Table of Contents

1 Introduction

This document shows examples of use of Autodesk.CodeChecking.Concrete named later “Component”.

Each use case includes paragraphs:
- Subject – short description of what we want to calculate and what data do we need
- Summary – detailed list of available results obtained for the list of data
- Data – example data
- Search for – searched results
- Results from Component – results obtained from Component
- Results (manual calculations) – results obtained using manual calculations with calculation path. Some examples (Case 14, Cases 16-18) compare Component results with results from the literature (design manuals) and other program.

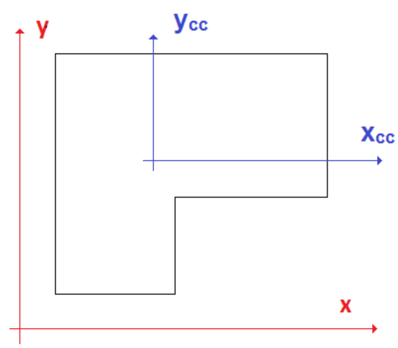
Examples below have physical sense for metric system units.
To see calculation methods, assumptions used by this Component and rules concerning units, please refer to the User Manual of the Code Checking Framework SDK.

Cases shown in this document are related to the C# example provided with this SDK.
Every case has its own implementation in the single *.cs file with the name corresponding to the example (e.g. file “Case01.cs“ corresponding to example Case 1). The project includes a dialog which allows selection of examples, run one or all of the examples and presents the results of calculation.

Main symbols used in this document:

x – the first axis of the initial coordinate system (x, y)

y – the second axis of the initial coordinate system (x, y)



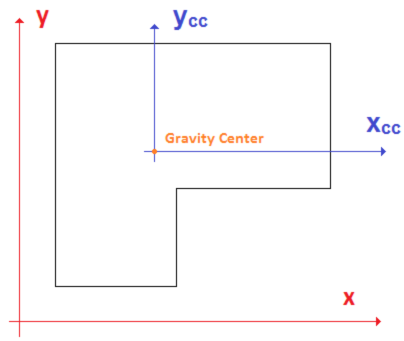
A_c – concrete gross section area or area of contour under compressive stress

x_{Cc} – concrete gross section gravity center first coordinate in (x,y) coordinate system

y_{Cc} – concrete gross section gravity center second coordinate in (x,y) coordinate system

x_{cc} - central axis of concrete gross section parallel to x axis or the first coordinate in (x_{cc}, y_{cc}) coordinate system

y_{cc} - central axis of concrete gross section parallel to y axis or the second coordinate in (x_{cc}, y_{cc}) coordinate system



I_{cx} – moment of inertia of concrete gross section with respect to x_{cc} axis

I_{cy} – moment of inertia of concrete gross section with respect to y_{cc} axis

A_s – reinforcement bars area

x_s – reinforcement bars gravity center first coordinate in (x,y) coordinate system

y_s – reinforcement bars gravity center second coordinate in (x,y) coordinate system

I_{sx} – moment of inertia of reinforcement bars with respect to x_{cc} axis

I_{sy} – moment of inertia of reinforcement bars with respect to y_{cc} axis

A_{eff} – reduced cross section area

x_c – reduced cross section gravity center first coordinate in (x,y) coordinate system

y_c – reduced cross section gravity center second coordinate in (x,y) coordinate system

I_{effx} – moment of inertia of reduced cross section with respect to x axis

I_{effy} – moment of inertia of reduced cross section with respect to y axis

N – internal axial force in reduced cross section

M_x – moment of internal forces in reduced section with respect to x_{cc} axis

M_y – moment of internal forces in reduced section with respect to y_{cc} axis

N_s – internal axial force in reinforcing bars

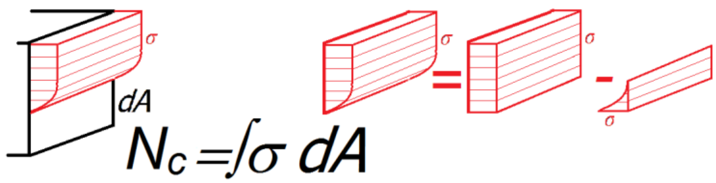
M_{xs} – moment of internal forces in reinforcing bars with respect to x_{cc} axis

M_{ys} – moment of internal forces in reinforcing bars with respect to y_{cc} axis

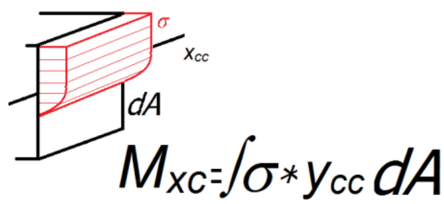
x_{cg} – center of gravity of stresses in the concrete with respect to x_{cc} axis

y_{cg} – center of gravity of stresses in the concrete with respect to y_{cc} axis

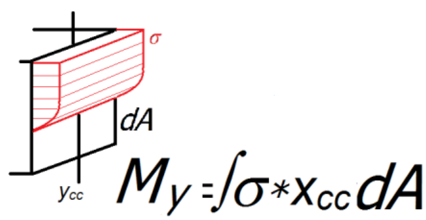
N_c –internal axial force in the concrete part of a section, calculated as a volume of solid of stresses as shown below:



M_{xc} – moment of internal forces in the concrete part of a section with respect to x_{cc} axis



M_{yc} – moment of internal forces in the concrete part of a section with respect to y_{cc} axis



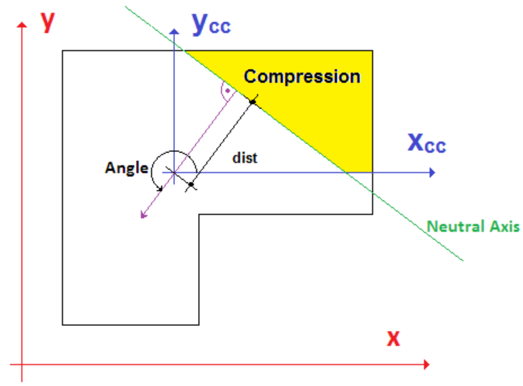
Eps_bot – a strain in the most tensioned point of concrete section

Eps_top – a strain in the most compressed point of concrete section

Eps_sbot – a strain in the most tensioned rebar

Eps_stop – a strain in the most compressed rebar

Angle – angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber



dist – distance between neutral axis and a gravity center of concrete gross section

N – internal axial force in cross section at the failure

M_{xf} – moment of internal forces in section with respect to x_{cc} axis at the failure

M_{yf} – moment of internal forces in section with respect to y_{cc} axis at the failure

sigma, sigma_i, Sigma, Sigma_max, Sigma_min – stress in rebar or in concrete, description in the text

eps, eps_i – stress in rebar or in concrete, description in the text

2 Case 1: Cross section characteristics

2.1 Subject:

Read cross section geometry, positions of reinforcing bars inside the section, parameters of concrete and steel and give cross section characteristics

2.2 Summary:

This sample gives as results the cross section characteristics.
Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- cross section characteristics:
    - Concrete section gross area (A_c) with the coordinates of its gravity center (x_{cc}, y_{cc})
    - Moments of inertia (I_{cx}, I_{cy})
    - Reinforcing bars area (A_s) with coordinates of its gravity center (x_s, y_s)
    - Moments of inertia (I_{sx}, I_{sy})
    - Reduced cross section characteristics (A_{eff}, x_c, y_c, I_{effx}, I_{effy}).

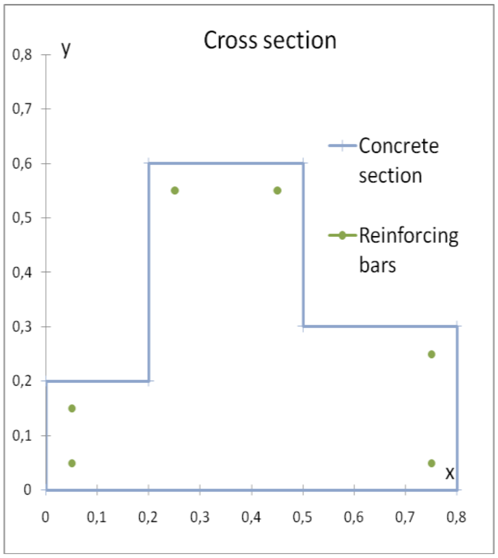
2.3 Data:

- Geometry:
    - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.2
x = 0.2	y = 0.2
x = 0.2	y = 0.6
x = 0.5	y = 0.6
x = 0.5	y = 0.3
x = 0.8	y = 0.3
x = 0.8	y = 0.0

    - Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.010
x = 0.75	y = 0.05	ϕ = 0.012
x = 0.75	y = 0.25	ϕ = 0.020
x = 0.45	y = 0.55	ϕ = 0.050
x = 0.25	y = 0.55	ϕ = 0.020
x = 0.05	y = 0.15	ϕ = 0.015



- Concrete parameters:
  Modulus of elasticity E_c = 30e9
- Steel parameters:
  Modulus of elasticity E_s = 200e9

2.4 Search for:

- Concrete gross section characteristics: A_c, x_{Cc}, y_{Cc}, I_{cx}, I_{cy},
- Reinforcing bars characteristics: A_s, x_s, y_s, I_{sx}, I_{sy},
- Reduced cross section characteristics: A_{eff}, x_c, y_c, I_{effx}, I_{effy}.

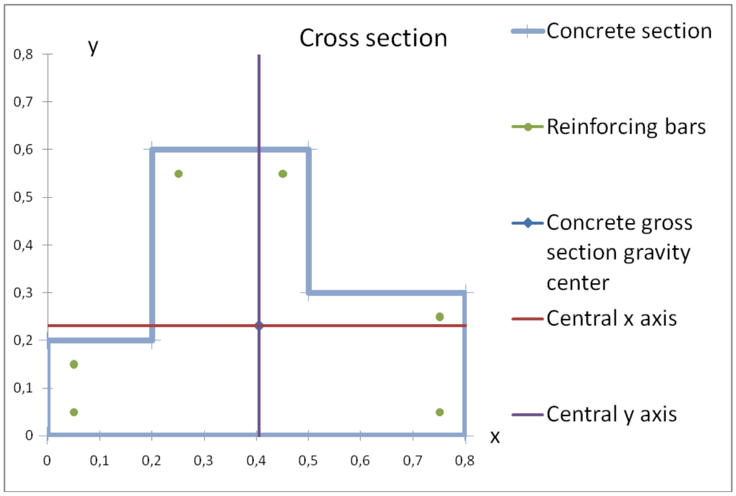
2.5 Results from Component

- Concrete gross section characteristics:
  A_c = 0.310000
  x_{Cc} = 0.40483871
  y_{Cc} = 0.23064516
  I_{cx} = 0.008342
  I_{cy} = 0.011826

- Reinforcing bars characteristics:
  A_s = 0.002960
  x_s = 0.43758291
  y_s = 0.46191298
  I_{sx} = 0.000240
  I_{sy} = 0.000095

- Reduced cross section characteristics:
  A_{eff} = 0.326774
  x_c = 0.40651956
  y_c = 0.24251681
  I_{effx} = 0.009701

$I_{effy} = 0.012362$



2.6 Results (manual calculations):

- Concrete gross section characteristics:

  Area:

  $A_c = 0.2 \cdot 0.2 + 0.3 \cdot 0.6 + 0.3 \cdot 0.3 = \underline{0.31}$

  Static moments:

  $S_{cx} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.3 + 0.3 \cdot 0.3 \cdot 0.15 = \underline{0.0715}$

  $S_{cy} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.35 + 0.3 \cdot 0.3 \cdot 0.65 = \underline{0.1255}$

  Gravity center:

  $x_{Cc} = S_{cy} / A_c = \underline{0.40483871}$

  $y_{Cc} = S_{cx} / A_c = \underline{0.23064516}$

  Moments of inertia:

  $I_x = 0.2 \cdot 0.2^3 / 12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.3 \cdot 0.6^3 / 12 + 0.3 \cdot 0.6 \cdot 0.3^2 + 0.3 \cdot 0.3^3 / 12 + 0.3 \cdot 0.3 \cdot 0.15^2 = \underline{0.024833}$

  $I_y = 0.2 \cdot 0.2^3 /12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.6 \cdot 0.3^3 /12 + 0.3 \cdot 0.6 \cdot 0.35^2 + 0.3 \cdot 0.3^3 /12 + 0.3 \cdot 0.3 \cdot 0.65^2 = \underline{0.062633}$

  Central moments of inertia:

  $I_{cx} = I_x - A_c \cdot y_{Cc}^2 = 0\underline{.008342}$

  $I_{cy} = I_y - A_c \cdot x_{Cc}^2 = 0\underline{.011826}$

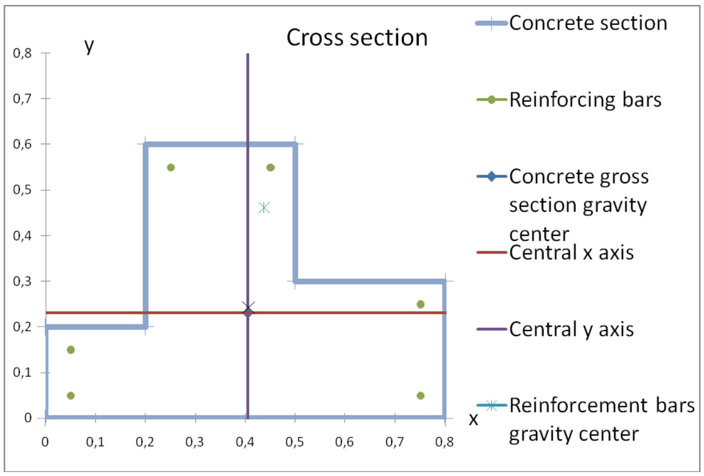
- Reinforcing bars characteristics:

i	x	y	ϕ	$A_s = \pi \cdot \phi^2 / 4$	Static moment $S_{sy} = A_s \cdot x$	Static moment $S_{sx} = A_s \cdot y$	Coordinates in x_{cc}, y_{cc} system $x_{cc} = x - x_{Cc}$	$y_{cc} = y - y_{Cc}$	$I_{sx} = \pi \cdot \phi^4 / 64 + A_s \cdot y_{cc}^2$	$I_{sy} = \pi \cdot \phi^4 / 64 + A_s \cdot x_{cc}^2$
1	0,05	0,05	0,01	0,000079	0,000004	0,000004	-0,3548	-0,1806	0,000003	0,000010
2	0,75	0,05	0,012	0,000113	0,000085	0,000006	0,3452	-0,1806	0,000004	0,000013
3	0,75	0,25	0,02	0,000314	0,000236	0,000079	0,3452	0,0194	0,000000	0,000037
4	0,45	0,55	0,05	0,001963	0,000884	0,001080	0,0452	0,3194	0,000201	0,000004
5	0,25	0,55	0,02	0,000314	0,000079	0,000173	-0,1548	0,3194	0,000032	0,000008
6	0,05	0,15	0,015	0,000177	0,000009	0,000027	-0,3548	-0,0806	0,000001	0,000022
			Total:	0,002960	0,001295	0,001367			0,000240	0,000095

Area:
$A_s = \underline{0.002960}$
Static moments:
$S_{sx} = 0.001367$
$S_{sy} = 0.001295$
Gravity center:
$x_s = S_{sy} / A = \underline{0.43758291}$
$y_s = S_{sx} / A = 0\underline{.46191298}$
Moments of inertia:
$I_{sx} = \underline{0.000240}$
$I_{sy} = \underline{0.000095}$

- Reduced cross section characteristics:

  Area:
  $A_{eff} = A_c + A_s \cdot (E_s / E_c - 1) = 0\underline{.326774}$
  Static moments:
  $S_x = S_{cx} + S_{sx} \cdot (E_s / E_c - 1) = 0.079246$
  $S_y = S_{cy} + S_{sy} \cdot (E_s / E_c - 1) = 0.132838$
  Gravity center:
  $x_c = S_y / A = \underline{0.4065}$
  $y_c = S_x / A = \underline{0.2425}$
  Moments of inertia:
  $I_{effx} = I_{cx} + I_{sx} \cdot (E_s / E_c - 1) = \underline{0.009701}$
  $I_{effy} = I_{cx} + I_{sx} \cdot (E_s / E_c - 1) = \underline{0.012362}$



3 Case 2: Calculation of internal forces for given state of strain in the section and rectangular model of concrete

3.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

3.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete section with symmetric reinforcement (Case 2a and Case 2b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 2c)
- A rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 2a)
- A section under unidirectional bending with a part of section under compression (Case 2b and Case 2c)
- A state of strain with ultimate compressive strain in concrete (Case 2a and Case 2b)
- A strain below the ultimate compressive strain in concrete (Case 2c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force (N)
    - Moment with respect to x_{cc} axis (M_x)
    - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

3.3 Case 2a

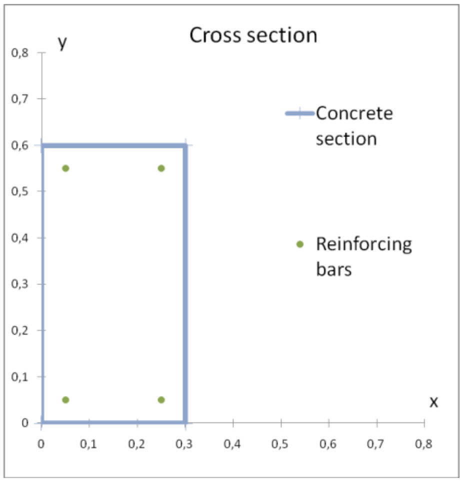
3.3.1 Data:

- Geometry:
    - Concrete section:

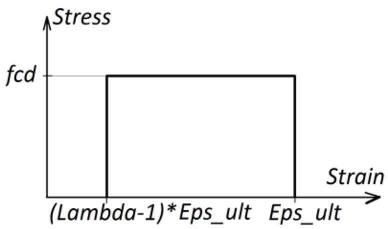
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

- Reinforcing bars:

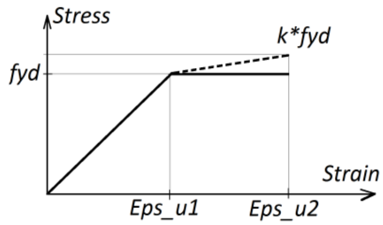
x = 0.05	y = 0.05	ϕ = 0.012
x = 0.05	y = 0.55	ϕ = 0.012
x = 0.25	y = 0.55	ϕ = 0.012
x = 0.25	y = 0.05	ϕ = 0.012



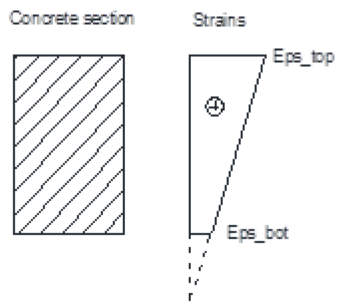
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Bottom strain Eps_bot = 0.0005
  - Top strain Eps_top = 0.0035
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



3.3.2 Search for :

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, (x_{cg}, y_{cg}), N_c, M_{xc} , M_{yc}
- Results for reduced section:N, M_x, M_y

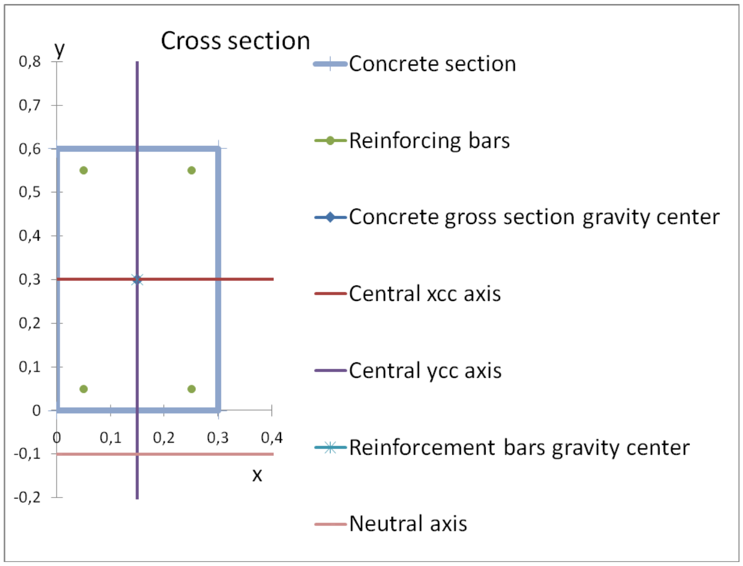
3.3.3 Results from Component

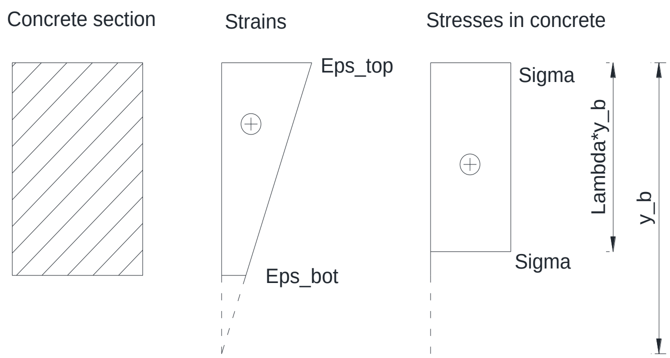
- Results for reinforcement bars:
  N_s = 147.03e3
  M_{xs} = -19.79e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.1680
  (x_{cg}, y_{cg}) = (0.00000000,0.02005400)
  N_c = 3350.95e3
  M_{xc} = -67.20e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = 3497.98e3
  M_x = -86.99e3
  M_y = 0.00e3

3.3.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.7
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{Cc} = 0.15
  y_{Cc} = 0.30

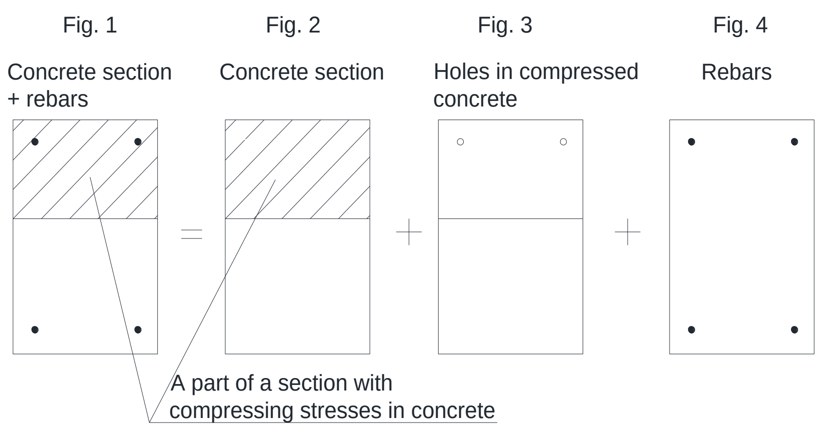
  Distance between reinforcing bar and neutral axis:
  d= y – h + y_b

  Stress in bar i:
  If |eps_i| ≤ f_{yd} / E_s then
  sigma_i = eps_i / Eps_u1 • f_{yd}
  else
  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs} = - N • y_{cc}	M_{ys} = N • x_{cc}
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

N_s = <u>147.03e3</u>
M_{xs} = -<u>19.79e3</u>
M_{ys} = <u>0.00e3</u>

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing below).



Reduction of forces carried by the concrete section is calculated as follows:

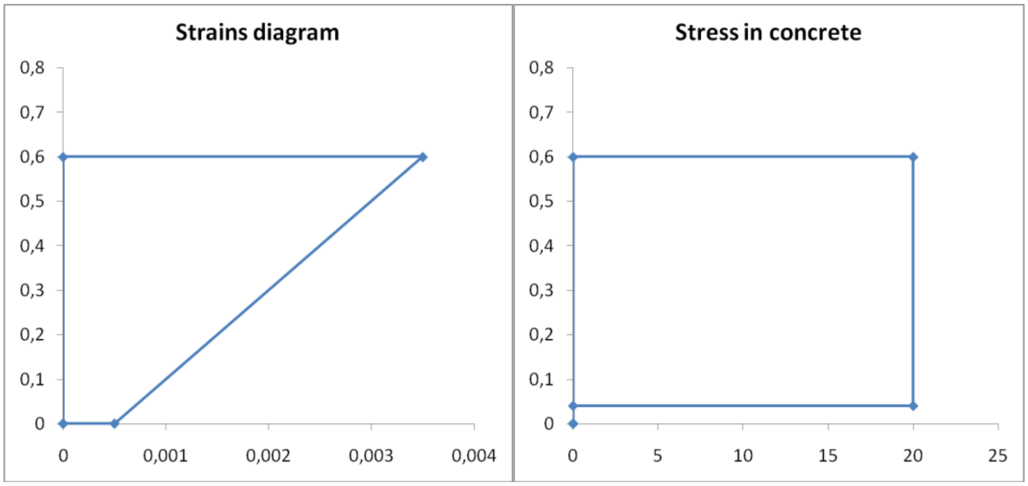
i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • Eps_top / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc} = N_{sc} • x_{cc}
1	0,05	0,05	0,012	-0,1	-0,25	0,000113097	2262	565	-226
2	0,05	0,55	0,012	-0,1	0,25	0,000113097	2262	-565	-226
3	0,25	0,55	0,012	0,1	0,25	0,000113097	2262	-565	226
4	0,25	0,05	0,012	0,1	-0,25	0,000113097	2262	565	226
						Sum of forces:	9048	0	0

Reduction forces in concrete:

N_{sc} = 9.05e3
M_{xsc} = 0.00e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see fig. 3 on drawing above) in central axes coordinate system:
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc}_i • Area_i) / sum(Area_i) = 0.00

- Results for concrete section:



Height of compressed part of the section:
Lambda • y_b = 0.56

Area of contour under compressive stress:
A_c = 0.3 • 0.56 = <u>0.168</u>

Stress in concrete:
Sigma = Eps_top / Eps_ult • f_{cd} = 20e6

Center of gravity of stresses in the concrete in central axes coordinate system:
x_{cg} = ((0.3 / 2- x_{Cc}) • A_c • Sigma - x_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = <u>0</u>
y_{cg} = ((0.6-0.56 / 2-y_{Cc}) • A_c • Sigma-y_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = <u>0.020054</u>

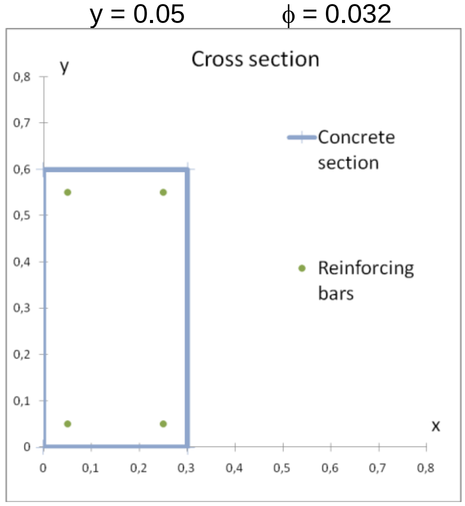
Internal forces in concrete:
N_c = A_c • Sigma - N_{sc} = <u>3350.95e3</u>
M_{xc} = - N_c • y_{cg} = -<u>67.20e3</u>
M_{yc} = N_c • x_{cg} = 0<u>.00e3</u>

- Results for reduced section:
N = N_c + N_s = <u>3497.98e3</u>
M_x = M_{xc} + M_{xs} = <u>-86.99e3</u>
M_y = M_{yc} + M_{ys} = <u>0.00e3</u>

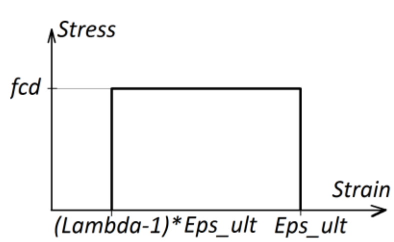
3.4 Case 2b

3.4.1 Data:

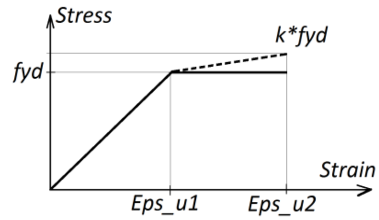
- Geometry:
  - Concrete section:
    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:
    x = 0.05 y = 0.05 ϕ = 0.032
    x = 0.05 y = 0.55 ϕ = 0.032
    x = 0.25 y = 0.55 ϕ = 0.032
    x = 0.25 y = 0.05 ϕ = 0.032



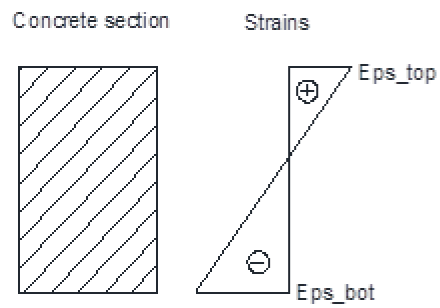
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Bottom strain Eps_bot = - 0.022029
  - Top strain Eps_top = 0.0035
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



3.4.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

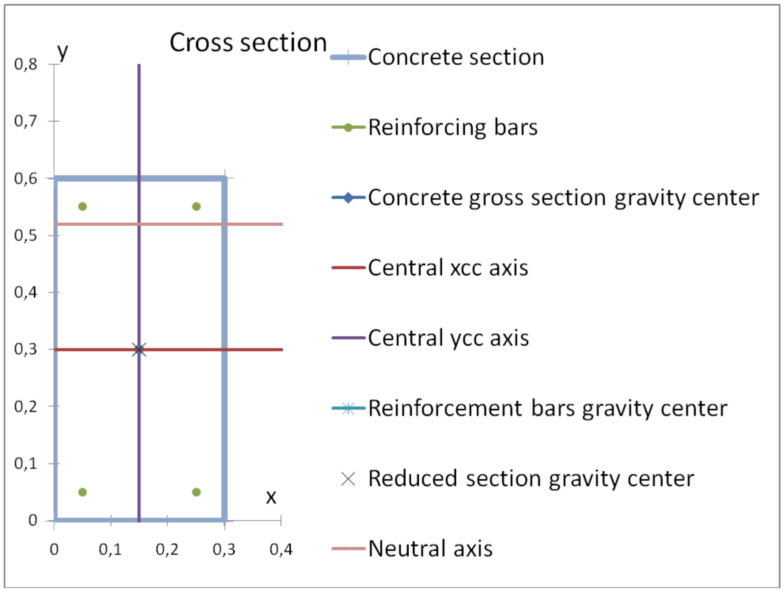
3.4.3 Results from Component

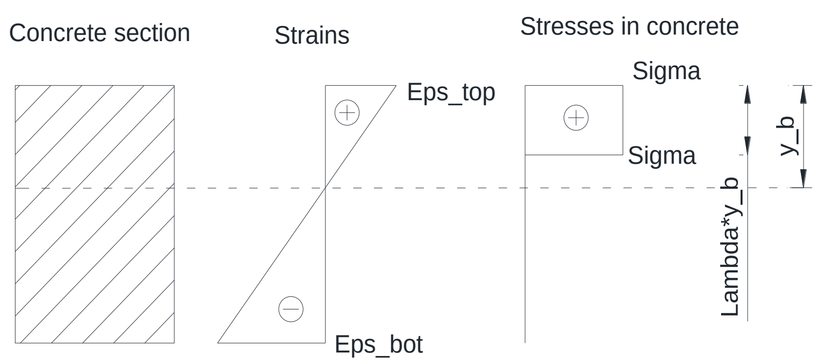
- Results for reinforcement bars:
  N_s = -362.69e3
  M_{xs} = -311.45e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.01974
  x_{cg} = 0.00000000
  y_{cg} = 0.26861271
  N_c = 362.68e3
  M_{xc} = -97.42e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = -0.01e3
  M_x = -408.87e3
  M_y = 0.00e3

3.4.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.0822604

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y – h + y_b

  Stress in bar i:

  If |eps_i| ≤ f_{yd} / E_s
  then

$sigma\_i = eps\_i / Eps\_u1 \cdot f_{yd}$

else

$sigma\_i = f_{yd} + (|eps\_i| - Eps\_u1) / (Eps\_u2 - Eps\_u1) \cdot (k - 1) \cdot f_{yd})$

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d/ y_b • Eps_top	Stresses Sigma / 10e6	Force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs} = - N • y_{cc}	M_{ys} = N • x_{cc}
1	0,05	0,05	0,032	-0,468	-0,01990	-500	-402124	-0,10	-0,25	-100531	40212
2	0,05	0,55	0,032	0,032	0,00137	275	220784	-0,10	0,25	-55196	-22078
3	0,25	0,55	0,032	0,032	0,00137	275	220784	0,10	0,25	-55196	22078
4	0,25	0,05	0,032	-0,468	-0,01990	-500	-402124	0,10	-0,25	-100531	-40212
						Sum of forces in bars:	-362680			-311454	0

N_s = 362.68e3
M_{xs} = -311.45e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd}$ • Eps_top / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc} = N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,1	-0,25				
2	0,05	0,55	0,032	-0,1	0,25	0,000804248	16085	-4021	-1608
3	0,25	0,55	0,032	0,1	0,25	0,000804248	16085	-4021	1608
4	0,25	0,05	0,032	0,1	-0,25				
						Sum of forces :	32170	-8042	0

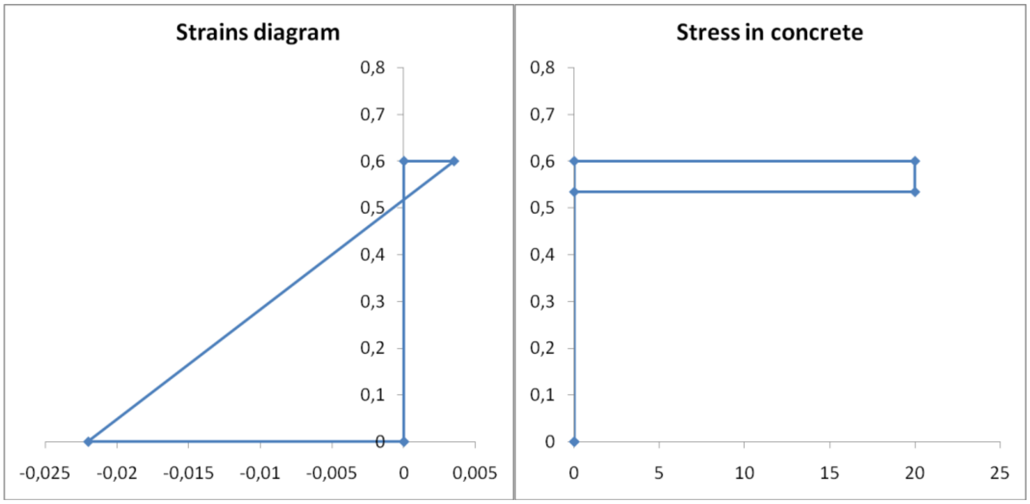
Reduction forces in concrete:

N_{sc} = 32.17e3
M_{xsc} = -8.04e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in Case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc}_i • Area_i) / sum(Area_i)=0.00
y_{scg} = sum(y_{cc}_i • Area_i) / sum(Area_i)=0.25

- Results for concrete section:



Height of compressed part of the section:

Lambda • y_b = 0.065808351

Area of contour under compressive stress:

A_c = 0.3 • 0.064608= <u>0.01974</u>

Stress in concrete:

Sigma = Eps_top / Eps_ult • f_{cd} = 20e6

Center of gravity of stresses in the concrete in central axes coordinate system:

x_{cg} = ((0.3 / 2 – x_{Cc}) • A_c • Sigma - x_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = <u>0</u>

y_{cg} = ((0.6 - Lambda • y_b / 2 – y_{Cc}) • A_c • Sigma - y_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = <u>0.268612</u>

Internal forces in concrete:

N_c = A_c • Sigma - N_{sc} = <u>362.68e3</u>

M_{xc} = - N_c • y_{cg} = -<u>97.42e3</u>

M_{yc} = N_c • x_{cg} = 0<u>.00e3</u>

- Results for reduced section:

  N = N_c + N_s = <u>0.00e3</u>

  M_x = M_{xc} + M_{xs} = <u>-408.87e3</u>

  M_y = M_{yc} + M_{ys} = <u>0.00e3</u>

3.5 Case 2c

3.5.1 Data:

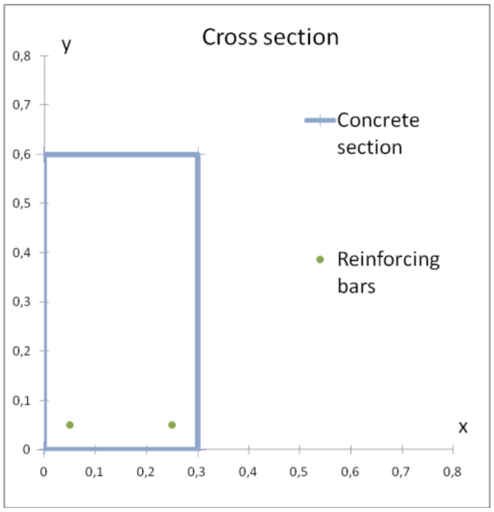
- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0

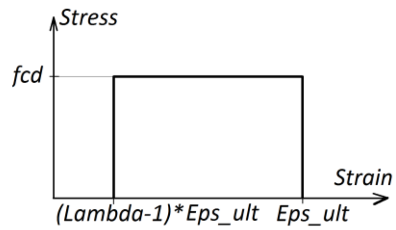
    x = 0.0 y = 0.6

x = 0.3 y = 0.6
x = 0.3 y = 0.0

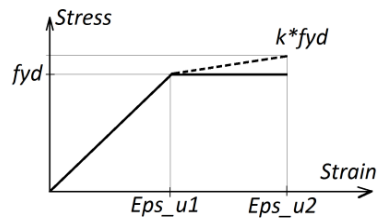
- Reinforcing bars:
  x = 0.05 y = 0.05 ϕ = 0.032
  x = 0.25 y = 0.05 ϕ = 0.032



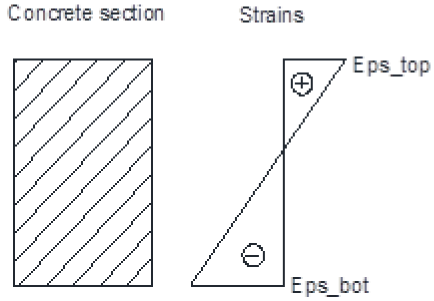
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
    - Top strain Eps_top = 0.0015
    - Bottom strain Eps_bot = -0.002
    - Neutral axis angle Angle = π • 3 / 2



3.5.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

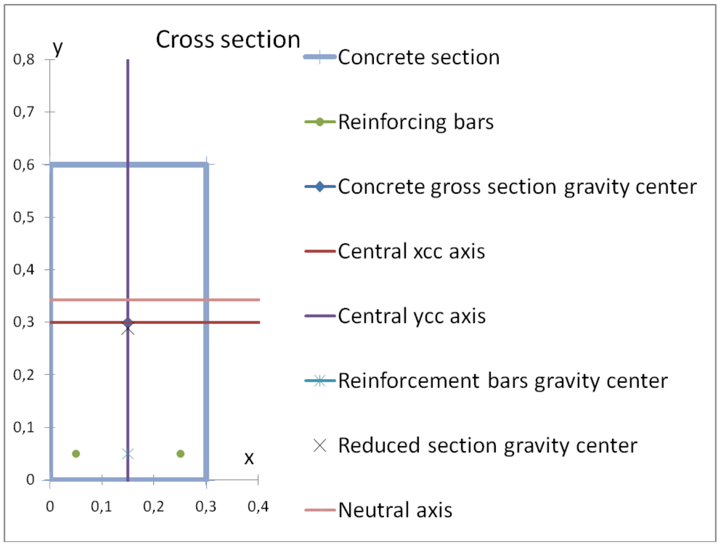
3.5.3 Results from Component

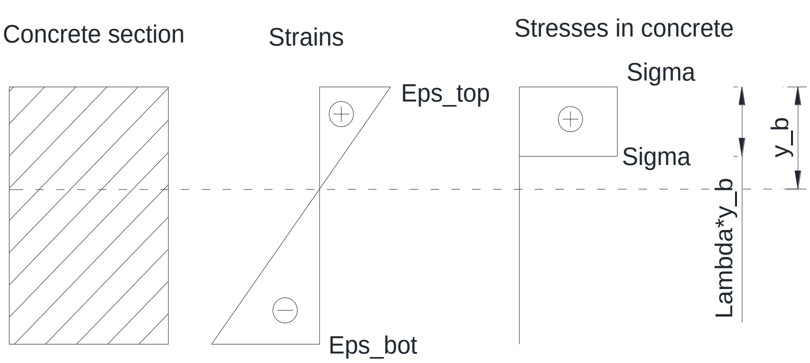
- Results for reinforcement bars:
  N_s = -549.57e3
  M_{xs} = -137.39e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.0617
  x_{cg} = 0.000000
  y_{cg} = 0.197143
  N_c = 528.98e3
  M_{xc} = -104.28e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = -20.59e3
  M_x = -241.68e3
  M_y = 0.00e3

3.5.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.257143
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{Cc} = 0.15
  y_{Cc}= 0.30

  Distance between reinforcing bar and neutral axis:
  d = y - h + y_b

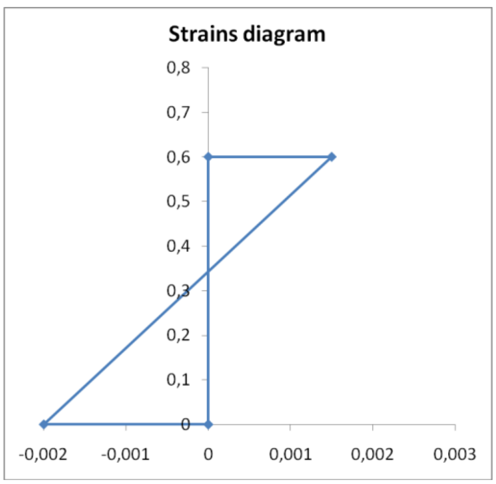
  Stress in bar i:
  If |eps_i | ≤ f_{yd} / E_s then
  sigma_i = eps_i/ Eps_u1 • f_{yd}
  else

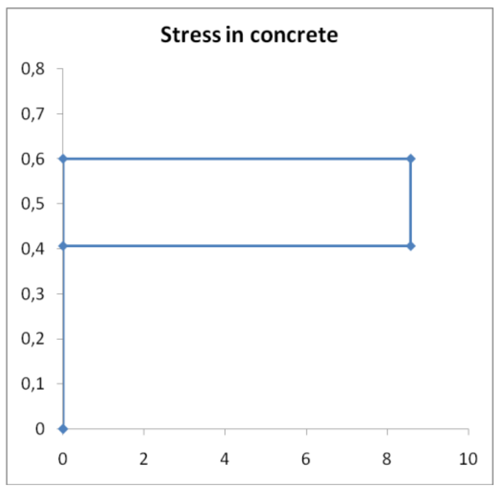
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc} y_{cc}		M_{xs} = - N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
						Sum of forces in bars:	-549569			-137392	0

N_s = -<u>549.57e3</u>
M_{xs} = <u>-137.39e3</u>
M_{ys} = <u>0.00e3</u>

- Results for concrete section:





Height of compressed part of the section:
Lambda • y_b = 0.205714
Area of contour under compressive stress:
A_c = 0.3 • 0.205714 = <u>0.0617</u>
Stress in concrete:
Sigma = Eps_bot/Eps_ult • f_{cd} = 8.571e6

Center of gravity of stresses in the concrete in central axes coordinate system:
x_{cg} = (0.3/2-x_{Cc}) • A_c • Sigma/(A_c • Sigma) = <u>0</u>
y_{cg} = (0.6- Lambda • y_b/2-y_{Cc}) • A_c • Sigma/(A_c • Sigma) = <u>0.197143</u>
Internal forces in concrete:
N_c = A_c • Sigma = <u>528.98e3</u>
M_{xc} = - N_c • y_{cg} = <u>-104.28e3</u>
M_{yc} = N_c • x_{cg} = <u>0.00e3</u>

- Results for reduced section:
  N = N_c+N_s = <u>-20.59e3</u>
  M_x = M_{xc}+M_{xs} = <u>-241.68e3</u>

$M_y = M_{yc}+M_{ys} = \underline{0.00e3}$

4 Case 3: Calculation of internal forces for given state of strain in the section and linear model of concrete

4.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

4.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 3a and Case 3b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 3c)
- A linear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 3a)
- A section under unidirectional bending with a part of section under compression (Case 3b and Case 3c)
- A state of strain with ultimate compressive strain in concrete (Case 3a and Case 3b)
- A strain below the ultimate compressive strain in concrete (Case 3c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force (N)
    - Moment with respect to x_{cc} axis (M_x)
    - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

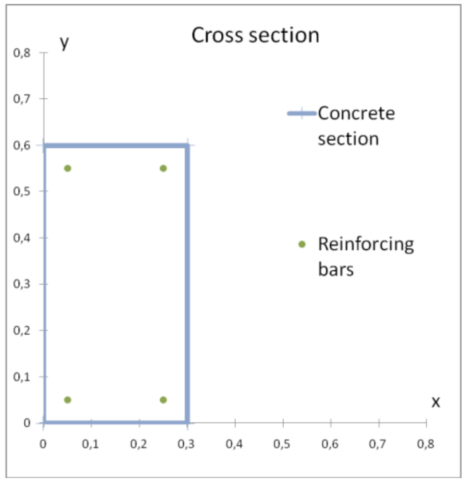
4.3 Case 3a

4.3.1 Data:

- Geometry:
    - Concrete section:

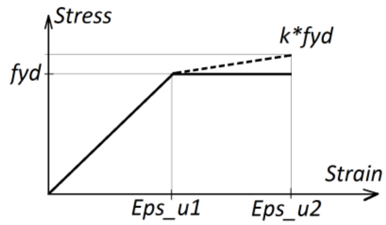
x = 0.0 y = 0.0
x = 0.0 y = 0.6
x = 0.3 y = 0.6
x = 0.3 y = 0.0

- Reinforcing bars:
  x = 0.05 y = 0.05 ϕ = 0.012
  x = 0.05 y = 0.55 ϕ = 0.012
  x = 0.25 y = 0.55 ϕ = 0.012
  x = 0.25 y = 0.05 ϕ = 0.012

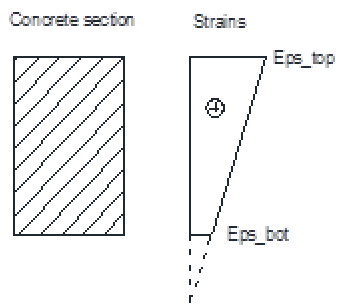


- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: linear
  - Strain ultimate limit Eps_ult = 0.0035

- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = 0.0005
  - Neutral axis angle Angle = π • 3 / 2



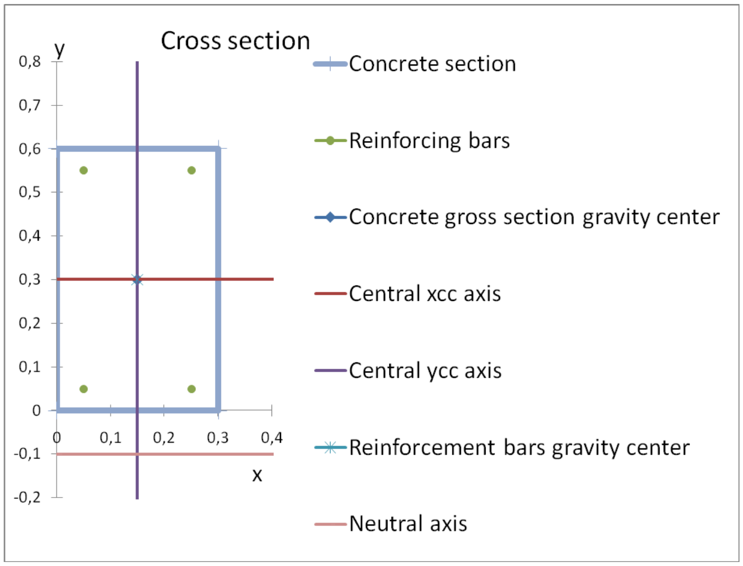
4.3.2 Search for:

