

# betaMC: Staging

Ivan Jacob Agaloos Pesigan

## 1 Monte Carlo Simulation

```
# Fit the regression model
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
# Generate the sampling distribution of parameter estimates
mc <- MC(object, R = 20000, type = "mvn", seed = 42)
```

## 2 Standardized Regression Slopes

```
out <- BetaMC(mc)
# Methods -----
print(out)

#> Call:
#> BetaMC(object = mc)
#>
#> Standardized regression slopes
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0765 20000 0.2416 0.2869 0.3374 0.6365 0.6756 0.7320
#> PCTGRT  0.3915 0.0768 20000 0.1410 0.1965 0.2395 0.5401 0.5853 0.6407
#> PCTSUPP 0.2632 0.0744 20000 0.0307 0.0765 0.1194 0.4104 0.4615 0.5077

summary(out)

#> Call:
#> BetaMC(object = mc)
#>
#> Standardized regression slopes
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0765 20000 0.2416 0.2869 0.3374 0.6365 0.6756 0.7320
#> PCTGRT  0.3915 0.0768 20000 0.1410 0.1965 0.2395 0.5401 0.5853 0.6407
#> PCTSUPP 0.2632 0.0744 20000 0.0307 0.0765 0.1194 0.4104 0.4615 0.5077
```

```

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4951451 0.3914887 0.2632477

vcov(out)

#>              NARTIC      PCTGRT      PCTSUPP
#> NARTIC    0.005855414 -0.003342667 -0.002187467
#> PCTGRT   -0.003342667  0.005902455 -0.001669725
#> PCTSUPP  -0.002187467 -0.001669725  0.005541865

confint(out)

#>              2.5 %      97.5 %
#> NARTIC    0.3373784 0.6364844
#> PCTGRT    0.2395242 0.5401378
#> PCTSUPP   0.1194307 0.4103937

```

### 3 Multiple Correlation

```

out <- RSqMC(mc)
# Methods -----
print(out)

#> Call:
#> RSqMC(object = mc)
#>
#> R-squared and adjusted R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> rsq 0.8045 0.0563 20000 0.5072 0.5978 0.6622 0.8820 0.9042 0.9313
#> adj 0.7906 0.0604 20000 0.4720 0.5691 0.6381 0.8735 0.8973 0.9264

summary(out)

#> Call:
#> RSqMC(object = mc)
#>
#> R-squared and adjusted R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> rsq 0.8045 0.0563 20000 0.5072 0.5978 0.6622 0.8820 0.9042 0.9313
#> adj 0.7906 0.0604 20000 0.4720 0.5691 0.6381 0.8735 0.8973 0.9264

```

```

coef(out)

#>      rsq      adj
#> 0.8045263 0.7905638

vcov(out)

#>      rsq      adj
#> rsq 0.003174789 0.003401559
#> adj 0.003401559 0.003644528

confint(out)

#>      2.5 %    97.5 %
#> rsq 0.6622208 0.8819643
#> adj 0.6380937 0.8735331

```

## 4 Semipartial Correlation

```

out <- SCorMC(mc)
# Methods -----
print(out)

#> Call:
#> SCorMC(object = mc)
#>
#> Semipartial correlations
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4312 0.0780 20000 0.1737 0.2271 0.2668 0.5723 0.6268 0.6996
#> PCTGRT  0.3430 0.0739 20000 0.1047 0.1586 0.1944 0.4850 0.5355 0.6011
#> PCTSUPP 0.2385 0.0696 20000 0.0286 0.0660 0.1029 0.3757 0.4265 0.4790

summary(out)

#> Call:
#> SCorMC(object = mc)
#>
#> Semipartial correlations
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4312 0.0780 20000 0.1737 0.2271 0.2668 0.5723 0.6268 0.6996
#> PCTGRT  0.3430 0.0739 20000 0.1047 0.1586 0.1944 0.4850 0.5355 0.6011
#> PCTSUPP 0.2385 0.0696 20000 0.0286 0.0660 0.1029 0.3757 0.4265 0.4790

