

## Table of Contents

1	Introduction .....	9
2	Case 1: Cross section characteristics .....	13
2.1	Subject: .....	13
2.2	Summary: .....	13
2.3	Data: .....	13
2.4	Search for: .....	14
2.5	Results from Component .....	14
2.6	Results (manual calculations): .....	15
3	Case 2: Calculation of internal forces for given state of strain in the section and rectangular model of concrete .....	17
3.1	Subject: .....	17
3.2	Summary: .....	17
3.3	Case 2a .....	17
3.3.1	Data: .....	17
3.3.2	Search for : .....	19
3.3.3	Results from Component .....	19
3.3.4	Results (manual calculations): .....	19
3.4	Case 2b .....	23
3.4.1	Data: .....	23
3.4.2	Search for: .....	24
3.4.3	Results from Component .....	24
3.4.4	Results (manual calculations): .....	24
3.5	Case 2c .....	27
3.5.1	Data: .....	27
3.5.2	Search for: .....	29
3.5.3	Results from Component .....	29
3.5.4	Results (manual calculations): .....	29
4	Case 3: Calculation of internal forces for given state of strain in the section and linear model of concrete .....	33

4.1	Subject: .....	33
4.2	Summary: .....	33
4.3	Case 3a .....	33
4.3.1	Data:.....	33
4.3.2	Search for: .....	35
4.3.3	Results from Component.....	35
4.3.4	Results (manual calculations): .....	36
4.4	Case 3b.....	39
4.4.1	Data:.....	39
4.4.2	Search for: .....	41
4.4.3	Results from Component.....	41
4.4.4	Results (manual calculations): .....	41
4.5	Case 3c .....	45
4.5.1	Data:.....	45
4.5.2	Search for: .....	47
4.5.3	Results from Component.....	47
4.5.4	Results (manual calculations): .....	47
5	Case 4: Calculation of internal forces for given state of strain in the section and bilinear model of concrete .....	51
5.1	Subject: .....	51
5.2	Summary: .....	51
5.3	Case 4a .....	51
5.3.1	Data:.....	51
5.3.2	Search for: .....	53
5.3.3	Results from Component.....	53
5.3.4	Results (manual calculations): .....	54
5.4	Case 4b .....	57
5.4.1	Data:.....	57
5.4.2	Search for: .....	59
5.4.3	Results from Component.....	59
5.4.4	Results (manual calculations): .....	59

5.5	Case 4c .....	63
5.5.1	Data:.....	63
5.5.2	Search for:.....	65
5.5.3	Results from Component.....	65
5.5.4	Results (manual calculations): .....	65
6	Case 5: Calculation of internal forces for given state of strain in the section and parabolic-rectangular model of concrete .....	69
6.1	Subject: .....	69
6.2	Summary:.....	69
6.3	Case 5a .....	69
6.3.1	Data:.....	69
6.3.2	Search for:.....	71
6.3.3	Results from Component.....	71
6.3.4	Results (manual calculations): .....	71
6.4	Case 5b.....	75
6.4.1	Data:.....	75
6.4.2	Search for:.....	77
6.4.3	Results from Component.....	77
6.4.4	Results (manual calculations): .....	77
6.5	Case 5c .....	81
6.5.1	Data:.....	81
6.5.2	Search for:.....	83
6.5.3	Results from Component.....	83
6.5.4	Results (manual calculations): .....	83
7	Case 6: Calculation of internal forces for given state of strain in the section and power-rectangular model of concrete.....	91
7.1	Subject: .....	91
7.2	Summary:.....	91
7.3	Case 6a .....	91
7.3.1	Data:.....	91
7.3.2	Search for: .....	93

7.3.3	Results from Component .....	93
7.3.4	Results (manual calculations): .....	93
7.4	Case 6b .....	100
7.4.1	Data:.....	100
7.4.2	Search for: .....	101
7.4.3	Results from Component .....	101
7.4.4	Results (manual calculations): .....	102
7.5	Case 6c .....	108
7.5.1	Data:.....	108
7.5.2	Search for: .....	110
7.5.3	Results from Component .....	110
7.5.4	Results (manual calculations): .....	111
8	Case 7 Calculation of internal forces for given state of strain in the section and inclined branch model of steel .....	118
8.1	Subject: .....	118
8.2	Summary: .....	118
8.3	Data:.....	118
8.4	Search for:.....	120
8.5	Results from Component .....	120
8.6	Results (manual calculations): .....	120
9	Case 8: Calculation of internal forces in asymmetric section for given state of strain with horizontal neutral axis.....	124
9.1	Subject: .....	124
9.2	Summary: .....	124
9.3	Data:.....	124
9.4	Search for:.....	126
9.5	Results from Component .....	126
9.6	Results (manual calculations): .....	126
10	Case 9: Calculation of internal forces in symmetric section for given state of strain with inclined neutral axis.....	132
10.1	Subject: .....	132

10.2	Summary: .....	132
10.3	Data: .....	132
10.4	Search for: .....	134
10.5	Results from Component .....	134
10.6	Results (manual calculations): .....	134
11	Case 10: Calculation of capacity state in symmetric section for bending moment Mx.....	146
11.1	Subject: .....	146
11.2	Summary: .....	146
11.3	Case 10a .....	147
11.3.1	Data: .....	147
11.3.2	Search for: .....	148
11.3.3	Results from Component .....	148
11.3.4	Results (manual calculations): .....	149
11.4	Case 10b.....	152
11.4.1	Data: .....	152
11.4.2	Search for: .....	153
11.4.3	Results from Component .....	153
11.4.4	Results (manual calculations): .....	154
12	Case 11: Calculation of capacity state in symmetric section for bending moment My.....	155
12.1	Subject: .....	155
12.2	Summary: .....	155
12.3	Case 11a .....	156
12.3.1	Data: .....	156
12.3.2	Search for: .....	157
12.3.3	Results from Component .....	157
12.3.4	Results (manual calculations): .....	158
12.4	Case 11b.....	161
12.4.1	Data: .....	161
12.4.2	Search for: .....	163
12.4.3	Results from Component .....	163

12.4.4	Results (manual calculations): .....	163
12.5	Case 11c .....	164
12.5.1	Data:.....	164
12.5.2	Search for:.....	165
12.5.3	Results from Component.....	166
12.5.4	Results (manual calculations): .....	166
13	Case 12: Calculation of capacity state in symmetric section for bending moment Mx with axial force	167
13.1	Subject: .....	167
13.2	Summary:.....	167
13.3	Case 12a .....	168
13.3.1	Data:.....	168
13.3.2	Search for:.....	169
13.3.3	Results from Component.....	169
13.3.4	Results (manual calculations): .....	170
13.4	Case 12b.....	174
13.4.1	Data:.....	174
13.4.2	Search for:.....	175
13.4.3	Results from Component.....	175
13.4.4	Results (manual calculations): .....	176
13.5	Case 12c .....	176
13.5.1	Data:.....	176
13.5.2	Search for:.....	178
13.5.3	Results from Component.....	178
13.5.4	Results (manual calculations): .....	178
14	Case 13: Calculation of capacity state in symmetric section for bending moment My with axial force	180
14.1	Subject: .....	180
14.2	Summary:.....	180
14.3	Case 13a .....	181
14.3.1	Data:.....	181

14.3.2	Search for: .....	182
14.3.3	Results from Component .....	182
14.3.4	Results (manual calculations): .....	183
14.4	Case 13b .....	186
14.4.1	Data: .....	186
14.4.2	Search for: .....	187
14.4.3	Results from Component .....	187
14.4.4	Results (manual calculations): .....	188
15	Case 14: Calculation of capacity state in symmetric section for bidirectional bending with axial force	189
15.1	Subject: .....	189
15.2	Summary: .....	189
15.3	Data: .....	190
15.4	Search for: .....	191
15.5	Results from Component .....	191
16	Case 15: Calculation of capacity state in asymmetric section for bidirectional bending with axial force	193
16.1	Subject: .....	193
16.2	Summary: .....	193
16.3	Case 15a .....	194
16.3.1	Data: .....	194
16.3.2	Search for: .....	195
16.3.3	Results from Component .....	196
16.3.4	Results (manual calculations): .....	196
16.4	Case 15b .....	204
17	Case 16: Calculation of capacity state in symmetric section for fixed axial force .....	205
17.1	Subject: .....	205
17.2	Summary: .....	205
17.3	Case 16a .....	206
17.3.1	Data: .....	206
17.3.2	Search for: .....	207

17.3.3	Results from Component .....	207
17.4	Case 16b .....	208
17.4.1	Data: .....	208
17.4.2	Search for: .....	209
17.4.3	Results from Component .....	209
18	Case 17: Calculation of capacity state in symmetric section for fixed moment .....	211
18.1	Subject: .....	211
18.2	Summary: .....	211
18.3	Case 17a .....	212
18.3.1	Data: .....	212
18.3.2	Search for: .....	213
18.3.3	Results from Component .....	213
18.4	Case 17b .....	214
18.4.1	Data: .....	214
18.4.2	Search for: .....	215
18.4.3	Results from Component .....	215
19	Case 18: Calculation of capacity state in symmetric section for fixed negative (tensioning) axial force	216
19.1	Subject: .....	216
19.2	Summary: .....	216
19.3	Case 18a .....	217
19.3.1	Data: .....	217
19.3.2	Search for: .....	218
19.3.3	Results from Component .....	218
19.4	Case 18b .....	220
19.4.1	Data: .....	220
19.4.2	Search for: .....	221
19.4.3	Results from Component .....	221

# 1 Introduction

This document shows examples of use of Autodesk.CodeChecking.Concrete named later “Component”.

Each use case includes paragraphs:

- Subject – short description of what we want to calculate and what data do we need
- Summary – detailed list of available results obtained for the list of data
- Data – example data
- Search for – searched results
- Results from Component – results obtained from Component
- Results (manual calculations) – results obtained using manual calculations with calculation path. Some examples (Case 14, Cases 16-18) compare Component results with results from the literature (design manuals) and other program.

Examples below have physical sense for metric system units.

To see calculation methods, assumptions used by this Component and rules concerning units, please refer to the User Manual of the Code Checking Framework SDK.

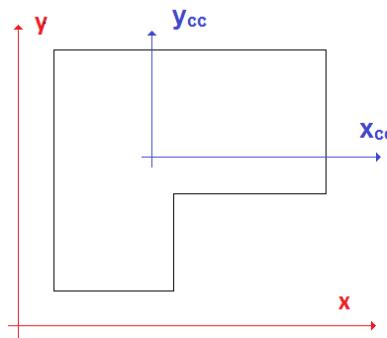
Cases shown in this document are related to the C# example provided with this SDK.

Every case has its own implementation in the single \*.cs file with the name corresponding to the example ( e.g. file “Case01.cs“ corresponding to example Case 1). The project includes a dialog which allows selection of examples, run one or all of the examples and presents the results of calculation.

Main symbols used in this document:

$x$  – the first axis of the initial coordinate system ( $x, y$ )

$y$  – the second axis of the initial coordinate system ( $x, y$ )



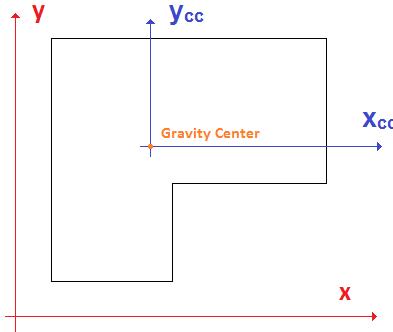
$A_c$  – concrete gross section area or area of contour under compressive stress

$x_{cc}$  – concrete gross section gravity center first coordinate in ( $x, y$ ) coordinate system

$y_{cc}$  – concrete gross section gravity center second coordinate in ( $x, y$ ) coordinate system

$x_{cc}$  - central axis of concrete gross section parallel to  $x$  axis or the first coordinate in ( $x_{cc}, y_{cc}$ ) coordinate system

$y_{cc}$  - central axis of concrete gross section parallel to y axis or the second coordinate in  $(x_{cc}, y_{cc})$  coordinate system



$I_{cx}$  – moment of inertia of concrete gross section with respect to  $x_{cc}$  axis

$I_{cy}$  – moment of inertia of concrete gross section with respect to  $y_{cc}$  axis

$A_s$  – reinforcement bars area

$x_s$  – reinforcement bars gravity center first coordinate in  $(x,y)$  coordinate system

$y_s$  – reinforcement bars gravity center second coordinate in  $(x,y)$  coordinate system

$I_{sx}$  – moment of inertia of reinforcement bars with respect to  $x_{cc}$  axis

$I_{sy}$  – moment of inertia of reinforcement bars with respect to  $y_{cc}$  axis

$A_{eff}$  – reduced cross section area

$x_c$  – reduced cross section gravity center first coordinate in  $(x,y)$  coordinate system

$y_c$  – reduced cross section gravity center second coordinate in  $(x,y)$  coordinate system

$I_{effx}$  – moment of inertia of reduced cross section with respect to x axis

$I_{effy}$  – moment of inertia of reduced cross section with respect to y axis

$N$  – internal axial force in reduced cross section

$M_x$  – moment of internal forces in reduced section with respect to  $x_{cc}$  axis

$M_y$  – moment of internal forces in reduced section with respect to  $y_{cc}$  axis

$N_s$  – internal axial force in reinforcing bars

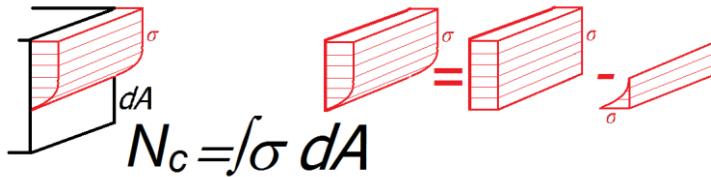
$M_{xs}$  – moment of internal forces in reinforcing bars with respect to  $x_{cc}$  axis

$M_{ys}$  – moment of internal forces in reinforcing bars with respect to  $y_{cc}$  axis

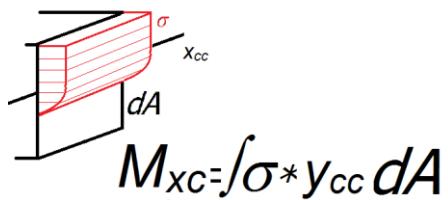
$x_{cg}$  – center of gravity of stresses in the concrete with respect to  $x_{cc}$  axis

$y_{cg}$  – center of gravity of stresses in the concrete with respect to  $y_{cc}$  axis

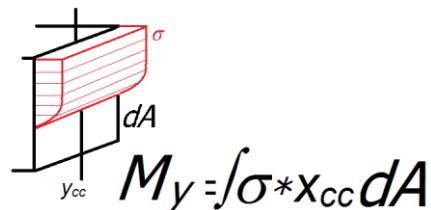
$N_c$  – internal axial force in the concrete part of a section, calculated as a volume of solid of stresses as shown below:



$M_{xc}$  – moment of internal forces in the concrete part of a section with respect to  $x_{cc}$  axis



$M_{yc}$  – moment of internal forces in the concrete part of a section with respect to  $y_{cc}$  axis



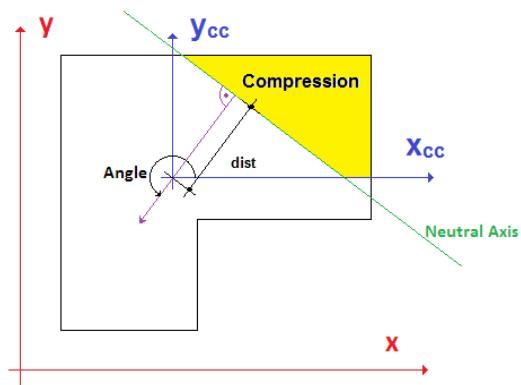
Eps\_bot – a strain in the most tensioned point of concrete section

Eps\_top – a strain in the most compressed point of concrete section

Eps\_sbot – a strain in the most tensioned rebar

Eps\_stop – a strain in the most compressed rebar

Angle – angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber



dist – distance between neutral axis and a gravity center of concrete gross section

N – internal axial force in cross section at the failure

$M_{xf}$  – moment of internal forces in section with respect to  $x_{cc}$  axis at the failure

$M_{yf}$  – moment of internal forces in section with respect to  $y_{cc}$  axis at the failure

sigma, sigma\_i, Sigma, Sigma\_max, Sigma\_min – stress in rebar or in concrete, description in the text

eps, eps\_i – stress in rebar or in concrete, description in the text

## 2 Case 1: Cross section characteristics

### 2.1 Subject:

Read cross section geometry, positions of reinforcing bars inside the section, parameters of concrete and steel and give cross section characteristics

### 2.2 Summary:

This sample gives as results the cross section characteristics.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- cross section characteristics:
  - Concrete section gross area ( $A_c$ ) with the coordinates of its gravity center ( $x_{cc}$ ,  $y_{cc}$ )
  - Moments of inertia ( $I_{cx}$ ,  $I_{cy}$ )
  - Reinforcing bars area ( $A_s$ ) with coordinates of its gravity center ( $x_s$ ,  $y_s$ )
  - Moments of inertia ( $I_{sx}$ ,  $I_{sy}$ )
  - Reduced cross section characteristics ( $A_{eff}$ ,  $x_c$ ,  $y_c$ ,  $I_{effx}$ ,  $I_{effy}$ ).

### 2.3 Data:

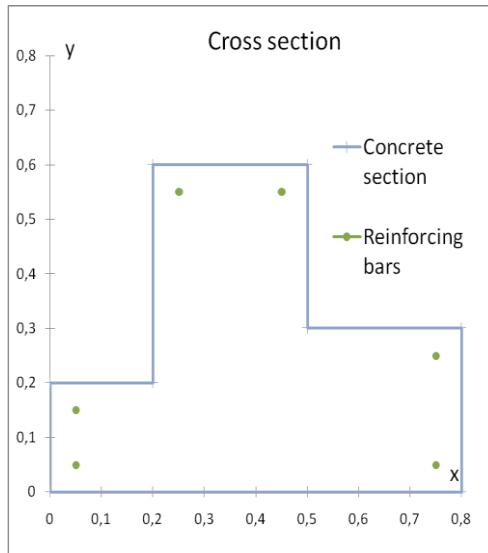
- Geometry:

- Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.2$
$x = 0.2$	$y = 0.2$
$x = 0.2$	$y = 0.6$
$x = 0.5$	$y = 0.6$
$x = 0.5$	$y = 0.3$
$x = 0.8$	$y = 0.3$
$x = 0.8$	$y = 0.0$

- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.010$
$x = 0.75$	$y = 0.05$	$\phi = 0.012$
$x = 0.75$	$y = 0.25$	$\phi = 0.020$
$x = 0.45$	$y = 0.55$	$\phi = 0.050$
$x = 0.25$	$y = 0.55$	$\phi = 0.020$
$x = 0.05$	$y = 0.15$	$\phi = 0.015$



- Concrete parameters:  
Modulus of elasticity  $E_c = 30e9$
- Steel parameters:  
Modulus of elasticity  $E_s = 200e9$

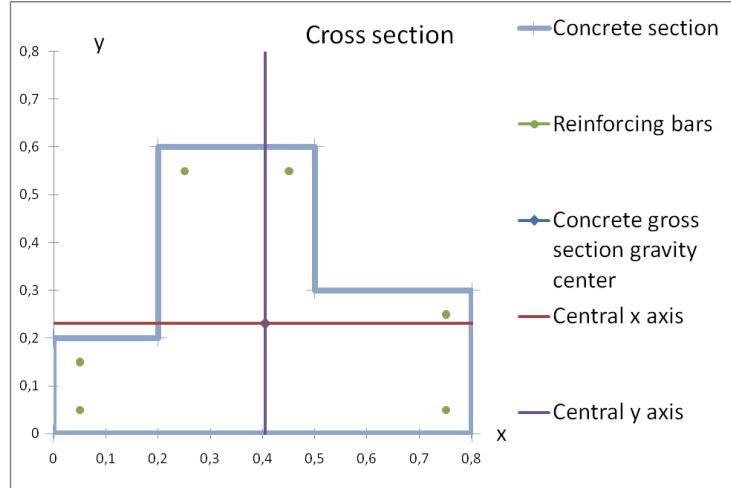
## 2.4 Search for:

- Concrete gross section characteristics:  $A_c, x_{Cc}, y_{Cc}, I_{cx}, I_{cy}$ ,
- Reinforcing bars characteristics:  $A_s, x_s, y_s, I_{sx}, I_{sy}$ ,
- Reduced cross section characteristics:  $A_{eff}, x_c, y_c, I_{effx}, I_{effy}$ .

## 2.5 Results from Component

- Concrete gross section characteristics:  
 $A_c = 0.310000$   
 $x_{Cc} = 0.40483871$   
 $y_{Cc} = 0.23064516$   
 $I_{cx} = 0.008342$   
 $I_{cy} = 0.011826$
- Reinforcing bars characteristics:  
 $A_s = 0.002960$   
 $x_s = 0.43758291$   
 $y_s = 0.46191298$   
 $I_{sx} = 0.000240$   
 $I_{sy} = 0.000095$
- Reduced cross section characteristics:  
 $A_{eff} = 0.326774$   
 $x_c = 0.40651956$   
 $y_c = 0.24251681$   
 $I_{effx} = 0.009701$

$$I_{effy} = 0.012362$$



## 2.6 Results (manual calculations):

- Concrete gross section characteristics:

Area:

$$A_c = 0.2 \cdot 0.2 + 0.3 \cdot 0.6 + 0.3 \cdot 0.3 = 0.31$$

Static moments:

$$S_{cx} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.3 + 0.3 \cdot 0.3 \cdot 0.15 = 0.0715$$

$$S_{cy} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.35 + 0.3 \cdot 0.3 \cdot 0.65 = 0.1255$$

Gravity center:

$$x_{Cc} = S_{cy} / A_c = 0.40483871$$

$$y_{Cc} = S_{cx} / A_c = 0.23064516$$

Moments of inertia:

$$I_x = 0.2 \cdot 0.2^3 / 12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.3 \cdot 0.6^3 / 12 + 0.3 \cdot 0.6 \cdot 0.3^2 + 0.3 \cdot 0.3^3 / 12 + 0.3 \cdot 0.3 \cdot 0.15^2 = 0.024833$$

$$I_y = 0.2 \cdot 0.2^3 / 12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.6 \cdot 0.3^3 / 12 + 0.3 \cdot 0.6 \cdot 0.35^2 + 0.3 \cdot 0.3^3 / 12 + 0.3 \cdot 0.3 \cdot 0.65^2 = 0.062633$$

Central moments of inertia:

$$I_{cx} = I_x - A_c \cdot y_{Cc}^2 = 0.008342$$

$$I_{cy} = I_y - A_c \cdot x_{Cc}^2 = 0.011826$$

- Reinforcing bars characteristics:

i	x	y	$\phi$	$A_s = \pi \cdot \phi^2 / 4$	Static moment $S_{sy} = A_s \cdot x$	Static moment $S_{sx} = A_s \cdot y$	Coordinates in $x_{cc}$ , $y_{cc}$ system $x_{cc} = x - x_{Cc}$ $y_{cc} = y - y_{Cc}$	$I_{sx} = \pi \cdot \phi^4 / 64 + A_s \cdot y_{cc}^2$	$I_{sy} = \pi \cdot \phi^4 / 64 + A_s \cdot x_{cc}^2$
1	0,05	0,05	0,01	0,000079	0,000004	0,000004	-0,3548	-0,1806	0,000003
2	0,75	0,05	0,012	0,000113	0,000085	0,000006	0,3452	-0,1806	0,000004
3	0,75	0,25	0,02	0,000314	0,000236	0,000079	0,3452	0,0194	0,000000
4	0,45	0,55	0,05	0,001963	0,000884	0,001080	0,0452	0,3194	0,000201
5	0,25	0,55	0,02	0,000314	0,000079	0,000173	-0,1548	0,3194	0,000032
6	0,05	0,15	0,015	0,000177	0,000009	0,000027	-0,3548	-0,0806	0,000001
			Total:	0,002960	0,001295	0,001367			0,000240
									0,000095

Area:

$$A_s = 0,002960$$

Static moments:

$$S_{sx} = 0,001367$$

$$S_{sy} = 0,001295$$

Gravity center:

$$x_s = S_{sy} / A = 0,43758291$$

$$y_s = S_{sx} / A = 0,46191298$$

Moments of inertia:

$$I_{sx} = 0,000240$$

$$I_{sy} = 0,000095$$

- Reduced cross section characteristics:

Area:

$$A_{eff} = A_c + A_s \cdot (E_s / E_c - 1) = 0,326774$$

Static moments:

$$S_x = S_{cx} + S_{sx} \cdot (E_s / E_c - 1) = 0,079246$$

$$S_y = S_{cy} + S_{sy} \cdot (E_s / E_c - 1) = 0,132838$$

Gravity center:

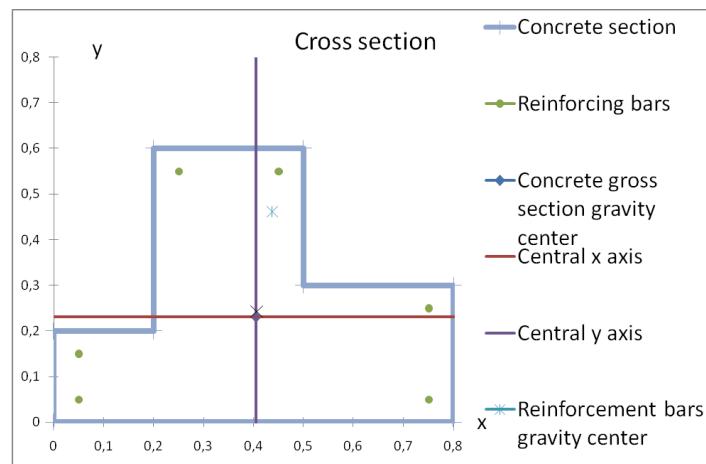
$$x_c = S_y / A = 0,4065$$

$$y_c = S_x / A = 0,2425$$

Moments of inertia:

$$I_{effx} = I_{cx} + I_{sx} \cdot (E_s / E_c - 1) = 0,009701$$

$$I_{effy} = I_{cy} + I_{sy} \cdot (E_s / E_c - 1) = 0,012362$$



### 3 Case 2: Calculation of internal forces for given state of strain in the section and rectangular model of concrete

#### 3.1 *Subject:*

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

#### 3.2 *Summary:*

This sample gives as results the internal forces for:

- A rectangular concrete section with symmetric reinforcement (Case 2a and Case 2b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 2c)
- A rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 2a)
- A section under unidirectional bending with a part of section under compression (Case 2b and Case 2c)
- A state of strain with ultimate compressive strain in concrete (Case 2a and Case 2b)
- A strain below the ultimate compressive strain in concrete (Case 2c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

---

#### 3.3 *Case 2a*

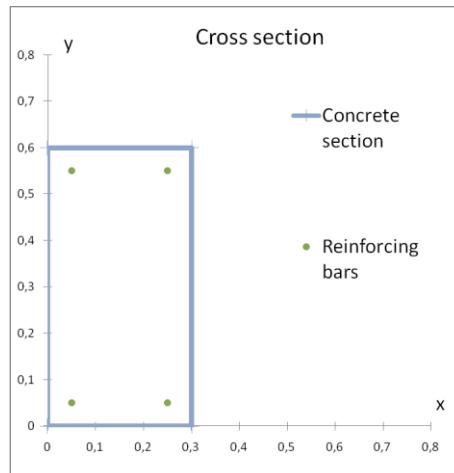
##### 3.3.1 *Data:*

- Geometry:
  - Concrete section:

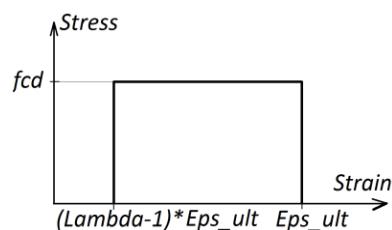
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

- Reinforcing bars:

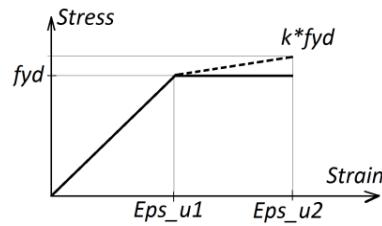
$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$



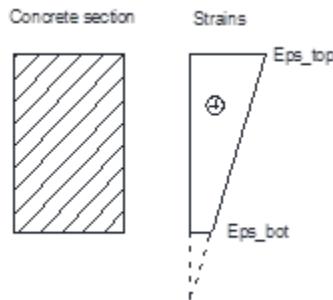
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Strains:
  - Bottom strain  $Eps\_bot = 0.0005$
  - Top strain  $Eps\_top = 0.0035$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



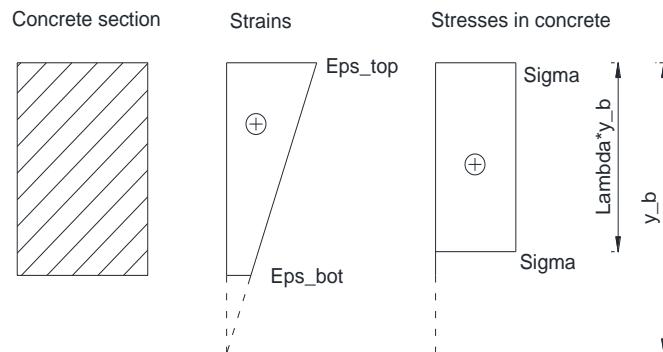
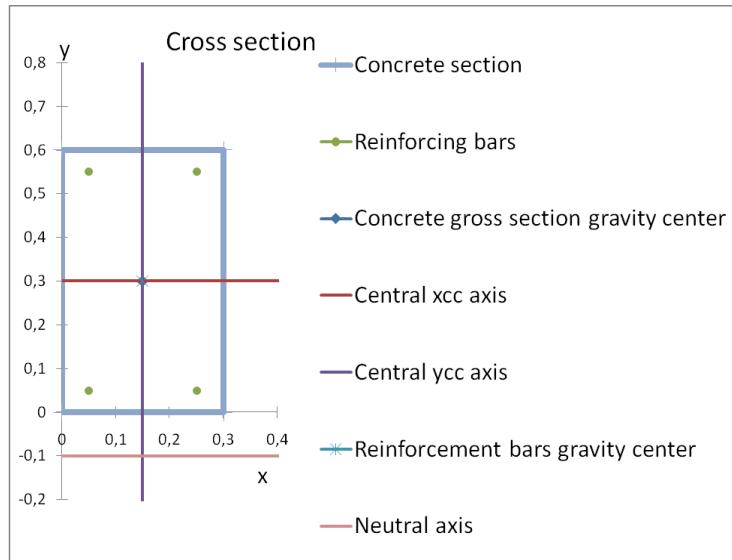
### 3.3.2 Search for :

- Results for reinforcement bars:  $N_s, M_{xs}, M_{ys}$
- Results for concrete section:  $A_c, (x_{cg}, y_{cg}), N_c, M_{xc}, M_{yc}$
- Results for reduced section:  $N, M_x, M_y$

### 3.3.3 Results from Component

- Results for reinforcement bars:  
 $N_s = 147.03e3$   
 $M_{xs} = -19.79e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1680$   
 $(x_{cg}, y_{cg}) = (0.00000000, 0.02005400)$   
 $N_c = 3350.95e3$   
 $M_{xc} = -67.20e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 3497.98e3$   
 $M_x = -86.99e3$   
 $M_y = 0.00e3$

### 3.3.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.7$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|Eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = Eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|Eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	φ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • φ^2 / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = - N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1 -0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1 0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1 0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1 -0,25	4241	1696
					Sum of forces in bars:		147027		-19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing below).

Fig. 1

Concrete section + rebars

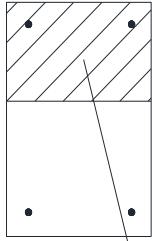


Fig. 2

Concrete section

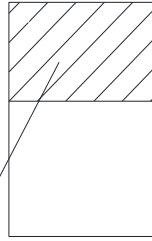


Fig. 3

Holes in compressed concrete

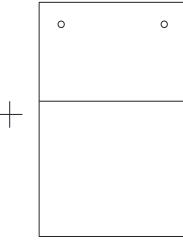
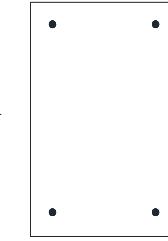


Fig. 4

Rebars



A part of a section with compressing stresses in concrete

Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	φ	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars	Force N_sc in concrete in place of bars $N_{sc} = f_{cd} \cdot Eps_{top} / Eps_{ult} \cdot Area$	M_xsc = - N_{sc} • y_cc	M_ysc = N_{sc} • x_cc
1	0,05	0,05	0,012	-0,1 -0,25	0,000113097	2262	565	-226
2	0,05	0,55	0,012	-0,1 0,25	0,000113097	2262	-565	-226
3	0,25	0,55	0,012	0,1 0,25	0,000113097	2262	-565	226
4	0,25	0,05	0,012	0,1 -0,25	0,000113097	2262	565	226
				Sum of forces:		9048	0	0

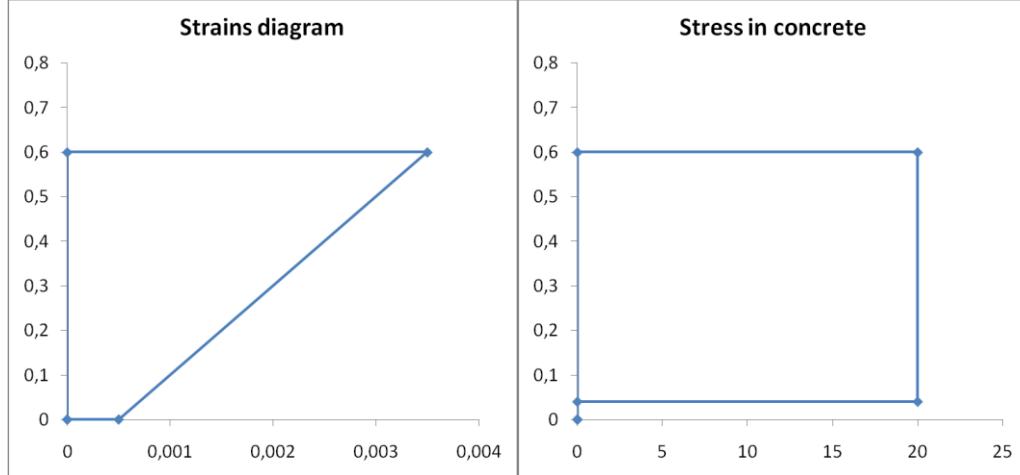
Reduction forces in concrete:

$$\begin{aligned}N_{sc} &= 9.05e3 \\M_{xsc} &= 0.00e3 \\M_{ysc} &= 0.00e3\end{aligned}$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see fig. 3 on drawing above) in central axes coordinate system:

$$\begin{aligned}x_{scg} &= \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00 \\y_{scg} &= \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00\end{aligned}$$

- Results for concrete section:



Height of compressed part of the section:

$$\Lambda \cdot y_b = 0.56$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.56 = 0.168$$

Stress in concrete:

$$\Sigma = Eps\_top / Eps\_ult \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$\begin{aligned}x_{cg} &= ((0.3 / 2 - x_{cc}) \cdot A_c \cdot \Sigma - x_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0 \\y_{cg} &= ((0.6 - 0.56 / 2 - y_{cc}) \cdot A_c \cdot \Sigma - y_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0.020054\end{aligned}$$

Internal forces in concrete:

$$\begin{aligned}N_c &= A_c \cdot \Sigma - N_{sc} = 3350.95e3 \\M_{xc} &= -N_c \cdot y_{cg} = -67.20e3 \\M_{yc} &= N_c \cdot x_{cg} = 0.00e3\end{aligned}$$

- Results for reduced section:

$$\begin{aligned}N &= N_c + N_s = 3497.98e3 \\M_x &= M_{xc} + M_{xs} = -86.99e3 \\M_y &= M_{yc} + M_{ys} = 0.00e3\end{aligned}$$

## 3.4 Case 2b

### 3.4.1 Data:

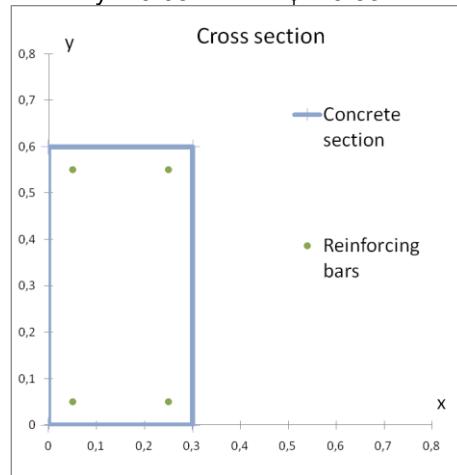
- Geometry:

- Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

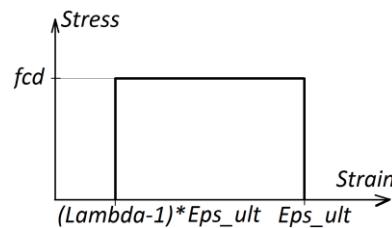
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



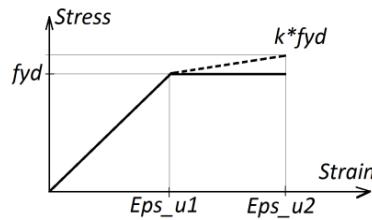
- Concrete parameters:

- Design strength  $f_{cd} = 20e6$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$