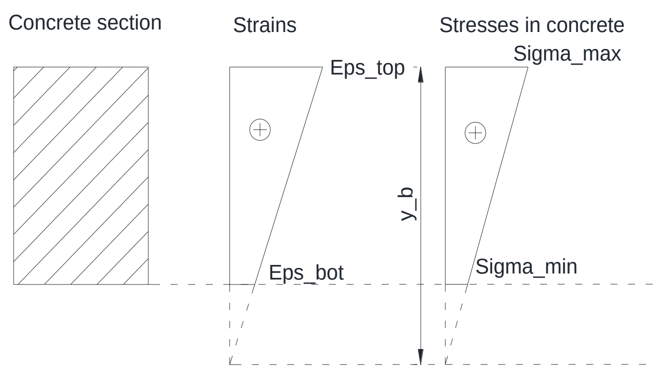
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

4.3.3 Results from Component

- Results for reinforcement bars:
  N_s = 147.03e3
  M_{xs} = -19.79e3
  M_{ys} = 0.00e3
- Results for concrete section:
  A_c = 0.1800
  x_{cg} = 0.00000000
  y_{cg} = 0.07479528
  N_c = 2051.97e3
  M_{xc} = -153.48e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 2199.00e3
  M_x = -173.27e3
  M_y = 0.00e3

4.3.4 Results (manual calculations):





- Results for reinforcement bars:

  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.7

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y - h + y_b

  Stress in bar i:

  If |eps_i | ≤ f_{yd} / E_s then

  sigma_i = eps_i / Eps_u1 • f_{yd}

  else

  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses Sigma / 10e6	Force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs} = - N • y_{cc}	M_{ys} = N • x_{cc}
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

N_s = <u>147.03e3</u>
M_{xs} = -<u>19.79e3</u>
M_{ys} = <u>0.00e3</u>

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd}$ • eps / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc} = N_{sc} • x_{cc}
1	0,05	0,05	0,012	0,00075	-0,1	-0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1	0,25	0,000113097	485	121	48
3	0,25	0,55	0,012	0,00325	0,1	0,25	0,000113097	2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1	-0,25	0,000113097	2100	-525	-210
							Sum of forces:	5170	-808	0

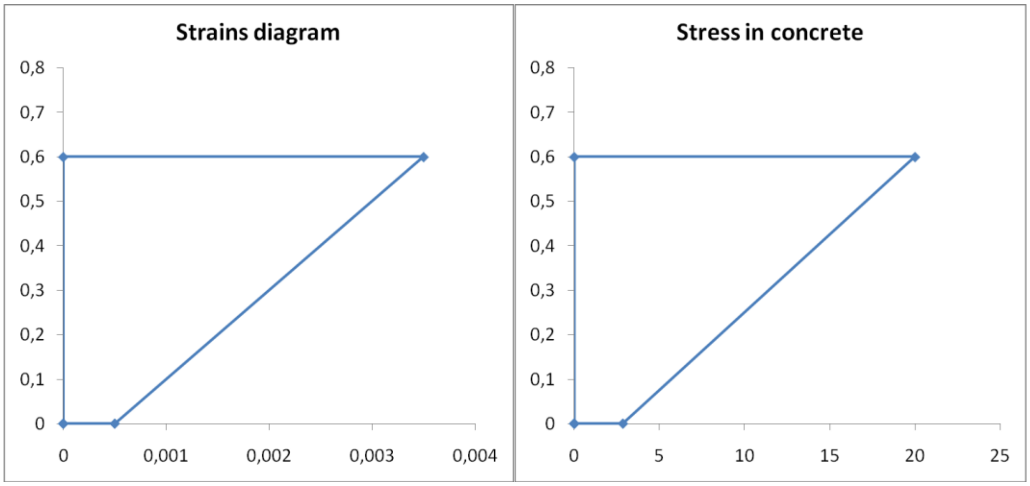
Reduction forces in concrete:

N = 5.17e3
M_{xsc} = -0.81e3
M_{ysc} = 0.000e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00000
y_{scg} = sum(y_{cc}_i • Area_i) / sum(Area_i) = 0.15625

- Results for concrete section:



Height of compressed part of the section:

0.6000

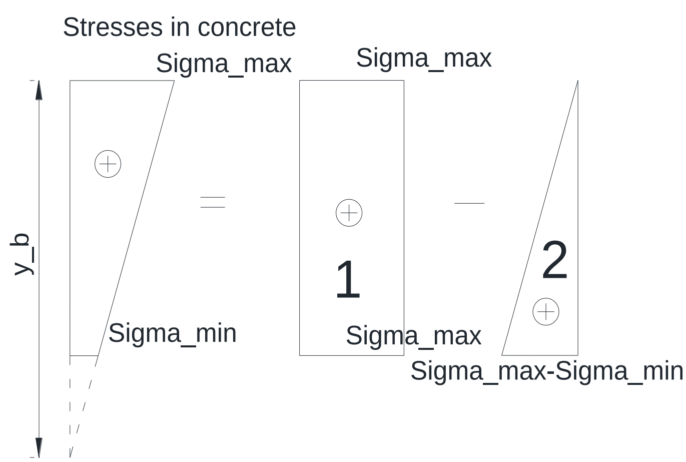
Area of contour under compressive stress:

A_c = 0.3 • 0.6000 = <u>0.1800</u>

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

$Sigma\_max = f_{cd} \cdot Eps\_top / Eps\_c1 = 20.000e6$
$Sigma\_min = f_{cd} \cdot Eps\_bot / Eps\_c1 = 2.857e6$

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \cdot Sigma\_max = 3.600e6$
$N_{c2} = A_c \cdot (Sigma\_max - Sigma\_min) / 2 = 1.543e6$
$N_{c'} = N_{c1} - N_{c2} = 2.057e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
$y_{c1} = 0$
$x_{c2} = 0$
$y_{c2} = - 0.3 + 0.6 / 3 = - 0.1$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.000000}$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.074795}$

Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{2051.97e3}$
$M_{xc} = - N_c \cdot y_{cg} = -\underline{153.48e3}$
$M_{yc} = N_{c'} \cdot x_{cg} = \underline{0.00e3}$

- Results for reduced section:

  $N = N_c + N_s = \underline{2199.00e3}$
  $M_x = M_{xc} + M_{xs} = \underline{-173.27e3}$
  $M_y = M_{yc} + M_{ys} = 0\underline{.00e3}$

4.4 Case 3b

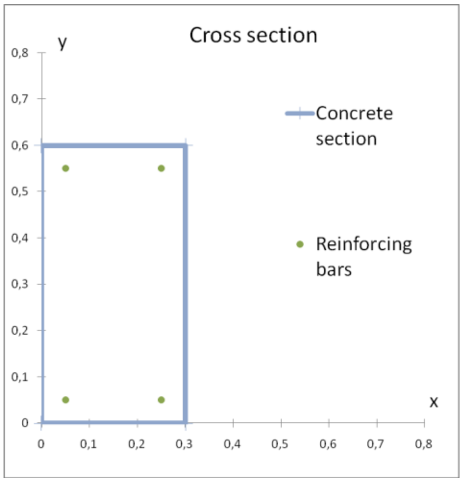
4.4.1 Data:

- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

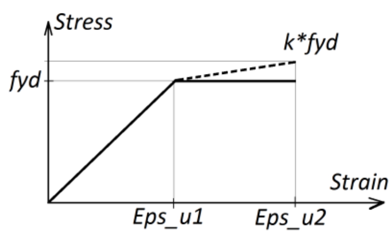
  - Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.032
x = 0.05	y = 0.55	ϕ = 0.032
x = 0.25	y = 0.55	ϕ = 0.032
x = 0.25	y = 0.05	ϕ = 0.032

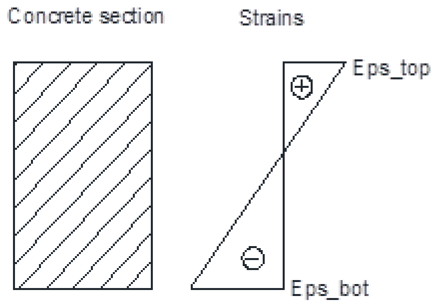


- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: linear
  - Strain ultimate limit Eps_ult = 0.0035

- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = - 0.01857592
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



4.4.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c,M_{xc}, M_{yc}
- Results for reduced section: N, M_x ,M_y

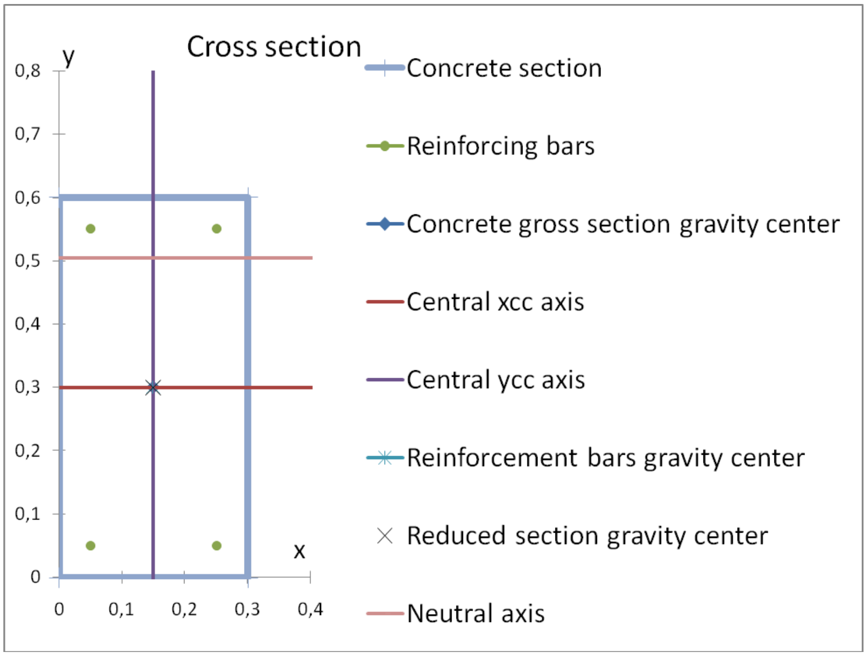
4.4.3 Results from Component

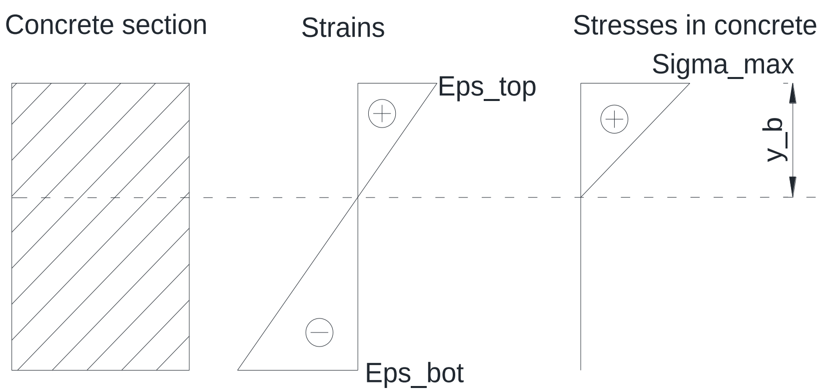
- Results for reinforcement bars:
  N_s = -270.12e3
  M_{xs} = -334.59e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.0285
  x_{cg} = 0.00000000
  y_{cg} = 0.26932464
  N_c = 270.12e3
  M_{xc} = -72.75e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = 0.00e3
  M_x = -407.34e3
  M_y = 0.00e3

4.4.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.095126
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{Cc} = 0.15
  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
  d = y – h + y_b

  Stress in bar i:

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs} = - N • y_{cc}	M_{ys} = N • x_{cc}
1	0,05	0,05	0,032	-0,455	-0,01674	-500	-402124	-0,10	-0,25	-100531	40212
2	0,25	0,05	0,032	-0,455	-0,01674	-500	-402124	0,10	-0,25	-100531	-40212
3	0,25	0,55	0,032	0,045	0,00166	332	267065	0,10	0,25	-66766	26706
4	0,05	0,55	0,032	0,045	0,00166	332	267065	-0,10	0,25	-66766	-26706
						Sum of forces in bars:	-270118			-334594	0

If |eps_i| ≤ f_{yd} / E_s then

sigma_i = eps_i / Eps_u1 • f_{yd}

else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

N_s = -270.12e3
M_{xs} = -334.594e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • eps / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,01674	-0,1	-0,25				
2	0,25	0,05	0,032	-0,01674	0,1	-0,25				
3	0,25	0,55	0,032	0,00166	0,1	0,25	0,000804248	7630	-1908	763
4	0,05	0,55	0,032	0,00166	-0,1	0,25	0,000804248	7630	-1908	-763
							Sum of forces :	15261	-3815	0

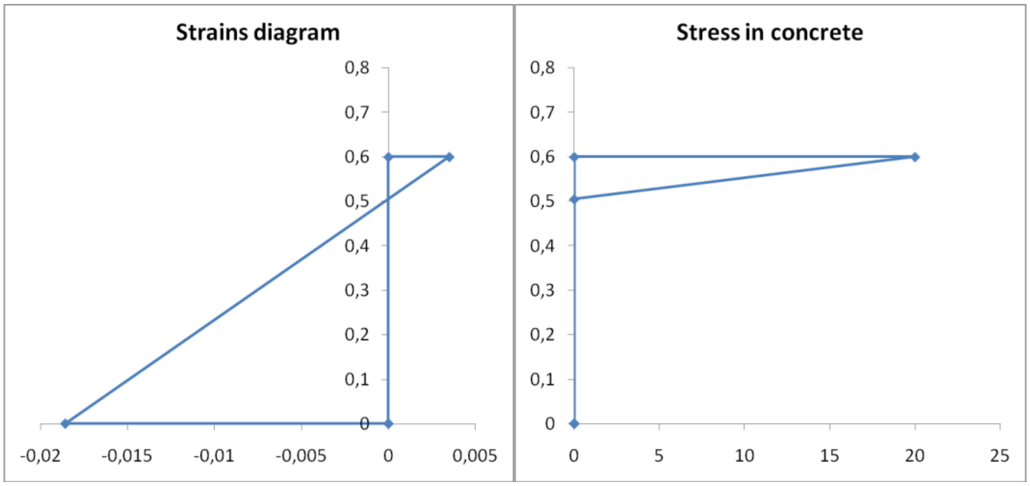
Reduction forces in concrete:

N_{sc} = 15.26e3
M_{xsc} = -3.82e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc}_i • Area_i) / sum(Area_i) = 0.25

- Results for concrete section:



Height of compressed part of the section:

$y\_b = 0.095126$

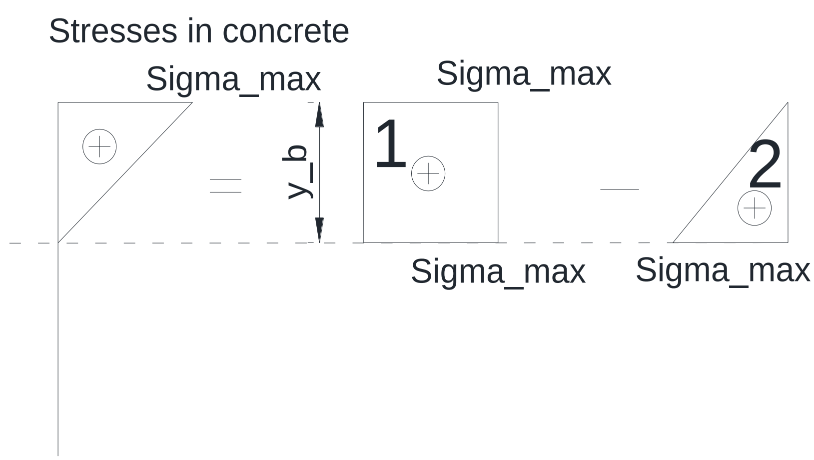
Area of contour under compressive stress:

$A_c = 0.3 \cdot y\_b = 0.0285$

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \bullet x_{cg}$

Sigma_max = f_{cd} • Eps_top / Eps_ult = 20.000e6

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \bullet$ Sigma_max = 0.5708e6

$N_{c2} = A_c \bullet$ Sigma_max / 2 = 0.2854e6

$N_{c'} = N_{c1} - N_{c2} = 0.28538e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$

y_{c1} = 0.3 -y_b / 2= 0.25243687

$x_{c2} = 0$

y_{c2} = 0.3-y_b • 2/3= 0.23658249

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \bullet N_{c1} - x_{c2} \bullet N_{c2} - x_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0\underline{.00000000}$

$y_{cg} = (y_{c1} \bullet N_{c1} - y_{c2} \bullet N_{c2} - y_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0\underline{.26932464}$

Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = 2\underline{70.12e3}$

$M_{xc} = - N_c \bullet y_{cg} = -\underline{72.75e3}$

$M_{yc} = N_{c'} . x_{cg} = 0\underline{.00e3}$

- Results for reduced section:

  $N = N_c + N_s = 0\underline{.00e3}$

  $M_x = M_{xc} + M_{xs} = -\underline{407.34e3}$

  $M_y = M_{yc} + M_{ys} = 0\underline{.00e3}$

4.5 Case 3c

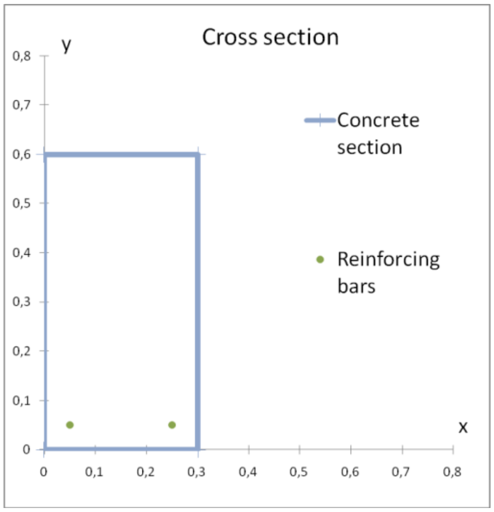
4.5.1 Data:

- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

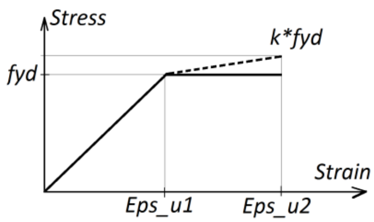
  - Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.032
x = 0.25	y = 0.05	ϕ = 0.032



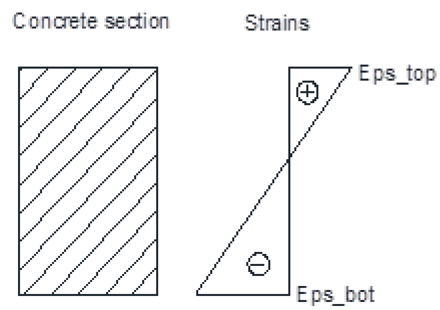
- Concrete parameters:
    - Design strength f_{cd} = 20e6
    - Modulus of elasticity E_c = 30e9
    - Strain-stress model: linear
    - Strain ultimate limit Eps_ult= 0.0035

- Steel parameters:
    - Design strength f_{yd} = 500e6
    - Hardening factor k= 1.0
    - Modulus of elasticity E_s = 200e9
    - Strain ultimate limit Eps_u2 = 0.075



Strains:

Top strain Eps_top = 0.0015
Bottom strain Eps_bot = -0.002
Neutral axis angle Angle = $\pi \cdot 3 / 2$



4.5.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

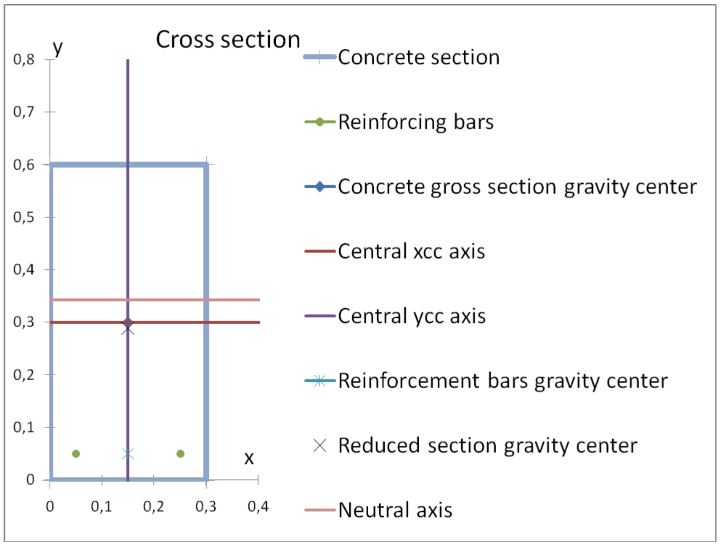
4.5.3 Results from Component

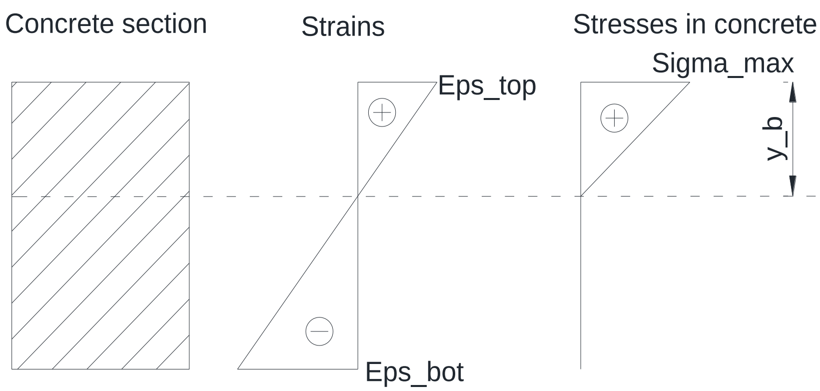
- Results for reinforcement bars:
  N_s = -549.57e3
  M_{xs}= -137.39e3
  M_{ys}= 0.00e3

- Results for concrete section:
  A_c = 0.0771
  x_{cg} = 0.000000
  y_{cg} = 0.21428571
  N_c = 330.61e3
  M_{xc} = -70.85e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = -218.96e3
  M_x = -208.24e3
  M_y = 0.00e3

4.5.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.257143
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{cc} = 0.15
  y_{cc}= 0.30

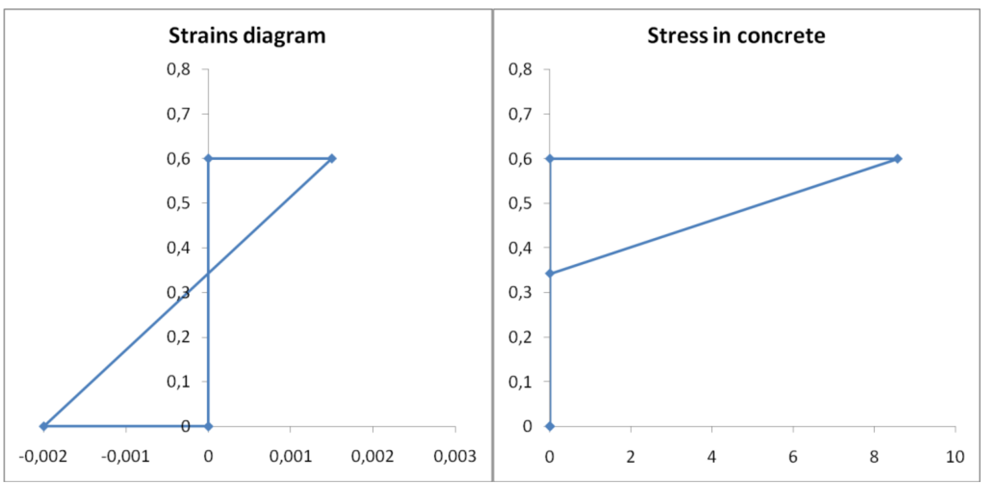
  Distance between reinforcing bar and neutral axis:
  d = y - h + y_b

  Stress in bar i:
  If |eps_i| ≤ f_{yd} / E_s then
  sigma_i = eps_i / Eps_u1 • f_{yd}
  else
  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
						Sum of forces in bars:	-549569			-137392	0

N_s = -549.57e3
M_{xs} = -137.39e3
M_{ys} = 0.00e3

- Results for concrete section:



Height of compressed part of the section:

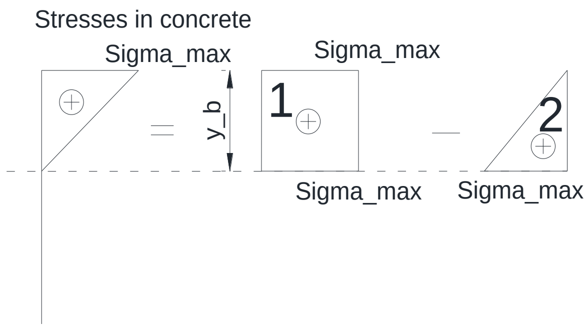
y_b = 0.257143

Area of contour under compressive stress:

A_c = 0.3 • y_b = 0.0771

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_c'$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$
$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} • Eps_top / Eps_ult = 8.571e6

Axial force in compressed part of concrete gross section

N_{c1} = A_c • Sigma_max = 0.6612e6
N_{c2} = A_c • Sigma_max / 2 = 0.3306e6
$N_{c'} = N_{c1} - N_{c2} = 0.3306e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
y_{c1} = 0.3 - y_b / 2 = 0.17142857
$x_{c2} = 0$
y_{c2} = 0.3 - y_b • 2/3= 0.12857143

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.00000000$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.21428571$

Internal forces in concrete:

$N_c = N_{c'} = 330.61e3$
$M_{xc} = - N_c \cdot y_{cg} = -70.85e3$
$M_{yc} = N_{c'} \cdot x_{cg} = 0.00e3$

- Results for reduced section:

$N = N_c + N_s = -218.96e3$
$M_x = M_{xc} + M_{xs} = -208.24e3$
$M_y = M_{yc} + M_{ys} = 0.00e3$

5 Case 4: Calculation of internal forces for given state of strain in the section and bilinear model of concrete

5.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

5.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 4a and Case 4b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 4c)
- A bilinear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 4a)
- A section under unidirectional bending with a part of section under compression (Case 4b and Case 4c)
- A state of strain with ultimate compressive strain in concrete (Case 4a and Case 4b)
- A strain below the ultimate compressive strain in concrete (Case 4c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force (N)
  - Moment with respect to x_{cc} axis (M_x)
  - Moment with respect to y_{cc} axis (M_y)

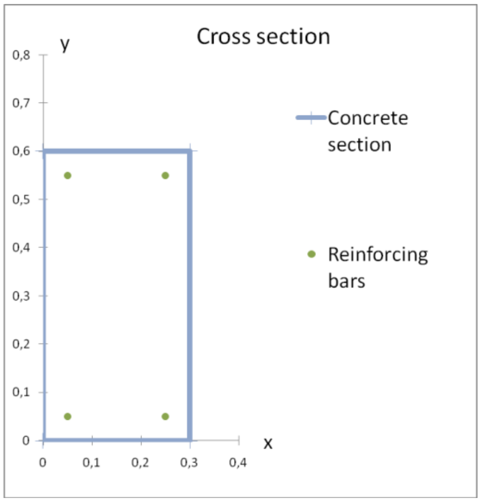
For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

5.3 Case 4a

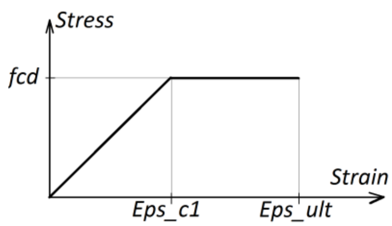
5.3.1 Data:

- Geometry:

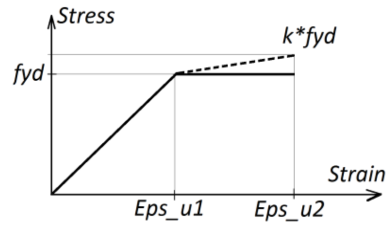
- Concrete section:
  x = 0.0 y = 0.0
  x = 0.0 y = 0.6
  x = 0.3 y = 0.6
  x = 0.3 y = 0.0
- Reinforcing bars:
  x = 0.05 y = 0.05 ϕ = 0.016
  x = 0.25 y = 0.05 ϕ = 0.016
  x = 0.25 y = 0.55 ϕ = 0.016
  x = 0.05 y = 0.55 ϕ = 0.016



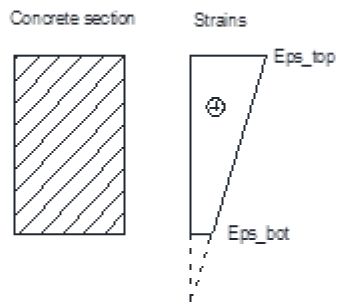
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k= 1.0
  - Modulus of elasticity E_s = 205e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = 0.0005
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



5.3.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

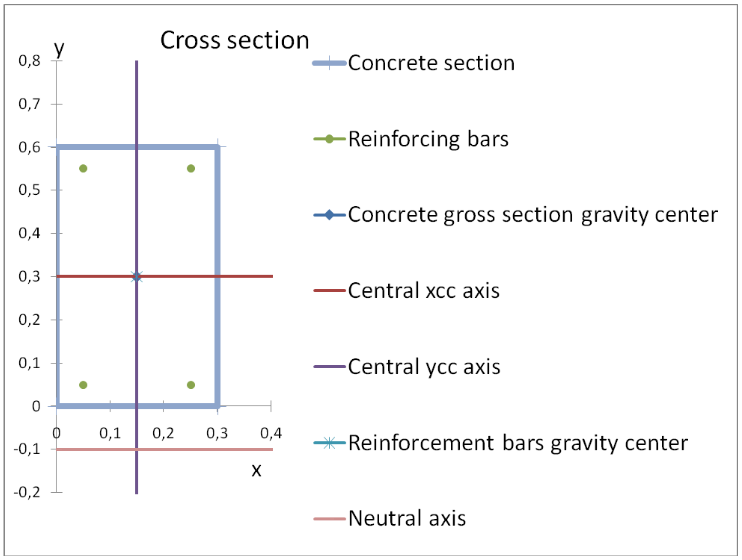
5.3.3 Results from Component

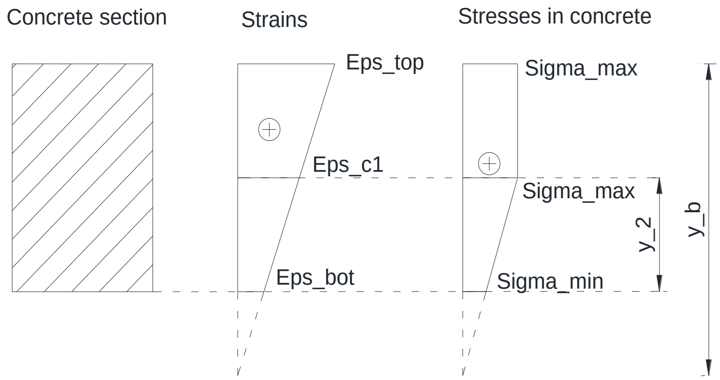
- Results for reinforcement bars:
  N_s = 222.68e3
  M_{xs}= -24.76e3
  M_{ys}= 0.00e3

- Results for concrete section:
  A_c = 0.1800
  x_{cg} = 0.00000000
  y_{cg} = 0.04589775
  N_c = 4370.91e3
  M_{xc} = -200.62e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = 4593.59e3
  M_x = -225.37e3
  M_y = 0.00e3

5.3.4 Results (manual calculations):





- Results for reinforcement bars:

  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.7

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y - h + y_b

  Stress in bar i:

  If |eps_i| ≤ f_{yd} / E_s then

sigma_i = eps_i / Eps_u1 • f_{yd}
else
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,016	0,150	0,00075	154	30913	-0,10	-0,25	7728	-3091
2	0,25	0,05	0,016	0,150	0,00075	154	30913	0,10	-0,25	7728	3091
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	222676			-24756	0

N_s = 222.68e3
M_{xs}= -24.76e3
M_{ys}= 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

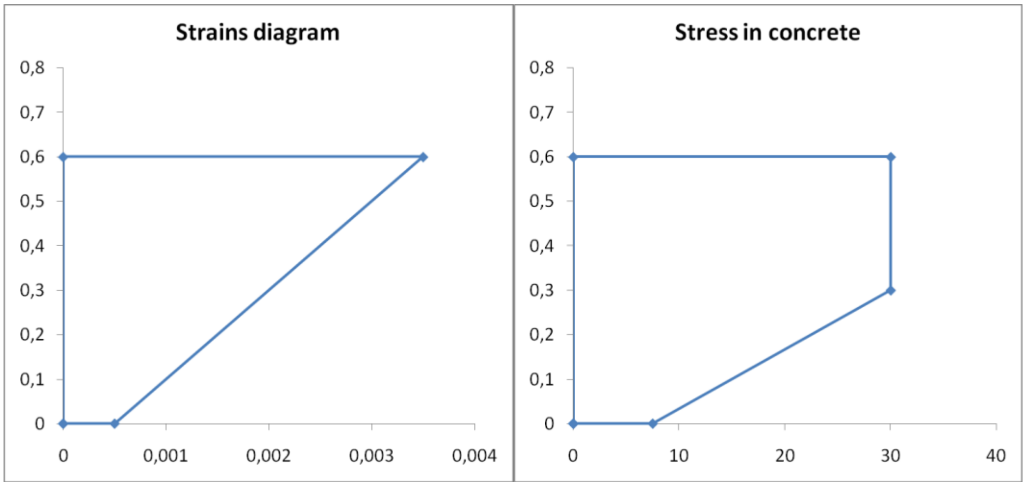
if eps ≤ Eps_c1 then N_{sc} = f_{cd} • eps / Eps_c1 • Area
if eps > Eps_c1 then N_{sc} = f_{cd} • Area

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	2262	565	-226
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	2262	565	226
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	16588	-1885	0

Reduction forces in concrete:
N_{sc} = 16.59e3
M_{xsc} = -1.89e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00000
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.11364

- Results for concrete section:



Height of compressed part of the section:

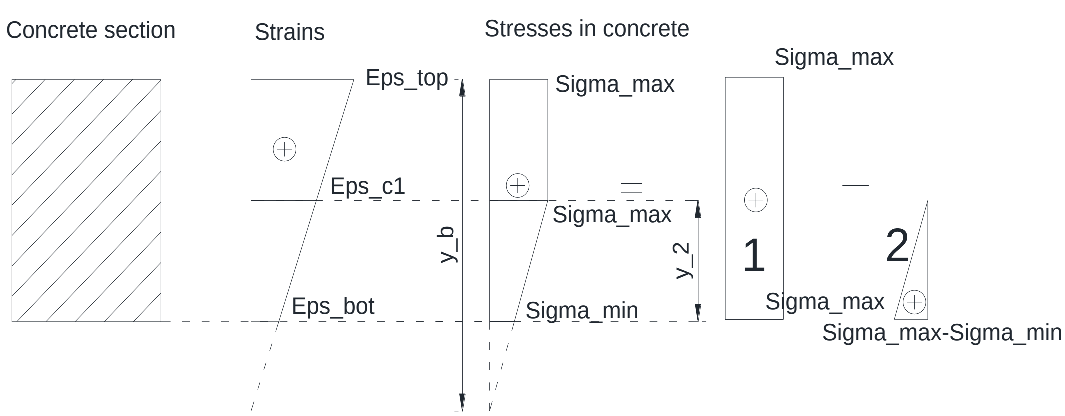
$h = 0.6000$

Area of contour under compressive stress:

$A_c = 0.3 \cdot 0.6000 = 0.\underline{1800}$

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_c' - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations.

$M_{xc} = - N_c \cdot y_{cg}$
$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6

Sigma_min = f_{cd} • Eps_bot/Eps_c1) = 7.500e6

Distance between gravity center of contour and neutral axis:

delta = - (0.6 / 2 - y_b) = 0.4
Angle = 3 / 2 • π = 4.712

Position of axis with stresses equal Eps_c1 in central axes coordinate system:

delta2 = - delta + MIN(y_b; Eps_c1 / Eps_top • y_b) = 0
Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

y_2 = 0.3 - delta2 = 0.3

Axial force in compressed part of concrete gross section

$N_{c1} = A_c$ • Sigma_max = 5.400e6
N_{c2} = 0.3 • y_2 • (Sigma_max - Sigma_min) / 2= 1.0125e6
$N_{c'} = N_{c1} - N_{c2}$ = 4.3875e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
$y_{c1} = 0$
$x_{c2} = 0$
y_{c2} = - 0.3 + y_2 / 3 = - 0.2

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0\underline{.000000}$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0\underline{.045898}$

Internal forces in concrete:

$N_c = N_c' - N_{sc} = \underline{4370.91e3}$
$M_{xc} = - N_c \cdot y_{cg} = -\underline{200.62e3}$
$M_{yc} = N_{c'} \cdot x_{cg} = 0\underline{.00e3}$

- Results for reduced section:

  $N = N_c + N_s = 4\underline{593.59e3}$
  $M_x = M_{xc} + M_{xs} = -\underline{225.37e3}$
  $M_y = M_{yc} + M_{ys} = 0\underline{.00e3}$

5.4 *Case 4b*

5.4.1 Data:

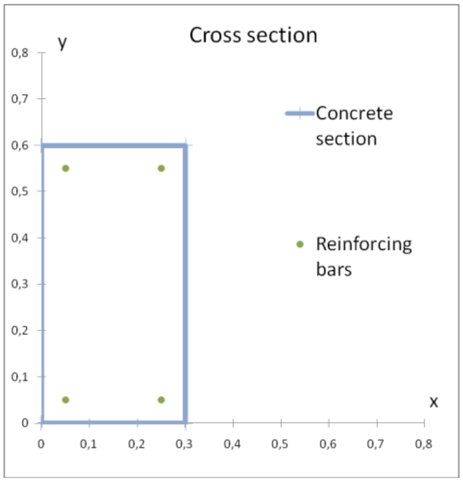
- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

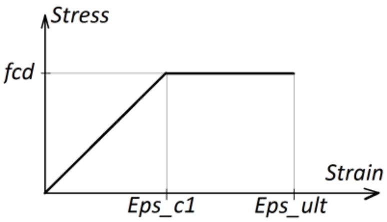
  - Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.032
x = 0.05	y = 0.55	ϕ = 0.032
x = 0.25	y = 0.55	ϕ = 0.032

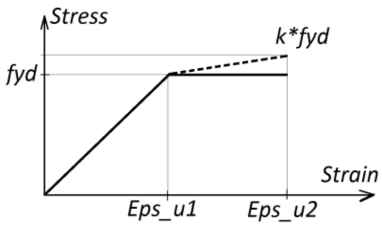
x = 0.25 y = 0.05 ϕ = 0.032



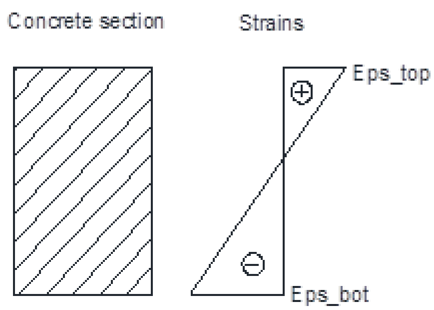
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity Ec = 32e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength fyd = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 205e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = -0.02936558
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



5.4.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

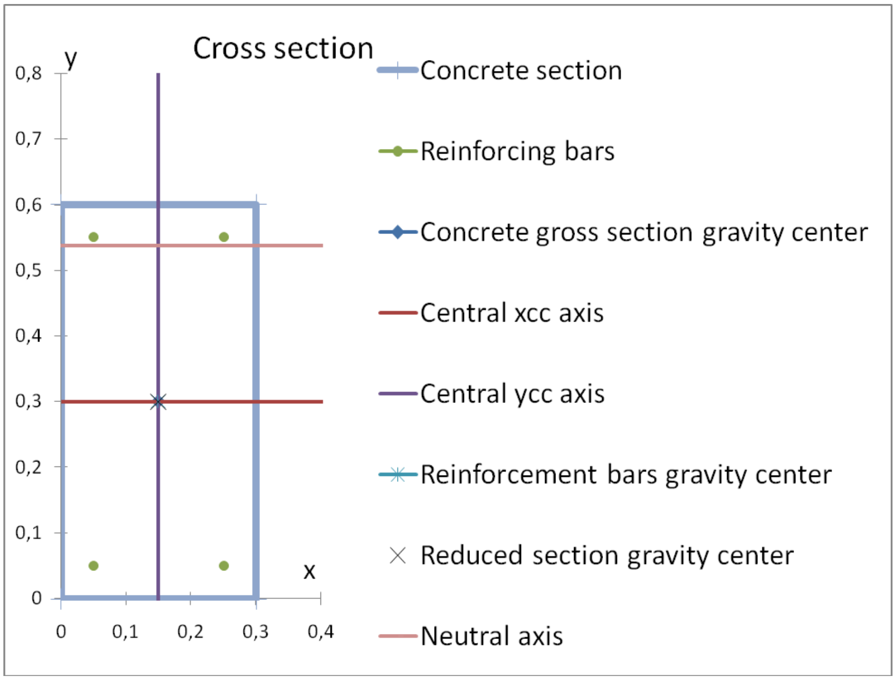
5.4.3 Results from Component

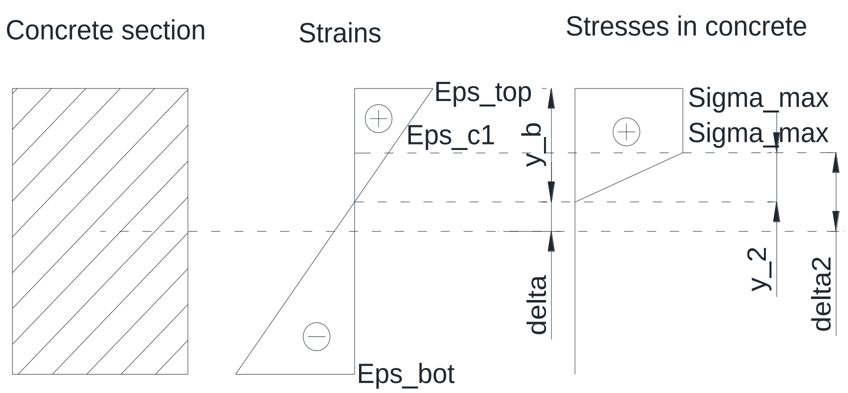
- Results for reinforcement bars:
  N_s = -392.40e3
  M_{xs}= -223.60e3
  M_{ys}= 0.00e3

- Results for concrete section:
  A_c = 0.0192
  x_{cg} = 0.00000000
  y_{cg} = 0.27717785
  N_c = 392.40e3
  M_{xc} = -108.76e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = 0.00e3
  M_x = -332.36e3
  M_y = 0.00e3

5.4.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.0639

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y - h + y_b

  Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then
sigma_i = eps_i / Eps_u1 • f_{yd}
else
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,486	-0,02663	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,486	-0,02663	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,014	0,00076	156	125500	0,10	0,25	-31375	12550
4	0,05	0,55	0,032	0,014	0,00076	156	125500	-0,10	0,25	-31375	-12550
						Sum of forces in bars:	-392398			-223600	0

N_s = -392.40e3
M_{xs} = -223.60e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

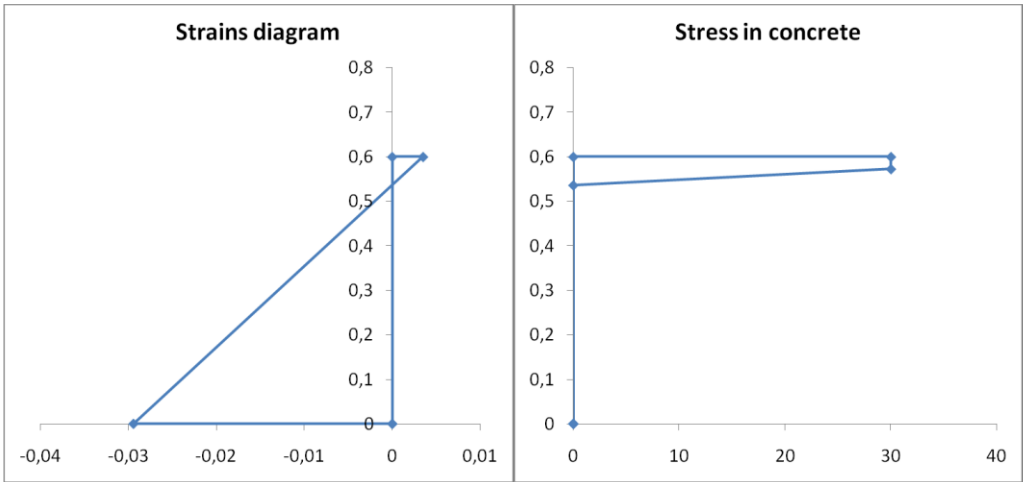
if eps ≤ Eps_c1 N_{sc} = f_{cd} • eps / Eps_c1 • Area
if eps > Eps_c1 N_{sc} = f_{cd} • Area

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,02663	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02663	0,10	-0,25				
3	0,25	0,55	0,032	0,00076	0,10	0,25	0,000804248	9183	-2296	918
4	0,05	0,55	0,032	0,00076	-0,10	0,25	0,000804248	9183	-2296	-918
							Sum of forces:	18366	-4591	0

Reduction forces in concrete:
N_{sc} = 18.37e3
M_{xsc} = - 4.59e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.25

- Results for concrete section:



Height of compressed part of the section:

$y\_b = 0.0639$

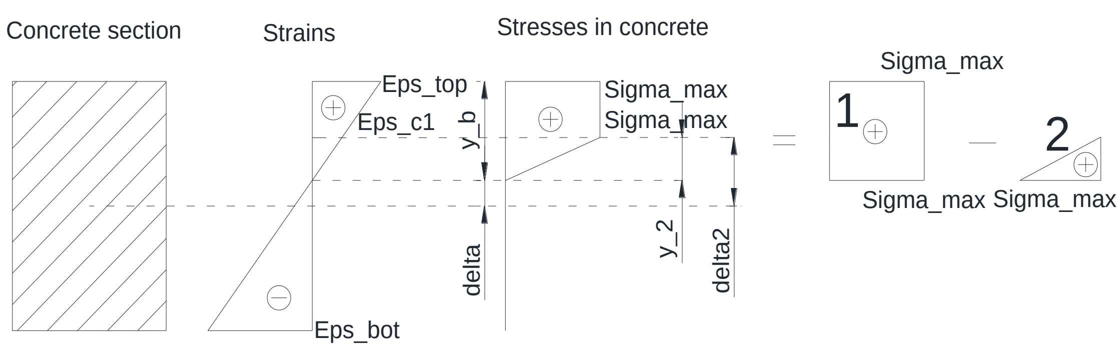
Area of contour under compressive stress:

$A_c = 0.3 \cdot y\_b = 0.0192$

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_c' - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6

Distance between gravity center of contour and neutral axis:

Delta = 0.6 / 2-y_b = 0.2361

Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.2726
Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

y_2 = delta2 - delta = 0.0365

Axial force in compressed part of concrete gross section

$N_{c1} = A_c$ • Sigma_max = 0.5751e6
N_{c2} = 0.3 • y_2 • Sigma_max / 2 = 0.1643e6
$N_{c'} = N_{c1} - N_{c2}$ = 0.41076e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
y_{c1} = delta + y_b / 2 = 0.26805
$x_{c2} = 0$
y_{c2} = delta + y_2 / 3 = 0.24827

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.277178$

Internal forces in concrete:

$N_c = N_c' - N_{sc}$ = 392.40e3
$M_{xc} = - N_c \cdot y_{cg}$ = - 108.76e3
$M_{yc} = N_{c'} \cdot x_{cg}$ = 0.00e3