```

```
coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4311525 0.3430075 0.2384789

vcov(out)

#>              NARTIC          PCTGRT          PCTSUPP
#> NARTIC    0.0060913992 -0.001277676 -0.0009384268
#> PCTGRT   -0.0012776761  0.005459685 -0.0007390450
#> PCTSUPP  -0.0009384268 -0.000739045  0.0048375376

confint(out)

#>           2.5 %    97.5 %
#> NARTIC  0.2667500 0.5722613
#> PCTGRT  0.1944016 0.4850499
#> PCTSUPP 0.1029251 0.3756988
```

## 5 Improvement in R-Squared

```
out <- DeltaRSqMC(mc)
# Methods -----
print(out)

#> Call:
#> DeltaRSqMC(object = mc)
#>
#> Improvement in R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.1859 0.0663 20000 0.0302 0.0516 0.0712 0.3275 0.3929 0.4895
#> PCTGRT  0.1177 0.0509 20000 0.0110 0.0252 0.0378 0.2353 0.2868 0.3613
#> PCTSUPP 0.0569 0.0339 20000 0.0008 0.0044 0.0106 0.1411 0.1819 0.2295

summary(out)

#> Call:
#> DeltaRSqMC(object = mc)
#>
#> Improvement in R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.1859 0.0663 20000 0.0302 0.0516 0.0712 0.3275 0.3929 0.4895
#> PCTGRT  0.1177 0.0509 20000 0.0110 0.0252 0.0378 0.2353 0.2868 0.3613
#> PCTSUPP 0.0569 0.0339 20000 0.0008 0.0044 0.0106 0.1411 0.1819 0.2295
```

```

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.1858925 0.1176542 0.0568722

vcov(out)

#>              NARTIC          PCTGRT          PCTSUPP
#> NARTIC    0.0043960905 -0.0007302909 -0.0003683045
#> PCTGRT   -0.0007302909  0.0025911799 -0.0002265225
#> PCTSUPP  -0.0003683045 -0.0002265225  0.0011498248

confint(out)

#>              2.5 %      97.5 %
#> NARTIC    0.07115559 0.3274830
#> PCTGRT    0.03779197 0.2352734
#> PCTSUPP   0.01059357 0.1411496

```

## 6 Squared Partial Correlation

```

out <- PCorMC(mc)
# Methods -----
print(out)

#> Call:
#> PCorMC(object = mc)
#>
#> Squared partial correlations
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4874 0.1058 20000 0.1210 0.1801 0.2408 0.6530 0.7077 0.7756
#> PCTGRT  0.3757 0.1082 20000 0.0520 0.1001 0.1474 0.5688 0.6381 0.7061
#> PCTSUPP 0.2254 0.0992 20000 0.0035 0.0198 0.0472 0.4273 0.4982 0.5763

summary(out)

#> Call:
#> PCorMC(object = mc)
#>
#> Squared partial correlations
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4874 0.1058 20000 0.1210 0.1801 0.2408 0.6530 0.7077 0.7756
#> PCTGRT  0.3757 0.1082 20000 0.0520 0.1001 0.1474 0.5688 0.6381 0.7061
#> PCTSUPP 0.2254 0.0992 20000 0.0035 0.0198 0.0472 0.4273 0.4982 0.5763

```

```
coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4874382 0.3757383 0.2253739

vcov(out)

#>              NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.0111975007 0.0006762126 0.0002985984
#> PCTGRT  0.0006762126 0.0117076719 0.0002499549
#> PCTSUPP 0.0002985984 0.0002499549 0.0098451610

confint(out)

#>              2.5 %      97.5 %
#> NARTIC  0.24076178 0.6529756
#> PCTGRT  0.14741748 0.5687977
#> PCTSUPP 0.04717967 0.4273490
```

