# Package 'cTMed'

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confint.ctmeddelta

Delta Method Confidence Intervals

# Description

Delta Method Confidence Intervals

## Usage

```
## S3 method for class 'ctmeddelta'
confint(object, parm = NULL, level = 0.95, ...)
```

# Arguments

object Object of class ctmeddelta.

parm a specification of which parameters are to be given confidence intervals, either

a vector of numbers or a vector of names. If missing, all parameters are consid-

ered.

level the confidence level required.

... additional arguments.

#### Value

Returns a data frame of confidence intervals.

# Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
```

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```
-0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
confint(delta, level = 0.95)
# Range of time intervals ------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
confint(delta, level = 0.95)
```

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# **Description**

Monte Carlo Method Confidence Intervals

## Usage

```
## S3 method for class 'ctmedmc'
confint(object, parm = NULL, level = 0.95, ...)
```

# **Arguments**

object Object of class ctmedmc.

parm a specification of which parameters are to be given confidence intervals, either

a vector of numbers or a vector of names. If missing, all parameters are consid-

ered.

level the confidence level required.

... additional arguments.

#### Value

Returns a data frame of confidence intervals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
```

```
-0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
)
confint(mc, level = 0.95)
# Range of time intervals ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
 R = 100L # use a large value for R in actual research
confint(mc, level = 0.95)
```

DeltaBeta

Delta Method Sampling Variance-Covariance Matrix for the Elements of the Matrix of Lagged Coefficients Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function computes the delta method sampling variance-covariance matrix for the elements of the matrix of lagged coefficients  $\beta$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\Phi$ .

## Usage

DeltaBeta(phi, vcov\_phi\_vec, delta\_t, ncores = NULL, tol = 0.01)

# **Arguments**

Numeric matrix. The drift matrix  $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.

vcov\_phi\_vec Numeric matrix. The sampling variance-covariance matrix of vec  $(\Phi)$ .

delta\_t Vector of positive numbers. Time interval  $(\Delta t)$ .

Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when the length of delta\_t is long.

tol Numeric. Smallest possible time interval to allow.

## **Details**

See Total().

## **Delta Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . By the multivariate central limit theory, the function  $\mathbf{g}$  using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ , that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{oldsymbol{ heta}}
ight)pprox\mathcal{N}\left(\mathbf{g}\left(oldsymbol{ heta}
ight),\mathbf{J}\hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)\mathbf{J}'
ight).$$

#### Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaBeta").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta\_t Time interval.

jacobian Jacobian matrix.

est Estimated elements of the matrix of lagged coefficients.

vcov Sampling variance-covariance matrix of estimated elements of the matrix of lagged coefficients.

# Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

# See Also

```
Other Continuous Time Mediation Functions: DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
),
  nrow = 3</pre>
```

```
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
 data = c(
   0.00843, 0.00040, -0.00151,
   -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
   -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
DeltaBeta(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1
# Range of time intervals -----
delta <- DeltaBeta(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5
)
plot(delta)
# Methods ------
# DeltaBeta has a number of methods including
# print, summary, confint, and plot
```

```
print(delta)
summary(delta)
confint(delta, level = 0.95)
plot(delta)
```

DeltaBetaStd

Delta Method Sampling Variance-Covariance Matrix for the Elements of the Standardized Matrix of Lagged Coefficients Over a Specific Time Interval or a Range of Time Intervals

# Description

This function computes the delta method sampling variance-covariance matrix for the elements of the standardized matrix of lagged coefficients  $\beta$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\Phi$  and process noise covariance matrix  $\Sigma$ .

# Usage

```
DeltaBetaStd(phi, sigma, vcov_theta, delta_t, ncores = NULL, tol = 0.01)
```

# Arguments

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
vcov_theta	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\Phi\right)$ and $\operatorname{vech}\left(\Sigma\right)$
delta_t	Numeric. Time interval ( $\Delta t$ ).
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
tol	Numeric. Smallest possible time interval to allow.

# Details

See TotalStd().

# **Delta Method:**

Let  $\theta$  be a vector that combines  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise and  $\operatorname{vech}(\Sigma)$ , that is, the unique elements of the  $\Sigma$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be a vector that combines  $\operatorname{vec}(\hat{\Phi})$  and  $\operatorname{vech}(\hat{\Sigma})$ . By the multivariate central limit theory, the function  $\mathbf{g}$  using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ , that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), \mathbf{J}\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right) \mathbf{J}'\right).$$

# Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaBetaStd").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta\_t Time interval.

jacobian Jacobian matrix.

est Estimated elements of the matrix of lagged coefficients.

**vcov** Sampling variance-covariance matrix of estimated elements of the matrix of lagged coefficients.

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
sigma <- matrix(</pre>
 data = c(
   0.24455556, 0.02201587, -0.05004762,
    0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
 ),
 nrow = 3
vcov_theta <- matrix(</pre>
 data = c(
    0.00843, 0.00040, -0.00151, -0.00600, -0.00033,
    0.00110, 0.00324, 0.00020, -0.00061, -0.00115,
    0.00011, 0.00015, 0.00001, -0.00002, -0.00001,
    0.00040, 0.00374, 0.00016, -0.00022, -0.00273,
    -0.00016, 0.00009, 0.00150, 0.00012, -0.00010,
    -0.00026, 0.00002, 0.00012, 0.00004, -0.00001,
    -0.00151, 0.00016, 0.00389, 0.00103, -0.00007,
    -0.00283, -0.00050, 0.00000, 0.00156, 0.00021,
    -0.00005, -0.00031, 0.00001, 0.00007, 0.00006,
    -0.00600, -0.00022, 0.00103, 0.00644, 0.00031,
    -0.00119, -0.00374, -0.00021, 0.00070, 0.00064,
    -0.00015, -0.00005, 0.00000, 0.00003, -0.00001,
    -0.00033, -0.00273, -0.00007, 0.00031, 0.00287,
    0.00013, -0.00014, -0.00170, -0.00012, 0.00006,
     0.00014, \ -0.00001, \ -0.00015, \ 0.00000, \ 0.00001, \\
    0.00110, -0.00016, -0.00283, -0.00119, 0.00013,
    0.00297, 0.00063, -0.00004, -0.00177, -0.00013,
    0.00005, 0.00017, -0.00002, -0.00008, 0.00001,
    0.00324, 0.00009, -0.00050, -0.00374, -0.00014,
    0.00063, 0.00495, 0.00024, -0.00093, -0.00020,
    0.00006, -0.00010, 0.00000, -0.00001, 0.00004,
    0.00020, 0.00150, 0.00000, -0.00021, -0.00170,
    -0.00004, 0.00024, 0.00214, 0.00012, -0.00002,
```

```
-0.00004, 0.00000, 0.00006, -0.00005, -0.00001,
   -0.00061, 0.00012, 0.00156, 0.00070, -0.00012,
   -0.00177, -0.00093, 0.00012, 0.00223, 0.00004,
   -0.00002, -0.00003, 0.00001, 0.00003, -0.00013,
   -0.00115, -0.00010, 0.00021, 0.00064, 0.00006,
   -0.00013, -0.00020, -0.00002, 0.00004, 0.00057,
   0.00001, -0.00009, 0.00000, 0.00000, 0.00001,
   0.00011, -0.00026, -0.00005, -0.00015, 0.00014,
   0.00005, 0.00006, -0.00004, -0.00002, 0.00001,
   0.00012, 0.00001, 0.00000, -0.00002, 0.00000,
   0.00015, 0.00002, -0.00031, -0.00005, -0.00001,
   0.00017, -0.00010, 0.00000, -0.00003, -0.00009,
   0.00001, 0.00014, 0.00000, 0.00000, -0.00005,
   0.00001, 0.00012, 0.00001, 0.00000, -0.00015,
   -0.00002, 0.00000, 0.00006, 0.00001, 0.00000,
   0.00000, 0.00000, 0.00010, 0.00001, 0.00000,
   -0.00002, 0.00004, 0.00007, 0.00003, 0.00000,
   -0.00008, -0.00001, -0.00005, 0.00003, 0.00000,
   -0.00002, 0.00000, 0.00001, 0.00005, 0.00001,
   -0.00001, -0.00001, 0.00006, -0.00001, 0.00001,
   0.00001, 0.00004, -0.00001, -0.00013, 0.00001,
   0.00000, -0.00005, 0.00000, 0.00001, 0.00012
 ),
 nrow = 15
)
# Specific time interval -------
DeltaBetaStd(
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1
# Range of time intervals ------
delta <- DeltaBetaStd(</pre>
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1:5
plot(delta)
# Methods ------
# DeltaBetaStd has a number of methods including
# print, summary, confint, and plot
print(delta)
summary(delta)
confint(delta, level = 0.95)
plot(delta)
```

14 DeltaIndirectCentral

DeltaIndirectCentral Delta Method Sampling Variance-Covariance Matrix for the Indirect Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function computes the delta method sampling variance-covariance matrix for the indirect effect centrality over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\Phi$ .

## Usage

DeltaIndirectCentral(phi, vcov\_phi\_vec, delta\_t, ncores = NULL, tol = 0.01)

## **Arguments**

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system.
vcov_phi_vec	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\Phi\right)$ .
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when the length of delta_t is long.
tol	Numeric. Smallest possible time interval to allow.

#### **Details**

See IndirectCentral() more details.

# **Delta Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . By the multivariate central limit theory, the function g using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ ,

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that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{oldsymbol{ heta}}
ight)pprox\mathcal{N}\left(\mathbf{g}\left(oldsymbol{ heta}
ight),\mathbf{J}\hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)\mathbf{J}'
ight).$$

#### Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaIndirectCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta\_t Time interval.

jacobian Jacobian matrix.

est Estimated indirect effect centrality.

vcov Sampling variance-covariance matrix of estimated indirect effect centrality.

## Author(s)

Ivan Jacob Agaloos Pesigan

## References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

# See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

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```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
 data = c(
   0.002704274, -0.001475275, 0.000949122,
   -0.001619422, 0.000885122, -0.000569404,
   0.00085493, -0.000465824, 0.000297815,
   -0.001475275, 0.004428442, -0.002642303,
   0.000980573, -0.00271817, 0.001618805,
   -0.000586921, 0.001478421, -0.000871547,
   0.000949122, -0.002642303, 0.006402668,
   -0.000697798, 0.001813471, -0.004043138,
   0.000463086, -0.001120949, 0.002271711,
   -0.001619422, 0.000980573, -0.000697798,
   0.002079286, -0.001152501, 0.000753,
   -0.001528701, 0.000820587, -0.000517524,
   0.000885122, -0.00271817, 0.001813471,
   -0.001152501, 0.00342605, -0.002075005,
   0.000899165, -0.002532849, 0.001475579,
   -0.000569404, 0.001618805, -0.004043138,
   0.000753, -0.002075005, 0.004984032,
   -0.000622255, 0.001634917, -0.003705661,
    \hbox{\tt 0.00085493, -0.000586921, 0.000463086,} \\
   -0.001528701, 0.000899165, -0.000622255,
   0.002060076, -0.001096684, 0.000686386,
   -0.000465824, 0.001478421, -0.001120949,
   0.000820587, -0.002532849, 0.001634917,
   -0.001096684, 0.003328692, -0.001926088,
   0.000297815, -0.000871547, 0.002271711,
   -0.000517524, 0.001475579, -0.003705661,
   0.000686386, -0.001926088, 0.004726235
 ),
 nrow = 9
)
# Specific time interval ------
DeltaIndirectCentral(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1
# Range of time intervals -------
delta <- DeltaIndirectCentral(</pre>
```

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DeltaMed

Delta Method Sampling Variance-Covariance Matrix for the Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

# **Description**

This function computes the delta method sampling variance-covariance matrix for the total, direct, and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$ .

# Usage

