The Infinite Actuary's
Exam FM
Sample Exam 2

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Take this sample exam under strict exam conditions. Start a timer for 3 hours and stop immediately when the timer is done. Do not stop the clock when you go to the bathroom. Do not look at your notes. Do not look at the answer key.

This exam contains 35 questions. Do not spend too much time on any one question. Choose the best available answer for each question.

1. An investor purchases a 10-year 1000 par value bond to yield 5% annually. The bond pays 3% annual coupons. The investor reinvests the coupons in an account paying 4% effective annual interest.

   Find the annualized yield rate for the investor over the ten-year period.

   A. 4.47% B. 4.57% C. 4.67% D. 4.77% E. 4.87%

2. Today you decide to buy a new BMW to celebrate passing Exam FM. To pay for the car you take out a loan with the following details:

   (i) The cost of the car is \$50,000.

   (ii) You will make monthly payments of \$1454.06 with the first payment due one month from today.

   (iii) The APR (nominal annual rate convertible monthly) is 3%.

   Calculate the amount of principal paid in the 15th car payment.

   A. 1356 B. 1366 C. 1376 D. 1386 E. 1396

3. Which of the following are true?

1. Modified duration is greater for bonds with higher par values.
2. Modified duration is greater for equivalent yields compounded more frequently.
3. The modified duration of a single cashflow is the time remaining until the cashflow.

A. 1 only
B. 2 only
C. 3 only
D. 2 and 3 only
E. The correct answer is not given by A, B, C or D.

4. Your company is considering an investment in two projects:

   A. Requires an investment of 500 now and 500 one year from now. Company will receive 1300 two years from now.

   B. Requires an investment of 700 one year from now. Company will receive 900 two years from now.

   At an annual effective rate of i the net present value of the two projects are equal. Determine i.

   A. 0.1100 B. 0.1150 C. 0.1155 D. 0.1160 E. 0.1165

5. An annuity-immediate has annual payments of \$7000, \$5000 and \$3000.

   Determine the convexity of the annuity at $i = 5\%$.

   A. 4.7 B. 4.8 C. 4.9 D. 5.0 E. 5.1

6. Susie begins saving \$100 at the end of each month from her salary in an account starting in January. The account pays interest at an effective monthly rate of 0.73%. At the end of each December, Susie gets a Christmas bonus of \$1000, which she also saves in the same account.

   What is the approximate value of Susie's account after 5 years of such savings?

   A. \$12,500 B. \$13,000 C. \$13,500 D. \$14,000 E. \$14,500

7. A 4-year 100 par value bond with 5% annual coupons redeems at par. The table contains values from the zero-coupon yield curve.

Term	Yield
1	3%
2	4%
3	5%
4	5.5%

Find the price of the bond.

A. 97.5 B. 98 C. 98.5 D. 99 E. 99.5

8. You are given the following information about an n-year bond:

(i) The par value is 100

(ii) It pays annual coupons of 4.4%

(iii) The redemption value is 105

(iv) The adjustment to book value in the next-to-last coupon is 1.42

(v) The annual yield rate i of the bond is less than 30%

Find i.

A. 5.4% B. 5.7% C. 6.0% D. 6.3% E. 6.6%

9. A company has one liability, due at time 6, with a present value of 1000. Two assets are available:

   - a zero-coupon bond expiring at time T and
   - a zero-coupon bond expiring at time $2T$.

   If the company invests equal amounts in each bond and full immunization is achieved, find T.

   A. 4 B. 4.5 C. 5 D. 5.5 E. No value of T can achieve full immunization

10. Ellen enters into a deferred swap on January 1, 2017. The first swap period will begin two years later, and once the swap begins, it will last for two years. Ellen has agreed to pay a level swap rate of 4.4055%, and she will receive the floating market rate. The notional amount of the swap is 100,000, and annual swap payments will be made.

    On January 1, 2018, the spot rates are:

Term (in years)	Spot rate
1	2.5%
2	3.2%
3	3.7%
4	4.1%
5	4.3%

    Find the market value of the swap for Ellen on January 1, 2018.

    A. -350 B. -200 C. -50 D. 100 E. 250

11. Deposits are made into a fund at the beginning of every year for five years. Seven and one-half years after the final deposit, annual withdrawals begin of an amount equal to the size of the original deposits.

The fund earns an annual effective rate of 8%.

How many full withdrawals can be made?

A. 18 B. 19 C. 20 D. 21 E. 22

12. Bob loans \$10,000 to Jim at a nominal annual rate of 8%, convertible monthly.

You are given the following information:

1. Jim makes payments at the beginning of each month, starting one month after the loan is made.
2. Jim's first payment is \$7. Each subsequent payment is \$7 more than the previous one. This pattern continues through the first 59 payments.
3. Jim's 60th payment pays off the remaining outstanding balance and any interest accrued during the 60th period.
4. Bob reinvests each payment in a bank account paying 6% nominal annual interest, convertible monthly.

