0.2(x-5)} dx}{\int_{10}^{\infty} 0.2 e^{-0.2(x-5)} dx} \right]^2 .$$

Performing integration (using integration by parts) produces the answer of 25.

An alternative solution is to first determine the density function for the conditional distribution. It is

$$f(y) = \frac{0.2e^{-0.2(y-5)}}{\int_{10}^{\infty} 0.2e^{-0.2(x-5)} dx} = \frac{0.2e^{-0.2(y-5)}}{-e^{-0.2(x-5)}\Big|_{10}^{\infty}} = \frac{0.2e^{-0.2(y-5)}}{e^{-0.2(5)}} = 0.2e^{-0.2(y-10)},\ y > 10 .$$

Then note that Y – 10 has an exponential distraction with mean 5. Subtracting a constant does not change the variance, so the variance of Y is also 25.

221. Solution: C

Let X and Y represent the annual profits for companies A and B, respectively and m represent the common mean and s the standard deviation of Y. Let Z represent the standard normal random variable.

Then because X's standard deviation is one-half its mean,

$$P(X < 0) = P\left(\frac{X - m}{0.5m} < \frac{0 - m}{0.5m} \right) = P(Z < -2) = 0.0228.$$

Therefore company B's probability of a loss is 0.9(0.0228) = 0.02052. Then,

$0.02052 = P(Y < 0) = P\left(\frac{Y - m}{s} < \frac{0 - m}{s} \right) = P(Z < -m/s)$. From the tables, –2.04 = –m/s and therefore $s = m/2.04$. The ratio of the standard deviations is $(m/2.04)/(0.5m) = 0.98$.

222. Solution: B

One approach is to take derivatives of the mgf and set them equal to zero. This yields a mean of 0.45 + 0.35(2) + 0.15(3) + 0.05(4) = 1.8 and a second moment of 0.45 + 0.35(4) + 0.15(9) + 0.05(16) = 4. The variance is 4 – 3.24 = 0.76 and the standard deviation is 0.87.

Alternatively, it can be recognized that this mgf corresponds to a discrete random variable with probabilities 0.45, 0.35, 0.15, and 0.05 at 1, 2, 3, and 4, respectively. The same formulas result.

223. Solution: B

Y is a normal random variable with mean 1.04(100) + 5 = 109 and standard deviation 1.04(25) = 26. The average of 25 observations has mean 109 and standard deviation 26/5 = 5.2. The requested probability is

$$P(100 < \text{sample mean} < 110) = P\left(\frac{100-109}{5.2} = -1.73 < Z < \frac{110-109}{5.2} = 0.19\right)$$
$$= 0.5753 - (1-0.9582) = 0.5335.$$

224. Solution: B

The density function is constant, c. It is determined from

$$1 = \int_0^1 \int_0^{1-x^2} c\,dy\,dx = \int_0^1 c(1-x^2)dx = c(x - x^3/3)\Big|_0^1 = c(2/3) \Rightarrow c = 1.5.$$
$$E(XY) = \int_0^1 \int_0^{1-x^2} 1.5xy\,dy\,dx = \int_0^1 0.75xy^2\Big|_0^{1-x^2} dx = \int_0^1 0.75x(1-x^2)^2 dx$$
$$= \int_0^1 0.75(x - 2x^3 + x^5)dx = 0.75[1/2 - 2(1/4) + 1/6] = 0.125.$$

225. Solution: E

The possible events are (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), and (3,3). The probabilities (without c) sum to 0 + 2 + 4 + 6 + 3 + 5 + 7 + 6 + 8 + 9 = 50. Therefor c = 1/50. The number of tornadoes with fewer than 50 million in losses is $Y – X$. The expected value is (1/50)[0(0) + 1(2) + 2(4) + 3(6) + 0(3) + 1(5) + 2(7) + 0(6) + 1(8) + 0(9)]=55/50 = 1.1.

226. Solution: D

Consider three cases, one for each result of the first interview.
Independent (prob 0.5): Expected absolute difference is (4/9)(0) +(5/9)(1) = 5/9
Republican (prob =0.3): Expected absolute difference is (2/9)(0) + (5/9)(1) + (2/9)(2) = 1
Democrat (prob = 0.2): Expected absolute difference is (3/9)(0) + (5/9)(1) + (1/9)(2) = 7/9.
The unconditional expectation is 0.5(5/9) + 0.3(1) + 0.2(7/9) = 6.6/9 = 11/15.

Alternatively, the six possible outcomes can be listed along with their probabilities and absolute differences.

227. Solution: C

Let $Z = XY$. Let a, b, and c be the probabilities that Z takes on the values 0, 1, and 2, respectively. We have $b = p(1,1)$ and $c = p(1,,2)$ and thus $3b = c$. And because the probabilities sum to 1, $a = 1 – b – c = 1 – 4b$. *Then,* $E(Z) = b + 2c = 7b$, $E(Z*Z) = b + 4c = 13b$. Then,

$$Var(Z) = 13b - 49b^2$$

$$(d/db)Var(Z) = 13 - 98b = 0 \Rightarrow b = 13/98.$$

The probability that either X or Y is zero is the same as the probability that Z is 0 which is $a = 1 – 4b = 46/98 = 23/49$.

228. Solution: B

The marginal density of X at 1/3 is $\int_{1/3}^{1} 24(1/3)(1-y)dy = 16/9$. The conditional density of Y given $X = 1/3$ is $\dfrac{24(1/3)(1-y)}{16/9} = 4.5(1-y), 1/3 < y < 1$. The mean is

$$4.5\int_{1/3}^{1} y(1-y)dy = 2.25y^2 - 1.5y^3\Big|_{1/3}^{1} = 5/9.$$

229. Solution: C

Let J and K be the random variables for the number of severe storms in each city.

$$P(J = j \mid K = 5) = \frac{P(K = 5 \mid J = j)P(J = j)}{P(K = 5)}$$

$$P(K = 5 \mid J = 3) = 1/6,\ P(J = 3) = \binom{5}{3}0.6^3 0.4^2 = 0.3456$$

$$P(K = 5 \mid J = 4) = 1/3,\ P(J = 4) = \binom{5}{4}0.6^4 0.4^1 = 0.2592$$

$$P(K = 5 \mid J = 5) = 1/2,\ P(J = 5) = \binom{5}{5}0.6^5 0.4^0 = 0.07776$$

$$P(K = 5) = (1/6)(0.3456) + (1/3)(0.2592) + (1/2)(0.07776) = 0.18288$$

$$P(J = 3 \mid K = 5) = \frac{(1/6)(0.3456)}{0.18288} = 0.31496$$

$$P(J = 4 \mid K = 5) = \frac{(1/3)(0.2592)}{0.18288} = 0.47244$$

$$P(J = 5 \mid K = 5) = \frac{(1/2)(0.07776)}{0.18288} = 0.21260$$

$$E(J \mid K = 5) = 3(0.31496) + 4(0.47244) + 5(0.21260) = 3.89764.$$

230. Solution: C

$$F_Y(y) = F(1, y) = y + y^2 - y^3 \Rightarrow f_Y(y) = 1 + 2y - 3y^2$$
$$E(Y) = \int_0^1 y(1+2y-3y^2)dy = 1/2 + 2(1/3) - 3(1/4) = 5/12 = 0.417.$$

231. Solution: B

Given $N + S = 2$, there are 3 possibilities $(N,S) = (2,0), (1,1), (0,2)$ with probabilities 0.12, 0.18, and 0.10 respectively.
The associated conditional probabilities are

P($N = 0$ | $N + S = 2$) = 0.10/0.40 = 0.25,
P($N = 1$ | $N + S = 2$) = 0.18/0.40 = 0.45,
P($N = 2$ | N + S = 2) = 0.12/0.40 = 0.30.

The mean is 0.25(0) + 0.45(1) + 0.30(2) = 1.05.
The second moment is 0.25(0) + 0.45(1) + 0.30(4) = 1.65.
The variance is 1.65 – (1.05)(1.05) = 0.5475.

232. Solution: A

$$F_X(x) = F(x,100) = \frac{100x(x+100)}{2,000,000} = \frac{100x^2 + 10,000x}{2,000,000} \Rightarrow f_X(x) = \frac{x}{10,000} + \frac{1}{200}$$
$$E(X) = \int_0^{100} \frac{x^2}{10,000} + \frac{x}{200} dx = \left. \frac{x^3}{30,000} + \frac{x^2}{400} \right|_0^{100} = 58.33$$
$$E(X^2) = \int_0^{100} \frac{x^3}{10,000} + \frac{x^2}{200} dx = \left. \frac{x^4}{40,000} + \frac{x^3}{600} \right|_0^{100} = 4166.67$$
$$Var(X) = 4166.67 - 58.33^2 = 764.$$

233. Solution: B
The marginal distribution is
$f_X(x) = \int_0^\infty f(x,y)dy = 0.65e^{-0.5x} - 0.30e^{-x} - 0.15e^{-0.5x} + 0.30e^{-x} = 0.5e^{-0.5x}$. This is an exponential distribution with a mean of 1/0.5 = 2. The standard deviation is equal to the mean.