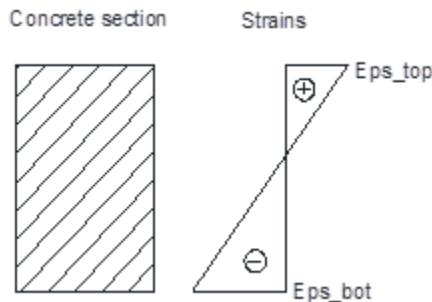


- Steel parameters:

- Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Strains:
  - Bottom strain  $Eps\_bot = -0.022029$
  - Top strain  $Eps\_top = 0.0035$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



### 3.4.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 3.4.3 Results from Component

- Results for reinforcement bars:

$$\begin{aligned}N_s &= -362.69e3 \\M_{xs} &= -311.45e3 \\M_{ys} &= 0.00e3\end{aligned}$$

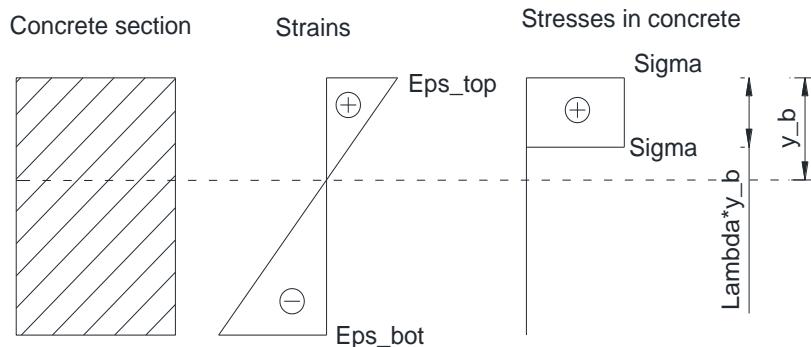
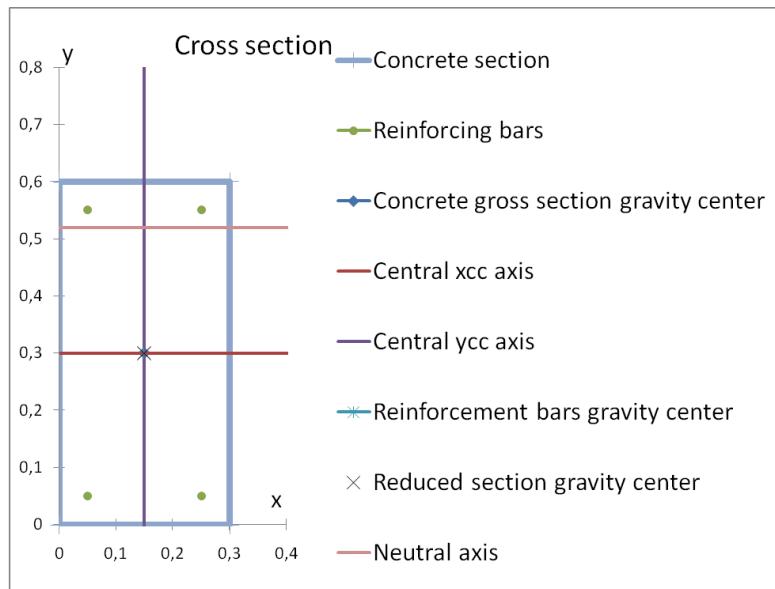
- Results for concrete section:

$$\begin{aligned}A_c &= 0.01974 \\x_{cg} &= 0.00000000 \\y_{cg} &= 0.26861271 \\N_c &= 362.68e3 \\M_{xc} &= -97.42e3 \\M_{yc} &= 0.00e3\end{aligned}$$

- Results for reduced section:

$$\begin{aligned}N &= -0.01e3 \\M_x &= -408.87e3 \\M_y &= 0.00e3\end{aligned}$$

### 3.4.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.0822604$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$   
then

```

    else      sigma_i = eps_i / Eps_u1 * f_yd
    sigma_i = f_yd + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) * (k - 1) * f_yd

```

i	x	y	φ	Distance d from neutral axis	Strain eps = d/y_b * Eps_top	Stresses Sigma / 10e6	Force in bar N = sigma * π * φ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = - N * y_cc	M_ys = N * x_cc
1	0,05	0,05	0,032	-0,468	-0,01990	-500	-402124	-0,10 -0,25	- 100531	40212
2	0,05	0,55	0,032	0,032	0,00137	275	220784	-0,10 0,25	-55196	-22078
3	0,25	0,55	0,032	0,032	0,00137	275	220784	0,10 0,25	-55196	22078
4	0,25	0,05	0,032	-0,468	-0,01990	-500	-402124	0,10 -0,25	- 100531	-40212
						Sum of forces in bars:	-362680		- 311454	0

$$N_s = 362.68e3$$

$$M_{xs} = -311.45e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	φ	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars	Force N_sc in concrete in place of bars N_sc = f_cd * Eps_top / Eps_ult * Area	M_xsc = - N_sc * y_cc	M_ysc = N_sc * x_cc
1	0,05	0,05	0,032	-0,1 -0,25				
2	0,05	0,55	0,032	-0,1 0,25	0,000804248	16085	-4021	-1608
3	0,25	0,55	0,032	0,1 0,25	0,000804248	16085	-4021	1608
4	0,25	0,05	0,032	0,1 -0,25				
					Sum of forces :	32170	-8042	0

Reduction forces in concrete:

$$N_{sc} = 32.17e3$$

$$M_{xsc} = -8.04e3$$

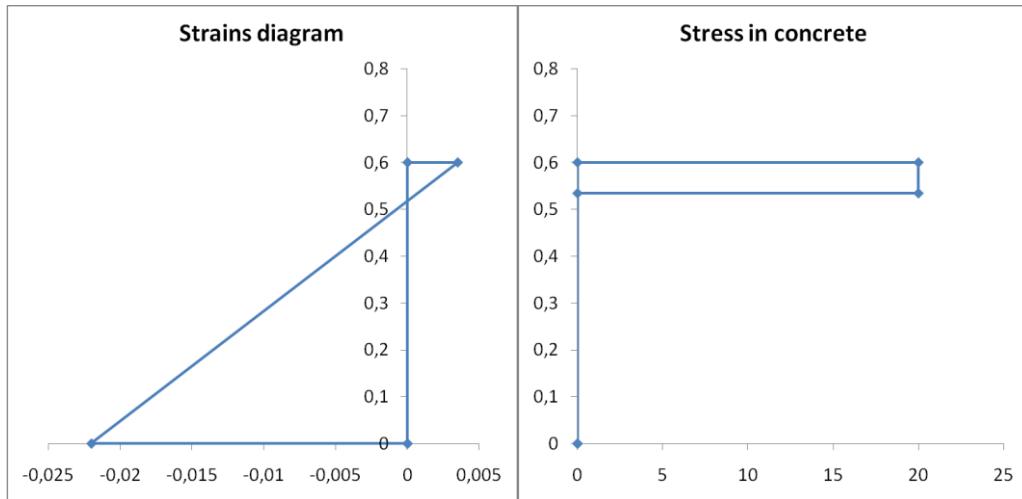
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in Case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} * \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00$$

$$y_{scg} = \text{sum}(y_{cc\_i} * \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$\Lambda \cdot y_b = 0.065808351$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.064608 = 0.01974$$

Stress in concrete:

$$\Sigma = Eps\_top / Eps\_ult \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.3 / 2 - x_{cc}) \cdot A_c \cdot \Sigma - x_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0$$

$$y_{cg} = ((0.6 - \Lambda \cdot y_b / 2 - y_{cc}) \cdot A_c \cdot \Sigma - y_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0.268612$$

Internal forces in concrete:

$$N_c = A_c \cdot \Sigma - N_{sc} = 362.68e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -97.42e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00e3$$

$$M_x = M_{xc} + M_{xs} = -408.87e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 3.5 Case 2c

### 3.5.1 Data:

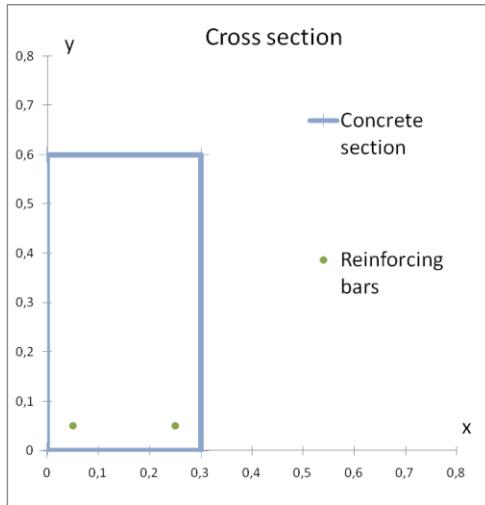
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$

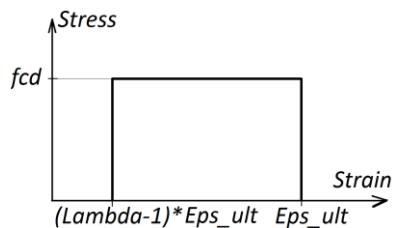
$$\begin{array}{ll} x = 0.3 & y = 0.6 \\ x = 0.3 & y = 0.0 \end{array}$$

- Reinforcing bars:

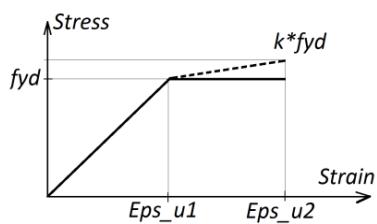
$$\begin{array}{lll} x = 0.05 & y = 0.05 & \phi = 0.032 \\ x = 0.25 & y = 0.05 & \phi = 0.032 \end{array}$$



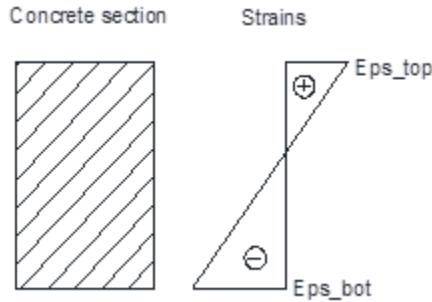
- Concrete parameters:
  - Design strength  $f_{cd} = 20\text{e}6$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 30\text{e}9$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Strains:
  - Top strain  $Eps\_top = 0.0015$
  - Bottom strain  $Eps\_bot = -0.002$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



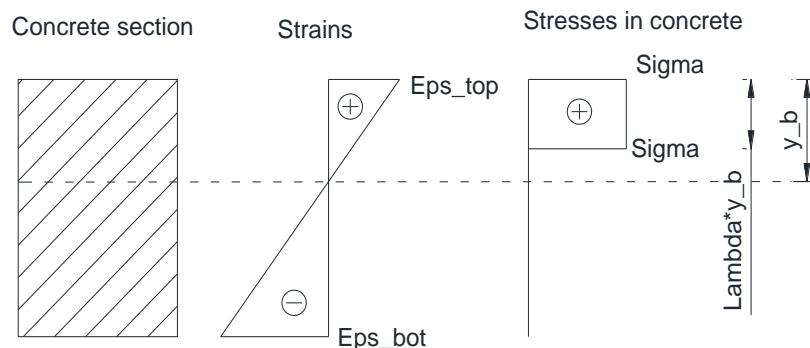
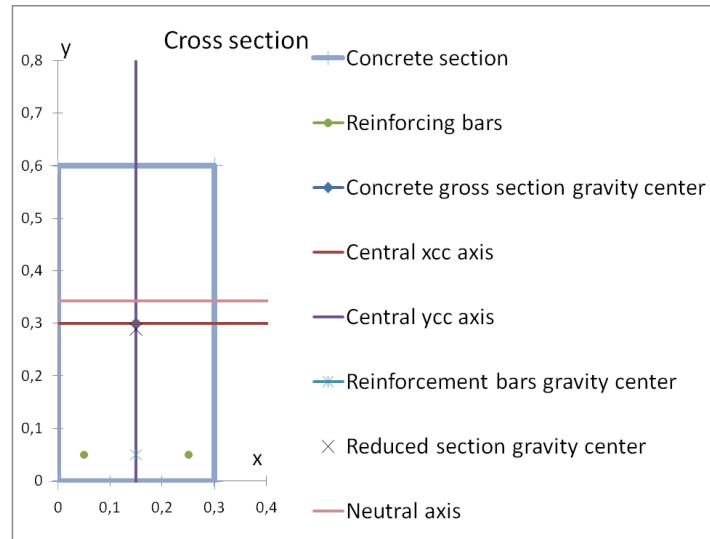
### 3.5.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 3.5.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -549.57e3$   
 $M_{xs} = -137.39e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0617$   
 $x_{cg} = 0.000000$   
 $y_{cg} = 0.197143$   
 $N_c = 528.98e3$   
 $M_{xc} = -104.28e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = -20.59e3$   
 $M_x = -241.68e3$   
 $M_y = 0.00e3$

### 3.5.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.257143$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

```
If |eps_i| ≤ f_yd / E_s then
    sigma_i = eps_i / Eps_u1 * f_yd
else
```

$$\sigma_i = f_{yd} + (|\epsilon_i - \epsilon_u|) / (\epsilon_u - \epsilon_i) \cdot (k - 1) \cdot f_{yd}$$

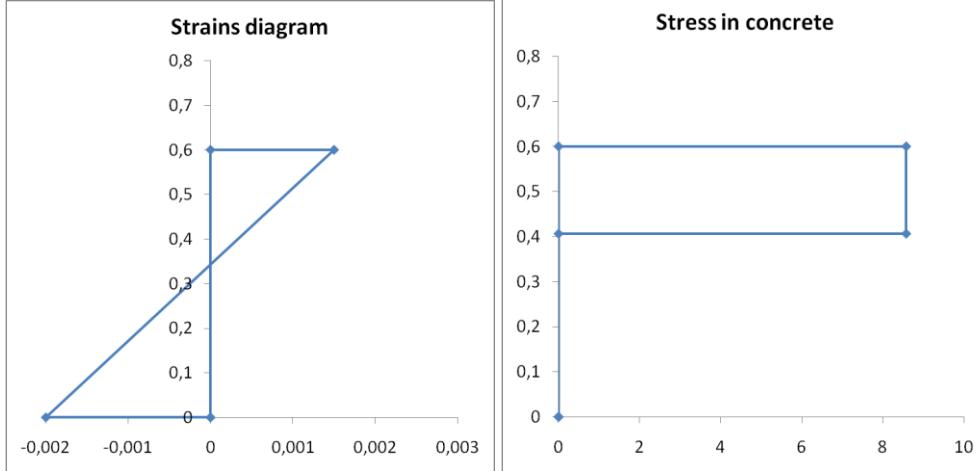
i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses σ / 10e6	force in bar N = σ • π • ϕ^2 / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = - N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10 -0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10 -0,25	-68696	-27478
						Sum of forces in bars:	-549569		- 137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$\Lambda \cdot y_b = 0.205714$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.205714 = 0.0617$$

Stress in concrete:

$$\sigma = E_s \cdot \epsilon = 20.59e3 \cdot 0.0617 = 8.571e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (0.3/2 - x_{cc}) \cdot A_c \cdot \sigma / (A_c \cdot \sigma) = 0$$

$$y_{cg} = (0.6 - \Lambda \cdot y_b / 2 - y_{cc}) \cdot A_c \cdot \sigma / (A_c \cdot \sigma) = 0.197143$$

Internal forces in concrete:

$$N_c = A_c \cdot \sigma = 528.98e3$$

$$M_{xc} = - N_c \cdot y_{cg} = -104.28e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = -20.59e3$$

$$M_x = M_{xc} + M_{xs} = -241.68e3$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00e3}$$

## 4 Case 3: Calculation of internal forces for given state of strain in the section and linear model of concrete

### 4.1 *Subject:*

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 4.2 *Summary:*

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 3a and Case 3b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 3c)
- A linear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 3a)
- A section under unidirectional bending with a part of section under compression (Case 3b and Case 3c)
- A state of strain with ultimate compressive strain in concrete (Case 3a and Case 3b)
- A strain below the ultimate compressive strain in concrete (Case 3c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

---

### 4.3 Case 3a

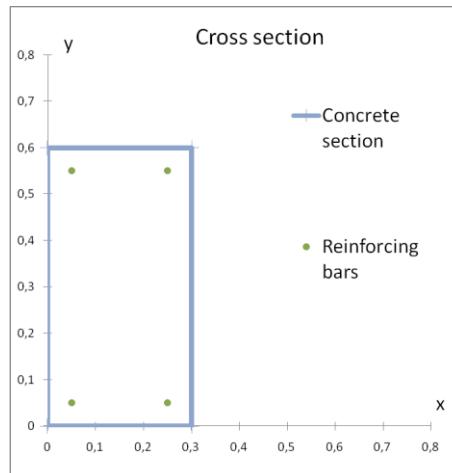
#### 4.3.1 Data:

- Geometry:
  - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

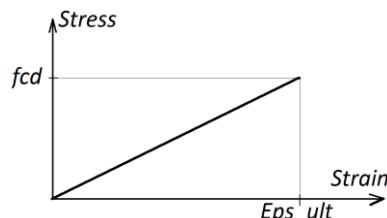
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$



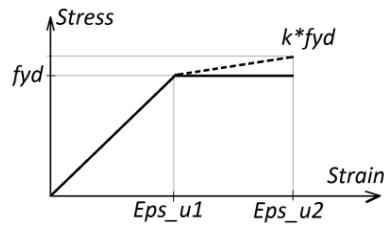
- Concrete parameters:

- Design strength  $f_{cd} = 20e6$
- Modulus of elasticity  $E_c = 30e9$
- Strain-stress model: linear
- Strain ultimate limit  $Eps\_ult = 0.0035$

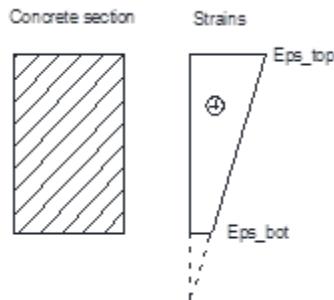


- Steel parameters:

- Design strength  $f_{yd} = 500e6$
- Hardening factor  $k = 1.0$
- Modulus of elasticity  $E_s = 200e9$
- Strain ultimate limit  $Eps_u2 = 0.075$



- Strains:
  - Top strain  $Eps\_top = 0.0035$
  - Bottom strain  $Eps\_bot = 0.0005$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



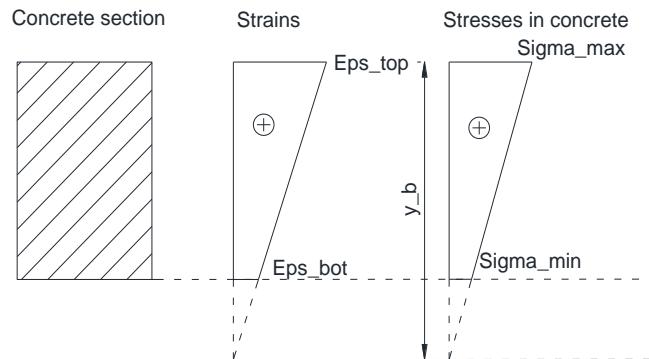
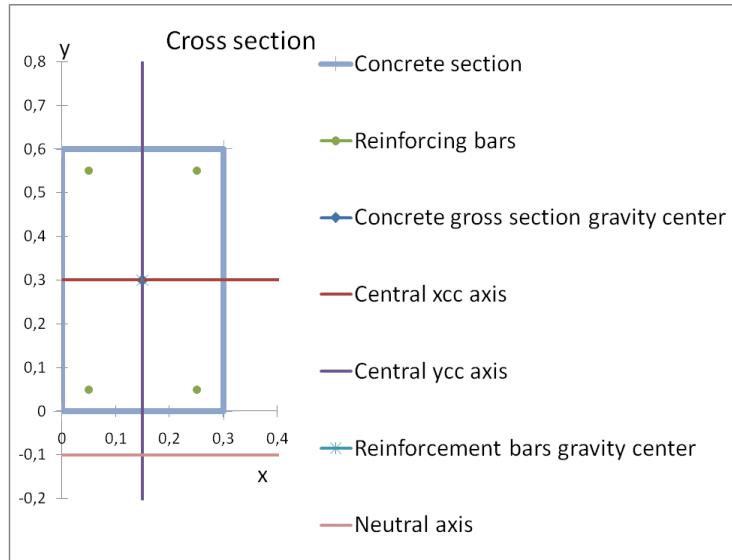
#### 4.3.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### 4.3.3 Results from Component

- Results for reinforcement bars:  
 $N_s = 147.03e3$   
 $M_{xs} = -19.79e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1800$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.07479528$   
 $N_c = 2051.97e3$   
 $M_{xc} = -153.48e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 2199.00e3$   
 $M_x = -173.27e3$   
 $M_y = 0.00e3$

#### 4.3.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -\epsilon_{top} \cdot h / (\epsilon_{bot} - \epsilon_{top}) = 0.7$$

$$\epsilon_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar  $i$ :

If  $|\epsilon_i| \leq f_{yd} / E_s$  then

$$\sigma_i = \epsilon_i / \epsilon_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses Sigma / 10e6	Force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_cc y_cc	M_xs = - N • y_cc	M_ys = N • x_cc	
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
					Sum of forces in bars:		147027			-19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars	Force N_sc in concrete in place of bars N_sc = f_cd • eps / Eps_ult • Area	M_xsc = - N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,012	0,00075	-0,1 -0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1 0,25	0,000113097	485	121	48
3	0,25	0,55	0,012	0,00325	0,1 0,25	0,000113097	2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1 -0,25	0,000113097	2100	-525	-210
					Sum of forces:		5170	-808	0

Reduction forces in concrete:

$$N = 5.17e3$$

$$M_{xsc} = -0.81e3$$

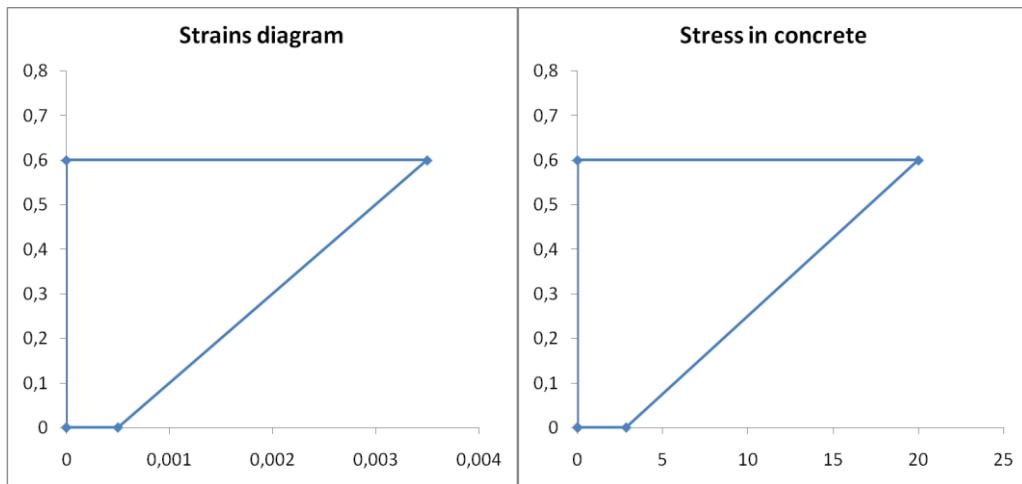
$$M_{ysc} = 0.000e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00000$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.15625$$

- Results for concrete section:



Height of compressed part of the section:

0.6000

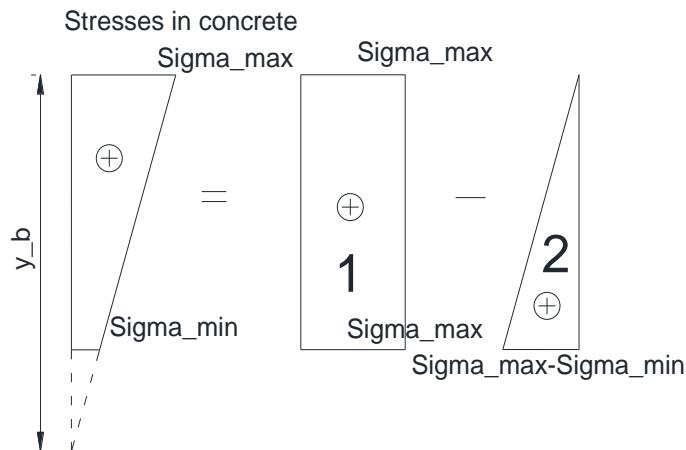
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = \underline{0.1800}$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{\max} = f_{cd} \cdot Eps\_top / Eps\_c1 = 20.000e6$$

$$\Sigma_{\min} = f_{cd} \cdot Eps\_bot / Eps\_c1 = 2.857e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \Sigma_{\max} = 3.600e6$$

$$N_{c2} = A_c \cdot (\Sigma_{\max} - \Sigma_{\min}) / 2 = 1.543e6$$

$$N_c = N_{c1} - N_{c2} = 2.057e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -0.3 + 0.6 / 3 = -0.1$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.074795$$

Internal forces in concrete:

$$N_c = N_c - N_{sc} = 2051.97e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -153.48e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 2199.00e3$$

$$M_x = M_{xc} + M_{xs} = -173.27e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

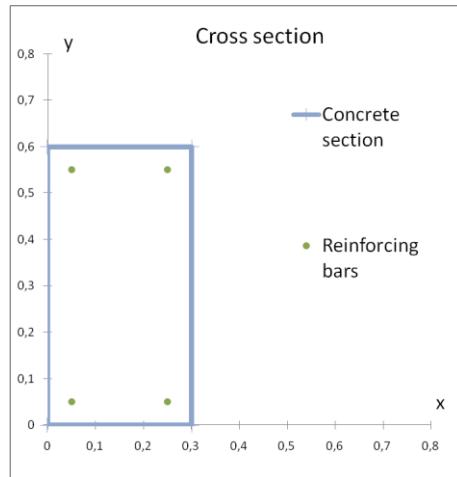
## 4.4 Case 3b

### 4.4.1 Data:

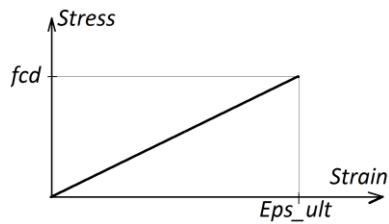
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

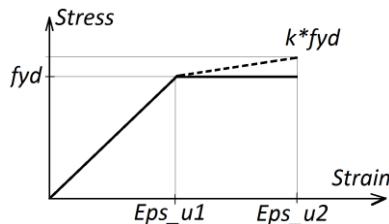
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.05	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$



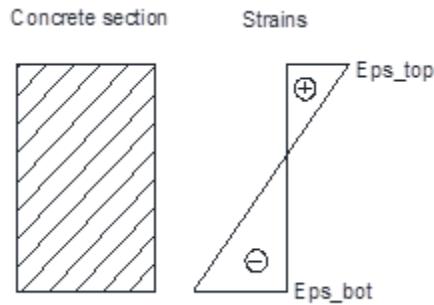
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: linear
  - Strain ultimate limit  $Eps\_ult = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $Eps\_u2 = 0.075$



- Strains:
  - Top strain  $Eps\_top = 0.0035$
  - Bottom strain  $Eps\_bot = -0.01857592$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



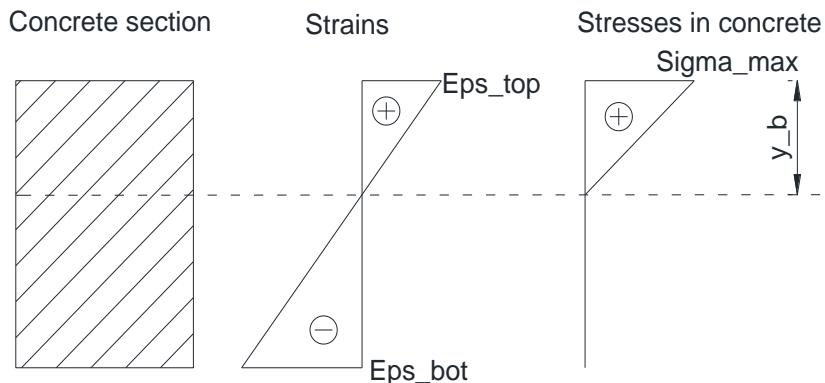
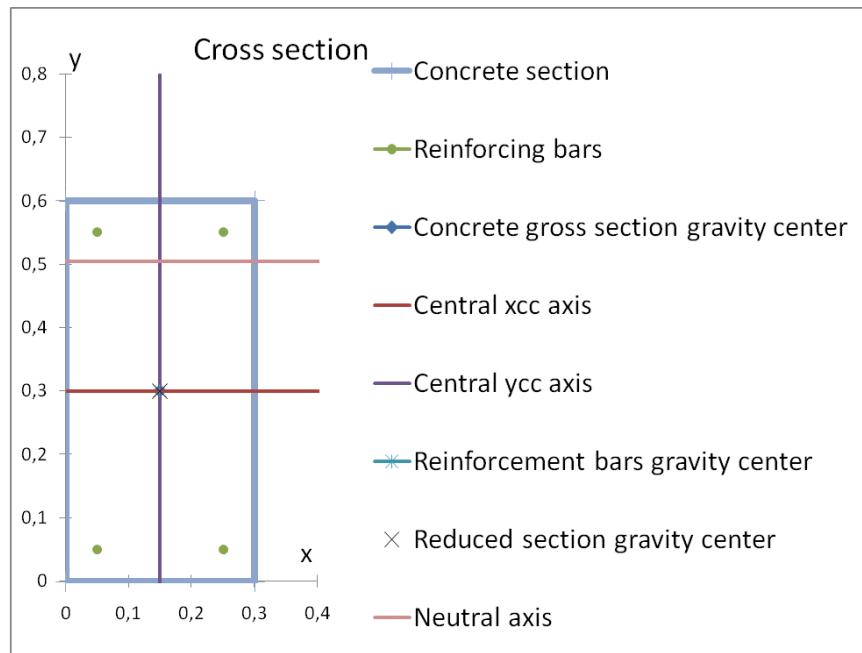
#### 4.4.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### 4.4.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -270.12e3$   
 $M_{xs} = -334.59e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0285$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.26932464$   
 $N_c = 270.12e3$   
 $M_{xc} = -72.75e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 0.00e3$   
 $M_x = -407.34e3$   
 $M_y = 0.00e3$

#### 4.4.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.095126$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • ϕ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = - N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,032	-0,455	-0,01674	-500	-402124	-0,10 -0,25	100531	40212
2	0,25	0,05	0,032	-0,455	-0,01674	-500	-402124	0,10 -0,25	100531	-40212
3	0,25	0,55	0,032	0,045	0,00166	332	267065	0,10 0,25	-66766	26706
4	0,05	0,55	0,032	0,045	0,00166	332	267065	-0,10 0,25	-66766	-26706
						Sum of forces in bars:	-270118		334594	0

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

$$N_s = -270.12e3$$

$$M_{xs} = -334.594e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars $N_{sc} = f_{cd} \cdot \sigma / Eps_{ult} \cdot Area$	$M_{xsc} = - N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,01674	-0,1 -0,25				
2	0,25	0,05	0,032	-0,01674	0,1 -0,25				
3	0,25	0,55	0,032	0,00166	0,1 0,25	0,000804248	7630	-1908	763
4	0,05	0,55	0,032	0,00166	-0,1 0,25	0,000804248	7630	-1908	-763
						Sum of forces :	15261	-3815	0

Reduction forces in concrete:

$$N_{sc} = 15.26e3$$

$$M_{xsc} = -3.82e3$$

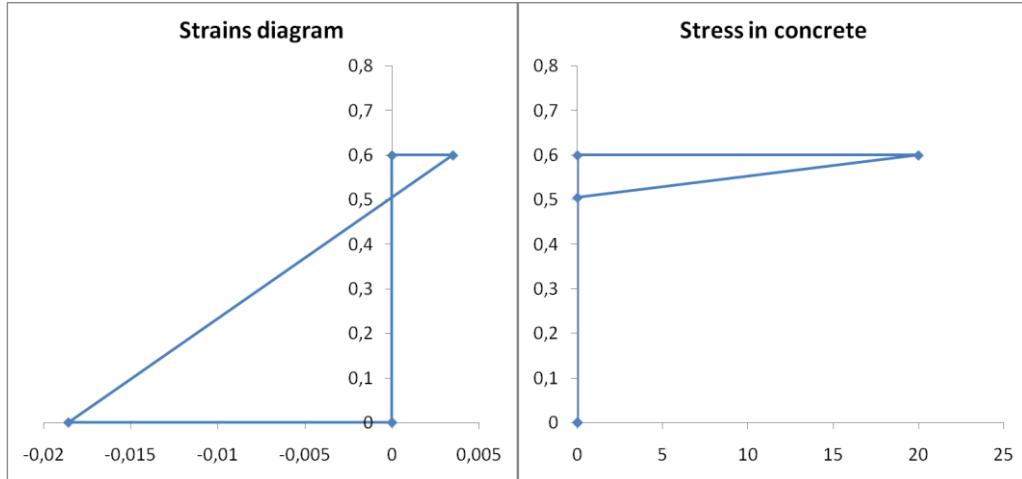
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

$$y_{scg} = \sum(y_{cc\_i} \cdot Area_i) / \sum(Area_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0,095126$$

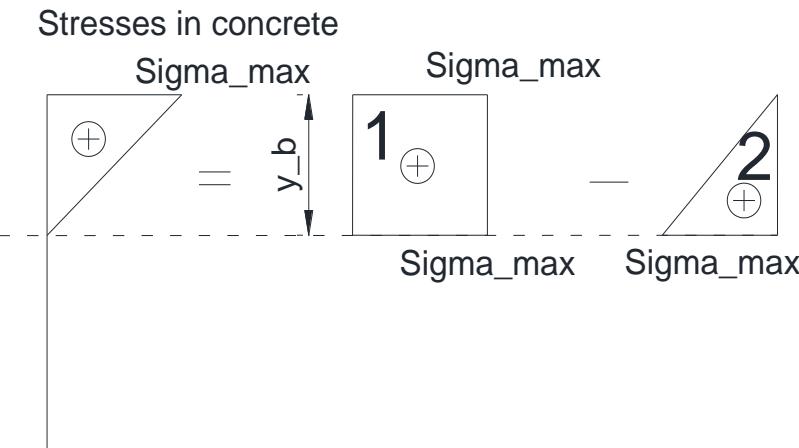
Area of contour under compressive stress:

$$A_c = 0,3 \cdot y_b = 0,0285$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c' - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot Eps\_top / Eps\_ult = 20.000e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 0.5708e6$$

$$N_{c2} = A_c \cdot \text{Sigma\_max} / 2 = 0.2854e6$$

$$N_c' = N_{c1} - N_{c2} = 0.28538e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 \cdot y_b / 2 = 0.25243687$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 \cdot y_b \cdot 2/3 = 0.23658249$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.26932464$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 270.12e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -72.75e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00e3$$

$$M_x = M_{xc} + M_{xs} = -407.34e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 4.5 Case 3c

### 4.5.1 Data:

- Geometry:

- Concrete section:

$$x = 0.0 \quad y = 0.0$$

$$x = 0.0 \quad y = 0.6$$

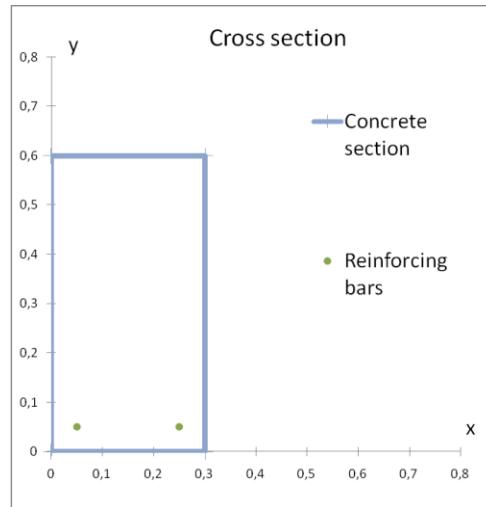
$$x = 0.3 \quad y = 0.6$$

$$x = 0.3 \quad y = 0.0$$

- Reinforcing bars:

$$x = 0.05 \quad y = 0.05 \quad \phi = 0.032$$

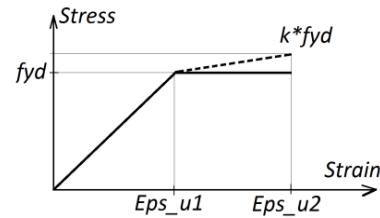
$$x = 0.25 \quad y = 0.05 \quad \phi = 0.032$$



- Concrete parameters:
  - Design strength  $f_{cd} = 20\text{e}6$
  - Modulus of elasticity  $E_c = 30\text{e}9$
  - Strain-stress model: linear
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0035$

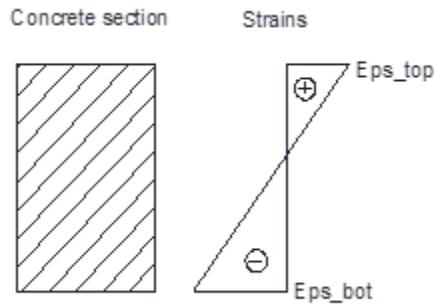


- Steel parameters:
  - Design strength  $f_{yd} = 500\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.075$



Strains:

Top strain  $\text{Eps\_top} = 0.0015$   
 Bottom strain  $\text{Eps\_bot} = -0.002$   
 Neutral axis angle Angle =  $\pi \cdot 3 / 2$



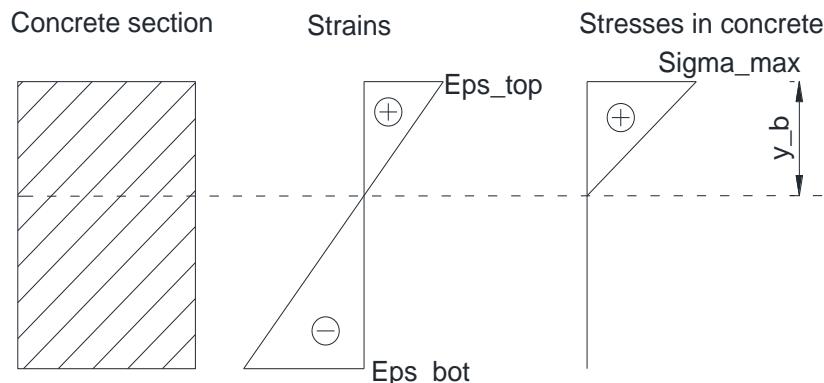
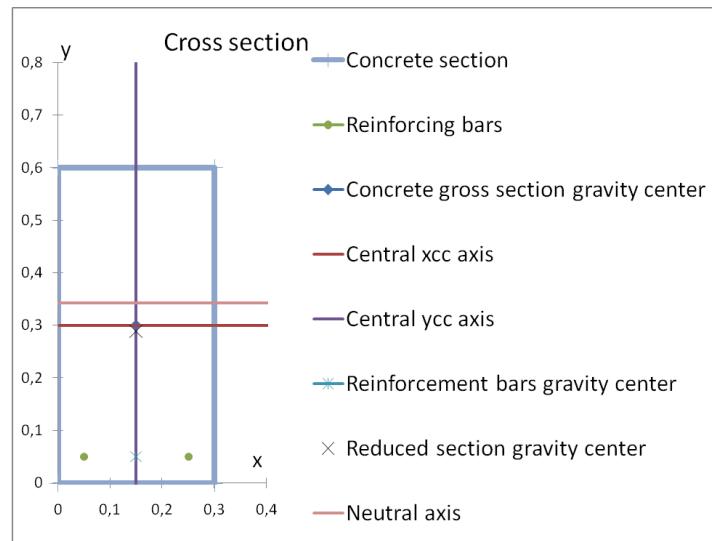
#### 4.5.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### 4.5.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -549.57e3$   
 $M_{xs} = -137.39e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0771$   
 $x_{cg} = 0.000000$   
 $y_{cg} = 0.21428571$   
 $N_c = 330.61e3$   
 $M_{xc} = -70.85e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = -218.96e3$   
 $M_x = -208.24e3$   
 $M_y = 0.00e3$

#### 4.5.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.257143$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{cc} = 0.15$$

$$y_{cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|Eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = Eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|Eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

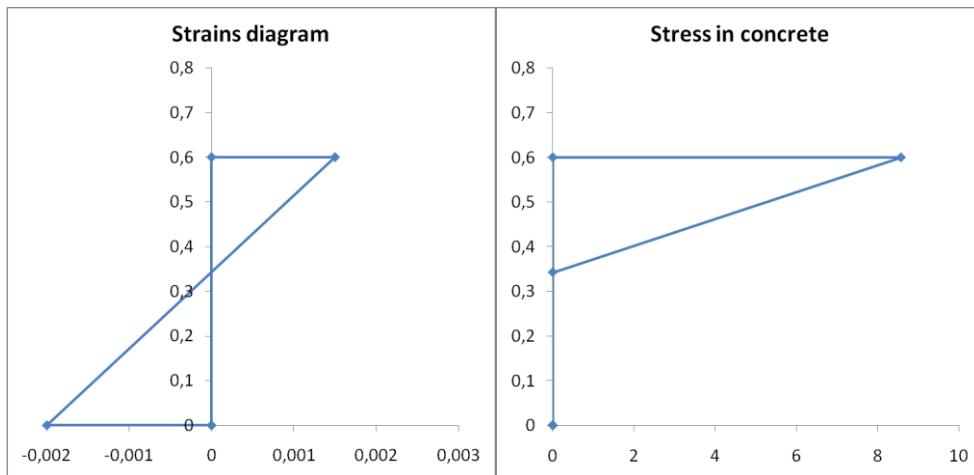
i	x	y	$\phi$	Distance d from neutral axis	Strain $\epsilon = d / y_b \cdot E_{top}$	Stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc} \quad y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10 0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10 0,25	-68696	-27478
						Sum of forces in bars:	-549569		137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

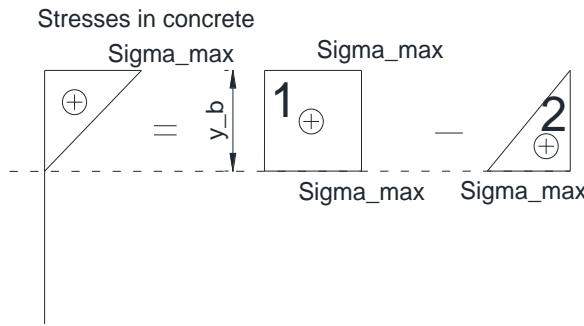
$$y_b = 0.257143$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$\begin{aligned} N_{c'} &= N_{c1} - N_{c2} \\ N_c &= N_{c'} \end{aligned}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot Eps\_top / Eps\_ult = 8.571e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 0.6612e6$$

$$N_{c2} = A_c \cdot \text{Sigma\_max} / 2 = 0.3306e6$$

$$N_c = N_{c1} - N_{c2} = 0.3306e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 - y_b / 2 = 0.17142857$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 - y_b \cdot 2/3 = 0.12857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.21428571$$

Internal forces in concrete:

$$N_c = N_c = 330.61e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -70.85e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = -218.96e3$$

$$M_x = M_{xc} + M_{xs} = -208.24e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 5 Case 4: Calculation of internal forces for given state of strain in the section and bilinear model of concrete

### 5.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 5.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 4a and Case 4b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 4c)
- A bilinear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 4a)
- A section under unidirectional bending with a part of section under compression (Case 4b and Case 4c)
- A state of strain with ultimate compressive strain in concrete (Case 4a and Case 4b)
- A strain below the ultimate compressive strain in concrete (Case 4c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

---

### 5.3 Case 4a

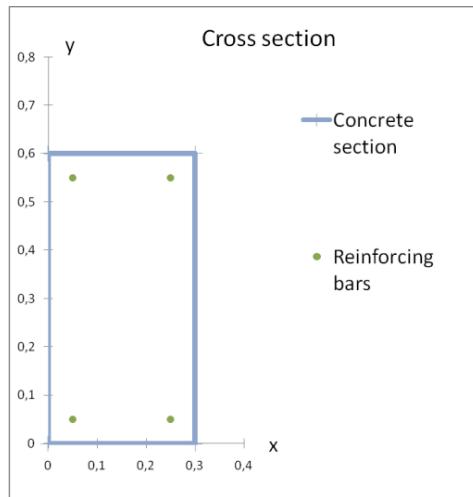
#### 5.3.1 Data:

- Geometry:

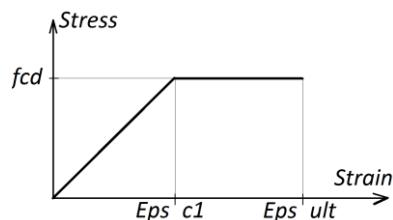
- Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
- Reinforcing bars:
 

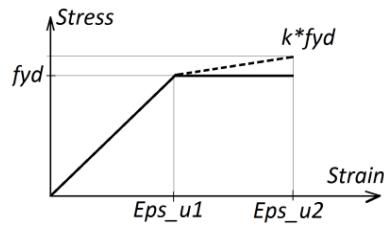
x = 0.05	y = 0.05	$\phi = 0.016$
x = 0.25	y = 0.05	$\phi = 0.016$
x = 0.25	y = 0.55	$\phi = 0.016$
x = 0.05	y = 0.55	$\phi = 0.016$



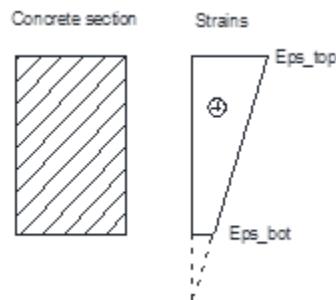
- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $Eps\_ult = 0.0035$
  - Strain relation change over  $Eps\_c1 = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400e6$
  - Hardening factor  $k= 1.0$
  - Modulus of elasticity  $E_s = 205e9$
  - Strain ultimate limit  $Eps\_u2 = 0.1$



- Strains:
  - Top strain  $Eps\_top = 0.0035$
  - Bottom strain  $Eps\_bot = 0.0005$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



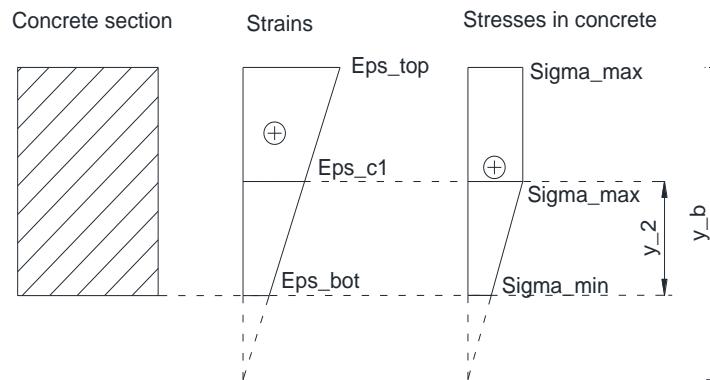
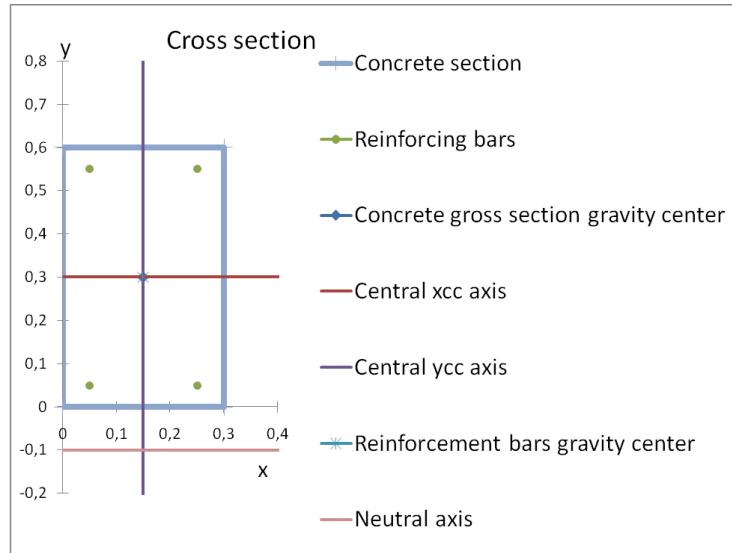
### 5.3.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 5.3.3 Results from Component

- Results for reinforcement bars:  
 $N_s = 222.68e3$   
 $M_{xs} = -24.76e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1800$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.04589775$   
 $N_c = 4370.91e3$   
 $M_{xc} = -200.62e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 4593.59e3$   
 $M_x = -225.37e3$   
 $M_y = 0.00e3$

### 5.3.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.7$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then}$$

```

sigma_i = eps_i / Eps_u1 * f_yd
else
sigma_i = f_yd + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) * (k - 1) * f_yd

```

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = -N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,016	0,150	0,00075	154	30913	-0,10 -0,25	7728	-3091
2	0,25	0,05	0,016	0,150	0,00075	154	30913	0,10 -0,25	7728	3091
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10 0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10 0,25	-20106	-8042
						Sum of forces in bars:	222676		-24756	0

$$N_s = 222.68e3$$

$$M_{xs} = -24.76e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if  $\text{eps} \leq \text{Eps\_c1}$  then  $N_{sc} = f_{cd} \cdot \text{eps} / \text{Eps\_c1} \cdot \text{Area}$   
 if  $\text{eps} > \text{Eps\_c1}$  then  $N_{sc} = f_{cd} \cdot \text{Area}$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars N_sc	Force in concrete in place of bars M_xsc = -N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,016	0,00075	-0,1 -0,25	0,000201062	2262	565 -226
2	0,25	0,05	0,016	0,00075	0,1 -0,25	0,000201062	2262	565 226
3	0,25	0,55	0,016	0,00325	0,1 0,25	0,000201062	6032	-1508 603
4	0,05	0,55	0,016	0,00325	-0,1 0,25	0,000201062	6032	-1508 -603
						Sum of forces:	16588	-1885 0

Reduction forces in concrete:

$$N_{sc} = 16.59e3$$

$$M_{xsc} = -1.89e3$$

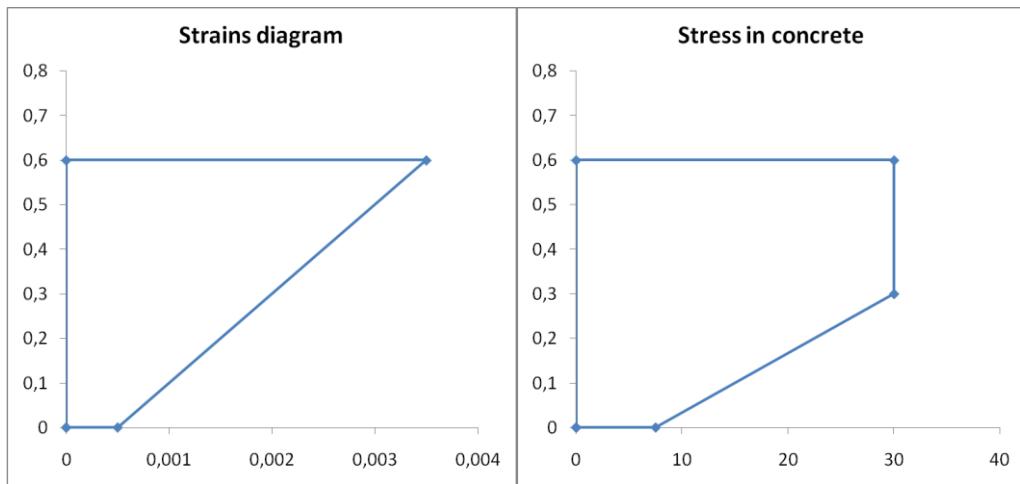
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.00000$$

$$y_{scg} = \sum(y_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.11364$$

- Results for concrete section:



Height of compressed part of the section:

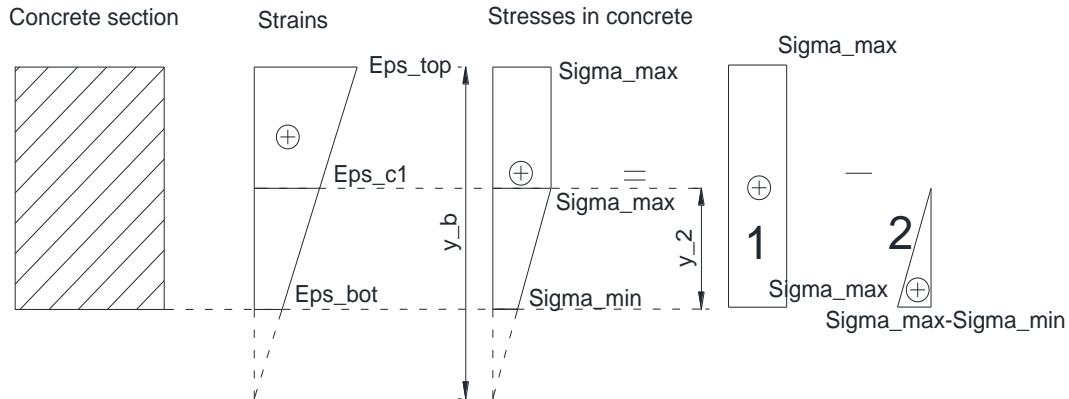
$$h = 0.6000$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$\begin{aligned} N_c' &= N_{c1} - N_{c2} \\ N_c &= N_c' - N_{sc} \end{aligned}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$\begin{aligned} x_{cg} &= (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) \\ y_{cg} &= (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) \end{aligned}$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$\begin{aligned} M_{xc} &= -N_c \cdot y_{cg} \\ M_{yc} &= N_c \cdot x_{cg} \end{aligned}$$

$$\sigma_{max} = f_{cd} = 30.000e6$$

$$\Sigma_{min} = f_{cd} \cdot Eps\_bot/Eps\_c1) = 7.500e6$$

Distance between gravity center of contour and neutral axis:

$$\delta = - (0.6 / 2 - y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Position of axis with stresses equal  $Eps\_c1$  in central axes coordinate system:

$$\delta_2 = - \delta + \text{MIN}(y_b; Eps\_c1 / Eps\_top \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = 0.3 - \delta_2 = 0.3$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \Sigma_{max} = 5.400e6$$

$$N_{c2} = 0.3 \cdot y_2 \cdot (\Sigma_{max} - \Sigma_{min}) / 2 = 1.0125e6$$

$$N_c' = N_{c1} - N_{c2} = 4.3875e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = - 0.3 + y_2 / 3 = - 0.2$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.045898$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 4370.91e3$$

$$M_{xc} = - N_c \cdot y_{cg} = -200.62e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 4593.59e3$$

$$M_x = M_{xc} + M_{xs} = -225.37e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 5.4 Case 4b

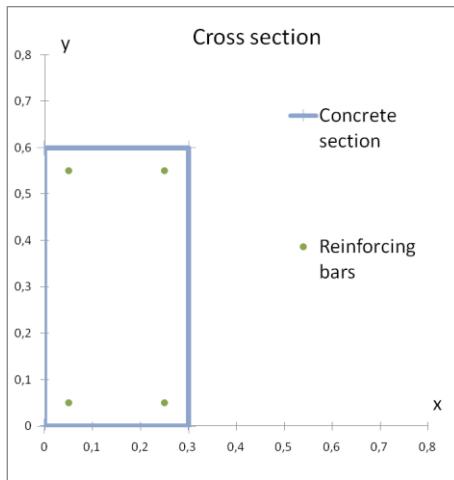
### 5.4.1 Data:

- Geometry:
  - Concrete section:
 

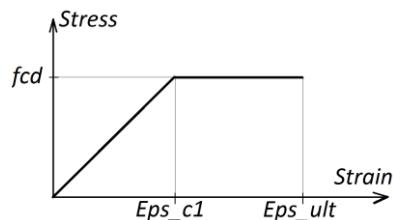
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.55$	$\phi = 0.032$

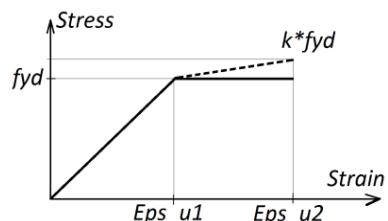
$$x = 0.25 \quad y = 0.05 \quad \phi = 0.032$$



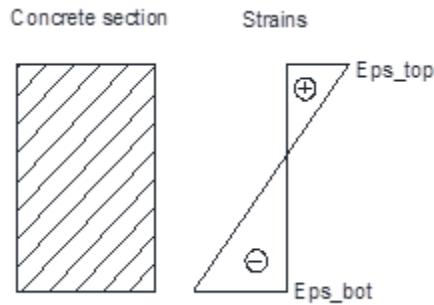
- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$
  - Strain relation change over  $\epsilon_{c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_yd = 400e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 205e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



- Strains:
  - Top strain  $\epsilon_{top} = 0.0035$
  - Bottom strain  $\epsilon_{bot} = -0.02936558$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



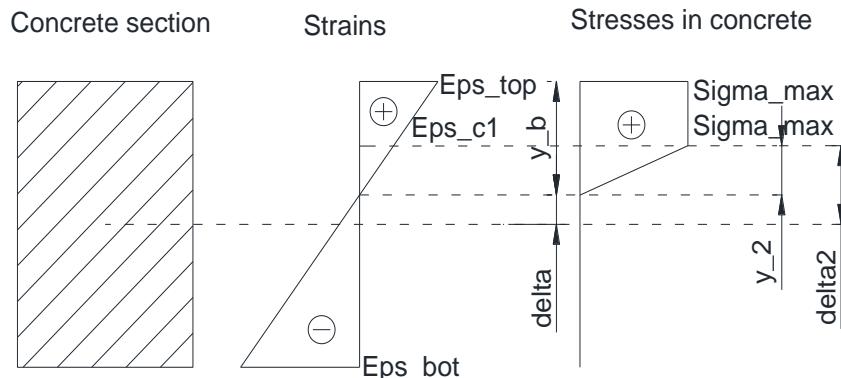
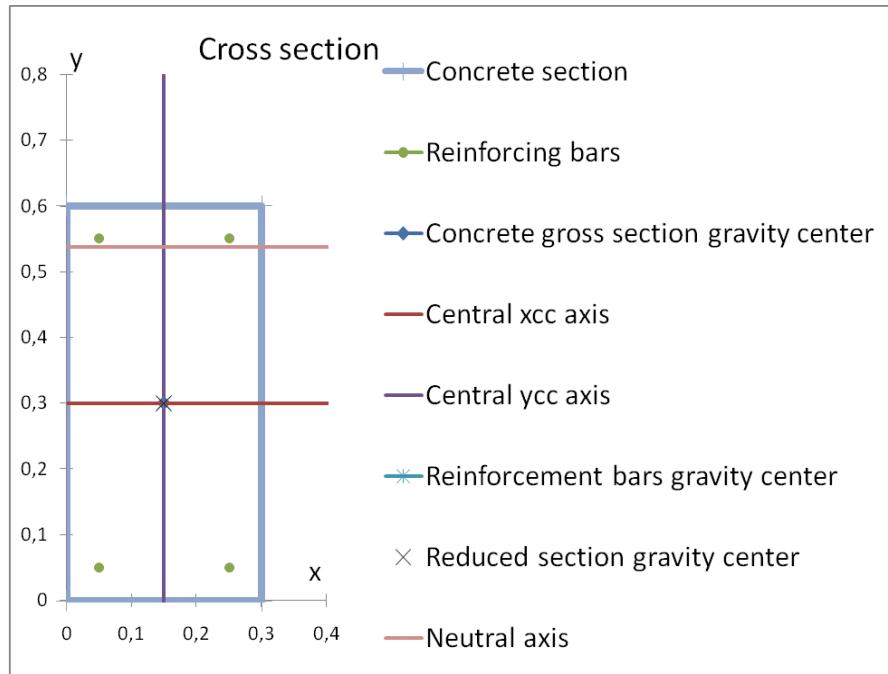
#### 5.4.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### 5.4.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -392.40e3$   
 $M_{xs} = -223.60e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0192$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.27717785$   
 $N_c = 392.40e3$   
 $M_{xc} = -108.76e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 0.00e3$   
 $M_x = -332.36e3$   
 $M_y = 0.00e3$

#### 5.4.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0,6$

$$y_b = -\epsilon_{top} \cdot h / (\epsilon_{bot} - \epsilon_{top}) = 0,0639$$

$$\epsilon_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0,15$$

$$y_{Cc} = 0,30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

```

If |eps_i| ≤ f_yd / E_s then
    sigma_i = eps_i / Eps_u1 * f_yd
else
    sigma_i = f_yd + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) * (k - 1) * f_yd

```

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = -N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,032	-0,486	-0,02663	-400	-321699	-0,10 -0,25	-80425	32170
2	0,25	0,05	0,032	-0,486	-0,02663	-400	-321699	0,10 -0,25	-80425	-32170
3	0,25	0,55	0,032	0,014	0,00076	156	125500	0,10 0,25	-31375	12550
4	0,05	0,55	0,032	0,014	0,00076	156	125500	-0,10 0,25	-31375	-12550
						Sum of forces in bars:	-392398		-	223600 0

$$N_s = -392.40e3$$

$$M_{xs} = -223.60e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if  $\text{eps} \leq \text{Eps\_c1}$   $N_{sc} = f_{cd} \cdot \text{eps} / \text{Eps\_c1} \cdot \text{Area}$   
 if  $\text{eps} > \text{Eps\_c1}$   $N_{sc} = f_{cd} \cdot \text{Area}$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_sc	M_xsc = -N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,032	-0,02663	-0,10 -0,25				
2	0,25	0,05	0,032	-0,02663	0,10 -0,25				
3	0,25	0,55	0,032	0,00076	0,10 0,25	0,000804248	9183	-2296	918
4	0,05	0,55	0,032	0,00076	-0,10 0,25	0,000804248	9183	-2296	-918
						Sum of forces:	18366	-4591	0

Reduction forces in concrete:

$$N_{sc} = 18.37e3$$

$$M_{xsc} = -4.59e3$$

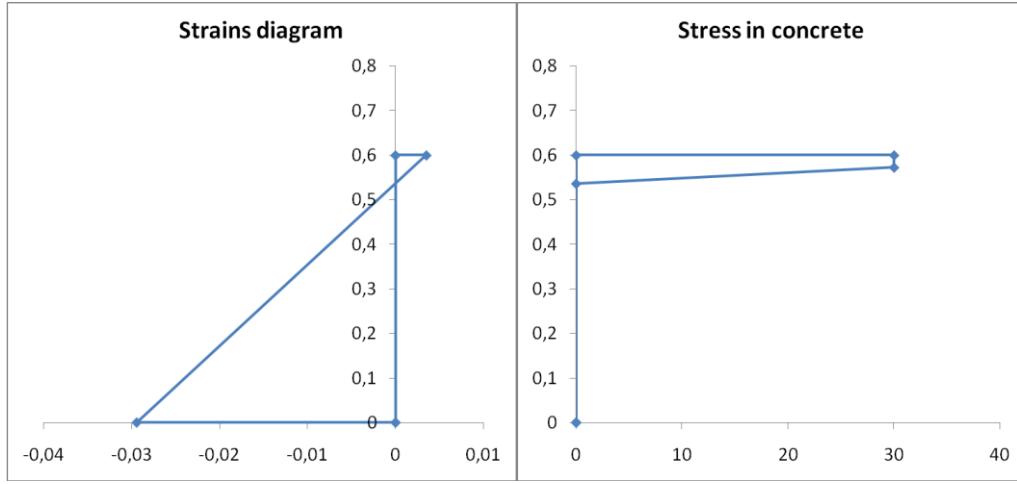
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0639$$

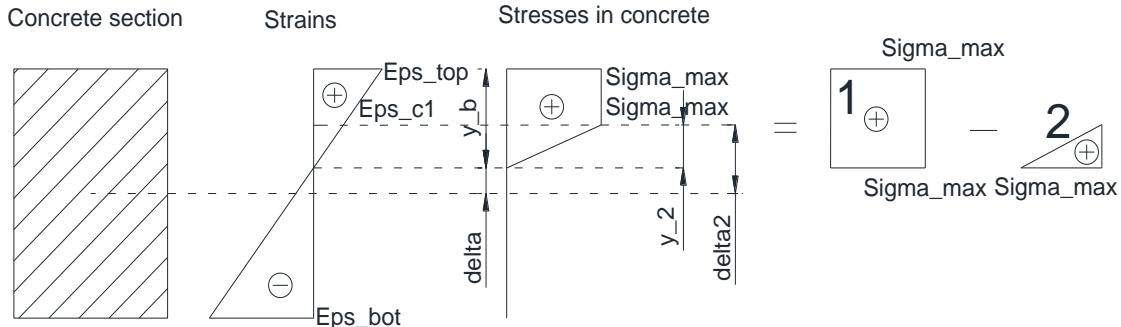
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0192$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c' - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{max} = f_{cd} = 30.000e6$$

Distance between gravity center of contour and neutral axis:

$$\Delta = 0.6 / 2 - y_b = 0.2361$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps\_c1 if Eps\_top ≥ Eps\_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps\_top < Eps\_c1.;

$$\text{delta2} = \text{delta} + \text{MIN}(\text{y}_b; \text{Eps}_c1 / \text{Eps}_\text{top} \cdot \text{y}_b) = 0.2726$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$\text{y}_2 = \text{delta2} - \text{delta} = 0.0365$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma}_\text{max} = 0.5751e6$$

$$N_{c2} = 0.3 \cdot y_2 \cdot \text{Sigma}_\text{max} / 2 = 0.1643e6$$

$$N_c' = N_{c1} - N_{c2} = 0.41076e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.26805$$

$$x_{c2} = 0$$

$$y_{c2} = \text{delta} + y_2 / 3 = 0.24827$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.277178$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 392.40e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -108.76e3$$

$$M_{yc} = N_c' \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00e3$$

$$M_x = M_{xc} + M_{xs} = -332.36e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

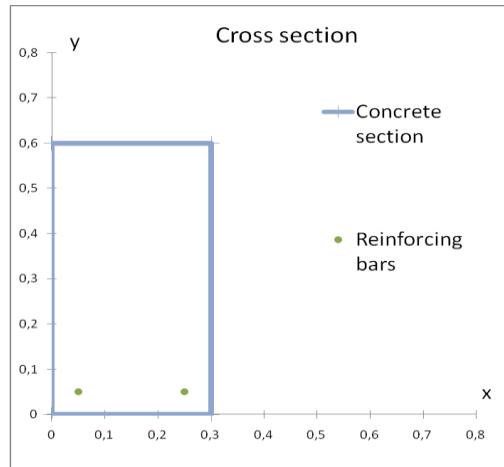
## 5.5 Case 4c

### 5.5.1 Data:

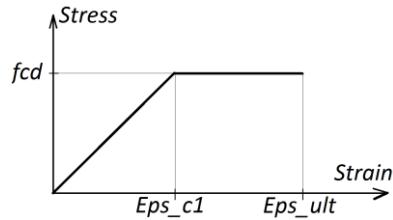
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

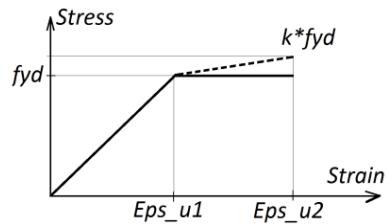
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



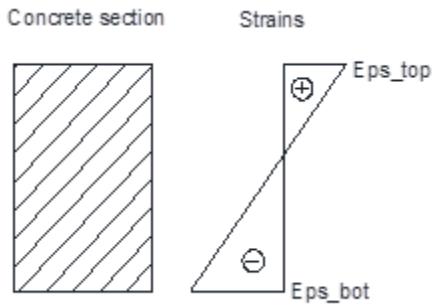
- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0035$
  - Strain relation change over  $\text{Eps\_c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 205\text{e}9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.1$



- Strains:
  - Top strain  $\text{Eps\_top} = 0.0015$
  - Bottom strain  $\text{Eps\_bot} = -0.002$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



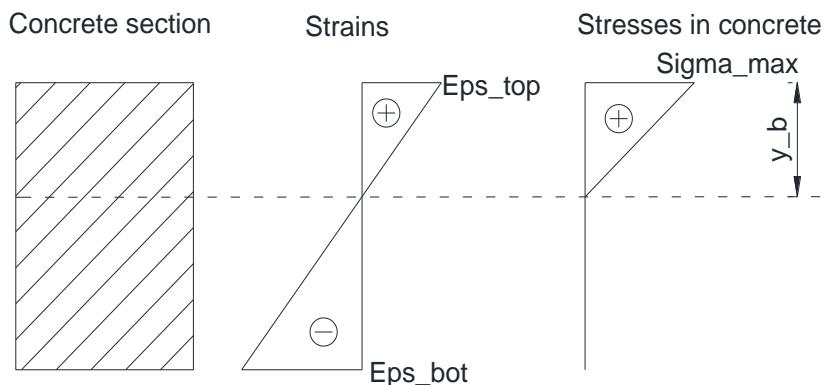
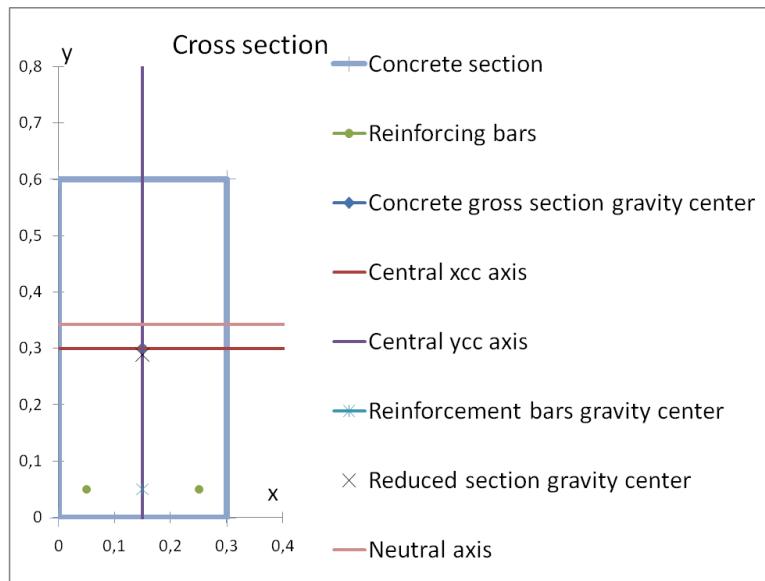
### 5.5.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 5.5.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -563.31e3$   
 $M_{xs} = -140.83e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0771$   
 $x_{cg} = 0.000000$   
 $y_{cg} = 0.21428571$   
 $N_c = 867.86e3$   
 $M_{xc} = -185.97e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 304.55e3$   
 $M_x = -326.80e3$   
 $M_y = 0.00e3$

### 5.5.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.257143$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then  
 $\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_u) / (\epsilon_u - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

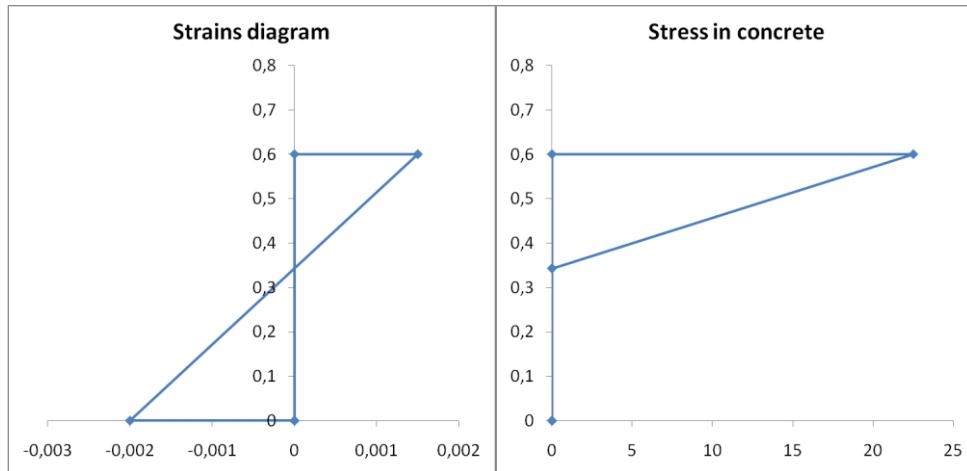
i	x	y	$\phi$	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot \epsilon_{top}$	stresses $\sigma = \epsilon \cdot 10^6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-350	-281654	-0,10    -0,25	-70414	28165
2	0,25	0,05	0,032	-0,293	-0,00171	-350	-281654	0,10    -0,25	-70414	-28165
						Sum of forces in bars:	-563309		-	140827    0

$$N_s = -563.31e3$$

$$M_{xs} = -140.83e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

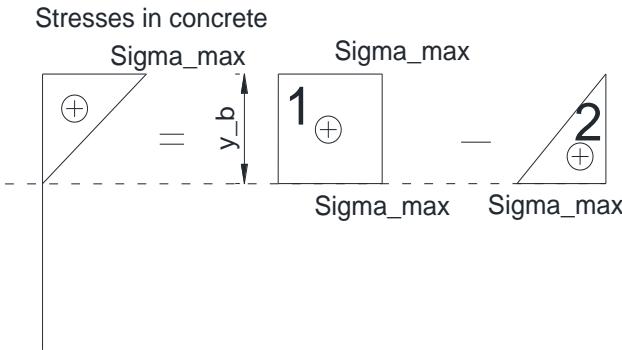
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c'$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{max} = f_{cd} \cdot Eps\_top / Eps\_c1 = 22.500e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \Sigma_{max} = 1.7357e6$$

$$N_{c2} = A_c \cdot \Sigma_{max} / 2 = 0.8679e6$$

$$N_c' = N_{c1} - N_{c2} = 0.8679e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 - y_b / 2 = 0.17142857$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 - y_b \cdot 2/3 = 0.12857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.21428571$$

Internal forces in concrete:

$$N_c = N_c' = 867.86e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -185.97e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 304.55e3$$

$$M_x = M_{xc} + M_{xs} = -326.80e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 6 Case 5: Calculation of internal forces for given state of strain in the section and parabolic-rectangular model of concrete

### 6.1 *Subject:*

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 6.2 *Summary:*

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 5a and Case 5b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 5c)
- A parabolic-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 5a)
- A section under unidirectional bending with a part of section under compression (Case 5b and Case 5c)
- A state of strain with ultimate compressive strain in concrete (Case 5a and Case 5b)
- A strain below the ultimate compressive strain in concrete (Case 5c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