```
DeltaMed(phi, vcov_phi_vec, delta_t, from, to, med, ncores = NULL, tol = 0.01)
```

# **Arguments**

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system.
vcov_phi_vec	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\mathbf{\Phi}\right)$ .
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when the length of delta_t is long.
tol	Numeric. Smallest possible time interval to allow.

DeltaMed

## **Details**

See Total(), Direct(), and Indirect() for more details.

#### **Delta Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . By the multivariate central limit theory, the function  $\mathbf{g}$  using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\mathbf{\Gamma}\mathbf{J}'\right)$$

The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ , that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), \mathbf{J}\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right) \mathbf{J}'\right).$$

## Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaMed").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta\_t Time interval.

jacobian Jacobian matrix.

est Estimated total, direct, and indirect effects.

vcov Sampling variance-covariance matrix of the estimated total, direct, and indirect effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

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#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ).
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
```

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```
-0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
DeltaMed(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
)
# Range of time intervals ------
delta <- DeltaMed(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
)
plot(delta)
# Methods -----
# DeltaMed has a number of methods including
# print, summary, confint, and plot
print(delta)
summary(delta)
confint(delta, level = 0.95)
plot(delta)
```

DeltaMedStd

Delta Method Sampling Variance-Covariance Matrix for the Standardized Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function computes the delta method sampling variance-covariance matrix for the standardized total, direct, and indirect effects of the independent variable X on the dependent variable Y through

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mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$  and process noise covariance matrix  $\mathbf{\Sigma}$ .

# Usage

```
DeltaMedStd(
   phi,
   sigma,
   vcov_theta,
   delta_t,
   from,
   to,
   med,
   ncores = NULL,
   tol = 0.01
)
```

# Arguments

phi	Numeric matrix. The drift matrix $(\Phi)$ . phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
vcov_theta	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\Phi\right)$ and $\operatorname{vech}\left(\Sigma\right)$
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
tol	Numeric. Smallest possible time interval to allow.

# **Details**

See TotalStd(), DirectStd(), and IndirectStd() for more details.

## **Delta Method:**

Let  $\theta$  be a vector that combines  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise and  $\operatorname{vech}(\Sigma)$ , that is, the unique elements of the  $\Sigma$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be a vector that combines  $\operatorname{vec}(\hat{\Phi})$  and  $\operatorname{vech}(\hat{\Sigma})$ . By the multivariate central limit theory, the function  $\mathbf{g}$  using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ , that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), \mathbf{J}\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right) \mathbf{J}'\right).$$

# Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaMedStd").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta t Time interval.

**jacobian** Jacobian matrix.

est Estimated total, direct, and indirect effects.

vcov Sampling variance-covariance matrix of the estimated total, direct, and indirect effects.

# Author(s)

Ivan Jacob Agaloos Pesigan

# References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

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#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
sigma <- matrix(</pre>
 data = c(
   0.24455556, 0.02201587, -0.05004762,
   0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
 ),
 nrow = 3
vcov_theta <- matrix(</pre>
 data = c(
    0.00843, 0.00040, -0.00151, -0.00600, -0.00033,
    0.00110, 0.00324, 0.00020, -0.00061, -0.00115,
    0.00011, 0.00015, 0.00001, -0.00002, -0.00001,
    0.00040, 0.00374, 0.00016, -0.00022, -0.00273,
    -0.00016, 0.00009, 0.00150, 0.00012, -0.00010,
    -0.00026, 0.00002, 0.00012, 0.00004, -0.00001,
    -0.00151, 0.00016, 0.00389, 0.00103, -0.00007,
    -0.00283, -0.00050, 0.00000, 0.00156, 0.00021,
    -0.00005, -0.00031, 0.00001, 0.00007, 0.00006,
    -0.00600, -0.00022, 0.00103, 0.00644, 0.00031,
    -0.00119, -0.00374, -0.00021, 0.00070, 0.00064,
    -0.00015, -0.00005, 0.00000, 0.00003, -0.00001,
    -0.00033, -0.00273, -0.00007, 0.00031, 0.00287,
    0.00013, -0.00014, -0.00170, -0.00012, 0.00006,
     0.00014, \ -0.00001, \ -0.00015, \ 0.00000, \ 0.00001, \\
    0.00110, -0.00016, -0.00283, -0.00119, 0.00013,
    0.00297, 0.00063, -0.00004, -0.00177, -0.00013,
    0.00005, 0.00017, -0.00002, -0.00008, 0.00001,
    0.00324, 0.00009, -0.00050, -0.00374, -0.00014,
    0.00063, 0.00495, 0.00024, -0.00093, -0.00020,
    0.00006, -0.00010, 0.00000, -0.00001, 0.00004,
    0.00020, 0.00150, 0.00000, -0.00021, -0.00170,
    -0.00004, 0.00024, 0.00214, 0.00012, -0.00002,
```

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```
-0.00004, 0.00000, 0.00006, -0.00005, -0.00001,
   -0.00061, 0.00012, 0.00156, 0.00070, -0.00012,
   -0.00177, -0.00093, 0.00012, 0.00223, 0.00004,
   -0.00002, -0.00003, 0.00001, 0.00003, -0.00013,
   -0.00115, -0.00010, 0.00021, 0.00064, 0.00006,
   -0.00013, -0.00020, -0.00002, 0.00004, 0.00057,
   0.00001, -0.00009, 0.00000, 0.00000, 0.00001,
   0.00011, -0.00026, -0.00005, -0.00015, 0.00014,
   0.00005, 0.00006, -0.00004, -0.00002, 0.00001,
   0.00012, 0.00001, 0.00000, -0.00002, 0.00000,
   0.00015, 0.00002, -0.00031, -0.00005, -0.00001,
   0.00017, -0.00010, 0.00000, -0.00003, -0.00009,
   0.00001, 0.00014, 0.00000, 0.00000, -0.00005,
   0.00001, 0.00012, 0.00001, 0.00000, -0.00015,
   -0.00002, 0.00000, 0.00006, 0.00001, 0.00000,
   0.00000, 0.00000, 0.00010, 0.00001, 0.00000,
   -0.00002, 0.00004, 0.00007, 0.00003, 0.00000,
   -0.00008, -0.00001, -0.00005, 0.00003, 0.00000,
   -0.00002, 0.00000, 0.00001, 0.00005, 0.00001,
   -0.00001, -0.00001, 0.00006, -0.00001, 0.00001,
   0.00001, 0.00004, -0.00001, -0.00013, 0.00001,
   0.00000, -0.00005, 0.00000, 0.00001, 0.00012
 ),
 nrow = 15
)
# Specific time interval -------
DeltaMedStd(
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
)
# Range of time intervals ------
delta <- DeltaMedStd(</pre>
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
)
plot(delta)
# Methods ------
# DeltaMedStd has a number of methods including
# print, summary, confint, and plot
print(delta)
```

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```
summary(delta)
confint(delta, level = 0.95)
plot(delta)
```

DeltaTotalCentral

Delta Method Sampling Variance-Covariance Matrix for the Total Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

# **Description**

This function computes the delta method sampling variance-covariance matrix for the total effect centrality over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\Phi$ .

## Usage

DeltaTotalCentral(phi, vcov\_phi\_vec, delta\_t, ncores = NULL, tol = 0.01)

#### **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
vcov_phi_vec	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\Phi\right)$ .
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when the length of delta_t is long.
tol	Numeric. Smallest possible time interval to allow.

## **Details**

See TotalCentral() more details.

#### **Delta Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . By the multivariate central limit theory, the function  $\mathbf{g}$  using  $\hat{\theta}$  as input can be expressed as:

$$\sqrt{n}\left(\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{g}\left(\boldsymbol{\theta}\right)\right) \xrightarrow{\mathrm{D}} \mathcal{N}\left(0, \mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

where **J** is the matrix of first-order derivatives of the function **g** with respect to the elements of  $\theta$  and  $\Gamma$  is the asymptotic variance-covariance matrix of  $\hat{\theta}$ .

From the former, we can derive the distribution of  $g(\hat{\theta})$  as follows:

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right) \approx \mathcal{N}\left(\mathbf{g}\left(\boldsymbol{\theta}\right), n^{-1}\mathbf{J}\boldsymbol{\Gamma}\mathbf{J}'\right)$$

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The uncertainty associated with the estimator  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is, therefore, given by  $n^{-1}\mathbf{J}\Gamma\mathbf{J}'$ . When  $\Gamma$  is unknown, by substitution, we can use the estimated sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ , that is,  $\hat{\mathbb{V}}\left(\hat{\boldsymbol{\theta}}\right)$  for  $n^{-1}\Gamma$ . Therefore, the sampling variance-covariance matrix of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  is given by

$$\mathbf{g}\left(\hat{\boldsymbol{ heta}}\right) pprox \mathcal{N}\left(\mathbf{g}\left(oldsymbol{ heta}
ight), \mathbf{J}\hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight) \mathbf{J}'
ight).$$

#### Value

Returns an object of class ctmeddelta which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DeltaTotalCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

delta\_t Time interval.

jacobian Jacobian matrix.

est Estimated total effect centrality.

vcov Sampling variance-covariance matrix of estimated total effect centrality.

## Author(s)

Ivan Jacob Agaloos Pesigan

# References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

DeltaTotalCentral 27

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
vcov_phi_vec <- matrix(</pre>
 data = c(
   0.00843, 0.00040, -0.00151,
   -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
    -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
DeltaTotalCentral(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1
)
# Range of time intervals ------
delta <- DeltaTotalCentral(</pre>
```

28 Direct

Direct

Direct Effect of X on Y Over a Specific Time Interval

## **Description**

This function computes the direct effect of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$ .

# Usage

```
Direct(phi, delta_t, from, to, med)
```

## **Arguments**

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system.
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in $\mathtt{phi}$ .
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.

# **Details**

The direct effect of the independent variable X on the dependent variable Y relative to some mediator variables  $\mathbf{m}$  is given by

$$\operatorname{Direct}_{\Delta t_{i,j}} = \exp\left(\Delta t \mathbf{D} \mathbf{\Phi} \mathbf{D}\right)_{i,j}$$

where  $\Phi$  denotes the drift matrix,  $\mathbf{D}$  a diagonal matrix where the diagonal elements corresponding to mediator variables  $\mathbf{m}$  are set to zero and the rest to one, i the row index of Y in  $\Phi$ , j the column index of X in  $\Phi$ , and  $\Delta t$  the time interval.

Direct 29

## **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}\right)$$

where  $\mathbf{y}_{i,t}$ ,  $\eta_{i,t}$ , and  $\varepsilon_{i,t}$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\eta_{i,t}$  a vector of latent random variables, and  $\varepsilon_{i,t}$  a vector of random measurement errors, at time t and individual t.  $\nu$  denotes a vector of intercepts,  $\Lambda$  a matrix of factor loadings, and  $\Theta$  the covariance matrix of  $\varepsilon$ .

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i,t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i,t}, \quad ext{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}
ight)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$ . The dynamic structure is given by

$$\mathrm{d} \boldsymbol{\eta}_{i,t} = \left( \boldsymbol{\iota} + \boldsymbol{\Phi} \boldsymbol{\eta}_{i,t} \right) \mathrm{d} t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d}W$  is a Wiener process or Brownian motion, which represents random fluctuations.

## Value

Returns an object of class ctmedeffect which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("Direct").

output The direct effect.

# Author(s)

Ivan Jacob Agaloos Pesigan

# References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

30 DirectStd

## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

# **Examples**

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
delta_t <- 1
Direct(
  phi = phi,
  delta_t = delta_t,
  from = "x",
  to = "y",
  med = "m"
phi <- matrix(</pre>
  data = c(
    -6, 5.5, 0, 0,
    1.25, -2.5, 5.9, -7.3,
    0, 0, -6, 2.5,
    5, 0, 0, -6
  ),
  nrow = 4
)
colnames(phi) <- rownames(phi) <- paste0("y", 1:4)</pre>
Direct(
  phi = phi,
  delta_t = delta_t,
  from = "y2",
  to = "y4",
  med = c("y1", "y3")
)
```

DirectStd

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# **Description**

This function computes the standardized direct effect of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$  and process noise covariance matrix  $\mathbf{\Sigma}$ .