234. Solution: A

Because the territories are evenly distributed, the probabilities can be averaged. Thus the probability of a 100 claim is 0.80, of a 500 claim is 0.13, and of a 1000 claim as 0.07. The mean is 0.80(100) + 0.13(500) + 0.07(1000) = 215. The second moment is 0.80(10,000) + 0.13(250,000) + 0.07(1,000,000) = 110,500. The variance is 110,500 – (215)(215) = 64,275. The standard deviation is 253.53.

235. Solution: D

With each load of coal having mean 1.5 and standard deviation 0.25, twenty loads have a mean of 20(1.5) = 30 and a variance of 20(0.0625) = 1.25. The total amount removed is normal with mean 4(7.25) = 29 and standard deviation 4(0.25) = 1. The difference is normal with mean 30 – 29 = 1 and standard deviation sqrt(1.25 + 1) = 1.5. If *D* is that difference,

$$P(D > 0) = P\left(Z > \frac{0-1}{1.5} = -0.67\right) = 0.7486.$$

236. Solution: C

The probability needs to be calculated for each total number of claims.
0: 0.5(0.2) = 0.10
1: 0.5(0.3) + 0.3(0.2) = 0.21
2: 0.5(0.4) + 0.3(0.3) + 0.2(0.2) = 0.33
3: 0.5(0.1) + 0.3(0.4) + 0.2(0.3) + 0.0(0.2) = 0.23
At this point there is only 0.13 probability remaining, so the mode must be at 2.

237. Solution: B

Let *X* represent the number of policyholders who undergo radiation.
Let *Y* represent the number of policyholders who undergo chemotherapy.
X and *Y* are independent and binomially distributed with 15 trials each and with "success" probabilities 0.9 and 0.4, respectively.

The variances are 15(0.9)(0.1) = 1.35 and 15(0.4)(0.6) = 3.6.

The total paid is 2*X* + 3*Y* and so the variance is 4(1.35) + 9(3.6) = 37.8.

238. Solution: C

Let X and Y represent the number of selected patients with early stage and advanced stage cancer, respectively. We need to calculate $E(Y \mid X \geq 1)$.

From conditioning on whether or not $X \geq 1$, we have
$E(Y) = P[X = 0]E(Y \mid X = 0) + P[X \geq 1]E(Y \mid X \geq 1)$.

Observe that $P[X = 0] = (1-0.2)^6 = (0.8)^6$, $P[X \geq 1] = 1 - P[X = 0] = 1-(0.8)^6$, and $E(Y) = 6(0.1) = 0.6$. Also, note that if none of the 6 selected patients have early stage cancer, then each of the 6 selected patients would independently have conditional probability $\frac{0.1}{1-0.2} = \frac{1}{8}$ of having late stage cancer, so $E(Y \mid X = 0) = 6(1/8) = 0.75$.

Therefore,

$$E(Y \mid X \geq 1) = \frac{E(Y) - P[X = 0]E(Y \mid X = 0)}{P[X \geq 1]} = \frac{0.6 - (0.8)^6(0.75)}{1-(0.8)^6} = 0.547.$$

239. Solution: A

Because there must be two smaller values and one larger value than X, X cannot be 1, 2, or 12. If X is 3, there is one choice for the two smallest of the four integers and nine choices for the largest integer. If X is 4, there are three choices for the two smallest of the four integers and eight choices for the largest integer. In general, if $X = x$, there are $(x - 1)$ choose (2) choices for the two smallest integers and $12 - x$ choices for the largest integer. The total number of ways of choosing 4 integers from 12 integers is 12 choose 4 which is 12!/(4!8!) = 495. So the probability that $X = x$ is:

$$\frac{\binom{x-1}{2}(12-x)}{495} = \frac{(x-1)(x-2)(12-x)}{990}.$$

240. Solution: A

We have

$$0.95 = P(X < k \mid X > 10,000) = \frac{P(X < k) - P(X \le 10,000)}{1 - P(\mathrm{X} \le 10,000)}$$

$$0.95[1 - P(X \le 10,000)] = 0.9582 - P(X \le 10,000)$$

$$P(X \le 10,000) = \frac{0.9582 - 0.95}{1 - 0.95} = 0.164$$

$$0.164 = \Phi\left(\frac{10,000 - 12,000}{c}\right).$$

The z-value that corresponds to 0.164 is between –0.98 and –0.97. Interpolating leads to z = –0.978. Then,

$$0.164 = \Phi\left(\frac{10,000 - 12,000}{c}\right) \Rightarrow -0.978 = \frac{-2,000}{c} \Rightarrow c = 2045.$$

241. Solution: B

Before applying the deductible, the median is 500 and the 20th percentile is 200. After applying the deductible, the median payment is 500 – 250 = 250 and the 20th percentile is max(0, 200 – 250) = 0. The difference is 250.

242. Solution: E

Let X and Y represent the annual profits for companies A and B, respectively.

We are given that X and Y have a bivariate normal distribution, the correlation coefficient is $\rho = 0.8$, X has mean $\mu_X = 2000$ and standard deviation $\sigma_X = 1000$, and Y has mean $\mu_Y = 3000$ and standard deviation $\sigma_Y = 500$.

In general for a bivariate normal distribution, given that $X = x$, Y is normally distributed with mean $\mu_Y + \frac{\rho\sigma_Y}{\sigma_X}\left(x - \mu_X\right)$ and standard deviation $\sigma_Y\sqrt{1-\rho^2}$.

So given that company A's annual profit is 2300, company B's annual profit is normally distributed with mean $3000 + \frac{0.8(500)}{1000}\left(2300 - 2000\right) = 3120$ and standard deviation $500\sqrt{1-(0.8)^2} = 300$.

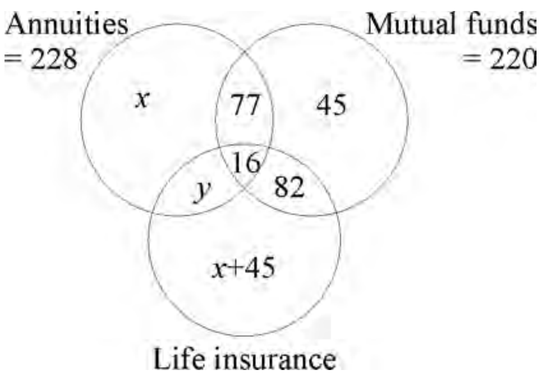
Therefore, given that company A's annual profit is 2300, the probability that company B's profit is at most 3900 is $\mathrm{P}\left[Z \le \frac{3900 - 3120}{300}\right] = \mathrm{P}\left[Z \le 2.6\right] = 0.9953$.

243. Solution: B

32 own L/A/H
55 own L/H so 55 – 32 = 23 own L/H/notA
96 own A/H so 96 – 32 = 64 own A/H/notL
207 own H so 207 – 32 – 23 – 64 = 88 own H only
L only = X, A only = X + 76
88 + X + (X + 76) = 270 so X = 53 so L only = 53, A only = 129
129 + 64 + 32 + L/A/notH = 243 so L/A/notH = 18
Total clients = 53 + 129 + 88 + 18 + 64 + 23 + 32 = 407

Alternatively, a Venn diagram could be used to guide the calculations.

244. Solution: D



Then 290 = 45 + 45 + x + x, thus x = 100.
Also 228 = 100 + y + 16 + 77, thus y = 35.
Total clients = 145 + 82 + 16 + 35 + 100 + 77 + 45 = 500.

245. Solution: C

If k is the number of days of hospitalization, then the insurance payment $g(k)$ is

$$g(k) = \begin{cases} 100k, & k = 1, 2, 3 \\ 300 + 50(k-3), & k = 4, 5. \end{cases}$$

Thus, the expected payment is

$$\sum_{k=1}^{5} g(k) p_k = 100 p_1 + 200 p_2 + 300 p_3 + 350 p_4 + 400 p_5$$

$$= 100(5/15) + 200(4/15) + 300(3/15) + 350(2/15) + 400(1/15) = 220.$$

246. Solution: D

Let A, B, and C be the sets of policies in the portfolio on three-bedroom homes, one-story homes, and two-bath homes, respectively. We are asked to calculate $1000 - n(A \cup B \cup C)$, where $n(D)$ denotes the number of elements of the set D. Then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= 130 + 280 + 150 - 40 - 30 - 50 + 10 = 450.$$

The answer is 1000 – 450 = 550.

247. Solution: B

We seek the number of ways to select 4 individuals from 7 and choose one selected member as subcommittee chair. (The existence of a subcommittee secretary is irrelevant.) There are (7 choose 4) = 7(6)(5)(3)/4! = 35 ways to form a collection of 4 individuals from 7. For each of them, there are 4 ways to assign a chair. The product, 140, is the number of different ways to form a subcommittee of 4 individuals and assign a chair and thus is the maximum number without repetition.