How much money does Bob have in the bank account immediately after depositing Jim's final payment?

A. \$14,400 B. \$14,500 C. \$14,600 D. \$14,700 E. \$14,800

13. A \$10,000 loan is to be repaid by equal payments at the end of each year for ten years. If $i = 0.08$ for the first five years and $i = 0.06$ for the last five years, find the outstanding balance just before the seventh payment.

A. 5050 B. 5100 C. 5250 D. 5300 E. 5350

14. An investor deposits 100 in an investment account on January 1. The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit or Withdrawal	Deposit	Withdrawal
March 1	110		50
July 1	X	60	
October 1	120		

The balance at the end of the year is 120. What is the smallest value of X such that the time-weighted return is non-negative?

A. 50 B. 55 C. 60 D. 65 E. Internal balances do not matter for time-weighted return

15. A 100 par value bond with semi-annual coupons is purchased for 86.82 to yield a nominal annual rate of 5% convertible semiannually. The annual coupon rate is 4%. The bond redeems for C at the end of 30 years.

Find the redemption value C.

A. 95 B. 100 C. 105 D. 110 E. 115

16. Bob takes out a 30-year loan with level monthly payments at a nominal annual rate of 5%, convertible monthly. After 10 years, he refinances the loan to a 20-year loan with level monthly payments at interest rate i. The monthly payment after refinancing is 10% less than before refinancing. Find i, expressed as a nominal annual rate convertible monthly.

A. 2.24% B. 2.58% C. 3.02% D. 3.77% E. 4.10%

17. An amount X is invested in a fund earning simple interest of i. Which of the following are true?

1. The effective rate of interest is constant for each period.
2. The amount of interest is constant for each period.
3. The equivalent force of interest is $\ln(1+i)$.

A. 1 only
B. 2 only
C. 3 only
D. 2 and 3 only
E. The correct answer is not given by A, B, C or D.

18. You are given the following prices for $100 zero-coupon bonds:

Term	Price
1	95.24
2	89.00
3	81.63

R is the swap rate in a 3-year swap contract with a constant notional amount.

Determine R.

A. 5.7% B. 6.1% C. 6.5% D. 6.9% E. 7.2%

19. A company wishes to achieve full immunization of their liabilities.

- They have two liability cashflows in the future: 1000 at time 3, 1000 at time 4.
- Two assets are available: zero-coupon bonds expiring at times 2 and 6.
- The annual effective interest rate used to value the assets and the liabilities is 5%.

Find the sum of the face values of the zero-coupon bonds needed to achieve full immunization.

A. 1992 B. 2000 C. 2008 D. 2016 E. 2024

20. A loan of L is being repaid using $2n$ semi-annual payments, with the payments made at the end of each six-month period.

- The first n payments are in the amount of $2R$.
- The second n payments are in the amount of R.

The loan has an annual interest rate of i convertible semi-annually.

Which expression below gives the principal repaid by the n payments of $2R$?

A. $2R\, a_{\overline{n}|i/2}$

B. $2R\, s_{\overline{n}|i/2} - R\, a_{\overline{n}|i/2}$

C. $L - R\, a_{\overline{n}|i/2}$

D. $L(1+\frac{i}{2})^n - R\, a_{\overline{n}|i/2}$

E. $2R\, s_{\overline{n}|i/2}$

21. Which of the following is true for Redington immunization?

I. Matching the convexities of asset cashflows and liability cashflows is a required condition.

II. A flat yield curve with parallel shifts is assumed for immunization protection to work as expected.

III. Asset cashflows occurring both before and after each liability cashflow is a required condition.

A. I only
B. II only
C. III only
D. I and II only
E. None of the above

22. Fly-by-Night Airlines must pay a liability of $100,000 at the end of 5 years. The following zero-coupon bonds are available for investment:

Maturity	Annual Effective Yield
1 Year	3.0%
2 Years	3.5%
3 Years	4.0%
4 Years	4.2%

Assume the yield curve will not change over the next 5 years. What is the minimum cost for Fly-by-Night Airlines to exactly match its liability?

A. $81,349 B. $82,355 C. $82,989 D. $86,261 E. Not enough information.

23. A father is arranging some payments to his two sons. He has \$10,000, which is being split between two accounts.

    1. The first account pays 5% effective annual interest. Its interest will accumulate, and at the end of each year the accumulated interest will be paid in a single payment. The payments will be made alternately to the first son and the second son, beginning with the first son. The payments will continue indefinitely.

    2. The second account earns 5% effective annual interest and will be given entirely to the second son in one payment ten years from now.

    Find the amount that should be deposited right now in the second account to produce equal present values for the two sons. The present values should be calculated using an annual effective rate of 6%.