## Usage

```
DirectStd(phi, sigma, delta_t, from, to, med)
```

# **Arguments**

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in $\mathtt{phi}$ .
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.

#### **Details**

The standardized direct effect of the independent variable X on the dependent variable Y relative to some mediator variables  $\mathbf{m}$  is given by

$$\mathrm{Direct}_{\Delta t_{i,j}}^{*} = \mathbf{S} \left( \exp \left( \Delta t \mathbf{D} \mathbf{\Phi} \mathbf{D} \right)_{i,j} \right) \mathbf{S}^{-1}$$

where  $\Phi$  denotes the drift matrix,  $\mathbf{D}$  a diagonal matrix where the diagonal elements corresponding to mediator variables  $\mathbf{m}$  are set to zero and the rest to one, i the row index of Y in  $\Phi$ , j the column index of X in  $\Phi$ ,  $\mathbf{S}$  a diagonal matrix with model-implied standard deviations on the diagonals, and  $\Delta t$  the time interval.

#### Value

Returns an object of class ctmedeffect which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("DirectStd").

output The direct effect.

#### Author(s)

Ivan Jacob Agaloos Pesigan

32 DirectStd

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
sigma <- matrix(</pre>
  data = c(
    0.24455556, 0.02201587, -0.05004762,
    0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
  ),
  nrow = 3
)
delta_t <- 1
DirectStd(
  phi = phi,
  sigma = sigma,
  delta_t = delta_t,
  from = "x",
  to = "y",
  med = "m"
```

ExpCov

33

ExpCov

Model-Implied State Covariance Matrix

# **Description**

The function returns the model-implied state covariance matrix for a particular time interval  $\Delta t$ given by

$$\operatorname{vec}\left(\operatorname{Cov}\left(\boldsymbol{\eta}\right)\right) = \left(\mathbf{J} - \boldsymbol{\beta}_{\Delta t} \otimes \boldsymbol{\beta}_{\Delta t}\right)^{-1} \operatorname{vec}\left(\boldsymbol{\Psi}_{\Delta t}\right)$$

where

$$\begin{split} \boldsymbol{\beta}_{\Delta t} &= \exp \left( \Delta t \boldsymbol{\Phi} \right), \\ \boldsymbol{\Psi}_{\Delta t} &= \boldsymbol{\Phi}^{\#} \left( \exp \left( \Delta t \boldsymbol{\Phi} \right) - \mathbf{J} \right) \operatorname{vec} \left( \boldsymbol{\Sigma} \right), \quad \text{and} \\ \boldsymbol{\Phi}^{\#} &= \left( \boldsymbol{\Phi} \otimes \mathbf{I} \right) + \left( \mathbf{I} \otimes \boldsymbol{\Phi} \right). \end{split}$$

Note that I and J are identity matrices.

# Usage

ExpCov(phi, sigma, delta\_t)

# **Arguments**

Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names phi pertaining to the variables in the system.

sigma Numeric matrix. The process noise covariance matrix  $(\Sigma)$ .

delta\_t Numeric. Time interval ( $\Delta t$ ).

#### **Details**

## **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}\right)$$

where  $y_{i,t}$ ,  $\eta_{i,t}$ , and  $\varepsilon_{i,t}$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $y_{i,t}$ represents a vector of observed random variables,  $\eta_{i,t}$  a vector of latent random variables, and  $\varepsilon_{i,t}$  a vector of random measurement errors, at time t and individual i.  $\nu$  denotes a vector of intercepts,  $\Lambda$  a matrix of factor loadings, and  $\Theta$  the covariance matrix of  $\varepsilon$ .

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i,t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i,t}, \quad ext{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\mathbf{\Theta}^{\frac{1}{2}}\right)\left(\mathbf{\Theta}^{\frac{1}{2}}\right)' = \mathbf{\Theta}$ . The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

34 ExpMean

## Value

Returns a numeric matrix.

## Author(s)

Ivan Jacob Agaloos Pesigan

# See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

# **Examples**

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
sigma <- matrix(</pre>
  data = c(
    0.24, 0.02, -0.05,
    0.02, 0.07, 0.02,
    -0.05, 0.02, 0.08
  ),
  nrow = 3
)
delta_t <- 1
ExpCov(
  phi = phi,
  sigma = sigma,
  delta_t = delta_t
)
```

ExpMean

ExpMean 35

# **Description**

The function returns the model-implied state mean vector for a particular time interval  $\Delta t$  given by

Mean 
$$(\boldsymbol{\eta}) = (\mathbf{I} - \boldsymbol{\beta}_{\Delta t})^{-1} \boldsymbol{\alpha}_{\Delta t}$$

where

$$oldsymbol{eta}_{\Delta t} = \exp\left(\Delta t oldsymbol{\Phi}\right),$$
 $oldsymbol{lpha}_{\Delta t} = oldsymbol{\Phi}^{-1} \left(oldsymbol{eta}_{\Delta t} - \mathbf{I}\right) oldsymbol{\iota}.$ 

Note that I is an identity matrix.

# Usage

ExpMean(phi, iota, delta\_t)

# **Arguments**

phi Numeric matrix. The drift matrix  $(\Phi)$ , phi should have row and column names

pertaining to the variables in the system.

iota Numeric vector. An unobserved term that is constant over time ( $\iota$ ).

delta\_t Numeric. Time interval ( $\Delta t$ ).

## **Details**

#### **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where  $\mathbf{y}_{i,t}$ ,  $\eta_{i,t}$ , and  $\varepsilon_{i,t}$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\eta_{i,t}$  a vector of latent random variables, and  $\varepsilon_{i,t}$  a vector of random measurement errors, at time t and individual i.  $\nu$  denotes a vector of intercepts,  $\Lambda$  a matrix of factor loadings, and  $\Theta$  the covariance matrix of  $\varepsilon$ .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$ . The dynamic structure is given by

$$\mathrm{d}oldsymbol{\eta}_{i,t} = \left(oldsymbol{\iota} + oldsymbol{\Phi}oldsymbol{\eta}_{i,t}
ight)\mathrm{d}t + oldsymbol{\Sigma}^{rac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d} W$  is a Wiener process or Brownian motion, which represents random fluctuations.

## Value

Returns a numeric matrix.

36 Indirect

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

# **Examples**

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
iota <-c(.5, .3, .4)
delta_t <- 1
ExpMean(
  phi = phi,
  iota = iota,
  delta_t = delta_t
)
```

Indirect

Indirect Effect of X on Y Through M Over a Specific Time Interval

# Description

This function computes the indirect effect of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$ .

## Usage

```
Indirect(phi, delta_t, from, to, med)
```

Indirect 37

### **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ . phi should have row and column names
	pertaining to the variables in the system.
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.

## **Details**

The indirect effect of the independent variable X on the dependent variable Y relative to some mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  is given by

$$Indirect_{\Delta t_{i,j}} = \exp(\Delta t \mathbf{\Phi})_{i,j} - \exp(\Delta t \mathbf{D_m} \mathbf{\Phi} \mathbf{D_m})_{i,j}$$

where  $\Phi$  denotes the drift matrix,  $\mathbf{D_m}$  a matrix where the off diagonal elements are zeros and the diagonal elements are zero for the index/indices of mediator variables  $\mathbf{m}$  and one otherwise, i the row index of Y in  $\Phi$ , j the column index of X in  $\Phi$ , and  $\Delta t$  the time interval.

## **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} oldsymbol{\eta}_{i.t} + oldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad oldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Theta}
ight)$$

where  $\mathbf{y}_{i,t}$ ,  $\eta_{i,t}$ , and  $\varepsilon_{i,t}$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\eta_{i,t}$  a vector of latent random variables, and  $\varepsilon_{i,t}$  a vector of random measurement errors, at time t and individual t.  $\nu$  denotes a vector of intercepts,  $\Lambda$  a matrix of factor loadings, and  $\Theta$  the covariance matrix of  $\varepsilon$ .

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i,t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i,t}, \quad ext{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}
ight)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\mathbf{\Theta}^{\frac{1}{2}}\right)\left(\mathbf{\Theta}^{\frac{1}{2}}\right)' = \mathbf{\Theta}$ . The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d}W$  is a Wiener process or Brownian motion, which represents random fluctuations.

## Value

Returns an object of class ctmedeffect which is a list with the following elements:

call Function call.

args Function arguments.

**fun** Function used ("Indirect").

output The indirect effect.

38 Indirect

### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
delta_t <- 1
Indirect(
  phi = phi,
  delta_t = delta_t,
  from = "x",
  to = "y",
  med = "m"
)
phi <- matrix(</pre>
  data = c(
    -6, 5.5, 0, 0,
    1.25, -2.5, 5.9, -7.3,
    0, 0, -6, 2.5,
    5, 0, 0, -6
  ),
  nrow = 4
)
```

```
colnames(phi) <- rownames(phi) <- paste0("y", 1:4)
Indirect(
  phi = phi,
  delta_t = delta_t,
  from = "y2",
  to = "y4",
  med = c("y1", "y3")
)</pre>
```

IndirectCentral

Indirect Effect Centrality

# **Description**

**Indirect Effect Centrality** 

# Usage

```
IndirectCentral(phi, delta_t, tol = 0.01)
```

# **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
tol	Numeric. Smallest possible time interval to allow.

# **Details**

Indirect effect centrality is the sum of all possible indirect effects between different pairs of variables in which a specific variable serves as the only mediator.

# Value

Returns an object of class ctmedmed which is a list with the following elements:

```
call Function call.args Function arguments.fun Function used ("IndirectCentral").output A matrix of indirect effect centrality.
```

# Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Specific time interval ------
IndirectCentral(
 phi = phi,
 delta_t = 1
)
# Range of time intervals ------
indirect_central <- IndirectCentral(</pre>
 phi = phi,
 delta_t = 1:30
plot(indirect_central)
# Methods -----
# IndirectCentral has a number of methods including
# print, summary, and plot
indirect_central <- IndirectCentral(</pre>
 phi = phi,
 delta_t = 1:5
)
```

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```
print(indirect_central)
summary(indirect_central)
plot(indirect_central)
```

IndirectStd	Standardized Indirect Effect of X on Y Through M Over a Specific Time
	Interval

## **Description**

This function computes the standardized indirect effect of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$  and process noise covariance matrix  $\mathbf{\Sigma}$ .

## Usage

```
IndirectStd(phi, sigma, delta_t, from, to, med)
```

# Arguments

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.

## **Details**

The standardized indirect effect of the independent variable X on the dependent variable Y relative to some mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  is given by

$$Indirect^*_{\Delta t_{i,j}} = Total^*_{\Delta t} - Direct^*_{\Delta t}$$

where  $\operatorname{Total}_{\Delta t}^*$  and  $\operatorname{Direct}_{\Delta t}^*$  are standardized total and direct effects for time interval  $\Delta t$ .