248. Solution: D

$$P(2\text{ red and }2\text{ blue transferred}\,|\,\text{blue drawn}) = \frac{P(2\text{ red and }2\text{ blue transferred and blue drawn})}{P(\text{blue drawn})}$$

$$P(2\text{ red and }2\text{ blue transferred and blue drawn}) = \frac{\binom{8}{2}\binom{6}{2}}{\binom{14}{4}} \times \frac{2}{4} = \frac{28(15)}{1001} \times \frac{2}{4} = \frac{840}{4004}$$

$$P(\text{blue drawn}) = \frac{\binom{8}{0}\binom{6}{4}}{\binom{14}{4}} \times \frac{4}{4} + \frac{\binom{8}{1}\binom{6}{3}}{\binom{14}{4}} \times \frac{3}{4} + \frac{\binom{8}{2}\binom{6}{2}}{\binom{14}{4}} \times \frac{2}{4} + \frac{\binom{8}{3}\binom{6}{1}}{\binom{14}{4}} \times \frac{1}{4} = \frac{60 + 480 + 840 + 336}{4004} = \frac{1716}{4004}$$

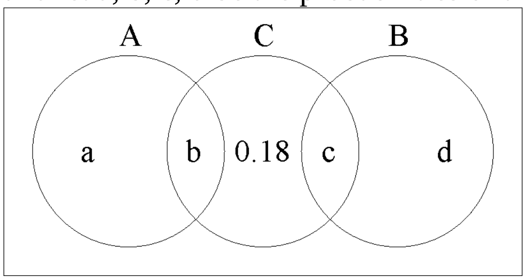
$$P(2\text{ red and }2\text{ blue transferred}\,|\,\text{blue drawn}) = \frac{840}{1716} = 0.49.$$

249. Solution: D

If a policy is of Type A, the probability that the two claims are equal is (0.4)(0.4) + (0.3)(0.3) + (0.2)(0.2) + (0.1)(0.1) = 0.16 + 0.09 + 0.04 + 0.01 = 0.30.
If a policy is of Type B, the probability that the two claims are equal is 4(0.25)(0.25) = 0.25.
Therefore, the probability that a randomly selected policy has equal claims is 0.70(0.30) + 0.30(0.25) = 0.285.
If four policies are selected, the desired probability is the probability that a binomial random variable with n = 4 and p = 0.285 is 1. This is 4(0.285)(1 – 0.285)^3 = 0.417.

250. Solution: A

Let A = event that person wants life policy P
B = event that person wants life policy Q
C = event that person wants the health policy
and let a, b, c, d be the probabilities of the regions as shown.



i) is reflected by no intersection of A and B
iv) is reflected by the 0.18 in the diagram
ii) implies a + b = 2(c + d)
iii) implies b + c + 0.18 = 0.45 or b + c = 0.27
v) implies P([A or B] and C) = P(A or B)P(C) or b + c = (a + b + c + d)(0.45)

So 0.27 = (a + d + 0.27)(0.45) and then a + d = 0.33.
The desired probability is a + 0.18 + d = 0.33 + 0.18 = 0.51.

251. Solution: B

The state will receive 800,000($1) = $800,000 in revenue, and will lose money if there are 2 or more winning tickets sold. The player's entry can be viewed as fixed. The probability the lottery randomly selects those same six numbers is from a hypergeometric distribution and is

$$\frac{\binom{6}{6}\binom{24}{0}}{\binom{30}{6}} = \frac{1(1)}{\frac{30!}{6!(24!)}} = \frac{6(5)(4)(3)(2)(1)}{30(29)(28)(27)(26)(25)} = \frac{1}{593,775}.$$

The number of winners has a binomial distribution with n = 800,000 and p = 1/593,775. The desired probability is

$$\Pr(2 \text{ or more winners}) = 1 - \Pr(0 \text{ winners}) - \Pr(1 \text{ winner})$$

$$= 1 - \binom{800,000}{0}\left(\frac{1}{593,775}\right)^0\left(\frac{593,774}{593,775}\right)^{800,000} - \binom{800,000}{1}\left(\frac{1}{593,775}\right)^1\left(\frac{593,774}{593,775}\right)^{799,999}$$

$$= 1 - 0.2599 - 0.3502 = 0.39.$$

252. Solution: E

The number that have errors is a binomial random variable with p = 0.03 and n = 100. Let X be the number that have errors. Then,

$$\Pr(\text{number that are error-free} \le 95) = \Pr(X \ge 5) = 1 - P(0) - P(1) - P(2) - P(3) - P(4)$$

$$= 1 - \binom{100}{0}(0.03)^0(0.97)^{100} - \binom{100}{1}(0.03)^1(0.97)^{99} - \binom{100}{2}(0.03)^2(0.97)^{98} - \binom{100}{3}(0.03)^3(0.97)^{97}$$

$$-\binom{100}{4}(0.03)^4(0.97)^{96} = 0.1821.$$

Or, the Poisson approximation can be used. Then, $\lambda = 3$ and

$$P(X \ge 5) = 1 - \frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!} - \frac{e^{-3}3^2}{2!} - \frac{e^{-3}3^3}{3!} - \frac{e^{-3}3^4}{4!} = 1 - e^{-3}\left(1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24}\right) = 0.1847.$$

253. Solution: D

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$

$$= 0.2 + 0.1 + 0.3 - 0.2(0.1) - 0 - 0.1(0.3) + 0 = 0.55.$$

254. Solution: A

The probability a union of three events equals the sum of their probabilities if and only if they are mutually exclusive, that is, no two of them can both occur.

Events A and B cannot both occur since no thefts in the first three years would imply no thefts in the second year, thus precluding the possibility of at least 1 theft in the second year.
Events A and E cannot both occur since no thefts in the first three years would imply no thefts in the third year, thus precluding the possibility of at least 1 theft in the third year.
Events B and E cannot both occur since it is impossible to experience both no thefts and at least 1 theft in the second year.
Thus, events A, B, and E satisfy the desired condition.

255. Solution: D

Consider the two mutually exclusive events “first envelope correct” and “first envelope incorrect.” The probability of the first event is 1/4 and meets the requirement of at least one correct. For the 3/4 of the time the first envelope is incorrect, there are now 3 more envelopes to fill. Of the six permutations, three will place one letter correctly. The total probability is 1/4 + 3/4(3/6) = 5/8.

256. Solution: C

The deductible is exceeded for 4, 5 or 6 office visits. Therefore, the requested probability is 0.02/(0.04 + 0.02 + 0.01) = 0.286.

257. Solution: C

Let A be the event that part A is working after one year and B be the event that part B is working after one year. Then,

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A) + P(B) - P(A \text{ or } B)}{P(A)} = \frac{0.8 + 0.6 - 0.9}{0.8} = 5/8.$$

258. Solution: D

$$\frac{\binom{2}{1}\binom{4}{2}\binom{7}{3}}{\binom{13}{6}} = 0.245.$$

259. Solution: E

The maximum number of draws needed is 5. This can only happen if the first four draws produce four different colors. The first draw can be any sock. The second draw must be one of the 6 (of 7 remaining) that are different. The third draw must be one of the 4 (of 6) that are different from the first two. The fourth draw must be one of the 2 (of 5) that are different. The probability all of this happens is 1(6/7)(4/6)(2/5) = 0.2286.

260. Solution: E

Define the events as follows:
A = applies for a mortgage
S = initially spoke to an attendant
R = call returned the same day
N = call returned the next day
Then, using Bayes' Theorem,

$$P(S \mid A) = \frac{P(A \mid S)P(S)}{P(A \mid S)P(S) + P(A \mid R)P(R) + P(A \mid N)P(N)}$$

$$= \frac{0.8(0.6)}{0.8(0.6) + 0.6(0.4)(0.75) + 0.4(0.4)(0.25)} = 0.69.$$

261. Solution: A

Define the events as follows:
C = files a claim
N = no lifting
M = moderate lifting
H = heavy lifting
Then, using Bayes' Theorem,

$$P(M \text{ or } H \mid C) = 1 - P(N \mid C) = 1 - \frac{P(C \mid N)P(N)}{P(C \mid N)P(N) + P(C \mid \mathrm{M})P(\mathrm{M}) + P(C \mid \mathrm{H})P(\mathrm{H})}$$

$$= 1 - \frac{0.05(0.4)}{0.05(0.4) + 0.08(0.5) + 0.2(0.1)} = 1 - 0.25 = 0.75.$$

262. Solution: E

From the Law of Total Probability, the required probability is

$$\sum_{k=0}^{\infty} P(0 \text{ accidents with an uninsured driver} \mid k \text{ accidents}) P(k \text{ accidents})$$

$$= \sum_{k=0}^{\infty} (0.75)^k \frac{e^{-5}5^k}{k!} = \frac{e^{-5}}{e^{-3.75}} \sum_{k=0}^{\infty} \frac{e^{-3.75}(3.75)^k}{k!} = e^{-1.25} = 0.287.$$

263. Solution: B

From the binomial distribution formula, the probability P that a given patient tests positive for at least 2 of these 3 risk factors is $P = \binom{3}{2} p^2(1-p)^{3-2} + \binom{3}{3} p^3(1-p)^{3-3} = 3p^2(1-p) + p^3$.