- Results for reduced section:

  $N = N_c + N_s$ = 0.00e3
  $M_x = M_{xc} + M_{xs}$ = -332.36e3
  $M_y = M_{yc} + M_{ys}$ = 0.00e3

5.5 Case 4c

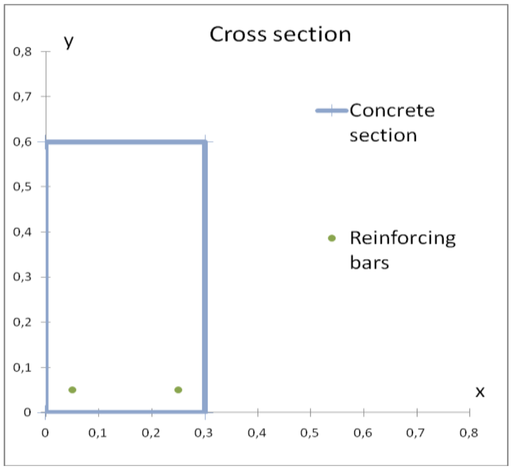
5.5.1 Data:

- Geometry:
  - Concrete section:

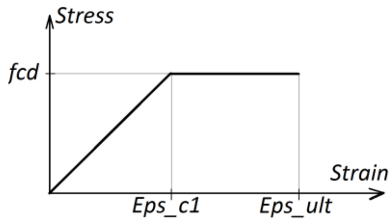
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

- Reinforcing bars:

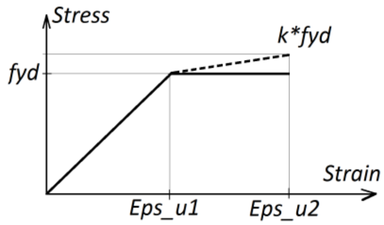
x = 0.05	y = 0.05	ϕ = 0.032
x = 0.25	y = 0.05	ϕ = 0.032



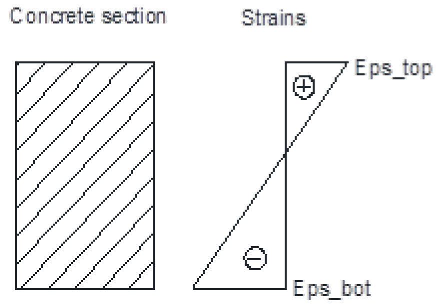
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength fyd = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 205e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0015
  - Bottom strain Eps_bot = -0.002
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



5.5.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

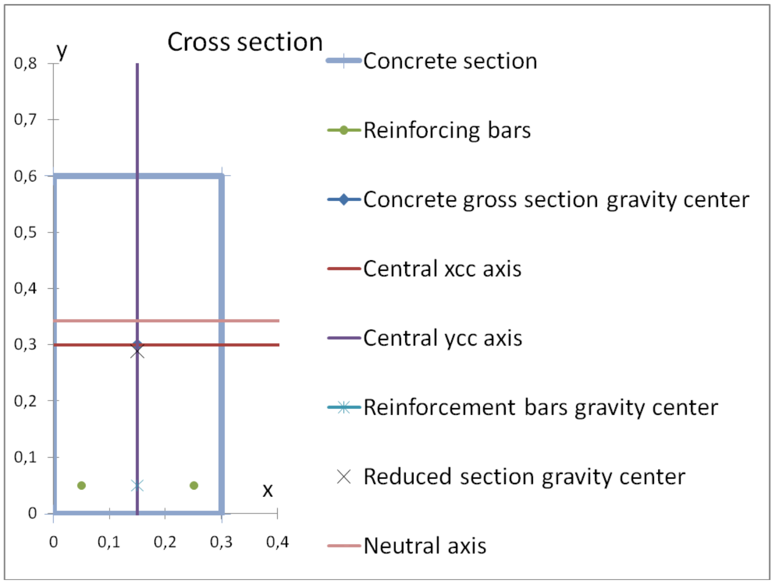
5.5.3 Results from Component

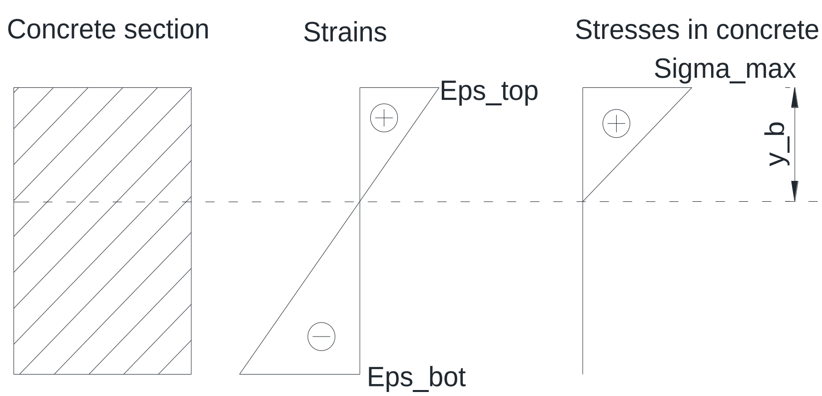
- Results for reinforcement bars:
  N_s = -563.31e3
  M_{xs}= -140.83e3
  M_{ys}= 0.00e3

- Results for concrete section:
  A_c = 0.0771
  x_{cg} = 0.000000
  y_{cg} = 0.21428571
  N_c = 867.86e3
  M_{xc} = -185.97e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = 304.55e3
  M_x = -326.80e3
  M_y = 0.00e3

5.5.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  - height of a section: h = 0.6
  - y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.257143
  - Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  - x_{Cc} = 0.15
  - y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
  - d = y - h + y_b

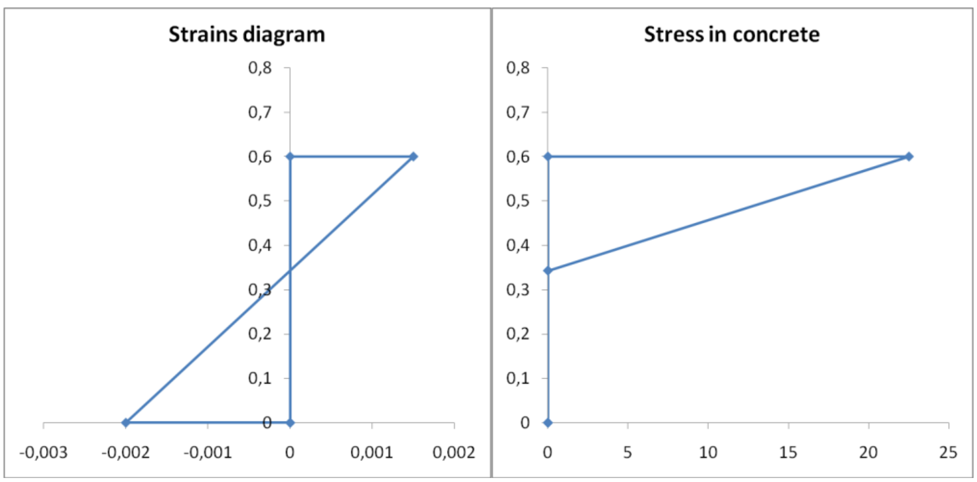
  Stress in bar i:
  - If |eps_i| ≤ f_{yd} / E_s then
    - sigma_i = eps_i / Eps_u1 • f_{yd}
  - else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,293	-0,00171	-350	-281654	-0,10	-0,25	-70414	28165
2	0,25	0,05	0,032	-0,293	-0,00171	-350	-281654	0,10	-0,25	-70414	-28165
						Sum of forces in bars:	-563309			- 140827	0

N_s = -563.31e3
M_{xs} = -140.83e3
M_{ys} = 0.00e3

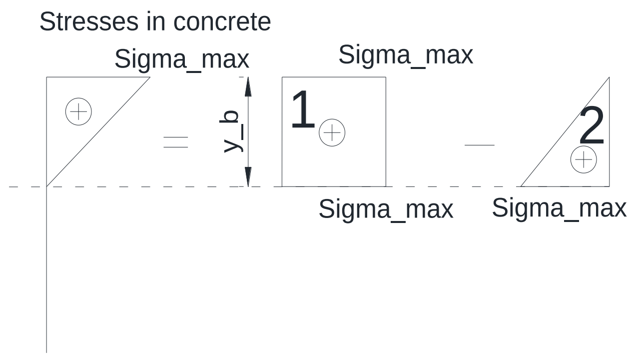
- Results for concrete section:



Height of compressed part of the section:
y_b = 0.257143
Area of contour under compressive stress:
A_c = 0.3 • y_b = 0.0771

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_{c'}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg}= (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$
$y_{cg}= (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$
$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} • Eps_top / Eps_c1 = 22.500e6

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \cdot$ Sigma_max = 1.7357e6
$N_{c2} = A_c \cdot$ Sigma_max / 2 = 0.8679e6
$N_{c'} = N_{c1} - N_{c2}$ = 0.8679e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
y_{c1} = 0.3 - y_b / 2 = 0.17142857
$x_{c2} = 0$
y_{c2} = 0.3-y_b • 2/3 = 0.12857143

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.\underline{00000000}$
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.\underline{21428571}$

Internal forces in concrete:

$N_c = N_{c'} = 8\underline{67.86e3}$
$M_{xc} = - N_c \cdot y_{cg} = -\underline{185.97e3}$
$M_{yc} = N_{c'} \cdot x_{cg} = 0\underline{.00e3}$

- Results for reduced section:

$N = N_c + N_s = 3\underline{04.55e3}$
$M_x = M_{xc} + M_{xs} = -\underline{326.80e3}$
$M_y = M_{yc} + M_{ys} = 0\underline{.00e3}$

6 Case 5: Calculation of internal forces for given state of strain in the section and parabolic-rectangular model of concrete

6.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

6.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 5a and Case 5b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 5c)
- A parabolic-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 5a)
- A section under unidirectional bending with a part of section under compression (Case 5b and Case 5c)
- A state of strain with ultimate compressive strain in concrete (Case 5a and Case 5b)
- A strain below the ultimate compressive strain in concrete (Case 5c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force (N)
    - Moment with respect to x_{cc} axis (M_x)
    - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

6.3 Case 5a

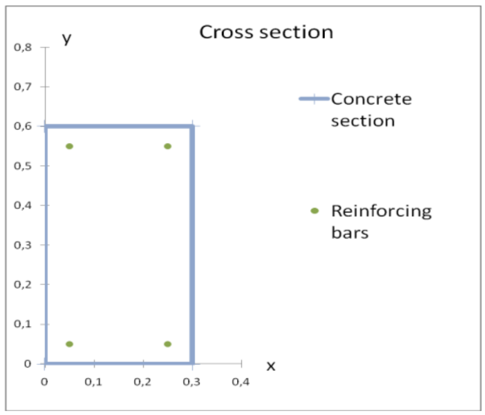
6.3.1 Data:

- Geometry:
    - Concrete section:

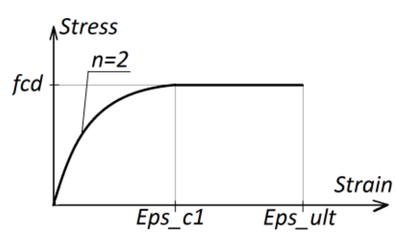
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

- Reinforcing bars:

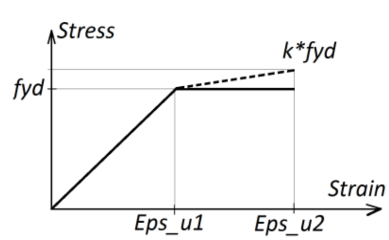
x = 0.05	y = 0.05	ϕ = 0.016
x = 0.05	y = 0.55	ϕ = 0.016
x = 0.25	y = 0.55	ϕ = 0.016
x = 0.25	y = 0.05	ϕ = 0.016



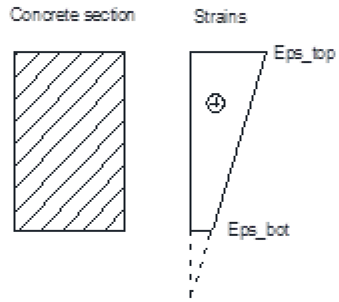
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = 0.0005
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



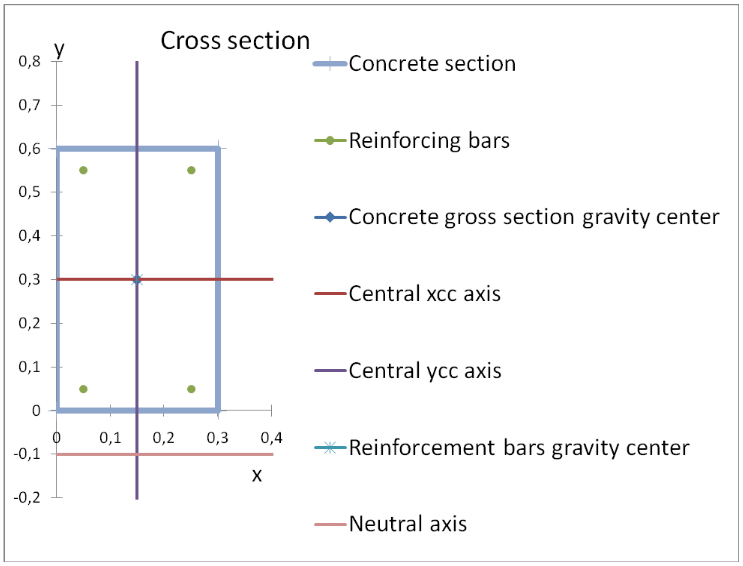
6.3.2 Search for:

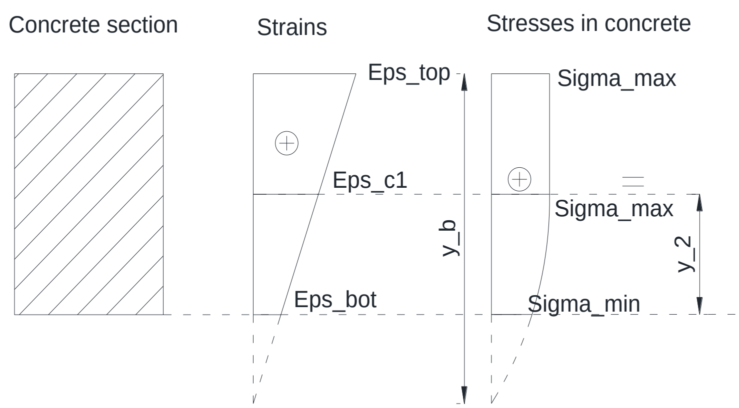
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

6.3.3 Results from Component

- Results for reinforcement bars:
  N_s = 221.17e3
  M_{xs}= -25.13e3
  M_{ys}= 0.00e3
- Results for concrete section:
  A_c = 0.1800
  x_{cg} = 0.00000000
  y_{cg} =0.02312688
  N_c = 4874.33e3
  M_{xc} = -112.73e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 5095.50e3
  M_x = -137.86e3
  M_y = 0.00e3

6.3.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = - Eps_top • h / (Eps_bot - Eps_top) = 0.7
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{Cc} = 0.15
  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
  d = y - h + y_b

  Stress in bar i:
  If |eps_i| ≤ f_{yd} / E_s then
  sigma_i = eps_i / Eps_u1 • f_{yd}
  else
  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 /4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= - N • v	M_{ys}= N • x_{cc}
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10	-0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10	-0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	221168			-25133	0

N_s = 221.17e3
M_{xs} = -25.13e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

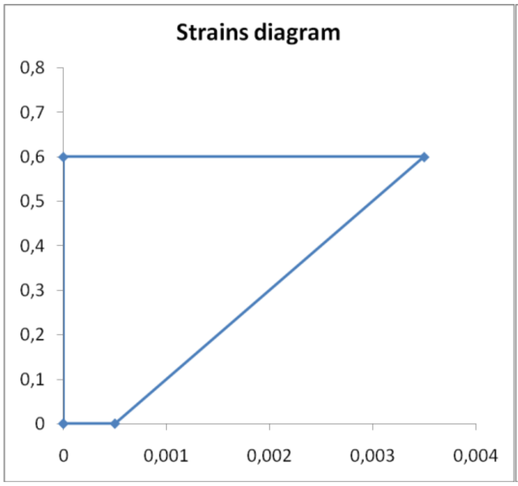
if eps ≤ Eps_c1 then $N_{sc} = f_{cd} \cdot (1-(1-eps/Eps\_c1)^2) \cdot$ Area
if eps > Eps_c1 then $N_{sc} = f_{cd} \cdot$ Area

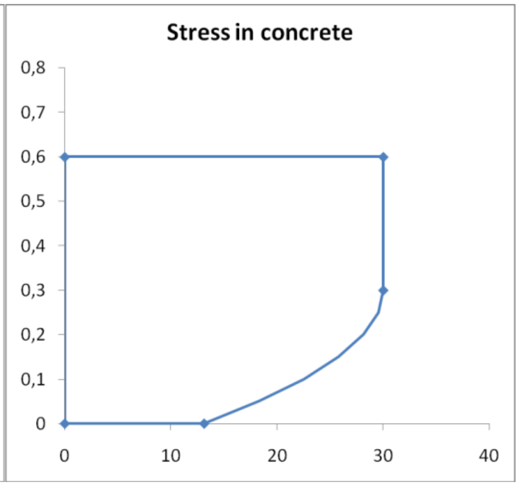
i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc} = N_{sc} • x_{cc}
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	3676	919	-226
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	3676	919	226
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	19415	-1178	0

Reduction forces in concrete:
N_{sc} = 19.42e3
M_{xsc} = -1.18e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00000
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.06068

- Results for concrete section:





Height of compressed part of the section:

0.6000

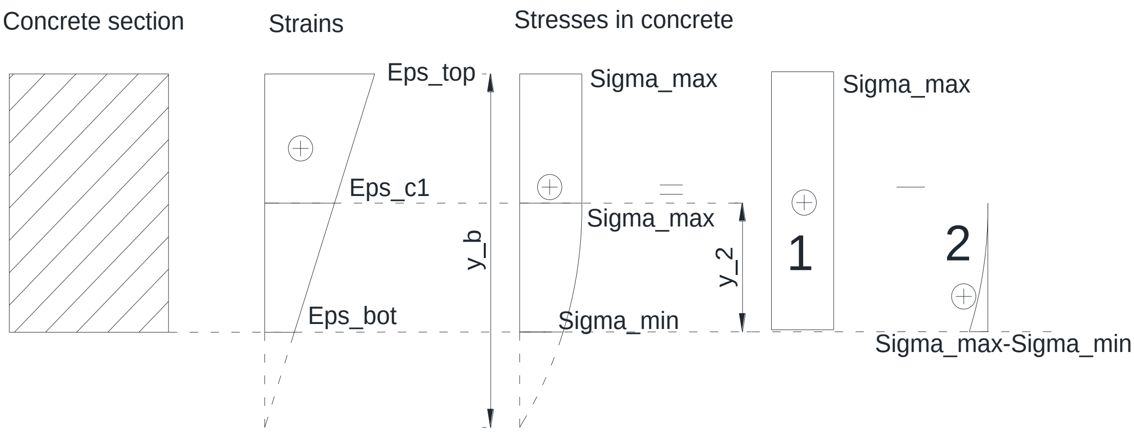
Area of contour under compressive stress:

A_c = 0.3 • 0.6000 = 0.1800

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction)

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_c' - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6

Sigma_min = f_{cd} • (1 - (1 - Eps_bot / Eps_c1) ^{2}) = 13.125e6

Distance between gravity center of contour and neutral axis:

Delta = - (0.6 / 2 - y_b) = 0.4
Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = - delta + MIN(y_b; Eps_c1 / Eps_top • y_b) = 0
Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

y_2 = 0.3 - delta2 = 0.3

Axial force in compressed part of concrete gross section

N_{c1} = A_c • Sigma_max = 5.400e6
N_{c2} = 0.3 • 1 / 3 • y_2 • (Sigma_max - Sigma_min) = 0.50625e6
$N_{c'}$ = N_{c1} - N_{c2} = 4.89375e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

x_{c1} = 0
y_{c1} = 0
x_{c2} = 0
y_{c2} = - 0.3 + y_2 / 4= - 0.225

Center of gravity of stresses in the concrete in central axes coordinate system:

x_{cg} = (x_{c1} • N_{c1} - x_{c2} • N_{c2} - x_{scg} • N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000
y_{cg} = (y_{c1} • N_{c1} - y_{c2} • N_{c2} - y_{scg} • N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.023127

Internal forces in concrete:

N_c = $N_{c'}$ - N_{sc} = 4874.33e3
M_{xc} = - N_c • y_{cg} = -112.73e3
M_{yc} = $N_{c'}$ • x_{cg} = 0.00e3

- Results for reduced section:

  N = N_c + N_s = 5095.50e3
  M_x = M_{xc} + M_{xs}= -137.86e3
  M_y = M_{yc} + M_{ys}= 0.00e3

6.4 Case 5b

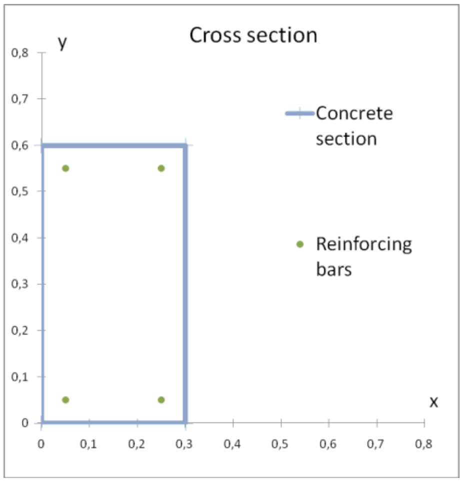
6.4.1 Data:

- Geometry:
  - Concrete section:

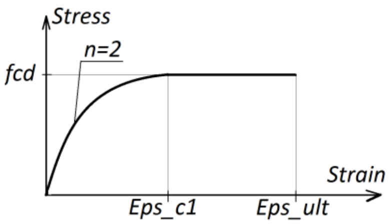
    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:

    x = 0.05 y = 0.05 ϕ = 0.032

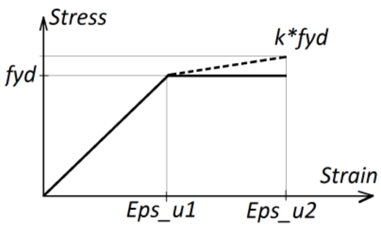
x = 0.05	y = 0.55	ϕ = 0.032
x = 0.25	y = 0.55	ϕ = 0.032
x = 0.25	y = 0.05	ϕ = 0.032



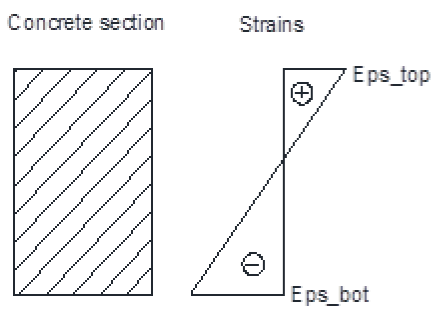
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity Ec = 32e9
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength fyd = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = -0.03034666
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



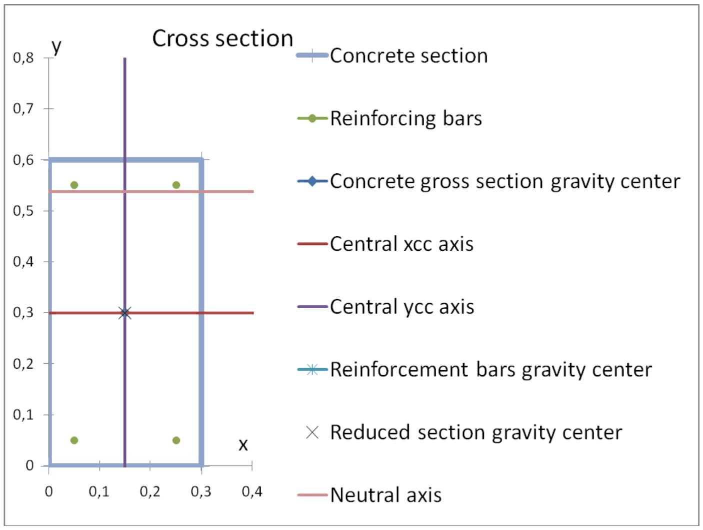
6.4.2 Search for:

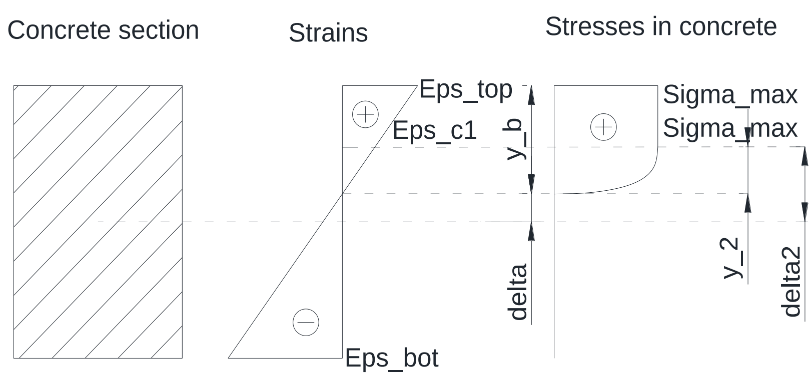
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

6.4.3 Results from Component

- Results for reinforcement bars:
  N_s = -424.82e3
  M_{xs}= -215.49e3
  M_{ys}= 0.00e3
- Results for concrete section:
  A_c = 0.0186
  x_{cg} = 0.00000000
  y_{cg} = 0.27574146
  N_c = 424.82e3
  M_{xc} = -117.14e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 0.00e3
  M_x = -332.63e3
  M_y = 0.00e3

6.4.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.0620

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y - h + y_b

  Stress in bar i:

  If |eps_i| ≤ f_{yd} / E_s then

  sigma_i = eps_i / Eps_u1 • f_{yd}

  else

  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,488	-0,02753	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02753	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00068	136	109288	0,10	0,25	-27322	10929
4	0,05	0,55	0,032	0,012	0,00068	136	109288	-0,10	0,25	-27322	-109290
						Sum of forces in bars:	-424821			-215494	0

N_s = -424.82e3
M_{xs} = -215.49e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

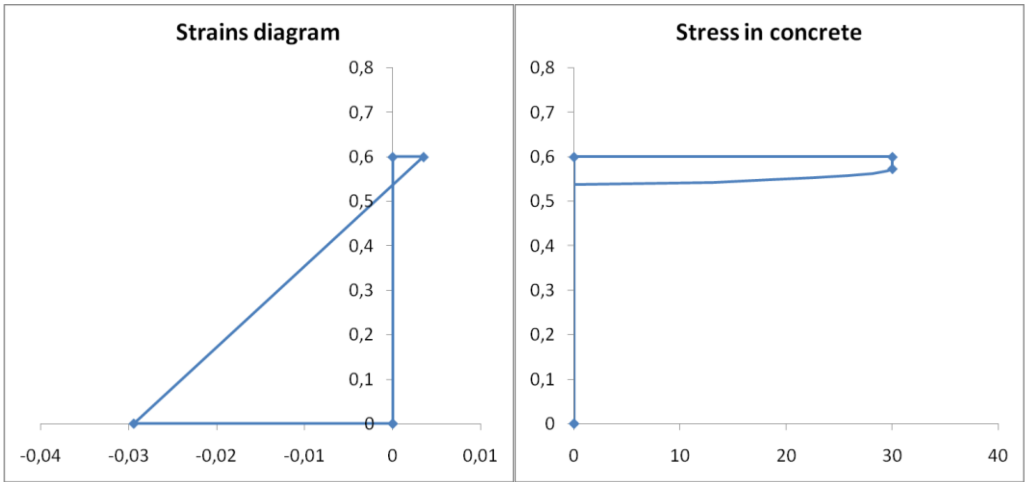
if eps ≤ Eps_c1 then $N_{sc} = f_{cd} \cdot (1-(1-eps/Eps\_c1)^2) \cdot Area$
if eps > Eps_c1 then $N_{sc} = f_{cd} \cdot Area$

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc} = N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,02753	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02753	0,10	-0,25				
3	0,25	0,55	0,032	0,00068	0,10	0,25	0,000804248	13609	-3402	1361
4	0,05	0,55	0,032	0,00068	-0,10	0,25	0,000804248	13609	-3402	-1361
							Sum of forces:	27217	-6804	0

Reduction forces in concrete:
N_{sc} = 27.22e3
M_{xsc} = -6.80e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.25

- Results for concrete section:



Height of compressed part of the section:

y_b = 0.0620

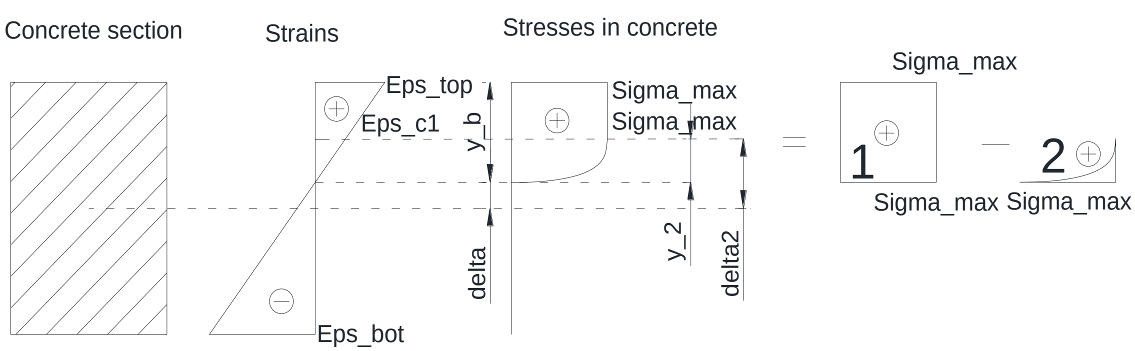
Area of contour under compressive stress:

A_c = 0.3 • y_b = 0.0186

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations.

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6

Distance between gravity center of contour and neutral axis:

delta= 0.6 / 2-y_b = 0.2380

Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.2734

Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

y_2 = delta2 - delta = 0.0354

Axial force in compressed part of concrete gross section

$N_{c1} = A_c$ • Sigma_max = 0.558401e6

N_{c2} = 0.3 • y_2 • Sigma_max / 3 = 0.106362e6

$N_{c'} = N_{c1} - N_{c2}$ = 0.452039e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$

y_{c1} = delta + y_b / 2= 0.268978

$x_{c2} = 0$

y_{c2} = delta+y_2/3= 0.246819

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.000000

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.275741

Internal forces in concrete:

$N_c = N_{c'} - N_{sc}$ = 424.82e3

$M_{xc} = - N_c \cdot y_{cg}$ = -117.14e3

$M_{yc} = N_{c'} \cdot x_{cg}$ = 0.00e3

- Results for reduced section:

  $N = N_c + N_s$ = 0.00e3

  $M_x = M_{xc} + M_{xs}$ = -332.63e3

  $M_y = M_{yc} + M_{ys}$ = 0.00e3

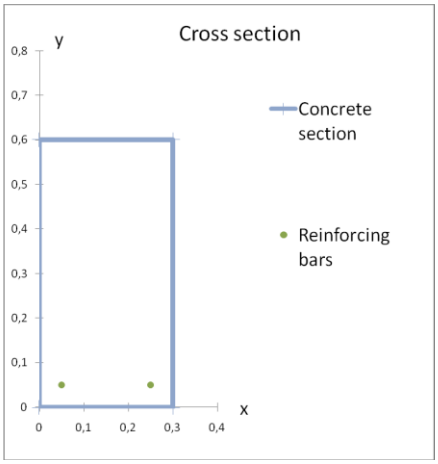
6.5 Case 5c

6.5.1 Data:

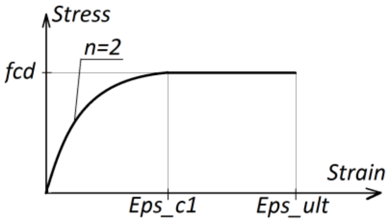
- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:

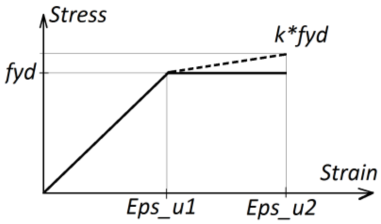
    x = 0.05 y = 0.05 ϕ = 0.032
    x = 0.25 y = 0.05 ϕ = 0.032



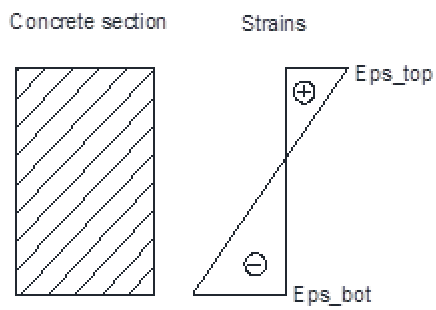
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0015
  - Bottom strain Eps_bot = -0.002
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



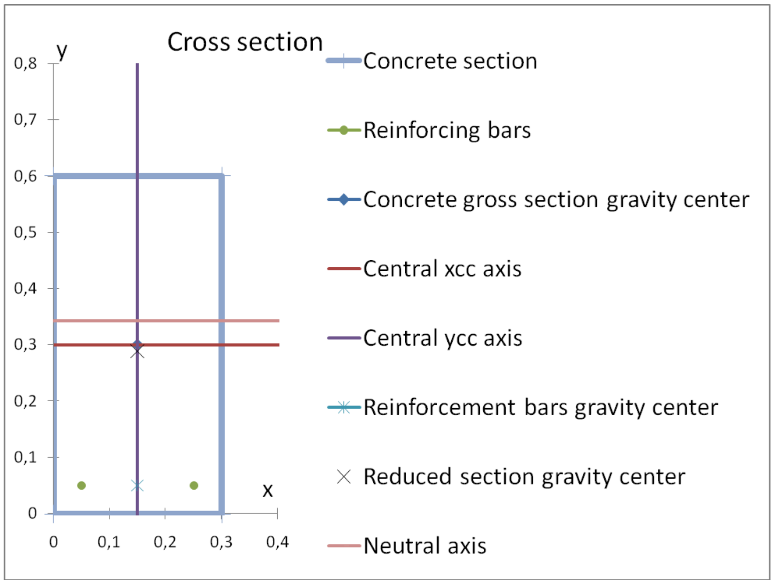
6.5.2 Search for:

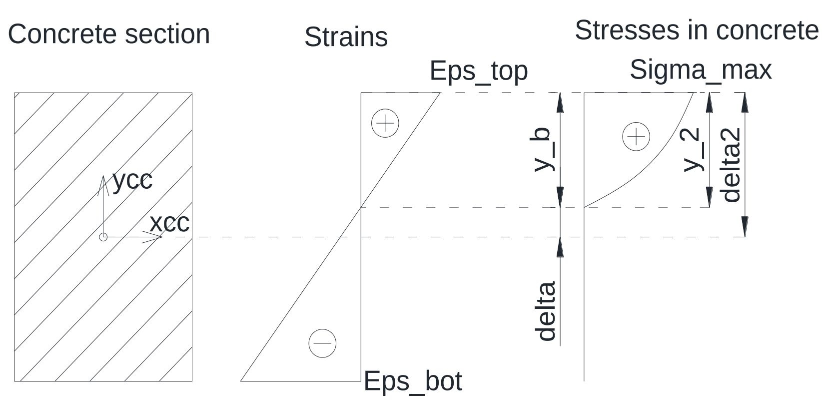
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

6.5.3 Results from Component

- Results for reinforcement bars:
  N_s = -549.57e3
  M_{xs} = -137.39e3
  M_{ys} = 0.00e3
- Results for concrete section:
  A_c = 0.0771
  x_{cg} = 0.000000
  y_{cg} = 0.20714286
  N_c = 1301.79e3
  M_{xc} = -269.66e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 752.22e3
  M_x = -407.05e3
  M_y = 0.00e3

6.5.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  - height of a section: h = 0.6
  - y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.257143
  - Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  - x_{Cc} = 0.15
  - y_{Cc} = 0.30

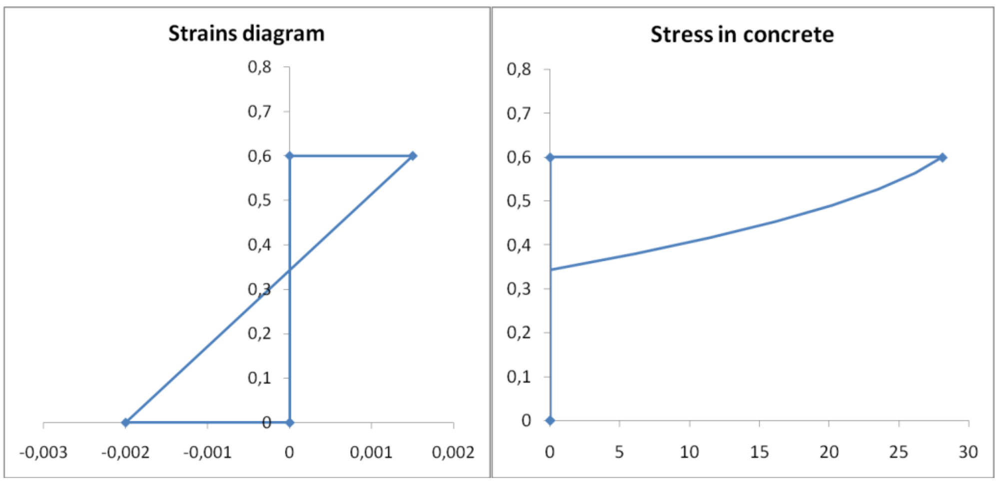
  Distance between reinforcing bar and neutral axis:
  - d = y - h + y_b

  Stress in bar i:
  - If |eps_i| ≤ f_{yd} / E_s then
    - sigma_i = eps_i / Eps_u1 • f_{yd}
  - else
    - sigma_i = fyd + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • fyd)

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
						Sum of forces in bars:	-549569			-137392	0

N_s = -549.57e3
M_{xs}= -137.39e3
M_{ys}= 0.00e3

- Results for concrete section:



Height of compressed part of the section:

y_b = 0.257143

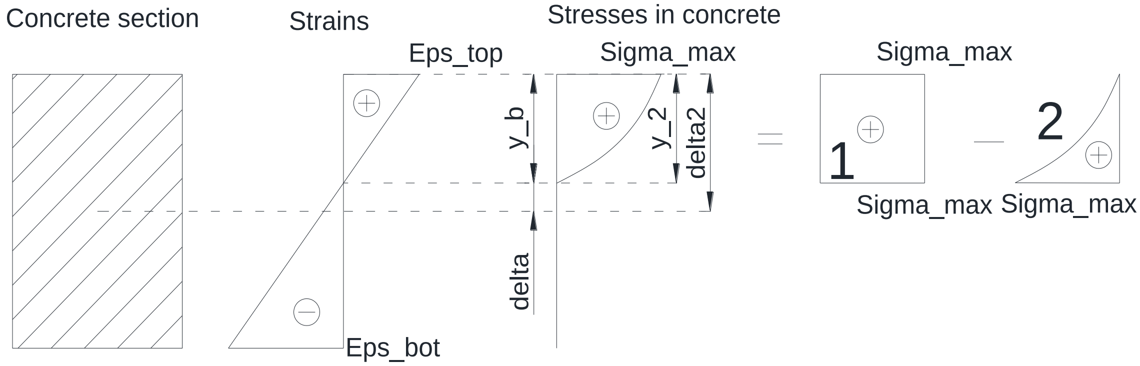
Area of contour under compressive stress:

$A_c = 0.3 \cdot y\_b = 0.0771$

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = $f_{cd} \cdot (1-(1-\text{Eps\_top}/\text{Eps\_c1})^2)$ = 28.125e6

Distance between gravity center of contour and neutral axis:

delta= 0.6 / 2-y_b = 0.042857

Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.3000

Angle = 3 / 2 • π= 4.712

Height of the part of the section where concrete stress is not constant:

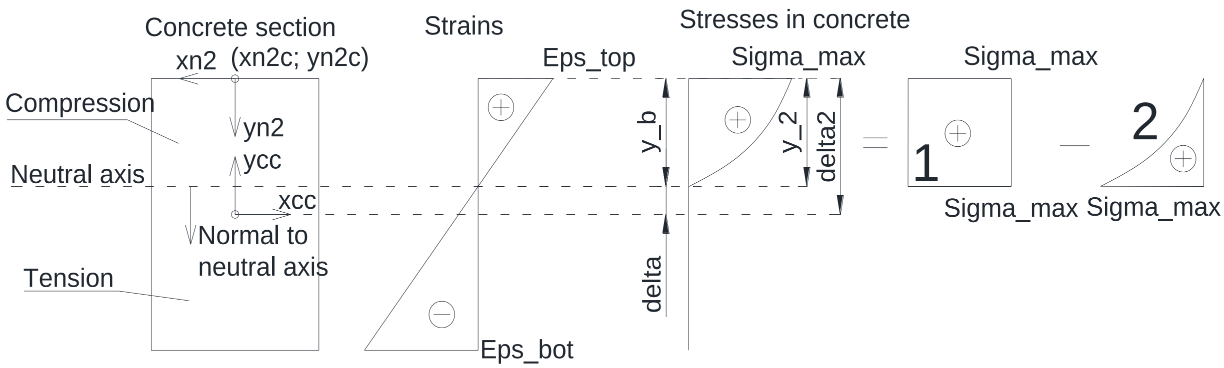
y_2 = delta2 - delta = 0.257143

Axial force in compressed part of concrete gross section

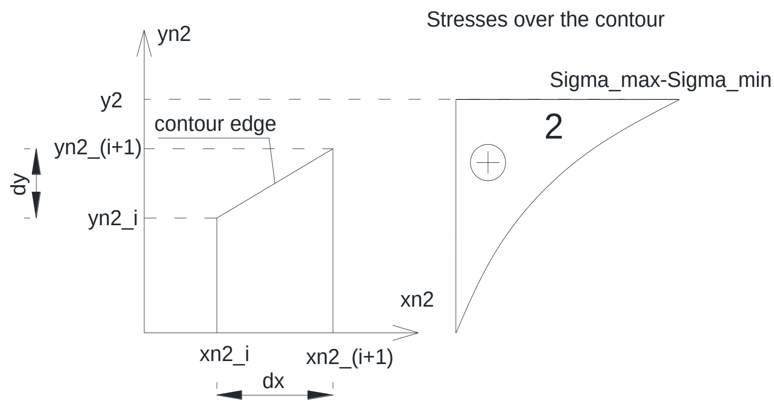
$N_{c1} = A_c \cdot$ Sigma_max = 2.169643e6

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2}, y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}; y_{n2c}) given in central axis coordinate system.