## Value

Returns an object of class ctmedeffect which is a list with the following elements:

```
call Function call.args Function arguments.fun Function used ("IndirectStd").output The indirect effect.
```

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### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
sigma <- matrix(</pre>
  data = c(
    0.24455556, 0.02201587, -0.05004762,
    0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
  ),
  nrow = 3
)
delta_t <- 1
IndirectStd(
  phi = phi,
  sigma = sigma,
  delta_t = delta_t,
  from = "x",
  to = "y",
  med = "m"
)
```

MCBeta 43

MCBeta	Monte Carlo Sampling Distribution for the Elements of the Matrix of
	Lagged Coefficients Over a Specific Time Interval or a Range of Time
	Intervals

# Description

This function generates a Monte Carlo method sampling distribution for the elements of the matrix of lagged coefficients  $\beta$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model drift matrix  $\Phi$ .

# Usage

```
MCBeta(
   phi,
   vcov_phi_vec,
   delta_t,
   R,
   test_phi = TRUE,
   ncores = NULL,
   seed = NULL,
   tol = 0.01
)
```

# **Arguments**

ph1	Numeric matrix. The drift matrix ( $\Phi$ ), phi should have row and column names pertaining to the variables in the system.
vcov_phi_vec	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\mathbf{\Phi}\right)$ .
delta_t	Numeric. Time interval ( $\Delta t$ ).
R	Positive integer. Number of replications.
test_phi	Logical. If test_phi = TRUE, the function tests the stability of the generated drift matrix $\Phi$ . If the test returns FALSE, the function generates a new drift matrix $\Phi$ and runs the test recursively until the test returns TRUE.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
seed	Random seed.
tol	Numeric. Smallest possible time interval to allow.

### **Details**

44

See Total().

### **Monte Carlo Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

**MCBeta** 

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$ , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$ , can be generated by using the simulated estimates to calculate  $\mathbf{g}$ . The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100\left(1-\alpha\right)\%$  are the confidence intervals.

### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCBeta").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est A vector of total, direct, and indirect effects.

thetahatstar A matrix of Monte Carlo total, direct, and indirect effects.

# Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

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## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
    -0.00374, -0.00014, 0.00063,
    0.00495, 0.00024, -0.00093,
    0.00020, 0.00150, 0.00000,
    -0.00021, -0.00170, -0.00004,
    0.00024, 0.00214, 0.00012,
    -0.00061, 0.00012, 0.00156,
    0.00070, -0.00012, -0.00177,
    -0.00093, 0.00012, 0.00223
  ),
  nrow = 9
)
```

```
# Specific time interval ------
MCBeta(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 R = 100L # use a large value for R in actual research
# Range of time intervals ------
mc <- MCBeta(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 R = 100L # use a large value for R in actual research
)
plot(mc)
# Methods ------
# MCBeta has a number of methods including
# print, summary, confint, and plot
print(mc)
summary(mc)
confint(mc, level = 0.95)
plot(mc)
```

MCBetaStd

Monte Carlo Sampling Distribution for the Elements of the Standardized Matrix of Lagged Coefficients Over a Specific Time Interval or a Range of Time Intervals

# Description

This function generates a Monte Carlo method sampling distribution for the elements of the standardized matrix of lagged coefficients  $\beta$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model drift matrix  $\Phi$  and process noise covariance matrix  $\Sigma$ .

# Usage

```
MCBetaStd(
   phi,
   sigma,
   vcov_theta,
   delta_t,
   R,
   test_phi = TRUE,
   ncores = NULL,
```

```
seed = NULL,
tol = 0.01
)
```

## **Arguments**

phi Numeric matrix. The drift matrix  $(\Phi)$ , phi should have row and column names pertaining to the variables in the system. sigma Numeric matrix. The process noise covariance matrix  $(\Sigma)$ . vcov\_theta Numeric matrix. The sampling variance-covariance matrix of vec  $(\Phi)$  and vech  $(\Sigma)$ delta\_t Numeric. Time interval ( $\Delta t$ ). R Positive integer. Number of replications. test\_phi Logical. If test\_phi = TRUE, the function tests the stability of the generated drift matrix  $\Phi$ . If the test returns FALSE, the function generates a new drift matrix  $\Phi$  and runs the test recursively until the test returns TRUE. Positive integer. Number of cores to use. If ncores = NULL, use a single core. ncores Consider using multiple cores when number of replications R is a large value. Random seed. seed

Numeric. Smallest possible time interval to allow.

## **Details**

tol

See TotalStd().

### **Monte Carlo Method:**

Let  $\theta$  be a vector that combines  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise and  $\operatorname{vech}(\Sigma)$ , that is, the unique elements of the  $\Sigma$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be a vector that combines  $\operatorname{vec}(\hat{\Phi})$  and  $\operatorname{vech}(\hat{\Sigma})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$ , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$ , can be generated by using the simulated estimates to calculate  $\mathbf{g}$ . The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100\left(1-\alpha\right)\%$  are the confidence intervals.

### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCBetaStd").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est A vector of total, direct, and indirect effects.

thetahatstar A matrix of Monte Carlo total, direct, and indirect effects.

### Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

## See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
sigma <- matrix(
  data = c(</pre>
```

```
0.24455556, 0.02201587, -0.05004762,
   0.02201587, 0.07067800, 0.01539456,
   -0.05004762, 0.01539456, 0.07553061
 ),
 nrow = 3
)
vcov_theta <- matrix(
 data = c(
   0.00843, 0.00040, -0.00151, -0.00600, -0.00033,
   0.00110, 0.00324, 0.00020, -0.00061, -0.00115,
   0.00011, 0.00015, 0.00001, -0.00002, -0.00001,
   0.00040, 0.00374, 0.00016, -0.00022, -0.00273,
   -0.00016, 0.00009, 0.00150, 0.00012, -0.00010,
   -0.00026, 0.00002, 0.00012, 0.00004, -0.00001,
   -0.00151, 0.00016, 0.00389, 0.00103, -0.00007,
   -0.00283, -0.00050, 0.00000, 0.00156, 0.00021,
   -0.00005, -0.00031, 0.00001, 0.00007, 0.00006,
   -0.00600, -0.00022, 0.00103, 0.00644, 0.00031,
   -0.00119, -0.00374, -0.00021, 0.00070, 0.00064,
   -0.00015, -0.00005, 0.00000, 0.00003, -0.00001,
   -0.00033, -0.00273, -0.00007, 0.00031, 0.00287,
   0.00013, -0.00014, -0.00170, -0.00012, 0.00006,
   0.00014, -0.00001, -0.00015, 0.00000, 0.00001,
   0.00110, -0.00016, -0.00283, -0.00119, 0.00013,
   0.00297, 0.00063, -0.00004, -0.00177, -0.00013,
   0.00005, 0.00017, -0.00002, -0.00008, 0.00001,
   0.00324, 0.00009, -0.00050, -0.00374, -0.00014,
   0.00063, 0.00495, 0.00024, -0.00093, -0.00020,
   0.00006, -0.00010, 0.00000, -0.00001, 0.00004,
   0.00020, 0.00150, 0.00000, -0.00021, -0.00170,
   -0.00004, 0.00024, 0.00214, 0.00012, -0.00002,
   -0.00004, 0.00000, 0.00006, -0.00005, -0.00001,
   -0.00061, 0.00012, 0.00156, 0.00070, -0.00012,
   -0.00177, -0.00093, 0.00012, 0.00223, 0.00004,
   -0.00002, -0.00003, 0.00001, 0.00003, -0.00013,
   -0.00115, -0.00010, 0.00021, 0.00064, 0.00006,
   -0.00013, -0.00020, -0.00002, 0.00004, 0.00057,
   0.00001, -0.00009, 0.00000, 0.00000, 0.00001,
   0.00011, -0.00026, -0.00005, -0.00015, 0.00014,
   0.00005, 0.00006, -0.00004, -0.00002, 0.00001,
   0.00012, 0.00001, 0.00000, -0.00002, 0.00000,
   0.00015, 0.00002, -0.00031, -0.00005, -0.00001,
   0.00017, -0.00010, 0.00000, -0.00003, -0.00009,
   0.00001, 0.00014, 0.00000, 0.00000, -0.00005,
   0.00001, 0.00012, 0.00001, 0.00000, -0.00015,
   -0.00002, 0.00000, 0.00006, 0.00001, 0.00000,
   0.00000, 0.00000, 0.00010, 0.00001, 0.00000,
   -0.00002, 0.00004, 0.00007, 0.00003, 0.00000,
   -0.00008, -0.00001, -0.00005, 0.00003, 0.00000,
   -0.00002, 0.00000, 0.00001, 0.00005, 0.00001,
   -0.00001, -0.00001, 0.00006, -0.00001, 0.00001,
   0.00001, 0.00004, -0.00001, -0.00013, 0.00001,
   0.00000, -0.00005, 0.00000, 0.00001, 0.00012
```

```
),
 nrow = 15
)
# Specific time interval ------
MCBetaStd(
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1,
 R = 100L # use a large value for R in actual research
# Range of time intervals -----
mc <- MCBetaStd(</pre>
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1:5,
 R = 100L # use a large value for R in actual research
)
plot(mc)
# Methods -----
# MCBetaStd has a number of methods including
# print, summary, confint, and plot
print(mc)
summary(mc)
confint(mc, level = 0.95)
plot(mc)
```

MCIndirectCentral

Monte Carlo Sampling Distribution of Indirect Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

# **Description**

This function generates a Monte Carlo method sampling distribution of the indirect effect centrality at a particular time interval  $\Delta t$  using the first-order stochastic differential equation model drift matrix  $\Phi$ .

# Usage

```
MCIndirectCentral(
  phi,
  vcov_phi_vec,
  delta_t,
  R,
```

```
test_phi = TRUE,
  ncores = NULL,
 seed = NULL.
  tol = 0.01
)
```

## **Arguments**

phi Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names pertaining to the variables in the system. Numeric matrix. The sampling variance-covariance matrix of  $\text{vec}(\Phi)$ . vcov\_phi\_vec Numeric. Time interval ( $\Delta t$ ). delta\_t Positive integer. Number of replications. test\_phi Logical. If test\_phi = TRUE, the function tests the stability of the generated drift matrix  $\Phi$ . If the test returns FALSE, the function generates a new drift matrix  $\Phi$  and runs the test recursively until the test returns TRUE. Positive integer. Number of cores to use. If ncores = NULL, use a single core. ncores Consider using multiple cores when number of replications R is a large value. Random seed. seed

tol Numeric. Smallest possible time interval to allow.

#### **Details**

See IndirectCentral() for more details.

## **Monte Carlo Method:**

Let  $\theta$  be vec  $(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be vec  $(\hat{\Phi})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$  , can be generated by using the simulated estimates to calculate g. The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100 (1 - \alpha) \%$  are the confidence intervals.

### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCIndirectCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est A vector of indirect effect centrality.

thetahatstar A matrix of Monte Carlo indirect effect centrality.

### Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
set.seed(42)
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
vcov_phi_vec <- matrix(</pre>
```

```
data = c(
   0.00843, 0.00040, -0.00151,
   -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
   -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval -------
MCIndirectCentral(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 R = 100L # use a large value for R in actual research
)
# Range of time intervals ------
mc <- MCIndirectCentral(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 R = 100L # use a large value for R in actual research
)
plot(mc)
# Methods ------
# MCIndirectCentral has a number of methods including
# print, summary, confint, and plot
print(mc)
```