Using the geometric distribution formula with probability of success $P = 3p^2(1-p) + p^3$, the probability that exactly n patients are tested is

$$(1-P)^{n-1}P = \left[1 - 3p^2(1-p) - p^3\right]^{n-1}\left[3p^2(1-p) + p^3\right].$$

264. Solution: B

For there to be more than three calls before one completed survey all that is required is the first three calls not result in a completed survey. This probability is $(1-0.25)^3 = 0.42$.

265. Solution: B

For a given x, there are $x - 1$ choices for the smaller of the four integers and $12 - x$ choices for the two larger integers. Thus, there are $(x-1)\binom{12-x}{2} = \frac{(x-1)(12-x)(11-x)}{2}$ triples that satisfy the event. The total number of possible draws is $\binom{12}{4} = 495$ and the probability is

$\frac{(x-1)(12-x)(11-x)}{2}\frac{1}{495} = \frac{x-1)(12-x)(11-x)}{990}$.

266. Solution: D

The cumulative distribution function for the exponential distribution is
$F(x) = 1 - e^{-\lambda x} = 1 - e^{-x/\mu} = 1 - e^{-x/100}$, $\mathrm{x} > 0$.

From the given probability data,

$$\begin{aligned}
&F(50) - F(40) = F(r) - F(60) \\
&1 - e^{-50/100} - \left(1 - e^{-40/100}\right) = 1 - e^{-r/100} - \left(1 - e^{-60/100}\right) \\
&e^{-40/100} - e^{-50/100} = e^{-60/100} - e^{-r/100} \\
&e^{-r/100} = e^{-60/100} - e^{-40/100} + e^{-50/100} = 0.4850 \\
&-r/100 = \ln(0.4850) = -0.7236 \\
&r = 72.36.
\end{aligned}$$

267. Solution: B

The desired event is equivalent to the time of the next accident being between 365 and 730 days from now. The probability is
$F(730) - F(365) = 1 - e^{-730/200} - \left(1 - e^{-365/200}\right) = e^{-1.825} - e^{-3.65} = 0.1352.$
Note that the problem provides no information about the distribution of the time to subsequent accidents, but that information is not needed. With nothing given, anything can be assumed. If the time to subsequent accidents has the same exponential distribution and the times are independent, then the number of accidents in each 365 day period is Poisson with mean 1.825. Then the required probability is $e^{-1.825}(1 - e^{-1.825}) = 0.1352$.

268. Solution: C

$$\Pr(Z \le 0.72) = 0.7642 = \Pr(X \le 2000) = \Pr[Z \le (2000 - \mu) / \sigma]$$
$$0.72 = (2000 - \mu) / \sigma$$
$$\Pr(Z \le 1.32) = 0.9066 = \Pr(X \le 3000) = \Pr[Z \le (3000 - \mu) / \sigma]$$
$$1.32 = (3000 - \mu) / \sigma$$
$$1.32 / 0.72 = (3000 - \mu) / (2000 - \mu)$$
$$1.8333(2000 - \mu) = 3000 - \mu$$
$$\mu = [1.8333(2000) - 3000] / (1.8333 - 1) = 800$$
$$\sigma = (3000 - \mu) / 1.32 = 1666.67$$
$$\Pr(X \le 1000) = \Pr[Z \le (1000 - 800) / 1666.67] = \Pr(Z \le 0.12) = 0.5478.$$

269. Solution: B

$$P(X > V) = 1 - P(X \le V) = 1 - F(V) = 1 - \left(1 - \frac{1}{10} e^{-\frac{V-V}{V}}\right) = 0.10.$$

270. Solution: E

The given mean of 5 years corresponds to the pdf $f(t) = 0.2e^{-0.2t}$ and the cumulative distribution function $F(t) = 1 - e^{-0.2t}$. The conditional pdf is

$$g(t) = \frac{f(t)}{F(10)} = \frac{0.2e^{-0.2t}}{1 - e^{-2}}, \; 0 < t < 10.$$

The conditional mean is (using integration by parts)

$$E(T \mid T < 10) = \int_0^{10} t g(t) dt = \int_0^{10} t \frac{0.2e^{-0.2t}}{1 - e^{-2}} dt = 0.2313 \int_0^{10} t e^{-0.2t} dt$$
$$= 0.2313 \left[t(-5e^{-0.2t}) \Big|_0^{10} - \int_0^{10} -5e^{-0.2t}) dt \right] = 0.2323 \left[-6.7668 + 0 - 25e^{-0.2t} \Big|_0^{10} \right]$$
$$= 0.2313[-6.7668 - 3.3834 + 25] = 3.435.$$

271. Solution: D

$$F(x) = \int_0^x 2e^{-2y} dy = -e^{-2y} \Big|_0^x = 1 - e^{-2x}$$
$$P[X \le 0.5 \mid X \le 1.0] = \frac{P[X \le 0.5]}{P[X \le 1.0]} = \frac{F(0.5)}{F(1.0)} = \frac{1 - e^{-1}}{1 - e^{-2}} = 0.731.$$

272. Solution: B

If E and F are independent, so are E and the complement of F. Then,
$P(\text{exactly one}) = P(E \cap F^c) + P(E^c \cap F) = 0.84(0.35) + 0.16(0.65) = 0.398.$

273. Solution: E

Let M and N be the random variables for the number of claims in the first and second month. Then

$$P[M+N>3 \mid M<2] = 1 - P[M+N \le 3 \mid M<2] = 1 - \frac{P[M+N \le 3, M<2]}{P[M<2]}$$

$$= 1 - \frac{\begin{array}{c} P[M=0,N=0]+P[M=1,N=0]+P[M=0,N=1]+P[M=1,N=1] \\ +P[M=0,N=2]+P[M=1,N=2]+P[M=0,N=3] \end{array}}{P[M=0]+P[M=1]}$$

$$= 1 - \frac{(2/3)(2/3)+(2/9)(2/3)+(2/3)(2/9)+(2/9)(2/9)+(2/3)(2/27)+(2/9)(2/27)+(2/3)(2/81)}{2/3+2/9}$$

$$= 1 - \frac{0.87243}{0.88889} = 0.0185.$$

274. Solution: C

Let X = number of patients tested, which is geometrically distributed with constant "success" probability, say p.

$$P[X \ge n] = P[\text{first } n-1 \text{ patients do not have apnea}] = (1-p)^{n-1}.$$

Therefore,

$$r = P[X \ge 4] = (1-p)^3$$

$$P[X \ge 12 \mid X \ge 4] = \frac{P[X \ge 12]}{P[X \ge 4]} = \frac{(1-p)^{11}}{(1-p)^3} = (1-p)^8 = \left[(1-p)^3\right]^{\frac{8}{3}} = r^{\frac{8}{3}}$$

275. Solution: D

The number of defects has a binomial distribution with n = 100 and p = 0.02.

$$P[X=2 \mid X \le 2] = \frac{P[X=2]}{P[X \le 2]} = \frac{\binom{100}{2}(0.02)^2(0.98)^{98}}{\binom{100}{0}(0.02)^0(0.98)^{100} + \binom{100}{1}(0.02)^1(0.98)^{99} + \binom{100}{2}(0.02)^2(0.98)^{98}}$$

$$= \frac{0.27341}{0.13262+0.27065+0.27341} = 0.404.$$

276. Solution: A

The town experiences one tornado every 0.8 years on average, which is the mean of the exponential distribution. The median is found from

$$0.5 = P[X \le m] = 1 - e^{-m/0.8}$$
$$\ln(0.5) = -m / 0.8$$
$$m = -0.8\ln(0.5) = 0.55.$$

277. Solution: A

Let X = the amount of a loss. Ignoring the deductible, the median loss is the solution to

$0.5 = P[X > m] = \int_m^\infty 0.25e^{-0.25x}dx = 0 - (-e^{-0.25m}) = e^{-0.25m}$ which is $m = -4(\ln 0.5) = 2.77$.

Because 2.77 > 1, the loss exceeds 2.77 if and only if the claim payment exceeds 2.77 – 1 = 1.77, which is therefore the median claim payment.