---

### 6.3 *Case 5a*

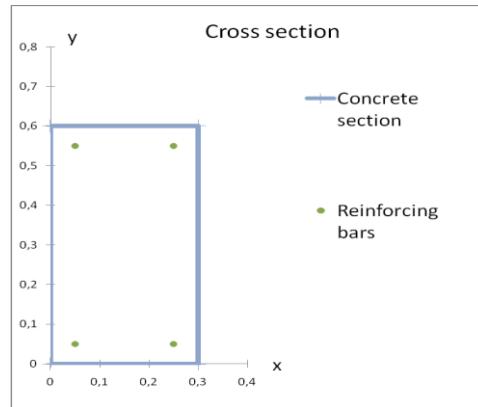
#### 6.3.1 *Data:*

- Geometry:
  - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0

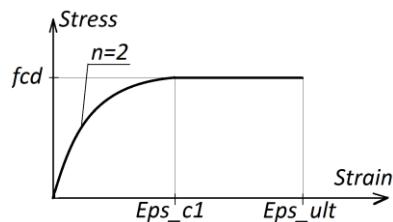
- Reinforcing bars:

x = 0.05	y = 0.05	$\phi = 0.016$
x = 0.05	y = 0.55	$\phi = 0.016$
x = 0.25	y = 0.55	$\phi = 0.016$
x = 0.25	y = 0.05	$\phi = 0.016$



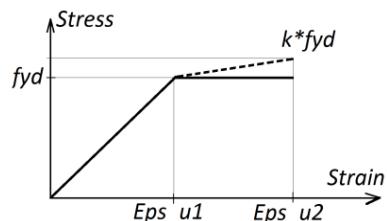
- Concrete parameters:

- Design strength  $f_{cd} = 30e6$
- Modulus of elasticity  $E_c = 32e9$
- Strain-stress model: parabolic-rectangular
- Strain ultimate limit  $Eps\_ult = 0.0035$
- Strain relation change over  $Eps\_c1 = 0.0020$

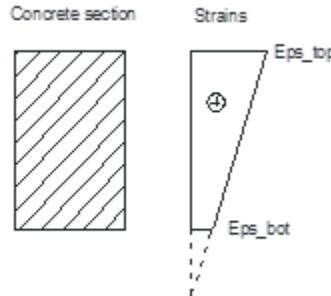


- Steel parameters:

- Design strength  $f_{yd} = 400e6$
- Hardening factor  $k = 1.0$
- Modulus of elasticity  $E_s = 200e9$
- Strain ultimate limit  $Eps\_u2 = 0.1$



- Strains:
  - Top strain  $\text{Eps\_top} = 0.0035$
  - Bottom strain  $\text{Eps\_bot} = 0.0005$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



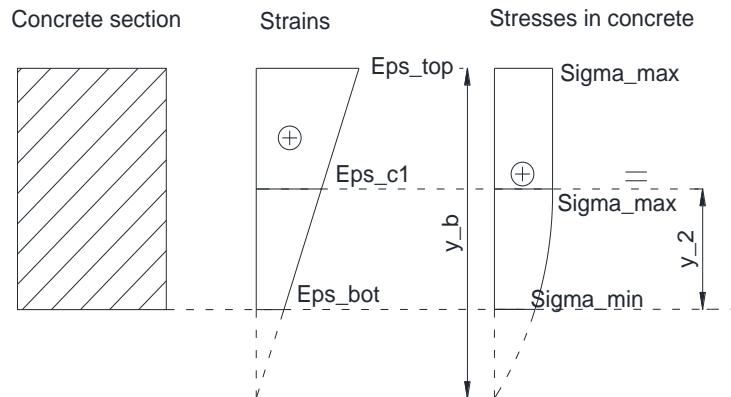
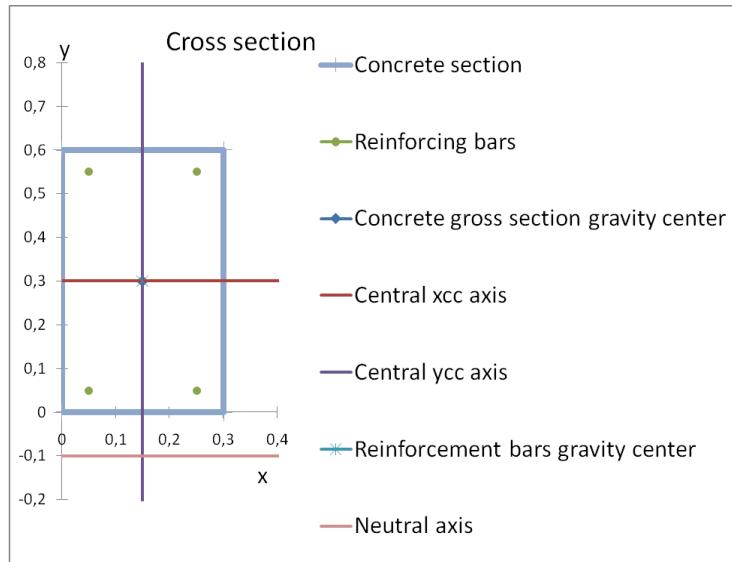
### 6.3.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 6.3.3 Results from Component

- Results for reinforcement bars:  
 $N_s = 221.17e3$   
 $M_{xs} = -25.13e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1800$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.02312688$   
 $N_c = 4874.33e3$   
 $M_{xc} = -112.73e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 5095.50e3$   
 $M_x = -137.86e3$   
 $M_y = 0.00e3$

### 6.3.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

$$\text{height of a section: } h = 0.6$$

$$y_b = -\epsilon_{top} \cdot h / (\epsilon_{bot} - \epsilon_{top}) = 0.7$$

$$\epsilon_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|\epsilon_i| \leq f_{yd} / E_s$  then

$$\sigma_i = \epsilon_i / \epsilon_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot E_{top}$	stresses $\sigma = \epsilon \cdot \frac{\pi}{10e6}$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot v$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10    -0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10    -0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10    0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10    0,25	-20106	-8042
						Sum of forces in bars:	221168		-25133	0

$$N_s = 221.17e3$$

$$M_{xs} = -25.13e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon \leq \epsilon_{c1} \text{ then } N_{sc} = f_{cd} \cdot (1 - (\epsilon - \epsilon_{c1})^2) \cdot \text{Area}$$

$$\text{if } \epsilon > \epsilon_{c1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	$\phi$	strain $\epsilon$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars $N_{sc}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,016	0,00075	-0,1    -0,25	0,000201062	3676	919	-226
2	0,25	0,05	0,016	0,00075	0,1    -0,25	0,000201062	3676	919	226
3	0,25	0,55	0,016	0,00325	0,1    0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1    0,25	0,000201062	6032	-1508	-603
						Sum of forces:	19415	-1178	0

Reduction forces in concrete:

$$N_{sc} = 19.42e3$$

$$M_{xsc} = -1.18e3$$

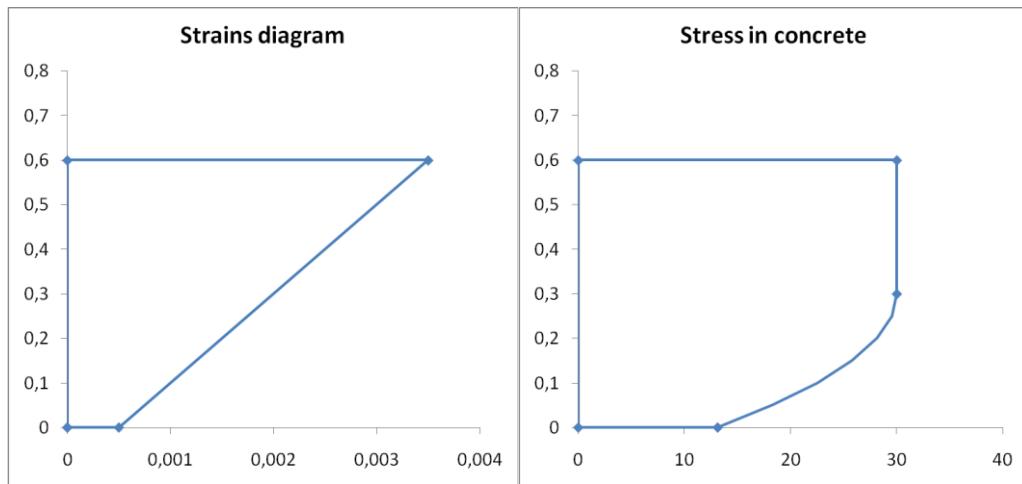
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.000000$$

$$y_{scg} = \sum(y_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.06068$$

- Results for concrete section:



Height of compressed part of the section:

0.6000

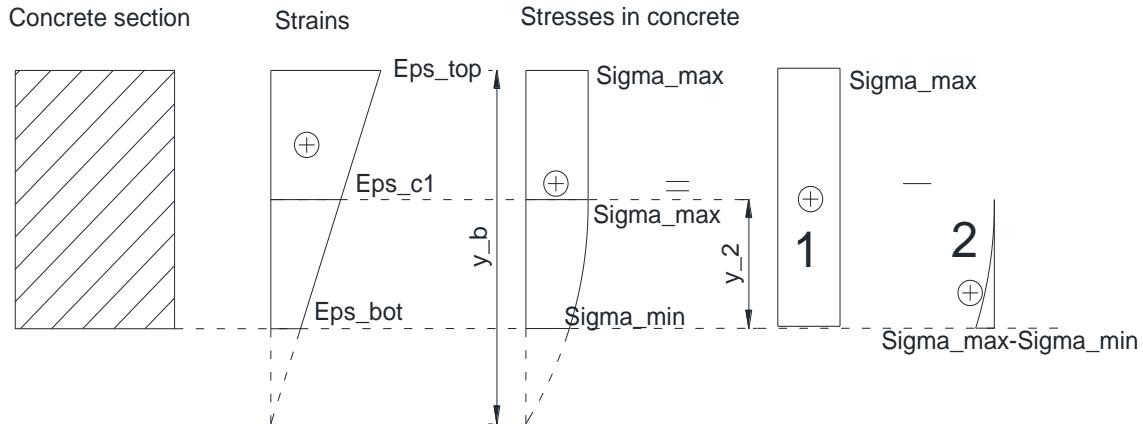
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction)

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma \sigma_{max} = f_{cd} = 30.000e6$$

$$\text{Sigma\_min} = f_{cd} \cdot (1 - (1 - Eps\_bot / Eps\_c1)^2) = 13.125e6$$

Distance between gravity center of contour and neutral axis:

$$\Delta = -(0.6 / 2 - y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $Eps\_c1$  if  $Eps\_top \geq Eps\_c1$  or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $Eps\_top < Eps\_c1$ :

$$\text{delta2} = -\Delta + \text{MIN}(y_b; Eps\_c1 / Eps\_top \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = 0.3 - \text{delta2} = 0.3$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 5.400e6$$

$$N_{c2} = 0.3 \cdot 1 / 3 \cdot y_2 \cdot (\text{Sigma\_max} - \text{Sigma\_min}) = 0.50625e6$$

$$N_c' = N_{c1} - N_{c2} = 4.89375e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -0.3 + y_2 / 4 = -0.225$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.023127$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 4874.33e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -112.73e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 5095.50e3$$

$$M_x = M_{xc} + M_{xs} = -137.86e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 6.4 Case 5b

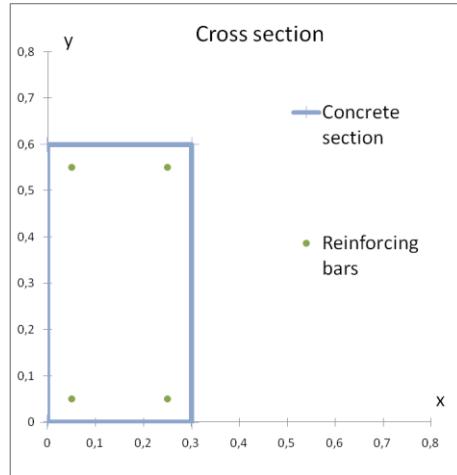
### 6.4.1 Data:

- Geometry:
  - Concrete section:
 

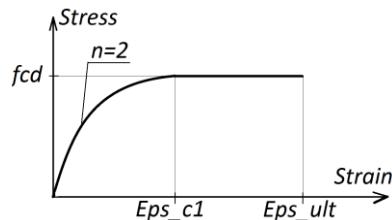
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
------------	------------	----------------

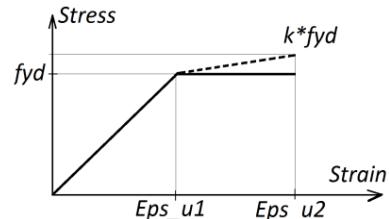
$$\begin{array}{lll}
 x = 0.05 & y = 0.55 & \phi = 0.032 \\
 x = 0.25 & y = 0.55 & \phi = 0.032 \\
 x = 0.25 & y = 0.05 & \phi = 0.032
 \end{array}$$



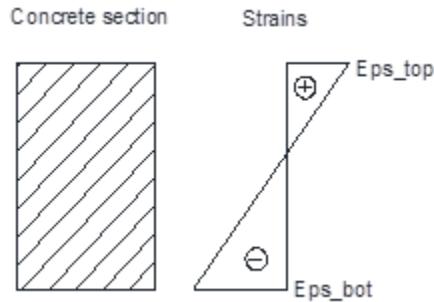
- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$
  - Strain relation change over  $\epsilon_{c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



- Strains:
  - Top strain  $\epsilon_{top} = 0.0035$
  - Bottom strain  $\epsilon_{bot} = -0.03034666$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



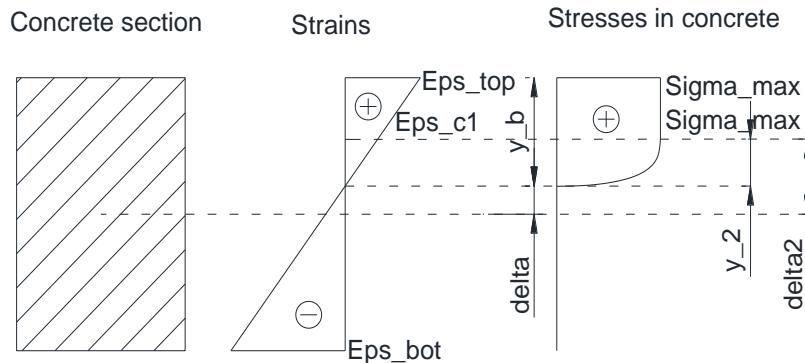
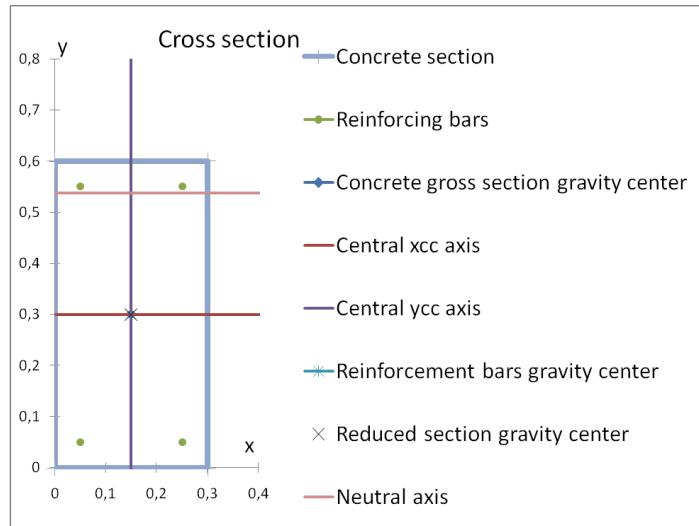
#### 6.4.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### 6.4.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -424.82e3$   
 $M_{xs} = -215.49e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0186$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.27574146$   
 $N_c = 424.82e3$   
 $M_{xc} = -117.14e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 0.00e3$   
 $M_x = -332.63e3$   
 $M_y = 0.00e3$

#### 6.4.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.0620$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	φ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses sigma / 10e6	Force in bar N = sigma • π • φ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = -N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,032	-0,488	-0,02753	-400	-321699	-0,10 -0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02753	-400	-321699	0,10 -0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00068	136	109288	0,10 0,25	-27322	10929
4	0,05	0,55	0,032	0,012	0,00068	136	109288	-0,10 0,25	-27322	-109290
						Sum of forces in bars:	-424821		-	215494 0

$$N_s = -424.82e3$$

$$M_{xs} = -215.49e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \text{eps} \leq \text{Eps\_c1} \text{ then } N_{sc} = f_{cd} \cdot (1 - (\text{eps}/\text{Eps\_c1})^2) \cdot \text{Area}$$

$$\text{if } \text{eps} > \text{Eps\_c1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	φ	Strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars N_sc	Force in concrete in place of bars N_sc	M_xsc = -N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,032	-0,02753	-0,10 -0,25				
2	0,25	0,05	0,032	-0,02753	0,10 -0,25				
3	0,25	0,55	0,032	0,00068	0,10 0,25	0,000804248	13609	-3402	1361
4	0,05	0,55	0,032	0,00068	-0,10 0,25	0,000804248	13609	-3402	-1361
						Sum of forces:	27217	-6804	0

Reduction forces in concrete:

$$N_{sc} = 27.22e3$$

$$M_{xsc} = -6.80e3$$

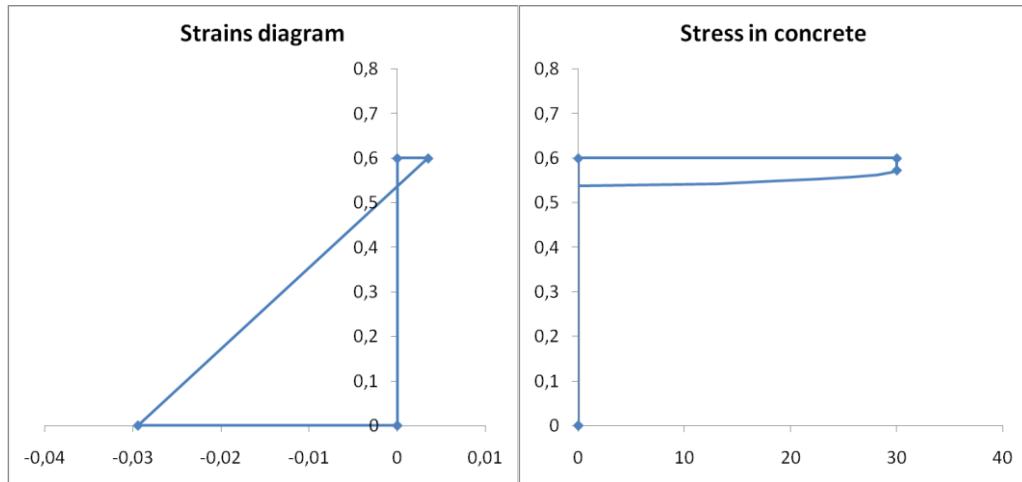
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0620$$

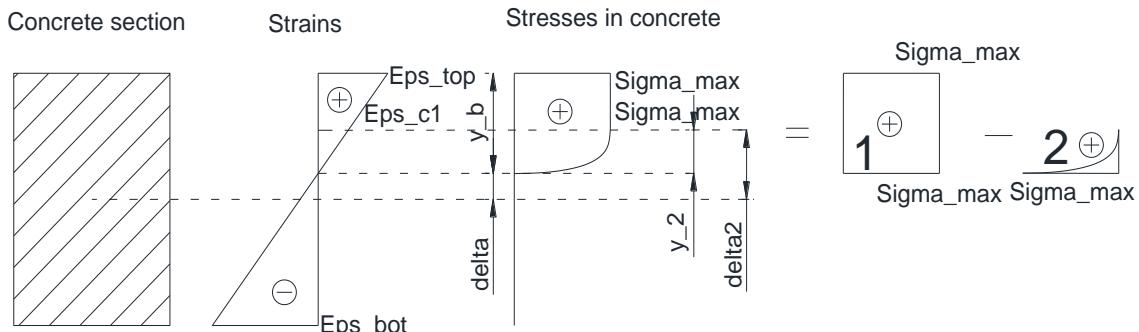
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0186$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{max} = f_{cd} = 30.000e6$$

Distance between gravity center of contour and neutral axis:

$$\delta = 0.6 / 2 - y_b = 0.2380$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps\_c1 if Eps\_top ≥ Eps\_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps\_top < Eps\_c1.;

$$\text{delta2} = \text{delta} + \text{MIN}(\text{y}_b; \text{Eps}_c1 / \text{Eps}_top \cdot \text{y}_b) = 0.2734$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$\text{y}_2 = \text{delta2} - \text{delta} = 0.0354$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 0.558401e6$$

$$N_{c2} = 0.3 \cdot y_2 \cdot \text{Sigma\_max} / 3 = 0.106362e6$$

$$N_c' = N_{c1} - N_{c2} = 0.452039e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.268978$$

$$x_{c2} = 0$$

$$y_{c2} = \text{delta} + y_2 / 3 = 0.246819$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.275741$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 424.82e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -117.14e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00e3$$

$$M_x = M_{xc} + M_{xs} = -332.63e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

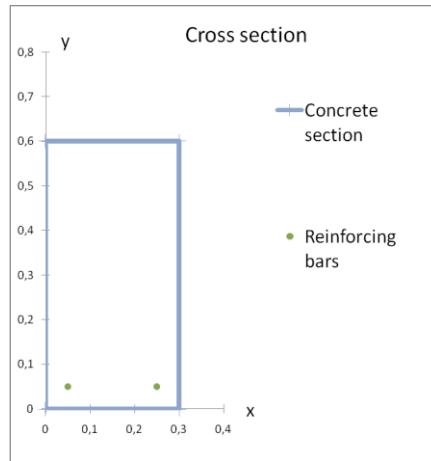
## 6.5 Case 5c

### 6.5.1 Data:

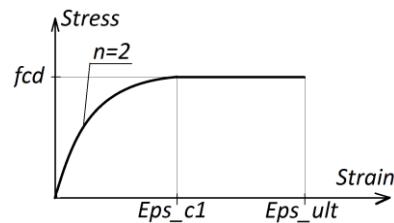
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

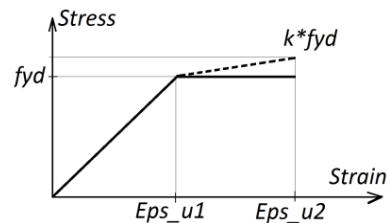
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



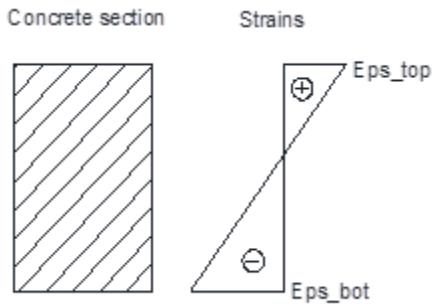
- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0035$
  - Strain relation change over  $\text{Eps\_c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.1$



- Strains:
  - Top strain  $\text{Eps\_top} = 0.0015$
  - Bottom strain  $\text{Eps\_bot} = -0.002$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



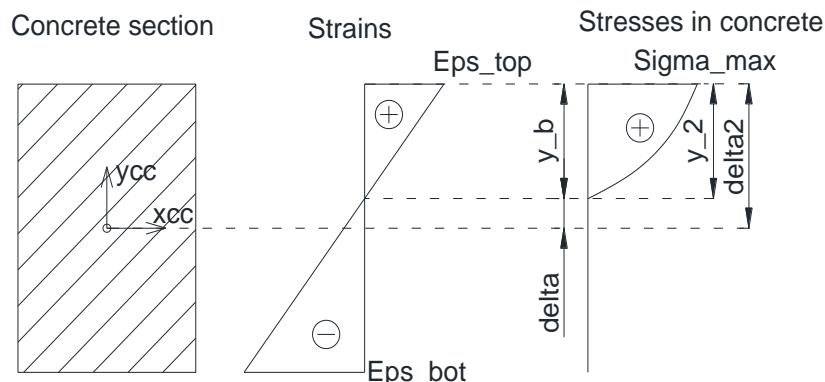
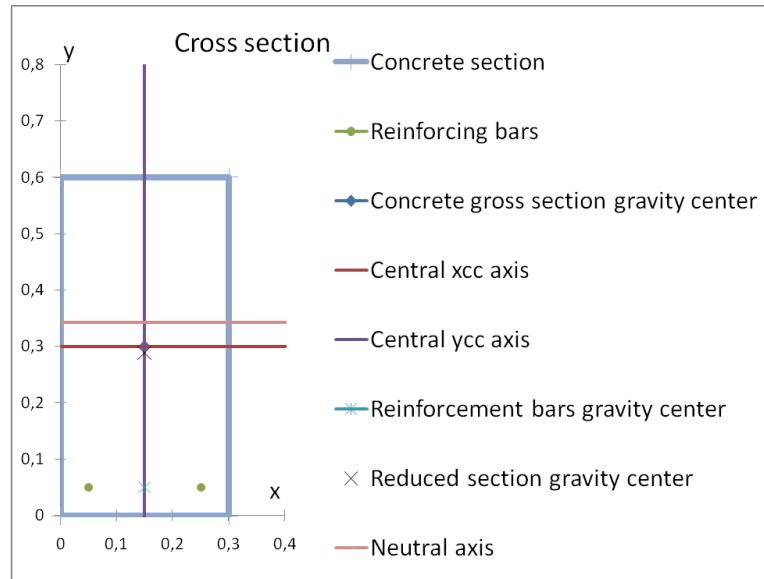
### 6.5.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 6.5.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -549.57e3$   
 $M_{xs} = -137.39e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0771$   
 $x_{cg} = 0.000000$   
 $y_{cg} = 0.20714286$   
 $N_c = 1301.79e3$   
 $M_{xc} = -269.66e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 752.22e3$   
 $M_x = -407.05e3$   
 $M_y = 0.00e3$

### 6.5.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.257143$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

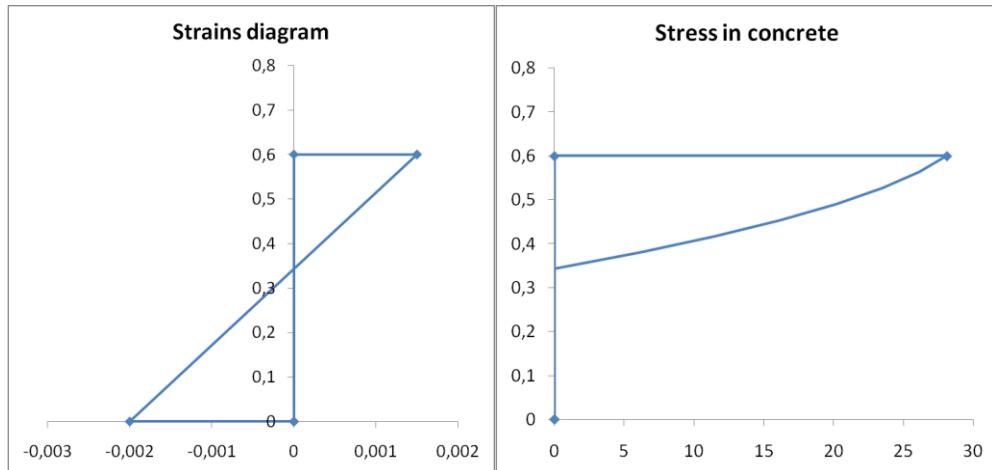
i	x	y	$\phi$	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot E_{top}$	stresses $\sigma = \epsilon \cdot 10^6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10    -0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10    -0,25	-68696	-27478
						Sum of forces in bars:	-549569		137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

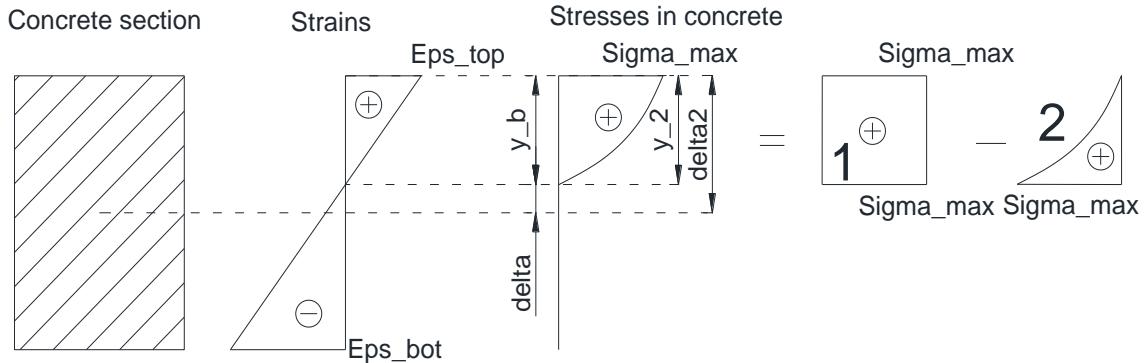
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c'$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot (1 - (1 - \text{Eps\_top}/\text{Eps\_c1})^2) = 28.125e6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.042857$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $\text{Eps\_c1}$  if  $\text{Eps\_top} \geq \text{Eps\_c1}$  or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $\text{Eps\_top} < \text{Eps\_c1}$ :

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; \text{Eps\_c1} / \text{Eps\_top} \cdot y_b) = 0.3000$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

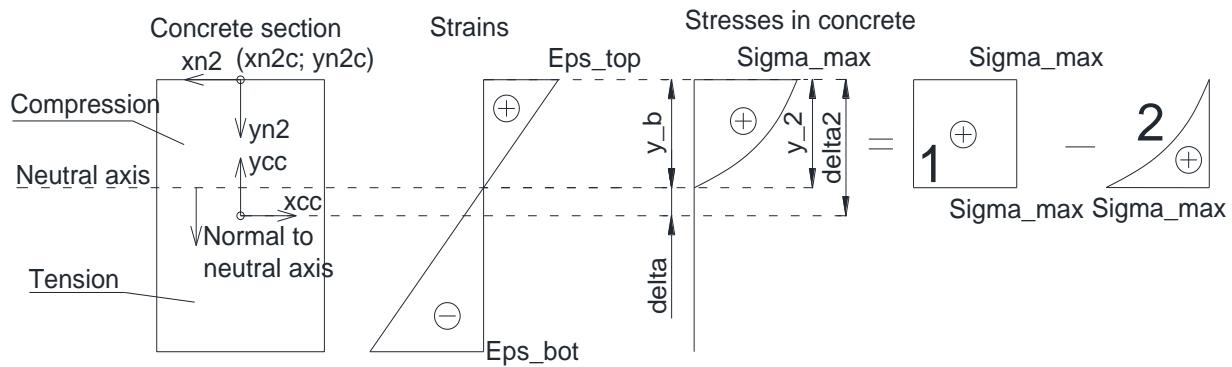
$$y_2 = \text{delta2} - \text{delta} = 0.257143$$

Axial force in compressed part of concrete gross section

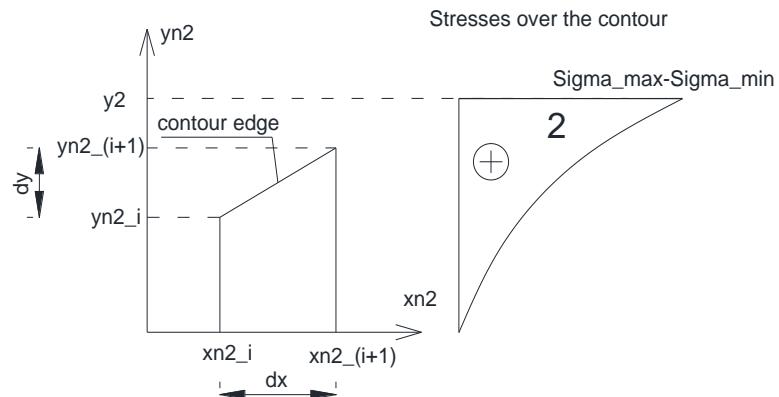
$$N_{c1} = A_c \cdot \text{Sigma\_max} = 2.169643e6$$

To calculate  $N_{c2}$  we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system  $x_{n2}, y_{n2}$  (see picture below), where  $x_{n2}$  is an axis parallel to neutral axis, in distance  $\text{delta2}$  from the gravity center of contour of the section, and it crosses axis  $y_{n2}$  in point  $(x_{n2c}; y_{n2c})$  given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in  $(x_{n2}, y_{n2})$  coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (x <sub>n2</sub> , y <sub>n2</sub> ) system						N <sub>c2</sub>	Static moment of stress solid MS <sub>yn2,zn2</sub>	Static moment of stress solid MS <sub>xn2,zn2</sub>
	x	y	x <sub>cc</sub>	y <sub>cc</sub>	x <sub>n2</sub>	y <sub>n2</sub>	dx	dy	A	B			
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,00	0,00	0,00
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,87	0,00	0,16
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,00	0,00	0,00
4	0,00	0,60	-0,15	0,30	0,15	0,00	0,30	-	0,00	0,00	0,00	0,00	0,00
	$N_{c2} = \text{sum}(N_{c2,i}) =$										0,87		
	C= 0,34												
	D= 0,26												
	gravity center of stress solid 2 in (x <sub>n2</sub> ; y <sub>n2</sub> ) coordinates system												x <sub>c2'</sub> 0,0000
													y <sub>c2'</sub> 0,1821
	gravity center of stress solid 2 in central axes ( x <sub>cc</sub> ; y <sub>cc</sub> ) coordinates system												x <sub>c2</sub> 0,0000
													y <sub>c2</sub> 0,1179

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{(i+1)} - x_i$$

$$dy = y_{(i+1)} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2(i+1)} + y_{n2(i+1)} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot Eps_c1 / Eps_top$$

$$D = \text{MIN}(Eps_c1; Eps_top) \cdot y_b / Eps_top$$

$$n = 2 \text{ (power of stress-strain relation curve)}$$

$$dSigma = \text{Sigma\_max} - \text{Sigma\_min}$$

$$N_{c2\_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2\_i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2} - (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2}) - (x_{n2\_i} \cdot (i+1) - x_{n2\_i}) \cdot (1 - D / C)^{n+1}) + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2) + B \cdot (x_{n2\_i} \cdot (i+1) - x_{n2\_i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2\_i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + dSigma) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2\_i})$$

$$MS_{yn2,zn2\_i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2) + (-f_{cd} + dSigma) \cdot B / 2 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2\_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2\_i} \cdot (i+1) \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2} - x_{n2\_i} \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+3} - (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+3})) - 0.5 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot (x_{n2\_i} \cdot (i+1)^3 - x_{n2\_i}^3) + B / 2 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2\_i})$$

$$MS_{xn2,zn2\_i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2\_i} \cdot (i+1) - x_{n2\_i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2\_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2\_i} \cdot (i+1) \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2} - y_{n2\_i} \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2\_i} \cdot (i+1) + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2\_i} \cdot (i+1)^3 - x_{n2\_i}^3) + A \cdot B / 2 \cdot (x_{n2\_i} \cdot (i+1)^2 - x_{n2\_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2\_i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.867857e6$$

$$N_c' = N_{c1} - N_{c2} = 1.301786e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.171428571$$

$$x_{c2} = 0$$

$$y_{c2} = 0.117857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.207143$$

Internal forces in concrete:

$$N_c = N_c' = 1301.79e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -269.66e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 752.22e3$$

$$M_x = M_{xc} + M_{xs} = -407.05e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 7 Case 6: Calculation of internal forces for given state of strain in the section and power-rectangular model of concrete

### 7.1 *Subject:*

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 7.2 *Summary:*

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 6a and Case 6b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 6c)
- A power-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 6a)
- A section under unidirectional bending with a part of section under compression (Case 6b and Case 6c)
- A state of strain with ultimate compressive strain in concrete (Case 6a and Case 6b)
- A strain below the ultimate compressive strain in concrete (Case 6c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

---

### 7.3 *Case 6a*

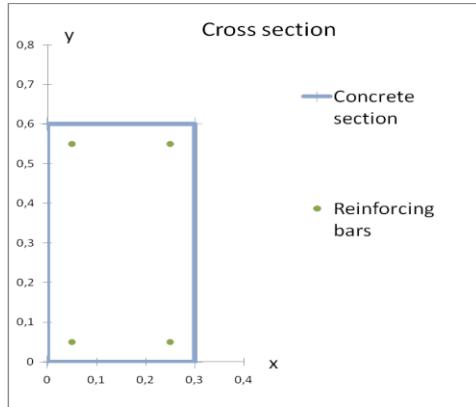
#### 7.3.1 *Data:*

- Geometry:
  - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

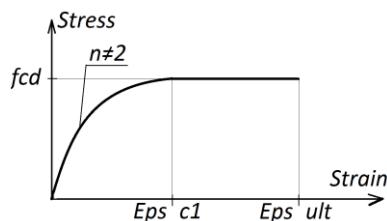
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$



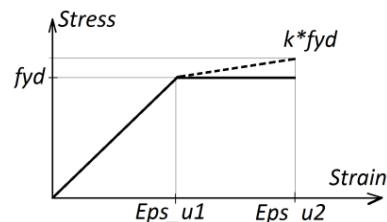
- Concrete parameters:

- Design strength  $f_{cd} = 30e6$
- Modulus of elasticity  $E_c = 32e9$
- Strain-stress model: power-rectangular
- Power:  $n = 1.4$
- Strain ultimate limit  $Eps\_ult = 0.0035$
- Strain relation change over  $Eps\_c1 = 0.0020$

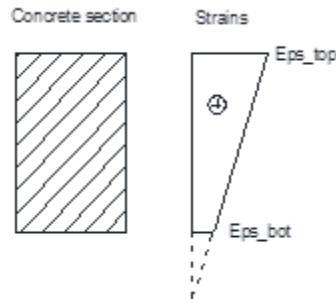


- Steel parameters:

- Design strength  $f_{yd} = 400e6$
- Hardening factor  $k = 1.0$
- Modulus of elasticity  $E_s = 200e9$
- Strain ultimate limit  $Eps\_u2 = 0.1$



- Strains:
  - Top strain  $\text{Eps\_top} = 0.0035$
  - Bottom strain  $\text{Eps\_bot} = 0.0005$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



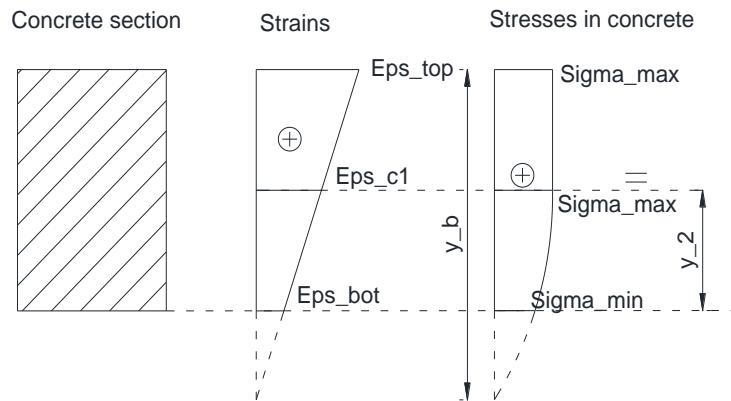
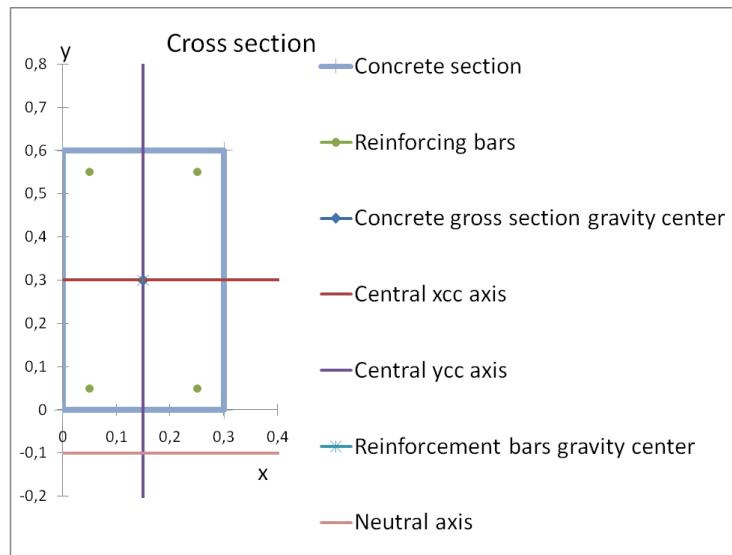
### 7.3.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 7.3.3 Results from Component

- Results for reinforcement bars:  
 $N_s = 221.17e3$   
 $M_{xs} = -25.13e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1800$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.03405827$   
 $N_c = 4630.08e3$   
 $M_{xc} = -157.69e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 4851.25e3$   
 $M_x = -182.83e3$   
 $M_y = 0.00e3$

### 7.3.4 Results (manual calculations):



- Results for reinforcement bars:  
Distance between maximum compression corner of the section and neutral axis:  
height of a section:  $h = 0.6$   
 $y_b = -\epsilon_{top} \cdot h / (\epsilon_{bot} - \epsilon_{top}) = 0.7$   
 $\epsilon_{u1} = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

```
If  $|\epsilon_i| \leq f_{yd} / E_s$  then
     $\sigma_i = \epsilon_i / \epsilon_{u1} \cdot f_{yd}$ 
else
     $\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$ 
```

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • ϕ^2 / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = - N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10 -0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10 -0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10 0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10 0,25	-20106	-8042
						Sum of forces in bars:	221168		-25133	0

$$N_s = 221.17e3$$

$$M_{xs} = -25.13e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if  $\text{eps} \leq \text{Eps\_c1}$  then  $N_{sc} = f_{cd} \cdot (1 - (1 - \text{eps}/\text{Eps\_c1})^n) \cdot \text{Area}$   
 if  $\text{eps} > \text{Eps\_c1}$  then  $N_{sc} = f_{cd} \cdot \text{Area}$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars N_sc	Force in concrete in place of bars N_sc	M_xsc = - N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,016	0,00075	-0,1 -0,25	0,000201062	2908	727	-291
2	0,25	0,05	0,016	0,00075	0,1 -0,25	0,000201062	2908	727	291
3	0,25	0,55	0,016	0,00325	0,1 0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1 0,25	0,000201062	6032	-1508	-603
						Sum of forces:	17880	-1562	0

Reduction forces in concrete:

$$N_{sc} = 17.88e3$$

$$M_{xsc} = -1.56e3$$

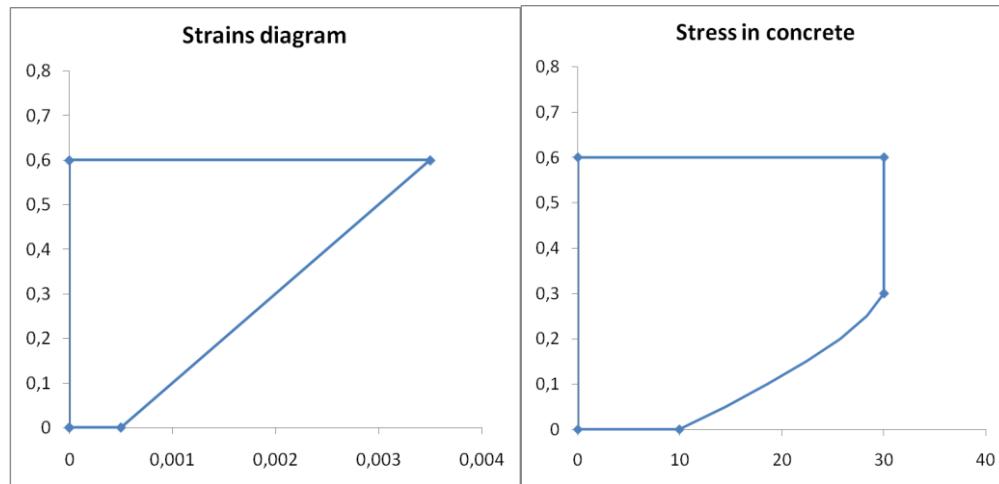
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.00000$$

$$y_{scg} = \sum(y_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.08736$$

- Results for concrete section:



Height of compressed part of the section:

$$h = 0.6000$$

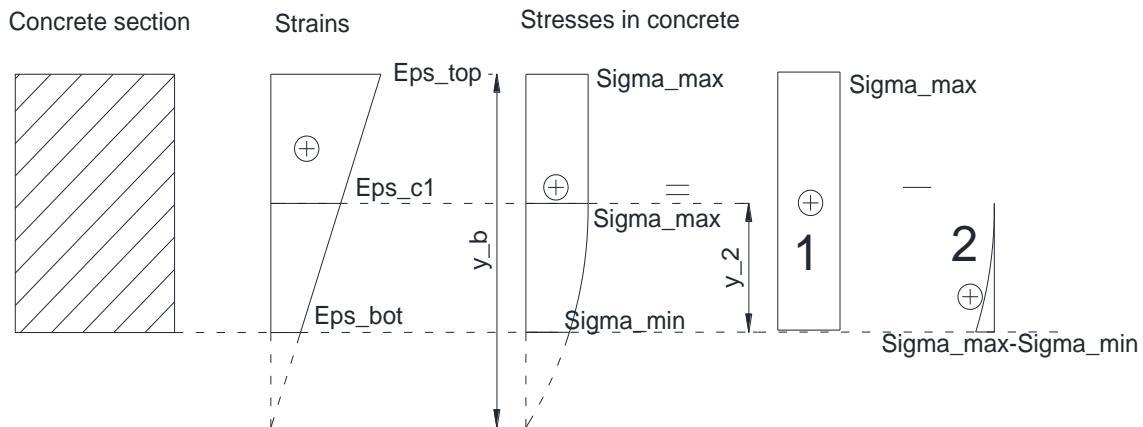
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a power ( $n = 1.4$ ) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c' - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{\max} = f_{cd} = 30.000e6$$

$$\Sigma_{\min} = f_{cd} \cdot (1 - (1 - Eps_{bot} / Eps_{c1})^n) = 9.946e6$$

Distance between gravity center of contour and neutral axis:

$$\delta = -(0.6 / 2 \cdot y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $Eps_{c1}$  if  $Eps_{top} \geq Eps_{c1}$
- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $Eps_{top} < Eps_{c1}$ :

$$\delta_2 = -\delta + \min(y_b; Eps_{c1} / Eps_{top} \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

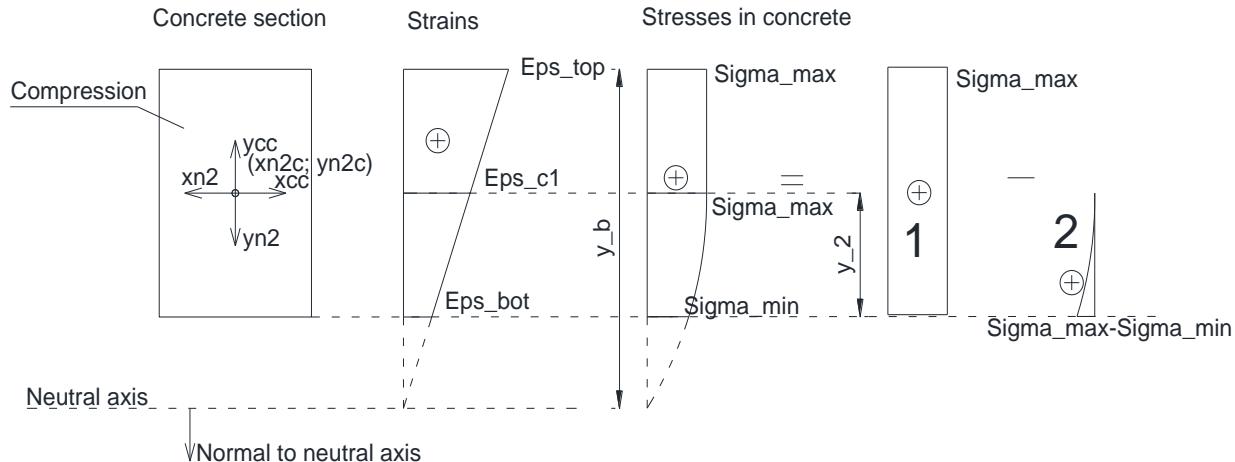
$$y_2 = 0.3 - \delta_2 = 0.3$$

Axial force in compressed part of concrete gross section

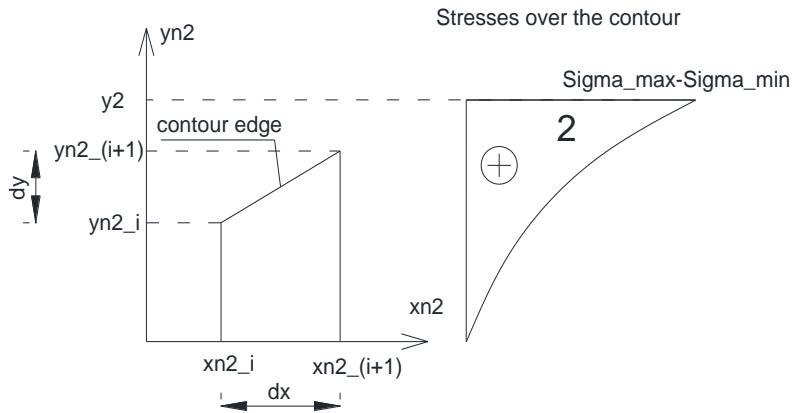
$$N_{c1} = A_c \cdot \Sigma_{\max} = 5.400e6$$

To calculate  $N_{c2}$  we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system  $x_{n2}$ ,  $y_{n2}$  (see picture below), where  $x_{n2}$  is an axis parallel to neutral axis, in distance  $\delta_2$  from the gravity center of contour of the section, and it crosses axis  $y_{n2}$  in point  $(x_{n2c}; y_{n2c})$  given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \delta_2)$$



Now we find coordinates of contour under stress solid 2 in  $(x_{n2}, y_{n2})$  coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in $(x_{n2}, y_{n2})$ system		dx	dy	A	B	$N_{c2}$	Static moment of stress solid	
	x	y	$x_{cc}$	$y_{cc}$	$x_{n2}$	$y_{n2}$						$MS_{yn2,zn2}$	$MS_{xn2,zn2}$
1	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	-0,30	0,00	0,00	0,00	0,00	0,00
2	0,00	0,30	-0,15	0,00	0,15	0,00	-0,30	0,00	0,00	0,00	0,00	0,00	0,00
3	0,30	0,30	0,15	0,00	-0,15	0,00	0,00	0,30	0,00	0,00	0,00	0,00	0,00
4	0,30	0,00	0,15	-0,30	-0,15	0,30	0,30	0,00	0,00	0,30	0,75	0,00	0,16
5	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00
										$N_{c2} = \text{sum}(N_{c2,i}) =$	0,75		
										C=	0,40		
										D=	0,40		
gravity center of stress solid 2 in $(x_{n2}; y_{n2})$ coordinates system												$x_{c2}'$	0,0000
												$y_{c2}'$	0,2118
gravity center of stress solid 2 in central axes $(x_{cc}; y_{cc})$ coordinates system												$x_{c2}$	0,0000
												$y_{c2}$	-0,2118

Where:

$$x_{cc} = x - x_{c1}$$

$$y_{cc} = y - y_{c1}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{(i+1)} - x_i$$

$$dy = y_{(i+1)} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2-(i+1)} + y_{n2-(i+1)} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot Eps_c1 / Eps_top$$

$$D = \text{MIN}(Eps_c1; Eps_top) \cdot y_b / Eps_top$$

$$n = 1.4 \text{ (power of stress-strain relation curve)}$$

$$dSigma = \text{Sigma\_max} - \text{Sigma\_min}$$

$$N_{c2-i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2-i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-y_{n2-(i+1)} + D) / C)^{n+2} - (1 - (-y_{n2-(i+1)} + D) / C)^{n+2}) - (x_{n2-(i+1)} - x_{n2-i}) \cdot (1 - D / C)^{n+1}) + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2) + B \cdot (x_{n2-(i+1)} - x_{n2-i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2-i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + dSigma) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2-i})$$

$$MS_{yn2,zn2-i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2) + (-f_{cd} + dSigma) \cdot B / 2 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2-i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2-(i+1)} \cdot (1 - (-y_{n2-(i+1)} + D) / C)^{n+2} - x_{n2-i} \cdot (1 - (-y_{n2-i} + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-y_{n2-(i+1)} + D) / C)^{n+3} - (1 - (-y_{n2-i} + D) / C)^{n+3})) - 0.5 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot (x_{n2-(i+1)}^3 - x_{n2-i}^3) + B / 2 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2-i})$$

$$MS_{xn2,zn2-i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2-(i+1)} - x_{n2-i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2-i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2-(i+1)} \cdot (1 - (-y_{n2-(i+1)} + D) / C)^{n+2} - y_{n2-i} \cdot (1 - (-y_{n2-i} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2-(i+1)} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2-i} + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2-(i+1)}^3 - x_{n2-i}^3) + A \cdot B / 2 \cdot (x_{n2-(i+1)}^2 - x_{n2-i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2-i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.752035e6$$

$$N_c' = N_{c1} - N_{c2} = 4.647965e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -0.211764706$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.034058$$

Internal forces in concrete:

$$\begin{aligned}N_c &= N_{c'} - N_{sc} = \underline{4630.08e3} \\M_{xc} &= -N_c \cdot y_{cg} = \underline{-157.69e3} \\M_{yc} &= N_{c'} \cdot x_{cg} = \underline{0.00e3}\end{aligned}$$

Results for reduced section:

$$\begin{aligned}N &= N_c + N_s = \underline{4851.25e3} \\M_x &= M_{xc} + M_{xs} = \underline{-182.83e3} \\M_y &= M_{yc} + M_{ys} = \underline{0.00e3}\end{aligned}$$

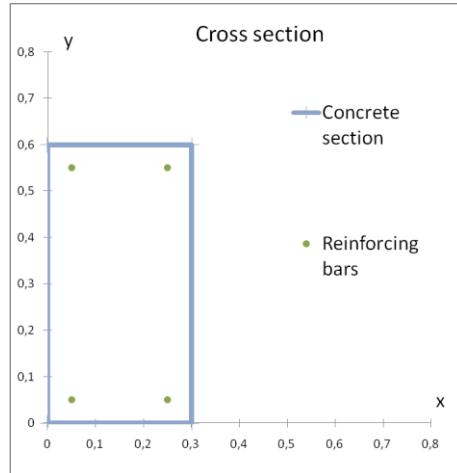
## 7.4 Case 6b

### 7.4.1 Data:

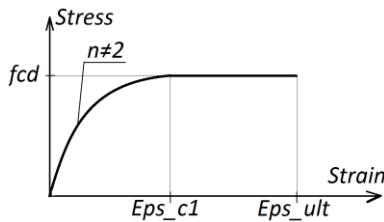
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
- Reinforcing bars:
 

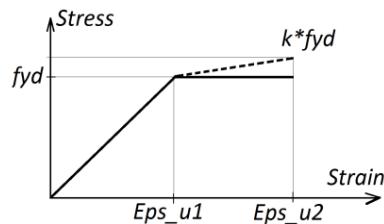
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$
x = 0.25	y = 0.55	$\phi = 0.032$
x = 0.05	y = 0.55	$\phi = 0.032$



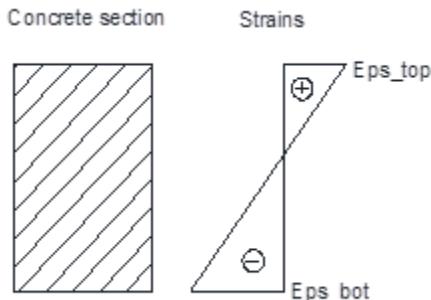
- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: power-rectangular
  - Power:  $n = 1.8$
  - Strain ultimate limit  $Eps_{ult} = 0.0035$
  - Strain relation change over  $Eps_{c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



- Strains:
  - Top strain  $\epsilon_{top} = 0.0035$
  - Bottom strain  $\epsilon_{bot} = -0.03034666$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



#### 7.4.2 Search for:

- Results for reinforcement bars:  $N_s, M_{xs}, M_{ys}$
- Results for concrete section:  $A_c, x_{cg}, y_{cg}, N_c, M_{xc}, M_{yc}$
- Results for reduced section:  $N, M_x, M_y$

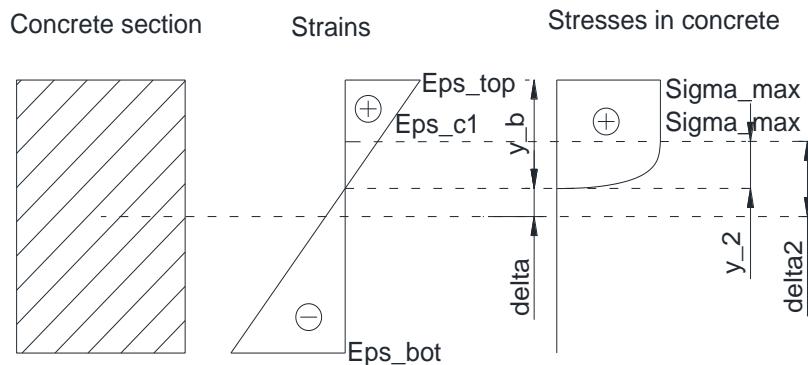
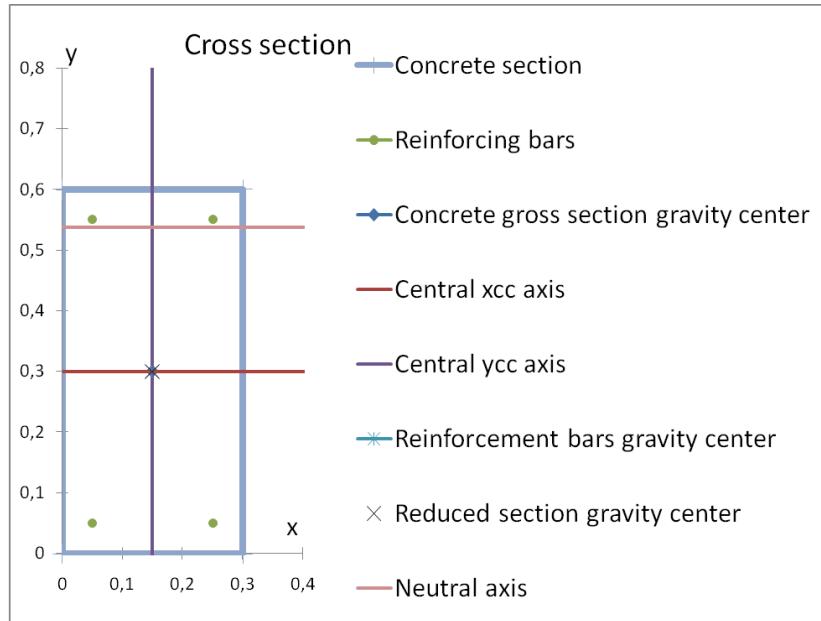
#### 7.4.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -420.68\text{e}3$   
 $M_{xs} = -216.53\text{e}3$   
 $M_{ys} = 0.00\text{e}3$
- Results for concrete section:  
 $A_c = 0.0187$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.27592078$

$$\begin{aligned}N_c &= 420.68e3 \\M_{xc} &= -116.08e3 \\M_{yc} &= 0.00e3\end{aligned}$$

- Results for reduced section:  
 $N = 0.00e3$   
 $M_x = -332.60e3$   
 $M_y = 0.00e3$

#### 7.4.4 Results (manual calculations):



- Results for reinforcement bars:  
 Distance between maximum compression corner of the section and neutral axis:  
 height of a section:  $h = 0.6$   
 $y_b = -Eps\_top \cdot h / (Eps\_bot - Eps\_top) = 0.0623$   
 $Eps\_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{cc} = 0.15$$

$$y_{cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_y d / E_s$  then

$$\sigma_i = eps_i / E_s \cdot f_y d$$

else

$$\sigma_i = f_y d + (|eps_i| - E_s) / (E_s - E_u) \cdot (k - 1) \cdot f_y d$$

i	x	y	$\phi$	Distance d from neutral axis	Strain $eps = d / y_b \cdot E_{top}$	Stresses $\sigma / 10e6$	Force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc} \quad y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,488	-0,02738	-400	-321699	-0,10 -0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02738	-400	-321699	0,10 -0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00069	138	111358	0,10 0,25	-27839	11136
4	0,05	0,55	0,032	0,012	0,00069	138	111358	-0,10 0,25	-27839	-11136
						Sum of forces in bars:	-420682		-	216528 0

$$N_s = -420.68e3$$

$$M_{xs} = -216.53e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if  $eps \leq Eps_c1$  then  $N_{sc} = f_{cd} \cdot (1 - (1 - eps / Eps_c1)^n) \cdot Area$   
 if  $eps > Eps_c1$  then  $N_{sc} = f_{cd} \cdot Area$

i	x	y	$\phi$	strain eps	Positions in central axes coordinate system $x_{cc} \quad y_{cc}$	Area of concrete with compression stress cut from the section in place of bars $N_{sc}$	Force in concrete in place of bars $N_{sc} \cdot y_{cc}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,02738	-0,10 -0,25				
2	0,25	0,05	0,032	-0,02738	0,10 -0,25				
3	0,25	0,55	0,032	0,00069	0,10 0,25	0,000804248	12898	-3224	1290
4	0,05	0,55	0,032	0,00069	-0,10 0,25	0,000804248	12898	-3224	-1290
						Sum of forces:	25796	-6449	0

Reduction forces in concrete:

$$N_{sc} = 25.79e3$$

$$M_{xsc} = -6.45e3$$

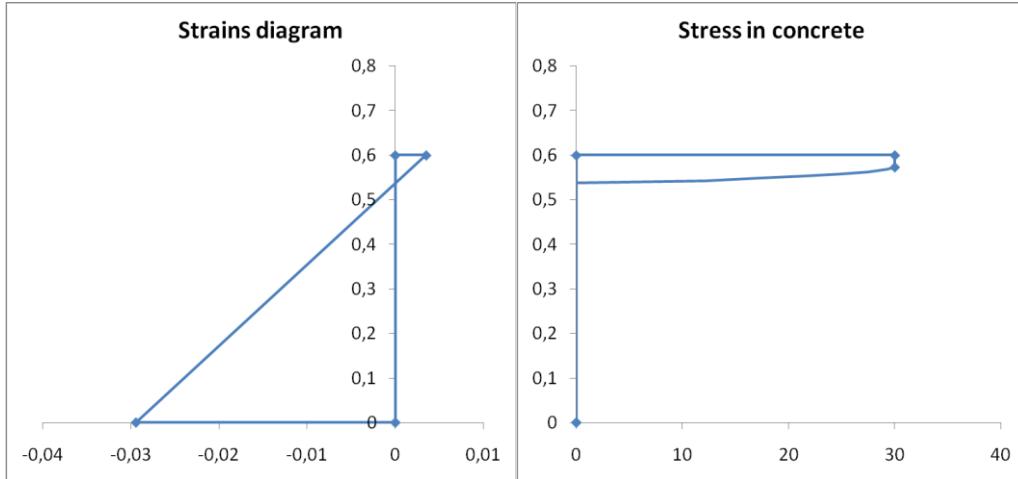
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.00$$

$$y_{scg} = \sum(y_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0623$$

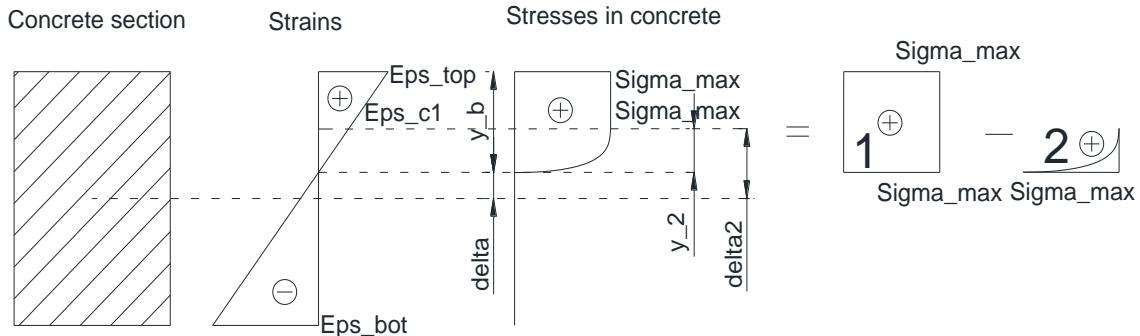
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0187$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a power ( $n = 1.8$ ) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c' - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} = 30.000\text{e}6$$

$$\text{Sigma\_min} = 0.000\text{e}6$$

Distance between gravity center of contour and neutral axis:

$$\delta = 0.6 / 2 - y_b = 0.237671$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $\epsilon_{c1}$  if  $\epsilon_{top} \geq \epsilon_{c1}$  or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $\epsilon_{top} < \epsilon_{c1}$ :

$$\delta_2 = \delta + \min(y_b; \epsilon_{c1} / \epsilon_{top} \cdot y_b) = 0.273288$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

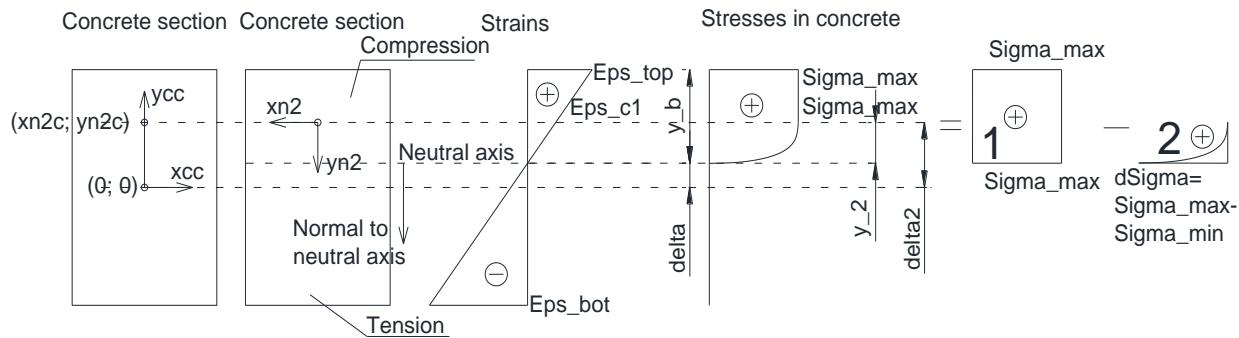
$$y_2 = \delta_2 - \delta = 0.0356165$$

Axial force in compressed part of concrete gross section

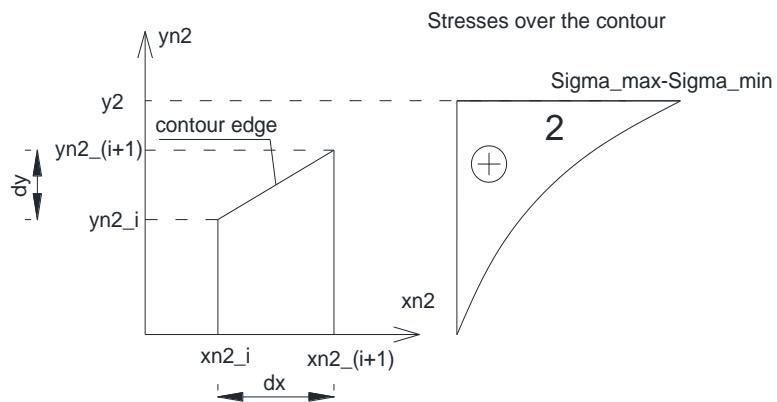
$$N_{c1} = A_c \cdot \text{Sigma\_max} = 0.560960\text{e}6$$

To calculate  $N_{c2}$  we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system  $x_{n2}, y_{n2}$  (see picture below), where  $x_{n2}$  is an axis parallel to neutral axis, in distance  $\delta_2$  from the gravity center of contour of the section, and it crosses axis  $y_{n2}$  in point  $(x_{n2c}; y_{n2c})$  given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \delta_2)$$



Now we find coordinates of contour under stress solid 2 in  $(x_{n2}, y_{n2})$  coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in $(x_{n2}, y_{n2})$ system		dx	dy	A	B	$N_{c2}$	Static moment of stress solid		
	x	y	$x_{cc}$	$y_{cc}$	$x_{n2}$	$y_{n2}$						$MS_{y_{n2}, z_{n2}}$	$MS_{x_{n2}, z_{n2}}$	
1	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,04	0,00	0,00	0,000	0,000	0,000	
2	0,30	0,54	0,15	0,24	-0,15	0,04	0,30	0,00	0,00	0,04	0,114	0,000	0,003	
3	0,00	0,54	-0,15	0,24	0,15	0,04	0,00	-0,04	0,00	0,00	0,000	0,000	0,000	
4	0,00	0,57	-0,15	0,27	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000	
5	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000	
										$N_{c2} = \text{sum}(N_{c2\_i}) =$	0,114			
										C=	0,03 6			
										D=	0,03 6			
gravity center of stress solid 2 in $(x_{n2}, y_{n2})$ coordinates system													$x_{c2}'$	0,0000
													$y_{c2}'$	0,0262
gravity center of stress solid 2 in central axes $(x_{cc}, y_{cc})$ coordinates system													$x_{c2}$	0,0000
													$y_{c2}$	0,2470

Where:

$$x_{cc} = x - x_{C_c}$$

$$y_{cc} = y - y_{C_c}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{(i+1)} - x_i$$

$$dy = y_{(i+1)} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2(i+1)} + y_{n2(i+1)} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot Eps_c1 / Eps_top$$

$$D = \text{MIN}(Eps_c1; Eps_top) \cdot y_b / Eps_top$$

$$n = 1.8 \text{ (power of stress-strain relation curve)}$$

$$dSigma = \text{Sigma\_max} - \text{Sigma\_min}$$

$$N_{c2\_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2\_i} = f_{cd} \cdot C / (n + 1) \cdot (C / (n + 2) / A \cdot ((1 - (-y_{n2\_i+1} + D) / C)^{n+2} - (1 - (-y_{n2\_i} + D) / C)^{n+1})^2 - (x_{n2\_i+1} - x_{n2\_i}) \cdot (1 - D / C)^{n+1}) + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2) + B \cdot (x_{n2\_i+1} - x_{n2\_i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2\_i} = (f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + dSigma) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2\_i})$$

$$MS_{yn2,zn2\_i} = f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2) + (-f_{cd} + dSigma) \cdot B / 2 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2\_i} = f_{cd} \cdot C^2 / ((n + 1) \cdot (n + 2) \cdot A) \cdot (x_{n2\_i+1} \cdot (1 - (-y_{n2\_i+1} + D) / C)^{n+2} - x_{n2\_i} \cdot (1 - (-y_{n2\_i} + D) / C)^{n+1}) - C / (n + 3) / A \cdot ((1 - (-y_{n2\_i+1} + D) / C)^{n+3} - (1 - (-y_{n2\_i} + D) / C)^{n+2}) - 0.5 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2) \cdot f_{cd} \cdot C / (n + 1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot (x_{n2\_i+1}^3 - x_{n2\_i}^3) + B / 2 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2\_i})$$

$$MS_{xn2,zn2\_i} = (f_{cd} \cdot C / (n + 1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n + 2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+1})) + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2\_i+1} - x_{n2\_i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2\_i} = f_{cd} \cdot C^2 / ((n + 1) \cdot (n + 2) \cdot A) \cdot (y_{n2\_i+1} \cdot (1 - (-y_{n2\_i+1} + D) / C)^{n+2} - y_{n2\_i} \cdot (1 - (-y_{n2\_i} + D) / C)^{n+1}) - C / (n + 3) \cdot (2 \cdot (1 - (-y_{n2\_i+1} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2\_i} + D) / C)^{n+2}) + A \cdot (1 - D / C)^{n+2} \cdot dx + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2\_i+1}^3 - x_{n2\_i}^3) + A \cdot B / 2 \cdot (x_{n2\_i+1}^2 - x_{n2\_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2\_i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.114482e6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.446478e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.268836$$

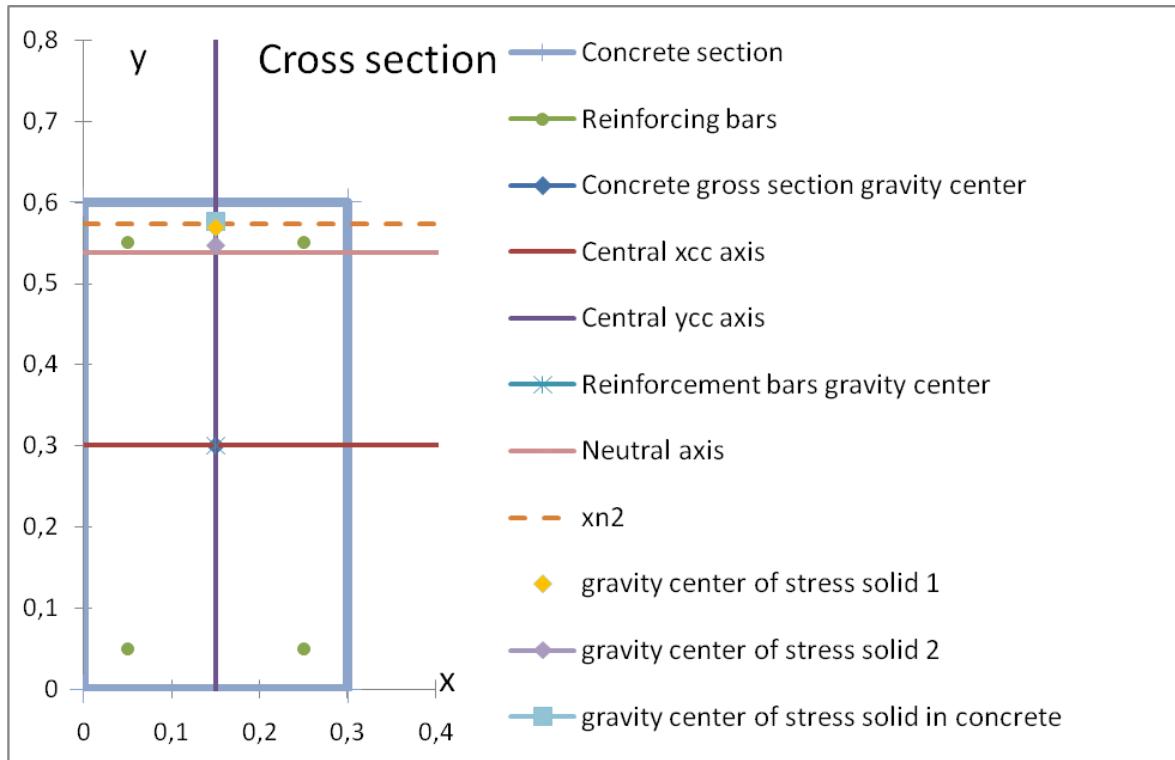
$$x_{c2} = 0$$

$$y_{c2} = 0.247043919$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.275921$$