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```
summary(mc)
confint(mc, level = 0.95)
plot(mc)
```

**MCMed** 

Monte Carlo Sampling Distribution of Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function generates a Monte Carlo method sampling distribution of the total, direct and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model drift matrix  $\mathbf{\Phi}$ .

## Usage

```
MCMed(
   phi,
   vcov_phi_vec,
   delta_t,
   from,
   to,
   med,
   R,
   test_phi = TRUE,
   ncores = NULL,
   seed = NULL,
   tol = 0.01
)
```

### **Arguments**

pertaining to the variables in the system.

vcov\_phi\_vec Numeric matrix. The sampling variance-covariance matrix of  $vec(\Phi)$ .

delta\_t Numeric. Time interval ( $\Delta t$ ).

from Character string. Name of the independent variable X in phi. to Character string. Name of the dependent variable Y in phi. med Character vector. Name/s of the mediator variable/s in phi.

R Positive integer. Number of replications.

test\_phi Logical. If test\_phi = TRUE, the function tests the stability of the generated

drift matrix  $\Phi$ . If the test returns FALSE, the function generates a new drift

matrix  $\Phi$  and runs the test recursively until the test returns TRUE.

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ncores Positive integer. Number of cores to use. If ncores = NULL, use a single core.

Consider using multiple cores when number of replications R is a large value.

seed Random seed.

Numeric. Smallest possible time interval to allow.

### **Details**

See Total(), Direct(), and Indirect() for more details.

### **Monte Carlo Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$ , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$ , can be generated by using the simulated estimates to calculate  $\mathbf{g}$ . The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100\left(1-\alpha\right)\%$  are the confidence intervals.

## Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCMed").

**output** A list with length of length(delta\_t).

Each element in the output list has the following elements:

**est** A vector of total, direct, and indirect effects.

thetahatstar A matrix of Monte Carlo total, direct, and indirect effects.

### Author(s)

Ivan Jacob Agaloos Pesigan

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### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
```

```
0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval -----
MCMed(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
)
# Range of time intervals ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
)
plot(mc)
# Methods -----
# MCMed has a number of methods including
# print, summary, confint, and plot
print(mc)
summary(mc)
confint(mc, level = 0.95)
```

MCMedStd

Monte Carlo Sampling Distribution of Standardized Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

# **Description**

This function generates a Monte Carlo method sampling distribution of the standardized total, direct and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model drift matrix  $\mathbf{\Phi}$  and process noise covariance matrix  $\mathbf{\Sigma}$ .

# Usage

```
MCMedStd(
    phi,
    sigma,
    vcov_theta,
    delta_t,
    from,
    to,
    med,
    R,
    test_phi = TRUE,
    ncores = NULL,
    seed = NULL,
    tol = 0.01
)
```

# **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ . phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
vcov_theta	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\mathbf{\Phi}\right)$ and $\operatorname{vech}\left(\mathbf{\Sigma}\right)$
delta_t	Numeric. Time interval ( $\Delta t$ ).
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.
R	Positive integer. Number of replications.
test_phi	Logical. If test_phi = TRUE, the function tests the stability of the generated drift matrix $\Phi$ . If the test returns FALSE, the function generates a new drift matrix $\Phi$ and runs the test recursively until the test returns TRUE.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
seed	Random seed.
tol	Numeric. Smallest possible time interval to allow.

### **Details**

See TotalStd(), DirectStd(), and IndirectStd() for more details.

#### **Monte Carlo Method:**

Let  $\theta$  be a vector that combines  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise and  $\operatorname{vech}(\Sigma)$ , that is, the unique elements of the  $\Sigma$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be a vector that combines  $\operatorname{vec}(\hat{\Phi})$  and  $\operatorname{vech}(\hat{\Sigma})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$ , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$ , can be generated by using the simulated estimates to calculate  $\mathbf{g}$ . The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100\left(1-\alpha\right)\%$  are the confidence intervals.

### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCMedStd").

output A list with length of length(delta\_t).

Each element in the output list has the following elements:

est A vector of total, direct, and indirect effects.

thetahatstar A matrix of Monte Carlo total, direct, and indirect effects.

# Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
sigma <- matrix(</pre>
 data = c(
   0.24455556, 0.02201587, -0.05004762,
   0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
 ),
 nrow = 3
vcov_theta <- matrix(</pre>
 data = c(
    0.00843, 0.00040, -0.00151, -0.00600, -0.00033,
    0.00110, 0.00324, 0.00020, -0.00061, -0.00115,
    0.00011, 0.00015, 0.00001, -0.00002, -0.00001,
    0.00040, 0.00374, 0.00016, -0.00022, -0.00273,
    -0.00016, 0.00009, 0.00150, 0.00012, -0.00010,
    -0.00026, 0.00002, 0.00012, 0.00004, -0.00001,
    -0.00151, 0.00016, 0.00389, 0.00103, -0.00007,
    -0.00283, -0.00050, 0.00000, 0.00156, 0.00021,
    -0.00005, -0.00031, 0.00001, 0.00007, 0.00006,
    -0.00600, -0.00022, 0.00103, 0.00644, 0.00031,
    -0.00119, -0.00374, -0.00021, 0.00070, 0.00064,
    -0.00015, -0.00005, 0.00000, 0.00003, -0.00001,
    -0.00033, -0.00273, -0.00007, 0.00031, 0.00287,
    0.00013, -0.00014, -0.00170, -0.00012, 0.00006,
     0.00014, \ -0.00001, \ -0.00015, \ 0.00000, \ 0.00001, \\
    0.00110, -0.00016, -0.00283, -0.00119, 0.00013,
    0.00297, 0.00063, -0.00004, -0.00177, -0.00013,
    0.00005, 0.00017, -0.00002, -0.00008, 0.00001,
    0.00324, 0.00009, -0.00050, -0.00374, -0.00014,
    0.00063, 0.00495, 0.00024, -0.00093, -0.00020,
    0.00006, -0.00010, 0.00000, -0.00001, 0.00004,
    0.00020, 0.00150, 0.00000, -0.00021, -0.00170,
    -0.00004, 0.00024, 0.00214, 0.00012, -0.00002,
```

```
-0.00004, 0.00000, 0.00006, -0.00005, -0.00001,
   -0.00061, 0.00012, 0.00156, 0.00070, -0.00012,
   -0.00177, -0.00093, 0.00012, 0.00223, 0.00004,
   -0.00002, -0.00003, 0.00001, 0.00003, -0.00013,
   -0.00115, -0.00010, 0.00021, 0.00064, 0.00006,
   -0.00013, -0.00020, -0.00002, 0.00004, 0.00057,
   0.00001, -0.00009, 0.00000, 0.00000, 0.00001,
   0.00011, -0.00026, -0.00005, -0.00015, 0.00014,
   0.00005, 0.00006, -0.00004, -0.00002, 0.00001,
   0.00012, 0.00001, 0.00000, -0.00002, 0.00000,
   0.00015, 0.00002, -0.00031, -0.00005, -0.00001,
   0.00017, -0.00010, 0.00000, -0.00003, -0.00009,
   0.00001, 0.00014, 0.00000, 0.00000, -0.00005,
   0.00001, 0.00012, 0.00001, 0.00000, -0.00015,
   -0.00002, 0.00000, 0.00006, 0.00001, 0.00000,
   0.00000, 0.00000, 0.00010, 0.00001, 0.00000,
   -0.00002, 0.00004, 0.00007, 0.00003, 0.00000,
   -0.00008, -0.00001, -0.00005, 0.00003, 0.00000,
   -0.00002, 0.00000, 0.00001, 0.00005, 0.00001,
   -0.00001, -0.00001, 0.00006, -0.00001, 0.00001,
   0.00001, 0.00004, -0.00001, -0.00013, 0.00001,
   0.00000, -0.00005, 0.00000, 0.00001, 0.00012
 ),
 nrow = 15
)
# Specific time interval -------
MCMedStd(
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
# Range of time intervals ------
mc <- MCMedStd(</pre>
 phi = phi,
 sigma = sigma,
 vcov_theta = vcov_theta,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
)
plot(mc)
# Methods -----
# MCMedStd has a number of methods including
```

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```
# print, summary, confint, and plot
print(mc)
summary(mc)
confint(mc, level = 0.95)
```

MCPhi

Generate Random Drift Matrices Using the Monte Carlo Method

# **Description**

This function generates random drift matrices  $\Phi$  using the Monte Carlo method.

## Usage

```
MCPhi(phi, vcov_phi_vec, R, test_phi = TRUE, ncores = NULL, seed = NULL)
```

# **Arguments**

phi Numeric matrix. The drift matrix ( $\Phi$ ), phi should have row and column names

pertaining to the variables in the system.

vcov\_phi\_vec Numeric matrix. The sampling variance-covariance matrix of vec  $(\Phi)$ .

R Positive integer. Number of replications.

test\_phi Logical. If test\_phi = TRUE, the function tests the stability of the generated

drift matrix  $\Phi$ . If the test returns FALSE, the function generates a new drift

matrix  $\Phi$  and runs the test recursively until the test returns TRUE.

ncores Positive integer. Number of cores to use. If ncores = NULL, use a single core.

Consider using multiple cores when number of replications R is a large value.

seed Random seed.

## **Details**

## **Monte Carlo Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

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## Value

Returns an object of class ctmedmc which is a list with the following elements:

```
call Function call.args Function arguments.fun Function used ("MCPhi").output A list simulated drift matrices.
```

### Author(s)

Ivan Jacob Agaloos Pesigan

## See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
MCPhi(
  phi = phi,
  vcov_phi_vec = 0.1 * diag(9),
  R = 100L # use a large value for R in actual research
phi <- matrix(</pre>
  data = c(
    -6, 5.5, 0, 0,
    1.25, -2.5, 5.9, -7.3,
    0, 0, -6, 2.5,
    5, 0, 0, -6
  ),
  nrow = 4
)
colnames(phi) <- rownames(phi) <- paste0("y", 1:4)</pre>
MCPhi(
  phi = phi,
```

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```
vcov_phi_vec = 0.1 * diag(16),
R = 100L, # use a large value for R in actual research
test_phi = FALSE
)
```

**MCTotalCentral** 

Monte Carlo Sampling Distribution of Total Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

# Description

This function generates a Monte Carlo method sampling distribution of the total effect centrality at a particular time interval  $\Delta t$  using the first-order stochastic differential equation model drift matrix  $\Phi$ .

# Usage

```
MCTotalCentral(
   phi,
   vcov_phi_vec,
   delta_t,
   R,
   test_phi = TRUE,
   ncores = NULL,
   seed = NULL,
   tol = 0.01
)
```

## **Arguments**

tol

phi	Numeric matrix. The drift matrix $(\Phi)$ . phi should have row and column names pertaining to the variables in the system.
vcov_phi_vec	Numeric matrix. The sampling variance-covariance matrix of $\operatorname{vec}\left(\mathbf{\Phi}\right)$ .
delta_t	Numeric. Time interval ( $\Delta t$ ).
R	Positive integer. Number of replications.
test_phi	Logical. If test_phi = TRUE, the function tests the stability of the generated drift matrix $\Phi$ . If the test returns FALSE, the function generates a new drift matrix $\Phi$ and runs the test recursively until the test returns TRUE.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
seed	Random seed.

Numeric. Smallest possible time interval to allow.

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### **Details**

See TotalCentral() for more details.