278. Solution: B

The payment random variable is $1000(X - 2)$ if positive, where X has a Poisson distribution with mean 1. The expected value is

$$1000\sum_{x=3}^{\infty}(x-2)\frac{e^{-1}}{x!} = 1000\left(\sum_{x=0}^{\infty}(x-2)\frac{e^{-1}}{x!} - \sum_{x=0}^{2}(x-2)\frac{e^{-1}}{x!}\right)$$
$$= 1000\left(1 - 2 - [-2e^{-1} - e^{-1}]\right) = 1000(-1 + 3e^{-1}) = 104.$$

Note the first sum splits into the expected value of X, which is 1, and 2 times the sum of the probabilities (also 1).

279. Solution: C

Let X be the number of employees who die. The expected cost to the company is

$$100P[Y=1] + 200P[Y=2] + 300P[Y=3] + 400P[Y>3]$$
$$= 100(2)e^{-2} + 200(2)e^{-2} + 300(4/3)e^{-2} + 400[1 - (1 + 2 + 2 + 4/3)e^{-2}]$$
$$= 400 - 1533.33e^{-2} = 192.$$

280. Solution: B

Let X be the number of burglaries. Then,

$$E(X \mid X \geq 2) = \frac{\sum_{x=2}^{\infty} xp(x)}{1 - p(0) - p(1)} = \frac{\sum_{x=0}^{\infty} xp(x) - (0)p(0) - (1)p(1)}{1 - p(0) - p(1)}$$

$$= \frac{1 - p(1)}{1 - p(0) - p(1)} = \frac{1 - e^{-1}}{1 - e^{-1} - e^{-1}} = 2.39.$$

281. Solution: A

The expected unreimbursed loss is

$$\int_0^d x \frac{1}{450} dx + \int_d^{450} d \frac{1}{450} dx = \frac{d^2}{900} + d \frac{450 - d}{450} = \frac{1}{900}(900d - d^2) = 56$$

$$d^2 - 900d + 50,400 = 0$$

$$d = \frac{900 \pm \sqrt{900^2 - 201,600}}{2} = \frac{900 - 780}{2} = 60.$$

282. Solution: D

Let X represent the loss due to the accident.
From the given information, the probability that X is in[0, b] is 0.75, which is larger than 0.5. So the median, 672, must lie in the interval [0, b].

Note that in a uniform distribution over an interval I, the probability of landing in an interval J is the length of the intersection of J and I, divided by the length of I.
Therefore, we have $0.5 = \mathrm{P}[X \leq 672] = 0.75\left(\frac{672 - 0}{b - 0}\right)$ which gives $b = 672\left(\frac{0.75}{0.5}\right) = 1008$.

From the law of total probability applied to means, the mean loss due to the accident is

$$\mathrm{E}(X) = \mathrm{P}[\text{minor acc.}]\mathrm{E}(X \mid \text{minor acc.}) + \mathrm{P}[\text{major acc.}]\mathrm{E}(X \mid \text{major acc.})$$

$$= 0.75\mathrm{E}(X \mid X \text{ is uniform on } [0, b]) + 0.25\mathrm{E}(X \mid X \text{ is uniform on } [b, 3b]).$$

$$= 0.75\left(\frac{0 + 1008}{2}\right) + 0.25\left(\frac{1008 + 3(1008)}{2}\right) = 882.$$

283. Solution: C

The claim amount distribution is a mixture distribution with 20% point mass at 0. To obtain the median, the remaining 30% probability is from the case where there is a non-zero payment. This corresponds to the 30/(1 – 0.2) = 37.5 percentile of the unconditional claim amount distribution. The 37.5 percentile of the standard normal distribution is at z = –0.3187 and thus the median is 1000 – 0.3187(400) = 873.

284. Solution: B

First, the *z*-score associated with the deductible is:

$$z = \frac{15,000 - 20,000}{4,500} = -1.11$$

Next, using the normal table, we find that the probability that a loss exceeds the deductible is:

$P(Z > -1.11) = 0.8665.$

The 95th percentile of losses that exceed the deductible is the 1 – 0.05(0.8665) = 0.9567 = 95.67th percentile of all losses.

The 95.67th percentile of all losses is between 1.71 and 1.72 standard deviations above the mean. To the nearest hundred, both of these correspond to a loss amount of 27,700.

285. Solution: C

The *z*-score corresponding to the 98th percentile is 2.054. The answer is 20 + 2.054(2) = 24.108.

286. Solution: C

Let *T* be the time of registration. Due to symmetry of the density function about 6.5. The constant of proportionality, *c*, can be solved from

$0.5 = \int_0^{6.5} c \frac{1}{t+1} dt = c \ln(t+1) \Big|_0^{6.5} = c \ln(7.5)$, which gives *c* = 0.5/ln(7.5).

Again using the symmetry, if 60th percentile of *T* is at *k*, then $P[T \le 13-k] = 0.4$. Thus,

$$0.4 = P[T \le 13-k] = \int_0^{13-k} \frac{0.5}{\ln(7.5)} \frac{1}{t+1} dt = \frac{0.5}{\ln(7.5)} \ln(14-k)$$

$$\ln(14-k) = 0.8 \ln(7.5) = 1.6119$$

$$14 - k = e^{1.6119} = 5.0124$$

$$k = 8.99.$$

287. Solution: E

The distribution function of *L* is $F(x) = 1 - e^{-\lambda x}$ and its variance is $1/\lambda^2$. We are given *F*(2) = 1.9*F*(1) and therefore,

$$1 - e^{-2\lambda} = 1.9(1 - e^{-\lambda})$$

$$(1 - e^{-\lambda})(1 + e^{-\lambda}) = 1.9(1 - e^{-\lambda})$$

$$e^{-\lambda} = 0.9$$

$$\lambda = -\ln(0.9) = 0.10536$$

$$Var(L) = 1/0.10536^2 = 90.1.$$

288. Solution: D

We have $Y = 0$ when $X < d$ and $Y = X - d$ otherwise. Then, noting that the second moment of an exponential random variable is twice the square of the mean,

$$E(Y) = \int_o^d 0(0.1e^{-0.1x})dx + \int_d^\infty (x-d)(0.1e^{-0.1x})dx = 0 + \int_0^\infty x(0.1e^{-0.1(x+d)})dx = e^{-0.1d}(10)$$

$$E(Y^2) = \int_o^d 0^2(0.1e^{-0.1x})dx + \int_d^\infty (x-d)^2(0.1e^{-0.1x})dx = 0 + \int_0^\infty x^2(0.1e^{-0.1(x+d)})dx = e^{-0.1d}(200)$$

$$\text{Var(Y)} = \text{e}^{-0.1d}(200) - [e^{-0.1d}(10)]^2 = 100[2e^{-0.1d} - e^{-0.2d}],$$

289. Solution: C

For the Poisson distribution the variance is equal to the mean and hence the second moment is the mean plus the square of the mean. Then,

$$E[X] = 0.1(1) + 0.5(2) + 0.4(10) = 5.1$$

$$E[X^2] = 0.1(1+1^2) + 0.5(2+2^2) + 0.4(10+10^2) = 47.2$$

$$Var(X) = 47.2 - 5.1^2 = 21.19.$$

290. Solution: C

Let X be the number of tornadoes and Y be the conditional distribution of X given that X is at least one. There are (at least) two ways to solve this problem. The first way is to begin with the probability function for Y and observe that starting the sums at zero adds nothing because that term is zero. Then note that the sums are the first and second moments of a regular Poisson distribution.

$$p(y) = P[Y = y] = P[X = y \mid X > 0] = \frac{P[X = y]}{P[X > 0]} = \frac{3^y e^{-3}/y!}{1-e^{-3}}, \quad y = 1, 2, \ldots$$

$$E(Y) = \frac{1}{1-e^{-3}}\sum_{y=1}^\infty y\frac{3^y e^{-3}}{y!} = \frac{1}{1-e^{-3}}\sum_{y=0}^\infty y\frac{3^y e^{-3}}{y!} = \frac{3}{1-e^{-3}}$$

$$E(Y^2) = \frac{1}{1-e^{-3}}\sum_{y=1}^\infty y^2\frac{3^y e^{-3}}{y!} = \frac{1}{1-e^{-3}}\sum_{y=0}^\infty y^2\frac{3^y e^{-3}}{y!} = \frac{3+3^2}{1-e^{-3}}$$

$$Var(Y) = \frac{12}{1-e^{-3}} - \left(\frac{3}{1-e^{-3}}\right)^2 = 2.6609.$$

The second way is to use formulas about conditional expectation based on the law of total probability.