## 7 Differences of Standardized Slopes

```
out <- DiffBetaMC(mc)
# Methods -----
print(out)

#> Call:
#> DiffBetaMC(object = mc)
#>
#> Differences of standardized regression slopes
#> type = "mvn"
#>              est      se      R   0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC-PCTGRT  0.1037 0.1358 20000 -0.3362 -0.2509 -0.1707 0.3615 0.4411 0.5050
#> NARTIC-PCTSUPP 0.2319 0.1256 20000 -0.2007 -0.1080 -0.0231 0.4715 0.5399 0.6130
#> PCTGRT-PCTSUPP 0.1282 0.1216 20000 -0.2626 -0.1843 -0.1138 0.3664 0.4371 0.5269

summary(out)

#> Call:
#> DiffBetaMC(object = mc)
#>
#> Differences of standardized regression slopes
#> type = "mvn"
#>              est      se      R   0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC-PCTGRT  0.1037 0.1358 20000 -0.3362 -0.2509 -0.1707 0.3615 0.4411 0.5050
#> NARTIC-PCTSUPP 0.2319 0.1256 20000 -0.2007 -0.1080 -0.0231 0.4715 0.5399 0.6130
#> PCTGRT-PCTSUPP 0.1282 0.1216 20000 -0.2626 -0.1843 -0.1138 0.3664 0.4371 0.5269
```

```

coef(out)

#>   NARTIC-PCTGRT NARTIC-PCTSUPP PCTGRT-PCTSUPP
#>      0.1036564      0.2318974      0.1282410

vcov(out)

#>               NARTIC-PCTGRT NARTIC-PCTSUPP PCTGRT-PCTSUPP
#> NARTIC-PCTGRT      0.018443203      0.009715823     -0.008727380
#> NARTIC-PCTSUPP      0.009715823      0.015772213      0.006056389
#> PCTGRT-PCTSUPP     -0.008727380      0.006056389      0.014783770

confint(out)

#>               2.5 %      97.5 %
#> NARTIC-PCTGRT  -0.17070611  0.3614905
#> NARTIC-PCTSUPP -0.02314239  0.4715098
#> PCTGRT-PCTSUPP -0.11383192  0.3664444

```

## 8 Monte Carlo Simulation - Multiple Imputation

```

nas1982_missing <- mice::ampute(nas1982, mech = "MCAR")$amp
# Fit the regression model
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982_missing)
# Generate the sampling distribution of parameter estimates
mi <- mice::mice(nas1982_missing, m = 100, seed = 42, print = FALSE)
mc <- MCMC(object, mi = mi, R = 20000, type = "mvn",
            seed = 42)

```

## 9 Standardized Regression Slopes

```

out <- BetaMC(mc)
# Methods -----
print(out)

#> Call:
#> BetaMC(object = mc)
#>
#> Standardized regression slopes
#> type = "mvn"
#>      est      se      R  0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.5222 0.0797 20000  0.2429 0.3071 0.3566 0.6695 0.7163 0.7660

```

```

#> PCTGRT  0.3797 0.0802 20000  0.1284 0.1713 0.2209 0.5335 0.5813 0.6378
#> PCTSUPP 0.2432 0.0830 20000 -0.0364 0.0258 0.0800 0.4050 0.4626 0.5333

summary(out)

#> Call:
#> BetaMC(object = mc)
#>
#> Standardized regression slopes
#> type = "mvn"
#>      est      se      R  0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.5222 0.0797 20000  0.2429 0.3071 0.3566 0.6695 0.7163 0.7660
#> PCTGRT  0.3797 0.0802 20000  0.1284 0.1713 0.2209 0.5335 0.5813 0.6378
#> PCTSUPP 0.2432 0.0830 20000 -0.0364 0.0258 0.0800 0.4050 0.4626 0.5333

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.5222438 0.3796524 0.2431948

vcov(out)

#>           NARTIC           PCTGRT           PCTSUPP
#> NARTIC  0.006357486 -0.003489733 -0.002427020
#> PCTGRT -0.003489733  0.006427931 -0.002064887
#> PCTSUPP -0.002427020 -0.002064887  0.006880782

confint(out)

#>           2.5 %      97.5 %
#> NARTIC  0.3566042 0.6694893
#> PCTGRT  0.2208700 0.5335096
#> PCTSUPP 0.0799574 0.4049745

```

## 10 Multiple Correlation

```

out <- RSqMC(mc)
# Methods -----
print(out)

#> Call:
#> RSqMC(object = mc)
#>
#> R-squared and adjusted R-squared
#> type = "mvn"