#### **Monte Carlo Method:**

Let  $\theta$  be  $\operatorname{vec}(\Phi)$ , that is, the elements of the  $\Phi$  matrix in vector form sorted column-wise. Let  $\hat{\theta}$  be  $\operatorname{vec}(\hat{\Phi})$ . Based on the asymptotic properties of maximum likelihood estimators, we can assume that estimators are normally distributed around the population parameters.

$$\hat{oldsymbol{ heta}} \sim \mathcal{N}\left(oldsymbol{ heta}, \mathbb{V}\left(\hat{oldsymbol{ heta}}
ight)
ight)$$

Using this distributional assumption, a sampling distribution of  $\hat{\theta}$  which we refer to as  $\hat{\theta}^*$  can be generated by replacing the population parameters with sample estimates, that is,

$$\hat{oldsymbol{ heta}}^* \sim \mathcal{N}\left(\hat{oldsymbol{ heta}}, \hat{\mathbb{V}}\left(\hat{oldsymbol{ heta}}
ight)
ight).$$

Let  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$  be a parameter that is a function of the estimated parameters. A sampling distribution of  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}\right)$ , which we refer to as  $\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right)$ , can be generated by using the simulated estimates to calculate  $\mathbf{g}$ . The standard deviations of the simulated estimates are the standard errors. Percentiles corresponding to  $100\left(1-\alpha\right)\%$  are the confidence intervals.

### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("MCTotalCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est A vector of total effect centrality.

thetahatstar A matrix of Monte Carlo total effect centrality.

# Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

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## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
    -0.00374, -0.00014, 0.00063,
    0.00495, 0.00024, -0.00093,
    0.00020, 0.00150, 0.00000,
    -0.00021, -0.00170, -0.00004,
    0.00024, 0.00214, 0.00012,
    -0.00061, 0.00012, 0.00156,
    0.00070, -0.00012, -0.00177,
    -0.00093, 0.00012, 0.00223
  ),
  nrow = 9
)
```

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```
# Specific time interval ------
MCTotalCentral(
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 R = 100L # use a large value for R in actual research
# Range of time intervals -----
mc <- MCTotalCentral(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 R = 100L # use a large value for R in actual research
plot(mc)
# Methods ------
# MCTotalCentral has a number of methods including
# print, summary, confint, and plot
print(mc)
summary(mc)
confint(mc, level = 0.95)
plot(mc)
```

Med

Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function computes the total, direct, and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$ .

## Usage

```
Med(phi, delta_t, from, to, med, tol = 0.01)
```

## **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.
tol	Numeric. Smallest possible time interval to allow.

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### **Details**

See Total(), Direct(), and Indirect() for more details.

# **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} oldsymbol{\eta}_{i,t} + oldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad oldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}
ight)$$

where  $\mathbf{y}_{i,t}$ ,  $\eta_{i,t}$ , and  $\varepsilon_{i,t}$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\eta_{i,t}$  a vector of latent random variables, and  $\varepsilon_{i,t}$  a vector of random measurement errors, at time t and individual i.  $\nu$  denotes a vector of intercepts,  $\Lambda$  a matrix of factor loadings, and  $\Theta$  the covariance matrix of  $\varepsilon$ .

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i.t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i.t}, \quad ext{with} \quad \mathbf{z}_{i.t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$ . The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d}W$  is a Wiener process or Brownian motion, which represents random fluctuations.

### Value

Returns an object of class ctmedmed which is a list with the following elements:

call Function call.

**args** Function arguments.

fun Function used ("Med").

**output** A matrix of total, direct, and indirect effects.

# Author(s)

Ivan Jacob Agaloos Pesigan

### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

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## See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Specific time interval ------
Med(
 phi = phi,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
)
# Range of time intervals ------
med <- Med(</pre>
 phi = phi,
 delta_t = 1:30,
 from = "x",
 to = "y",
 med = "m"
)
plot(med)
# Methods ------
# Med has a number of methods including
# print, summary, and plot
med <- Med(
 phi = phi,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
print(med)
summary(med)
plot(med)
```

70 MedStd

MedStd	Standardized Total, Direct, and Indirect Effects of X on Y Through M
	Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function computes the standardized total, direct, and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model's drift matrix  $\mathbf{\Phi}$  and process noise covariance matrix  $\mathbf{\Sigma}$ .

## Usage

```
MedStd(phi, sigma, delta_t, from, to, med, tol = 0.01)
```

# **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
delta_t	Numeric. Time interval $(\Delta t)$ .
from	Character string. Name of the independent variable $X$ in phi.
to	Character string. Name of the dependent variable $Y$ in phi.
med	Character vector. Name/s of the mediator variable/s in phi.
tol	Numeric. Smallest possible time interval to allow.

# **Details**

```
See TotalStd(), DirectStd(), and IndirectStd() for more details.
```

## Value

Returns an object of class ctmedmed which is a list with the following elements:

```
call Function call.args Function arguments.fun Function used ("MedStd").output A matrix of total, direct, and indirect effects.
```

# Author(s)

Ivan Jacob Agaloos Pesigan

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#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
sigma <- matrix(</pre>
 data = c(
   0.24455556, 0.02201587, -0.05004762,
   0.02201587, 0.07067800, 0.01539456,
   -0.05004762, 0.01539456, 0.07553061
 ),
 nrow = 3
)
# Specific time interval -------
MedStd(
 phi = phi,
 sigma = sigma,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
# Range of time intervals -------
med <- MedStd(</pre>
```

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```
phi = phi,
 sigma = sigma,
 delta_t = 1:30,
 from = "x",
 to = "y",
 med = "m"
)
plot(med)
# Methods ------
# MedStd has a number of methods including
# print, summary, and plot
med <- MedStd(</pre>
 phi = phi,
 sigma = sigma,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
print(med)
summary(med)
plot(med)
```

plot.ctmeddelta

Plot Method for an Object of Class ctmeddelta

# **Description**

Plot Method for an Object of Class ctmeddelta

## Usage

```
## S3 method for class 'ctmeddelta'
plot(x, alpha = 0.05, col = NULL, ...)
```

# Arguments

x	Object of class ctmeddelta.
alpha	Numeric. Significance level
col	Character vector. Optional argument. Character vector of colors.
	Additional arguments.

# Value

Displays plots of point estimates and confidence intervals.

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#### Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
 data = c(
   0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
    -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Range of time intervals -------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
```

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```
from = "x",
  to = "y",
  med = "m"
)
plot(delta)
```

plot.ctmedmc

Plot Method for an Object of Class ctmedmc

# **Description**

Plot Method for an Object of Class ctmedmc

# Usage

```
## S3 method for class 'ctmedmc'
plot(x, alpha = 0.05, col = NULL, ...)
```

# Arguments

X	Object of class ctmedmc.
alpha	Numeric. Significance level
col	Character vector. Optional argument. Character vector of colors.
	Additional arguments.

## Value

Displays plots of point estimates and confidence intervals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
set.seed(42)
phi <- matrix(
   data = c(
     -0.357, 0.771, -0.450,
     0.0, -0.511, 0.729,
     0, 0, -0.693
   ),
   nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
vcov_phi_vec <- matrix(
   data = c(</pre>
```

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```
0.00843, 0.00040, -0.00151,
   -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
   -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
    -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Range of time intervals ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
 R = 100L # use a large value for R in actual research
plot(mc)
```

plot.ctmedmed

Plot Method for an Object of Class ctmedmed

#### **Description**

Plot Method for an Object of Class ctmedmed

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#### Usage

```
## S3 method for class 'ctmedmed'
plot(x, col = NULL, legend_pos = "topright", ...)
```

# Arguments

x Object of class ctmedmed.

col Character vector. Optional argument. Character vector of colors.

legend\_pos Character vector. Optional argument. Legend position.

... Additional arguments.

#### Value

Displays plots of point estimates and confidence intervals.

## Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Range of time intervals ------
med <- Med(
 phi = phi,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
)
plot(med)
```

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plot.ctmedtraj

Plot Method for an Object of Class ctmedtraj

## **Description**

Plot Method for an Object of Class ctmedtraj

# Usage

```
## S3 method for class 'ctmedtraj'
plot(x, legend_pos = "topright", total = TRUE, ...)
```

## **Arguments**

x Object of class ctmedtraj.
 legend\_pos Character vector. Optional argument. Legend position.
 total Logical. If total = TRUE, include the total effect trajectory. If total = FALSE, exclude the total effect trajectory.
 ... Additional arguments.

#### Value

Displays trajectory plots of the effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")

traj <- Trajectory(
  mu0 = c(3, 3, -3),
    time = 150,
    phi = phi,
    med = "m"
)

plot(traj)</pre>
```

78 PosteriorBeta

PosteriorBeta	Posterior Sampling Distribution for the Elements of the Matrix of Lagged Coefficients Over a Specific Time Interval or a Range of Time Intervals

# Description

This function generates a posterior sampling distribution for the elements of the matrix of lagged coefficients  $\beta$  over a specific time interval  $\Delta t$  or a range of time intervals using the first-order stochastic differential equation model drift matrix  $\Phi$ .

## Usage

```
PosteriorBeta(phi, delta_t, ncores = NULL, tol = 0.01)
```

## **Arguments**

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
delta_t	Numeric. Time interval ( $\Delta t$ ).
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
tol	Numeric. Smallest possible time interval to allow.

# **Details**

```
See Total().
```

#### Value

Returns an object of class ctmedmc which is a list with the following elements:

```
call Function call.
```

args Function arguments.

fun Function used ("PosteriorBeta").

output A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est A vector of total, direct, and indirect effects.

thetahatstar A matrix of Monte Carlo total, direct, and indirect effects.

## Author(s)

Ivan Jacob Agaloos Pesigan

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#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ).
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
```

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```
-0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
phi <- MCPhi(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 R = 1000L
)$output
# Specific time interval ------
PosteriorBeta(
 phi = phi,
 delta_t = 1
)
# Range of time intervals ------
posterior <- PosteriorBeta(</pre>
 phi = phi,
 delta_t = 1:5
plot(posterior)
# Methods -----
# PosteriorBeta has a number of methods including
# print, summary, confint, and plot
print(posterior)
summary(posterior)
confint(posterior, level = 0.95)
plot(posterior)
```

PosteriorIndirectCentral

Posterior Distribution of the Indirect Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

# Description

This function generates a posterior distribution of the indirect effect centrality over a specific time interval  $\Delta t$  or a range of time intervals using the posterior distribution of the first-order stochastic differential equation model drift matrix  $\Phi$ .

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#### Usage

PosteriorIndirectCentral(phi, delta\_t, ncores = NULL, tol = 0.01)

#### **Arguments**

phi List of numeric matrices. Each element of the list is a sample from the posterior

distribution of the drift matrix ( $\Phi$ ). Each matrix should have row and column

names pertaining to the variables in the system.

delta\_t Numeric. Time interval ( $\Delta t$ ).

ncores Positive integer. Number of cores to use. If ncores = NULL, use a single core.

Consider using multiple cores when number of replications R is a large value.

tol Numeric. Smallest possible time interval to allow.

#### **Details**

See TotalCentral() for more details.

#### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("PosteriorIndirectCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est Mean of the posterior distribution of the total, direct, and indirect effects.

thetahatstar Posterior distribution of the total, direct, and indirect effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

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#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ).
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
 data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
    -0.00374, -0.00014, 0.00063,
    0.00495, 0.00024, -0.00093,
    0.00020, 0.00150, 0.00000,
    -0.00021, -0.00170, -0.00004,
    0.00024, 0.00214, 0.00012,
    -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
    -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
phi <- MCPhi(</pre>
```