$E(X) = E(X \mid X = 0)P[X = 0] + E(X \mid X > 0)P[X > 0]$

$3 = 0(e^{-3}) + E(X \mid X > 0)(1 - e^{-3})$

$E(X \mid X > 0) = \dfrac{3}{1 - e^{-3}} = 3.1572$

$E(X^2) = E(X^2 \mid X = 0)P[X = 0] + E(X^2 \mid X > 0)P[X > 0]$

$3 + 3^2 = 0(e^{-3}) + E(X^2 \mid X > 0)(1 - e^{-3})$

$E(X^2 \mid X > 0) = \dfrac{12}{1 - e^{-3}} = 12.6287$

$Var(X) = 12.6287 - 3.1572^2 = 2.6608.$

291. Solution: C

Let X represent individual expense. Then,

$Y = \begin{cases} 0, & 200 \le X \le 400 \\ X - 400, & 400 < X \le 900 \\ 500, & 900 < X \le 1200 \end{cases}$ and the density function of X is $f(x) = 0.001, \quad 200 \le x \le 1200.$

$$E(Y) = \int_{200}^{400} 0(0.001)dx + \int_{400}^{900} (\mathrm{x} - 400)(0.001)dx + \int_{900}^{1200} 500(0.001)dx$$

$$= 0 + 0.001\left.\frac{(x-400)^2}{2}\right|_{400}^{900} + 500(0.001)(1200 - 900) = 0 + 125 + 150 = 275$$

$$E(Y^2) = \int_{200}^{400} 0^2(0.001)dx + \int_{400}^{900} (\mathrm{x} - 400)^2(0.001)dx + \int_{900}^{1200} 500^2(0.001)dx$$

$$= 0 + 0.001\left.\frac{(x-400)^3}{3}\right|_{400}^{900} + 500^2(0.001)(1200 - 900) = 0 + 41,666.67 + 75,000 = 116,666.67$$

$Var(Y) = 116,666.67 - 275^2 = 41,041.67.$

292. Solution: D

Let X represent the loss.
The variance for a uniform distribution is the square of the interval length, divided by 12. Thus, $\text{Var}\ (X) = \dfrac{b^2}{12}$.
Let C represent the claim payment from the loss. Then $C = 0$ for $X < b/2$ and $C = X - b/2$, otherwise. Then,

$$E(C) = \int_0^{b/2} 0(1/b)dx + \int_{b/2}^{b}(x-b/2)(1/b)dx = 0 + (x-b/2)^2/(2b)\Big|_{b/2}^{b} = (b/2)^2/(2b) = b/8$$

$$E(C^2) = \int_0^{b/2} 0^2(1/b)dx + \int_{b/2}^{b}(x-b/2)^2(1/b)dx = 0 + (x-b/2)^3/(3b)\Big|_{b/2}^{b} = (b/2)^3/(3b) = b^2/24$$

$Var(C) = b^2/24 - (b/8)^2 = 5b^2/192.$
The ratio is $[5b^2/192]/[b^2/12] = 60/192 = 5/16.$

293. Solution: A

Let X be the profit random variable. Then, $0.05 = P(X < 0) = P(Z < -\mu/\sigma)$ and from the table, $-\mu/\sigma = -1.645.$ From the problem, $\sigma^2 = \mu^3$. Therefore, $-1.645 = -\mu/\mu^{3/2} = -\mu^{-1/2}$ and $\mu = 1/1.645^2 = 0.37.$ in billions, or 370 million.

294. Solution: A

$E[(X-1)^2] = E[X^2] - 2E[X] + 1 = 47$ so $E[X] = (61+1-47)/2 = 7.5$. The standard deviation is $\sqrt{E[X^2] - E[X]^2} = \sqrt{61 - 7.5^2} = 2.18.$

295. Solution: D

Let D be the number of diamonds selected and S be the number of spades. First obtain the hypergeometric probability $S = 0$:

$$P(S=0)=\frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}}=\frac{1(21)}{45}=\frac{7}{15}.$$

The required probability distribution is:

$$P(D=0\,|\,S=0)=\frac{P(D=0,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{0}\binom{3}{0}\binom{5}{2}}{\binom{10}{2}}=\frac{15}{7}\frac{1(1)(10)}{45}=\frac{10}{21}$$

$$P(D=1\,|\,S=0)=\frac{P(D=1,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{1}\binom{3}{0}\binom{5}{1}}{\binom{10}{2}}=\frac{15}{7}\frac{2(1)(5)}{45}=\frac{10}{21}$$

$$P(D=2\,|\,S=0)=\frac{P(D=2,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{2}\binom{3}{0}\binom{5}{0}}{\binom{10}{2}}=\frac{15}{7}\frac{1(1)(1)}{45}=\frac{1}{21}.$$

Then,

$$E(D\,|\,S=0)=0(10/21)+1(10/21)+2(1/21)=12/21=4/7$$

$$E(D^2\,|\,S=0)=0^2(10/21)+1^2(10/21)+2^2(1/21)=14/21=2/3$$

$$Var(D\,|\,S=0)=2/3-(4/7)^2=50/147=0.34.$$

296. Solution: A

Let X represent the loss and Y represent the claim payment. Note that $Y=\max\left(0,X-3\right)$.

Then,

$$M(t)=E(e^{tY})=\int_0^3 e^{t(0)}(0.1)dx+\int_3^{10}e^{t(x-3)}(0.1)dx=0.3+e^{t(x-3)}\frac{1}{t}(0.1)\Big|_3^{10}=0.3+0.1\frac{e^{7t}-1}{t}=\frac{3}{10}+\frac{e^{7t}-1}{10t}.$$

297. Solution: C

$M(t)M(5t)$ is the mgf of the sum of X and an independent random variable Y with the same distribution as $5X$.
$2M(t)$ is not an mgf because $M(0) = 1$ and thus $2M(0) = 2$. But the mgf at 0 must be 1, so this is not an mgf.
$e^t M(t)$ is the mgf of the sum of X and an independent random variable Y that takes on the value 1 with certainty.

298. Solution: A

Let X = # of accidents and Y = # of unreimbursed accidents.
Then $Y = \max(0, X - 1)$ and
$\Pr(Y=0) = \Pr(\max(0, X-1)=0) = \Pr(X<2) = e^{-0.1} + 0.1e^{-0.1} = 1.1e^{-0.1}$.
For $Y > 0$,

$$\Pr(Y=y) = \Pr(\max(0, X-1)=y) = \Pr(X=y+1) = \frac{(0.1)^{y+1}e^{-0.1}}{(y+1)!}.$$

299. Solution: B

$$\Pr(Y=k) = \int_{k-1}^{k} \lambda e^{-\lambda x}dx = -e^{-\lambda x}\Big|_{k-1}^{k} = e^{-(k-1)\lambda} - e^{-k\lambda} = e^{-k\lambda}(\mathrm{e}^{\lambda}-1) \text{ for } k = 1, 2, 3, \ldots$$

300. Solution: E

Let X and Y be the lifetime random variables for batteries A and B respectively. The joint density is $f(x,y) = m^{-2}e^{-(x+y)/m}$. Then,

$$0.33 = P(Y > X+1) = \int_0^\infty \int_{x+1}^\infty m^{-2}e^{-(x+y)/m}dydx = \int_0^\infty m^{-1}e^{-(x+y)/m}\Big|_{x+1}^\infty dx$$

$$= \int_0^\infty m^{-1}e^{-(2x+1)/m}dx = 0.5e^{-(2x+1)/m}\Big|_0^\infty = 0.5e^{-1/m}$$

$$0.66 = e^{-1/m}$$

$$\ln(0.66) = -1/m$$

$$m = -1/\ln(0.66) = 2.41.$$

Alternatively, using the memoryless property,

$$0.33 = P(Y > X+1) = P(Y > X+1 \text{ and } Y > X) = P(Y > X+1 \mid Y > X)P(Y > X)$$

$$= P(Y > 1)(0.5) = 0.5e^{-1/m}.$$

301. Solution: A

To be delayed over three minutes, either the car or the bus must arrive between 7:20 and 7:22. The probability for each is 2/15. The probability they both arrive in that interval is (2/15)(2/15). Thus, the probability of at least one being delayed is 2/15 + 2/15 – (2/15)(2/15) = 56/225 = 0.25.

302. Solution: A

$$
\begin{aligned}
&P(X \geq 80 \text{ and } Y \geq 80) = 1 - P(X < 80 \text{ or } Y < 80) \\
&= 1 - [P(X < 80) + P(Y < 80) - P(X < 80 \text{ and } Y < 80) \\
&= 1 - [F(80,100) + F(100,80) - F(80,80)] \\
&= 1 - [80(100)(180) + 100(80)(180) - 80(80)(160)] / 2,000,000 = 0.072.
\end{aligned}
$$

303. Solution: C

The probability that a skateboarder makes no more than two attempts is the probability of being injured on the first or second attempt, which is $p + (1-p)p = 2p - p^2$. Then,

$$
\begin{aligned}
&0.0441 = F(2,2) = (2p - p^2)^2 \\
&0.21 = 2p - p^2 \\
&p^2 - 2p + 1 = 0.79 \\
&(p-1)^2 = 0.79 \\
&p - 1 = \pm 0.88882 \\
&p = 0.11118.
\end{aligned}
$$

The probability that a skateboarder makes no more than one attempt is *p* while the probability of making no more than five attempts is the complement of having no injuries on the first five attempts. Hence,

$$F(1,5) = p[1-(1-p)^5] = 0.0495.$$

304. Solution: B

$$P(X = 3, Y = 3)) = F(3,3) - F(2,3) - F(3,2) + F(2,2) = 0.9360 - 0.8736 - 0.9300 + 0.8680 = 0.0004$$

305. Solution: D

Let X denote the number of deaths next year, and S denote life insurance payments next year. Then S = 50,000X, where $X \sim$ Bin(1000, 0.014). Therefore,

$$
\begin{aligned}
&E(S) = E(50,000X) = 50,000(1000)(0.014) = 700,000 \\
&Var(D) = Var(50,000X) = 50,000^2(1000)(0.014)(0.986) = 34,510,000,000 \\
&StdDev(S) = 185,769.
\end{aligned}
$$

The 99th percentile is 700,000+185,769(2.326)= 1,132,099, which rounds to 1,150,000.