```



```

#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> rsq 0.8029 0.0623 20000 0.4825 0.5843 0.6465 0.8912 0.9155 0.9510
#> adj 0.7889 0.0690 20000 0.4271 0.5397 0.6086 0.8795 0.9064 0.9458

summary(out)

#> Call:
#> RSqMC(object = mc)
#>
#> R-squared and adjusted R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> rsq 0.8029 0.0623 20000 0.4825 0.5843 0.6465 0.8912 0.9155 0.9510
#> adj 0.7889 0.0690 20000 0.4271 0.5397 0.6086 0.8795 0.9064 0.9458

coef(out)

#>      rsq      adj
#> 0.8029376 0.7888617

vcov(out)

#>      rsq      adj
#> rsq 0.003879851 0.004295550
#> adj 0.004295550 0.004755787

confint(out)

#>      2.5 %    97.5 %
#> rsq 0.6464860 0.8911611
#> adj 0.6086095 0.8794998

```

## 11 Semipartial Correlation

```

out <- SCorMC(mc)
# Methods -----
print(out)

#> Call:
#> SCorMC(object = mc)
#>
#> Semipartial correlations
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC 0.4601 0.0834 20000 0.1623 0.2356 0.2827 0.6114 0.6666 0.7289

```

```

#> PCTGRT  0.3312 0.0761 20000  0.0981 0.1395 0.1775 0.4768 0.5267 0.5962
#> PCTSUPP 0.2183 0.0764 20000 -0.0308 0.0229 0.0675 0.3656 0.4243 0.4986

summary(out)

#> Call:
#> SCorMC(object = mc)
#>
#> Semipartial correlations
#> type = "mvn"
#>      est      se      R  0.05%  0.5%  2.5% 97.5% 99.5% 99.95%
#> NARTIC  0.4601 0.0834 20000  0.1623 0.2356 0.2827 0.6114 0.6666 0.7289
#> PCTGRT  0.3312 0.0761 20000  0.0981 0.1395 0.1775 0.4768 0.5267 0.5962
#> PCTSUPP 0.2183 0.0764 20000 -0.0308 0.0229 0.0675 0.3656 0.4243 0.4986

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4601125 0.3311773 0.2183476

vcov(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.006961884 -0.0014003821 -0.0010411184
#> PCTGRT -0.001400382  0.0057883573 -0.0009937412
#> PCTSUPP -0.001041118 -0.0009937412  0.0058442999

confint(out)

#>      2.5 %      97.5 %
#> NARTIC  0.28274379 0.6113921
#> PCTGRT  0.17754247 0.4767866
#> PCTSUPP 0.06746644 0.3655848

```

## 12 Improvement in R-Squared

```

out <- DeltaRSqMC(mc)
# Methods -----
print(out)

#> Call:
#> DeltaRSqMC(object = mc)
#>
#> Improvement in R-squared
#> type = "mvn"

```

```

#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.2117 0.0752 20000 0.0263 0.0555 0.0799 0.3738 0.4444 0.5312
#> PCTGRT  0.1097 0.0504 20000 0.0096 0.0195 0.0315 0.2273 0.2774 0.3554
#> PCTSUPP 0.0477 0.0344 20000 0.0000 0.0007 0.0046 0.1337 0.1800 0.2486

summary(out)

#> Call:
#> DeltaRSqMC(object = mc)
#>
#> Improvement in R-squared
#> type = "mvn"
#>      est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.2117 0.0752 20000 0.0263 0.0555 0.0799 0.3738 0.4444 0.5312
#> PCTGRT  0.1097 0.0504 20000 0.0096 0.0195 0.0315 0.2273 0.2774 0.3554
#> PCTSUPP 0.0477 0.0344 20000 0.0000 0.0007 0.0046 0.1337 0.1800 0.2486

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.21170355 0.10967840 0.04767569

vcov(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.0056617763 -0.0008298234 -0.0003916251
#> PCTGRT -0.0008298234  0.0025434893 -0.0002680556
#> PCTSUPP -0.0003916251 -0.0002680556  0.0011860969

confint(out)

#>      2.5 %      97.5 %
#> NARTIC  0.07994405 0.3738003
#> PCTGRT  0.03152133 0.2273255
#> PCTSUPP 0.00455172 0.1336523