    A. \$130 B. \$260 C. \$680 D. \$1250 E. \$4935

24. Two perpetuities currently have the same price. The first perpetuity is an arithmetic increasing annuity paying 1 after 1 year, 2 after 2 years, and so on. The second perpetuity is a geometric increasing annuity which increases by 5% each year. Its first payment occurs after 1 year, and the payment amount is 4.

    The effective annual interest rate i is less than 20%.

    Find the current price of the perpetuities.

    A. 120 B. 240 C. 360 D. 480 E. 600

25. You are given the following forces of interest:

Year	Rate
1	5%
2	5%
3	6%
4	4%

$\$X$ is invested at time 0 and grows to \$1000 at time 4. Determine $\$X$.

A. \$808.76 B. \$812.31 C. \$814.42 D. \$818.73 E. \$822.78

26. Annual payments are made into a fund earning 7% annual effective interest. The payments are \$10,000 at the end of year one, increasing by \$500 per year in the second through the tenth years. After the tenth year, each payment increases by 3.5% over the prior payment.

Calculate the accumulated value of the fund at the end of 20 years, rounded to the nearest thousand.

A. 287,000 B. 357,000 C. 451,000 D. 495,000 E. 564,000

27. You are given the following information:

1. Michael takes out a loan on January 1, 2014.
2. He will repay the loan with 20 annual level payments.
3. The first payment occurs on January 1, 2015.
4. The 8th payment has equal parts interest and principal.

What is the effective annual interest rate on the loan?

A. 4.47% B. 5.48% C. 5.77% D. 6.12% E. 7.03%

28. A $1000 bond paying semiannual coupons at 10% is redeemable at the option of the seller on any coupon date on or after July 1, 2018. The bond is due on July 1, 2032. The purchase price was determined on July 1, 2007 to guarantee the purchaser a yield of at least 8% compounded semiannually.

    What is the maximum possible nominal annual yield convertible semiannually the bond purchaser can earn?

    A. 8% B. 8.2% C. 8.4% D. 8.6% E. 8.8%

29. Suppose account #1 earns simple interest at 8%, and account #2 earns interest at 6% compounded quarterly. What is the first year for which the effective rate of discount of account #1 is less than the effective rate of discount of account #2?

A. second year B. third year C. fourth year D. fifth year E. sixth year

30. You are given the following spot rates:

Year	Spot Rate
1	8%
2	9%
3	10%

Calculate the 1-year forward rate deferred two years.

A. 8% B. 9% C. 10% D. 11% E. 12%

31. You are given the following information about an n-year, 1000 par value bond:

(i) It is purchased for 1011.35 to yield an effective annual rate of i

(ii) It pays annual coupons, with annual coupon rate $r = i$

(iii) $v_i^n = 0.2838$

(iv) It has a redemption value of C.

Find C.

A. 1000 B. 1011 C. 1025 D. 1040 E. 1060

32. At a nominal discount rate of d convertible semiannually, an investment of \$100 immediately and \$200 ten years from now will accumulate to \$964.35 twenty years from now.

Calculate d.

A. 0.077 B. 0.078 C. 0.079 D. 0.080 E. 0.081

33. An annuity-immediate makes quarterly payments of 50, 75, 100, and 125 every year for 5 years. The nominal annual rate is 12% compounded quarterly. Which of the following is *not* an expression for the accumulated value of this annuity immediately following the last payment?

A. $25s_{\overline{20}|.03} + 25\,(Is)_{\overline{4}|.03}\ s_{\overline{5}|.1255}$
B. $\left[25s_{\overline{4}|.03} + 25\,(Is)_{\overline{4}|.03}\right]\ s_{\overline{5}|.1255}$
C. $\left[25a_{\overline{20}|.03} + 25\,(Ia)_{\overline{4}|.03}\ \ddot{a}_{\overline{5}|.1255}\right]\,(1.1255)^5$
D. $\left[150s_{\overline{4}|.03} - 25\,(Ds)_{\overline{4}|.03}\right]\ s_{\overline{5}|.1255}$
E. $\left[150\ddot{a}_{\overline{4}|.03} - 25\,(D\ddot{a})_{\overline{4}|.03}\right]\ \ddot{s}_{\overline{5}|.1255}$

34. Consider two bonds:

- Bond A: n-year term and price P_A
- Bond B: $3n$-year term and price P_B

Both bonds have the same par value, coupon rate, and yield-to-maturity. The present values of the redemption amounts are also the same for these two bonds.

Find $P_B - P_A$.

A. $Fr\, a_{\overline{3n}|i}$
B. $Fr\, a_{\overline{2n}|i}$
C. $Fr\, a_{\overline{2n}|i}\, v^n$
D. $Fr\, a_{\overline{n}|i}\, v^{2n}$
E. $Fr\, a_{\overline{n}|i}(1 + v^n)$

35. Which of the following are goals of the monetary policy of the Federal Open Market Committee?

I. Low unemployment

II. Moderate deflation

III. Moderate long-term interest rates

A. I and II only
B. I and III only
C. II and III only
D. I, II and III
E. None of the above