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```
phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 R = 1000L
)$output
# Specific time interval -------
PosteriorIndirectCentral(
 phi = phi,
 delta_t = 1
)
# Range of time intervals ------
posterior <- PosteriorIndirectCentral(</pre>
 phi = phi,
 delta_t = 1:5
# Methods ------
# PosteriorIndirectCentral has a number of methods including
# print, summary, confint, and plot
print(posterior)
summary(posterior)
confint(posterior, level = 0.95)
plot(posterior)
```

PosteriorMed

Posterior Distribution of Total, Direct, and Indirect Effects of X on Y Through M Over a Specific Time Interval or a Range of Time Intervals

## **Description**

This function generates a posterior distribution of the total, direct and indirect effects of the independent variable X on the dependent variable Y through mediator variables  $\mathbf{m}$  over a specific time interval  $\Delta t$  or a range of time intervals using the posterior distribution of the first-order stochastic differential equation model drift matrix  $\Phi$ .

# Usage

```
PosteriorMed(phi, delta_t, from, to, med, ncores = NULL, tol = 0.01)
```

# Arguments

phi	List of numeric matrices. Each element of the list is a sample from the posterior distribution of the drift matrix ( $\Phi$ ). Each matrix should have row and column names pertaining to the variables in the system.	
delta_t	Numeric. Time interval ( $\Delta t$ ).	
from	Character string. Name of the independent variable $X$ in phi.	

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to	Character string. Name of the dependent variable $Y$ in phi.	
med	Character vector. Name/s of the mediator variable/s in phi.	
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.	
tol	Numeric. Smallest possible time interval to allow.	

#### **Details**

See Total(), Direct(), and Indirect() for more details.

#### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("PosteriorMed").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

est Mean of the posterior distribution of the total, direct, and indirect effects.

thetahatstar Posterior distribution of the total, direct, and indirect effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

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```
phi <- matrix(</pre>
  data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
    0.00063, -0.00004, -0.00177,
    0.00324, 0.00009, -0.00050,
    -0.00374, -0.00014, 0.00063,
    0.00495, 0.00024, -0.00093,
    0.00020, 0.00150, 0.00000,
    -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
    -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
  ),
  nrow = 9
)
phi <- MCPhi(</pre>
  phi = phi,
  vcov_phi_vec = vcov_phi_vec,
  R = 1000L
)$output
# Specific time interval ------
PosteriorMed(
  phi = phi,
```

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```
delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
)
# Range of time intervals ------
posterior <- PosteriorMed(</pre>
 phi = phi,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
# PosteriorMed has a number of methods including
# print, summary, confint, and plot
print(posterior)
summary(posterior)
confint(posterior, level = 0.95)
plot(posterior)
```

PosteriorPhi

Extract the Posterior Samples of the Drift Matrix

# Description

The function extracts the posterior samples of the drift matrix from a fitted model from the ctsem::ctStanFit() function.

# Usage

PosteriorPhi(object)

#### **Arguments**

object

Object of class ctStanFit. Output of the ctsem::ctStanFit() function.

#### Value

Returns an object of class ctmedposteriorphi which is a list drift matrices sampled from the posterior distribution.

## Author(s)

Ivan Jacob Agaloos Pesigan

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#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd(), Trajectory()

PosteriorTotalCentral Posterior Distribution of the Total Effect Centrality Over a Specific Time Interval or a Range of Time Intervals

#### **Description**

This function generates a posterior distribution of the total effect centrality over a specific time interval  $\Delta t$  or a range of time intervals using the posterior distribution of the first-order stochastic differential equation model drift matrix  $\Phi$ .

#### Usage

```
PosteriorTotalCentral(phi, delta_t, ncores = NULL, tol = 0.01)
```

#### **Arguments**

phi	List of numeric matrices. Each element of the list is a sample from the posterior distribution of the drift matrix ( $\Phi$ ). Each matrix should have row and column names pertaining to the variables in the system.
delta_t	Numeric. Time interval ( $\Delta t$ ).
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of replications R is a large value.
tol	Numeric. Smallest possible time interval to allow.

#### **Details**

See TotalCentral() for more details.

#### Value

Returns an object of class ctmedmc which is a list with the following elements:

call Function call.

**args** Function arguments.

fun Function used ("PosteriorTotalCentral").

**output** A list the length of which is equal to the length of delta\_t.

Each element in the output list has the following elements:

**est** Mean of the posterior distribution of the total, direct, and indirect effects.

thetahatstar Posterior distribution of the total, direct, and indirect effects.

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#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), Total(), TotalCentral(), TotalStd(), Trajectory()
```

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
 data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
```

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```
-0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
phi <- MCPhi(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 R = 1000L
)$output
# Specific time interval -----
PosteriorTotalCentral(
 phi = phi,
 delta_t = 1
)
# Range of time intervals ------
posterior <- PosteriorTotalCentral(</pre>
 phi = phi,
 delta_t = 1:5
)
# Methods ------
# PosteriorTotalCentral has a number of methods including
# print, summary, confint, and plot
print(posterior)
summary(posterior)
confint(posterior, level = 0.95)
plot(posterior)
```

print.ctmeddelta

Print Method for Object of Class ctmeddelta

#### **Description**

Print Method for Object of Class ctmeddelta

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#### Usage

```
## S3 method for class 'ctmeddelta'
print(x, alpha = 0.05, digits = 4, ...)
```

#### **Arguments**

```
x an object of class ctmeddelta. 
alpha Numeric vector. Significance level \alpha. 
digits Integer indicating the number of decimal places to display. 
... further arguments.
```

#### Value

Prints a list of matrices of time intervals, estimates, standard errors, test statistics, p-values, and confidence intervals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
    -0.00050, 0.00000, 0.00156,
    -0.00600, -0.00022, 0.00103,
    0.00644, 0.00031, -0.00119,
    -0.00374, -0.00021, 0.00070,
    -0.00033, -0.00273, -0.00007,
    0.00031, 0.00287, 0.00013,
    -0.00014, -0.00170, -0.00012,
    0.00110, -0.00016, -0.00283,
    -0.00119, 0.00013, 0.00297,
```

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```
0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = x^*,
 to = "y",
 med = "m"
)
print(delta)
# Range of time intervals ------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
print(delta)
```

print.ctmedeffect

Print Method for Object of Class ctmedeffect

# Description

Print Method for Object of Class ctmedeffect

# Usage

```
## S3 method for class 'ctmedeffect'
print(x, digits = 4, ...)
```

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# **Arguments**

x an object of class ctmedeffect.digits Integer indicating the number of decimal places to display.... further arguments.

#### Value

Prints the effects.

## Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
delta_t <- 1
# Time Interval of One ------
## Total Effect ---------------
total_dt <- Total(</pre>
 phi = phi,
 delta_t = delta_t
print(total_dt)
## Direct Effect -------
direct_dt <- Direct(</pre>
 phi = phi,
 delta_t = delta_t,
 from = "x",
 to = "y",
 med = "m"
)
print(direct_dt)
## Indirect Effect ------
indirect_dt <- Indirect(</pre>
 phi = phi,
 delta_t = delta_t,
 from = "x",
 to = "y",
```

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```
med = "m"
)
print(indirect_dt)
```

print.ctmedmc

Print Method for Object of Class ctmedmc

## **Description**

Print Method for Object of Class ctmedmc

## Usage

```
## S3 method for class 'ctmedmc'
print(x, alpha = 0.05, digits = 4, ...)
```

#### **Arguments**

```
x an object of class ctmedmc. 
alpha Numeric vector. Significance level \alpha. 
digits Integer indicating the number of decimal places to display. 
... further arguments.
```

#### Value

Prints a list of matrices of time intervals, estimates, standard errors, number of Monte Carlo replications, and confidence intervals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
set.seed(42)
phi <- matrix(
   data = c(
     -0.357, 0.771, -0.450,
     0.0, -0.511, 0.729,
     0, 0, -0.693
   ),
   nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
vcov_phi_vec <- matrix(
   data = c(
     0.00843, 0.00040, -0.00151,</pre>
```

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```
-0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
   -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval -------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
 R = 100L # use a large value for R in actual research
print(mc)
# Range of time intervals ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = x^{*},
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
print(mc)
```

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print.ctmedmcphi

Print Method for Object of Class ctmedmcphi

## **Description**

Print Method for Object of Class ctmedmcphi

# Usage

```
## S3 method for class 'ctmedmcphi'
print(x, digits = 4, ...)
```

# Arguments

```
x an object of class ctmedmcphi.digits Integer indicating the number of decimal places to display.... further arguments.
```

#### Value

Prints a list of drift matrices.

# Author(s)

Ivan Jacob Agaloos Pesigan

```
set.seed(42)
phi <- matrix(
    data = c(
        -0.357, 0.771, -0.450,
        0.0, -0.511, 0.729,
        0, 0, -0.693
    ),
    nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
mc <- MCPhi(
    phi = phi,
    vcov_phi_vec = 0.1 * diag(9),
    R = 100L # use a large value for R in actual research
)
print(mc)</pre>
```

96 print.ctmedmed

print.ctmedmed

Print Method for Object of Class ctmedmed

#### **Description**

Print Method for Object of Class ctmedmed

## Usage

```
## S3 method for class 'ctmedmed'
print(x, digits = 4, ...)
```

#### Arguments

```
x an object of class ctmedmed.digits Integer indicating the number of decimal places to display.... further arguments.
```

#### Value

Prints a matrix of effects.

## Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Specific time interval ------
med <- Med(</pre>
 phi = phi,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
print(med)
# Range of time intervals ------
```

print.ctmedtraj 97

```
med <- Med(
    phi = phi,
    delta_t = 1:5,
    from = "x",
    to = "y",
    med = "m"
)
print(med)</pre>
```

print.ctmedtraj

Print Method for Object of Class ctmedtraj

# Description

Print Method for Object of Class ctmedtraj

# Usage

```
## S3 method for class 'ctmedtraj' print(x, ...)
```

# Arguments

x an object of class ctmedtraj.... further arguments.

## Value

Prints a data frame of simulated data.

## Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
traj <- Trajectory(
  mu0 = c(3, 3, -3),</pre>
```

98 summary.ctmeddelta

```
time = 150,
phi = phi,
med = "m"
)
print(traj)
```

summary.ctmeddelta

Summary Method for an Object of Class ctmeddelta

# Description

Summary Method for an Object of Class ctmeddelta

# Usage

```
## S3 method for class 'ctmeddelta'
summary(object, alpha = 0.05, ...)
```

## **Arguments**

object Object of class ctmeddelta. alpha Numeric vector. Significance level  $\alpha$ . ... additional arguments.