(x_{n2c}; y_{n2c}) = (0; delta2)



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system x	y	Coordinates in central axes system x_{cc}	y_{cc}	Coordinates in (x_{n2}, y_{n2}) system x_{n2}	y_{n2}	dx	dy	A	B	N_{c2}	Static moment of stress solid $MS_{yn2,zn2}$	Static moment of stress solid $MS_{xn2,zn2}$
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,00	0,00	0,00
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,87	0,00	0,16
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,00	0,00	0,00
4	0,00	0,60	-0,15	0,30	0,15	0,00	-0,30	0,00	0,00	0,00	0,00	0,00	0,00
										N_{c2} = sum($N_{c2\ i}$) =	0,87		
								C=	0,34				
								D=	0,26				
	gravity center of stress solid 2 in (x_{n2}; y_{n2}) coordinates system											x_{c2}'	0,0000
												y_{c2}'	0,1821
	gravity center of stress solid 2 in central axes (x_{cc} ; y_{cc}) coordinates system											x_{c2}	0,0000
												y_{c2}	0,1179

Where:
$x_{cc} = x - x_{Cc}$
$y_{cc} = y - y_{Cc}$
$x_{n2} = \cos(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$
$y_{n2} = -\sin(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$
dx = x_(i + 1) - x_i
dy = y_(i + 1) - y_i
A = dy/dx if dx ≠ 0 or A=0 if dx=0
B = -dy/dx • x_{n2}_(i + 1)+y_{n2}_(i + 1) if dx ≠ 0 or B = 0 if dx = 0
C = y_b • Eps_c1 / Eps_top
D = MIN(Eps_c1;Eps_top) • y_b / Eps_top
n = 2 (power of stress-strain relation curve)
dSigma =Sigma_max - Sigma_min
N_{c2}_i = 0 if A = 0 and B = 0 or

N_{c2}_i = f_{cd} • C / (n + 1) • (C / (n + 2) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - (1 - (-y_{n2}_i + D)/C)$^{n+2}$) - (x_{n2}_(i + 1) - x_{n2}_i) • (1 – D / C)$^{n+1}$) + (dSigma - f_{cd}) • (A / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B • (x_{n2}_(i + 1) - x_{n2}_i)) if A ≠ 0 or

N_{c2}_i = (f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$)) • dx + (-f_{cd} + dSigma) • B • dx if A = 0 and B ≠ 0

N_{c2} = sum(N_{c2_i})
$MS_{yn2,zn2}$_i = f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$) • 1 / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + (-f_{cd} + dSigma) • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) if A = 0 or

$MS_{yn2,zn2}$_i = f_{cd} • C^2 / ((n + 1) • (n + 2) • A) • (x_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ -x_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - (1 - (-y_{n2}_i + D) / C)$^{n+3}$)) - 0.5 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) • f_{cd} • C / (n + 1) • (1 – D / C)$^{n+1}$ + (-f_{cd} + dSigma) • (A / 3 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2)) if A ≠ 0

$MS_{yn2,zn2}$ = sum($MS_{yn2,zn2}$_i)
$MS_{xn2,zn2}$_i = (f_{cd} • C / (n + 1) • (B • (1-(-B + D) / C)$^{n+1}$ – C / (n + 2) • ((1 - (-B + D) / C)$^{n+2}$ - (1 – D / C)$^{n+2}$)) + (-f_{cd} + dSigma) / 2 • B^2) • (x_{n2}_(i + 1) - x_{n2}_i) if A = 0 or

$MS_{xn2,zn2}$_i = f_{cd} • C^2/((n + 1) • (n + 2) • A) • (y_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - y_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) • (2 • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - 2 • (1 - (-y_{n2}_i + D) / C)$^{n+3}$) + A • (1 – D / C)$^{n+2}$ • dx) + (dSigma - f_{cd}) • (A^2 / 6 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + A • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B^2 / 2 • dx) if A ≠ 0

$MS_{xn2,zn2}$ = sum($MS_{xn2,zn2}$_i)
$x_{c2}' = MS_{yn2,zn2} / N_{c2}$
$y_{c2}' = MS_{xn2,zn2} / N_{c2}$
$x_{c2} = x_{n2c} + \cos(Angle - \pi / 2) \cdot x_{c2}' - \sin(Angle - \pi / 2) \cdot y_{c2}'$
$y_{c2} = y_{n2c} + \sin(Angle - \pi / 2) \cdot x_{c2}' + \cos(Angle - \pi / 2) \cdot y_{c2}'$

N_{c2} = 0.867857e6
$N_{c'} = N_{c1} - N_{c2}$ = 1.301786e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

x_{c1} = 0
y_{c1} = delta + y_b / 2= 0.171428571
x_{c2} = 0
y_{c2} = 0.117857143

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$ = 0.000000
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$ = 0.207143

Internal forces in concrete:

$N_c = N_{c'}$ = 1301.79e3
$M_{xc} = - N_c \cdot y_{cg}$ = -269.66e3
$M_{yc} = N_{c'} \cdot x_{cg}$ = 0.00e3

- Results for reduced section:

  $N = N_c + N_s = 752.22e3$

  $M_x = M_{xc} + M_{xs} = -407.05e3$

  $M_y = M_{yc} + M_{ys} = 0.00e3$

7 Case 6: Calculation of internal forces for given state of strain in the section and power-rectangular model of concrete

7.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

7.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 6a and Case 6b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 6c)
- A power-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 6a)
- A section under unidirectional bending with a part of section under compression (Case 6b and Case 6c)
- A state of strain with ultimate compressive strain in concrete (Case 6a and Case 6b)
- A strain below the ultimate compressive strain in concrete (Case 6c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force (N)
    - Moment with respect to x_{cc} axis (M_x)
    - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

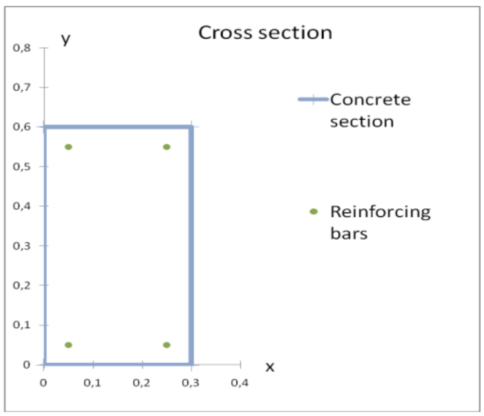
7.3 Case 6a

7.3.1 Data:

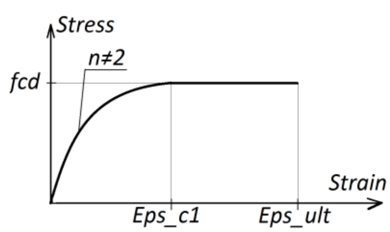
- Geometry:
    - Concrete section:

x = 0.0 y = 0.0
x = 0.0 y = 0.6
x = 0.3 y = 0.6
x = 0.3 y = 0.0

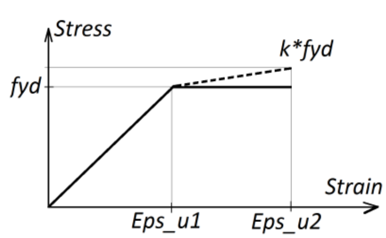
- Reinforcing bars:
  x = 0.05 y = 0.05 ϕ = 0.012
  x = 0.25 y = 0.05 ϕ = 0.012
  x = 0.25 y = 0.55 ϕ = 0.012
  x = 0.05 y = 0.55 ϕ = 0.012



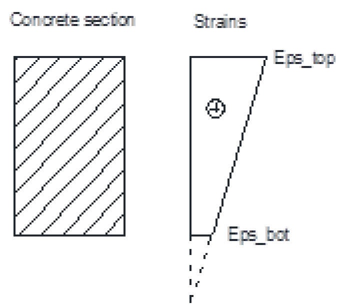
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: power-rectangular
  - Power: n = 1.4
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k= 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = 0.0005
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



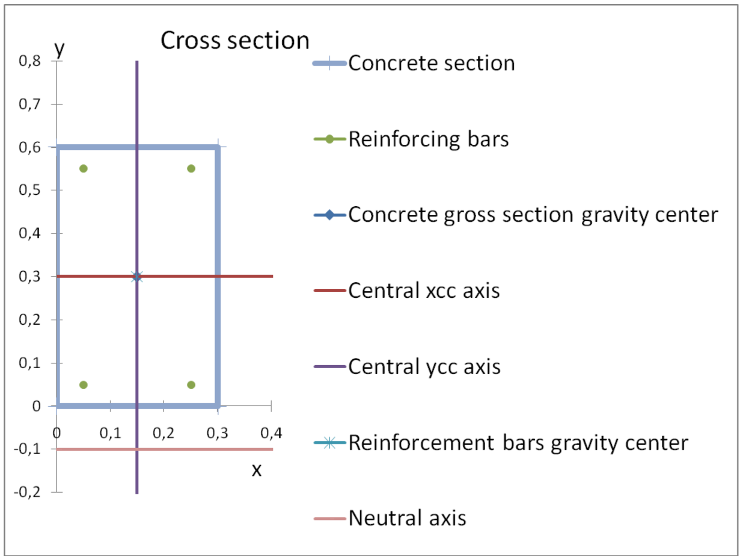
7.3.2 Search for:

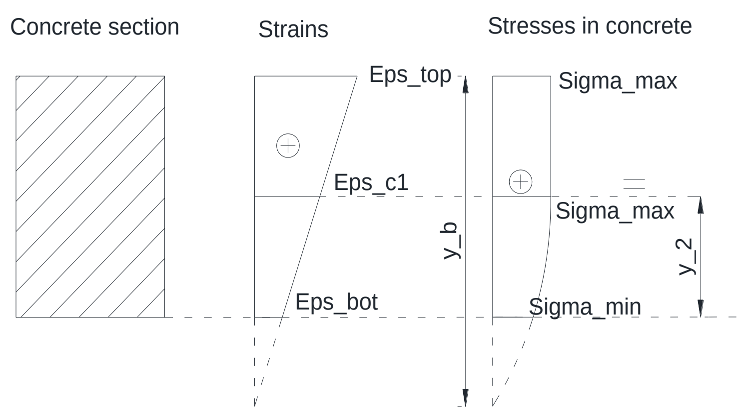
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

7.3.3 Results from Component

- Results for reinforcement bars:
  N_s = 221.17e3
  M_{xs} = -25.13e3
  M_{ys} = 0.00e3
- Results for concrete section:
  A_c = 0.1800
  x_{cg} = 0.00000000
  y_{cg} = 0.03405827)
  N_c = 4630.08e3
  M_{xc} = -157.69e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 4851.25e3
  M_x = -182.83e3
  M_y = 0.00e3

7.3.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:

  height of a section: h = 0.6

  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.7

  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

  x_{Cc} = 0.15

  y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:

  d = y - h + y_b

  Stress in bar i:

  If |eps_i| ≤ f_{yd} / E_s then

  sigma_i = eps_i / Eps_u1 • f_{yd}

  else

  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= - N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10	-0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10	-0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	221168			-25133	0

N_s = 221.17e3
M_{xs} = -25.13e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if eps ≤ Eps_c1 then $N_{sc} = f_{cd} \cdot (1-(1-eps/Eps\_c1)^n) \cdot Area$
if eps > Eps_c1 then $N_{sc} = f_{cd} \cdot Area$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	2908	727	-291
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	2908	727	291
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	17880	-1562	0

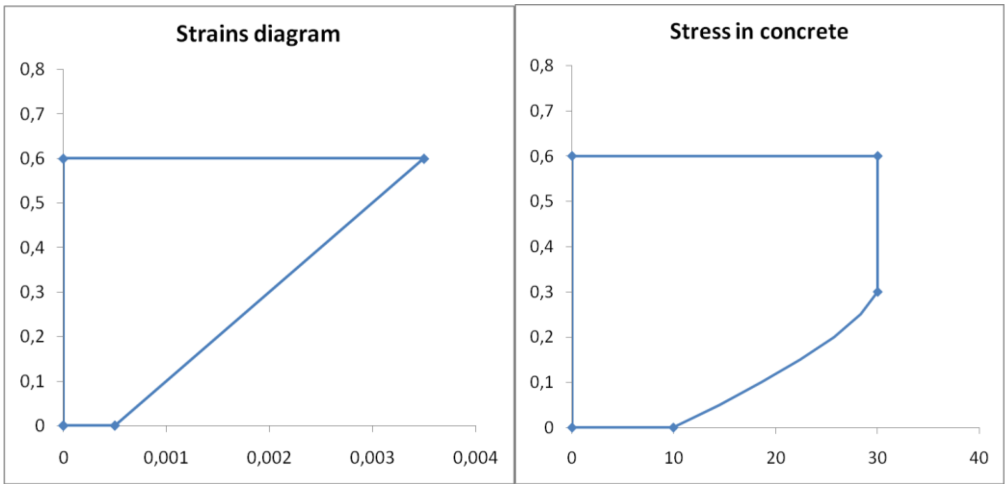
Reduction forces in concrete:

N_{sc} = 17.88e3
M_{xsc} = -1.56e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00000
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.08736

- Results for concrete section:



Height of compressed part of the section:

$h = 0.6000$

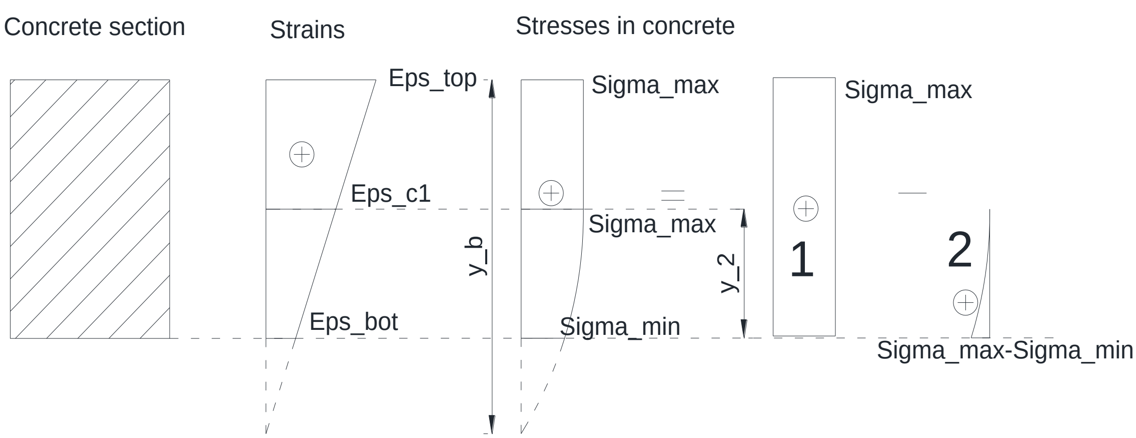
Area of contour under compressive stress:

$A_c = 0.3 \cdot 0.6000 = 0.1800$

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a power (n = 1.4) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6
Sigma_min = f_{cd} • (1-(1 - Eps_bot / Eps_c1)^n) = 9.946e6

Distance between gravity center of contour and neutral axis:
delta= -(0.6 / 2-y_b) = 0.4
Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

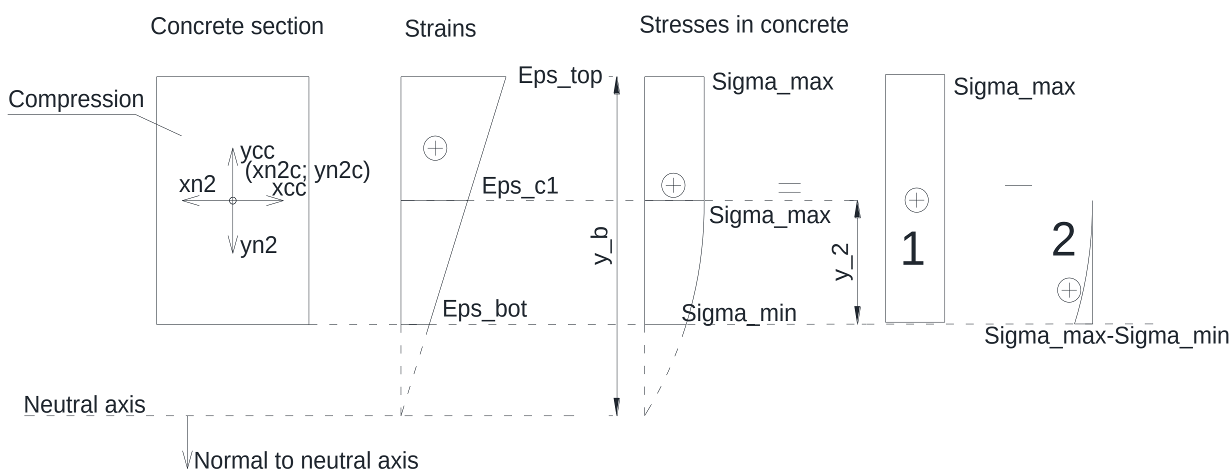
delta2 = - delta + MIN(y_b;Eps_c1 / Eps_top • y_b) = 0
Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:
y_2 = 0.3-delta2 = 0.3

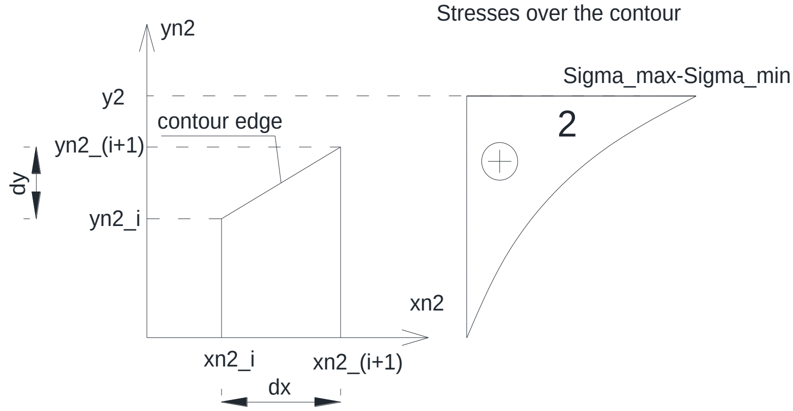
Axial force in compressed part of concrete gross section
$N_{c1} = A_c$ • Sigma_max = 5.400e6

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2}, y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}; y_{n2c}) given in central axis coordinate system.

(x_{n2c}; y_{n2c}) = (0; delta2)



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system: x	y	Coordinates in central axes system: x_{cc}	y_{cc}	Coordinates in (x_{n2}, y_{n2}) system: x_{n2}	y_{n2}	dx	dy	A	B	N_{c2}	Static moment of stress solid: $MS_{yn2,zn2}$	$MS_{xn2,zn2}$
1	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	-0,30	0,00	0,00	0,00	0,00	0,00
2	0,00	0,30	-0,15	0,00	0,15	0,00	-0,30	0,00	0,00	0,00	0,00	0,00	0,00
3	0,30	0,30	0,15	0,00	-0,15	0,00	0,00	0,30	0,00	0,00	0,00	0,00	0,00
4	0,30	0,00	0,15	-0,30	-0,15	0,30	0,30	0,00	0,00	0,30	0,75	0,00	0,16
5	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00
										N_{c2} = sum(N_{c2_i}) =	0,75		
								C=	0,40				
								D=	0,40				

gravity center of stress solid 2 in (x_{n2}; y_{n2}) coordinates system	x_{c2}'	0,0000
	y_{c2}'	0,2118
gravity center of stress solid 2 in central axes (x_{cc} ; y_{cc}) coordinates system	x_{c2}	0,0000
	y_{c2}	-0,2118

Where:
$x_{cc} = x - x_{Cc}$
$y_{cc} = y - y_{Cc}$
$x_{n2} = \cos(Angle - \pi / 2) \bullet (x_{cc} - x_{n2c}) + \sin(Angle - \pi / 2) \bullet (y_{cc} - y_{n2c})$
$y_{n2} = -\sin(Angle - \pi / 2) \bullet (x_{cc} - x_{n2c}) + \cos(Angle - \pi / 2) \bullet (y_{cc} - y_{n2c})$
dx = x_(i + 1) - x_i
dy = y_(i + 1) - y_i
A = dy/dx if dx ≠ 0 or A=0 if dx=0
B = -dy/dx • x_{n2}_(i + 1)+y_{n2}_(i + 1) if dx ≠ 0 or B = 0 if dx = 0
C = y_b • Eps_c1 / Eps_top
D = MIN(Eps_c1;Eps_top) • y_b / Eps_top
n = 1.4 (power of stress-strain relation curve)
dSigma =Sigma_max - Sigma_min
N_{c2}_i = 0 if A = 0 and B = 0 or

N_{c2}_i = f_{cd} • C / (n + 1) • (C / (n + 2) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - (1 - (-y_{n2}_i + D)/C)$^{n+2}$) - (x_{n2}_(i + 1) - x_{n2}_i) • (1 – D / C)$^{n+1}$) + (dSigma - f_{cd}) • (A / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B • (x_{n2}_(i + 1) - x_{n2}_i)) if A ≠ 0 or

N_{c2}_i = (f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$)) • dx + (-f_{cd} + dSigma) • B • dx if A = 0 and B ≠ 0

N_{c2} = sum(N_{c2_i})
$MS_{yn2,zn2}$_i = f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$) • 1 / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + (-f_{cd} + dSigma) • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) if A = 0 or

$MS_{yn2,zn2}$_i = f_{cd} • C^2 / ((n + 1) • (n + 2) • A) • (x_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ -x_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - (1 - (-y_{n2}_i + D) / C)$^{n+3}$)) - 0.5 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) • f_{cd} • C / (n + 1) • (1 – D / C)$^{n+1}$ + (-f_{cd} + dSigma) • (A / 3 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2)) if A ≠ 0

$MS_{yn2,zn2}$ = sum($MS_{yn2,zn2}$_i)
$MS_{xn2,zn2}$_i = (f_{cd} • C / (n + 1) • (B • (1-(-B + D) / C)$^{n+1}$ – C / (n + 2) • ((1 - (-B + D) / C)$^{n+2}$ - (1 – D / C)$^{n+2}$)) + (-f_{cd} + dSigma) / 2 • B^2) • (x_{n2}_(i + 1) - x_{n2}_i) if A = 0 or

$MS_{xn2,zn2}$_i = f_{cd} • C^2/((n + 1) • (n + 2) • A) • (y_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - y_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) • (2 • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - 2 • (1 - (-y_{n2}_i + D) / C)$^{n+3}$) + A • (1 – D / C)$^{n+2}$ • dx) + (dSigma - f_{cd}) • (A^2 / 6 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + A • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B^2 / 2 • dx) if A ≠ 0

$MS_{xn2,zn2}$= sum($MS_{xn2,zn2}$_i)
$x_{c2}' = MS_{yn2,zn2} / N_{c2}$
$y_{c2}' = MS_{xn2,zn2} / N_{c2}$
$x_{c2} = x_{n2c} + \cos(Angle - \pi / 2) \bullet x_{c2}' - \sin(Angle - \pi / 2) \bullet y_{c2}'$
$y_{c2} = y_{n2c} + \sin(Angle - \pi / 2) \bullet x_{c2}' + \cos(Angle - \pi / 2) \bullet y_{c2}'$

$N_{c2} = 0.752035e6$
$N_{c'} = N_{c1} - N_{c2} = 4.647965e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$
$y_{c1} = 0$

$x_{c2} = 0$
$y_{c2} = -0.211764706$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \bullet N_{c1} - x_{c2} \bullet N_{c2} - x_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.000000
$y_{cg} = (y_{c1} \bullet N_{c1} - y_{c2} \bullet N_{c2} - y_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.034058

Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{4630.08e3}$
$M_{xc} = - N_c \cdot y_{cg} = -\underline{157.69e3}$
$M_{yc} = N_{c'} \cdot x_{cg} = 0\underline{.00e3}$

Results for reduced section:
$N = N_c + N_s = 4\underline{851.25e3}$
$M_x = M_{xc} + M_{xs} = -\underline{182.83e3}$
$M_y = M_{yc}+M_{ys} = 0\underline{.00e3}$

7.4 Case 6b

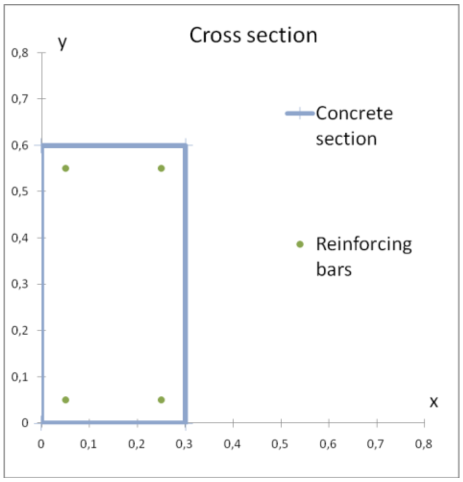
7.4.1 Data:

- Geometry:
  - Concrete section:

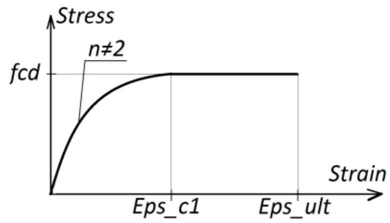
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

- Reinforcing bars:

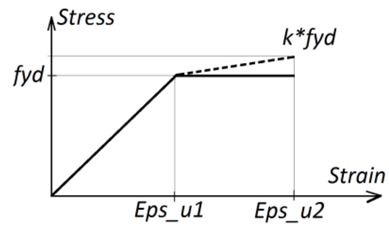
x = 0.05	y = 0.05	ϕ = 0.032
x = 0.25	y = 0.05	ϕ = 0.032
x = 0.25	y = 0.55	ϕ = 0.032
x = 0.05	y = 0.55	ϕ = 0.032



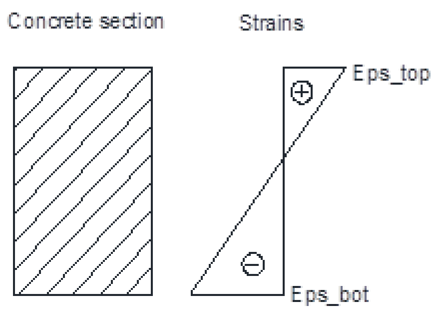
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: power-rectangular
  - Power: n = 1.8
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = -0.03034666
  - Neutral axis angle Angle = π • 3 / 2



7.4.2 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

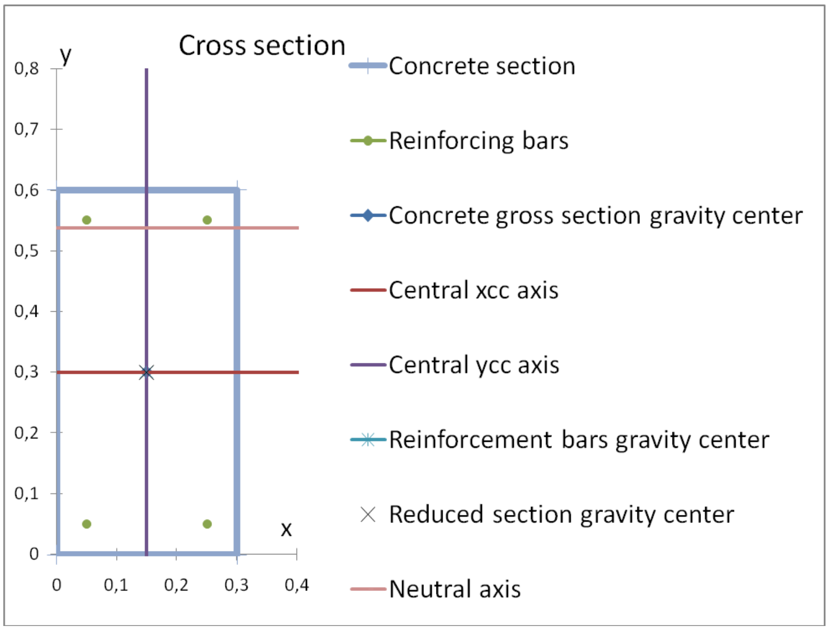
7.4.3 Results from Component

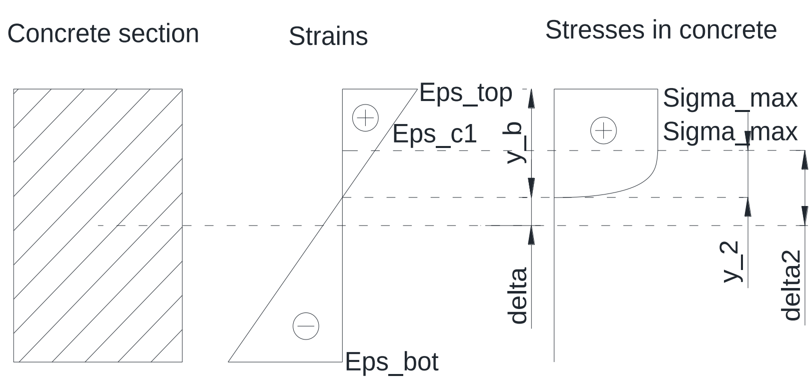
- Results for reinforcement bars:
  N_s = -420.68e3
  M_{xs} = -216.53e3
  M_{ys} = 0.00e3
- Results for concrete section:
  A_c = 0.0187
  x_{cg} = 0.00000000
  y_{cg} = 0.27592078

N_c = 420.68e3
M_{xc} = -116.08e3
M_{yc} = 0.00e3

- Results for reduced section:
  N = 0.00e3
  M_x = -332.60e3
  M_y = 0.00e3

7.4.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.0623
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:

$x_{Cc} = 0.15$
$y_{Cc} = 0.30$

Distance between reinforcing bar and neutral axis:

d = y - h + y_b

Stress in bar i:

If |eps_i| ≤ fyd / Es then
sigma_i = eps_i / Eps_u1 • f_{yd}
else
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,488	-0,02738	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02738	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00069	138	111358	0,10	0,25	-27839	11136
4	0,05	0,55	0,032	0,012	0,00069	138	111358	-0,10	0,25	-27839	-11136
						Sum of forces in bars:	-420682			-216528	0

N_s = -<u>420.68e3</u>
M_{xs} = -<u>216.53e3</u>
$M_{y\,s}$= 0<u>.00e3</u>

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if eps ≤ Eps_c1 then N_{sc} = f_{cd} • (1-(1-eps/Eps_c1)^n) • Area
if eps > Eps_c1 then N_{sc} = f_{cd} • Area

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,02738	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02738	0,10	-0,25				
3	0,25	0,55	0,032	0,00069	0,10	0,25	0,000804248	12898	-3224	1290
4	0,05	0,55	0,032	0,00069	-0,10	0,25	0,000804248	12898	-3224	-1290
							Sum of forces:	25796	-6449	0

Reduction forces in concrete:

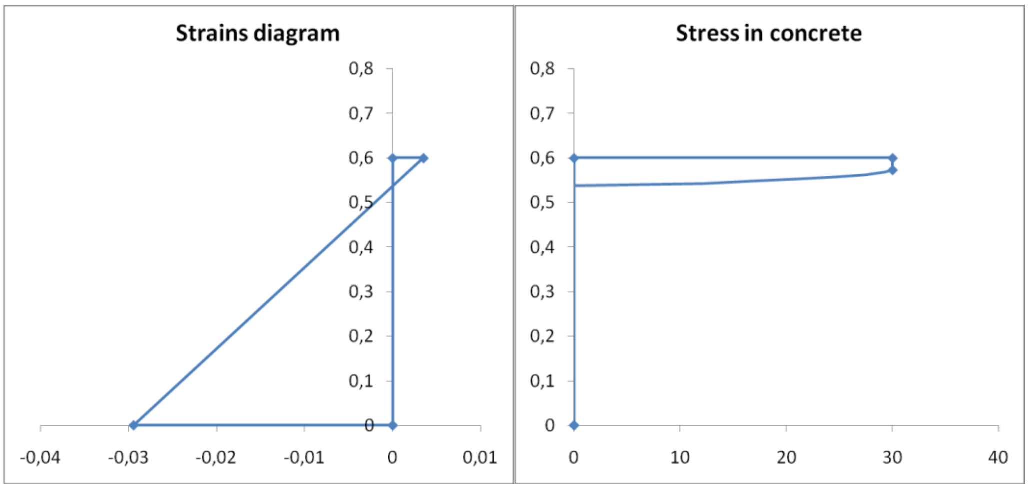
N_{sc} = 25.79e3
M_{xsc} = -6.45e3

M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.25

- Results for concrete section:



Height of compressed part of the section:

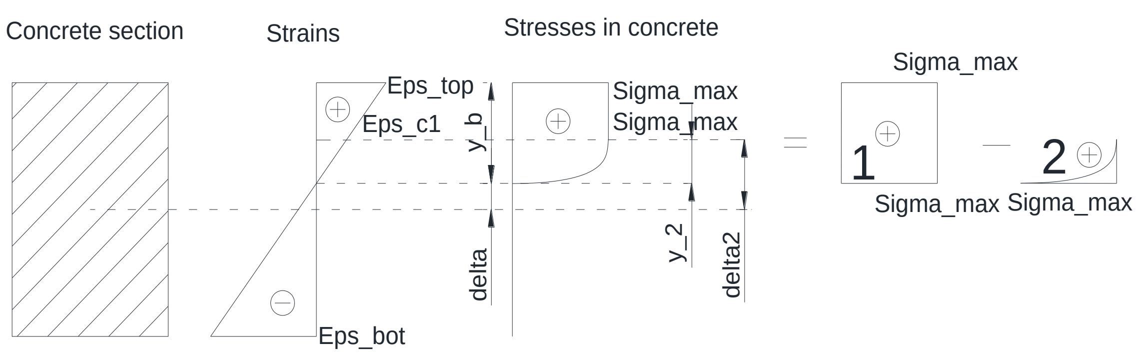
y_b = 0.0623

Area of contour under compressive stress:

A_c = 0.3 • y_b = 0.0187

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a power (n = 1.8) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 30.000e6
Sigma_min = 0.000e6

Distance between gravity center of contour and neutral axis:

delta = 0.6 / 2-y_b = 0.237671

Angle = 3 / 2 • π= 4.712

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.273288

Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

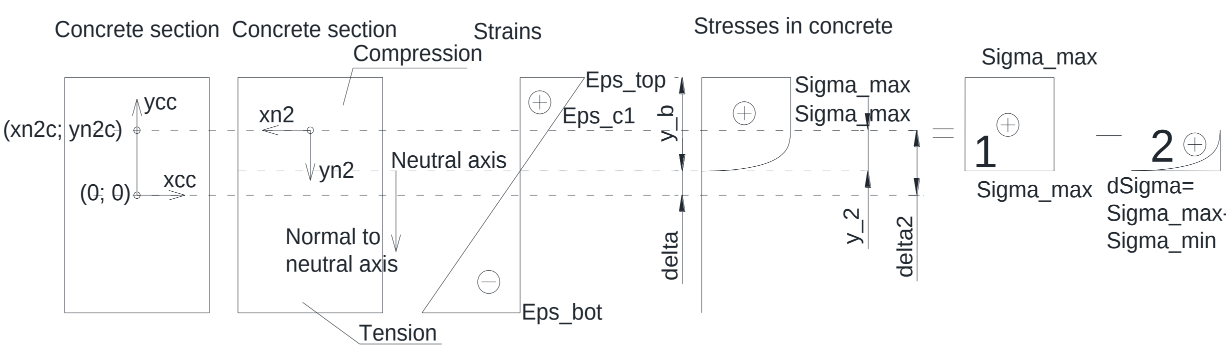
y_2 = delta2 - delta = 0.0356165

Axial force in compressed part of concrete gross section

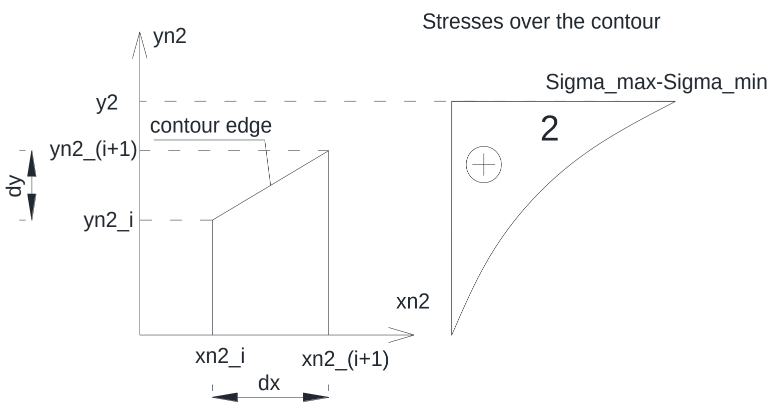
$N_{c1} = A_c$ • Sigma_max = 0.560960e6

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2}, y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}; y_{n2c}) given in central axis coordinate system.

(x_{n2c}; y_{n2c}) = (0; delta2)



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system x	y	Coordinates in central axes system x_{cc}	y_{cc}	Coordinates in (x_{n2}, y_{n2}) system x_{n2}	y_{n2}	dx	dy	A	B	N_{c2}	Static moment of stress solid $MS_{yn2,zn2}$	$MS_{xn2,zn2}$
1	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,04	0,00	0,00	0,000	0,000	0,000
2	0,30	0,54	0,15	0,24	-0,15	0,04	0,30	0,00	0,00	0,04	0,114	0,000	0,003
3	0,00	0,54	-0,15	0,24	0,15	0,04	0,00	-0,04	0,00	0,00	0,000	0,000	0,000
4	0,00	0,57	-0,15	0,27	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000
5	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000
										N_{c2} = sum(N_{c2}_i) =	0,114		
								C=	0,036				
								D=	0,036				

gravity center of stress solid 2 in (x_{n2}; y_{n2}) coordinates system	x_{c2}'	0,0000
	y_{c2}'	0,0262
gravity center of stress solid 2 in central axes (x_{cc}; y_{cc}) coordinates system	x_{c2}	0,0000
	y_{c2}	0,2470

Where:
$x_{cc} = x - x_{Cc}$
$y_{cc} = y - y_{Cc}$
$x_{n2} = \cos(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$
$y_{n2} = -\sin(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$
dx = x_(i + 1) - x_i
dy = y_(i + 1) - y_i
A = dy/dx if dx ≠ 0 or A=0 if dx=0
B = -dy/dx • x_{n2}_(i + 1)+y_{n2}_(i + 1) if dx ≠ 0 or B = 0 if dx = 0
C = y_b • Eps_c1 / Eps_top
D = MIN(Eps_c1;Eps_top) • y_b / Eps_top
n = 1.8 (power of stress-strain relation curve)
dSigma =Sigma_max - Sigma_min
N_{c2}_i = 0 if A = 0 and B = 0 or

N_{c2}_i = f_{cd} • C / (n + 1) • (C / (n + 2) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - (1 - (-y_{n2}_i + D)/C)$^{n+2}$) - (x_{n2}_(i + 1) - x_{n2}_i) • (1 – D / C)$^{n+1}$) + (dSigma - f_{cd}) • (A / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B • (x_{n2}_(i + 1) - x_{n2}_i)) if A ≠ 0 or

N_{c2}_i = (f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$)) • dx + (-f_{cd} + dSigma) • B • dx if A = 0 and B ≠ 0

N_{c2} = sum(N_{c2_i})
$MS_{yn2,zn2}$_i = f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$) • 1 / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + (-f_{cd} + dSigma) • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) if A = 0 or

$MS_{yn2,zn2}$_i = f_{cd} • C^2 / ((n + 1) • (n + 2) • A) • (x_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ -x_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - (1 - (-y_{n2}_i + D) / C)$^{n+3}$)) - 0.5 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) • f_{cd} • C / (n + 1) • (1 – D / C)$^{n+1}$ + (-f_{cd} + dSigma) • (A / 3 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2)) if A ≠ 0

$MS_{yn2,zn2}$ = sum($MS_{yn2,zn2}$_i)
$MS_{xn2,zn2}$_i = (f_{cd} • C / (n + 1) • (B • (1-(-B + D) / C)$^{n+1}$ – C / (n + 2) • ((1 - (-B + D) / C)$^{n+2}$ - (1 – D / C)$^{n+2}$)) + (-f_{cd} + dSigma) / 2 • B^2) • (x_{n2}_(i + 1) - x_{n2}_i) if A = 0 or

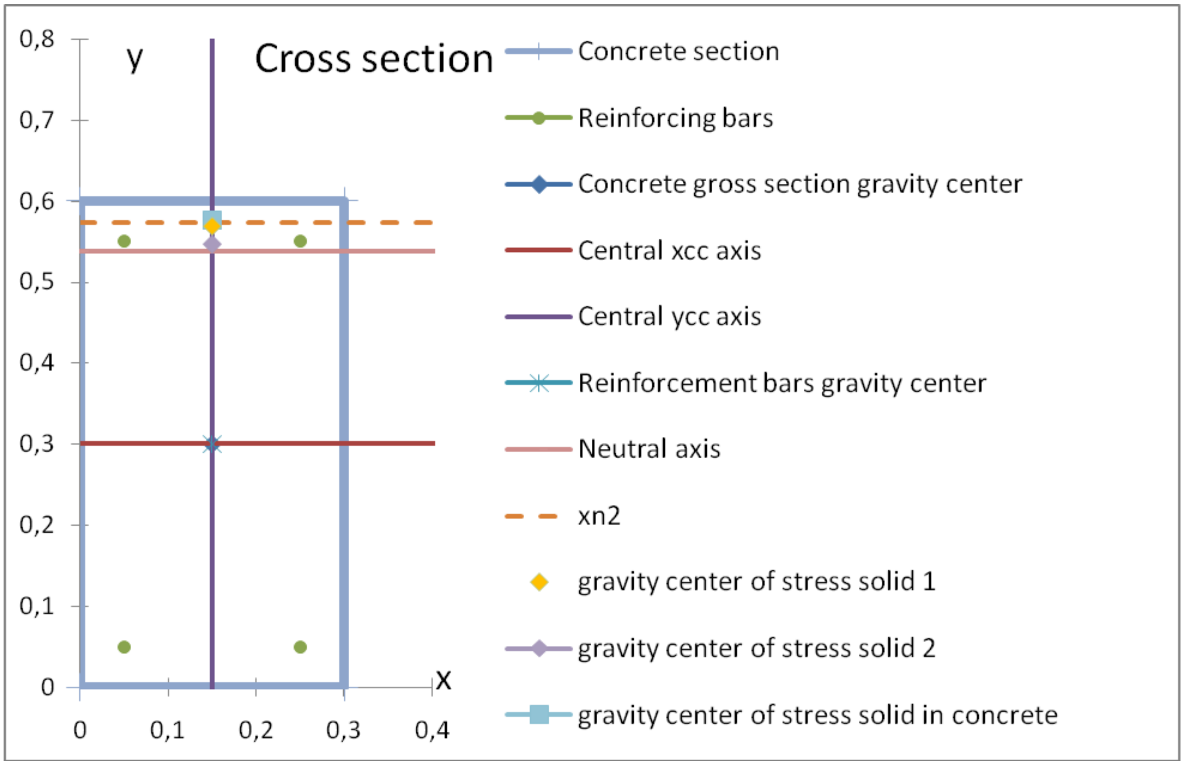
$MS_{xn2,zn2}$_i = f_{cd} • C^2/((n + 1) • (n + 2) • A) • (y_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - y_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) • (2 • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - 2 • (1 - (-y_{n2}_i + D) / C)$^{n+3}$) + A • (1 – D / C)$^{n+2}$ • dx) + (dSigma - f_{cd}) • (A^2 / 6 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + A • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B^2 / 2 • dx) if A ≠ 0

$MS_{xn2,zn2}$ = sum($MS_{xn2,zn2}$_i)
$x_{c2}' = MS_{yn2,zn2} / N_{c2}$
$y_{c2}' = MS_{xn2,zn2} / N_{c2}$
$x_{c2} = x_{n2c} + \cos(Angle - \pi / 2) \cdot x_{c2}' - \sin(Angle - \pi / 2) \cdot y_{c2}'$
$y_{c2} = y_{n2c} + \sin(Angle - \pi / 2) \cdot x_{c2}' + \cos(Angle - \pi / 2) \cdot y_{c2}'$

N_{c2} = 0.114482e6
$N_{c'} = N_{c1} - N_{c2}$ = 0.446478e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:
x_{c1} = 0
y_{c1} = delta + y_b / 2= 0.268836
x_{c2} = 0
y_{c2} = 0.247043919

Center of gravity of stresses in concrete in central axes coordinate system:
$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.000000
$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.275921



Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{420.68e3}$

$M_{xc} = - N_c \bullet y_{cg} = -\underline{116.08e3}$

$M_{yc} = N_{c'} \bullet x_{cg} = 0\underline{.00e3}$

- Results for reduced section:

  $N = N_c + N_s = 0\underline{.00e3}$

  $M_x = M_{xc} + M_{xs} = -\underline{332.60e3}$

  $M_y = M_{yc}+M_{ys} = M_{yc}+M_{ys} = 0\underline{.00e3}$

7.5 Case 6c

7.5.1 Data:

- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0

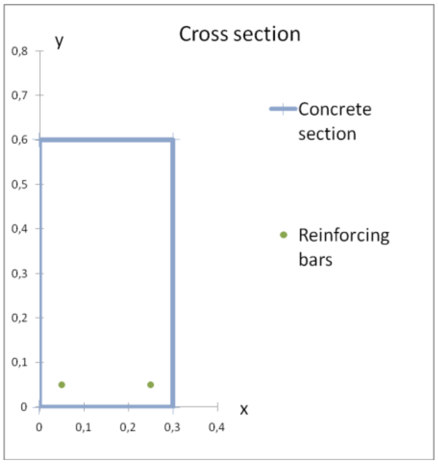
    x = 0.0 y = 0.6

    x = 0.3 y = 0.6

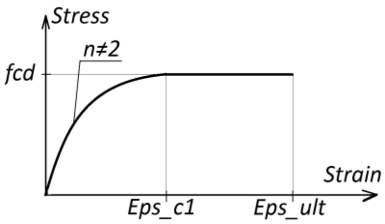
    x = 0.3 y = 0.0
  - Reinforcing bars:

    x = 0.05 y = 0.05 ϕ = 0.032

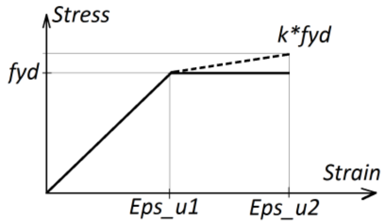
    x = 0.25 y = 0.05 ϕ = 0.032



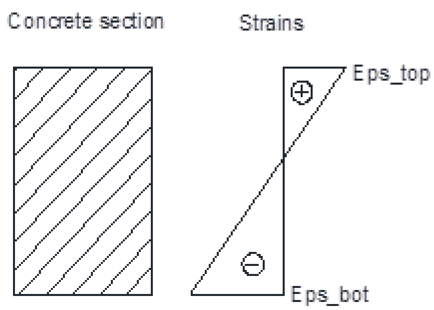
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: power-rectangular
  - Power: n = 1.2
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Top strain Eps_top = 0.0015
  - Bottom strain Eps_bot = -0.002
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



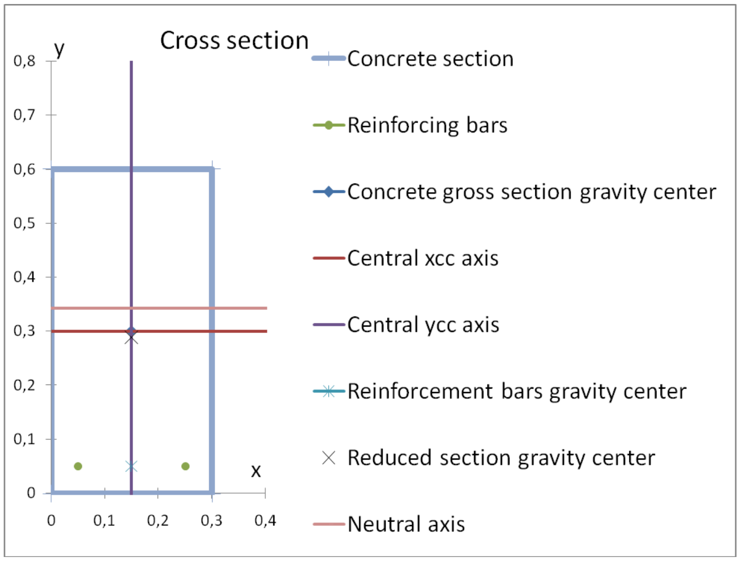
7.5.2 Search for:

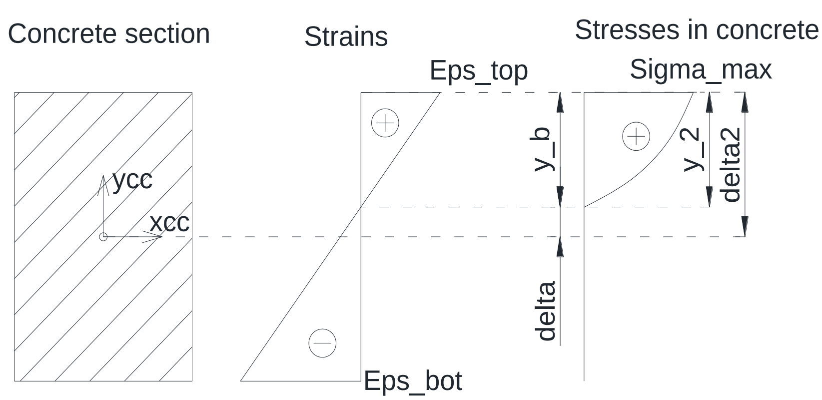
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

7.5.3 Results from Component

- Results for reinforcement bars:
  N_s = -549.57e3
  M_{xs} = -137.39e3
  M_{ys} = 0.00e3
- Results for concrete section:
  A_c = 0.0771
  x_{cg} = 0.000000
  y_{cg} = 0.21270852
  N_c = 978.12e3
  M_{xc} = -208.06e3
  M_{yc} = 0.00e3
- Results for reduced section:
  N = 428.55e3
  M_x = -345.45e3
  M_y = 0.00e3

7.5.4 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  - height of a section: h = 0.6
  - y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.257143
  - Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  - x_{Cc} = 0.15
  - y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
  - d = y - h + y_b

  Stress in bar i:
  - If |eps_i| ≤ f_{yd} / E_s then
    - sigma_i = eps_i / Eps_u1 • f_{yd}

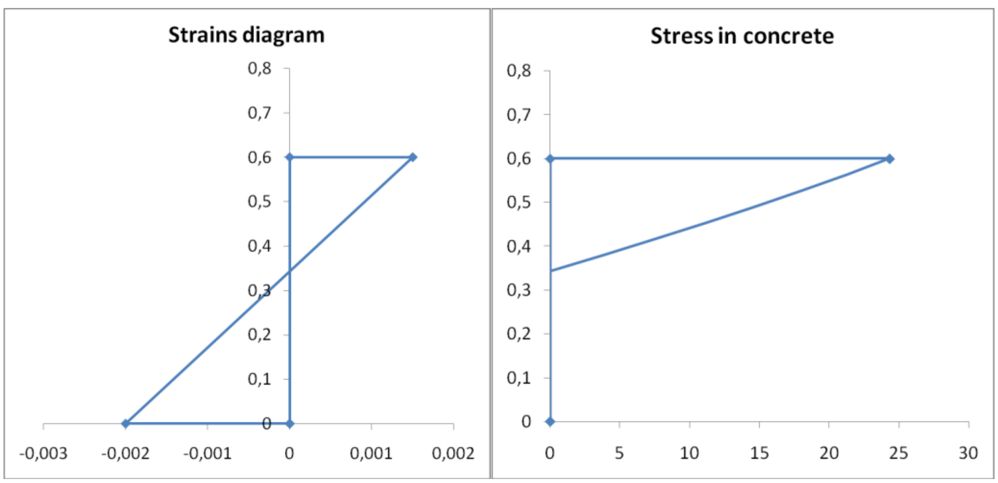
else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
						Sum of forces in bars:	-549569			-137392	0

N_s = -549.569e3
M_{xs} = -137.392e3
M_{ys} = 0.00e3

- Results for concrete section:



Height of compressed part of the section:

y_b = 0.257143

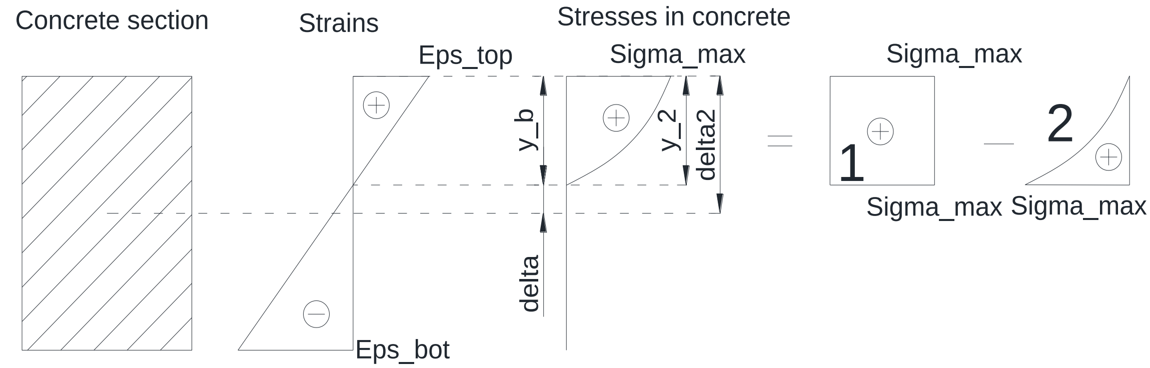
Area of contour under compressive stress:

A_c = 0.3 • y_b = 0.0771

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a power (n = 1.2) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \bullet N_{c1} - x_{c2} \bullet N_{c2}) / (N_{c1} - N_{c2})$

$y_{cg} = (y_{c1} \bullet N_{c1} - y_{c2} \bullet N_{c2}) / (N_{c1} - N_{c2})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \bullet y_{cg}$

$M_{yc} = N_c \bullet x_{cg}$

Sigma_max = $f_{cd} \bullet (1-(1-\text{Eps\_top}/\text{Eps\_c1})^{1,2})$ = 24.316e6

Sigma_min = 0.000e6

Distance between gravity center of contour and neutral axis:

delta = 0.6 / 2 - y_b = 0.042857

Angle = 3 / 2 • π = 4.712

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.3000

Angle = 3 / 2 • π = 4.712

Height of the part of the section where concrete stress is not constant:

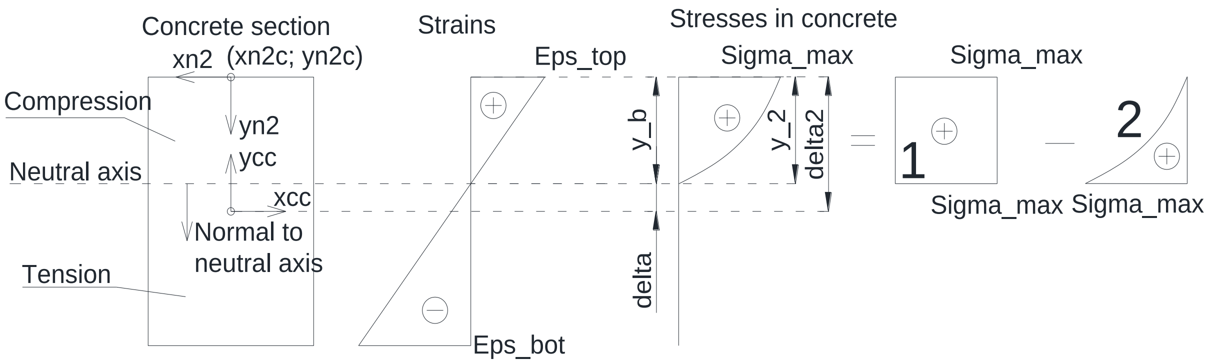
y_2 = delta2 – delta = 0.257143

Axial force in compressed part of concrete gross section

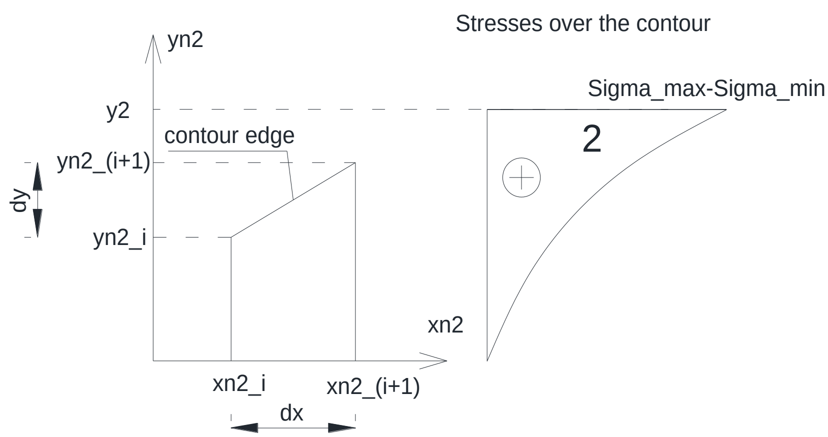
$N_{c1} = A_c \bullet$ Sigma_max = 1.875811e6

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2}, y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.