Internal forces in concrete:

$$\begin{aligned} N_c &= N_c' - N_{sc} = 420.68e3 \\ M_{xc} &= -N_c \cdot y_{cg} = -116.08e3 \\ M_{yc} &= N_c \cdot x_{cg} = 0.00e3 \end{aligned}$$

- Results for reduced section:
 
$$\begin{aligned} N &= N_c + N_s = 0.00e3 \\ M_x &= M_{xc} + M_{xs} = -332.60e3 \\ M_y &= M_{yc} + M_{ys} = M_{yc} + M_{ys} = 0.00e3 \end{aligned}$$

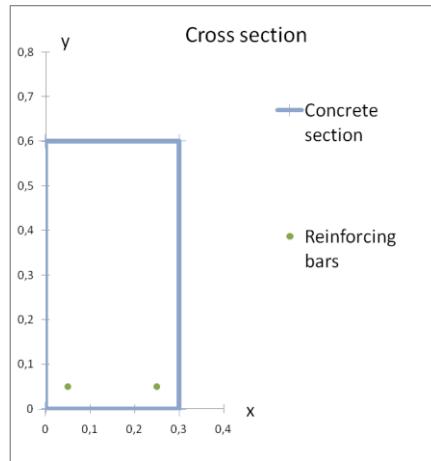
## 7.5 Case 6c

### 7.5.1 Data:

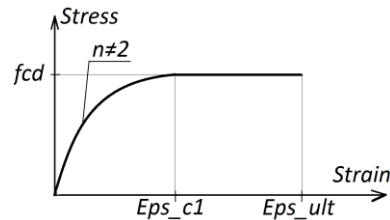
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

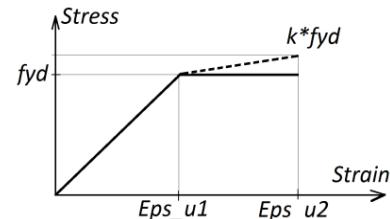
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



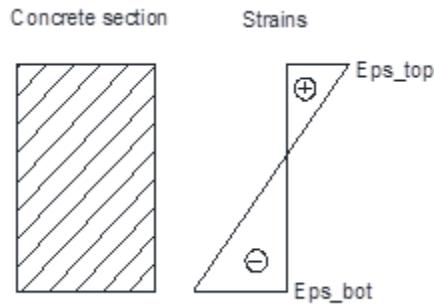
- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: power-rectangular
  - Power:  $n = 1.2$
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$
  - Strain relation change over  $\epsilon_{c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



- Strains:
  - Top strain  $\epsilon_{top} = 0.0015$
  - Bottom strain  $\epsilon_{bot} = -0.002$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



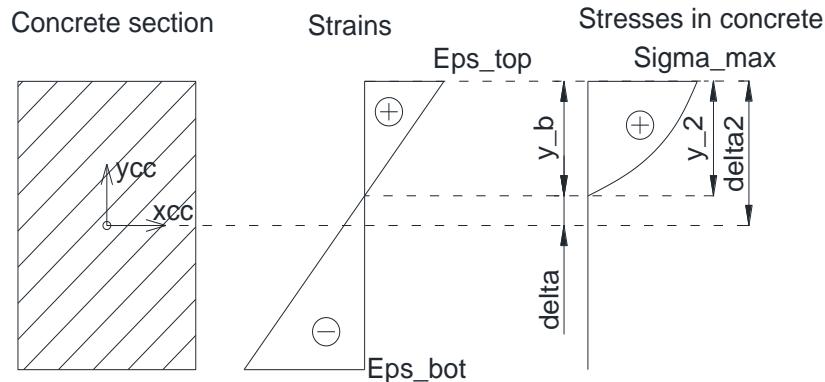
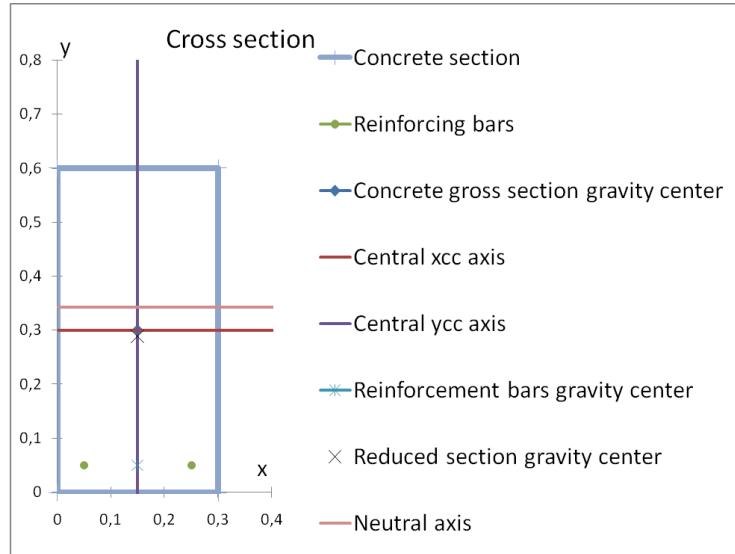
### 7.5.2 Search for:

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

### 7.5.3 Results from Component

- Results for reinforcement bars:  
 $N_s = -549.57e3$   
 $M_{xs} = -137.39e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.0771$   
 $x_{cg} = 0.000000$   
 $y_{cg} = 0.21270852$   
 $N_c = 978.12e3$   
 $M_{xc} = -208.06e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = 428.55e3$   
 $M_x = -345.45e3$   
 $M_y = 0.00e3$

### 7.5.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.257143$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

```
else
```

$$\sigma_i = f_{yd} + (|e_i| - E_{u1}) / (E_{u2} - E_{u1}) \cdot (k - 1) \cdot f_{yd}$$

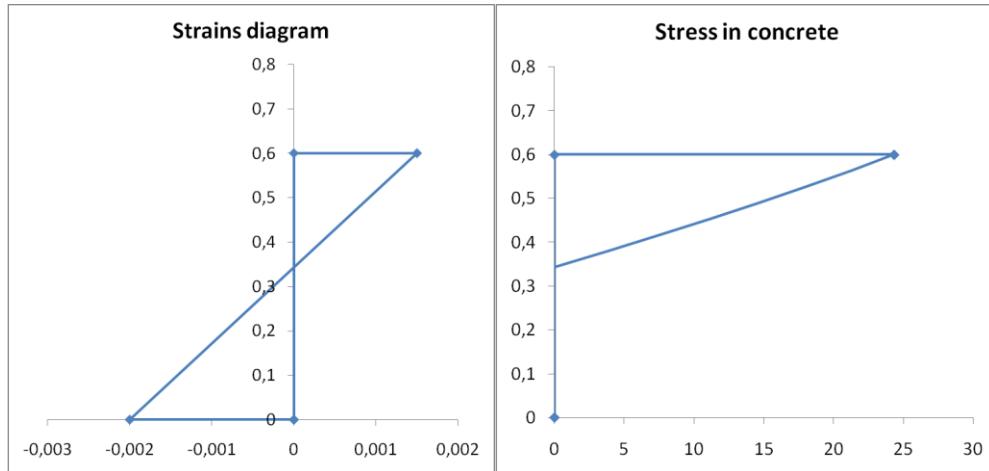
i	x	y	$\phi$	Distance d from neutral axis	Strain $e_i = d / y_b \cdot E_{top}$	Stresses $\sigma_i = \sigma / 10e6$	Force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc} \quad y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10    -0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10    -0,25	-68696	-27478
						Sum of forces in bars:	-549569		-137392	0

$$N_s = -549.569e3$$

$$M_{xs} = -137.392e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

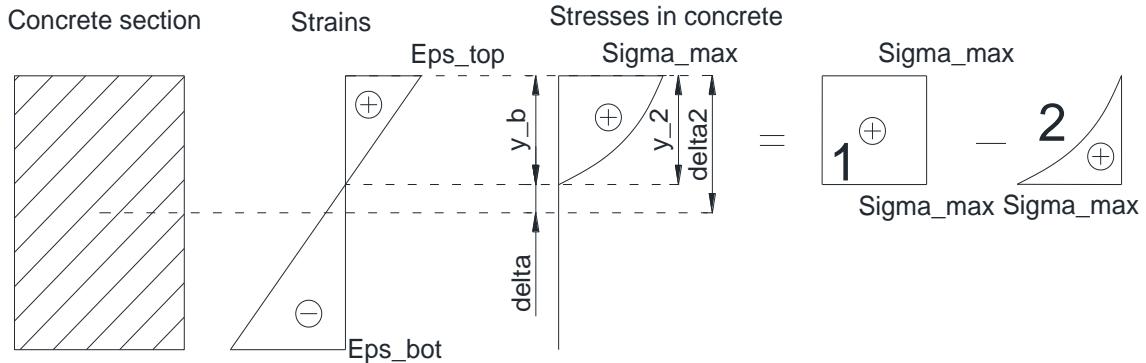
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a power ( $n = 1.2$ ) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot (1 - (1 - Eps_{top}/Eps_{c1})^{1/2}) = 24.316e6$$

$$\text{Sigma\_min} = 0.000e6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.042857$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $Eps_{c1}$  if  $Eps_{top} \geq Eps_{c1}$  or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $Eps_{top} < Eps_{c1}$ :

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; Eps_{c1} / Eps_{top} \cdot y_b) = 0.3000$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

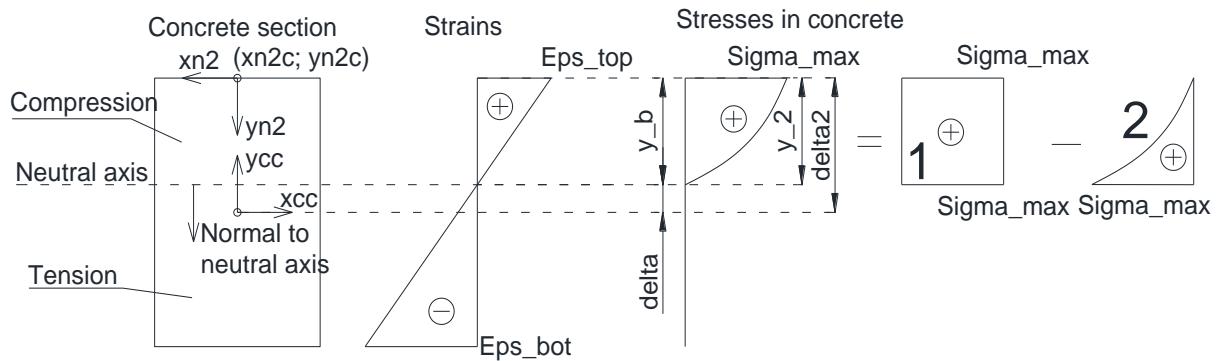
$$y_2 = \text{delta2} - \text{delta} = 0.257143$$

Axial force in compressed part of concrete gross section

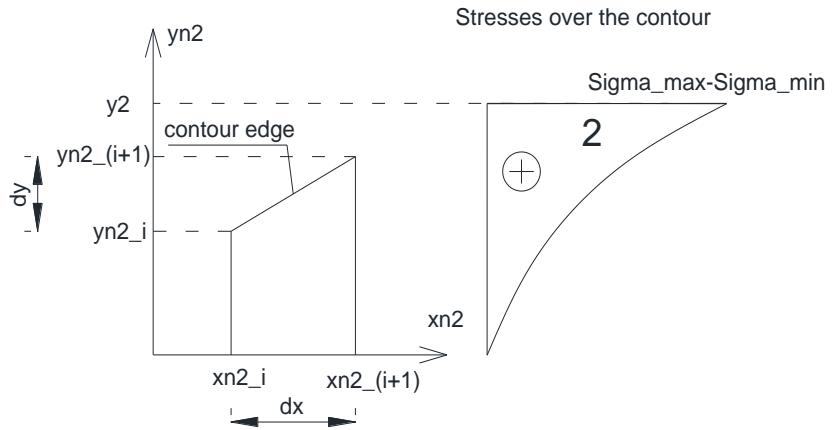
$$N_{c1} = A_c \cdot \text{Sigma\_max} = 1.875811e6$$

To calculate  $N_{c2}$  we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system  $x_{n2}$ ,  $y_{n2}$  (see picture below), where  $x_{n2}$  is an axis parallel to neutral axis, in distance  $\text{delta2}$  from the gravity center of contour of the section, and it crosses axis  $y_{n2}$  in point  $(x_{n2c}; y_{n2c})$  given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in  $(x_{n2}, y_{n2})$  coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in ( $x_{n2}$ , $y_{n2}$ ) system		dx	dy	A	B	$N_{c2}$	Static moment of stress solid $MS_{yn2,zn2}$	$MS_{xn2,zn2}$												
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,000	0,000	0,000												
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,898	0,000	0,156												
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,000	0,000	0,000												
4	0,00	0,60	-0,15	0,30	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000												
5	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000												
	$N_{c2} = \text{sum}(N_{c2\_i}) =$										0,898														
											0,34														
											3														
											0,25														
											7														
gravity center of stress solid 2 in ( $x_{n2}$ ; $y_{n2}$ ) coordinates system													$x_{c2}'$	0,0000											
gravity center of stress solid 2 in central axes ( $x_{cc}$ ; $y_{cc}$ ) coordinates system													$y_{c2}'$	0,1736											
													$x_{c2}$	0,0000											
													$y_{c2}$	0,1264											

Where:

$$x_{cc} = x - x_{c1}$$

$$y_{cc} = y - y_{c1}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{(i+1)} - x_i$$

$$dy = y_{(i+1)} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2_{(i+1)}} + y_{n2_{(i+1)}} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot Eps_c1 / Eps_top$$

$$D = \text{MIN}(Eps_c1; Eps_top) \cdot y_b / Eps_top$$

$$n = 1.2 \text{ (power of stress-strain relation curve)}$$

$$dSigma = \text{Sigma\_max} - \text{Sigma\_min}$$

$$N_{c2\_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$\begin{aligned} N_{c2\_i} &= f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - (1 - (-y_{n2\_i} + D) / C)^{n+2}) \\ &\quad - (x_{n2_{(i+1)}} - x_{n2\_i}) \cdot (1 - D / C)^{n+1}) + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2) + B \cdot \\ &\quad (x_{n2_{(i+1)}} - x_{n2\_i})) \text{ if } A \neq 0 \text{ or} \\ N_{c2\_i} &= (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + dSigma) \\ &\quad \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0 \end{aligned}$$

$$N_{c2} = \text{sum}(N_{c2\_i})$$

$$MS_{yn2,zn2\_i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2) + (-f_{cd} \\ + dSigma) \cdot B / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2) \text{ if } A = 0 \text{ or}$$

$$\begin{aligned} MS_{yn2,zn2\_i} &= f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2_{(i+1)}} \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - x_{n2\_i} \cdot \\ &\quad (1 - (-y_{n2\_i} + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-y_{n2_{(i+1)}} + D) / C)^{n+3} - (1 - (-y_{n2\_i} + D) / C)^{n+3}) \\ &\quad - 0.5 \cdot (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot \\ &\quad (x_{n2_{(i+1)}}^3 - x_{n2\_i}^3) + B / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2)) \text{ if } A \neq 0 \end{aligned}$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2\_i})$$

$$MS_{xn2,zn2\_i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) \\ + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2_{(i+1)}} - x_{n2\_i}) \text{ if } A = 0 \text{ or}$$

$$\begin{aligned} MS_{xn2,zn2\_i} &= f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2_{(i+1)}} \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - y_{n2\_i} \cdot \\ &\quad (1 - (-y_{n2\_i} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2\_i} + D) / C)^{n+3}) \\ &\quad + A \cdot (1 - D / C)^{n+2} \cdot dx) + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2_{(i+1)}}^3 - x_{n2\_i}^3) + A \cdot B / 2 \cdot \\ &\quad (x_{n2_{(i+1)}}^2 - x_{n2\_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0 \end{aligned}$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2\_i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.897687e6$$

$$N_c' = N_{c1} - N_{c2} = 0.978124e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.171429$$

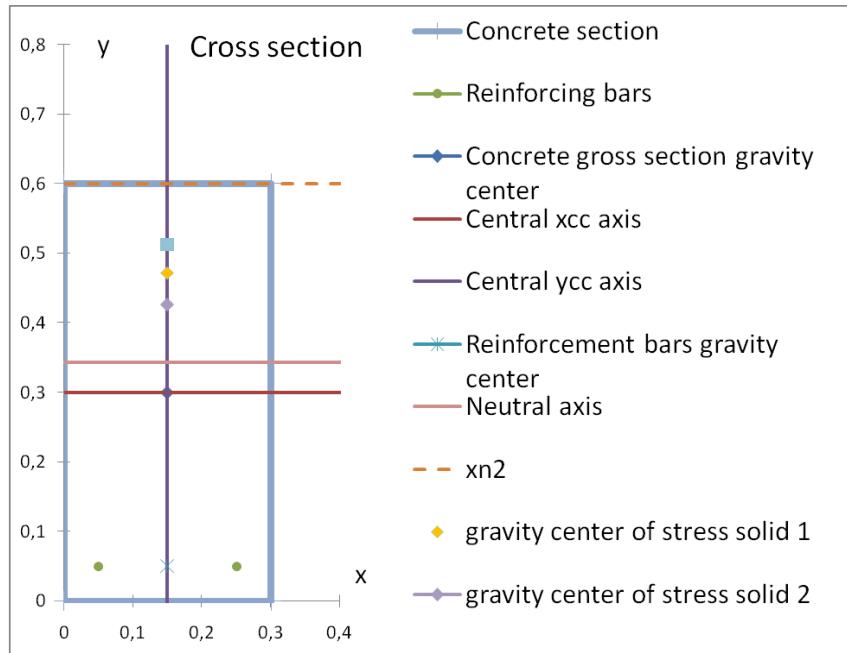
$$x_{c2} = 0$$

$$y_{c2} = 0.126450$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.212709$$



Internal forces in concrete:

$$\begin{aligned} N_c &= N_{c'} - N_{sc} = 978.12e3 \\ M_{xc} &= -N_c \cdot y_{cg} = -208.06e3 \\ M_{yc} &= N_c \cdot x_{cg} = 0.00e3 \end{aligned}$$

- Results for reduced section:

$$\begin{aligned} N &= N_c + N_s = 428.55e3 \\ M_x &= M_{xc} + M_{xs} = -345.45e3 \\ M_y &= M_{yc} + M_{ys} = 0.00e3 \end{aligned}$$

## 8 Case 7 Calculation of internal forces for given state of strain in the section and inclined branch model of steel

### 8.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 8.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with asymmetric reinforcement
- A linear model of concrete and inclined branch model of steel
- A section under unidirectional bending with a part of section under compression
- A state of strain with ultimate compressive strain in concrete

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

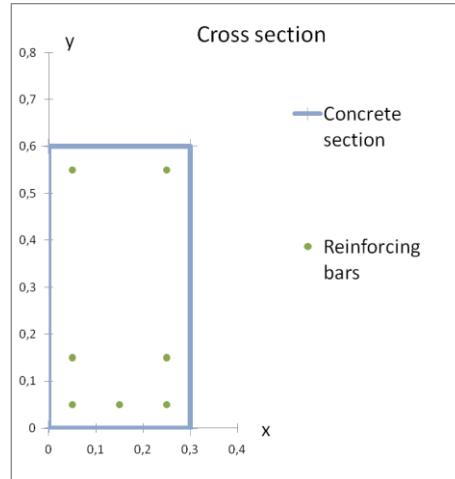
### 8.3 Data:

- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.15$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$

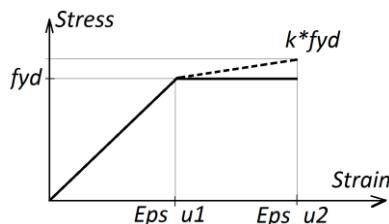
$x = 0.05$	$y = 0.15$	$\phi = 0.020$
$x = 0.25$	$y = 0.15$	$\phi = 0.020$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$



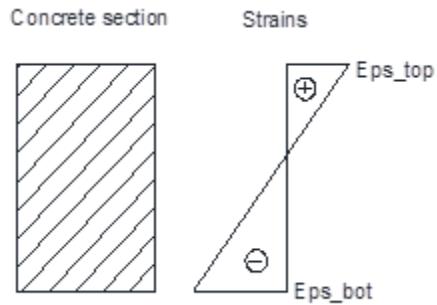
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: linear
  - Strain ultimate limit  $Eps\_ult = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.05$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $Eps\_u2 = 0.075$



- Strains:
  - Top strain  $Eps\_top = 0.0035$
  - Bottom strain  $Eps\_bot = -0.0070$
  - Neutral axis angle Angle =  $\pi \cdot 3 / 2$



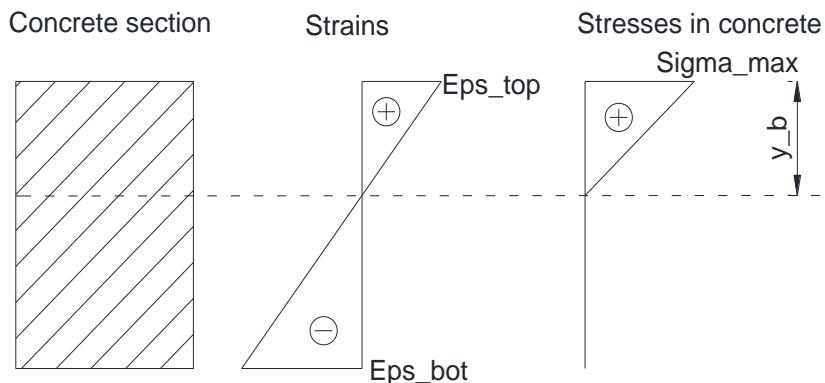
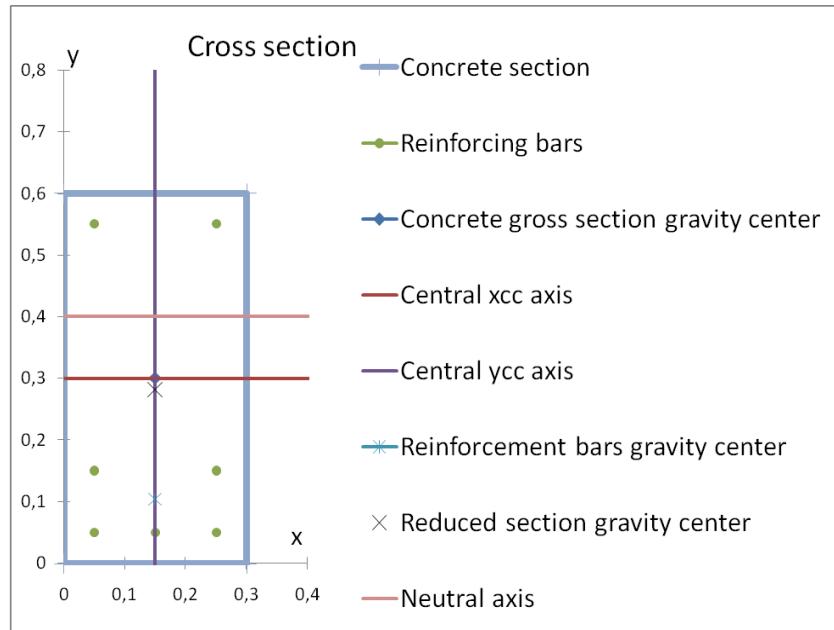
#### **8.4 Search for:**

- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

#### **8.5 Results from Component**

- Results for reinforcement bars:  
 $N_s = -1410.85e3$   
 $M_{xs} = -377.81e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.06$   
 $x_{cg} = 0.00000000$   
 $y_{cg} = 0.23323855$   
 $N_c = 596.61e3$   
 $M_{xc} = -139.15e3$   
 $M_{yc} = 0.00e3$
- Results for reduced section:  
 $N = -814.24e3$   
 $M_x = -516.96e3$   
 $M_y = 0.00e3$

#### **8.6 Results (manual calculations):**



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.2000$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

```
else
```

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot E_{top}$	stresses $\sigma = \epsilon \cdot 10^6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,350	-0,00613	-501,25	-403129	-0,10    -0,25	-100782	40313
2	0,15	0,05	0,032	-0,350	-0,00613	-501,25	-403129	0,00    -0,25	-100782	0
3	0,25	0,05	0,032	-0,350	-0,00613	-501,25	-403129	0,10    -0,25	-100782	-40313
4	0,05	0,15	0,02	-0,250	-0,00438	-500,65	-157283	-0,10    -0,15	-23592	15728
5	0,25	0,15	0,02	-0,250	-0,00438	-500,65	-157283	0,10    -0,15	-23592	-15728
6	0,05	0,55	0,012	0,150	0,00263	500,04	56554	-0,10    0,25	-14138	-5655
7	0,25	0,55	0,012	0,150	0,00263	500,04	56554	0,10    0,25	-14138	5655
					Sum of forces in bars:	-1410846			-377808	0

$$N_s = -1410.85e3$$

$$M_{xs} = -377.81e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	$\phi$	strain $\epsilon$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon / E_{ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,00613	-0,10    -0,25				
2	0,15	0,05	0,032	-0,00613	0,00    -0,25				
3	0,25	0,05	0,032	-0,00613	0,10    -0,25				
4	0,05	0,15	0,02	-0,00438	-0,10    -0,15				
5	0,25	0,15	0,02	-0,00438	0,10    -0,15				
6	0,05	0,55	0,012	0,00263	-0,10    0,25	0,000113097	1696	-424	-170
7	0,25	0,55	0,012	0,00263	0,10    0,25	0,000113097	1696	-424	170
					Sum of forces :	3393		-848	0

Reduction forces in concrete:

$$N_{sc} = 3.39e3$$

$$M_{xsc} = -0.85e3$$

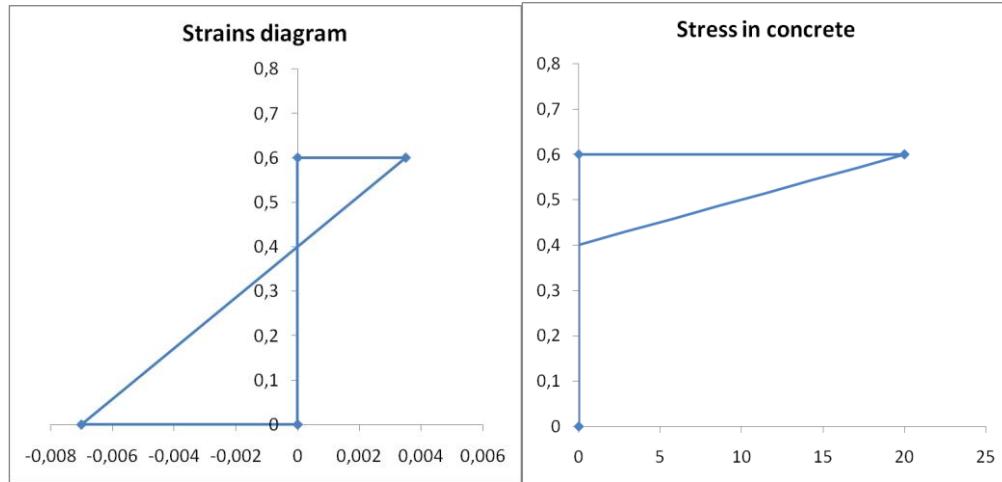
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.00$$

$$y_{scg} = \sum(y_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.20$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0600$$

Axial force in compressed part of concrete gross section

$$\Sigma_{max} = f_{cd} \cdot Eps_{top} / Eps_{ult} = 20.000e6$$

$$N_c = A_c \cdot \Sigma_{max}/2 = 0.6000e6$$

Center of gravity of stress solid in central axes coordinate system:

$$x_{cg}' = 0$$

$$y_{cg}' = 0.3 \cdot y_b \cdot 2/3 = 0.23658249$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{cg}' \cdot N_c - x_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{cg}' \cdot N_c - y_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0.233239$$

Internal forces in concrete:

$$N_c = N_c - N_{sc} = 596.61e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -139.15e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = -814.24e3$$

$$M_x = M_{xc} + M_{xs} = -516.96e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

## 9 Case 8: Calculation of internal forces in asymmetric section for given state of strain with horizontal neutral axis

### 9.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 9.2 Summary:

This sample gives as results the internal forces for:

- An asymmetric concrete shape section with asymmetric reinforcement
- A rectangular model of concrete and horizontal branch model of steel
- A state of strain with ultimate compressive strain in concrete and horizontal neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

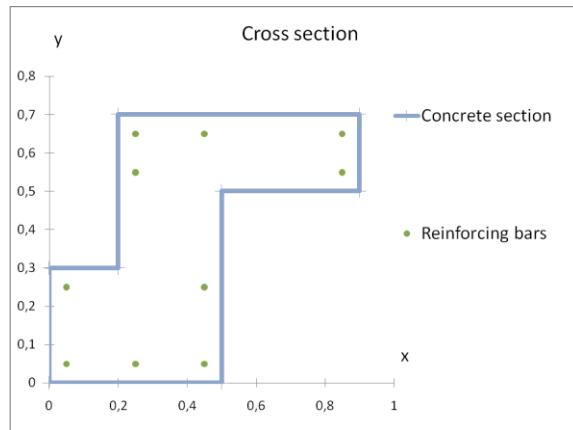
### 9.3 Data:

- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.2$	$y = 0.3$
$x = 0.2$	$y = 0.7$
$x = 0.9$	$y = 0.7$
$x = 0.9$	$y = 0.5$
$x = 0.5$	$y = 0.5$
$x = 0.5$	$y = 0.0$

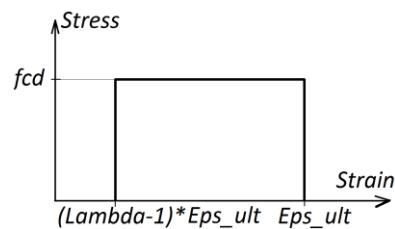
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.025$
$x = 0.45$	$y = 0.05$	$\phi = 0.025$
$x = 0.85$	$y = 0.55$	$\phi = 0.012$
$x = 0.85$	$y = 0.65$	$\phi = 0.012$
$x = 0.45$	$y = 0.65$	$\phi = 0.012$
$x = 0.25$	$y = 0.65$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.025$
$x = 0.05$	$y = 0.25$	$\phi = 0.012$
$x = 0.45$	$y = 0.25$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$



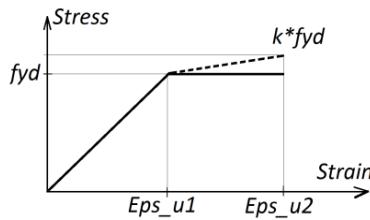
- Concrete parameters:

- Design strength  $f_{cd} = 50e6$
- Effective height reduction factor  $\Lambda = 0.8$
- Modulus of elasticity  $E_c = 37e9$
- Strain-stress model: rectangular
- Strain ultimate limit  $\epsilon_{ult} = 0.0035$

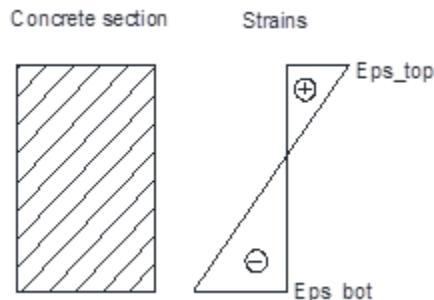


- Steel parameters:

- Design strength  $f_{yd} = 500e6$
- Hardening factor  $k = 1.0$
- Modulus of elasticity  $E_s = 200e9$
- Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Strains:
  - Bottom strain  $Eps\_bot = -0.00875$
  - Top strain  $Eps\_top = 0.00350$
  - Neutral axis angle  $\text{Angle} = \pi \cdot 3 / 2$



## 9.4 Search for:

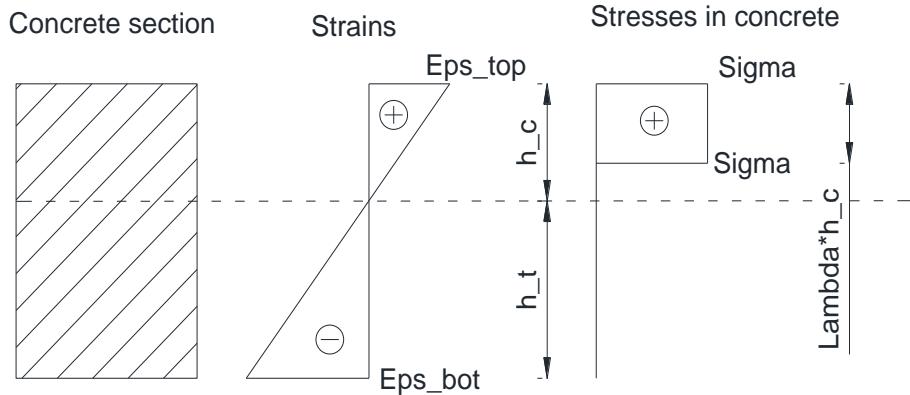
- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

## 9.5 Results from Component

- Results for reinforcement bars:  
 $N_s = -640.18e3$   
 $M_{xs} = -305.65e3$   
 $M_{ys} = 144.91e3$
- Results for concrete section:  
 $A_c = 0.1120$   
 $x_{cg} = 0.16295864$   
 $y_{cg} = 0.24719360$   
 $N_c = 5571.73e3$   
 $M_{xc} = -1377.29e3$   
 $M_{yc} = 907.96e3$
- Results for reduced section:  
 $N = 4931.55e3$   
 $M_x = -1682.94e3$   
 $M_y = 1052.87e3$

## 9.6 Results (manual calculations):

Position of neutral axis



Concrete gross section gravity center calculations:

i	Corners of concrete section		Area_c	Gravity center of contour corresponding to Area_c		Static moments	
	x	y		$x_{01}$	$y_{01}$	$S_{cy}$	$S_{cx}$
1	0	0	0	0,00	0,15	0	0
2	0	0,3	0,060	0,10	0,15	0,006	0,009
3	0,2	0,3	0	0,20	0,50	0	0
4	0,2	0,7	0,490	0,55	0,35	0,2695	0,1715
5	0,9	0,7	0	0,90	0,60	0	0
6	0,9	0,5	-0,20	0,70	0,25	-0,14	-0,05
7	0,5	0,5	0	0,50	0,25	0	0
8	0,5	0	0	0,25	0,00	0	0
	Total:		0,350			0,1355	0,1305

Where:

$edge\_i$  = edge between corners: i and  $i + 1$

$Area\_c\_i$  = Area between edge\_i and axes:  $x = x_i$ ,  $x = x_{(i+1)}$  and  $y = 0$

$$Area\_c\_i = 1/2 \cdot (y_{(i+1)} + y(i)) \cdot (x_{(i+1)} - x(i))$$

$$x_{01\_i} = x_{(i+1)} + (x_{(i+1)} - x(i)) \cdot (2/3 \cdot y_{(i+1)} + 1/3 \cdot y(i)) / (y_{(i+1)} + y(i)) \text{ if } y_{(i+1)} + y(i) \neq 0 \text{ or}$$

$$x_{01\_i} = (x_{(i+1)} + x(i)) / 2 \text{ if } y_{(i+1)} + y(i) = 0$$

$$y_{01\_i} = y_{(i+1)} / 2 + 0.5 \cdot (x_{01\_i} - x(i)) \cdot (y_{(i+1)} - y(i)) / (x_{(i+1)} - x(i)) \text{ if } x_{(i+1)} - x(i) \neq 0 \text{ or}$$

$$y_{01\_i} = (y_{(i+1)} + y(i)) / 2 \text{ if } x_{(i+1)} - x(i) = 0$$

$$S_{cy\_i} = Area\_c\_i \cdot x_{01\_i}$$

$$S_{cx\_i} = Area\_c\_i \cdot y_{01\_i}$$

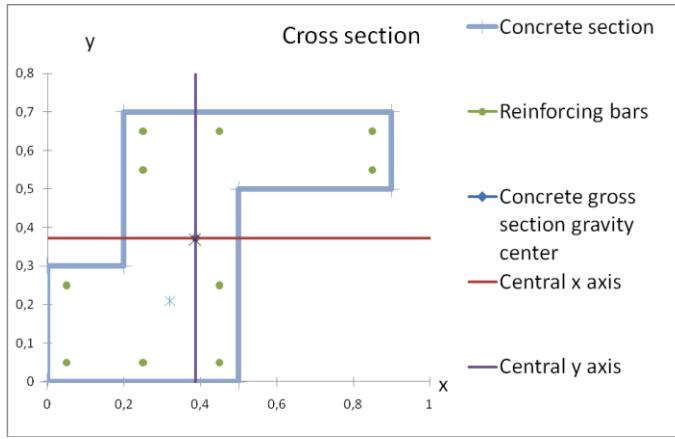
Concrete gross section area:

$$A_c = \sum(Area\_c\_i) = 0,350$$

Concrete gross section gravity center

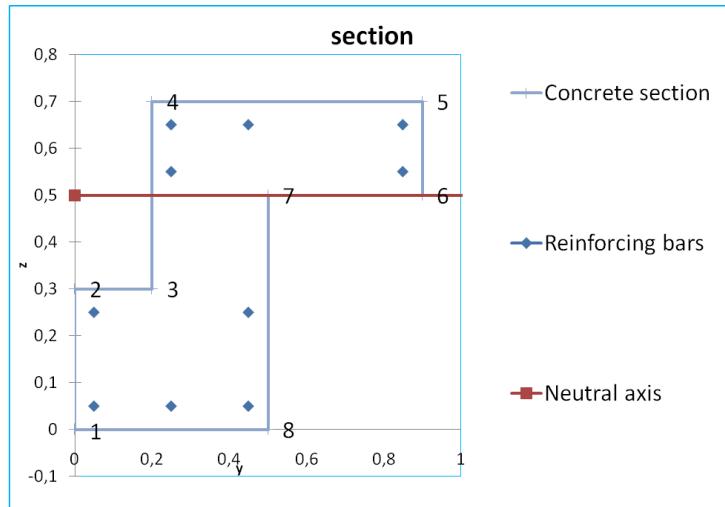
$$x_{Cc} = \sum(S_{cy\_i}) / A_c = 0,3871$$

$$y_{Cc} = \sum(S_{cx\_i}) / A_c = 0,3729$$



Equation of neutral axis in central axes coordinate system ( $x_{cc}$ ;  $y_{cc}$ )