306. Solution: E

Let X_k be the random change in month *k*. Then $E(X_k) = (0.5)(1.1) + 0.5(-0.9) = 0.1$ and $Var(X_k) = 0.5(1.1)^2 + 0.5(-0.9)^2 - (0.1)^2 = 1$. Let $S = \sum_{k=1}^{100} X_k$. Then, *E*(*S*) = 100(0.1) = 10 and *Var*(*S*) = 100(1) = 100. Finally,

$$P(100 + S > 91) = P(S > -9) \doteq P\left(Z > \frac{-9-10}{\sqrt{100}} = -1.9\right) = 0.9713.$$

307. Solution: B

Let *X* be the number of accidents. Then,

$$P[X = k \mid \lambda] = \frac{\lambda^k}{k!}e^{-\lambda}$$

$$P[X = 1] = \int_0^3 P[X = 1 \mid \lambda] f(\lambda) d\lambda = \int_0^3 \lambda e^{-\lambda}(1/3) d\lambda$$

$$= (1/3)(-\lambda e^{-\lambda} - e^{-\lambda})\Big|_0^3 = (1/3)(-3e^{-3} - e^{-3} + 0 + 1) = 0.267.$$

308. Solution: E

X has an exponential distribution with mean 8 and variance 64. The second moment is 128. The mean and second moment of *Z* are both 0.45. Then (using the independence of *X* and *Z*),

$$E(ZX) = E(Z)E(X) = 0.45(8)$$

$$E[(ZX)^2] = E(Z^2)E(X^2) = 0.45(128) = 57.6$$

$$Var(ZX) = 57.6 - 3.6^2 = 44.64.$$

309. Solution: D

Let *X* and *Y* be the coordinates of the resident. The distance to the origin is *X* + *Y*. The expected distance is

$$E(X+Y) = \int_0^1\int_0^1 (x+y)(1.5)(x^2+y^2)dydx = 1.5\int_0^1\int_0^1 (x^3 + xy^2 + x^2y + y^3)dydx$$

$$= 1.5\int_0^1 x^3y + xy^3/3 + x^2y^2/2 + y^4/4\Big|_0^1 dx$$

$$= 1.5\int_0^1 (x^3 + x/3 + x^2/2 + 1/4)dx$$

$$= 1.5(x^4/4 + x^2/6 + x^3/6 + x/4)\Big|_0^1$$

$$= 1.5(1/4 + 1/6 + 1/6 + 1/4) = 15/12 = 5/4.$$

Alternatively, by symmetry, *E*(*X*+*Y*) = *E*(*X*) + *E*(*Y*) = 2*E*(*X*) and then the appropriate integral can be evaluated.

310. Solution: B

Each (x,y) pair has probability 1/25. There are only three possible benefit amounts:
0: Occurs only for the pair (0.0) and so the probability is 1/25.
50: Occurs for the three pairs (0,1), (1,0), and (1,1) and so the probability is 3/25.
100: Occurs in all remaining cases and so the probability is 21/25.
The expected value is 0(1/25) + 50(3/25) + 100(21/25) = 2250/25 = 90.

311. Solution: C

Let X be the property damage loss and Y be the bodily injury loss. Because the original distribution is uniform, the conditional distribution is also uniform, but on the union of the regions $0<x<1$, $0<y<3$ and $1<x<3$, $0<y<1$. The area of the union is 1(3) + 2(1) = 5 and so the density is 0.2. Then,

$$\begin{aligned} E(Y) &= \int_0^1 \int_0^3 y(0.2)dydx + \int_1^3 \int_0^1 y(0.2)dydx \\ &= \int_0^1 0.9dx + \int_1^3 0.1dx = 0.9 + 0.2 = 1.1. \end{aligned}$$

312. Solution: B

The marginal distribution for the probability of a given number of hospitalizations can be calculated by adding the columns. Then $p(0) = 0.915$, $p(1) = 0.072$, $p(2) = 0.012$, and $p(3) = 0.001$. The expected value is 0.915(0) + 0.072(1) + 0.012(2) + 0.001(3) = 0.099.

313. Solution: B

Let Z be the number of accidents. Z has a binomial distribution with $n = 30$ and $p = 0.03$. Given Z, the number of major accidents, X, is binomial with $n = Z$ and $p = 0.01$ and $Y = Z – X$. Then,

$$\begin{aligned} M_{X,Y}(s,t) &= E(e^{sX+tY}) = E\left[E(e^{sX+tY}) \mid Z\right] = E\left[E(e^{(s-t)X+tZ}) \mid Z\right] \\ &= E\left[e^{tZ}M_{X|Z}(s-t)\right] = E\left[e^{tZ}(0.01e^{s-t}+0.99)^Z\right] \\ &= E\left[e^{tZ}e^{Z\ln(0.01e^{s-t}+0.99)}\right] = M_Z(t+\ln(0.01e^{s-t}+0.99)) \\ &= \left(0.03e^{t+\ln(0.01e^{s-t}+0.99)}+0.97\right)^{30} \\ &= \left(0.03e^t(0.01e^{s-t}+0.99)+0.97\right)^{30} \\ &= \left(0.0003e^s+0.0297e^t+0.97\right)^{30}. \end{aligned}$$

Alternatively, recognize that X, Y, and $30 – X – Y$ has a trinomial distribution with 30 trials and success probabilities 0.0003, 0.0297, and 097. The answer is the joint moment generating function for a trinomial distribution.

314. Solution: C

For any value of r, Y has a uniform conditional distribution. The range, and thus the variance, is largest when $r = 0$. At this value, the conditional distribution of Y is uniform on (–1, 1). The variance is the square of the range divided by 12, which is 2(2)/12 = 1/3.

315. Solution: E

Let S be the speed and X be the loss. Given S, X has an exponential distribution with mean 3X. Then, noting that the variance of an exponential random variable is the square of the mean, the variance of a uniform random variable is the square of the range divided by 12, and for any random variable the second moment is the variance plus the square of the mean:

$$\begin{aligned} Var(X) &= Var[E(X \mid S)] + E[Var(X \mid S) \\ &= Var[3S] + E(9S^2) \\ &= 9(20-5)^2/12 + 9[(20-5)^2/12 + 12.5^2] \\ &= 1743.75. \end{aligned}$$

316. Solution: C

The four possible outcomes for which $X + Y = 3$ are given below, with their probabilities.

$$(0,3): e^{-1.7}\frac{2.3^3 e^{-2.3}}{3!} = 2.0278e^{-4}$$

$$(1,2): \frac{1.7e^{-1.7}}{1!}\frac{2.3^2 e^{-2.3}}{2!} = 4.4965e^{-4}$$

$$(2,1): \frac{1.7^2 e^{-1.7}}{2!}\frac{2.3\ e^{-2.3}}{1!} = 3.3235e^{-4}$$

$$(3,0): \frac{1.7^3 e^{-1.7}}{3!}e^{-2.3} = 0.8188e^{-4}.$$

The conditional probabilities are found by dividing the above probabilities by their sum. They are, 0.1901, 0.4215, 0.3116, 0.0768, respectively. These apply to the $X – Y$ values of –3, –1, 1, and 3. The mean is –3(0.1901) –1(0.4215) + 1(0.3116) + 3(0.0768) = –0.4498. The second moment is 9(0.1901) + 1(0.4215) + 1(0.3116) + 9(0.0768) = 3.1352. The variance is 2.9329.

317. Solution D

Let X be the hurricane damage and Y the intensity. Then, noting that the variance of an exponential random variable is the square of its mean and the variance of a uniform random variable is the square of the range divided by 12,

$$\begin{aligned} Var(X) &= Var[E(X \mid Y)] + E[Var(X \mid Y)] \\ &= Var(Y) + E(Y^2) \\ &= (3-0)^2/12 + [(3-0)^2/12 + 1.5^2] \\ &= 3.75. \end{aligned}$$

318. Solution: A

The marginal distribution of X has probability 1/5 + a at 0, 2a + b at 1, and 1/5 + b at 2. Due to symmetry, the mean is 1 and so the variance is $(0-1)^2(1/5+a)+(1-0)^2(1/5+a)=2/5+2a$ which is minimized at a = 0. The marginal distribution of Y is the same as that of X and thus has the same variance, 2/5 + 0 = 2/5.