#### Value

Returns a data frame of effects, time intervals, estimates, standard errors, test statistics, p-values, and confidence intervals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")
vcov_phi_vec <- matrix(
  data = c(
    0.00843, 0.00040, -0.00151,</pre>
```

summary.ctmeddelta 99

```
-0.00600, -0.00033, 0.00110,
   0.00324, 0.00020, -0.00061,
   0.00040, 0.00374, 0.00016,
   -0.00022, -0.00273, -0.00016,
   0.00009, 0.00150, 0.00012,
   -0.00151, 0.00016, 0.00389,
   0.00103, -0.00007, -0.00283,
   -0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
summary(delta)
# Range of time intervals -----
delta <- DeltaMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
)
summary(delta)
```

100 summary.ctmedmc

summary.ctmedmc

Summary Method for an Object of Class ctmedmc

## **Description**

Summary Method for an Object of Class ctmedmc

# Usage

```
## S3 method for class 'ctmedmc'
summary(object, alpha = 0.05, ...)
```

## **Arguments**

```
object Object of class ctmedmc.  
alpha Numeric vector. Significance level \alpha.  
additional arguments.
```

#### Value

Returns a data frame of effects, time intervals, estimates, standard errors, number of Monte Carlo replications, and confidence intervals.

## Author(s)

Ivan Jacob Agaloos Pesigan

```
set.seed(42)
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
vcov_phi_vec <- matrix(</pre>
  data = c(
    0.00843, 0.00040, -0.00151,
    -0.00600, -0.00033, 0.00110,
    0.00324, 0.00020, -0.00061,
    0.00040, 0.00374, 0.00016,
    -0.00022, -0.00273, -0.00016,
    0.00009, 0.00150, 0.00012,
    -0.00151, 0.00016, 0.00389,
    0.00103, -0.00007, -0.00283,
```

summary.ctmedmed 101

```
-0.00050, 0.00000, 0.00156,
   -0.00600, -0.00022, 0.00103,
   0.00644, 0.00031, -0.00119,
   -0.00374, -0.00021, 0.00070,
   -0.00033, -0.00273, -0.00007,
   0.00031, 0.00287, 0.00013,
   -0.00014, -0.00170, -0.00012,
   0.00110, -0.00016, -0.00283,
   -0.00119, 0.00013, 0.00297,
   0.00063, -0.00004, -0.00177,
   0.00324, 0.00009, -0.00050,
   -0.00374, -0.00014, 0.00063,
   0.00495, 0.00024, -0.00093,
   0.00020, 0.00150, 0.00000,
   -0.00021, -0.00170, -0.00004,
   0.00024, 0.00214, 0.00012,
   -0.00061, 0.00012, 0.00156,
   0.00070, -0.00012, -0.00177,
   -0.00093, 0.00012, 0.00223
 ),
 nrow = 9
)
# Specific time interval ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m",
 R = 100L # use a large value for R in actual research
)
summary(mc)
# Range of time intervals ------
mc <- MCMed(</pre>
 phi = phi,
 vcov_phi_vec = vcov_phi_vec,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
 R = 100L # use a large value for R in actual research
)
summary(mc)
```

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#### **Description**

Summary Method for an Object of Class ctmedmed

#### Usage

```
## S3 method for class 'ctmedmed'
summary(object, digits = 4, ...)
```

#### **Arguments**

```
object an object of class ctmedmed.

digits Integer indicating the number of decimal places to display.

further arguments.
```

#### Value

Returns a matrix of effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Specific time interval ------
med <- Med(
 phi = phi,
 delta_t = 1,
 from = "x",
 to = "y",
 med = "m"
)
summary(med)
# Range of time intervals -----
med <- Med(
 phi = phi,
 delta_t = 1:5,
 from = "x",
 to = "y",
 med = "m"
```

```
)
summary(med)
```

```
summary.ctmedposteriorphi
```

Summary Method for Object of Class ctmedposteriorphi

# Description

Summary Method for Object of Class ctmedposteriorphi

#### Usage

```
## S3 method for class 'ctmedposteriorphi'
summary(object, ...)
```

# Arguments

```
object an object of class ctmedposteriorphi.
... further arguments.
```

#### Value

Returns a list of the posterior means (in matrix form) and covariance matrix.

# Author(s)

Ivan Jacob Agaloos Pesigan

summary.ctmedtraj

Summary Method for an Object of Class ctmedtraj

# **Description**

Summary Method for an Object of Class ctmedtraj

#### **Usage**

```
## S3 method for class 'ctmedtraj'
summary(object, ...)
```

# Arguments

```
object an object of class ctmedtraj.
... further arguments.
```

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## Value

Returns a data frame of simulated data.

# Author(s)

Ivan Jacob Agaloos Pesigan

## **Examples**

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
),
  nrow = 3
)
colnames(phi) <- rownames(phi) <- c("x", "m", "y")

traj <- Trajectory(
  mu0 = c(3, 3, -3),
  time = 150,
  phi = phi,
  med = "m"
)
summary(traj)</pre>
```

Total

Total Effect Matrix Over a Specific Time Interval

# **Description**

This function computes the total effects matrix over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\Phi$ .

# Usage

```
Total(phi, delta_t)
```

## **Arguments**

Phi Numeric matrix. The drift matrix  $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.

delta\_t Numeric. Time interval ( $\Delta t$ ).

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#### **Details**

The total effect matrix over a specific time interval  $\Delta t$  is given by

$$Total_{\Delta t} = \exp(\Delta t \mathbf{\Phi})$$

where  $\Phi$  denotes the drift matrix, and  $\Delta t$  the time interval.

#### **Linear Stochastic Differential Equation Model:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}
ight)$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\boldsymbol{\eta}_{i,t}$  a vector of latent random variables, and  $\boldsymbol{\varepsilon}_{i,t}$  a vector of random measurement errors, at time t and individual i.  $\boldsymbol{\nu}$  denotes a vector of intercepts,  $\boldsymbol{\Lambda}$  a matrix of factor loadings, and  $\boldsymbol{\Theta}$  the covariance matrix of  $\boldsymbol{\varepsilon}$ .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$ . The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where  $\iota$  is a term which is unobserved and constant over time,  $\Phi$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d}W$  is a Wiener process or Brownian motion, which represents random fluctuations.

#### Value

Returns an object of class ctmedeffect which is a list with the following elements:

call Function call.

args Function arguments.

**fun** Function used ("Total").

output The matrix of total effects.

## Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

106 TotalCentral

#### See Also

Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), TotalCentral(), TotalStd(), Trajectory()

## **Examples**

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
delta_t <- 1
Total(
  phi = phi,
  delta_t = delta_t
phi <- matrix(</pre>
  data = c(
    -6, 5.5, 0, 0,
    1.25, -2.5, 5.9, -7.3,
    0, 0, -6, 2.5,
    5, 0, 0, -6
  ),
  nrow = 4
colnames(phi) <- rownames(phi) <- paste0("y", 1:4)</pre>
Total(
  phi = phi,
  delta_t = delta_t
```

TotalCentral

Total Effect Centrality

## **Description**

Total Effect Centrality

#### Usage

```
TotalCentral(phi, delta_t, tol = 0.01)
```

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## Arguments

phi	Numeric matrix. The drift matrix $(\Phi)$ , phi should have row and column names pertaining to the variables in the system.
delta_t	Vector of positive numbers. Time interval $(\Delta t)$ .
tol	Numeric. Smallest possible time interval to allow.

#### **Details**

The total effect centrality of a variable is the sum of the total effects of a variable on all other variables at a particular time interval.

#### Value

Returns an object of class ctmedmed which is a list with the following elements:

```
call Function call.
```

args Function arguments.

fun Function used ("TotalCentral").

output A matrix of total effect centrality.

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalStd(), Trajectory()
```

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#### **Examples**

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ),
 nrow = 3
colnames(phi) <- rownames(phi) <- c("x", "m", "y")</pre>
# Specific time interval ------
TotalCentral(
 phi = phi,
 delta_t = 1
)
# Range of time intervals ------
total_central <- TotalCentral(</pre>
 phi = phi,
 delta_t = 1:30
plot(total_central)
# Methods ------
# TotalCentral has a number of methods including
# print, summary, and plot
total_central <- TotalCentral(</pre>
 phi = phi,
 delta_t = 1:5
print(total_central)
summary(total_central)
plot(total_central)
```

TotalStd

Standardized Total Effect Matrix Over a Specific Time Interval

#### **Description**

This function computes the standardized total effects matrix over a specific time interval  $\Delta t$  using the first-order stochastic differential equation model's drift matrix  $\Phi$  and process noise covariance matrix  $\Sigma$ .

## Usage

```
TotalStd(phi, sigma, delta_t)
```

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#### Arguments

phi	Numeric matrix. The drift matrix ( $\Phi$ ). phi should have row and column names
	pertaining to the variables in the system.
sigma	Numeric matrix. The process noise covariance matrix $(\Sigma)$ .
delta_t	Numeric. Time interval ( $\Delta t$ ).

#### **Details**

The standardized total effect matrix over a specific time interval  $\Delta t$  is given by

$$\operatorname{Total}_{\Delta t}^{*} = \mathbf{S} \left( \exp \left( \Delta t \mathbf{\Phi} \right) \right) \mathbf{S}^{-1}$$

where  $\Phi$  denotes the drift matrix, S a diagonal matrix with model-implied standard deviations on the diagonals and  $\Delta t$  the time interval.

#### Value

Returns an object of class ctmedeffect which is a list with the following elements:

call Function call.

args Function arguments.

fun Function used ("TotalStd").

output The matrix of total effects.

#### Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Bollen, K. A. (1987). Total, direct, and indirect effects in structural equation models. Sociological Methodology, 17, 37. doi:10.2307/271028

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 23 (1), 61–75. doi:10.1080/10705511.2014.973960

Ryan, O., & Hamaker, E. L. (2021). Time to intervene: A continuous-time approach to network analysis and centrality. Psychometrika, 87 (1), 214–252. doi:10.1007/s11336021097670

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), Trajectory()
```

Trajectory Trajectory

#### **Examples**

```
phi <- matrix(</pre>
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
sigma <- matrix(</pre>
  data = c(
    0.24455556, 0.02201587, -0.05004762,
    0.02201587, 0.07067800, 0.01539456,
    -0.05004762, 0.01539456, 0.07553061
  ),
  nrow = 3
)
delta_t <- 1
TotalStd(
  phi = phi,
  sigma = sigma,
  delta_t = delta_t
```

Trajectory

Simulate Trajectories of Variables

# Description

This function simulates trajectories of variables without measurement error or process noise. Total corresponds to the total effect and Direct corresponds to the portion of the total effect where the indirect effect is removed.

## Usage

```
Trajectory(mu0, time, phi, med)
```

# Arguments

mu0	Numeric vector. Initial values of the variables.
time	Positive integer. Number of time points.
phi	Numeric matrix. The drift matrix $(\Phi)$ . phi should have row and column names pertaining to the variables in the system.
med	Character vector. Name/s of the mediator variable/s in phi.

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#### Value

```
Returns an object of class ctmedtraj which is a list with the following elements:
```

call Function call.args Function arguments.fun Function used ("Trajectory").

output A data frame of simulated data.

#### See Also

```
Other Continuous Time Mediation Functions: DeltaBeta(), DeltaBetaStd(), DeltaIndirectCentral(), DeltaMed(), DeltaMedStd(), DeltaTotalCentral(), Direct(), DirectStd(), ExpCov(), ExpMean(), Indirect(), IndirectCentral(), IndirectStd(), MCBeta(), MCBetaStd(), MCIndirectCentral(), MCMed(), MCMedStd(), MCPhi(), MCTotalCentral(), Med(), MedStd(), PosteriorBeta(), PosteriorIndirectCentral(), PosteriorMed(), PosteriorPhi(), PosteriorTotalCentral(), Total(), TotalCentral(), TotalStd()
```

```
phi <- matrix(</pre>
 data = c(
   -0.357, 0.771, -0.450,
   0.0, -0.511, 0.729,
   0, 0, -0.693
 ).
 nrow = 3
)
colnames(phi) \leftarrow rownames(phi) \leftarrow c("x", "m", "y")
traj <- Trajectory(</pre>
 mu0 = c(3, 3, -3),
 time = 150,
 phi = phi,
 med = "m"
)
plot(traj)
# Methods ------
# Trajectory has a number of methods including
# print, summary, and plot
traj <- Trajectory(</pre>
 mu0 = c(3, 3, -3),
 time = 25,
 phi = phi,
 med = "m"
)
print(traj)
summary(traj)
plot(traj)
```

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