$y_{cc} = 0.127143$  (e.g. obtained in iteration process)



General equation of neutral axis in central axes coordinate system ( $x_{cc}$ ;  $y_{cc}$ )

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = 0$$

$$B = -0.450$$

$$C = 0.057214$$

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed corner:

$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	$x_{cc}$	$y_{cc}$		
1	0	0	-0,387	-0,373	tension	0,500
2	0	0,3	-0,387	-0,073	tension	0,200
3	0,2	0,3	-0,187	-0,073	tension	0,200
4	0,2	0,7	-0,187	0,327	compression	-0,200
5	0,9	0,7	0,513	0,327	compression	-0,200
6	0,9	0,5	0,513	0,127	compression	0,000
7	0,5	0,5	0,113	0,127	compression	0,000
8	0,5	0	0,113	-0,373	tension	0,500

Where:  
 $x_{cc} = x - x_{Cc}$   
 $y_{cc} = y - y_{Cc}$

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.2$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.5$$

Verification of used equation of neutral axis:

$$\text{Eps\_bot} = -h_t/h_c \cdot \text{Eps\_top} = 0.00875 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \pi + \pi / 2 = 3 / 2 \cdot \pi = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$\text{Eps\_u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If  $|\text{eps}_i| \leq f_{yd} / E_s$  then

$$\sigma_i = \text{eps}_i / \text{Eps\_u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\text{eps}_i| - \text{Eps\_u1}) / (\text{Eps\_u2} - \text{Eps\_u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	φ	Positions in central axes coordinate system		Distance d from neutral axis	strain eps = d/h_c • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • φ² / 4	M <sub>xs</sub> = -N • y <sub>cc</sub>	M <sub>ys</sub> = N • x <sub>cc</sub>
				x <sub>cc</sub>	y <sub>cc</sub>						
1	0,05	0,05	0,025	-0,34	-0,32	0,450	-0,00788	-500	-245437	-79241	82747
2	0,45	0,05	0,025	0,06	-0,32	0,450	-0,00788	-500	-245437	-79241	-15427
3	0,85	0,55	0,012	0,46	0,18	-0,050	0,00088	175	19792	-3506	9161
4	0,85	0,65	0,012	0,46	0,28	-0,150	0,00263	500	56549	-15672	26174
5	0,45	0,65	0,012	0,06	0,28	-0,150	0,00263	500	56549	-15672	3554
6	0,25	0,65	0,012	-0,14	0,28	-0,150	0,00263	500	56549	-15672	-7755
7	0,25	0,05	0,025	-0,14	-0,32	0,450	-0,00788	-500	-245437	-79241	33660
8	0,05	0,25	0,012	-0,34	-0,12	0,250	-0,00438	-500	-56549	-6947	19065
9	0,45	0,25	0,012	0,06	-0,12	0,250	-0,00438	-500	-56549	-6947	-3554
10	0,25	0,55	0,012	-0,14	0,18	-0,050	0,00088	175	19792	-3506	-2714
								Sum of forces in bars:			
									-640178	305646	144910

$$N_s = -640.18e3$$

$$M_{xs} = -305.65e3$$

$$M_{ys} = 144.91e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	φ	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force N <sub>sc</sub> in concrete in place of bars N <sub>sc</sub> = f <sub>cd</sub> • Eps_top / Eps_ult • Area	M <sub>xsc</sub> = -N <sub>sc</sub> • y <sub>cc</sub>	M <sub>ysc</sub> = N <sub>sc</sub> • x <sub>cc</sub>
				x <sub>cc</sub>	y <sub>cc</sub>				
1	0,05	0,05	0,025	-0,34	-0,32				
2	0,45	0,05	0,025	0,06	-0,32				
3	0,85	0,55	0,012	0,46	0,18	0,000113097	5655	-1002	2617
4	0,85	0,65	0,012	0,46	0,28	0,000113097	5655	-1567	2617
5	0,45	0,65	0,012	0,06	0,28	0,000113097	5655	-1567	355
6	0,25	0,65	0,012	-0,14	0,28	0,000113097	5655	-1567	-776
7	0,25	0,05	0,025	-0,14	-0,32				
8	0,05	0,25	0,012	-0,34	-0,12				
9	0,45	0,25	0,012	0,06	-0,12				
10	0,25	0,55	0,012	-0,14	0,18	0,000113097	5655	-1002	-776
				Total:		0,000565487	28274	-6705	4039

Reduction forces in concrete:

$$N_{sc} = 28.27e3$$

$$M_{xsc} = -6.71e3$$

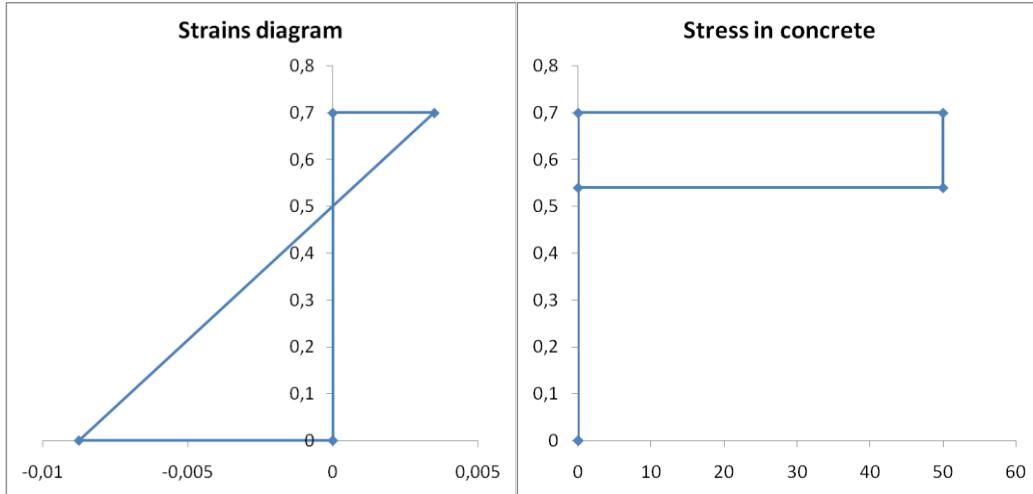
$$M_{ysc} = 4.04e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.1429$$

$$y_{scg} = \sum(y_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.2371$$

- Results for concrete section:



Height of compressed part of the section:

$$\Lambda \cdot h_c = 0.1600$$

Area of contour under compressive stress:

$$A_c = 0.7 \cdot 0.16 = 0.1120$$

Stress in concrete:

$$\Sigma = Eps\_top / Eps\_ult \cdot f_{cd} = 50e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.2 + 0.7 / 2 - x_{cc}) \cdot A_c \cdot \Sigma - x_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0.162959$$

$$y_{cg} = ((0.7 - 0.2 / 2 - y_{cc}) \cdot A_c \cdot \Sigma - y_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0.247194$$

Internal forces in concrete:

$$N_c = A_c \cdot \Sigma - N_{sc} = 5571.73e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -1377.29e3$$

$$M_{yc} = N_c \cdot x_{cg} = 907.96e3$$

- Results for reduced section:

$$N = N_c + N_s = 4931.55e3$$

$$M_x = M_{xc} + M_{xs} = -1682.94e3$$

$$M_y = M_{yc} + M_{ys} = 1052.87e3$$

## 10 Case 9: Calculation of internal forces in symmetric section for given state of strain with inclined neutral axis

### 10.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

### 10.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete shape section with symmetric reinforcement
- A bilinear model of concrete and horizontal branch model of steel
- A state of strain with compressive strain below the ultimate strain in concrete and inclined neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force ( $N$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_x$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_y$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

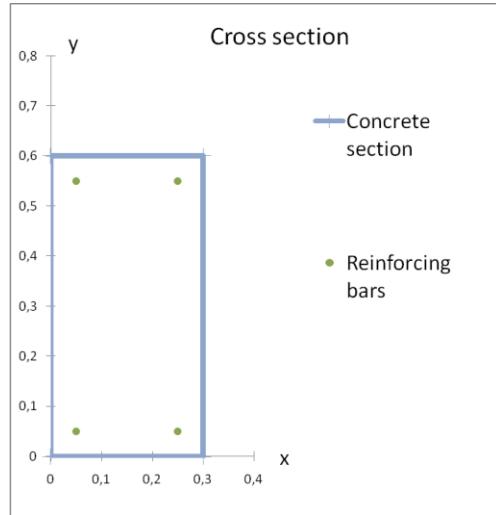
### 10.3 Data:

- Geometry:
  - Concrete section:
 

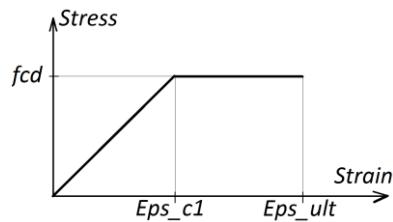
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
------------	------------	----------------

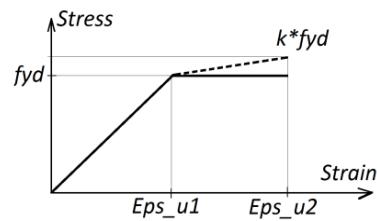
$$\begin{array}{lll}
 x = 0.25 & y = 0.05 & \phi = 0.012 \\
 x = 0.25 & y = 0.55 & \phi = 0.012 \\
 x = 0.05 & y = 0.55 & \phi = 0.012
 \end{array}$$



- Concrete parameters:
  - Design strength  $f_{cd} = 25e6$
  - Modulus of elasticity  $E_c = 28e9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0030$
  - Strain relation change over  $\text{Eps\_c1} = 0.0020$

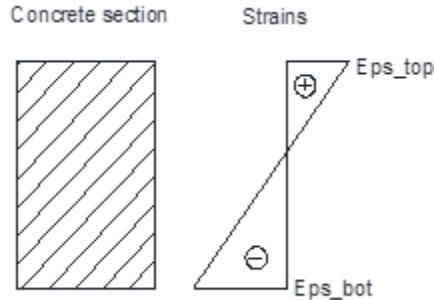


- Steel parameters:
  - Design strength  $f_{yd} = 550e6$
  - Hardening factor  $k= 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.1$



- Strains:
  - Bottom strain  $\text{Eps\_bot} = -0.00383423$
  - Top strain  $\text{Eps\_top} = 0.0025$

- Neutral axis angle Angle = 0.34906585



## 10.4 Search for:

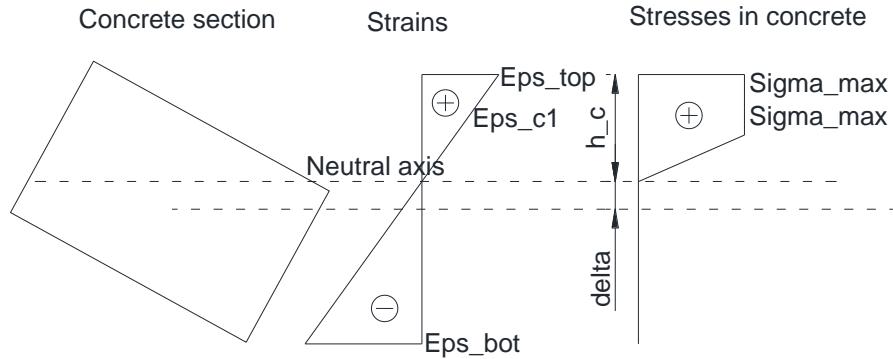
- Results for reinforcement bars:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:  $A_c$ ,  $x_{cg}$ ,  $y_{cg}$ ,  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$
- Results for reduced section:  $N$ ,  $M_x$ ,  $M_y$

## 10.5 Results from Component

- Results for reinforcement bars:  
 $N_s = -54.68e3$   
 $M_{xs} = 23.73e3$   
 $M_{ys} = -10.49e3$
- Results for concrete section:  
 $A_c = 0.0575$   
 $x_{cg} = -0.09851523$   
 $y_{cg} = -0.15819891$   
 $N_c = 591.85e3$   
 $M_{xc} = 93.63e3$   
 $M_{yc} = -58.31e3$
- Results for reduced section:  
 $N = 537.17e3$   
 $M_x = 117.36e3$   
 $M_y = -68.79e3$

## 10.6 Results (manual calculations):

Position of neutral axis



Concrete gross section gravity center

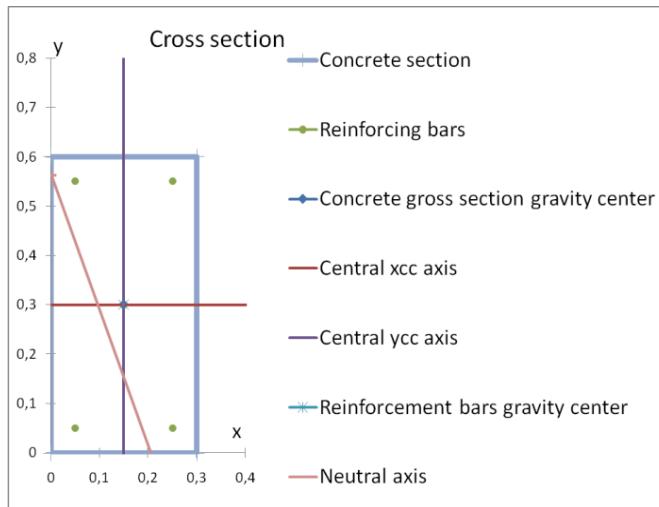
$$x_{cc} = 0.3 / 2 = 0.15$$

$$y_{cc} = 0.6 / 2 = 0.30$$

Equation of neutral axis in central axes coordinate system ( $x_{cc}; y_{cc}$ )

$$\text{Tangens}(\text{Angle} - \pi / 2) = -2.74748$$

$$y_{cc} = -2.74748 \cdot x_{cc} - 0.15 \text{ (e.g. obtained in iteration process)}$$



General equation of neutral axis in central axes coordinate system ( $x_{cc}; y_{cc}$ )

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = -0.412$$

$$B = -0.150$$

$$C = -0.0225$$

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed corner:

$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	$x_{cc}$	$y_{cc}$		
1	0	0	-0,150	-0,300	compression	-0,19225691
2	0	0,6	-0,150	0,300	tension	0,01295517
3	0,3	0,6	0,150	0,300	tension	0,29486296
4	0,3	0,0	0,150	-0,300	tension	0,08965087

Where:  
 $x_{cc} = x - x_{Cc}$   
 $y_{cc} = y - y_{Cc}$

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.19225691$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.29486296$$

Verification of used equation of neutral axis:

$$\text{Eps\_bot} = -h_t/h_c \cdot \text{Eps\_top} = -0.00383423 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \arctan(-2.74748) + \pi/2 = 0.34906585 = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$\text{Eps\_u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If  $|\text{eps}_i| \leq f_{yd} / E_s$  then

$$\sigma_i = \text{eps}_i / \text{Eps\_u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\text{eps}_i| - \text{Eps\_u1}) / (\text{Eps\_u2} - \text{Eps\_u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Positions in central axes coordinate system	Distance d from neutral axis	strain $\text{eps} = d/h_c \cdot \text{Eps\_top}$	stresses $\sigma = \sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,012	-0,10	-0,25	0,128	0,00167	333	37699	9425
2	0,25	0,05	0,012	0,10	-0,25	-0,060	-0,00078	-155	-17579	-4395

3	0,25	0,55	0,012	0,10	0,25	-0,231	-0,00300	-550	-62204	15551	-6220
4	0,05	0,55	0,012	-0,10	0,25	-0,043	-0,00056	-111	-12600	3150	1260
							Sum of forces in bars:		-54684	23731	-10488

$$N_s = -54.68e3$$

$$M_{xs} = 23.73e3$$

$$M_{ys} = -10.49e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon \leq \epsilon_{c1} \quad N_{sc} = f_{cd} \cdot \epsilon / \epsilon_{c1} \cdot \text{Area}$$

$$\text{if } \epsilon > \epsilon_{c1} \quad N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,012	0,00167	-0,10	-0,25	0,000113097		2356	589
2	0,25	0,05	0,012	-0,00078	0,10	-0,25				
3	0,25	0,55	0,012	-0,00300	0,10	0,25				
4	0,05	0,55	0,012	-0,00056	-0,10	0,25				
					Total:		0,000113097	2356	589	-236

Reduction forces in concrete:

$$N_{sc} = 2.36e3$$

$$M_{xsc} = 0.59e3$$

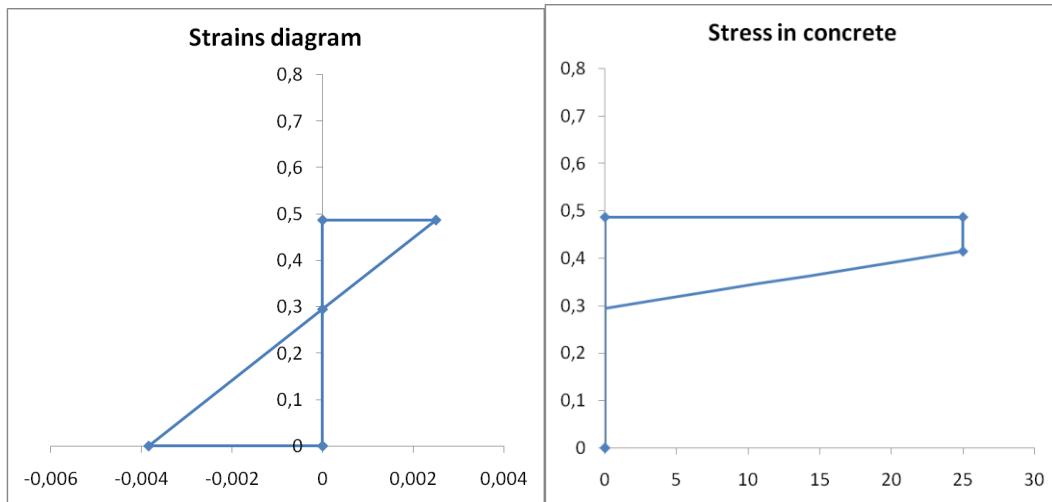
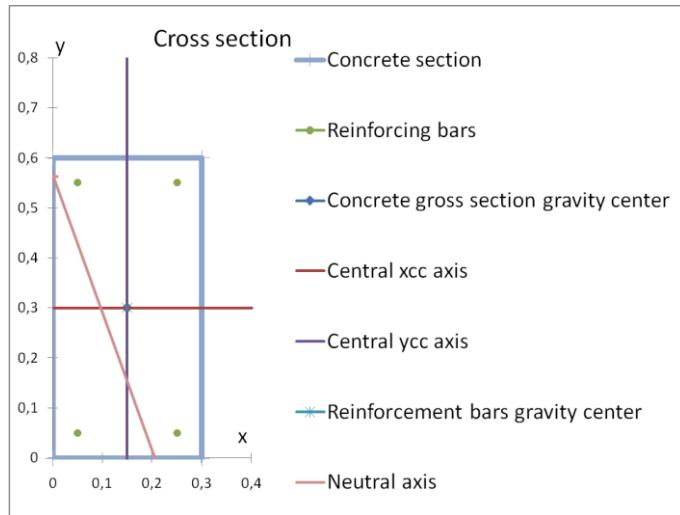
$$M_{ysc} = -0.24e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = -0.1000$$

$$y_{scg} = \sum(y_{cc\_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = -0.2500$$

- Results for concrete section:



Calculations of area of contour under compressive stress:

Calculations of points of intersection of neutral axis and the contour of concrete section:

i	Corners of concrete section	x	y	Corners of concrete section in central axes coordinate system	x <sub>cc</sub>	y <sub>cc</sub>	Equation of line containing edge_i	Intersection point of line containing edge_i and neutral axis	x <sub>cc</sub>	y <sub>cc</sub>	Is intersection on the contour?
1	0	0	-0,150	-0,300	x <sub>cc</sub> = -0,15	-0,15	0,2621				Yes
2	0	0,6	-0,150	0,300	y <sub>cc</sub> = 0,3	-0,1638		0,3			No
3	0,3	0,6	0,150	0,300	x <sub>cc</sub> = 0,15	0,15	-0,5621				No
4	0,3	0,0	0,150	-0,300	y <sub>cc</sub> = -0,3	0,0546		-0,3			Yes

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

edge\_i = edge between corners: i and i+1

New points created by intersections of lines containing edges and neutral axis and lying on the contour create an edge of contour of compressed part of the section.

i	Contour of compressed part of a section in central axes coordinate system		Contour of compressed part of a section in x y coordinate system		Area_c	Gravity center of contour corresponding to Area_c in (x; y) system		Static moments in (x; y) system	
	x_cc	y_cc	x	y		x_01	y_01	S_cv	S_cx
1	-0,150	-0,300	0,000	0,000	0	0,000	0,281	0,000000	0,000000
2	-0,150	0,262	0,000	0,562	0,0575	0,068	0,187	0,003922	0,010775
3	0,055	-0,300	0,205	0,000	0,0000	0,102	0,000	0,000000	0,000000
4	-0,150	-0,300	0	0	0	0,000	0,281	0,000000	0,000000
				Total:	0,0575			x1=	y1=
								-0,0818	-0,1126

Where:

$$x = x_{cc} + x_{Cc}$$

$$y = y_{cc} + y_{Cc}$$

edge\_i = edge between corners: i and i + 1

Area\_c\_i = Area between edge\_i and axes: x=x\_i, x=x\_(i + 1) and y = 0

$$\text{Area}_c_i = 1/2 \cdot (y_{(i+1)} + y_{(i)}) \cdot (x_{(i+1)} - x_{(i)})$$

$x_{01\_i} = x_{(i)} + (x_{(i+1)} - x_{(i)}) \cdot (2 / 3 \cdot y_{(i+1)} + 1 / 3 \cdot y_{(i)}) / (y_{(i)} + y_{(i+1)})$  if  $(y_{(i)} + y_{(i+1)}) \neq 0$  or

$$x_{01\_i} = (x_{(i+1)} + x_{(i)}) / 2 \text{ if } (y_{(i)} + y_{(i+1)}) = 0$$

$y_{01\_i} = y_{(i)} / 2 + 0.5 \cdot (x_{01\_i} - x_{(i)}) \cdot (y_{(i+1)} - y_{(i)}) / (x_{(i+1)} - x_{(i)})$  if  $x_{(i+1)} - x_{(i)} \neq 0$  or

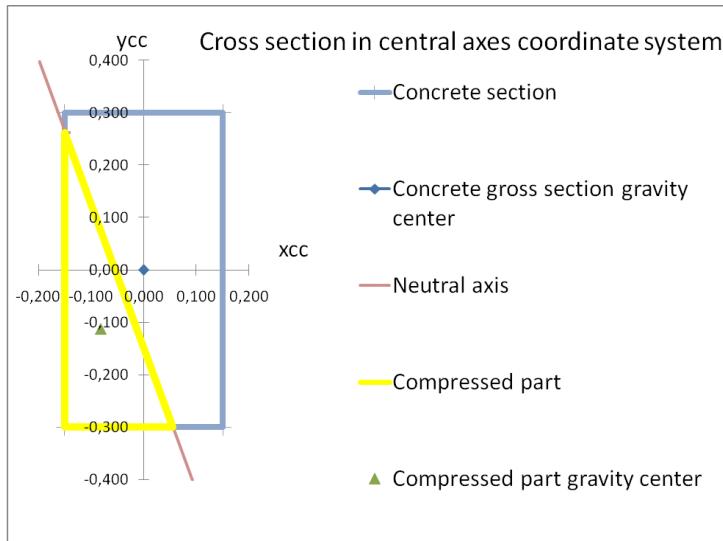
$$y_{01\_i} = (y_{(i+1)} + y_{(i)}) / 2 \text{ if } x_{(i+1)} - x_{(i)} = 0$$

$$S_{cv\_i} = \text{Area}_c_i \cdot x_{01\_i}$$

$$S_{cx\_i} = \text{Area}_c_i \cdot y_{01\_i}$$

$$x1 = \text{Sum}(S_{cv}) / \text{sum}(Area_c) - x_{cc}$$

$$y1 = \text{sum}(S_{cx}) / \text{sum}(Area_c) - y_{cc}$$



Compressed concrete area:

$$A_c = \text{sum}(\text{Area}_c)_i = 0.0575$$

Gravity center of compressed concrete contour (gross) in central axes coordinate system

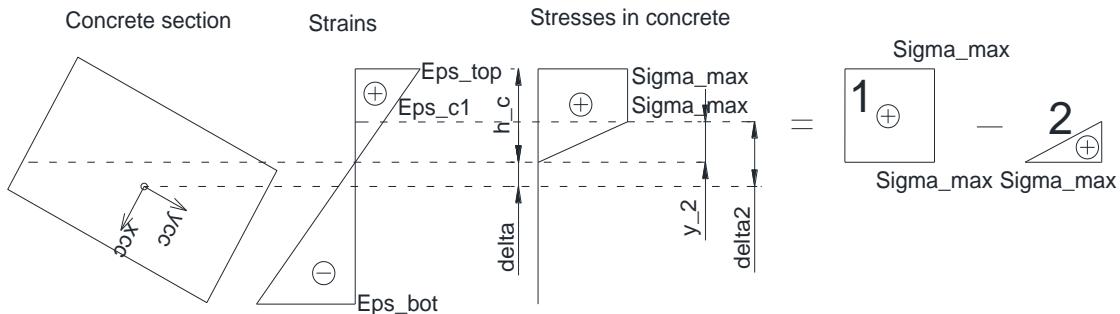
$$x_{c1} = \text{sum}(S_{cy})_i / A_c - x_{Cc} = -0.0818$$

$$y_{c1} = \text{sum}(S_{cx})_i / A_c - y_{Cc} = -0.1126$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c' - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\Sigma_{\max} = f_{cd} = 25.000e6$$

Distance between gravity center of contour and neutral axis:

$$\delta = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

for  $x_{cc} = y_{cc} = 0$  and coefficients A, B, C as defined in paragraph: Position of neutral axis, above

$$\delta = 0.0513$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal  $\epsilon_{c1}$  if  $\epsilon_{top} \geq \epsilon_{c1}$  or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if  $\epsilon_{top} < \epsilon_{c1}$ :

$$\delta_2 = \delta + \min(\epsilon_b; \epsilon_{c1} / \epsilon_{top} \cdot \epsilon_b) = 0.171352$$

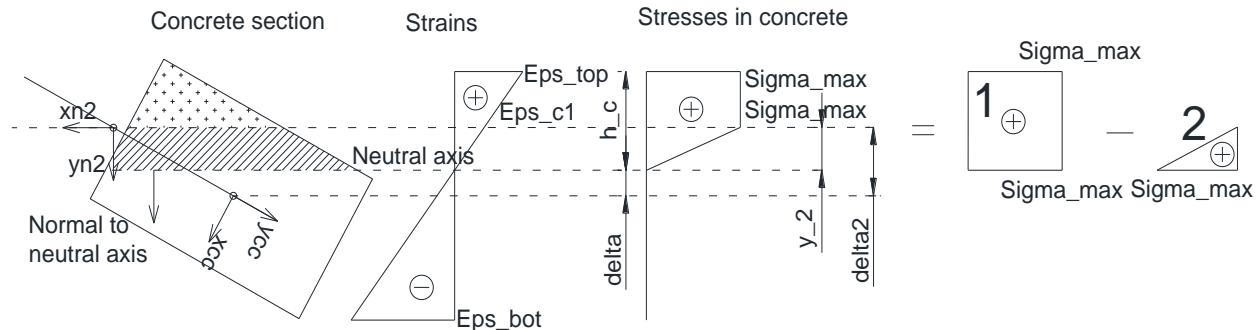
Height of the part of the section where concrete stress is not constant:

$$y_2 = \delta_2 - \delta = 0.120049$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \Sigma_{\max} = 1.437595e6$$

To calculate  $N_{c2}$  we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture above. To do this we define a new coordinate system  $x_{n2}$ ,  $y_{n2}$  (see picture below), where  $x_{n2}$  is an axis parallel to neutral axis, in distance  $\delta_2$  from the gravity center of contour of the section, and it crosses axis  $y_{n2}$  in point  $(x_{n2c}; y_{n2c})$  given in central axis coordinate system.



Equation of neutral axis in central axes coordinate system:

$$y_{cc} = -2.74748 \cdot x_{cc} - 0.15$$

Equation of  $x_{n2}$  axis in central axes coordinate system

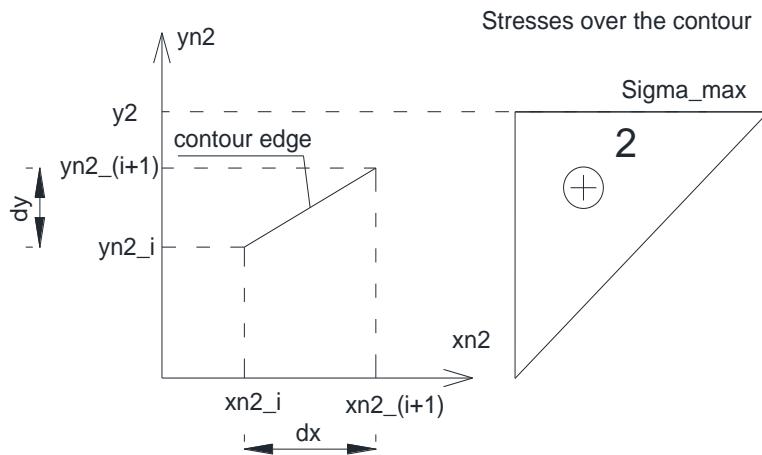
$$y_{nc} = -2.74748 \cdot x_{nc} - 0.15 - y_2 / \sin(\text{Angle}) = -2.74748 \cdot x_{nc} - 0.5010$$

$$\text{For } x_{n2c} = 0 \quad y_{n2c} = -0.5010$$

Now we find coordinates of contour under stress solid 2 in  $(x_{n2}, y_{n2})$  coordinates system and calculate an integral of stresses over this contour.

i	Contour of compressed part of a section		Contour of compressed part of a section in central axes coordinate system		Equation of line containing edge_i, in central axes coordinate system		Intersection point of line containing edge_i and x_n2 axis		Is intersection on the contour?
	x	y	x_cc	y_cc			x_cc	y_cc	
1	0	0	-0,150	-0,300	$x_{cc} = -0,15$		-0,15	-0,0889	Yes
2	0	0,562	-0,15	0,262	$y_{cc} = -2,7475 \cdot x_{cc} - 0,15$		-	-	No
3	0,205	0	0,055	-0,3	$y_{cc} = -0,3$		-0,0732	-0,3	Yes
4	0	0	-0,150	-0,300	$x_{cc} = -0,15$		-0,15	-0,0889	No

Where:  
 $x_{cc} = x - x_{cc}$   
 $y_{cc} = y - y_{cc}$   
edge\_i = edge between corners: i and i+1



i	Coordinates of contour below stress solid 2		Coordinates in central axes system		Coordinates in $(x_{n2}, y_{n2})$ system		dx	dy	A	B	$N_{c2}$	Static moments of stress solid 2 over the compressed part of concrete section	$MS_{vn2,zn2}$	$MS_{xn2,zn2}$
1	x	y	$x_{cc}$	$y_{cc}$	$x_{n2}$	$y_{n2}$								
1	0,000	0,562	-0,15	0,26	-0,77	0,12	0,60	0,00	0,00	0,12	0,701	-0,329	0,056	
2	0,205	0,000	0,05	-0,30	-0,17	0,12	-0,04	-0,12	2,75	0,59	-0,017	0,003	-0,001	
3	0,077	0,000	-0,07	-0,30	-0,21	0,00	-0,22	0,00	0,00	0,00	0,000	0,000	0,000	
4	0,000	0,211	-0,15	-0,09	-0,44	0,00	-0,33	0,12	-0,36	-0,16	-0,129	0,088	-0,008	
	$N_{c2} = \text{sum}(N_{c2\_i}) = 0,555$													
													$C = \frac{0,15}{4}$	
													$D = \frac{0,15}{4}$	
gravity center of stress solid 2 in $(x_{n2}; y_{n2})$ coordinates system												$x_{c2}'$	-0,4279	
												$y_{c2}'$	0,0853	
gravity center of stress solid 2 in central axes ( $x_{cc}; y_{cc}$ ) coordinates system												$x_{c2}$	-0,0662	
												$y_{c2}$	-0,0698	

Where:

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{n2(i+1)} - x_{n2i}$$

$$dy = y_{n2(i+1)} - y_{n2i}$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2(i+1)} + y_{n2(i+1)} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot Eps_c1 / Eps_top$$

$$D = \text{MIN}(Eps_c1; Eps_top) \cdot y_b / Eps_top$$

$$n = 1.2 \text{ (power of stress-strain relation curve)}$$

$$dSigma = \text{Sigma\_max} - \text{Sigma\_min}$$

$$N_{c2i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-y_{n2(i+1)} + D) / C)^{n+2} - (1 - (-y_{n2(i+1)} + D) / C)^{n+2}) - (x_{n2(i+1)} - x_{n2i}) \cdot (1 - D / C)^{n+1} + (dSigma - f_{cd}) \cdot (A / 2 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2) + B \cdot (x_{n2(i+1)} - x_{n2i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + dSigma) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2i})$$

$$MS_{yn2,zn2i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2) + (-f_{cd} + dSigma) \cdot B / 2 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2(i+1)} \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+2} - x_{n2i} \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+2} - C / (n+3) \cdot A \cdot ((1 - (-y_{n2(i+1)} + D) / C)^{n+3} - (1 - (-y_{n2(i+1)} + D) / C)^{n+3})) - 0.5 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + dSigma) \cdot (A / 3 \cdot (x_{n2(i+1)}^3 - x_{n2i}^3) + B / 2 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2i})$$

$$MS_{xn2,zn2i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + dSigma) / 2 \cdot B^2) \cdot (x_{n2(i+1)} - x_{n2i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2(i+1)} \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+2} - y_{n2i} \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2(i+1)} + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (dSigma - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2(i+1)}^3 - x_{n2i}^3) + A \cdot B / 2 \cdot (x_{n2(i+1)}^2 - x_{n2i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2i})$$

$$x_{c2'} = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2'} = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2'} - \sin(\text{Angle} - \pi / 2) \cdot y_{c2'}$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2'} + \cos(\text{Angle} - \pi / 2) \cdot y_{c2'}$$

$$N_{c2} = 0.554816e6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.882778e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = -0.0818$$

$$y_{c1} = -0.1126$$

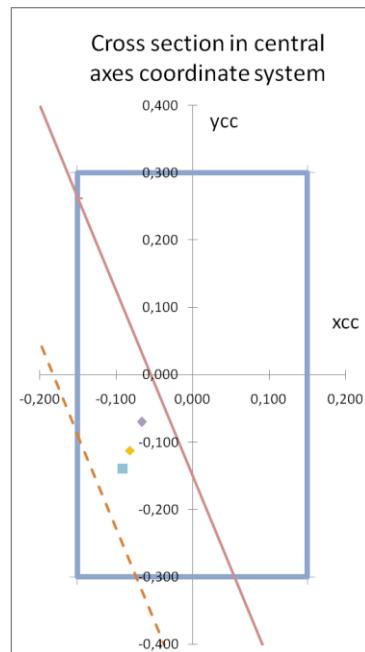
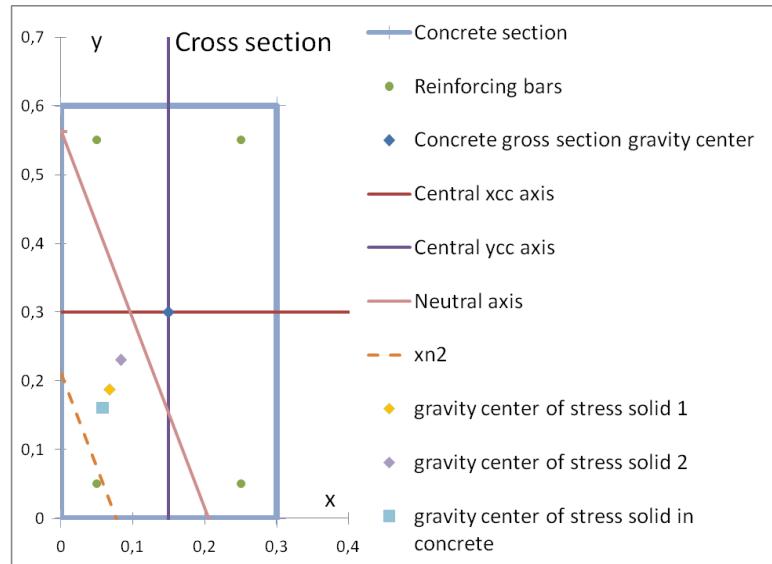
$$x_{c2} = -0.0662$$

$$y_{c2} = -0.0698$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.212709$$



Internal forces in concrete:

$$\begin{aligned} N_c &= N_{c'} - N_{sc} = 880.42e3 \\ M_{xc} &= -N_c \cdot y_{cg} = 122.62e3 \\ M_{yc} &= N_c \cdot x_{cg} = -80.64e3 \end{aligned}$$

- Results for reduced section:

$$\begin{aligned} N &= N_c + N_s = 825.74e3 \\ M_x &= M_{xc} + M_{xs} = 146.35e3 \\ M_y &= M_{yc} + M_{ys} = -91.12e3 \end{aligned}$$

## 11 Case 10: Calculation of capacity state in symmetric section for bending moment M<sub>x</sub>

### 11.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (0, M<sub>x</sub>, 0) and give state of strain and stress at the failure.