319. Solution: B

The conditional distribution is uniform over the triangle that connects (0,10), (0,0), and (10,0). The area of this triangle is 10(10)/2 = 50 and so the density function is 1/50. The calculations are:

$$E(X)=E(Y)=\int_0^{10}\int_0^{10-x}\frac{1}{50}xdydx=\int_0^{10}\frac{x(10-x)}{50}dx=\int_0^{10}\frac{10x-x^2}{50}dx=10-\frac{20}{3}=\frac{10}{3}$$

$$E(XY)=\int_0^{10}\int_0^{10-x}\frac{1}{50}xydydx=\int_0^{10}\frac{x(10-x)^2}{100}dx=\int_0^{10}\frac{x^3-20x^2+100x}{100}dx=25-\frac{200}{3}+50=\frac{25}{3}$$

$$Cov(X,Y)=E(XY)-E(X)E(Y)=\frac{25}{3}-\frac{10}{3}\frac{10}{3}=-\frac{25}{9}.$$

320. Solution: C

$$Cov(X,Y)=E(XY)-E(X)E(Y)=E(X^3)-E(X)E(X^2)$$
$$E(X)=E(X^3)=(1/3)(-1+0+1)=0$$
$$E(X^2)=(1/3)(1+0+1)=2/3$$
$$Cov(X,Y)=0-0(2/3)=0.$$

They are dependent, because

$$\Pr(X=0,Y=0)=\Pr(X=0,X^2=0)=\Pr(X=0)=1/3$$
$$\Pr(X=0)\Pr(Y=0)=(1/3)(1/3)=1/9\neq 1/3.$$

321. Solution: B

Let X and Y be the two independent losses and Z = min(X,Y). Then,

$$\Pr(Z>z)=\Pr(X>z\text{ and }Y>z)=\Pr(X>z)\Pr(Y>z)=e^{-z}e^{-z}=e^{-2z}$$
$$F_Z(z)=\Pr(Z\le z)=1-\Pr(Z>z)=1-e^{-2z},$$

which can be recognized as an exponential distribution with mean 1/2.

322. Solution: B

Let X and Y be the miles driven by the two cars. The total cost, is then C = 3(X/15 + Y/30) = 0.2X + 0.1Y. C has a normal distribution with mean 0.2(25) + 0.1(25) = 7.5 and variance 0.04(9) + 0.01(9) = 0.45. Then, $\Pr(C<7)=\Pr(Z<(7-7.5)/\sqrt{0.45}=-0.7454)=0.23$.

323. Solution: B

Let X denote the first estimate and Y the second. Then, $\Pr(X > 1.2Y) = \Pr(X - 1.2Y > 0)$. $W = X - 1.2Y$ has a normal distribution with mean 1(10b) – 1.2(10b) = –2b and variance $1^2 b^2 + 1.2^2 b^2 = 2.44b^2$. Then, $\Pr(W > 0) = \Pr(Z > (0 + 2b)/\sqrt{2.44b^2} = 1.280) = 0.100.$

324. Solution: C

Let μ be the common mean. Then the standard deviations of X and Y are 3μ and 4μ respectively. The mean and variance of (X + Y)/2 are then $(\mu + \mu)/2 = \mu$ and $[(3\mu)^2 + (4\mu)^2]/4 = 25\mu^2/4$ respectively. The coefficient of variation is $\frac{\sqrt{25\mu^2/4}}{\mu} = 5/2.$

325. Solution: D

Var(Z) = Var(3X + 2Y – 5) = 9Var(X) + 4Var(Y) = 9(3) + 4(4) = 43.

326. Solution: E

The mean is the weighted average of the three means: 0.1(20) + 0.3(15) + 0.6(10) = 12.5. The second moment is the weighted average of the three second moments (each of which is the square of the mean plus the mean, for a Poisson distribution): 0.1(420) + 0.3(240) + 0.6(110) = 180. The variance is the second moment minus the square of the mean, which is 23.75.

327. Solution: B

Let F be the number of fillings and R be the number of root canals. The total claim for a given policyholder, C, in a year is C = 50F + 0.7(500R) = 50F + 350R.

We have $E(F)$ = 0.6(0) + 0.2(1) + 0.15(2) + 0.05(3) = 0.65 and $E(R)$ = 0.8(0) + 0.2(1) = 0.2. Then, $E(C)$ = 50(0.65) + 350(0.2) = 102.50.

328. Solution: E

The probability generating function (pgf) of a random variable is the expectation of t to the power of the random variable. Furthermore, if the random variable takes on only non-negative integer values, evaluating this function at $t = 0$ gives the probability that the random variable is 0. Thus, $\mathrm{P}[X=0] = P_X(0) = e^{-0.2(1-0)} = e^{-0.2}$.

Also, $\mathrm{P}[Y=0] = \mathrm{P}[X=0] + \mathrm{P}[X=1]$, and for $y > 0$, $\mathrm{P}[Y=y] = \mathrm{P}[X=1+y]$.

Therefore, we have

$$\begin{aligned}
P_Y(t) = \sum_{y=0}^{\infty} t^y \mathrm{P}[Y=y] = \mathrm{P}[X=0] + \mathrm{P}[X=1] + \sum_{y=1}^{\infty} t^y \mathrm{P}[X=1+y] \\
&= \mathrm{P}[X=0] + \sum_{y=0}^{\infty} t^y \mathrm{P}[X=1+y] \\
&= \mathrm{P}[X=0] + \sum_{x=1}^{\infty} t^{x-1} \mathrm{P}[X=x] \\
&= \mathrm{P}[X=0] + \frac{1}{t}\sum_{x=1}^{\infty} t^{x} \mathrm{P}[X=x] \\
&= \mathrm{P}[X=0] + \frac{1}{t}\left(-\mathrm{P}[X=0] + \sum_{x=0}^{\infty} t^{x} \mathrm{P}[X=x]\right) \\
&= e^{-0.2} + \frac{1}{t}\left(-e^{-0.2} + e^{-0.2(1-t)}\right) = \frac{e^{-0.2}(t-1) + e^{-0.2(1-t)}}{t}
\end{aligned}$$

.

329. Solution: A

The coefficient of t^2 is the probability that there are two hurricanes. That is:

$$\frac{0.3^2 e^{-0.3}}{2!} = 0.03333682.$$

330. Solution: B

Due to the memoryless property of the exponential distribution, the distribution of the reimbursement given that there is a payment is exponential with the same parameter. Thus $0.5 = F(6000) = 1 - e^{-6000/\lambda}$ which implies that $\lambda = 8656.17$. The solution is

$$F(9000) - F(3000) = \left(1 - e^{-9000/8656.17}\right) - \left(1 - e^{-3000/8656.17}\right) = 0.35.$$

331. Solution: E

Using the formulas for the variance and mean of the uniform distribution:

$$E(X^2) = Var(X) + E(X)^2 = \frac{(100-a)^2}{12} + \left(\frac{100+a}{2}\right)^2 = \frac{100^2 - 200a + a^2 + 3(100)^2 + 600a + 3a^2}{12}$$

$$= \frac{40,000 + 400a + 4a^2}{12} = \frac{19,600}{3}$$

$$0 = 40,000 - 78,400 + 400a + 4a^2$$

$$0 = a^2 + 100a - 9,600$$

$$0 = (a-60)(a+160)$$

$$a = 60$$

Then, Y is uniform on the interval 1.25(60) = 75 to 100. The 80th percentile is 75 + 0.8(25) = 95.

332. Solution: C

$$P\left[Y > \frac{1}{3} \;\middle|\; X > \frac{2}{3}\right] = \frac{P(Y > 1/3, X > 2/3)}{P(X > 2/3)}$$

$$f(x) = \int_0^x 10x^2 y\,dy = 5x^2 y^2\Big|_0^x = 5x^4,\ 0 < x < 1$$

$$P(X > 2/3) = \int_{2/3}^1 5x^4 dx = x^5\Big|_{2/3}^1 = 1 - (2/3)^5 = 1 - 32/243 = 211/243$$

$$P(Y > 1/3, X > 2/3) = \int_{2/3}^1 \int_{1/3}^x 10x^2 y\,dy\,dx = \int_{2/3}^1 5x^2 y^2\Big|_{1/3}^x dx = \int_{2/3}^1 5x^4 - 5x^2/9\,dx$$

$$= x^5 - 5x^3/27\Big|_{2/3}^1 = 1 - 5/27 - 32/243 + 40/729 = 538/729$$

$$P\left[Y > \frac{1}{3} \;\middle|\; X > \frac{2}{3}\right] = \frac{538/729}{211/243} = \frac{538}{633} = 0.8499$$