$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (x_{n2}, y_{n2}) system							Static moment of stress solid	
i	x	y	x_{cc}	y_{cc}	x_{n2}	y_{n2}	dx	dy	A	B	N_{c2}	$MS_{yn2,zn2}$	$MS_{xn2,zn2}$
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,000	0,000	0,000
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,898	0,000	0,156
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,000	0,000	0,000
4	0,00	0,60	-0,15	0,30	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000
5	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000
										N_{c2} = sum(N_{c2}_i) =	0,898		
								C=	0,343				
								D=	0,257				
gravity center of stress solid 2 in (x_{n2}; y_{n2}) coordinates system												x_{c2}'	0,0000
												y_{c2}'	0,1736
gravity center of stress solid 2 in central axes (x_{cc} ; y_{cc}) coordinates system												x_{c2}	0,0000
												y_{c2}	0,1264

Where:

$x_{cc} = x - x_{Cc}$

$y_{cc} = y - y_{Cc}$

$x_{n2} = \cos(Angle - \pi / 2) \bullet (x_{cc} - x_{n2c}) + \sin(Angle - \pi / 2) \bullet (y_{cc} - y_{n2c})$

$y_{n2} = -\sin(Angle - \pi / 2) \bullet (x_{cc} - x_{n2c}) + \cos(Angle - \pi / 2) \bullet (y_{cc} - y_{n2c})$

dx = x_(i + 1) - x_i

dy = y_(i + 1) - y_i

A = dy/dx if dx ≠ 0 or A=0 if dx=0

B = -dy/dx • x_{n2}_(i + 1)+y_{n2}_(i + 1) if dx ≠ 0 or B = 0 if dx = 0

C = y_b • Eps_c1 / Eps_top

D = MIN(Eps_c1;Eps_top) • y_b / Eps_top

n = 1.2 (power of stress-strain relation curve)

dSigma =Sigma_max - Sigma_min

N_{c2}_i = 0 if A = 0 and B = 0 or

N_{c2}_i = f_{cd} • C / (n + 1) • (C / (n + 2) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - (1 - (-y_{n2}_i + D)/C)$^{n+2}$) - (x_{n2}_(i + 1) - x_{n2}_i) • (1 – D / C)$^{n+1}$) + (dSigma - f_{cd}) • (A / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B • (x_{n2}_(i + 1) - x_{n2}_i)) if A ≠ 0 or

N_{c2}_i = (f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$)) • dx + (-f_{cd} + dSigma) • B • dx if A = 0 and B ≠ 0

N_{c2} = sum(N_{c2_i})

$MS_{yn2,zn2}$_i = f_{cd} • C / (n + 1) • ((1 - (-B + D) / C)$^{n+1}$ - (1 – D / C)$^{n+1}$) • 1 / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + (-f_{cd} + dSigma) • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) if A = 0 or

$MS_{yn2,zn2}$_i = f_{cd} • C^2 / ((n + 1) • (n + 2) • A) • (x_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ -x_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) / A • ((1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - (1 - (-y_{n2}_i + D) / C)$^{n+3}$)) - 0.5 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) • f_{cd} • C / (n + 1) • (1 – D / C)$^{n+1}$ + (-f_{cd} + dSigma) • (A / 3 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2)) if A ≠ 0

$MS_{yn2,zn2}$ = sum($MS_{yn2,zn2}$_i)

$MS_{xn2,zn2}$_i = (f_{cd} • C / (n + 1) • (B • (1-(-B + D) / C)$^{n+1}$ – C / (n + 2) • ((1 - (-B + D) / C)$^{n+2}$ - (1 – D / C)$^{n+2}$)) + (-f_{cd} + dSigma) / 2 • B^2) • (x_{n2}_(i + 1) - x_{n2}_i) if A = 0 or

$MS_{xn2,zn2}$_i = f_{cd} • C^2/((n + 1) • (n + 2) • A) • (y_{n2}_(i + 1) • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+2}$ - y_{n2}_i • (1 - (-y_{n2}_i + D) / C)$^{n+2}$-C / (n + 3) • (2 • (1 - (-y_{n2}_(i + 1) + D) / C)$^{n+3}$ - 2 • (1 - (-y_{n2}_i + D) / C)$^{n+3}$) + A • (1 – D / C)$^{n+2}$ • dx) + (dSigma - f_{cd}) • (A^2 / 6 • (x_{n2}_(i + 1)^3 - x_{n2}_i^3) + A • B / 2 • (x_{n2}_(i + 1)^2 - x_{n2}_i^2) + B^2 / 2 • dx) if A ≠ 0

$MS_{xn2,zn2}$ = sum($MS_{xn2,zn2}$_i)

$x_{c2}' = MS_{yn2,zn2} / N_{c2}$

$y_{c2}' = MS_{xn2,zn2} / N_{c2}$

$x_{c2} = x_{n2c} + \cos(Angle - \pi / 2) \bullet x_{c2}' - \sin(Angle - \pi / 2) \bullet y_{c2}'$

$y_{c2} = y_{n2c} + \sin(Angle - \pi / 2) \bullet x_{c2}' + \cos(Angle - \pi / 2) \bullet y_{c2}'$

$N_{c2} = 0.897687e6$

$N_{c'} = N_{c1} - N_{c2} = 0.978124e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$

y_{c1} = delta + y_b / 2= 0.171429

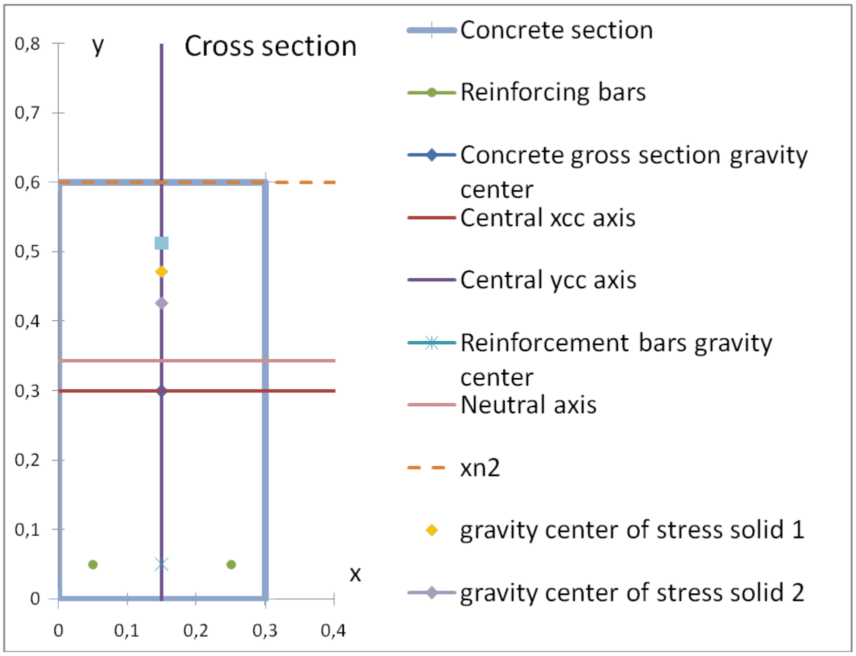
$x_{c2} = 0$

$y_{c2} = 0.126450$

Center of gravity of stresses in concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \bullet N_{c1} - x_{c2} \bullet N_{c2} - x_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.\underline{000000}$

$y_{cg} = (y_{c1} \bullet N_{c1} - y_{c2} \bullet N_{c2} - y_{scg} \bullet N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.\underline{212709}$



Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{978.12e3}$

$M_{xc} = - N_c \cdot y_{cg} = -\underline{208.06e3}$

$M_{yc} = N_{c'} \cdot x_{cg} = 0\underline{.00e3}$

- Results for reduced section:

  $N = N_c + N_s = 4\underline{28.55e3}$

  $M_x = M_{xc} + M_{xs} = -\underline{345.45e3}$

  $M_y = M_{yc}+M_{ys} = 0\underline{.00e3}$

8 Case 7 Calculation of internal forces for given state of strain in the section and inclined branch model of steel

8.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

8.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with asymmetric reinforcement
- A linear model of concrete and inclined branch model of steel
- A section under unidirectional bending with a part of section under compression
- A state of strain with ultimate compressive strain in concrete

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force (N)
    - Moment with respect to x_{cc} axis (M_x)
    - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

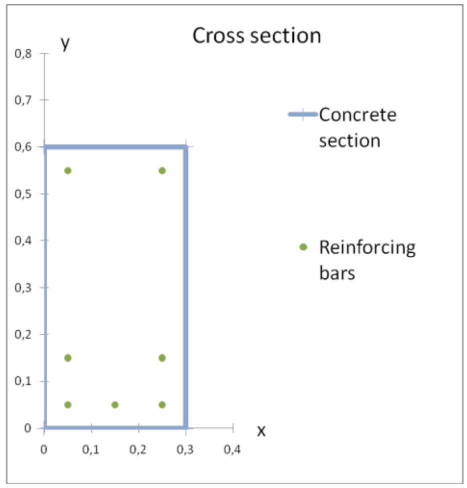
8.3 Data:

- Geometry:
    - Concrete section:

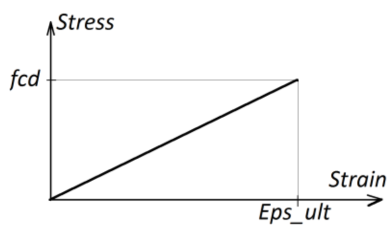
      x = 0.0 y = 0.0
      x = 0.0 y = 0.6
      x = 0.3 y = 0.6
      x = 0.3 y = 0.0
    - Reinforcing bars:

      x = 0.05 y = 0.05 ϕ = 0.032
      x = 0.15 y = 0.05 ϕ = 0.032
      x = 0.25 y = 0.05 ϕ = 0.032

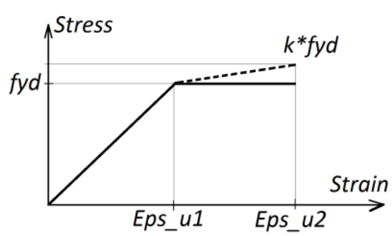
x = 0.05	y = 0.15	ϕ = 0.020
x = 0.25	y = 0.15	ϕ = 0.020
x = 0.05	y = 0.55	ϕ = 0.012
x = 0.25	y = 0.55	ϕ = 0.012



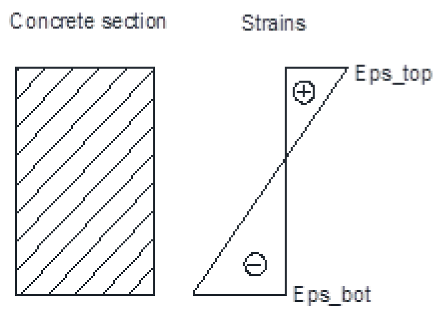
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: linear
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.05
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Top strain Eps_top = 0.0035
  - Bottom strain Eps_bot = -0.0070
  - Neutral axis angle Angle = $\pi \cdot 3 / 2$



8.4 Search for:

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

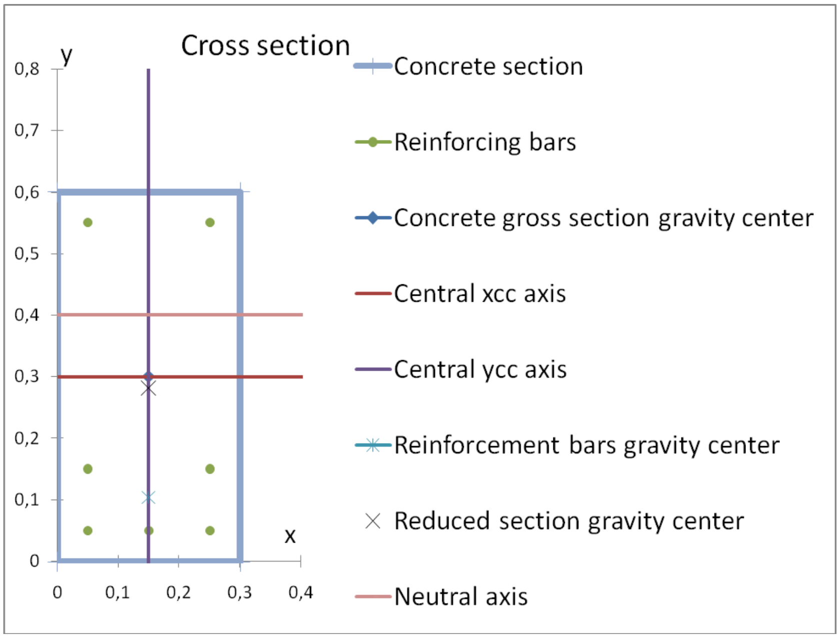
8.5 Results from Component

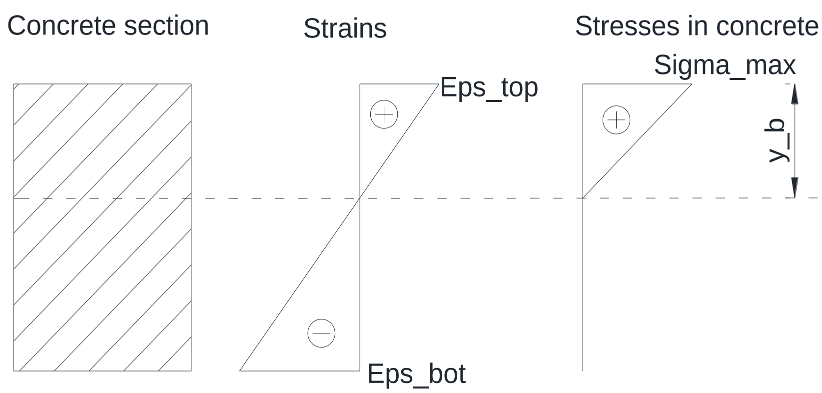
- Results for reinforcement bars:
  N_s = -1410.85e3
  M_{xs} = -377.81e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.06
  x_{cg} = 0.00000000
  y_{cg} = 0.23323855
  N_c = 596.61e3
  M_{xc} = -139.15e3
  M_{yc} = 0.00e3

- Results for reduced section:
  N = -814.24e3
  M_x = -516.96e3
  M_y = 0.00e3

8.6 Results (manual calculations):





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  - height of a section: h = 0.6
  - y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.2000
  - Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  - x_{Cc} = 0.15
  - y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
  - d = y - h + y_b

  Stress in bar i:
  - If |eps_i| ≤ fyd / Es then
    - sigma_i = eps_i / Eps_u1 • f_{yd}

else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,350	-0,00613	-501.25	-403129	-0,10	-0,25	-100782	40313
2	0,15	0,05	0,032	-0,350	-0,00613	-501.25	-403129	0,00	-0,25	-100782	0
3	0,25	0,05	0,032	-0,350	-0,00613	-501.25	-403129	0,10	-0,25	-100782	-40313
4	0,05	0,15	0,02	-0,250	-0,00438	-500.65	-157283	-0,10	-0,15	-23592	15728
5	0,25	0,15	0,02	-0,250	-0,00438	-500.65	-157283	0,10	-0,15	-23592	-15728
6	0,05	0,55	0,012	0,150	0,00263	500.04	56554	-0,10	0,25	-14138	-5655
7	0,25	0,55	0,012	0,150	0,00263	500.04	56554	0,10	0,25	-14138	5655
						Sum of forces in bars:	-1410846			-377808	0

N_s = -1410.85e3
M_{xs} = -377.81e3
M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • eps / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,032	-0,00613	-0,10	-0,25				
2	0,15	0,05	0,032	-0,00613	0,00	-0,25				
3	0,25	0,05	0,032	-0,00613	0,10	-0,25				
4	0,05	0,15	0,02	-0,00438	-0,10	-0,15				
5	0,25	0,15	0,02	-0,00438	0,10	-0,15				
6	0,05	0,55	0,012	0,00263	-0,10	0,25	0,000113097	1696	-424	-170
7	0,25	0,55	0,012	0,00263	0,10	0,25	0,000113097	1696	-424	170
							Sum of forces :	3393	-848	0

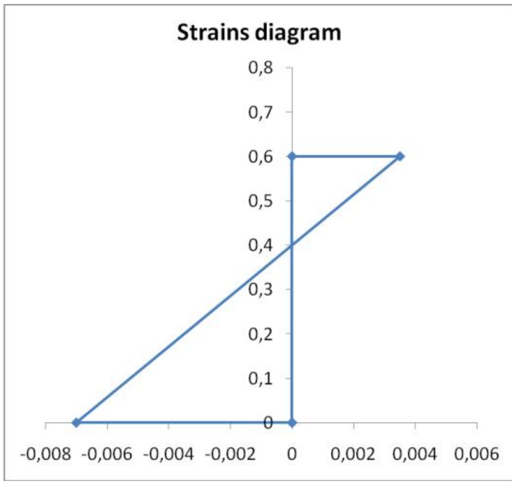
Reduction forces in concrete:

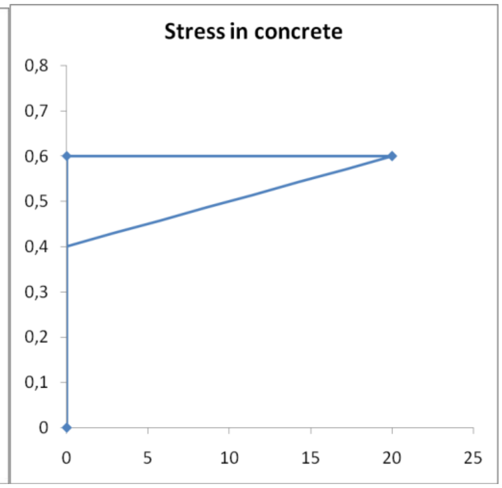
N_{sc} = 3.39e3
M_{xsc} = -0.85e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.25

- Results for concrete section:





Height of compressed part of the section:

$y\_b = 0.20$

Area of contour under compressive stress:

$A_c = 0.3 \cdot y\_b = 0.0600$

Axial force in compressed part of concrete gross section

$Sigma\_max = f_{cd} \cdot Eps\_top / Eps\_ult = 20.000e6$

$N_{c'} = A_c \cdot Sigma\_max/2 = 0.6000e6$

Center of gravity of stress solid in central axes coordinate system:

$x_c' = 0$

$y_c' = 0.3 - y\_b \cdot 2/3 = 0.23658249$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_c' \cdot N_{c'} - x_{scg} \cdot N_{sc}) / (N_{c'} - N_{sc}) = 0.000000$

$y_{cg} = (y_c' \cdot Nc1 - y_{scg} \cdot N_{sc}) / (N_{c'} - N_{sc}) = 0.233239$

Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = 596.61e3$

$M_{xc} = - N_c \cdot y_{cg} = -139.15e3$

$M_{yc} = N_{c'} \cdot x_{cg} = 0.00e3$

- Results for reduced section:

$N = N_c + N_s = -814.24e3$

$M_x = M_{xc} + M_{xs} = -516.96e3$

$M_y = M_{yc} + M_{ys} = 0.00e3$

9 Case 8: Calculation of internal forces in asymmetric section for given state of strain with horizontal neutral axis

9.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

9.2 Summary:

This sample gives as results the internal forces for:

- An asymmetric concrete shape section with asymmetric reinforcement
- A rectangular model of concrete and horizontal branch model of steel
- A state of strain with ultimate compressive strain in concrete and horizontal neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force (N)
  - Moment with respect to x_{cc} axis (M_x)
  - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

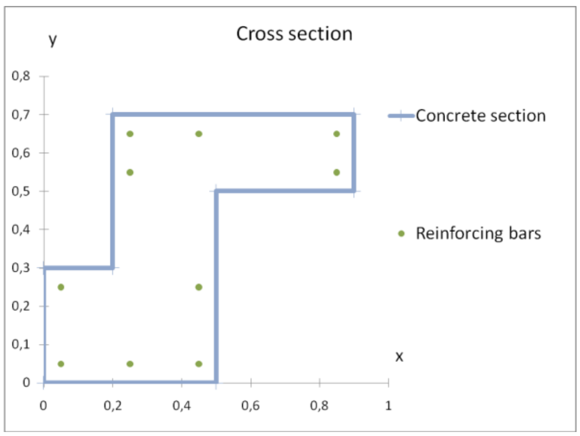
9.3 Data:

- Geometry:
  - Concrete section:

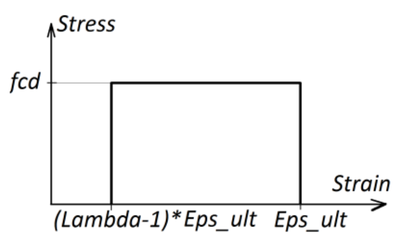
x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.2	y = 0.3
x = 0.2	y = 0.7
x = 0.9	y = 0.7
x = 0.9	y = 0.5
x = 0.5	y = 0.5
x = 0.5	y = 0.0

- Reinforcing bars:

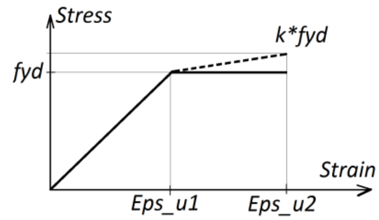
x = 0.05	y = 0.05	ϕ = 0.025
x = 0.45	y = 0.05	ϕ = 0.025
x = 0.85	y = 0.55	ϕ = 0.012
x = 0.85	y = 0.65	ϕ = 0.012
x = 0.45	y = 0.65	ϕ = 0.012
x = 0.25	y = 0.65	ϕ = 0.012
x = 0.25	y = 0.05	ϕ = 0.025
x = 0.05	y = 0.25	ϕ = 0.012
x = 0.45	y = 0.25	ϕ = 0.012
x = 0.25	y = 0.55	ϕ = 0.012



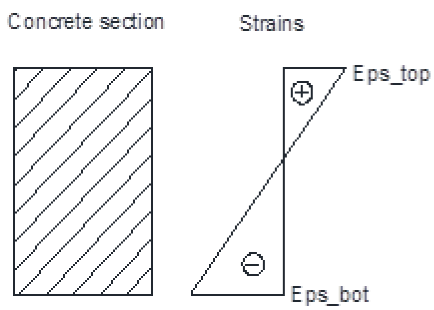
- Concrete parameters:
  - Design strength f_{cd} = 50e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 37e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Strains:
  - Bottom strain Eps_bot = -0.00875
  - Top strain Eps_top = 0.00350
  - Neutral axis angle Angle = π • 3 / 2



9.4 Search for:

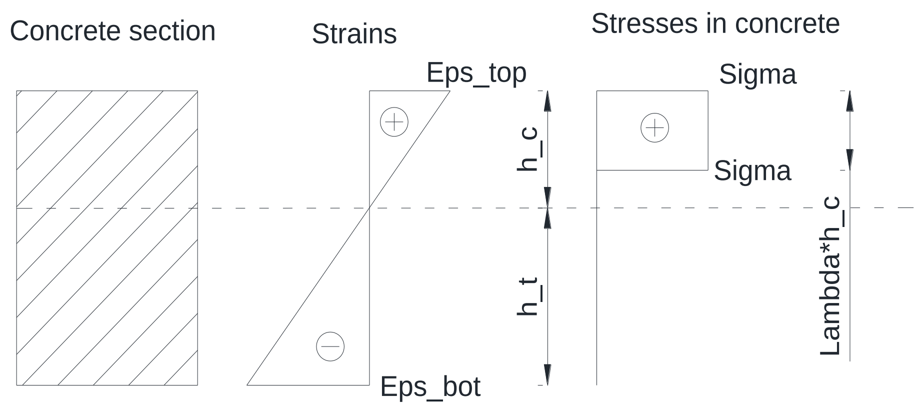
- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

9.5 Results from Component

- Results for reinforcement bars:
  N_s = -640.18e3
  M_{xs} = -305.65e3
  M_{ys} = 144.91e3
- Results for concrete section:
  A_c = 0.1120
  x_{cg} = 0.16295864
  y_{cg} = 0.24719360
  N_c = 5571.73e3
  M_{xc} = -1377.29e3
  M_{yc} = 907.96e3
- Results for reduced section:
  N = 4931.55e3
  M_x = -1682.94e3
  M_y = 1052.87e3

9.6 Results (manual calculations):

Position of neutral axis



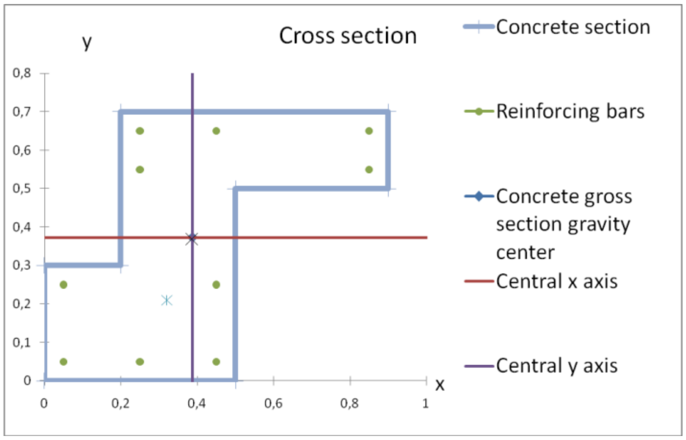
Concrete gross section gravity center calculations:

i	Corners of concrete section x	y	Area _c	Gravity center of contour corresponding to Area_c x_{01}	y_{01}	Static moments S_{cy}	S_{cx}
1	0	0	0	0,00	0,15	0	0
2	0	0,3	0,060	0,10	0,15	0,006	0,009
3	0,2	0,3	0	0,20	0,50	0	0
4	0,2	0,7	0,490	0,55	0,35	0,2695	0,1715
5	0,9	0,7	0	0,90	0,60	0	0
6	0,9	0,5	-0,20	0,70	0,25	-0,14	-0,05
7	0,5	0,5	0	0,50	0,25	0	0
8	0,5	0	0	0,25	0,00	0	0
		Total:	0,350			0,1355	0,1305

Where:
edge_i = edge between corners: i and i + 1
Area_c_i = Area between edge_i and axes: x = x_i, x = x_(i + 1) and y = 0
Area_c_i = 1/2 • (y_(i + 1) + y(i)) • (x_(i + 1) - x(i))
x_{01}_i = x_(i) + (x_(i + 1) - x_(i)) • (2 / 3 • y_(i + 1) + 1 / 3 • y_(i)) / (y_(i) + y_(i + 1)) if y_(i) + y_(i + 1) ≠ 0 or
x_{01} _i = (x_(i + 1) + x_(i)) / 2 if y_(i) + y_(i + 1) = 0
y_{01}_i = y_(i) / 2 + 0.5 • (x_{01}_i - x_(i)) • (y_(i + 1) - y_(i)) / (x_(i + 1) - x_(i)) if x_(i + 1) - x_(i) ≠ 0 or
y_{01}_i = (y_(i + 1) + y_(i)) / 2) if x_(i + 1) - x_(i) = 0
S_{cy}_i = Area_c_i • x_{01}_i
S_{cx}_i = Area_c_i • y_{01}_i

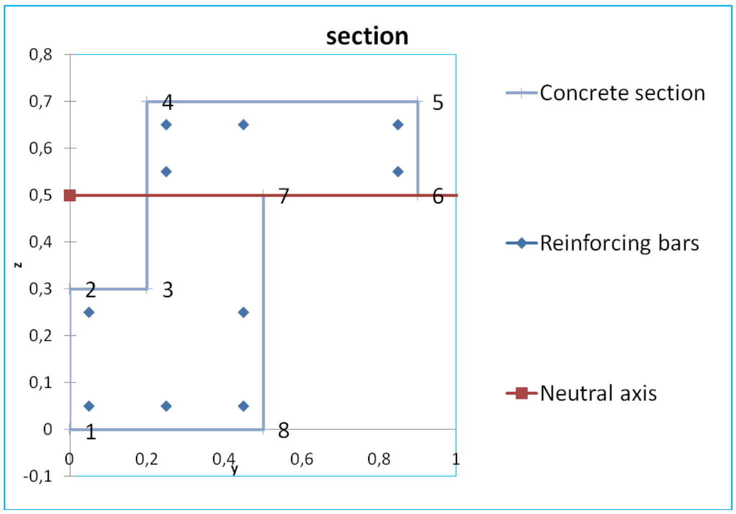
Concrete gross section area:
A_c = sum(Area_c_i) = 0.350

Concrete gross section gravity center
x_{Cc} = sum(S_{cy}_i) / A_c = 0.3871
y_{Cc} = Sum(S_{cx}_i) / A_c = 0.3729



Equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})

y_{cc} = 0.127143 (e.g. obtained in iteration process)



General equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})

$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$

A = 0

B = -0.450

C = 0.057214

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$dist = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

For compressed corner:

$dist = -(A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

i	Corners of concrete section x	y	Corners of concrete section in central axes coordinate system x_{cc}	y_{cc}	Corner under compression/tension	Distance between corner and neutral axis dist
1	0	0	-0,387	-0,373	tension	0,500
2	0	0,3	-0,387	-0,073	tension	0,200
3	0,2	0,3	-0,187	-0,073	tension	0,200
4	0,2	0,7	-0,187	0,327	compression	-0,200
5	0,9	0,7	0,513	0,327	compression	-0,200
6	0,9	0,5	0,513	0,127	compression	0,000
7	0,5	0,5	0,113	0,127	compression	0,000
8	0,5	0	0,113	-0,373	tension	0,500

Where:
$x_{cc} = x - x_{Cc}$
$y_{cc} = y - y_{Cc}$

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

h_c = |min(dist_i)| = 0.2

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

h_t = max(dist_i) = 0.5

Verification of used equation of neutral axis:

Eps_bot = -h_t/h_c • Eps_top = 0.00875 (OK)
and
neutral axis angle = $\pi + \pi / 2 = 3 / 2 \cdot \pi$ = Angle (OK)

- Results for reinforcement bars:

  Eps_u1 = f_{yd} / E_s

Distance between reinforcing bar and neutral axis:

For tensioned rebar:
$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$
For compressed rebar:
$d = -(A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then
sigma_i = eps_i / Eps_u1 • f_{yd}
else
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Distance d from neutral axis	strain eps = d/h_c • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,025	-0,34	-0,32	0,450	-0,00788	-500	-245437	-79241	82747
2	0,45	0,05	0,025	0,06	-0,32	0,450	-0,00788	-500	-245437	-79241	-15427
3	0,85	0,55	0,012	0,46	0,18	-0,050	0,00088	175	19792	-3506	9161
4	0,85	0,65	0,012	0,46	0,28	-0,150	0,00263	500	56549	-15672	26174
5	0,45	0,65	0,012	0,06	0,28	-0,150	0,00263	500	56549	-15672	3554
6	0,25	0,65	0,012	-0,14	0,28	-0,150	0,00263	500	56549	-15672	-7755
7	0,25	0,05	0,025	-0,14	-0,32	0,450	-0,00788	-500	-245437	-79241	33660
8	0,05	0,25	0,012	-0,34	-0,12	0,250	-0,00438	-500	-56549	-6947	19065
9	0,45	0,25	0,012	0,06	-0,12	0,250	-0,00438	-500	-56549	-6947	-3554
10	0,25	0,55	0,012	-0,14	0,18	-0,050	0,00088	175	19792	-3506	-2714
								Sum of forces in bars:	-640178	-305646	144910

N_s = -640.18e3
M_{xs} = -305.65e3
M_{ys} = 144.91e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • Eps_top / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,025	-0,34	-0,32				
2	0,45	0,05	0,025	0,06	-0,32				
3	0,85	0,55	0,012	0,46	0,18	0,000113097	5655	-1002	2617
4	0,85	0,65	0,012	0,46	0,28	0,000113097	5655	-1567	2617
5	0,45	0,65	0,012	0,06	0,28	0,000113097	5655	-1567	355
6	0,25	0,65	0,012	-0,14	0,28	0,000113097	5655	-1567	-776
7	0,25	0,05	0,025	-0,14	-0,32				
8	0,05	0,25	0,012	-0,34	-0,12				
9	0,45	0,25	0,012	0,06	-0,12				
10	0,25	0,55	0,012	-0,14	0,18	0,000113097	5655	-1002	-776
					Total:	0,000565487	28274	-6705	4039

Reduction forces in concrete:
N_{sc} = 28.27e3
M_{xsc} = -6.71e3

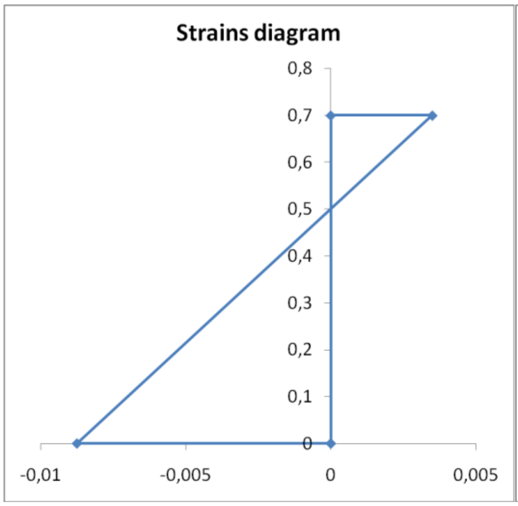
$M_{ysc} = 4.04e3$

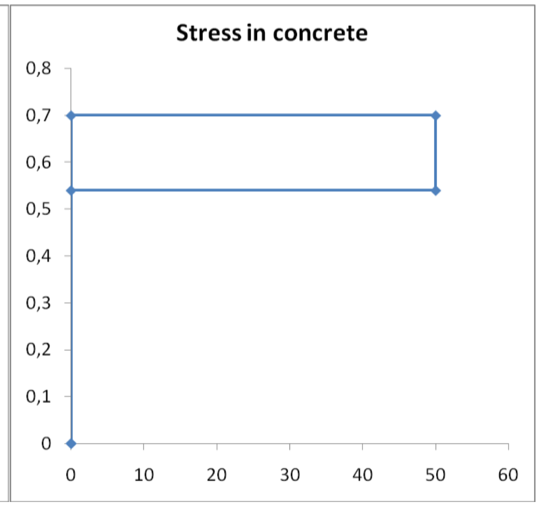
Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$x_{scg} = sum(x_{cc}\_i \cdot Area\_i) / sum(Area\_i) = 0.1429$

$y_{scg} = sum(y_{cc}\_i \cdot Area\_i) / sum(Area\_i) = 0.2371$

- Results for concrete section:





Height of compressed part of the section:

Lambda • h_c= 0.1600

Area of contour under compressive stress:

$A_c = 0.7 \cdot 0.16 = 0.1120$

Stress in concrete:

$Sigma = Eps\_top / Eps\_ult \cdot f_{cd} = 50e6$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = ((0.2 + 0.7 / 2 - x_{cc}) \cdot A_c \cdot Sigma - x_{scg} \cdot N_{sc}) / (A_c \cdot Sigma + N_{sc}) = 0.162959$

$y_{cg} = ((0.7 - 0.2 / 2 - y_{cc}) \cdot A_c \cdot Sigma - y_{scg} \cdot N_{sc}) / (A_c \cdot Sigma + N_{sc}) = 0.247194$

Internal forces in concrete:

$N_c = A_c \cdot Sigma - N_{sc} = 5571.73e3$

$M_{xc} = - N_c \cdot y_{cg} = -1377.29e3$

$M_{yc} = N_c \cdot x_{cg} = 907.96e3$

- Results for reduced section:

$N = N_c + N_s = 4931.55e3$

$M_x = M_{xc} + M_{xs} = -1682.94e3$

$M_y = M_{yc} + M_{ys} = 1052.87e3$

10 Case 9: Calculation of internal forces in symmetric section for given state of strain with inclined neutral axis

10.1 *Subject:*

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

10.2 *Summary:*

This sample gives as results the internal forces for:

- A symmetric concrete shape section with symmetric reinforcement
- A bilinear model of concrete and horizontal branch model of steel
- A state of strain with compressive strain below the ultimate strain in concrete and inclined neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force (N)
  - Moment with respect to x_{cc} axis (M_x)
  - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

10.3 *Data:*

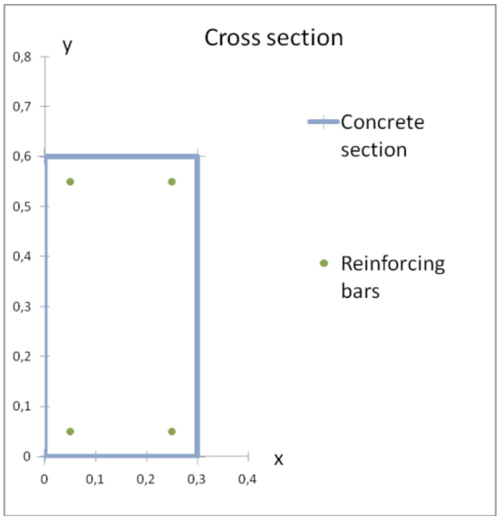
- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

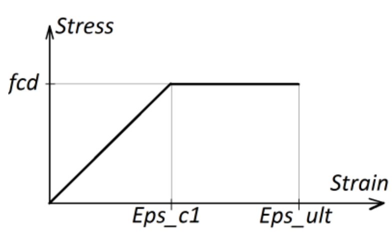
  - Reinforcing bars:

    x = 0.05 y = 0.05 ϕ = 0.012

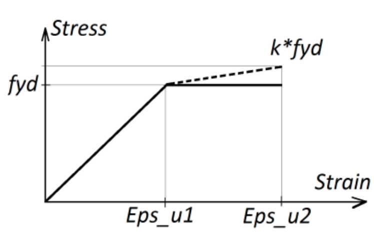
x = 0.25 y = 0.05 ϕ = 0.012
x = 0.25 y = 0.55 ϕ = 0.012
x = 0.05 y = 0.55 ϕ = 0.012



- Concrete parameters:
  - Design strength f_{cd} = 25e6
  - Modulus of elasticity E_c = 28e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0030
  - Strain relation change over Eps_c1 = 0.0020

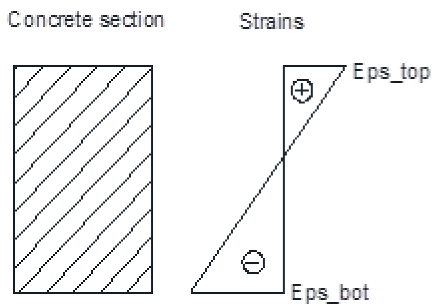


- Steel parameters:
  - Design strength f_{yd} = 550e6
  - Hardening factor k= 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Strains:
  - Bottom strain Eps_bot = -0.00383423
  - Top strain Eps_top = 0.0025

- Neutral axis angle Angle = 0.34906585



10.4 Search for:

- Results for reinforcement bars: Ns, M_{xs}, M_{ys}
- Results for concrete section: A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}
- Results for reduced section: N, M_x, M_y

10.5 Results from Component

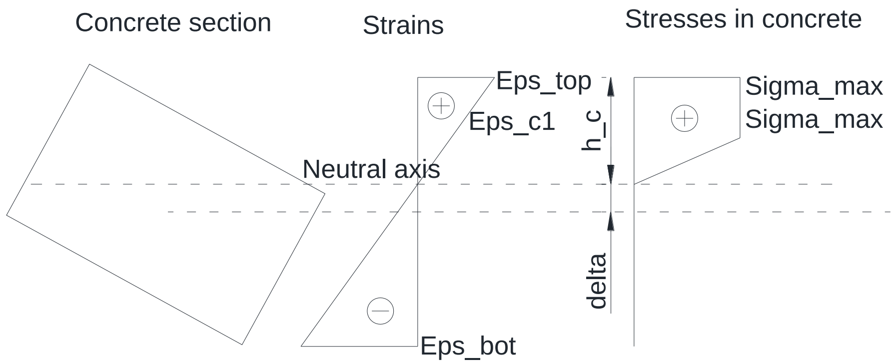
- Results for reinforcement bars:
  N_s = -54.68e3
  M_{xs} = 23.73e3
  M_{ys} = -10.49e3

- Results for concrete section:
  A_c = 0.0575
  x_{cg}= - 0.09851523
  y_{cg} = -0.15819891
  N_c = 591.85e3
  M_{xc} = 93.63e3
  M_{yc} = -58.31e3

- Results for reduced section:
  N = 537.17e3
  M_x = 117.36e3
  M_y = -68.79e3

10.6 Results (manual calculations):

Position of neutral axis



Concrete gross section gravity center

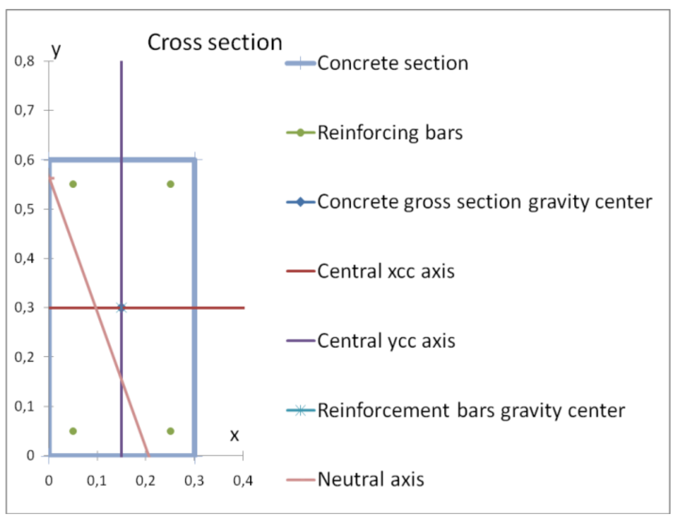
$x_{Cc} = 0.3 / 2 = 0.15$

$y_{Cc} = 0.6 / 2 = 0.30$

Equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})

Tangens(Angle - π / 2) = -2.74748

$y_{cc} = - 2.74748 \cdot x_{cc} - 0.15$ (e.g. obtained in iteration process)



General equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})

$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$

A = -0.412

B = -0.150

C = -0.0225

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$dist = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0,5}$

For compressed corner:

$dist = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0,5}$

i	Corners of concrete section x	y	Corners of concrete section in central axes coordinate system x_{cc}	y_{cc}	Corner under compression/tension	Distance between corner and neutral axis dist
1	0	0	-0,150	-0,300	compression	-0,19225691
2	0	0,6	-0,150	0,300	tension	0,01295517
3	0,3	0,6	0,150	0,300	tension	0,29486296
4	0,3	0,0	0,150	-0,300	tension	0,08965087

Where:
$x_{cc} = x - x_{Cc}$
$y_{cc} = y - y_{Cc}$

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

h_c = |min(dist_i)| = 0.19225691

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

h_t = max(dist_i) = 0.29486296

Verification of used equation of neutral axis:

Eps_bot = -h_t/h_c • Eps_top = -0.00383423 (OK)

and

neutral axis angle = arctan(-2.74748)+ π/2= 0.34906585= Angle (OK)

- Results for reinforcement bars:

  Eps_u1 = f_{yd} / E_s

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

For compressed rebar:

$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then

sigma_i = eps_i / Eps_u1 • f_{yd}

else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Distance d from neutral axis	strain eps = d/h_c • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,012	-0,10	-0,25	0,128	0,00167	333	37699	9425	-3770
2	0,25	0,05	0,012	0,10	-0,25	-0,060	-0,00078	-155	-17579	-4395	-1758

3	0,25	0,55	0,012	0,10	0,25	-0,231	-0,00300	-550	-62204	15551	-6220
4	0,05	0,55	0,012	-0,10	0,25	-0,043	-0,00056	-111	-12600	3150	1260
							Sum of forces in bars:		-54684	23731	-10488