```

## 13 Squared Partial Correlation

```

out <- PCorMC(mc)
# Methods -----
print(out)

#> Call:
#> PCorMC(object = mc)
#>

```

```

#> Squared partial correlations
#> type = "mvn"
#>           est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC   0.5188 0.1140 20000 0.1035 0.1886 0.2560 0.6982 0.7649 0.8286
#> PCTGRT   0.3592 0.1140 20000 0.0450 0.0805 0.1238 0.5668 0.6396 0.7394
#> PCTSUPP  0.1989 0.1091 20000 0.0002 0.0031 0.0198 0.4338 0.5249 0.6508

summary(out)

#> Call:
#> PCorMC(object = mc)
#>
#> Squared partial correlations
#> type = "mvn"
#>           est      se      R 0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC   0.5188 0.1140 20000 0.1035 0.1886 0.2560 0.6982 0.7649 0.8286
#> PCTGRT   0.3592 0.1140 20000 0.0450 0.0805 0.1238 0.5668 0.6396 0.7394
#> PCTSUPP  0.1989 0.1091 20000 0.0002 0.0031 0.0198 0.4338 0.5249 0.6508

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.5187942 0.3591718 0.1989111

vcov(out)

#>           NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.012985905 0.001649781 0.001538205
#> PCTGRT  0.001649781 0.012986822 0.000673645
#> PCTSUPP 0.001538205 0.000673645 0.011895553

confint(out)

#>           2.5 %    97.5 %
#> NARTIC  0.25602481 0.6981834
#> PCTGRT  0.12378559 0.5668399
#> PCTSUPP 0.01978582 0.4337912

```

## 14 Differences of Standardized Slopes

```

out <- DiffBetaMC(mc)
# Methods -----
print(out)

#> Call:

```

```

#> DiffBetaMC(object = mc)
#>
#> Differences of standardized regression slopes
#> type = "mvn"
#>           est      se      R  0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC-PCTGRT  0.1426 0.1406 20000 -0.3302 -0.2293 -0.1380 0.4091 0.4914 0.5748
#> NARTIC-PCTSUPP 0.2790 0.1345 20000 -0.1690 -0.0743  0.0070 0.5348 0.6173 0.7121
#> PCTGRT-PCTSUPP 0.1365 0.1321 20000 -0.3091 -0.2121 -0.1261 0.3935 0.4708 0.5531

summary(out)

#> Call:
#> DiffBetaMC(object = mc)
#>
#> Differences of standardized regression slopes
#> type = "mvn"
#>           est      se      R  0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC-PCTGRT  0.1426 0.1406 20000 -0.3302 -0.2293 -0.1380 0.4091 0.4914 0.5748
#> NARTIC-PCTSUPP 0.2790 0.1345 20000 -0.1690 -0.0743  0.0070 0.5348 0.6173 0.7121
#> PCTGRT-PCTSUPP 0.1365 0.1321 20000 -0.3091 -0.2121 -0.1261 0.3935 0.4708 0.5531

coef(out)

#>   NARTIC-PCTGRT NARTIC-PCTSUPP PCTGRT-PCTSUPP
#>      0.1425914      0.2790490      0.1364576

vcov(out)

#>           NARTIC-PCTGRT NARTIC-PCTSUPP PCTGRT-PCTSUPP
#> NARTIC-PCTGRT  0.019764884  0.010209352 -0.009555532
#> NARTIC-PCTSUPP  0.010209352  0.018092308  0.007882956
#> PCTGRT-PCTSUPP -0.009555532  0.007882956  0.017438488

confint(out)

#>           2.5 %    97.5 %
#> NARTIC-PCTGRT -0.137981901 0.4091218
#> NARTIC-PCTSUPP  0.006956886 0.5347634
#> PCTGRT-PCTSUPP -0.126070958 0.3935004

```

## References

R Core Team. (2025). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. Vienna, Austria. <https://www.R-project.org/>