N_s = -54.68e3
M_{xs} = 23.73e3
M_{ys} = -10.49e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if eps ≤ Eps_c1 $N_{sc} = f_{cd}$ • eps / Eps_c1 • Area
if eps > Eps_c1 $N_{sc} = f_{cd}$ • Area

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,012	0,00167	-0,10	-0,25	0,000113097	2356	589	-236
2	0,25	0,05	0,012	-0,00078	0,10	-0,25				
3	0,25	0,55	0,012	-0,00300	0,10	0,25				
4	0,05	0,55	0,012	-0,00056	-0,10	0,25				
					Total:		0,000113097	2356	589	-236

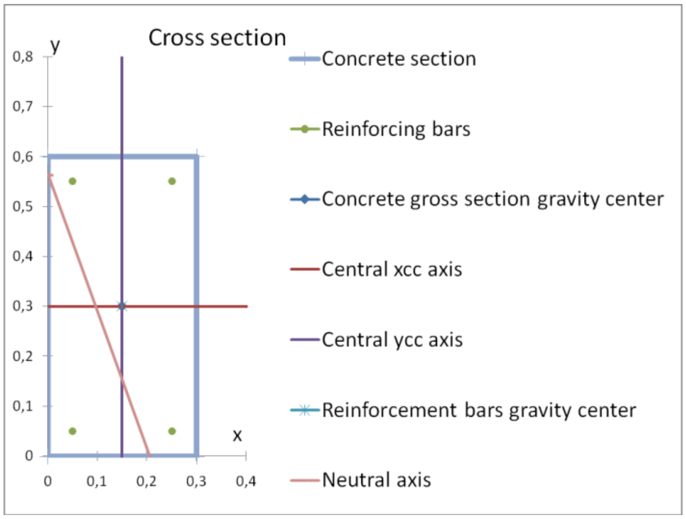
Reduction forces in concrete:

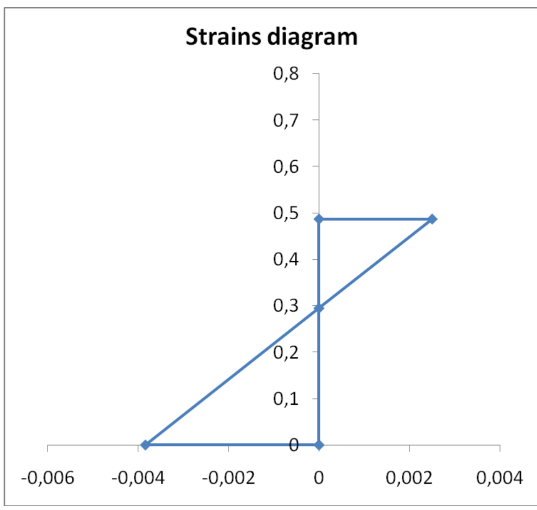
N_{sc} = 2.36e3
M_{xsc} = 0.59e3
M_{ysc} = -0.24e3

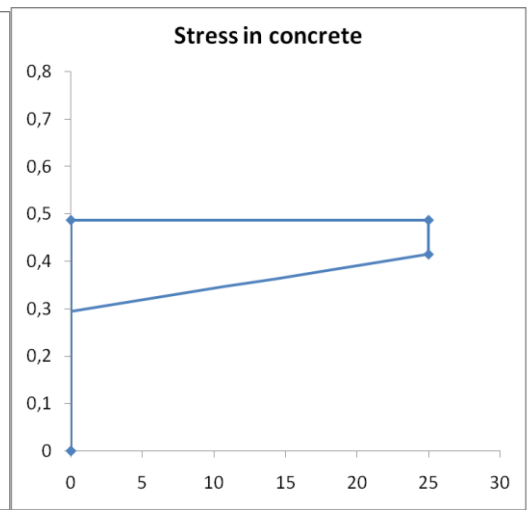
Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = -0.1000
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = -0.2500

- Results for concrete section:







Calculations of area of contour under compressive stress:

Calculations of points of intersection of neutral axis and the contour of concrete section:

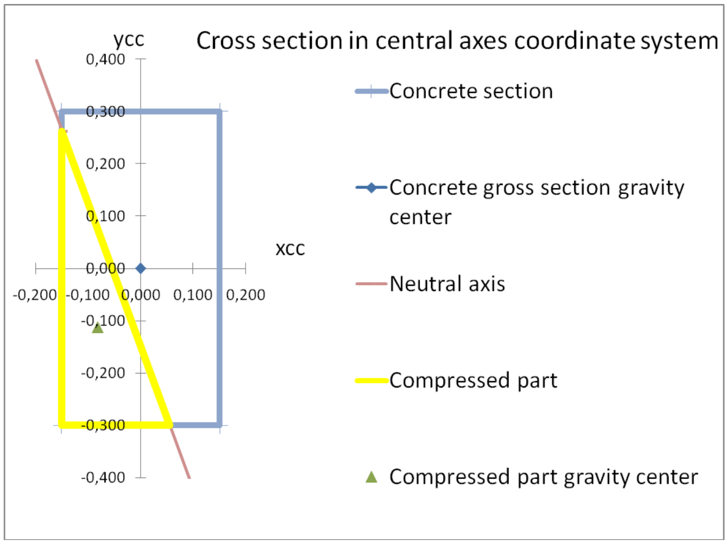
i	Corners of concrete section x	y	Corners of concrete section in central axes coordinate system x_{cc}	y_{cc}	Equation of line containing edge_i	Intersection point of line containing edge_i and neutral axis x_{cc}	y_{cc}	Is intersection on the contour?
1	0	0	-0,150	-0,300	x_{cc} = -0,15	-0,15	0,2621	Yes
2	0	0,6	-0,150	0,300	y_{cc} = 0,3	-0,1638	0,3	No
3	0,3	0,6	0,150	0,300	x_{cc} = 0,15	0,15	-0,5621	No
4	0,3	0,0	0,150	-0,300	y_{cc} = -0,3	0,0546	-0,3	Yes

Where:
x_{cc} =x- x_{Cc}
y_{cc} =y- y_{Cc}
edge_i = edge between corners: i and i+1

New points created by intersections of lines containing edges and neutral axis and lying on the contour create an edge of contour of compressed part of the section.

i	Contour of compressed part of a section in central axes coordinate system x_{cc}	y_{cc}	Contour of compressed part of a section in x y coordinate system x	y	Area_c	Gravity center of contour corresponding to Area_c in (x; y) system x_{01}	y_{01}	Static moments in (x; y) system S_{cy}	S_{cx}
1	-0,150	-0,300	0,000	0,000	0	0,000	0,281	0,000000	0,000000
2	-0,150	0,262	0,000	0,562	0,0575	0,068	0,187	0,003922	0,010775
3	0,055	-0,300	0,205	0,000	0,0000	0,102	0,000	0,000000	0,000000
4	-0,150	-0,300	0	0	0	0,000	0,281	0,000000	0,000000
				Total:	0,0575			x1=	y1=
								-0,0818	-0,1126

Where:
x= x_{cc} + x_{Cc}
y= y_{cc} + y_{Cc}
edge_i = edge between corners: i and i + 1
Area_c_i = Area between edge_i and axes: x=x_i, x=x_(i + 1) and y = 0
Area_c_i = 1/2 • (y_(i + 1) + y(i)) • (x_(i + 1) - x(i))
x_{01} _i = x_(i) + (x_(i + 1) - x_(i)) • (2 / 3 • y_(i + 1) + 1 / 3 • y_(i)) / (y_(i) + y_(i + 1)) if (y_(i) + y_(i + 1) ≠ 0 or
x_{01} _i = (x_(i + 1) + x_(i)) / 2 if (y_(i) + y_(i + 1) = 0
y_{01}_i = y_(i) / 2 + 0.5 • (x_{01}_i - x_(i)) • (y_(i + 1) - y_(i)) / (x_(i + 1) - x_(i)) if x_(i + 1) - x_(i) ≠ 0 or
y_{01}_i = (y_(i + 1) + y_(i))/2) if x_(i + 1) - x_(i) = 0
S_{cy} _i = Area_c_i • x_{01}_i
S_{cx} _i = Area_c_i • y_{01}_i
x1 = Sum(S_{cy}) / sum(Area_c) - x_{Cc}
y1 = sum(S_{cx}) / sum(Area_c) - y_{Cc}



Compressed concrete area:

A_c = sum(Area_c_i) = <u>0.0575</u>

Gravity center of compressed concrete contour (gross) in central axes coordinate system

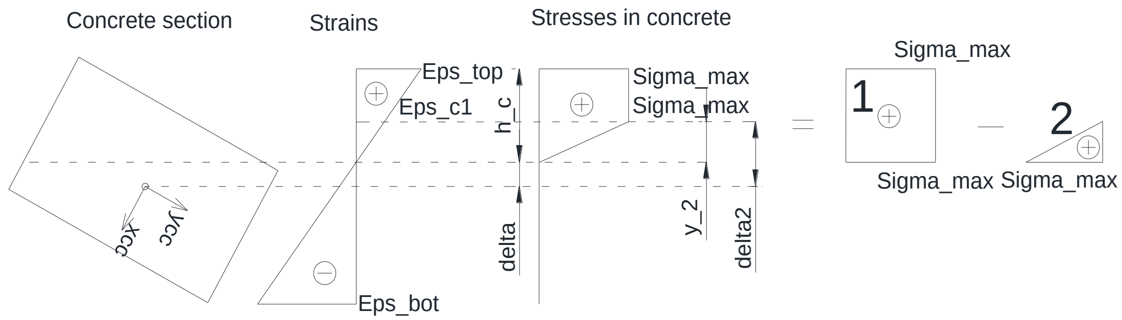
x_{c1} = sum(S_{cy}_i)/ A_c - x_{Cc} = -0.0818

y_{c1} = sum(S_{cx}_i)/ A_c - y_{Cc} = -0.1126

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$

$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} = 25.000e6

Distance between gravity center of contour and neutral axis:

delta = $(A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

for $x_{cc} = y_{cc} = 0$ and coefficients A, B, C as defined in paragraph: Position of neutral axis, above

delta = 0.0513

Distance between the gravity center of contour of the section and:
- the axis with strains equal Eps_c1 if Eps_top ≥ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

delta2 = delta + MIN (y_b; Eps_c1 / Eps_top • y_b) = 0.171352

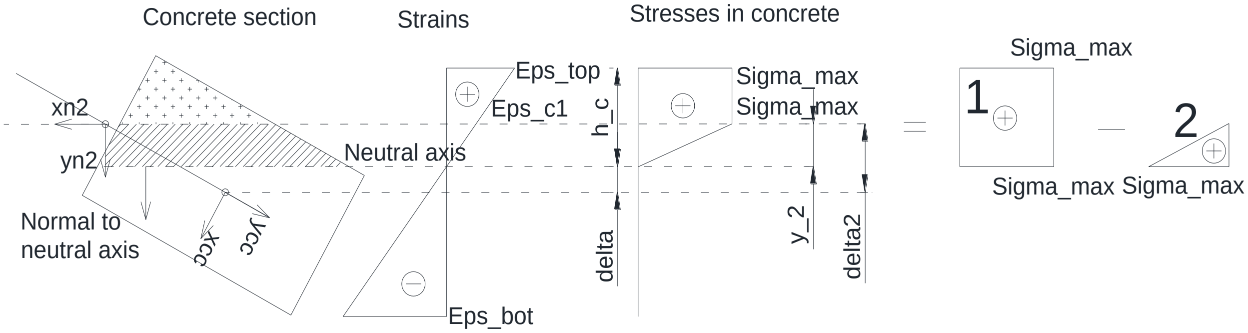
Height of the part of the section where concrete stress is not constant:

y_2 = delta2 - delta = 0.120049

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \cdot$ Sigma_max = 1.437595e6

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture above. To do this we define a new coordinate system x_{n2}, y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}; y_{n2c}) given in central axis coordinate system.



Equation of neutral axis in central axes coordinate system:

$y_{cc} = -2.74748 \cdot x_{cc} - 0.15$

Equation of x_{n2} axis in central axes coordinate system

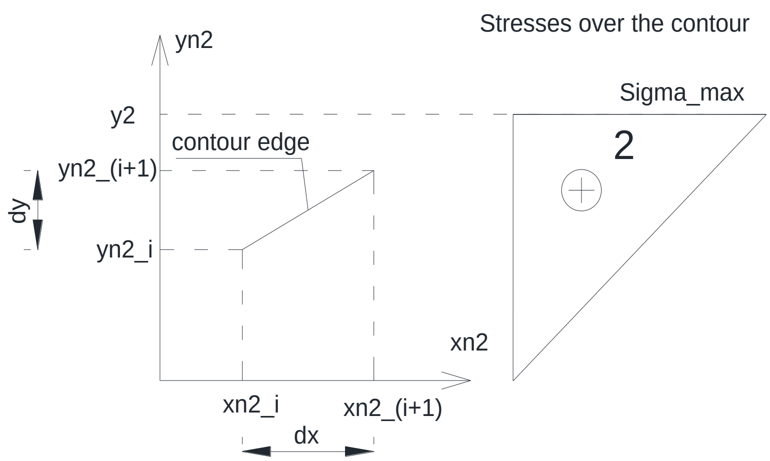
$y_{nc} = -2.74748 \cdot x_{nc}$ - 0.15-y_2/sin(Angle) = $-2.74748 \cdot x_{nc} - 0.5010$

For $x_{n2c} = 0$ $y_{n2c} = -0.5010$

Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.

i	Contour of compressed part of a section x	y	Contour of compressed part of a section in central axes coordinate system x_{cc}	y_{cc}	Equation of line containing edge_i, in central axes coordinate system	Intersection point of line containing edge_i and x_{n2} axis x_{cc}	y_{cc}	Is intersection on the contour?
1	0	0	-0,150	-0,300	x_{cc} = -0,15	-0,15	-0,0889	Yes
2	0	0,562	-0,15	0,262	y_{cc} = -2,7475 • x_{cc} -0,15	-	-	No
3	0,205	0	0,055	-0,3	y_{cc} = -0,3	-0,0732	-0,3	Yes
4	0	0	-0,150	-0,300	x_{cc} = -0,15	-0,15	-0,0889	No

Where:
x_{cc} =x- x_{Cc}
y_{cc} =y- y_{Cc}
edge_i = edge between corners: i and i+1



i	Coordinates of contour below stress solid 2		Coordinates in central axes system		Coordinates in (x_{n2}, y_{n2}) system							Static moments of stress solid 2 over the compressed part of concrete section	
	x	y	x_{cc}	y_{cc}	x_{n2}	y_{n2}	dx	dy	A	B	N_{c2}	$MS_{yn2,zn2}$	$MS_{xn2,zn2}$
1	0,000	0,562	-0,15	0,26	-0,77	0,12	0,60	0,00	0,00	0,12	0,701	-0,329	0,056
2	0,205	0,000	0,05	-0,30	-0,17	0,12	-0,04	-0,12	2,75	0,59	-0,017	0,003	-0,001
3	0,077	0,000	-0,07	-0,30	-0,21	0,00	-0,22	0,00	0,00	0,00	0,000	0,000	0,000
4	0,000	0,211	-0,15	-0,09	-0,44	0,00	-0,33	0,12	-0,36	-0,16	-0,129	0,088	-0,008
										N_{c2} = sum(N_{c2}_i) =	0,555		
								C=	0,154				
								D=	0,154				

gravity center of stress solid 2 in (x_{n2}; y_{n2}) coordinates system	x_{c2}'	-0,4279
	y_{c2}'	0,0853
gravity center of stress solid 2 in central axes (x_{cc} ; y_{cc}) coordinates system	x_{c2}	-0,0662
	y_{c2}	-0,0698

Where:

$x_{n2} = \cos(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$

$y_{n2} = -\sin(Angle - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(Angle - \pi / 2) \cdot (y_{cc} - y_{n2c})$

$dx = x_{n2}\_(i + 1) - x_{n2}\_i$

$dy = y_{n2}\_(i + 1) - y_{n2}\_i$

A = dy/dx if dx ≠ 0 or A=0 if dx=0

B = -dy/dx • $x_{n2}\_(i + 1)+y_{n2}\_(i + 1)$ if dx ≠ 0 or B = 0 if dx = 0

C = y_b • Eps_c1 / Eps_top

D = MIN(Eps_c1;Eps_top) • y_b / Eps_top

n = 1.2 (power of stress-strain relation curve)

dSigma =Sigma_max - Sigma_min

$N_{c2}\_i = 0$ if A = 0 and B = 0 or

$N_{c2}\_i = f_{cd} \cdot C / (n + 1) \cdot (C / (n + 2) / A \cdot ((1 - (-y_{n2}\_(i + 1) + D) / C)^{n + 2} - (1 - (-y_{n2}\_i + D)/C)^{n + 2}) - (x_{n2}\_(i + 1) - x_{n2}\_i) \cdot (1 – D / C)^{n + 1}) + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2) + B \cdot (x_{n2}\_(i + 1) - x_{n2}\_i))$ if A ≠ 0 or

$N_{c2}\_i = (f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n + 1} - (1 – D / C)^{n + 1})) \cdot dx + (-f_{cd} + dSigma) \cdot B \cdot dx$ if A = 0 and B ≠ 0

$N_{c2} = sum(N_{c2}\_i)$

$MS_{yn2,zn2}\_i = f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n + 1} - (1 – D / C)^{n + 1}) \cdot 1 / 2 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2) + (-f_{cd} + dSigma) \cdot B / 2 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2)$ if A = 0 or

$MS_{yn2,zn2}\_i = f_{cd} \cdot C^2 / ((n + 1) \cdot (n + 2) \cdot A) \cdot (x_{n2}\_(i + 1) \cdot (1 - (-y_{n2}\_(i + 1) + D) / C)^{n + 2} -x_{n2}\_i \cdot (1 - (-y_{n2}\_i + D) / C)^{n + 2} -C / (n + 3) / A \cdot ((1 - (-y_{n2}\_(i + 1) + D) / C)^{n + 3} - (1 - (-y_{n2}\_i + D) / C)^{n + 3})) - 0.5 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2) \cdot f_{cd} \cdot C / (n + 1) \cdot (1 – D / C)^{n + 1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot (x_{n2}\_(i + 1)^3 - x_{n2}\_i^3) + B / 2 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2))$ if A ≠ 0

$MS_{yn2,zn2} = sum(MS_{yn2,zn2}\_i)$

$MS_{xn2,zn2}\_i = (f_{cd} \cdot C / (n + 1) \cdot (B \cdot (1-(-B + D) / C)^{n + 1} – C / (n + 2) \cdot ((1 - (-B + D) / C)^{n + 2} - (1 – D / C)^{n + 2})) + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2}\_(i + 1) - x_{n2}\_i)$ if A = 0 or

$MS_{xn2,zn2}\_i = f_{cd} \cdot C^2/((n + 1) \cdot (n + 2) \cdot A) \cdot (y_{n2}\_(i + 1) \cdot (1 - (-y_{n2}\_(i + 1) + D) / C)^{n + 2} - y_{n2}\_i \cdot (1 - (-y_{n2}\_i + D) / C)^{n + 2} -C / (n + 3) \cdot (2 \cdot (1 - (-y_{n2}\_(i + 1) + D) / C)^{n + 3} - 2 \cdot (1 - (-y_{n2}\_i + D) / C)^{n + 3}) + A \cdot (1 – D / C)^{n + 2} \cdot dx) + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2}\_(i + 1)^3 - x_{n2}\_i^3) + A \cdot B / 2 \cdot (x_{n2}\_(i + 1)^2 - x_{n2}\_i^2) + B^2 / 2 \cdot dx)$ if A ≠ 0

$MS_{xn2,zn2} = sum(MS_{xn2,zn2}\_i)$

$x_{c2}' = MS_{yn2,zn2} / N_{c2}$

$y_{c2}' = MS_{xn2,zn2} / N_{c2}$

$x_{c2} = x_{n2c} + \cos(Angle - \pi / 2) \cdot x_{c2}' - \sin(Angle - \pi / 2) \cdot y_{c2}'$

$y_{c2} = y_{n2c} + \sin(Angle - \pi / 2) \cdot x_{c2}' + \cos(Angle - \pi / 2) \cdot y_{c2}'$

$N_{c2} = 0.554816e6$

$N_{c'} = N_{c1} - N_{c2} = 0.882778e6$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = -0.0818$

$y_{c1} = -0.1126$

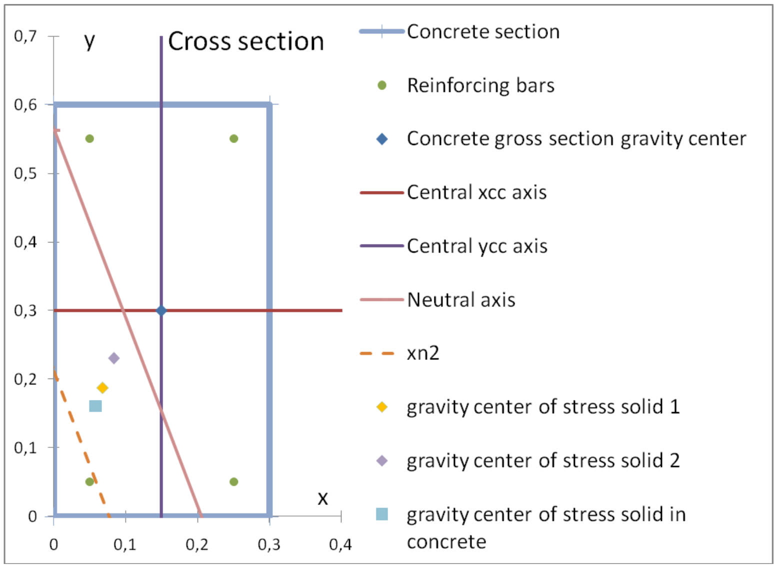
$x_{c2} = -0.0662$

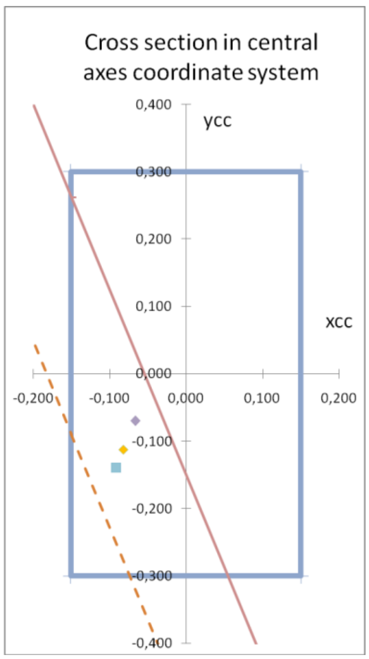
$y_{c2} = -0.0698$

Center of gravity of stresses in concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.212709$





Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{880.42e3}$

$M_{xc} = - N_c \cdot y_{cg} = \underline{122.62e3}$

$M_{yc} = N_{c'} \cdot x_{cg} = -\underline{80.64e3}$

- Results for reduced section:

$N = N_c + N_s = \underline{825.74e3}$

$M_x = M_{xc} + M_{xs} = \underline{146.35e3}$

$M_y = M_{yc} + M_{ys} = -\underline{91.12e3}$

11 Case 10: Calculation of capacity state in symmetric section for bending moment Mx

11.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (0, M_x, 0) and give state of strain and stress at the failure.

11.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 10a)
- A parabolic-rectangular model of concrete stress-strain relation (Case10b)
- A horizontal branch model of steel
- A simple bending in x direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

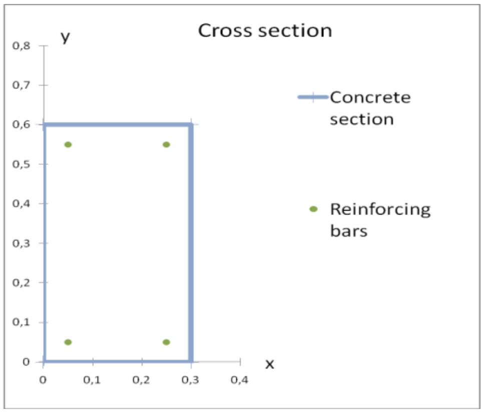
For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

11.3 Case 10a

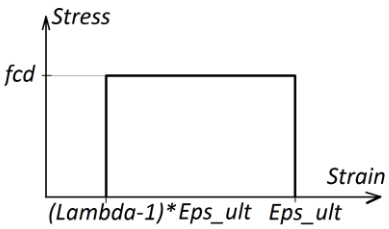
11.3.1 Data:

- Geometry:
    - Concrete section:
      x = 0.0 y = 0.0
      x = 0.0 y = 0.6
      x = 0.3 y = 0.6
      x = 0.3 y = 0.0

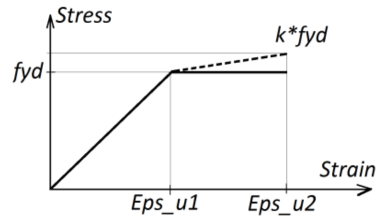
    - Reinforcing bars:
      x = 0.05 y = 0.05 ϕ = 0.020
      x = 0.25 y = 0.05 ϕ = 0.020



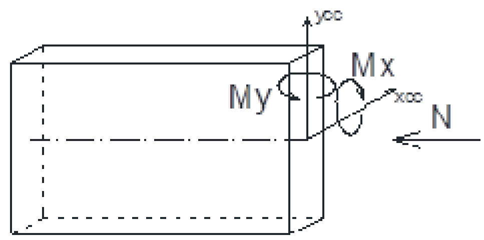
- Concrete parameters:
    - Design strength f_{cd} = 20e6
    - Effective height reduction factor Lambda= 0.9
    - Modulus of elasticity E_c = 30e9
    - Strain-stress model: rectangular
    - Strain ultimate limit Eps_ult= 0.0035



- Steel parameters:
    - Design strength fyd = 500e6
    - Hardening factor k= 1.0
    - Modulus of elasticity E_s = 200e9
    - Strain ultimate limit Eps_u2 = 0.1



- Forces:
  - N = 0.00
  - M_x = -50.00
  - M_y = 0.00



11.3.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

11.3.3 Results from Component

- Results for reduced section:
  N_f = 0.00
  M_{xf} = -164.56
  M_{yf} = 0.00
  dist = -0.24182
  Angle = 4.71239= 270deg
  Eps_top = 0.003500
  Eps_bot = -0.032597
  Eps_stop= -0.029589
  Eps_sbot= -0.029589

- Results for reinforcement bars:
  N_s = -314.16

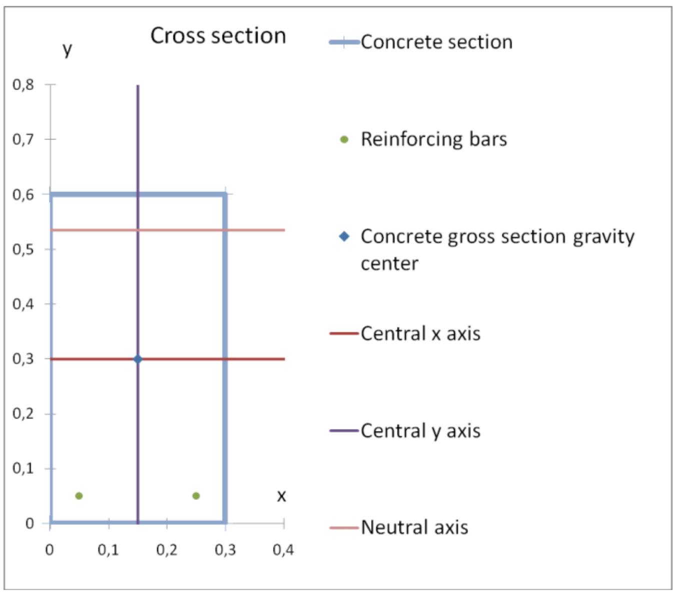
M_{xs} = -78.54
M_{ys} = 0.00

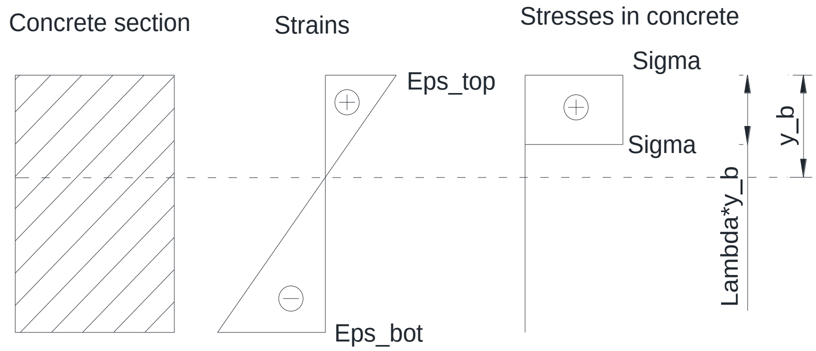
- Results for concrete section:
  A_c = 0.0157
  (x_{cg}, y_{cg}) = (0.00000000,0.27382035)
  N_c = 314.16
  M_{xc} = -86.02
  M_{yc} = 0.00

11.3.4 Results (manual calculations):

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Verification of safety factor:
alfa = M_{xf}/ M_x = 3.2912
N_f = alfa • N = 0 (OK)
M_{yf}= alfa • M_y = 0 (OK)





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
  height of a section: h = 0.6
  y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.05818
  Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
  x_{Cc} = 0.15
  y_{Cc} = 0.30

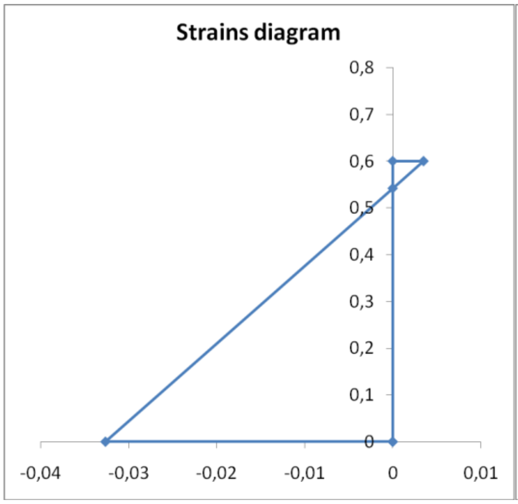
  Distance between reinforcing bar and neutral axis:
  d = y - h + y_b

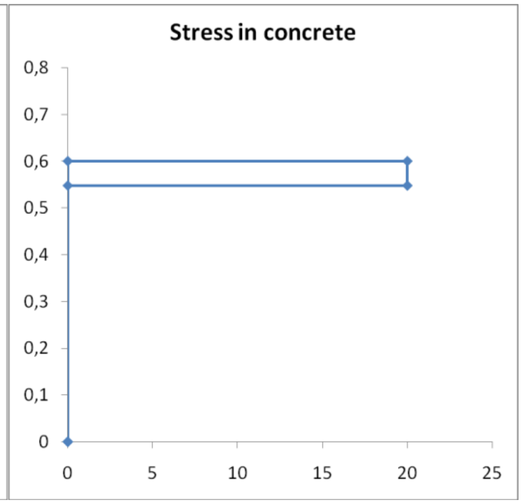
  Stress in bar i:
  If |eps_i| ≤ f_{yd} / E_s then
  sigma_i = eps_i / Eps_u1 • f_{yd}
  else
  sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Distance d from neutral axis	strain eps = d/h_c • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,02	-0,10	-0,25	0,492	-0,02959	-500	-157080	-39270	15708
2	0,25	0,05	0,02	0,10	-0,25	0,492	-0,02959	-500	-157080	-39270	-15708
							Sum of forces in bars:		-314159	-78540	0

N_s = 314.16e3
M_{xs} = -78.54e3
M_{ys} = 0.00e3
Eps_stop=max(eps) = -0,02959
Eps_sbot=min(eps) = -0,02959

- Results for concrete section:



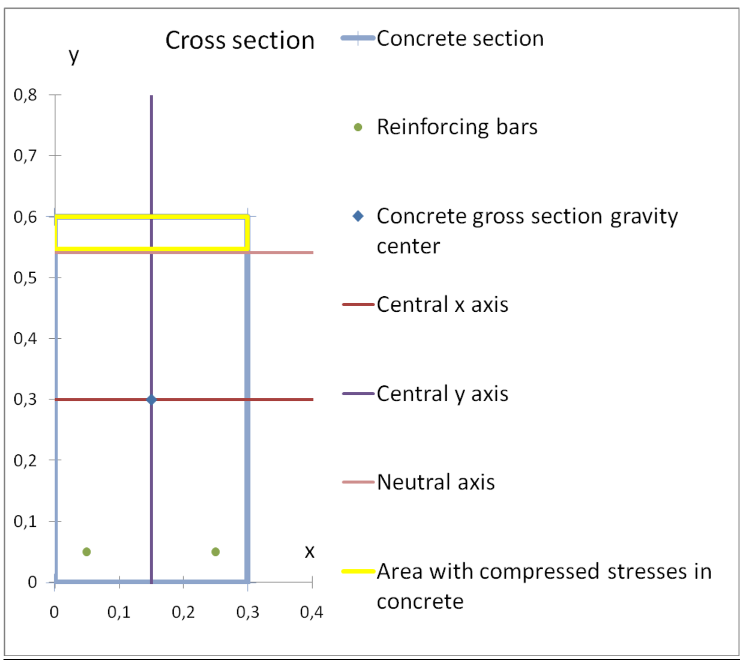


Height of compressed part of the section:

Lambda • y_b = 0.052362

Area of contour under compressive stress:

A_c = 0.3 • 0.052362= 0.0157086



Stress in concrete:

Sigma = Eps_top / Eps_ult • f_{cd} = 20e6

Center of gravity of stresses in the concrete in central axes coordinate system:

x_{cg} = 0.3 / 2- x_{Cc} = 0.0000

y_{cg} =h- Lambda • y_b/2- y_{Cc} = 0.273819

Internal forces in concrete:

$N_c = A_c \cdot Sigma = 314.17e3$

$M_{xc} = - N_c \cdot y_{cg} = -86.03e3$

$M_{yc} = N_c \cdot x_{cg} = 0.00e3$

- Results for reduced section:

  $N_f = N_c + N_s = 0.01e3$

  $M_{xf} = M_{xc} + M_{xs} = -164.57e3$

  $M_{yf} = M_{yc} + M_{ys} = 0.00e3$

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

11.4 Case 10b

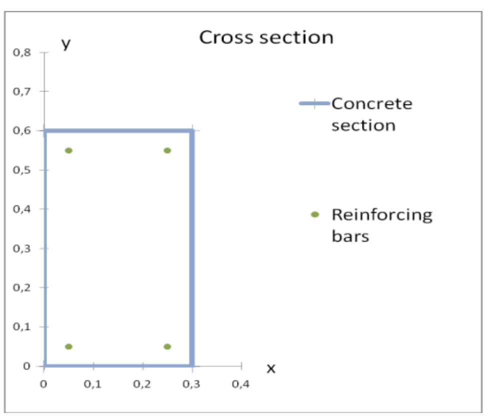
11.4.1 Data:

- Geometry:
  - Concrete section:

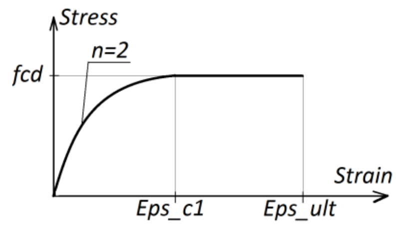
    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0

- Reinforcing bars:

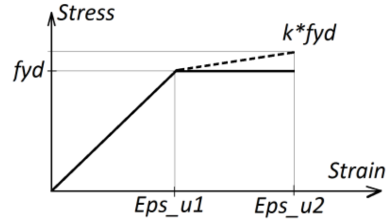
  x = 0.05 y = 0.05 ϕ = 0.032
  x = 0.05 y = 0.55 ϕ = 0.032
  x = 0.25 y = 0.55 ϕ = 0.032
  x = 0.25 y = 0.05 ϕ = 0.032



- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity Ec = 32e9
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020

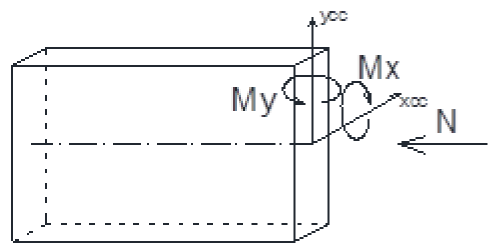


- Steel parameters:
    - Design strength fyd = 400e6
    - Hardening factor k= 1.0
    - Modulus of elasticity E_s = 200e9
    - Strain ultimate limit Eps_u2 = 0.1



Forces:

N = 0.00
M_x = -125.00
M_y = 0.00



11.4.2 Search for:

- Results for reduced section:
    - Forces: N_f, M_{xf}, M_{yf}
    - Neutral axis position: dist, Angle
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
    - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
    - Area of contour under compressive stress: A_c
    - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
    - Forces: N_c, M_{xc}, M_{yc}

11.4.3 Results from Component

- Results for reduced section:

N_f = 0.02

M_{xf} = -332.64
M_{yf} = 0.00
dist= -0.237955
Angle = 4.71239= 270deg
Eps_top = 0.003500
Eps_bot = -0.030346
Eps_stop = 0.000679
Eps_sbot = -0.027526

- Results for reinforcement bars:
  Ns = -424.81
  Mxs = -215.50
  Mys = 0.00

- Results for concrete section:
  Ac = 0.0186
  (xcg, ycg) = (0.00000000,0.27574117)
  Nc = 424.83
  Mxc = -117.14
  Myc = 0.00

11.4.4 Results (manual calculations):

Verification of safety factor:
alfa = M_{xf} / M_x = 2.66112
N_f = 0.02 ≅ alfa • N = 0 (OK)
M_{yf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 5b in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

12 Case 11: Calculation of capacity state in symmetric section for bending moment My

12.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (0, 0, M_y) and give state of strain and stress at the failure.

12.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 11a)
- A bilinear model of concrete stress-strain relation (Case11b)
- A power-rectangular model of concrete stress-strain relation (Case11c)
- A horizontal branch model of steel
- A simple bending in y direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

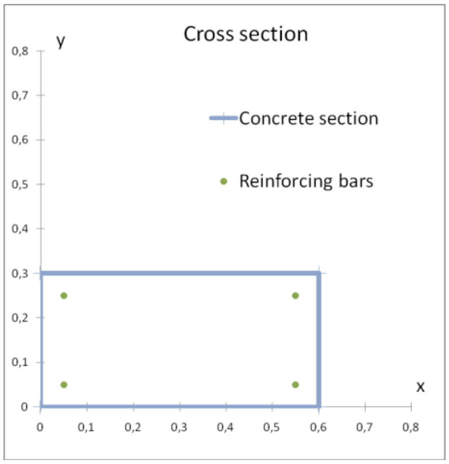
- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force at the failure related to acting forces (N_f = alpha • N)
    - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
    - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
      Where alpha is a coefficient called "safety factor".
- Neutral axis position:
    - Distance between the axis and a gravity center of concrete gross section (dist)
    - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_stop, Eps_sbot)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

12.3 Case 11a

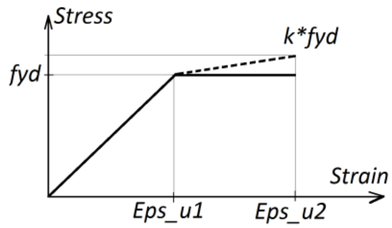
12.3.1 Data:

- Geometry:
  - Concrete section:
    x = 0.0 y = 0.0
    x = 0.0 y = 0.3
    x = 0.6 y = 0.3
    x = 0.6 y = 0.0
  - Reinforcing bars:
    x = 0.05 y = 0.05 ϕ = 0.032
    x = 0.05 y = 0.25 ϕ = 0.032
    x = 0.55 y = 0.25 ϕ = 0.032
    x = 0.55 y = 0.05 ϕ = 0.032

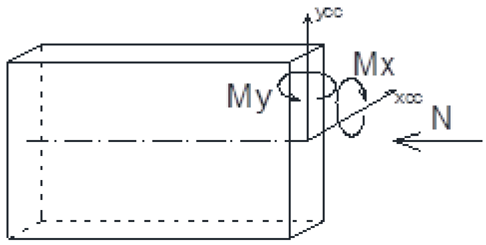


- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: linear
  - Strain ultimate limit Eps_ult = 0.0035

- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k= 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Forces:
  - N = 0.00 e3
  - M_x = 0.00 e3
  - M_y = -600.00 e3



12.3.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: Ns, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: Nc, M_{xc}, M_{yc}

12.3.3 Results from Component

- Results for reduced section:
  N_f = -0.10 e3
  M_{xf} = 0.00 e3
  M_{yf} = -407.32 e3
  dist = -0.204885
  Angle = 0.00000= 0.0000deg
  Eps_top = 0.003500
  Eps_bot = -0.018572
  Eps_stop = 0.001661
  Eps_sbot = -0.016733

- Results for reinforcement bars:
  N_s = -270.19 e3

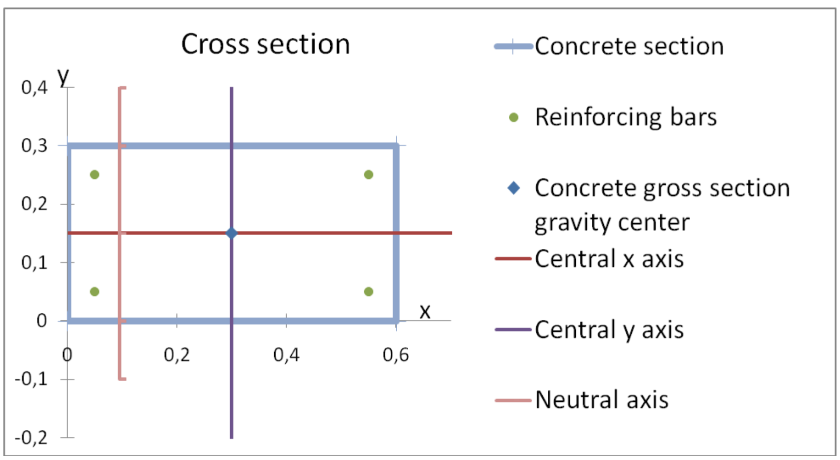
M_{xs} = 0.00 e3
M_{ys} = -334.58 e3

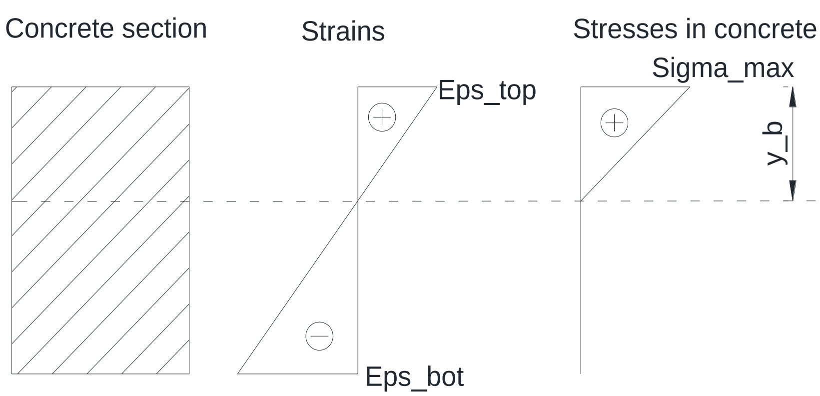
- Results for concrete section:
  A_c = 0.0285
  (x_{cg}, y_{cg}) = (-0.26932850,0.00000000)
  N_c = 270.09 e3
  M_{xc} = 0.00 e3
  M_{yc} = -72.74 e3

12.3.4 Results (manual calculations):

Verification of safety factor:
alfa = M_{yf}/ M_y = 0.67887
N_f = -0.10 ≅ alfa • N = 0 (OK)
M_{xf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.





Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
width of a section: b= 0.6

y_b = -Eps_top • b/(Eps_bot - Eps_top) = 0.095144
Eps_u1 = f_{yd} / E_s

Concrete gross section gravity center:

x_{Cc} = 0.30
y_{Cc} = 0.15

Distance between reinforcing bar and neutral axis:

d = y-b+y_b

Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then
sigma_i = eps_i / Eps_u1 • f_{yd}
else
sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,032	-0,045	0,00166	332	267120	-0,25	-0,10	26712	-66780
2	0,55	0,05	0,032	0,455	-0,01673	-500	-402124	0,25	-0,10	-40212	-100531
3	0,55	0,25	0,032	0,455	-0,01673	-500	-402124	0,25	0,10	40212	-100531
4	0,05	0,25	0,032	-0,045	0,00166	332	267120	-0,25	0,10	-26712	-66780
						Sum of forces in bars:	-270008			0	-334622

N_s = -270.01e3
M_{xs} = 0.00e3
M_{ys} = -334.62e3
Eps_stop = max(eps) = 0,00166
Eps_sbot = min(eps) = -0,01673

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • eps / Eps_ult • Area	M_{xsc} = -N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,032	0,00166	-0,25	-0,10	0,000804248	7632	763	-1908
2	0,55	0,05	0,032	-0,01673	0,25	-0,10				
3	0,55	0,25	0,032	-0,01673	0,25	0,10				
4	0,05	0,25	0,032	0,00166	-0,25	0,10	0,000804248	7632	-763	-1908
							Sum of forces :	15264	0	-3816

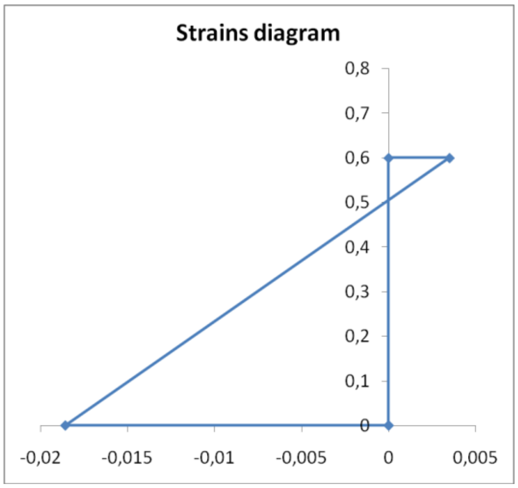
Reduction forces in concrete:

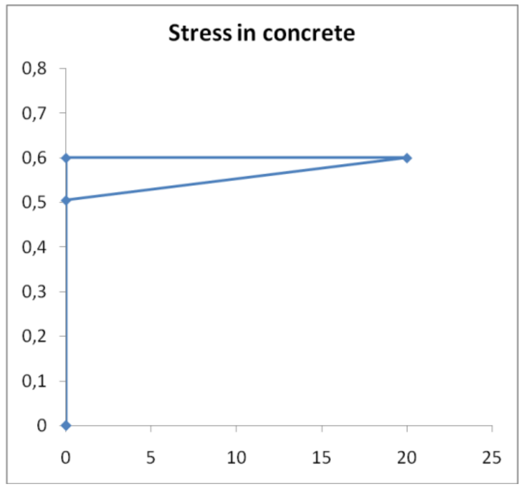
N_{sc} = 15.26e3
M_{xsc} = 0.00e3
M_{ysc} = -3.82e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = -0.25
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.00

- Results for concrete section:





Height of compressed part of the section:

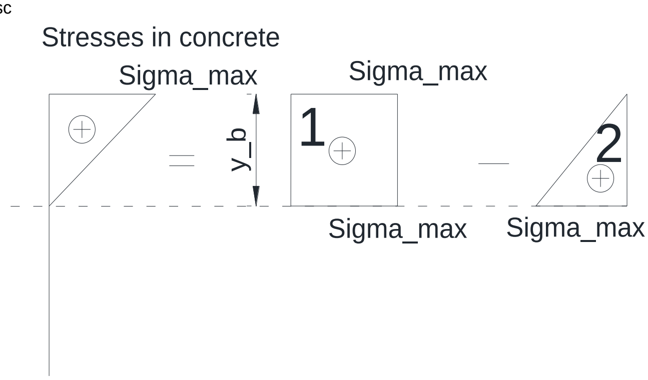
y_b = 0.095144

Area of contour under compressive stress:

A_c = 0.3 • y_b = 0.0285

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} • Eps_top / Eps_ult = 20.000e6

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \cdot$ Sigma_max = 0.5709e6

$N_{c2} = A_c \cdot$ Sigma_max / 2 = 0.2854e6

$N_{c'} = N_{c1} - N_{c2}$ = 0.285432e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

x_{c1} = -0.3+y_b/2 = -0.252428

$y_{c1} = 0$

x_{c2} = -0.3+y_b • 2/3 = -0.236570

$y_{c2} = 0$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.269318

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.000000

Internal forces in concrete:

$N_c = N_{c'} - N_{sc}$ = 270.17e3

$M_{xc} = - N_c \cdot y_{cg}$ = 0.00e3

$M_{yc} = N_{c'} \cdot x_{cg}$ = -72.76e3

- Results for reduced section:

  $N_f = N_c + N_s$ = 0.16e3

  $M_{xf} = M_{xc} + M_{xs}$ = 0.00e3

  $M_{yf} = M_{yc} + M_{ys}$ = -407.38e3

Small differences between results from Component and from manual calculations are below 0.1% and are due to precision of iterations.

12.4 Case 11b

12.4.1 Data:

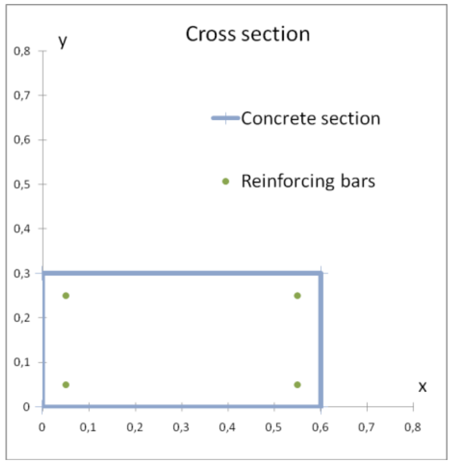
- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0

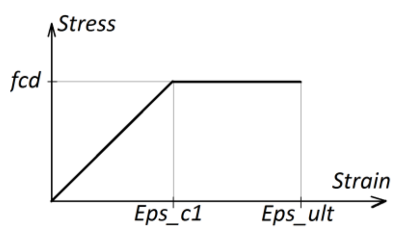
- Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.032
x = 0.05	y = 0.25	ϕ = 0.032

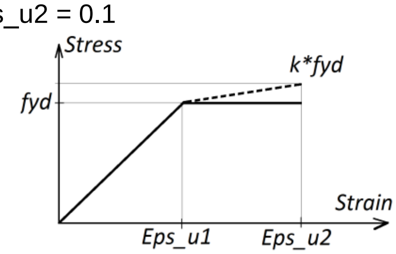
x = 0.55 y = 0.25 ϕ = 0.032
x = 0.55 y = 0.05 ϕ = 0.032



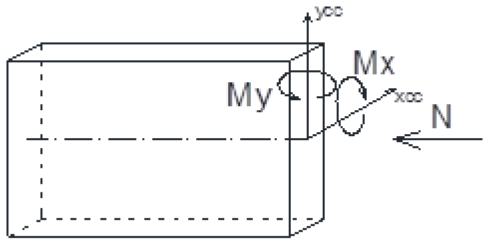
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1= 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 205e9
  - Strain ultimate limit Eps_u2 = 0.1



- Forces:
  - N = 0.00 e3
  - M_x = 0.00 e3
  - M_y = -10.00 e3



12.4.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: Ns, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: Nc, M_{xc}, M_{yc}

12.4.3 Results from Component

- Results for reduced section:
  N_f = 0.09 e3
  M_{xf} = 0.00 e3
  M_{yf} = -332.39 e3
  dist = -0.236099
  Angle = 0.000000= 0.000000 deg
  Eps_top = 0.003500
  Eps_bot = -0.029363
  Eps_stop = 0.000761
  Eps_sbot = -0.026625

- Results for reinforcement bars:
  N_s = -392.34e3
  M_{xs} = 0.00e3
  M_{ys}= -223.61e3

- Results for concrete section:
  A_c = 0.0192
  (x_{cg}, y_{cg}) = (-0.27717502,0.00000000)
  N_c = 392.42 e3
  M_{xc} = 0.00 e3
  M_{yc} = -108.77 e3

12.4.4 Results (manual calculations):

Verification of safety factor:

alfa = M_{yf}/ M_y = 33.239
N_f = 0.09 ≅ alfa • N = 0 (OK)
M_{xf} = alfa • M_x = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 4b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.03% and are due to precision of iterations.

12.5 Case 11c

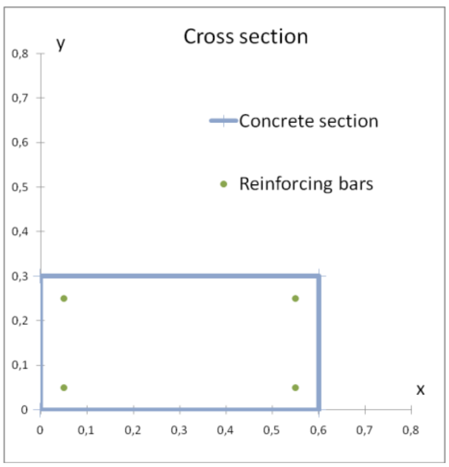
12.5.1 Data:

- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0

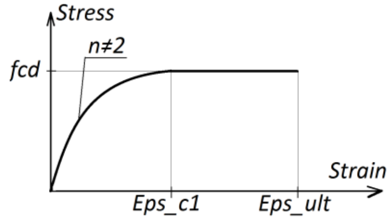
- Reinforcing bars:

x = 0.05	y = 0.05	ϕ = 0.032
x = 0.05	y = 0.25	ϕ = 0.032
x = 0.55	y = 0.25	ϕ = 0.032
x = 0.55	y = 0.05	ϕ = 0.032

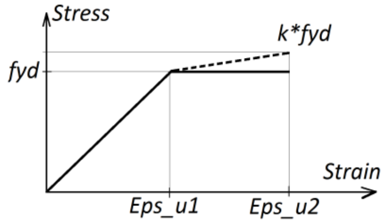


- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: power-rectangular
  - Power: n = 1.8

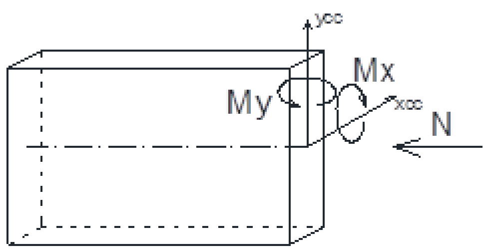
- Strain ultimate limit Eps_ult = 0.0035
- Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.1



- Forces:
  - N = 0.00 e3
  - M_x = 0.00 e3
  - M_y = -10.00 e3



12.5.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

12.5.3 Results from Component

- Results for reduced section:
  N_f = -0.16 e3
  M_{xf} = 0.00 e3
  M_{yf} = -332.56 e3
  dist = -0.237679
  Angle = 0.00000= 0.00000 deg
  Eps_top = 0.003500
  Eps_bot = -0.030196
  Eps_stop = 0.000692
  Eps_sbot = -0.027388

- Results for reinforcement bars:
  N_s = -420.79 e3
  M_{xs} = 0.00 e3
  M_{ys} = -216.50 e3

- Results for concrete section:
  A_c = 0.0187
  (x_{cg}, y_{cg}) = (-0.27591769,0.00000000)
  N_c = 420.64 e3
  M_{xc} = 0.00 e3
  M_{yc} = -116.06 e3

12.5.4 Results (manual calculations):

Verification of safety factor:
alfa = M_{yf}/ M_y = 33.256
N_f = -0.16 ≅ alfa • N = 0 (OK)
M_{xf} = alfa • M_x = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 6b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.05% and are due to precision of iterations.

13 Case 12: Calculation of capacity state in symmetric section for bending moment Mx with axial force

13.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N, Mx, 0) and give state of strain and stress at the failure.

13.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 12a)
- A bilinear model of concrete stress-strain relation (Case12b)
- A power-rectangular model of concrete stress-strain relation (Case12c)
- A horizontal branch model of steel
- A bending in x direction (M_x)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

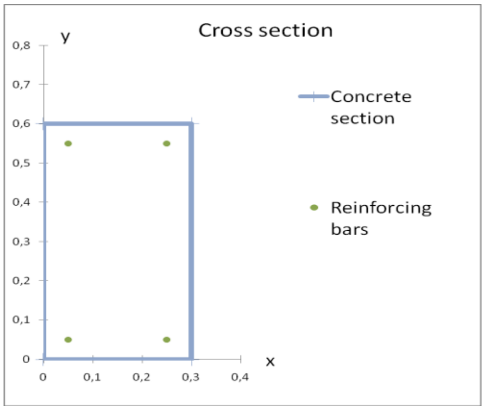
- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

13.3 Case 12a

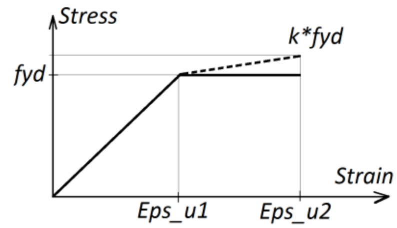
13.3.1 Data:

- Geometry:
  - Concrete section:
    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:
    x = 0.05 y = 0.05 ϕ = 0.012
    x = 0.05 y = 0.55 ϕ = 0.012
    x = 0.25 y = 0.55 ϕ = 0.012
    x = 0.25 y = 0.05 ϕ = 0.012

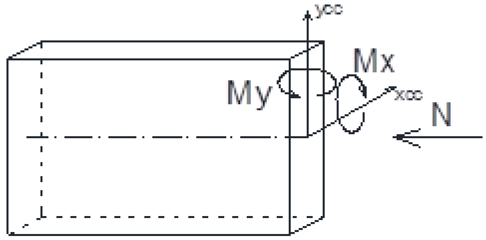


- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: linear
  - Strain ultimate limit Eps_ult = 0.0035

- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Forces:
  - N = 43979.98 e3
  - M_x = -3465.40 e3
  - M_y = 0.00 e3



13.3.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

13.3.3 Results from Component

- Results for reduced section:
  N_f = 2199.00 e3
  M_{xf} = -173.27 e3
  M_{yf} = 0.00 e3
  dist = 0.40000
  Angle = 4.712389= 270.00000deg
  Eps_top = 0.003500
  Eps_bot = 0.000500
  Eps_stop = 0.003250
  Eps_sbot = 0.000750

- Results for reinforcement bars:
  N_s = 147.03 e3
  M_{xs} = -19.79 e3
  M_{ys} = 0.00 e3

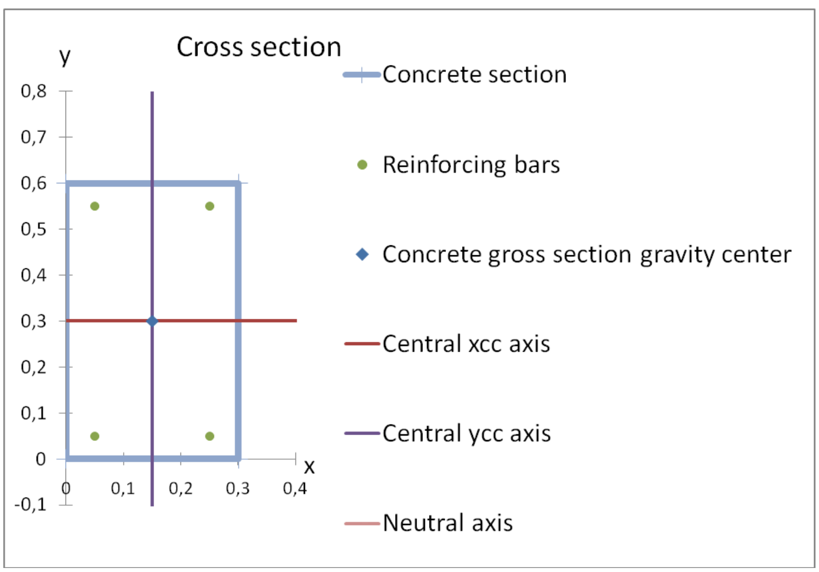
- Results for concrete section:
  A_c = 0.18
  (x_{cg}, y_{cg}) = (0.00000000,-0.07479510)
  N_c = 2051.97 e3
  M_{xc} = 153.48 e3
  M_{yc} = 0.00 e3

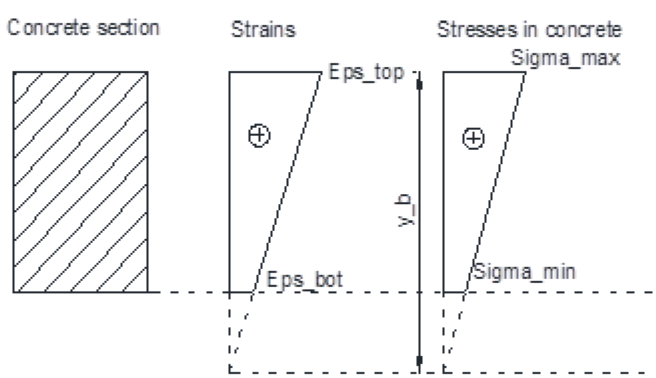
13.3.4 Results (manual calculations):

Verification of safety factor:

alfa = M_{xf}/ M_x = 0.05000
N_f = alfa • N = 2199.00 (OK)
M_{yf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.





- Results for reinforcement bars:
  Distance between maximum compression corner of the section and neutral axis:
    height of a section: h = 0.6
    y_b = -Eps_top • h / (Eps_bot - Eps_top) = 0.7
    Eps_u1 = f_{yd} / E_s

  Concrete gross section gravity center:
    x_{Cc} = 0.15
    y_{Cc} = 0.30

  Distance between reinforcing bar and neutral axis:
    d = y - h + y_b

  Stress in bar i:
    If |eps_i| ≤ f_{yd} / E_s then
      sigma_i = eps_i / Eps_u1 • f_{yd}
    else
      sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π x ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= - N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

  N_s = 147.03e3
  M_{xs} = -19.79e3
  M_{ys} = 0.00e3
  Eps_stop = max(eps) = 0.00325
  Eps_sbot = min(eps) = 0.00075

  Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • eps / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,012	0,00075	-0,1	-0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1	0,25	0,000113097	485	121	48

3	0,25	0,55	0,012	0,00325	0,1	0,25	0,000113097		2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1	-0,25	0,000113097		2100	-525	-210
							Sum of forces:		5170	-808	0

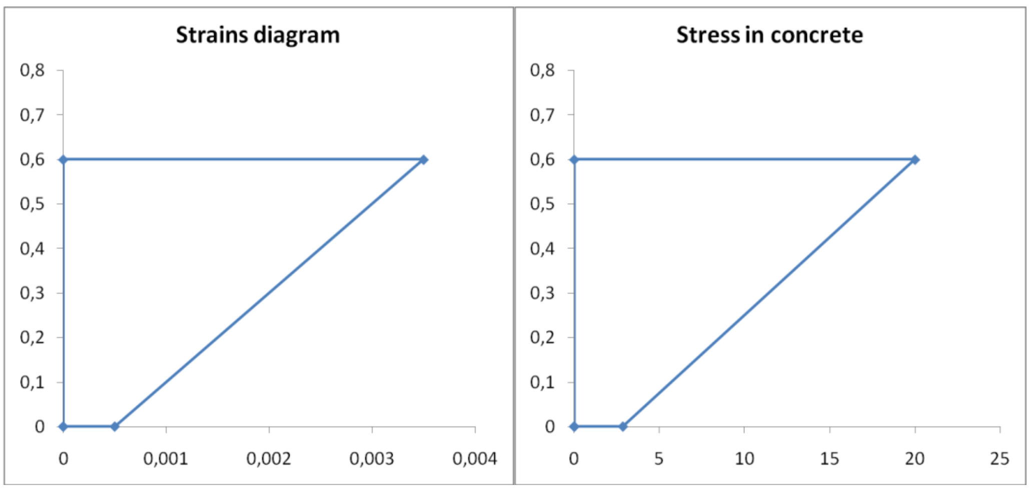
Reduction forces in concrete:

N_{sc} = 5.17e3
M_{xsc} = -0.81e3
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$_{xscg}$ = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00000
$_{yscg}$ = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.15625

- Results for concrete section:



Height of compressed part of the section:

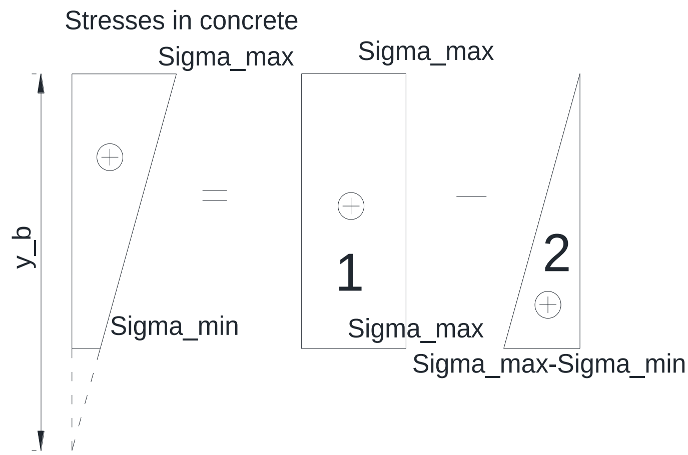
0.6000

Area of contour under compressive stress:

A_c = 0.3 • 0.6000= 0.1800

Axial force Nc in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$N_{c'} = N_{c1} - N_{c2}$
$N_c = N_{c'} - N_{sc}$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$M_{xc} = - N_c \cdot y_{cg}$

$M_{yc} = N_c \cdot x_{cg}$

Sigma_max = f_{cd} • Eps_top/Eps_c1= 20.000e6

Sigma_min = f_{cd} • Eps_bot/Eps_c1= 2.857e6

Axial force in compressed part of concrete gross section

$N_{c1} = A_c \cdot$ Sigma_max = 3.600e6

$N_{c2} = A_c \cdot$ (Sigma_max - Sigma_min) / 2 = 1.543e6

$N_{c'} = N_{c1} - N_{c2}$ = 2.057e6

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$x_{c1} = 0$

$y_{c1} = 0$

$x_{c2} = 0$

$y_{c2} = - y_{cc} + 0.6/3 = -0.1$

Center of gravity of stresses in the concrete in central axes coordinate system:

$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.000000

$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$ = 0.074795

Internal forces in concrete:

$N_c = N_{c'} - N_{sc}$ = 2051.97e3

$M_{xc} = - N_c \cdot y_{cg}$ = -153.48e3

$M_{yc} = N_{c'} \cdot x_{cg}$ = 0.00e3

- Results for reduced section:

$N_f = N_c + N_s$ = 2199.00e3

$M_{xf} = M_{xc} + M_{xs}$ = -173.27e3

$M_{yf} = M_{yc} + M_{ys}$ = 0.00e3

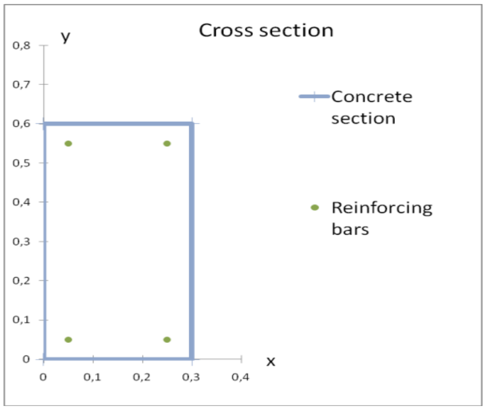
13.4 Case 12b

13.4.1 Data:

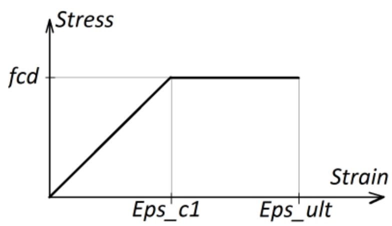
- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:

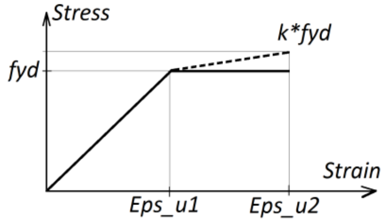
    x = 0.05 y = 0.05 ϕ = 0.016
    x = 0.05 y = 0.55 ϕ = 0.016
    x = 0.25 y = 0.55 ϕ = 0.016
    x = 0.25 y = 0.05 ϕ = 0.016



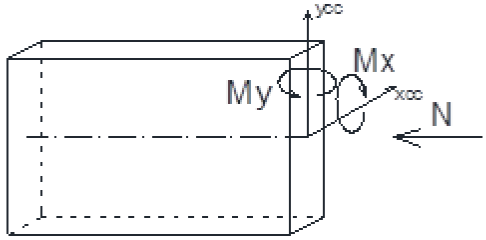
- Concrete parameters:
  - Design strength f_{cd} = 30e6
  - Modulus of elasticity E_c = 32e9
  - Strain-stress model: bilinear
  - Strain ultimate limit Eps_ult = 0.0035
  - Strain relation change over Eps_c1 = 0.0020



- Steel parameters:
  - Design strength f_{yd} = 400e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 205e9
  - Strain ultimate limit Eps_u2 = 0.1



- Forces:
    - N = 114.8397 e3
    - M_x = -5.634275 e3
    - M_y = 0.00 e3



13.4.2 Search for:

- Results for reduced section:
    - Forces: N_f, M_{xf}, M_{yf}
    - Neutral axis position: dist, Angle
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
    - Forces: Ns, M_{xs}, M_{ys}
- Results for concrete section:
    - Area of contour under compressive stress: A_c
    - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
    - Forces: Nc, M_{xc}, M_{yc}

13.4.3 Results from Component

- Results for reduced section:
  N_f = 4593.59 e3
  M_{xf} = -225.37 e3
  M_{yf} = 0.00 e3
  dist = 0.40000
  Angle = 4.71239= 270 deg
  Eps_top = 0.003500
  Eps_bot = 0.000500
  Eps_stop = 0.003250
  Eps_sbot = 0.000750

- Results for reinforcement bars:

N_s = 222.68 e3
M_{xs} = -24.76 e3
M_{ys} = 0.00 e3

- Results for concrete section:
  A_c = 0.1800
  (x_{cg}, y_{cg}) = (0.00000000, 0.04589755)
  N_c = 4370.92 e3
  M_{xc} = -200.61 e3
  M_{yc} = 0.00 e3

13.4.4 Results (manual calculations):

Verification of safety factor:
alfa = M_{xf}/ M_x = 40
N_f = alfa • N = 4593.59 (OK)
M_{yf}= alfa • M_y = 0 (OK)
We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 4a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

13.5 Case 12c

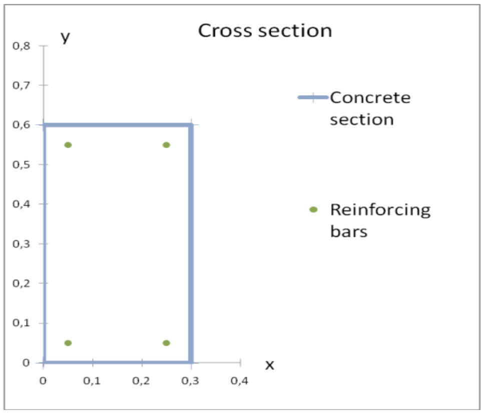
13.5.1 Data:

- Geometry:
  - Concrete section:

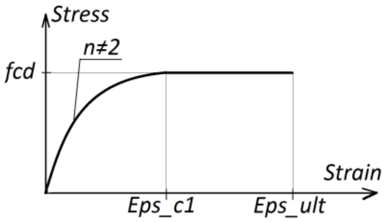
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

  - Reinforcing bars:

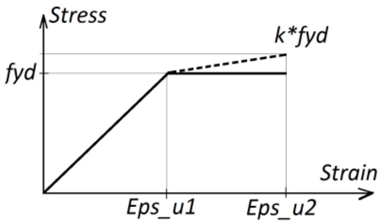
x = 0.05	y = 0.05	ϕ = 0.016
x = 0.05	y = 0.55	ϕ = 0.016
x = 0.25	y = 0.55	ϕ = 0.016
x = 0.25	y = 0.05	ϕ = 0.016



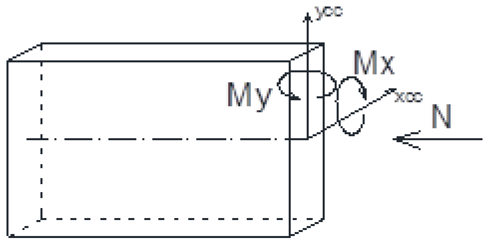
- Concrete parameters:
    - Design strength f_{cd} = 30e6
    - Modulus of elasticity E_c = 32e9
    - Strain-stress model: power-rectangular
    - Power: n = 1.4
    - Strain ultimate limit Eps_ult = 0.0035
    - Strain relation change over Eps_c1= 0.0020



- Steel parameters:
    - Design strength f_{yd} = 400e6
    - Hardening factor k = 1.0
    - Modulus of elasticity E_s = 200e9
    - Strain ultimate limit Eps_u2 = 0.1



- Forces:
    - N = 4.41023 e3
    - M_x = -0.1662045455 e3
    - M_y = 0.00 e3



13.5.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

13.5.3 Results from Component

- Results for reduced section:
  N_f = 4851.25e3
  M_{xf} = -182.83e3
  M_{yf} = 0.00e3
  dist = 0.40000
  Angle = 4.71239= 270deg
  Eps_top = 0.003500
  Eps_bot = 0.000500
  Eps_stop = 0.003250
  Eps_sbot = 0.000750

- Results for reinforcement bars:
  N_s = 221.17e3
  M_{xs} = -25.13e3
  M_{ys} = 0.00e3

- Results for concrete section:
  A_c = 0.1800
  (x_{cg}, y_{cg}) = (0.00000000, 0.03405830)
  N_c = 4630.08e3
  M_{xc} = -157.69e3
  M_{yc} = 0.00e3

13.5.4 Results (manual calculations):

Verification of safety factor:

alfa = M_{xf} / M_x = 1100

N_f = alfa • N = 4851.25 (OK)

M_{yf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 6a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

14 Case 13: Calculation of capacity state in symmetric section for bending moment My with axial force

14.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N, 0, M_y) and give state of strain and stress at the failure.

14.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 13a)
- A parabolic-rectangular model of concrete stress-strain relation (Case13b)
- A horizontal branch model of steel
- A bending in y direction (M_y)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
    - Axial force (N_s)
    - Moment with respect to x_{cc} axis (M_{xs})
    - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
    - Area of contour under compressive stress (A_c)
    - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
    - Axial force (N_c)
    - Moment with respect to x_{cc} axis (M_{xc})
    - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
    - Axial force at the failure related to acting forces (N_f = alpha • N)
    - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
    - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
      Where alpha is a coefficient called "safety factor".
- Neutral axis position:
    - Distance between the axis and a gravity center of concrete gross section (dist)
    - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

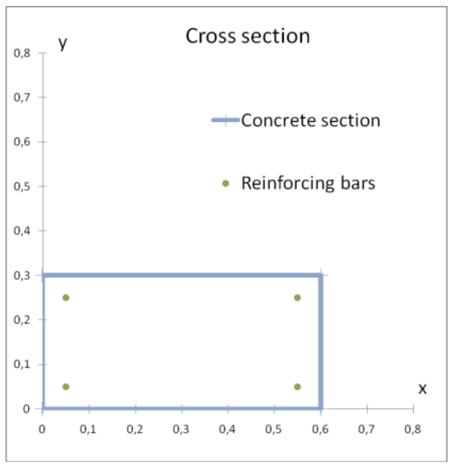
14.3 Case 13a

14.3.1 Data:

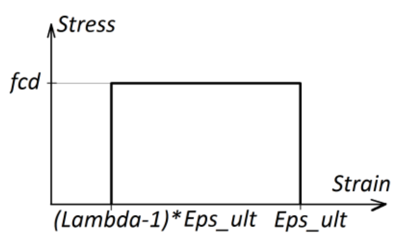
- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0
    x = 0.0 y = 0.3
    x = 0.6 y = 0.3
    x = 0.6 y = 0.0
  - Reinforcing bars:

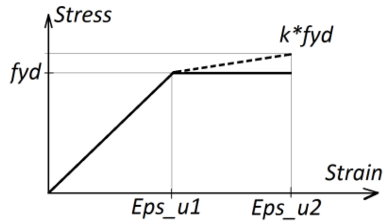
    x = 0.05 y = 0.05 ϕ = 0.012
    x = 0.05 y = 0.25 ϕ = 0.012
    x = 0.55 y = 0.25 ϕ = 0.012
    x = 0.55 y = 0.05 ϕ = 0.012



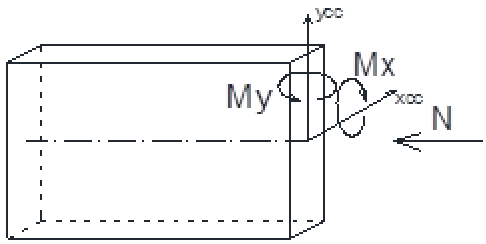
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 30e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 500e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Forces:
    - N = 3331.408571 e3
    - M_x = 0.00 e3
    - M_y = -82.84952381 e3



14.3.2 Search for:

- Results for reduced section:
    - Forces: N_f, M_{xf}, M_{yf}
    - Neutral axis position: dist, Angle
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
    - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
    - Area of contour under compressive stress: A_c
    - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
    - Forces: N_c, M_{xc}, M_{yc}

14.3.3 Results from Component

- Results for reduced section:
  N_f = 3497.98 e3
  M_{xf} = 0.00 e3
  M_{yf} = -86.99 e3
  dist = 0.400000
  Angle = 0.00000= 0.0000deg
  Eps_top = 0.003500
  Eps_bot = 0.000500
  Eps_stop = 0.003250
  Eps_sbot = 0.000750

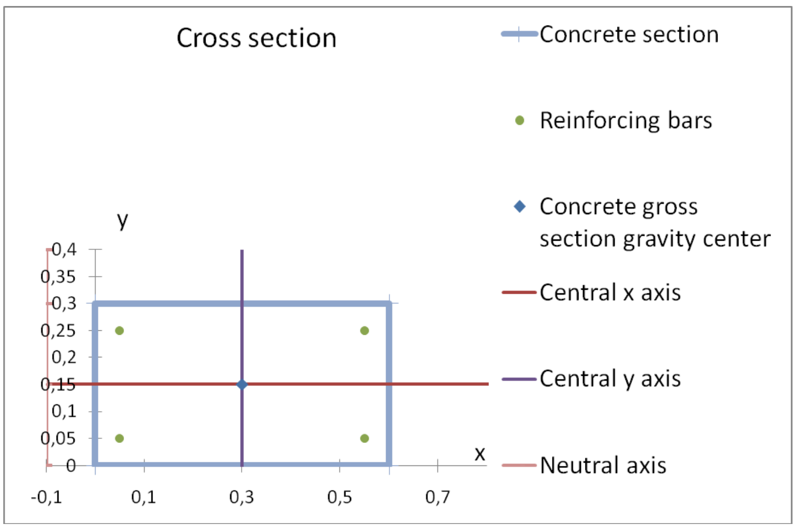
- Results for reinforcement bars:
  N_s = 147.03 e3
  M_{xs} = 0.00 e3
  M_{ys} = -19.79 e3

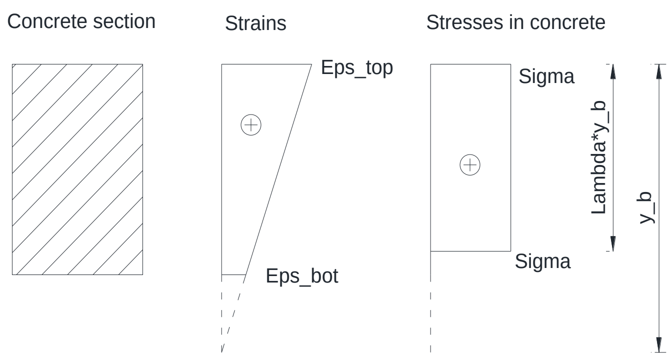
- Results for concrete section:
  A_c = 0.1680
  (x_{cg}, y_{cg}) = (-0.02005403, 0.00000000)
  N_c = 3350.95 e3
  M_{xc} = 0.00 e3
  M_{yc} = -67.20 e3

14.3.4 Results (manual calculations):

Verification of safety factor:
alfa = M_{yf} / M_y = 1.04998
N_f = 3497.98 ≅ alfa • N = 3497.90 (OK)
M_{xf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.





- Results for reinforcement bars:

  Distance between maximum compression corner of the section and neutral axis:
  width of a section: b= 0.6
  y_b = -Eps_top • b/(Eps_bot - Eps_top) = 0.7
  Eps_u1 = f_{yd} / E_s

Concrete gross section gravity center:

x_{Cc} = 0.30

y_{Cc} = 0.15

Distance between reinforcing bar and neutral axis:

d = y-b+y_b

Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then

sigma_i = eps_i / Eps_u1 • f_{yd}

else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_{cc}	y_{cc}	M_{xs}= - N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,012	0,150	0,00325	500	56549	-0,25	-0,10	5655	-14137
2	0,55	0,05	0,012	0,650	0,00075	150	16965	0,25	-0,10	1696	4241
3	0,55	0,25	0,012	0,650	0,00075	150	16965	0,25	0,10	-1696	4241
4	0,05	0,25	0,012	0,150	0,00325	500	56549	-0,25	0,10	-5655	-14137
						Sum of forces in bars:	147027			-19792	0

N_s = 147.03e3

M_{xs} = -19.79e3

M_{ys} = 0.00e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars N_{sc} = f_{cd} • Eps_top / Eps_ult • Area	M_{xsc} = - N_{sc} • y_{cc}	M_{ysc}= N_{sc} • x_{cc}
1	0,05	0,05	0,012	-0,25	-0,10	0,000113097	2262	226	-565
2	0,55	0,05	0,012	0,25	-0,10	0,000113097	2262	226	565
3	0,55	0,25	0,012	0,25	0,10	0,000113097	2262	-226	565
4	0,05	0,25	0,012	-0,25	0,10	0,000113097	2262	-226	-565
						Sum of forces:	9048	0	0

Reduction forces in concrete:

N_{sc} = 9.05e3

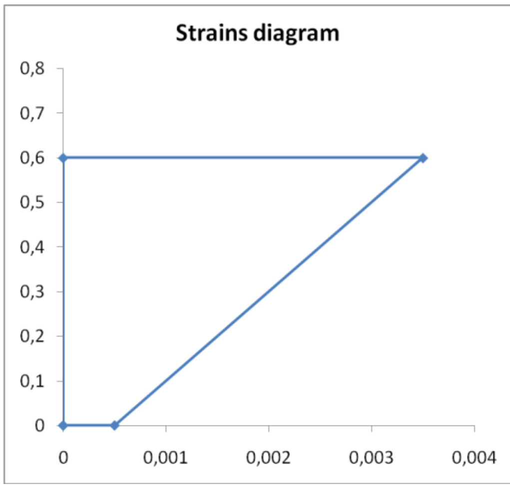
M_{xsc} = 0.00e3

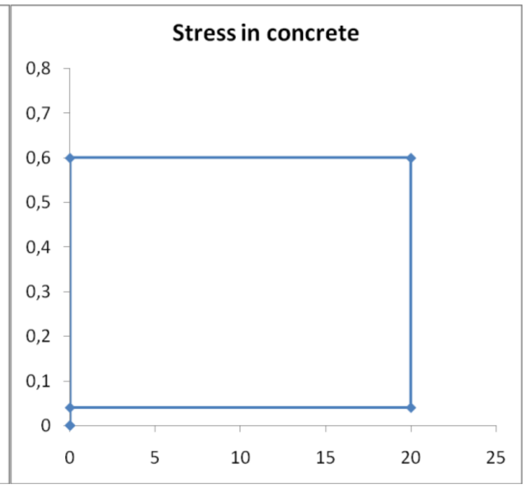
M_{ysc} = 0.00e3

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.00
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.00

- Results for concrete section:





Height of compressed part of the section:

Lambda • y_b = 0.56

Area of contour under compressive stress:

A_c = 0.3 • 0.56= 0.168

Stress in concrete:

Sigma = Eps_top / Eps_ult • f_{cd} = 20e6

Center of gravity of stresses in the concrete in central axes coordinate system:

x_{cg}= ((0.56 / 2- x_{cc}) • A_c • Sigma - x_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = -0.020054
y_{cg}= ((0.3 / 2- y_{cc}) • A_c • Sigma-y_{scg} • N_{sc}) / (A_c • Sigma + N_{sc}) = 0.000000

Internal forces in concrete:

N_c = A_c • Sigma - N_{sc} = 3350.95e3
M_{xc} = - N_c • y_{cg} = 0.00e3
M_{yc} = N_c • x_{cg} = -67.20e3

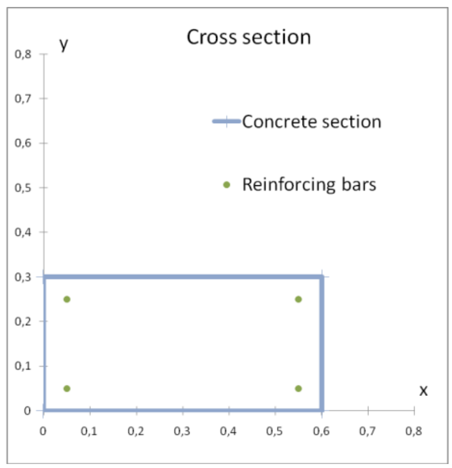
- Results for reduced section:

N_f = N_c + N_s = 3497.98e3
M_{xf} = M_{xc} + M_{xs} = 0.00e3
M_{yf} = M_{yc}+M_{ys} = -86.99e3

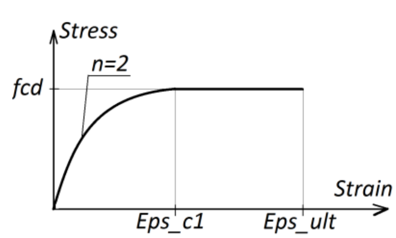
14.4 Case 13b

14.4.1 Data:

- Geometry:
    - Concrete section:
      x = 0.0 y = 0.0
      x = 0.0 y = 0.3
      x = 0.6 y = 0.3
      x = 0.6 y = 0.0
    - Reinforcing bars:
      x = 0.05 y = 0.05 ϕ = 0.016
      x = 0.05 y = 0.25 ϕ = 0.016
      x = 0.55 y = 0.25 ϕ = 0.016
      x = 0.55 y = 0.05 ϕ = 0.016

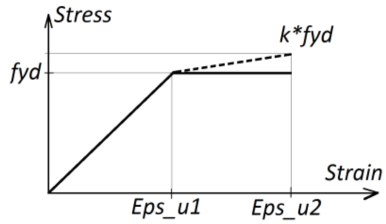


- Concrete parameters:
    - Design strength f_{cd} = 30e6
    - Modulus of elasticity E_c = 32e9
    - Strain-stress model: parabolic-rectangular
    - Strain ultimate limit Eps_ult = 0.0035
    - Strain relation change over Eps_c1 = 0.0020

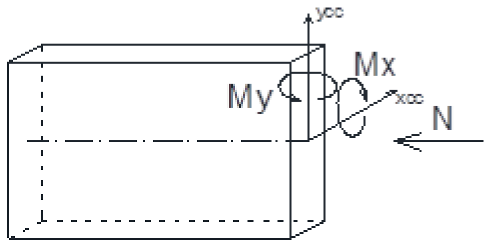


- Steel parameters:
    - Design strength f_{yd} = 400e6
    - Hardening factor k = 1.0
    - Modulus of elasticity E_s = 200e9

- Strain ultimate limit Eps_u2 = 0.1



- Forces:
  - N = 5095.50e3
  - M_x = 0.00e3
  - M_y = -137.86e3



14.4.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

14.4.3 Results from Component

- Results for reduced section:
  N_f = 5095.50 e3
  M_{xf} = 0.00 e3
  M_{yf} = -137.86 e3
  dist = 0.400000
  Angle = 0.00000= 0.0000deg
  Eps_top = 0.003500
  Eps_bot = 0.000500
  Eps_stop = 0.003250
  Eps_sbot = 0.000750
- Results for reinforcement bars:
  N_s = 221.17 e3
  M_{xs} = 0.00 e3
  M_{ys} = -25.13 e3

- Results for concrete section:
  A_c = 0.1800
  (x_{cg}, y_{cg}) = (-0.02312690, 0.000000000)
  N_c = 4874.33 e3
  M_{xc} = 0.00 e3
  M_{yc} = -112.73 e3

14.4.4 Results (manual calculations):

Verification of safety factor:

alfa = M_{yf} / M_y = 1.000
N_f = alfa • N = 5095.50 (OK)
M_{xf} = alfa • M_y = 0 (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 5a in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

15 Case 14: Calculation of capacity state in symmetric section for bidirectional bending with axial force

15.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N, Mx, My) and give state of strain and stress at the failure.

15.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x)
- A bending in y direction (M_y)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

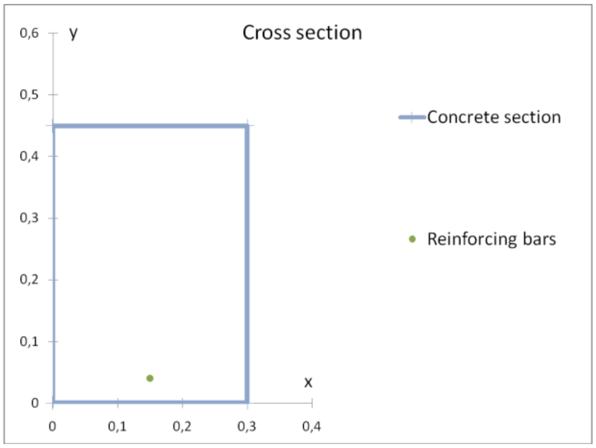
Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

15.3 Data:

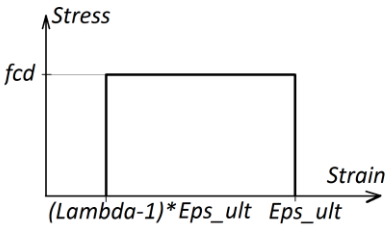
- Geometry:
  - Concrete section:

    x = 0.0 y = 0.0
    x = 0.0 y = 0.45
    x = 0.3 y = 0.45
    x = 0.3 y = 0.0
  - Reinforcing bars:

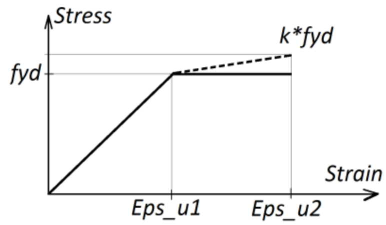
    x = 0.15 y = 0.04 area= 0.00053 ϕ = 0.025977239



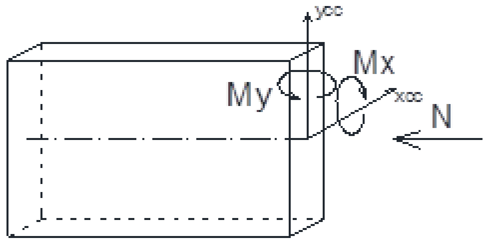
- Concrete parameters:
  - Design strength f_{cd} = 26.7 e6 • 0.8= 21.3 e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 35e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 310e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Forces:
  - N = 200.00 e3
  - M_x = -N • 0.48= -96.00 e3
  - M_y = -N • 0.12= 24.00 e3



15.4 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Extreme strains in concrete (Eps_top, Eps_bot)
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Force: N_c

15.5 Results from Component

- Results for reduced section:
  N_f = 199.77 e3
  M_{xf} = -95.95 e3
  M_{yf} = -24.00 e3
  Angle = 5.204135= 298.174942 deg
  dist = -0.120248
  Eps_top = 0.003500
  Eps_bot = -0.009152
  Eps_stop = -0.006659
  Eps_sbot = -0.006659

- Results for reinforcement bars:
  N_s = -164.41 e3
  M_{xs} = -30.41 e3
  M_{ys} = 0.00 e3

- Results for concrete section:
  A_c = 0.0170
  (x_{cg}, y_{cg}) = (-0.06589487, 0.17995110)
  N_c = 364.018 e3

M_{xc} = -65.53 e3
M_{yc} = -24.00 e3

Results from Component are consistent with results from book *"Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2"*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, example 6.18 on page 352., ISBN 83-7125-136-X. Calculations in the book are performend with smaller precision than in Component which causes some differences in results (about 1%).
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

16 Case 15: Calculation of capacity state in asymmetric section for bidirectional bending with axial force

16.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N, Mx, My) and give state of strain and stress at the failure.

16.2 Summary:

This sample gives as results the internal forces for:

- A asymmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 15a)
- A horizontal branch model of steel
- A bending in x direction (M_x)
- A bending in y direction (M_y)
- An axial force (N).

In the Case 15b can be seen a comparison of capacity forces for various models of concrete (rectangular, linear, bilinear, parabolic-rectangular and power-rectangular) obtained in Component and in manual calculations.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tension fiber.(Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)

- Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

16.3 Case 15a

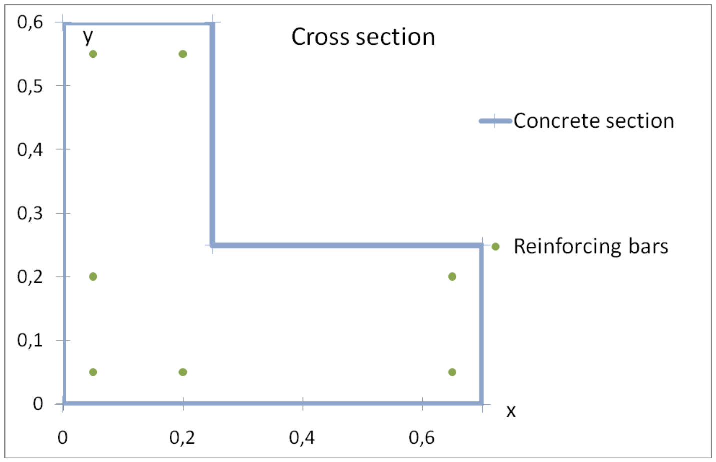
16.3.1 Data:

- Geometry:
  - Concrete section:

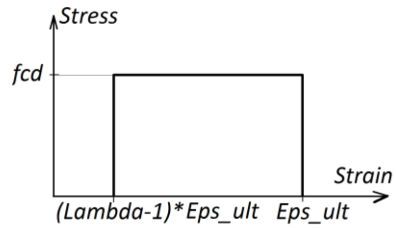
x = 0.00	y = 0.00
x = 0.00	y = 0.60
x = 0.25	y = 0.60
x = 0.25	y = 0.25
x = 0.70	y = 0.25
x = 0.70	y = 0.00

  - Reinforcing bars:

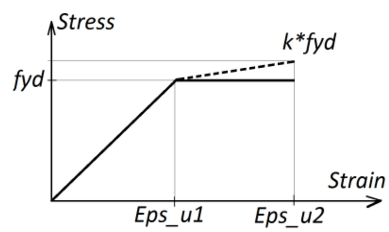
x = 0.05	y = 0.05	ϕ = 0.020
x = 0.05	y = 0.55	ϕ = 0.020
x = 0.20	y = 0.55	ϕ = 0.020
x = 0.20	y = 0.05	ϕ = 0.020
x = 0.65	y = 0.05	ϕ = 0.020
x = 0.65	y = 0.20	ϕ = 0.020
x = 0.05	y = 0.20	ϕ = 0.020



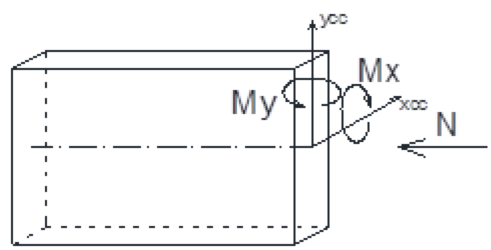
- Concrete parameters:
  - Design strength f_{cd} = 20e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 35e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 310e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.075



- Forces:
  - N = 72.4471 e3
  - M_x = -28.9825 e3
  - M_y = 2.5743 e3



16.3.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf}
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
  - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
  - Area of contour under compressive stress: A_c
  - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
  - Forces: N_c, M_{xc}, M_{yc}

16.3.3 Results from Component

- Results for reduced section:
  N_f = 725.91 e3
  M_{xf} = -290.09 e3
  M_{yf} = 25.68 e3
  Angle = 4.188995= 240.011739 deg
  dist= 0.095114
  Eps_top = 0.003500
  Eps_bot = -0,006888
  Eps_stop = 0.002399
  Eps_sbot = -0.005788

- Results for reinforcement bars:
  N_s = -108.39 e3
  M_{xs} = -101.21 e3
  M_{ys} = 31.32 e3

- Results for concrete section:
  A_c = 0.0427
  (x_{cg}, y_{cg}) = (-0.00676059, 0.22639273)
  N_c = 834.30 e3
  M_{xc} = -188.88 e3
  M_{yc} = -5.64 e3

16.3.4 Results (manual calculations):

Verification of safety factor:

M_{yf} / M_y = 9.98
M_{xf} / M_x = 10.01
N_f / N = 10.02
alfa = M_{yf} / My ≈ M_{xf} / Mx ≈ N_f / N (OK)

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Position of neutral axis

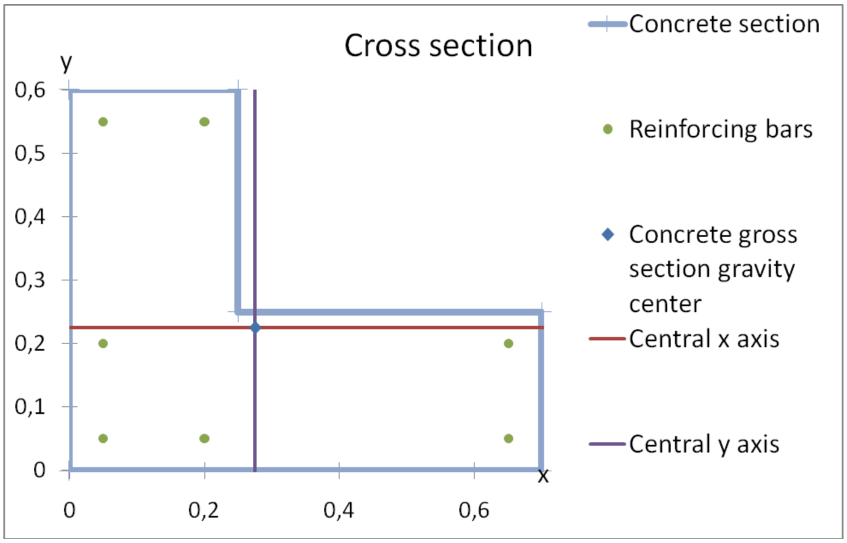
Concrete gross section gravity center calculations:

i	Corners of concrete section x	y	Area _c	Gravity center of contour corresponding to Area_c x_{01}	y_{01}	Static moments S_{cy}	S_{cx}
1	0	0	0,000	0,00	0,30	0	0
2	0	0,6	0,150	0,13	0,30	0,01875	0,045
3	0,25	0,6	0,000	0,25	0,43	0	0
4	0,25	0,25	0,113	0,48	0,13	0,05344	0,0141
5	0,7	0,25	0,000	0,70	0,13	0	0
6	0,7	0	0,000	0,35	0,00	0	0
		Total:	0,263			0,0722	0,0591

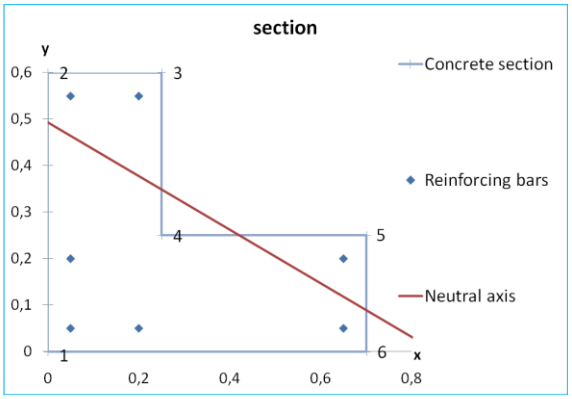
Where:
edge_i = edge between corners: i and i+1
Area_c_i = Area between edge_i and axes: x=x_i, x=x_(i+1) and y = 0
Area_c= 1/2 • (y_(i + 1) + y(i)) • (x_(i + 1)-x(i))
x_{01}_i= x_(i) + (x_(i + 1) - x_(i)) • (2 / 3 • y_(i + 1) + 1 / 3 • y_(i)) / (y_(i) + y_(i + 1)) if (y_(i) + y_(i + 1) ≠ 0 or
x_{01}_i = (x_(i + 1) + x_(i)) / 2 if (y_(i) + y_(i + 1) = 0
y_{01}_i = y_(i) / 2 + 0.5 • (x_{01}_i - x_(i)) • (y_(i + 1) - y_(i)) / (x_(i + 1) - x_(i)) if x_(i + 1) - x_(i) ≠ 0 or
y_{01}_i = (y_(i+1)+y_(i))/2) if x_(i + 1) - x_(i) = 0
S_{cy}_i = Area_c_i • x_{01}_i
S_{cx}_i = Area_c_i • y_{01}_i

Concrete gross section area:
A_c = sum(Area_c_i) = 0.263

Concrete gross section gravity center
x_{Cc} = sum(S_{cy}_i)/ A_c = 0.275
y_{Cc} = Sum(S_{cx}_i)/ A_c = 0.225



Equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})
y_{cc} = -0.577350316 • x_{cc} +0.110000027 (e.g. obtained in iteration process)



General equation of neutral axis in central axes coordinate system (x_{cc}; y_{cc})

$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$

A = -0.202

B = -0.350

C = 0.0385

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$dist = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0,5}$

For compressed corner:

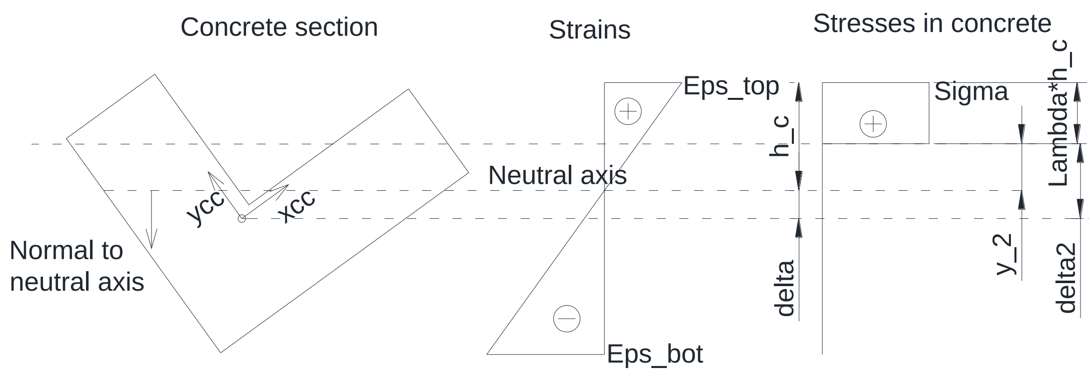
$dist = -(A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0,5}$

i	Corners of concrete section x	y	Corners of concrete section in central axes coordinate system x_{cc}	y_{cc}	Corner under compression/tension	Distance between corner and neutral axis dist
1	0	0	-0,275	-0,225	tension	0,428
2	0	0,6	-0,275	0,375	compression	-0,092
3	0,25	0,6	-0,025	0,375	compression	-0,217
4	0,25	0,25	-0,025	0,025	tension	0,086
5	0,7	0,25	0,425	0,025	compression	-0,139
6	0,7	0	0,425	-0,225	tension	0,078

Where:

$x_{cc} = x - x_{Cc}$

$y_{cc} = y - y_{Cc}$



Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

h_c = |min(dist_i)| = 0.217

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

h_t = max(dist_i) = 0.428

Verification of used equation of neutral axis:

Eps_bot = -h_t/h_c • Eps_top = -0.006903226 (OK)

and

neutral axis angle = arctan(-0.577350316)+ π/2+π= 4.18879= 240 deg= Angle (OK)

- Results for reinforcement bars:

  Eps_u1 = f_{yd} / E_s

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

For compressed rebar:

$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$

Stress in bar i:

If |eps_i| ≤ f_{yd} / E_s then

sigma_i = eps_i / Eps_u1 • f_{yd}

else

sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) • (k - 1) • f_{yd})

i	x	y	ϕ	Positions in central axes coordinate system x_{cc}	y_{cc}	Distance d from neutral axis	strain eps = d/h_c • Eps_top	stresses sigma / 10e6	force in bar N_s = sigma • π • ϕ^2 / 4	M_{xs}= -N • y_{cc}	M_{ys}= N • x_{cc}
1	0,05	0,05	0,02	-0,23	-0,18	0,359	-0,00580	-310	-97389	-17043	21913
2	0,05	0,55	0,02	-0,23	0,33	-0,074	0,00119	238	74685	-24273	-16804
3	0,2	0,55	0,02	-0,08	0,33	-0,149	0,00240	310	97389	-31652	-7304
4	0,2	0,05	0,02	-0,08	-0,18	0,284	-0,00459	-310	-97389	-17043	7304

5	0,65	0,05	0,02	0,38	-0,18	0,059	-0,00096	-191	-60114	-10520	-22543
6	0,65	0,2	0,02	0,38	-0,03	-0,071	0,00114	228	71535	1788	26826
7	0,05	0,2	0,02	-0,23	-0,03	0,229	-0,00370	-310	-97389	-2435	21913
							Sum of forces in bars:		-108673	-101177	31304

N_s = -108.67 e3
M_{xs} = -101.18 e3
M_{ys} = 31.30 e3

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if eps ≤ Eps_c1 $N_{sc} = f_{cd}$ • eps / Eps_c1 • Area
if eps > Eps_c1 $N_{sc} = f_{cd}$ • Area

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc}	y_{cc}	Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,02	-0,00580	-0,23	-0,18				
2	0,05	0,55	0,02	0,00119	-0,23	0,33	0,000314159	6283	-2042	-1414
3	0,2	0,55	0,02	0,00240	-0,08	0,33	0,000314159	6283	-2042	-471
4	0,2	0,05	0,02	-0,00459	-0,08	-0,18				
5	0,65	0,05	0,02	-0,00096	0,38	-0,18				
6	0,65	0,2	0,02	0,00114	0,38	-0,03	0,000314159	6283	157	2356
7	0,05	0,2	0,02	-0,00370	-0,23	-0,03				
						Total:	0,000942478	18850	-3927	471

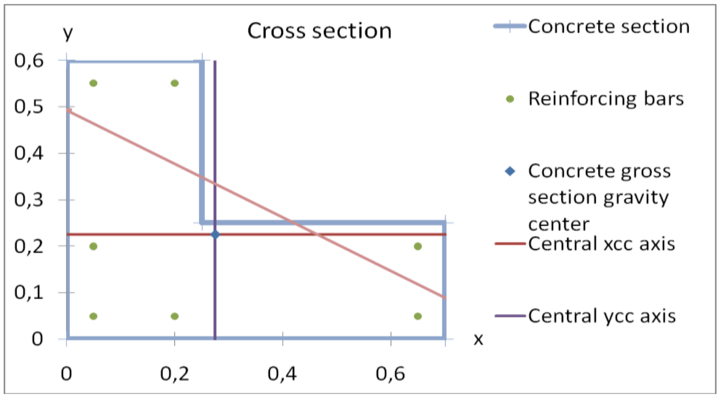
Reduction forces in concrete:

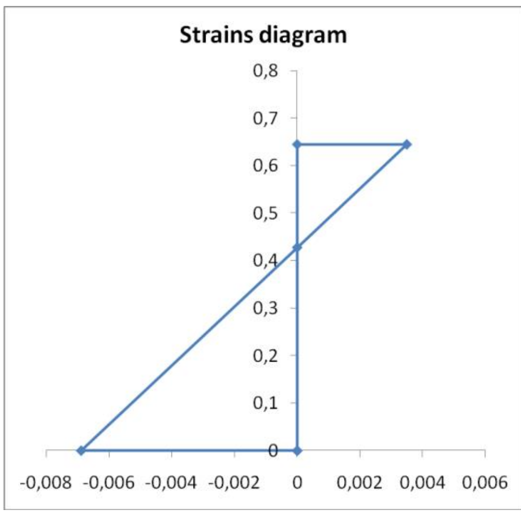
N_{sc} = 18.850 e3
M_{xsc} = -3.927 e3
M_{ysc} = 0.471 e3

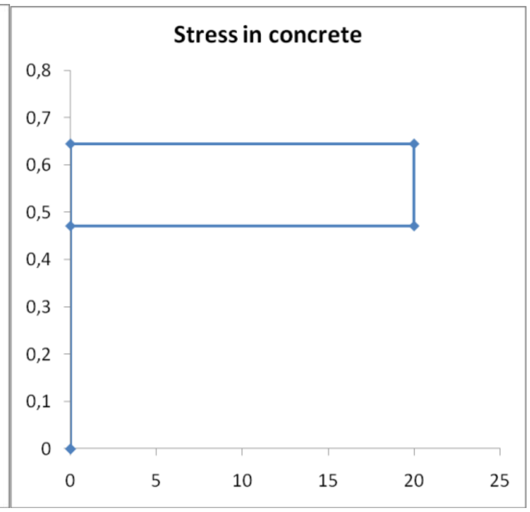
Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

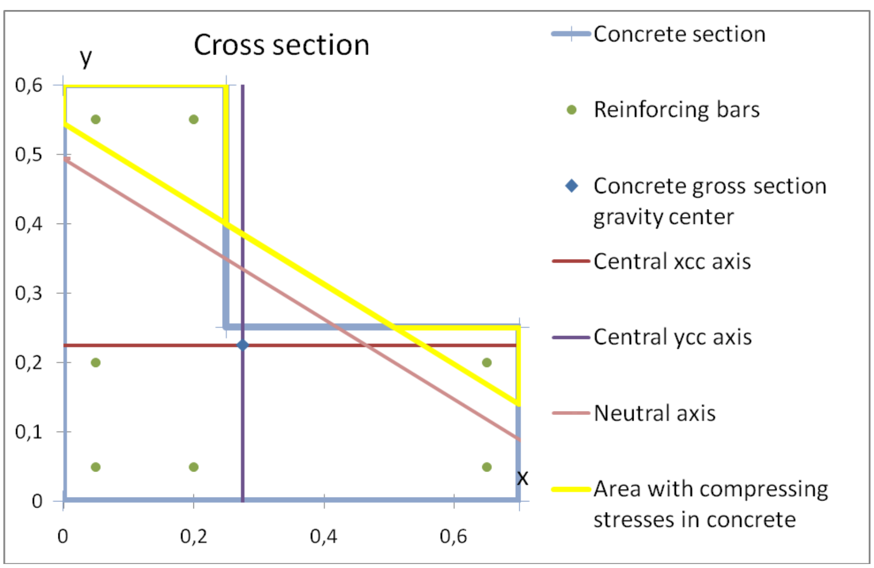
x_{scg} = sum(x_{cc} _i • Area_i) / sum(Area_i) = 0.0250
y_{scg} = sum(y_{cc} _i • Area_i) / sum(Area_i) = 0.2083

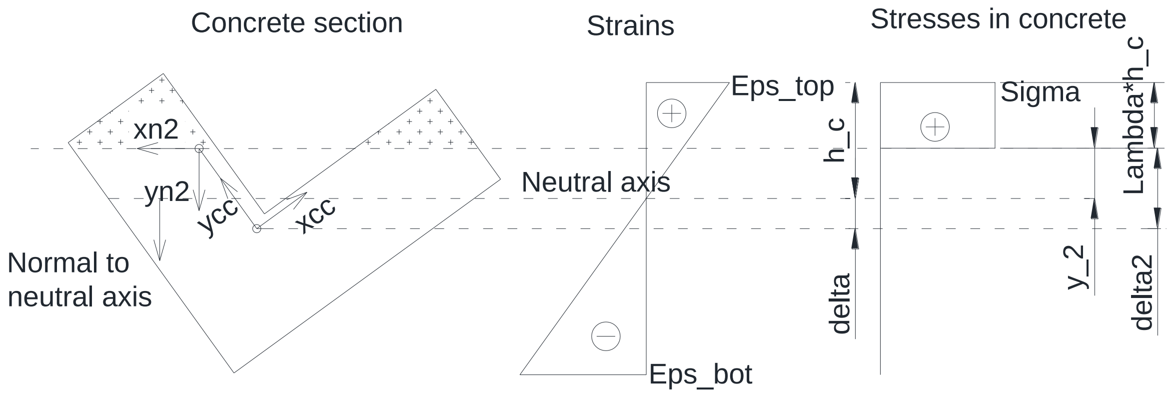
- Results for concrete section:


Cross section
y
0,6
0,5
0,4
0,3
0,2
0,1
0
0
0,2
0,4
0,6
x
Concrete section
Reinforcing bars
Concrete gross section gravity center
Central xcc axis
Central ycc axis


Strains diagram
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,008
-0,006
-0,004
-0,002
0
0,002
0,004
0,006


Stress in concrete
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
5
10
15
20
25


Cross section
y
0,6
0,5
0,4
0,3
0,2
0,1
0
0
0,2
0,4
0,6
x
Concrete section
Reinforcing bars
Concrete gross section gravity center
Central xcc axis
Central ycc axis
Neutral axis
Area with compressing stresses in concrete



Sigma = f_{cd} = 20.000 e6
Distance between gravity center of contour and neutral axis:

$$delta = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2+B^2)^{0.5}$$

for $x_{cc} = y_{cc} = 0$ and coefficients A, B, C as defined in paragraph: Position of neutral axis, above.

delta = 0.09526

Distance between the gravity center of contour of the section and area with compressive stresses in concrete:

delta2 = delta+ (1-Lambda) • h_c= 0.138862

To calculate a sum of stresses over the contour with compressive stresses in concrete we define a new coordinate system x_{n2}, y_{n2} (see picture above), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}; y_{n2c}) given in central axis coordinate system.

Equation of neutral axis in central axes coordinate system:

$$y_{cc} = -0.577350316 \cdot x_{cc} + 0.110000027$$

Equation of x_{n2} axis in central axes coordinate system

$$y_{nc} = -0.577350316 \cdot x_{nc} + delta2/\cos(Angle) = -0.577350316 \cdot x_{nc} + 0.16011327$$

For x_{n2c} = 0 and y_{n2c} = 0.16011327

Now we find coordinates of contour under stress solid in (x_{cc}, y_{cc}) coordinates system.

i	Contour x	Contour y	Contour in central axes coordinate system x_{cc}	Contour in central axes coordinate system y_{cc}	Equation of line containing edge_i, in central axes coordinate system	Intersection point of line containing edge_i and x_{n2} axis x_{cc}	Intersection point of line containing edge_i and x_{n2} axis y_{cc}	Is intersection on the contour?
1	0	0	-0,275	-0,225	x_{cc} = -0,275	-0,275	0,3189	Yes
2	0	0,6	-0,275	0,375	y_{cc} = 0,375	-0,3722	0,375	No
3	0,25	0,6	-0,025	0,375	x_{cc} = -0,025	-0,025	0,1745	Yes
4	0,25	0,25	-0,025	0,025	y_{cc} = 0,025	0,2340	0,025	Yes
5	0,7	0,25	0,425	0,025	x_{cc} = 0,425	0,425	-0,0853	Yes
6	0,7	0	0,425	-0,225	y_{cc} = -0,225	0,6670	-0,225	No
7	0	0	-0,275	-0,225	x_{cc} = -0,275	-0,275	0,3189	Yes

Where:
x_{cc} = x- x_{Cc}
y_{cc} = y- y_{Cc}
edge_i = edge between corners: i and i+1

Now we calculate area and gravity center of contour under compressive stresses in concrete in (x_{cc}, y_{cc}) coordinates system:

i	Contour with compressive stresses in concrete x_{cc}	y_{cc}	Contour with compressive stresses in concrete x	y	Area _c	Gravity center of contour corresponding to Area_c x_{01}	y_{01}	Static moments S_{cy}	S_{cx}
1	-0,275	0,375	0,000	0,600	0,1500	0,125	0,3	0,019	0,045
2	-0,025	0,375	0,250	0,600	0,0000	0,25	0,6	0	0
3	-0,025	0,175	0,250	0,400	0,0841	0,370	0,165	0,031	0,014
4	0,234	0,025	0,509	0,250	0,0477	0,605	0,125	0,029	0,006
5	0,425	0,025	0,700	0,250	0,0000	0,7	0,25	0	0
6	0,425	-0,085	0,700	0,140	-0,2393	0,281	0,191	-0,067	-0,046
7	-0,275	0,319	0,000	0,544	0,0000	0	0,544	0	0
8	-0,275	0,375	0,000	0,000	0,0000	0,125	0,3	0	0
				Total	0,0426			0,0115	0,0192

Where:
edge_i = edge between corners: i and i+1
Area_c_i = Area between edge_i and axes: x=x_i, x=x_(i+1) and y = 0
Area_c_i = 1/2 • (y_(i + 1) + y(i)) • (x_(i + 1) - x(i))
x_{01}_i = x_(i) + (x_(i + 1) - x_(i)) • (2 / 3 • y_(i + 1) + 1 / 3 • y_(i)) / (y_(i) + y_(i + 1)) if (y_(i) + y_(i + 1) ≠ 0 or
x_{01}_i = (x_(i + 1) + x_(i)) / 2 if (y_(i) + y_(i + 1) = 0
y_{01}_i = y_(i) / 2 + 0.5 • (x_{01}_i - x_(i)) • (y_(i + 1) - y_(i)) / (x_(i + 1) - x_(i)) if x_(i + 1) - x_(i) ≠ 0 or
y_{01}_i = (y_(i + 1)+y_(i))/2) if x_(i + 1) - x_(i) = 0
S_{cy}_i = Area_c_i • x_{01}_i - y_{cc}
S_{cx}_i = Area_c_i • y_{01}_i - x_{cc}

Compressed concrete area:

A_c = sum(Area_c_i) = <u>0.0426</u>

Gravity center of compressed concrete contour (gross) in central axes coordinate system

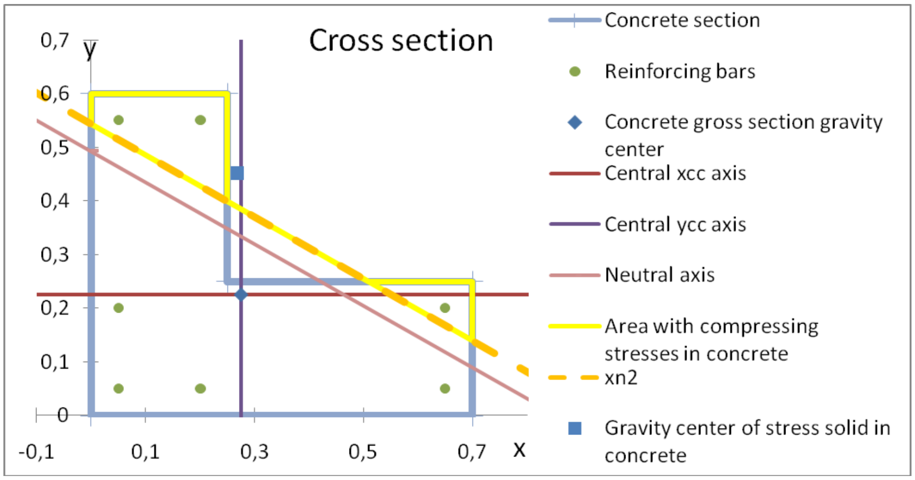
x_c = sum(S_{cy}_i)/ A_c – x_{Cc} = -0.0060
y_c = Sum(S_{cx}_i)/ A_c – y_{Cc} = -0.2260

Axial force in compressed part of concrete gross section

$N_{c1} = A_c$ • Sigma = 0.851993 e6

Center of gravity of stresses in concrete in central axes coordinate system:

$x_{cg} = (x_c \cdot N_c - x_{scg} \cdot N_{sc}) / (N_c - N_{sc})$ = -<u>0.006674</u>
$y_{cg} = (y_c \cdot N_c - y_{scg} \cdot N_{sc}) / (N_c - N_{sc})$ = 0<u>.226430</u>



Internal forces in concrete:

$N_c = N_{c'} - N_{sc} = \underline{833.14\ e3}$

$M_{xc} = -\ N_c \bullet y_{cg} = -\underline{188.65\ e3}$

$M_{yc} = N_{c'} \bullet x_{cg} = -\underline{5.56e3}$

- Results for reduced section:

  $N_f = N_c + N_s = 7\underline{24.47\ e3}$

  $M_{xf} = M_{xc} + M_{xs} = -\underline{289.83\ e3}$

  $M_{yf} = M_{yc} + M_{ys} = 2\underline{5.74\ e3}$

Small differences between results from Component and from manual calculations are below 1.5% and are due to precision of iterations.

16.4 Case 15b

Comparison of capacity forces for various models of concrete (obtained in Component / in manual calculations) for example as in Case 15a:

stress-strain model of concrete behavior:	rectangular	linear	bilinear	parabolic-rectangular	power-rectangular
parameters of concrete	as in Case 15a f_{cd} = 20e6	Eps_ult= 0,0035 f_{cd} = 20e6/0.8= 25e6	Eps_ult= 0,0035 Eps_c1= 0,0020 f_{cd} = 20e6/0.8= 25e6	Eps_ult= 0,0035 Eps_c1= 0,0020 f_{cd} = 20e6/0.8= 25e6	Eps_ult= 0,0035 Eps_c1= 0,0020 n = 1.5 f_{cd} = 20e6/0.8= 25e6
N_f / e3	725.91 / 724.47	658.70 / 657.37	737.18 / 736.31	760.14 / 759.21	751.24 / 750.30
M_{xf} / e3	-290.09 / -289.83	-263.20 / -262.98	-294.66 / -294.56	-303.83/ -303.72	-300.26 / -300.16
M_{yf} / e3	25.68 / 25.74	23.29 / 23.35	26.24 / 26.17	27.06 / 26.98	26.74 / 26.66

17 Case 16: Calculation of capacity state in symmetric section for fixed axial force

17.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N) and give state of strain and stress at the failure for fixed axial force N and M_x or M_y = 0.

17.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 16a)
- A bending in y direction (M_y) (Case 16b)
- A fixed axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

17.3 Case 16a

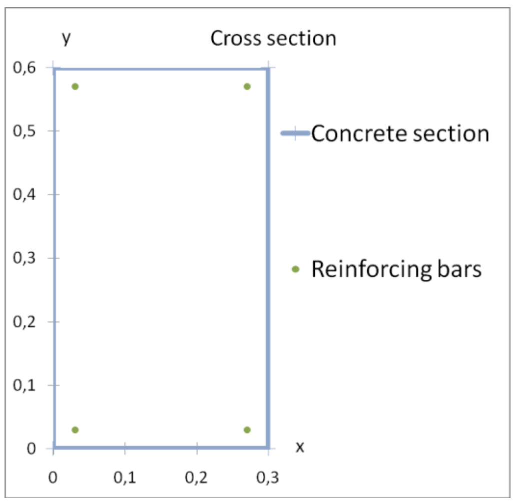
17.3.1 Data:

- Geometry:
  - Concrete section:

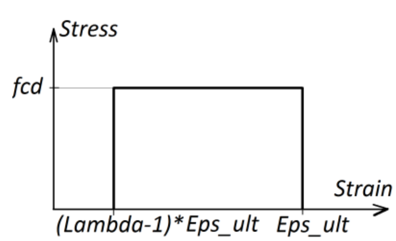
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

  - Reinforcing bars:

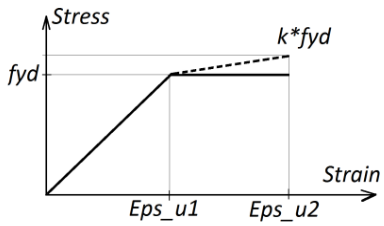
x = 0.03	y = 0.03	ϕ = 0.04
x = 0.03	y = 0.57	ϕ = 0.04
x = 0.27	y = 0.57	ϕ = 0.04
x = 0.27	y = 0.03	ϕ = 0.04



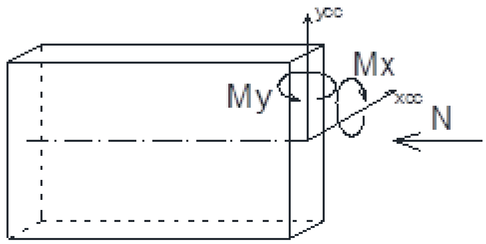
- Concrete parameters:
  - Design strength f_{cd} = 21.4 e6 • 0.8 = 17.12 e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 32 e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 310e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.025



- Forces:
  - N = 678.00e3
  - Mx
  - My



17.3.2 Search for:

- Results for reduced section:
  - Forces: N_f = 678.00e3, M_{xf}, M_{yf}
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_top, Eps_bot)

17.3.3 Results from Component

- Results for reduced section:
  N_f = 677.98 e3
  M_{xf} = -574.80 e3
  M_{yf} = 0.00 e3
  Eps_top = 0.003500
  Eps_bot = -0.008467
  Eps_stop = 0.002902
  Eps_sbot = -0.007869

Results from Component are consistent with a graph from book *"Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2"*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 360., ISBN 83-7125-136-X.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

17.4 Case 16b

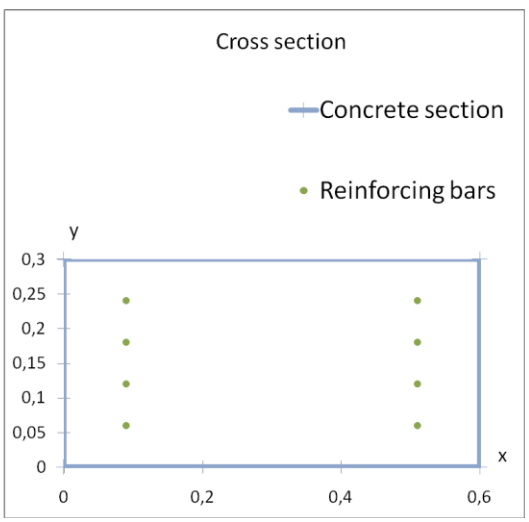
17.4.1 Data:

- Geometry:
  - Concrete section:

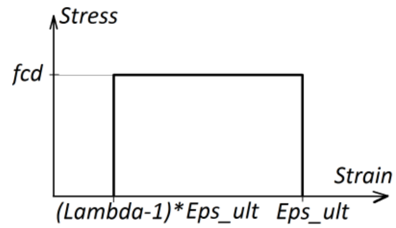
    x = 0.0 y = 0.0
    x = 0.0 y = 0.3
    x = 0.6 y = 0.3
    x = 0.6 y = 0.0

  - Reinforcing bars:

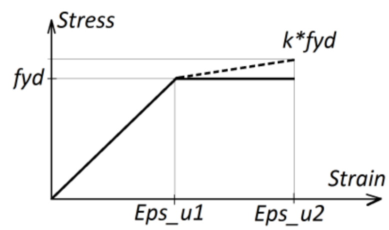
    x = 0.09 y = 0.06 ϕ = 0.036
    x = 0.09 y = 0.12 ϕ = 0.036
    x = 0.09 y = 0.18 ϕ = 0.036
    x = 0.09 y = 0.24 ϕ = 0.036
    x = 0.51 y = 0.06 ϕ = 0.036
    x = 0.51 y = 0.12 ϕ = 0.036
    x = 0.51 y = 0.18 ϕ = 0.036
    x = 0.51 y = 0.24 ϕ = 0.036



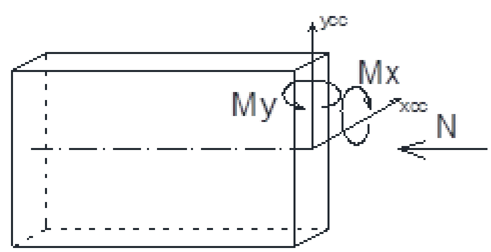
- Concrete parameters:
  - Design strength f_{cd} = 21.4 e6 • 0.8= 17.12 e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 32 e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd} = 420e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.025



- Forces:
  - N = 1700.00e3
  - Mx
  - My



17.4.2 Search for:

- Results for reduced section:
  - Forces: N_f = 1700.00e3, M_{xf}, M_{yf}
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_top, Eps_bot)

17.4.3 Results from Component

- Results for reduced section:
  N_f = 1700.07 e3
  M_{xf} = 0.00 e3
  M_{yf} = 859.56 e3
  Eps_top = 0.003500
  Eps_bot = -0.002581

Eps_stop = 0.002588
Eps_sbot = -0.001669

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

18 Case 17: Calculation of capacity state in symmetric section for fixed moment

18.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (Mx or My) and give state of strain and stress at the failure for fixed moment (Mx or My) and another moment= 0.

18.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 17a) with fixed moment.
- A bending in y direction (M_y) (Case 17b) with fixed moment.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

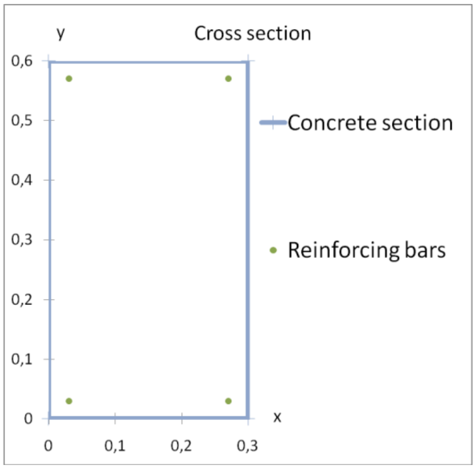
- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y) Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

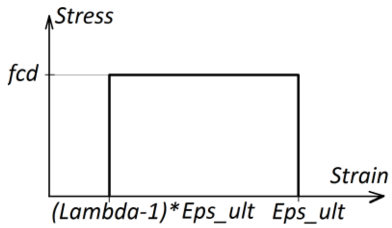
18.3 Case 17a

18.3.1 Data:

- Geometry:
    - Concrete section:
      x = 0.0 y = 0.0
      x = 0.0 y = 0.6
      x = 0.3 y = 0.6
      x = 0.3 y = 0.0
    - Reinforcing bars:
      x = 0.03 y = 0.03 ϕ = 0.04
      x = 0.03 y = 0.57 ϕ = 0.04
      x = 0.27 y = 0.57 ϕ = 0.04
      x = 0.27 y = 0.03 ϕ = 0.04

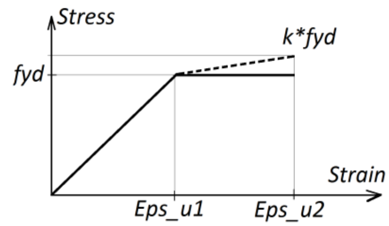


- Concrete parameters:
    - Design strength f_{cd} = 21.4 e6 • 0.8 = 17.12 e6
    - Effective height reduction factor Lambda = 0.8
    - Modulus of elasticity E_c = 32 e9
    - Strain-stress model: rectangular
    - Strain ultimate limit Eps_ult = 0.0035

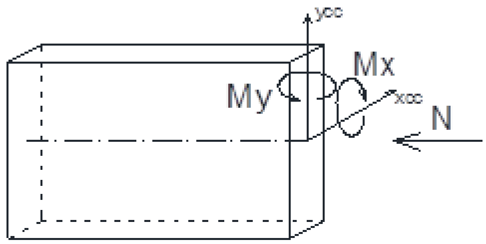


- Steel parameters:
    - Design strength f_{yd} = 310e6
    - Hardening factor k = 1.0

- Modulus of elasticity E_s = 200e9
- Strain ultimate limit Eps_u2 = 0.025



- Forces:
  - N
  - M_x = 184.90e3
  - M_y



18.3.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf} = 184.90 e3, M_{yf}
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_top, Eps_bot)

18.3.3 Results from Component

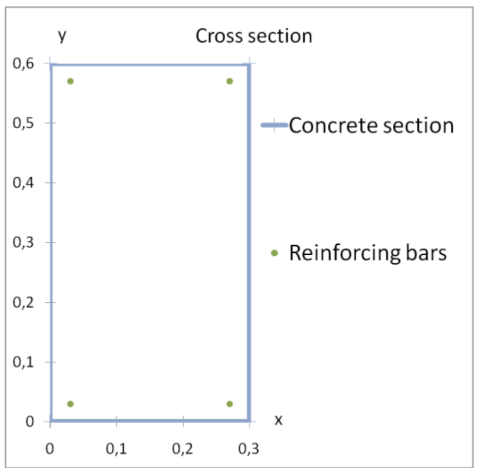
- Results for reduced section:
  N_f = 3876.03 e3
  M_{xf} = 184.90 e3
  M_{yf} = 0.00 e3
  Eps_top = 0.003500
  Eps_bot = 0.000486
  Eps_stop = 0.003349
  Eps_sbot = 0.000637

Results from Component are consistent with a graph from book *"Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2"*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 1%.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

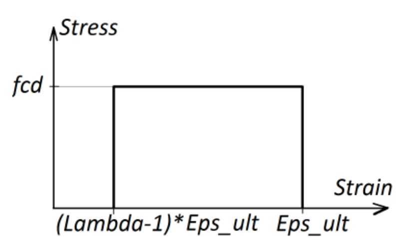
18.4 Case 17b

18.4.1 Data:

- Geometry:
  - Concrete section:
    x = 0.0 y = 0.0
    x = 0.0 y = 0.6
    x = 0.3 y = 0.6
    x = 0.3 y = 0.0
  - Reinforcing bars:
    x = 0.03 y = 0.03 ϕ = 0.032
    x = 0.03 y = 0.57 ϕ = 0.032
    x = 0.27 y = 0.57 ϕ = 0.032
    x = 0.27 y = 0.03 ϕ = 0.032

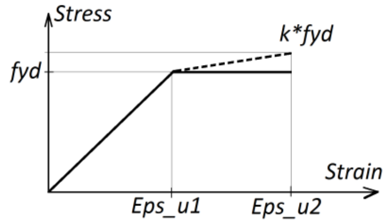


- Concrete parameters:
  - Design strength f_{cd} = 21.4 e6 • 0.8 = 17.12 e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 32 e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035

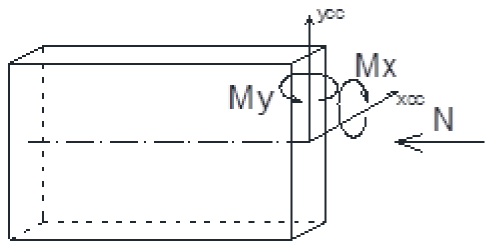


- Steel parameters:
  - Design strength f_{yd} = 310e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9

  - Strain ultimate limit Eps_u2 = 0.025



- Forces:
  - N
  - Mx
  - M_y = 138.67e3



18.4.2 Search for:

- Results for reduced section:
  - Forces: N_f, M_{xf}, M_{yf} = 138.67e3
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_top, Eps_bot)

18.4.3 Results from Component

- Results for reduced section:
  N_f = 2826.85 e3
  M_{xf} = 0.00 e3
  M_{yf} = 138.67 e3
  Eps_top = 0.003500
  Eps_bot = -0.000204
  Eps_stop = 0.003130
  Eps_sbot = 0.000167

Results from Component are consistent with a graph from book *"Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2"*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 363., ISBN 83-7125-136-X. Differences between results are smaller than 2%.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

19 Case 18: Calculation of capacity state in symmetric section for fixed negative (tensioning) axial force

19.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (negative N) and give state of strain and stress at the failure for fixed axial force N and Mx or $M_y = 0$.

19.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 18a)
- A bending in y direction (M_y) (Case 18b)
- A fixed negative axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N_s)
  - Moment with respect to x_{cc} axis (M_{xs})
  - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
  - Area of contour under compressive stress (A_c)
  - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
  - Axial force (N_c)
  - Moment with respect to x_{cc} axis (M_{xc})
  - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
  - Axial force at the failure related to acting forces (N_f = alpha • N)
  - Moment at the failure related to acting forces, with respect to x_{cc} axis (M_{xf} = alpha • M_x)
  - Moment at the failure related to acting forces, with respect to y_{cc} axis (M_{yf} = alpha • M_y)
    Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_stop, Eps_sbot)

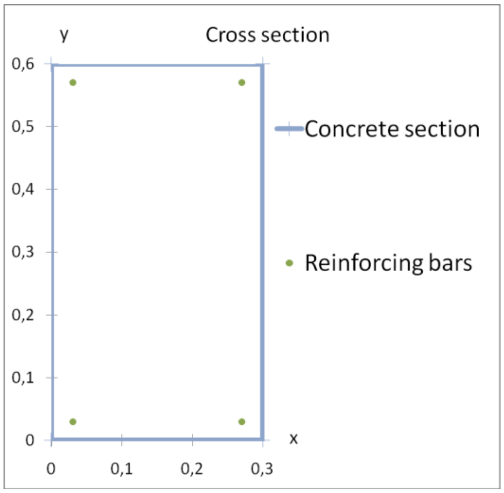
Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

19.3 Case 18a

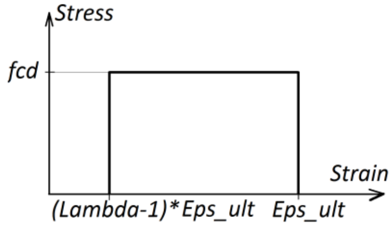
19.3.1 Data:

- Geometry:
    - Concrete section:
      x = 0.0 y = 0.0
      x = 0.0 y = 0.6
      x = 0.3 y = 0.6
      x = 0.3 y = 0.0

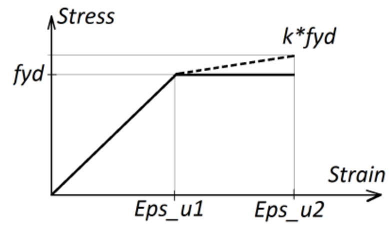
- Reinforcing bars:
  x = 0.03 y = 0.03 ϕ = 0.04
  x = 0.03 y = 0.57 ϕ = 0.04
  x = 0.27 y = 0.57 ϕ = 0.04
  x = 0.27 y = 0.03 ϕ = 0.04



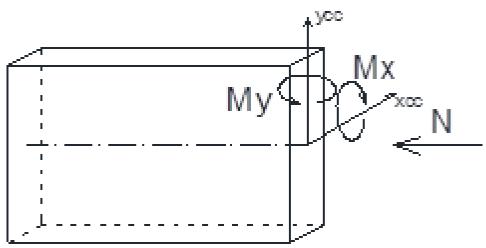
- Concrete parameters:
    - Design strength f_{cd} = 21.4 e6 • 0.8 = 17.12 e6
    - Effective height reduction factor Lambda = 0.8
    - Modulus of elasticity E_c = 32 e9
    - Strain-stress model: rectangular
    - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
    - Design strength f_{yd} = 310e6
    - Hardening factor k = 1.0
    - Modulus of elasticity E_s = 200e9
    - Strain ultimate limit Eps_u2 = 0.010



- Forces:
    - N = -493.06 e3
    - Mx
    - My



19.3.2 Search for:

- Results for reduced section:
    - Forces: N_f = -493.06 e3, M_{xf}, M_{yf}
    - Extreme strains in concrete (Eps_top, Eps_bot)
    - Extreme strains in steel (Eps_top, Eps_bot)

19.3.3 Results from Component

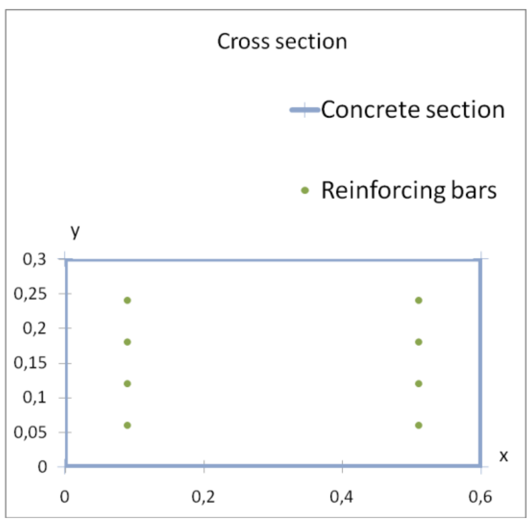
- Results for reduced section:
  N_f = -493.05 e3
  M_{xf} = -288.16 e3
  M_{yf} = 0.00 e3
  Eps_top = 0.001044
  Eps_bot = -0.010581
  Eps_stop = 0.000463
  Eps_sbot = -0.010000

Results from Component are consistent with a graph from book *"Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2"*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 3%.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

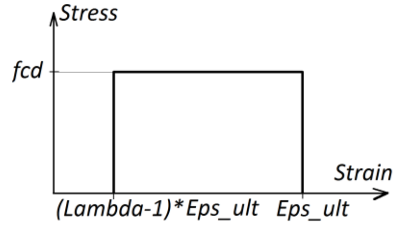
19.4 Case 18b

19.4.1 Data:

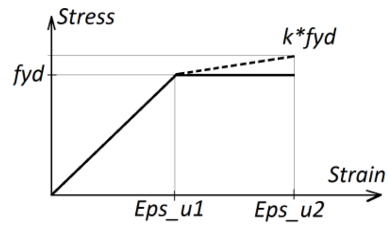
- Geometry:
  - Concrete section:
    x = 0.0 y = 0.0
    x = 0.0 y = 0.3
    x = 0.6 y = 0.3
    x = 0.6 y = 0.0
  - Reinforcing bars:
    x = 0.09 y = 0.06 ϕ = 0.036
    x = 0.09 y = 0.12 ϕ = 0.036
    x = 0.09 y = 0.18 ϕ = 0.036
    x = 0.09 y = 0.24 ϕ = 0.036
    x = 0.51 y = 0.06 ϕ = 0.036
    x = 0.51 y = 0.12 ϕ = 0.036
    x = 0.51 y = 0.18 ϕ = 0.036
    x = 0.51 y = 0.24 ϕ = 0.036



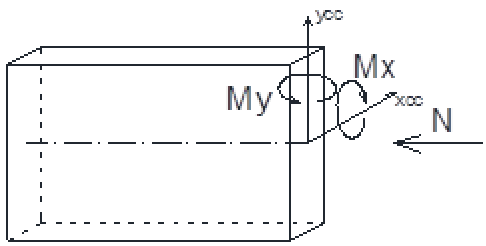
- Concrete parameters:
  - Design strength f_{cd} = 21.4 e6 • 0.8 = 17.12 e6
  - Effective height reduction factor Lambda = 0.8
  - Modulus of elasticity E_c = 32 e9
  - Strain-stress model: rectangular
  - Strain ultimate limit Eps_ult = 0.0035



- Steel parameters:
  - Design strength f_{yd}= 420e6
  - Hardening factor k = 1.0
  - Modulus of elasticity E_s = 200e9
  - Strain ultimate limit Eps_u2 = 0.010



- Forces:
  - N = -862.85 e3
  - Mx
  - My



19.4.2 Search for:

- Results for reduced section:
  - Forces: N_f = -862.85 e3, M_{xf}, M_{yf}
  - Extreme strains in concrete (Eps_top, Eps_bot)
  - Extreme strains in steel (Eps_top, Eps_bot)

19.4.3 Results from Component

- Results for reduced section:
  N_f = -862.80 e3
  M_{xf} = 0.00 e3
  M_{yf} = 554.30 e3
  Eps_top = 0.002908
  Eps_bot = -0.012278
  Eps_stop = 0.000630

Eps_sbot = -0.010000

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.
These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.