### 11.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 10a)
- A parabolic-rectangular model of concrete stress-strain relation (Case 10b)
- A horizontal branch model of steel
- A simple bending in x direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force (N<sub>s</sub>)
  - Moment with respect to x<sub>cc</sub> axis (M<sub>xs</sub>)
  - Moment with respect to y<sub>cc</sub> axis (M<sub>ys</sub>)
- Results for concrete section:
  - Area of contour under compressive stress (A<sub>c</sub>)
  - Center of gravity of stresses in the concrete (x<sub>cg</sub>, y<sub>cg</sub>)
  - Axial force (N<sub>c</sub>)
  - Moment with respect to x<sub>cc</sub> axis (M<sub>xc</sub>)
  - Moment with respect to y<sub>cc</sub> axis (M<sub>yc</sub>)
- Results for reduced section:
  - Axial force at the failure related to acting forces (N<sub>f</sub> = alpha • N)
  - Moment at the failure related to acting forces, with respect to x<sub>cc</sub> axis (M<sub>xf</sub> = alpha • M<sub>x</sub>)
  - Moment at the failure related to acting forces, with respect to y<sub>cc</sub> axis (M<sub>yf</sub> = alpha • M<sub>y</sub>)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete (Eps\_top, Eps\_bot)
  - Extreme strains in steel (Eps\_stop, Eps\_sbot)

For these results, x<sub>cc</sub> and y<sub>cc</sub> are central axes of concrete gross section parallel to x and y axes.

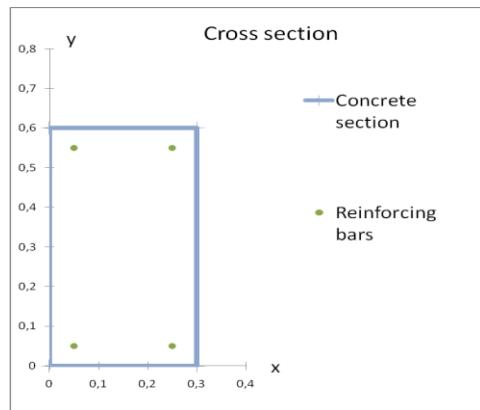
## 11.3 Case 10a

### 11.3.1 Data:

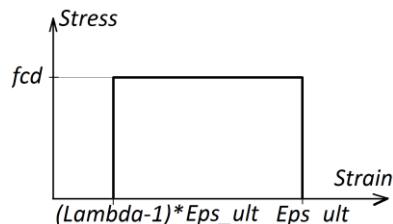
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

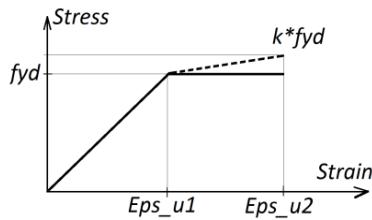
x = 0.05	y = 0.05	$\phi = 0.020$
x = 0.25	y = 0.05	$\phi = 0.020$



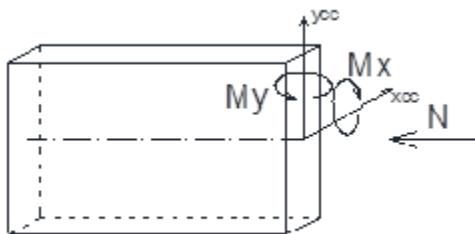
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Effective height reduction factor  $\Lambda = 0.9$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_yd = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



- Forces:
  - $N = 0.00$
  - $M_x = -50.00$
  - $M_y = 0.00$



### 11.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop$ ,  $Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete: ( $x_{cg}$ ,  $y_{cg}$ )
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 11.3.3 Results from Component

- Results for reduced section:
 

$N_f = 0.00$   
 $M_{xf} = -164.56$   
 $M_{yf} = 0.00$   
 $dist = -0.24182$   
 $Angle = 4.71239 = 270\text{deg}$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = -0.032597$   
 $Eps\_stop = -0.029589$   
 $Eps\_sbot = -0.029589$
- Results for reinforcement bars:
 

$N_s = -314.16$

$$\begin{aligned}M_{xs} &= -78.54 \\M_{ys} &= 0.00\end{aligned}$$

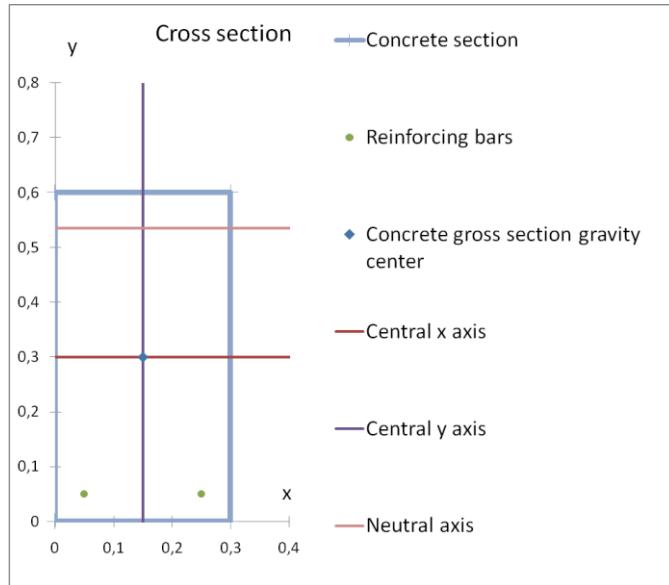
- Results for concrete section:  
 $A_c = 0.0157$   
 $(x_{cg}, y_{cg}) = (0.00000000, 0.27382035)$   
 $N_c = 314.16$   
 $M_{xc} = -86.02$   
 $M_{yc} = 0.00$

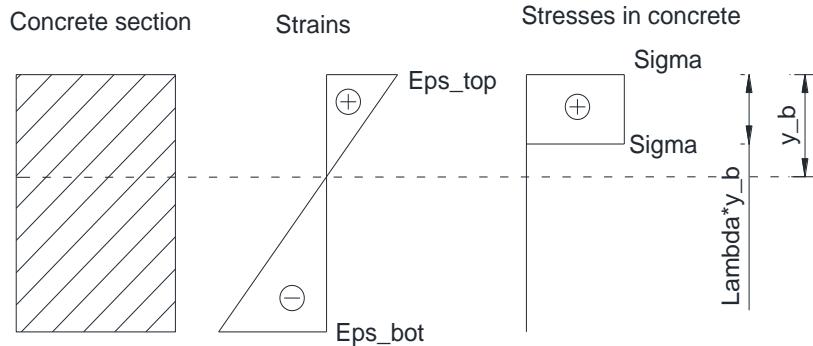
#### 11.3.4 Results (manual calculations):

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Verification of safety factor:

$$\begin{aligned}\alpha &= M_x / M_{xi} = 3.2912 \\N_f &= \alpha \cdot N = 0 \text{ (OK)} \\M_{yf} &= \alpha \cdot M_y = 0 \text{ (OK)}\end{aligned}$$





- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.05818$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Distance d from neutral axis $eps = d/h_c \cdot Eps_{top}$	strain $eps = d/h_c \cdot Eps_{top}$	stressessigma / 10e6	force in bar $N = sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,02	-0,10    -0,25	0,492	-0,02959	-500	-157080	-39270	15708
2	0,25	0,05	0,02	0,10    -0,25	0,492	-0,02959	-500	-157080	-39270	-15708
Sum of forces in bars:									-314159	-78540

$$N_s = 314.16e3$$

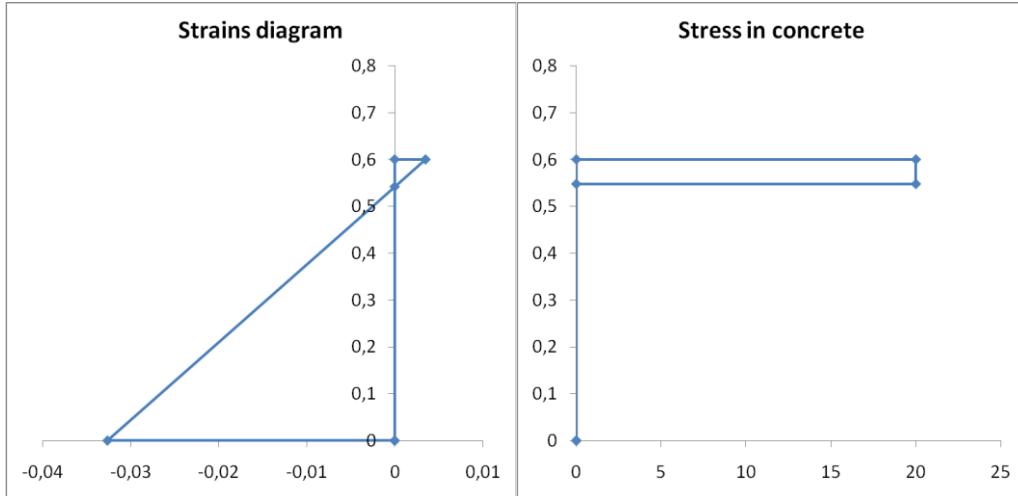
$$M_{xs} = -78.54e3$$

$$M_{ys} = 0.00e3$$

$$Eps_{stop} = \max(eps) = -0,02959$$

$$Eps_{sbot} = \min(eps) = -0,02959$$

- Results for concrete section:

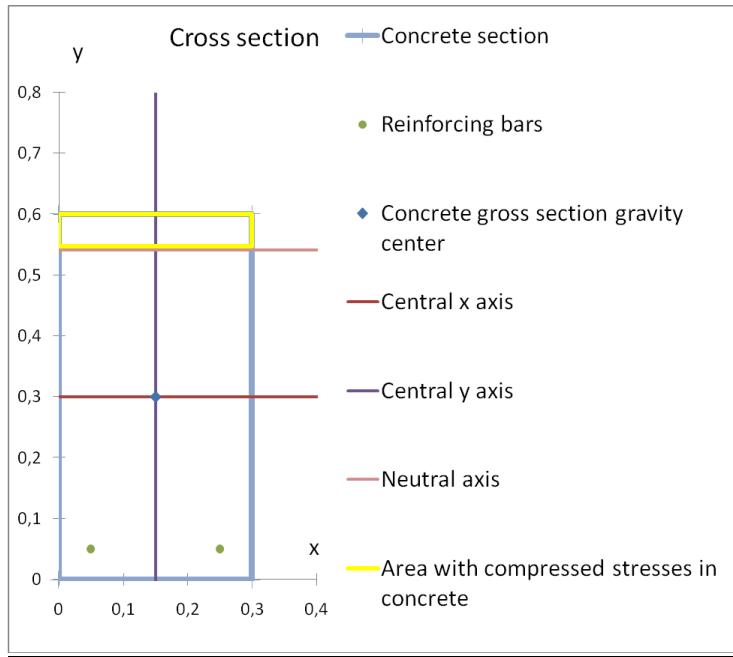


Height of compressed part of the section:

$$\Lambda \cdot y_b = 0.052362$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.052362 = 0.0157086$$



Stress in concrete:

$$\Sigma = E_{top} / E_{ult} \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = 0.3 / 2 - x_{cc} = 0.0000$$

$$y_{cg} = h - \Lambda \cdot y_b / 2 - y_{cc} = 0.273819$$

Internal forces in concrete:

$$\begin{aligned} N_c &= A_c \cdot \text{Sigma} = 314.17e3 \\ M_{xc} &= -N_c \cdot y_{cg} = -86.03e3 \\ M_{yc} &= N_c \cdot x_{cg} = 0.00e3 \end{aligned}$$

- Results for reduced section:

$$\begin{aligned} N_f &= N_c + N_s = 0.01e3 \\ M_{xf} &= M_{xc} + M_{xs} = -164.57e3 \\ M_{yf} &= M_{yc} + M_{ys} = 0.00e3 \end{aligned}$$

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

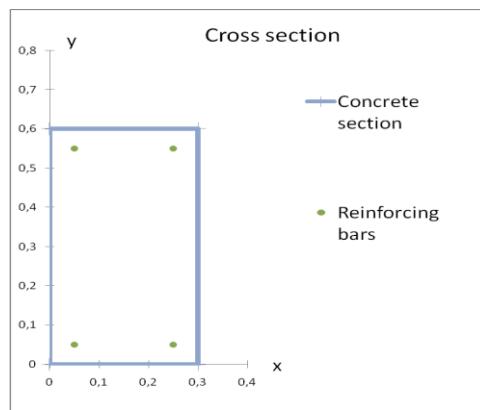
## 11.4 Case 10b

### 11.4.1 Data:

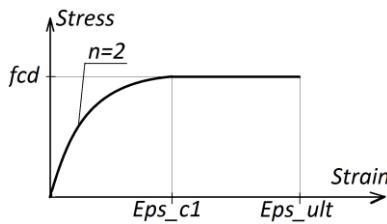
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
- Reinforcing bars:
 

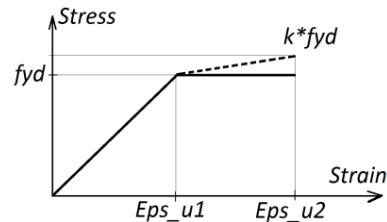
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.05	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$



- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$
  - Strain relation change over  $\epsilon_{c1} = 0.0020$

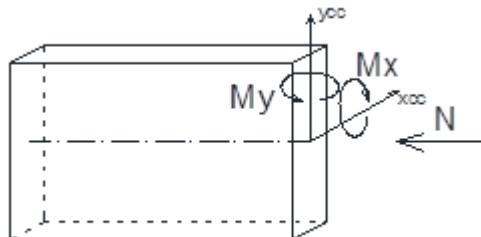


- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.1$



Forces:

$$\begin{aligned} N &= 0.00 \\ M_x &= -125.00 \\ M_y &= 0.00 \end{aligned}$$



#### 11.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

#### 11.4.3 Results from Component

- Results for reduced section:  
 $N_f = 0.02$

M<sub>xf</sub> = -332.64  
M<sub>yf</sub> = 0.00  
dist = -0.237955  
Angle = 4.71239 = 270deg  
Eps\_top = 0.003500  
Eps\_bot = -0.030346  
Eps\_stop = 0.000679  
Eps\_sbot = -0.027526

- Results for reinforcement bars:  
N<sub>s</sub> = -424.81  
M<sub>xs</sub> = -215.50  
M<sub>ys</sub> = 0.00
- Results for concrete section:  
A<sub>c</sub> = 0.0186  
(x<sub>cg</sub>, y<sub>cg</sub>) = (0.00000000, 0.27574117)  
N<sub>c</sub> = 424.83  
M<sub>xc</sub> = -117.14  
M<sub>yc</sub> = 0.00

#### 11.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\text{alfa} &= M_{xf} / M_x = 2.66112 \\ N_f &= 0.02 \leq \text{alfa} \cdot N = 0 \text{ (OK)} \\ M_{yf} &= \text{alfa} \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 5b in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

## 12 Case 11: Calculation of capacity state in symmetric section for bending moment $M_y$

### 12.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (0, 0,  $M_y$ ) and give state of strain and stress at the failure.

### 12.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 11a)
- A bilinear model of concrete stress-strain relation (Case 11b)
- A power-rectangular model of concrete stress-strain relation (Case 11c)
- A horizontal branch model of steel
- A simple bending in y direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )

Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

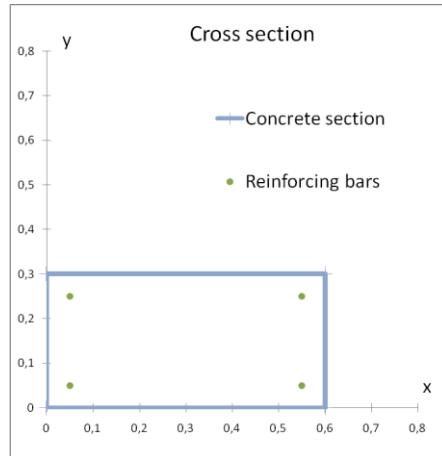
## 12.3 Case 11a

### 12.3.1 Data:

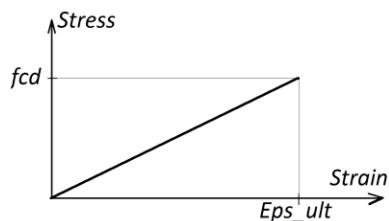
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
  - Reinforcing bars:
 

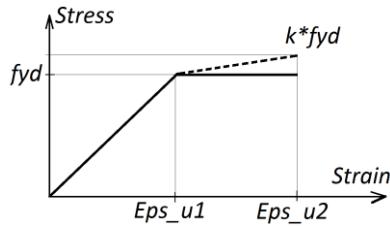
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.05	y = 0.25	$\phi = 0.032$
x = 0.55	y = 0.25	$\phi = 0.032$
x = 0.55	y = 0.05	$\phi = 0.032$



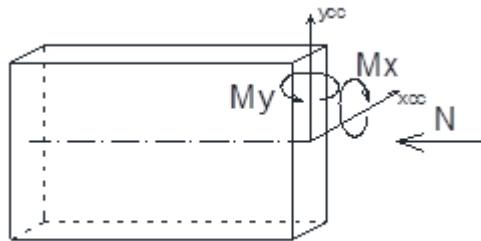
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: linear
  - Strain ultimate limit  $Eps\_ult = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k= 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $Eps_u2 = 0.075$



- Forces:
  - $N = 0.00 \text{ e3}$
  - $M_x = 0.00 \text{ e3}$
  - $M_y = -600.00 \text{ e3}$



### 12.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop$ ,  $Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 12.3.3 Results from Component

- Results for reduced section:  
 $N_f = -0.10 \text{ e3}$   
 $M_{xf} = 0.00 \text{ e3}$   
 $M_{yf} = -407.32 \text{ e3}$   
 $dist = -0.204885$   
 $Angle = 0.00000 = 0.0000\text{deg}$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = -0.018572$   
 $Eps\_stop = 0.001661$   
 $Eps\_sbot = -0.016733$
- Results for reinforcement bars:  
 $N_s = -270.19 \text{ e3}$

$$\begin{aligned}M_{xs} &= 0.00 \text{ e3} \\M_{ys} &= -334.58 \text{ e3}\end{aligned}$$

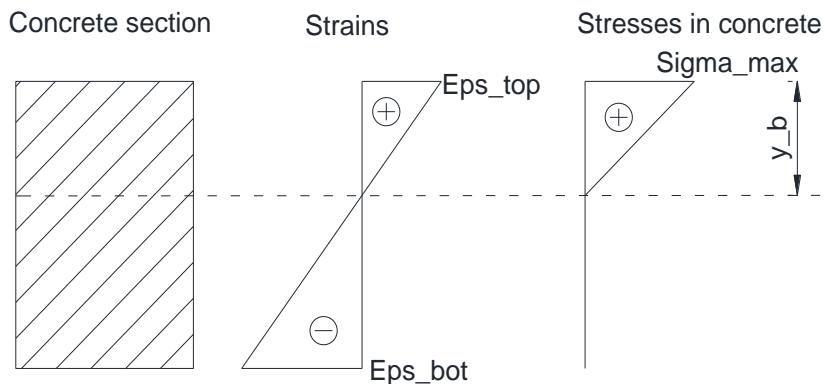
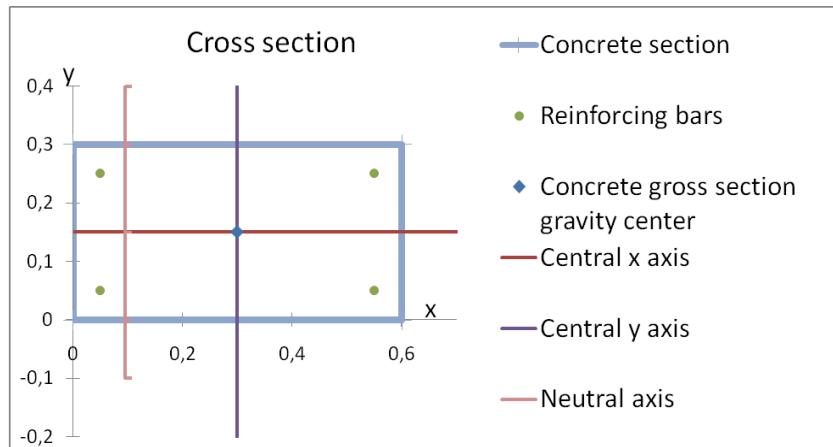
- Results for concrete section:  
 $A_c = 0.0285$   
 $(x_{cg}, y_{cg}) = (-0.26932850, 0.00000000)$   
 $N_c = 270.09 \text{ e3}$   
 $M_{xc} = 0.00 \text{ e3}$   
 $M_{yc} = -72.74 \text{ e3}$

### 12.3.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha_f &= M_y / M_y = 0.67887 \\N_f &= -0.10 \leq \alpha_f \cdot N = 0 \text{ (OK)} \\M_xf &= \alpha_f \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:  
width of a section:  $b = 0.6$

$$y_b = -Eps\_top \cdot b / (Eps\_bot - Eps\_top) = 0.095144$$

$$Eps\_u1 = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.30$$

$$y_{Cc} = 0.15$$

Distance between reinforcing bar and neutral axis:

$$d = y_b + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then}$$

$$\sigma_i = eps_i / Eps\_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps\_u1) / (Eps\_u2 - Eps\_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	φ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses σ / 10e6	force in bar N = σ • π • φ² / 4	Positions in central axes coordinate system x_cc y_cc	M_xs = -N • y_cc	M_ys = N • x_cc
1	0,05	0,05	0,032	-0,045	0,00166	332	267120	-0,25 -0,10	26712	-66780
2	0,55	0,05	0,032	0,455	-0,01673	-500	-402124	0,25 -0,10	-40212	-100531
3	0,55	0,25	0,032	0,455	-0,01673	-500	-402124	0,25 0,10	40212	-100531
4	0,05	0,25	0,032	-0,045	0,00166	332	267120	-0,25 0,10	-26712	-66780
						Sum of forces in bars:	-270008		0	-334622

$$N_s = -270.01e3$$

$$M_{xs} = 0.00e3$$

$$M_{ys} = -334.62e3$$

$$Eps\_stop = \max(eps) = 0,00166$$

$$Eps\_sbot = \min(eps) = -0,01673$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	φ	strain eps	Positions in central axes coordinate system x_cc y_cc	Area of concrete with compression stress cut from the section in place of bars N_sc = f_cd • eps / Eps_ult • Area	Force N_sc in concrete in place of bars N_sc = f_cd • eps / Eps_ult • Area	M_xsc = -N_sc • y_cc	M_ysc = N_sc • x_cc
1	0,05	0,05	0,032	0,00166	-0,25 -0,10	0,000804248	7632	763	-1908
2	0,55	0,05	0,032	-0,01673	0,25 -0,10				
3	0,55	0,25	0,032	-0,01673	0,25 0,10				
4	0,05	0,25	0,032	0,00166	-0,25 0,10	0,000804248	7632	-763	-1908
						Sum of forces :	15264	0	-3816

Reduction forces in concrete:

$$N_{sc} = 15.26e3$$

$$M_{xsc} = 0.00e3$$

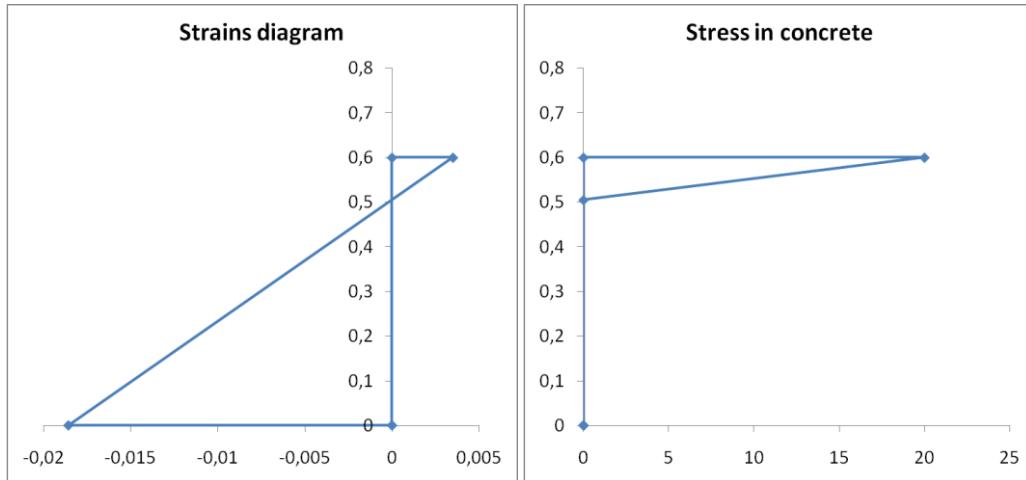
$$M_{ysc} = -3.82e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = -0.25$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.095144$$

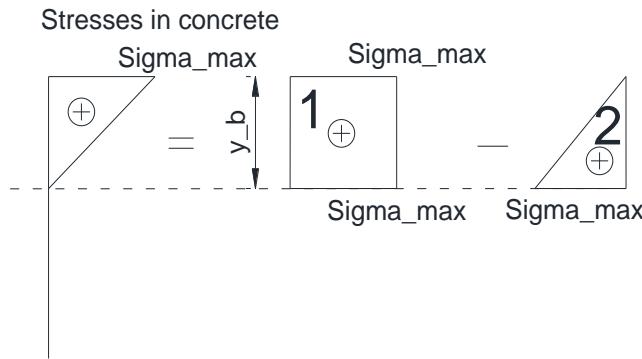
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0285$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of

stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot Eps\_top / Eps\_ult = 20.000e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 0.5709e6$$

$$N_{c2} = A_c \cdot \text{Sigma\_max} / 2 = 0.2854e6$$

$$N_c' = N_{c1} - N_{c2} = 0.285432e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = -0.3 + y_b / 2 = -0.252428$$

$$y_{c1} = 0$$

$$x_{c2} = -0.3 + y_b \cdot 2/3 = -0.236570$$

$$y_{c2} = 0$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.269318$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 270.17e3$$

$$M_{xc} = -N_c \cdot y_{cg} = 0.00e3$$

$$M_{yc} = N_c' \cdot x_{cg} = -72.76e3$$

- Results for reduced section:

$$N_f = N_c + N_s = 0.16e3$$

$$M_{xf} = M_{xc} + M_{xs} = 0.00e3$$

$$M_{yf} = M_{yc} + M_{ys} = -407.38e3$$

Small differences between results from Component and from manual calculations are below 0.1% and are due to precision of iterations.

## 12.4 Case 11b

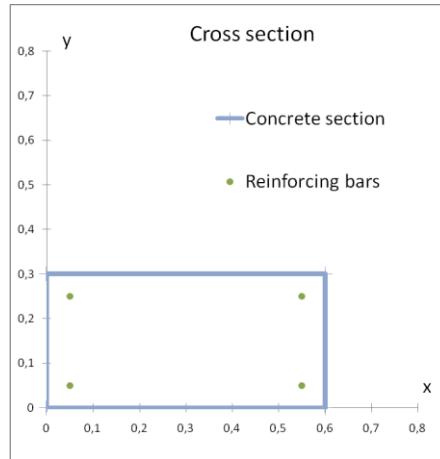
### 12.4.1 Data:

- Geometry:
  - Concrete section:
 

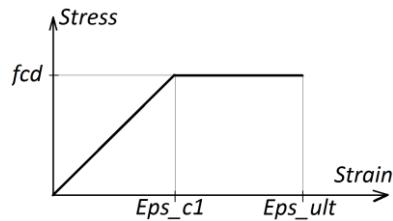
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.6$	$y = 0.3$
$x = 0.6$	$y = 0.0$
- Reinforcing bars:
 

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.25$	$\phi = 0.032$

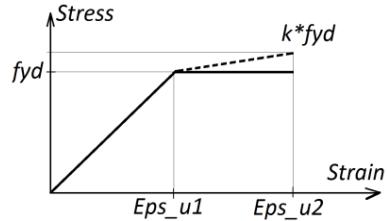
$$\begin{array}{lll} x = 0.55 & y = 0.25 & \phi = 0.032 \\ x = 0.55 & y = 0.05 & \phi = 0.032 \end{array}$$



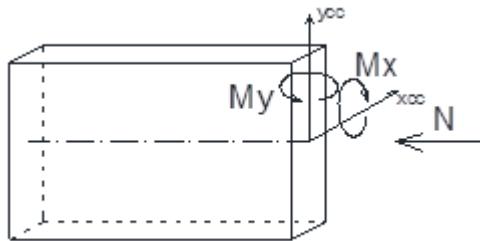
- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0035$
  - Strain relation change over  $\text{Eps\_c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 205\text{e}9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.1$



- Forces:
  - $N = 0.00 \text{ e}3$
  - $M_x = 0.00 \text{ e}3$
  - $M_y = -10.00 \text{ e}3$



#### 12.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

#### 12.4.3 Results from Component

- Results for reduced section:  
 $N_f = 0.09 \text{ e}3$   
 $M_{xf} = 0.00 \text{ e}3$   
 $M_{yf} = -332.39 \text{ e}3$   
 $\text{dist} = -0.236099$   
 $\text{Angle} = 0.000000 = 0.000000 \text{ deg}$   
 $\epsilon_{top} = 0.003500$   
 $\epsilon_{bot} = -0.029363$   
 $\epsilon_{stop} = 0.000761$   
 $\epsilon_{sbot} = -0.026625$
- Results for reinforcement bars:  
 $N_s = -392.34\text{e}3$   
 $M_{xs} = 0.00\text{e}3$   
 $M_{ys} = -223.61\text{e}3$
- Results for concrete section:  
 $A_c = 0.0192$   
 $(x_{cg}, y_{cg}) = (-0.27717502, 0.00000000)$   
 $N_c = 392.42 \text{ e}3$   
 $M_{xc} = 0.00 \text{ e}3$   
 $M_{yc} = -108.77 \text{ e}3$

#### 12.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned} \alpha &= M_{yf}/M_y = 33.239 \\ N_f &= 0.09 \geq \alpha \cdot N = 0 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_x = 0 \text{ (OK)} \end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 4b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.03% and are due to precision of iterations.

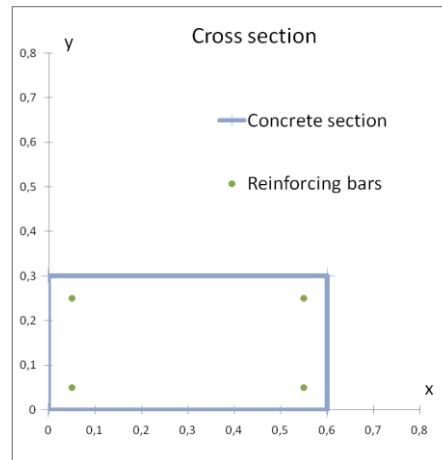
## 12.5 Case 11c

### 12.5.1 Data:

- Geometry:
  - Concrete section:
 

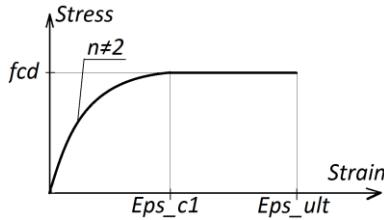
x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
- Reinforcing bars:
 

x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.05	y = 0.25	$\phi = 0.032$
x = 0.55	y = 0.25	$\phi = 0.032$
x = 0.55	y = 0.05	$\phi = 0.032$

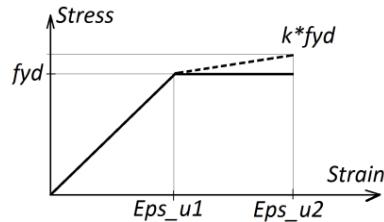


- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: power-rectangular
  - Power:  $n = 1.8$

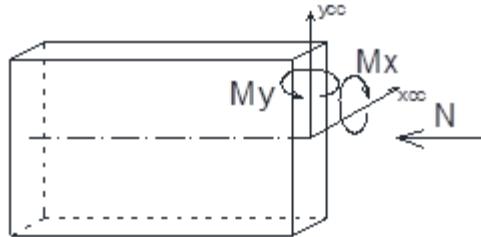
- Strain ultimate limit  $Eps\_ult = 0.0035$
- Strain relation change over  $Eps\_c1 = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $Eps_u2 = 0.1$



- Forces:
  - $N = 0.00 e3$
  - $M_x = 0.00 e3$
  - $M_y = -10.00 e3$



### 12.5.2 Search for:

- Results for reduced section:
  - Forces:  $N_f, M_{xf}, M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top, Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop, Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s, M_{xs}, M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c, M_{xc}, M_{yc}$

### 12.5.3 Results from Component

- Results for reduced section:

$N_f = -0.16 \text{ e3}$   
 $M_{xf} = 0.00 \text{ e3}$   
 $M_{yf} = -332.56 \text{ e3}$   
 $\text{dist} = -0.237679$   
 $\text{Angle} = 0.00000 = 0.00000 \text{ deg}$   
 $\text{Eps_top} = 0.003500$   
 $\text{Eps_bot} = -0.030196$   
 $\text{Eps_stop} = 0.000692$   
 $\text{Eps_sbot} = -0.027388$

- Results for reinforcement bars:

$N_s = -420.79 \text{ e3}$   
 $M_{xs} = 0.00 \text{ e3}$   
 $M_{ys} = -216.50 \text{ e3}$

- Results for concrete section:

$A_c = 0.0187$   
 $(x_{cg}, y_{cg}) = (-0.27591769, 0.00000000)$   
 $N_c = 420.64 \text{ e3}$   
 $M_{xc} = 0.00 \text{ e3}$   
 $M_{yc} = -116.06 \text{ e3}$

### 12.5.4 Results (manual calculations):

Verification of safety factor:

$\alpha = M_y / M_y = 33.256$   
 $N_f = -0.16 \leq \alpha \cdot N = 0 \text{ (OK)}$   
 $M_{xf} = \alpha \cdot M_x = 0 \text{ (OK)}$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 6b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.05% and are due to precision of iterations.

## 13 Case 12: Calculation of capacity state in symmetric section for bending moment $M_x$ with axial force

### 13.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $N$ ,  $M_x$ , 0) and give state of strain and stress at the failure.

### 13.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 12a)
- A bilinear model of concrete stress-strain relation (Case 12b)
- A power-rectangular model of concrete stress-strain relation (Case 12c)
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ )
- An axial force ( $N$ ).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )
  - Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

For these results,  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

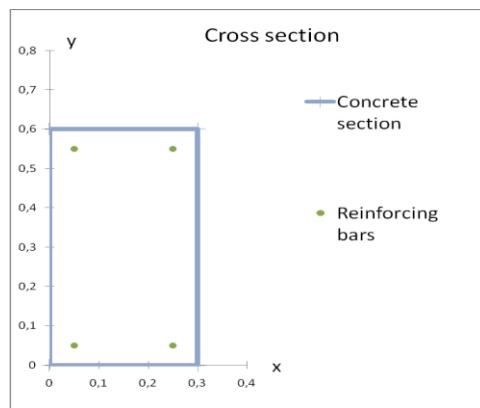
## 13.3 Case 12a

### 13.3.1 Data:

- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

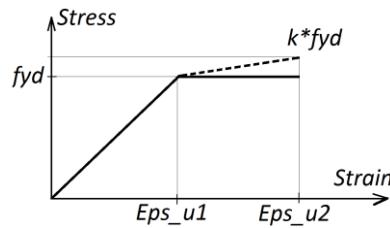
x = 0.05	y = 0.05	$\phi = 0.012$
x = 0.05	y = 0.55	$\phi = 0.012$
x = 0.25	y = 0.55	$\phi = 0.012$
x = 0.25	y = 0.05	$\phi = 0.012$



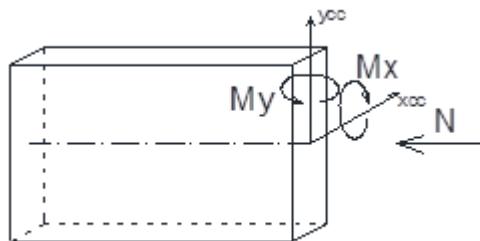
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: linear
  - Strain ultimate limit  $Eps\_ult = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $Eps\_u2 = 0.075$



- Forces:
  - $N = 43979.98 \text{ e3}$
  - $M_x = -3465.40 \text{ e3}$
  - $M_y = 0.00 \text{ e3}$



### 13.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop$ ,  $Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete: ( $x_{cg}$ ,  $y_{cg}$ )
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 13.3.3 Results from Component

- Results for reduced section:
 

$N_f = 2199.00 \text{ e3}$   
 $M_{xf} = -173.27 \text{ e3}$   
 $M_{yf} = 0.00 \text{ e3}$   
 $dist = 0.40000$   
 $Angle = 4.712389 = 270.00000\text{deg}$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = 0.000500$   
 $Eps\_stop = 0.003250$   
 $Eps\_sbot = 0.000750$

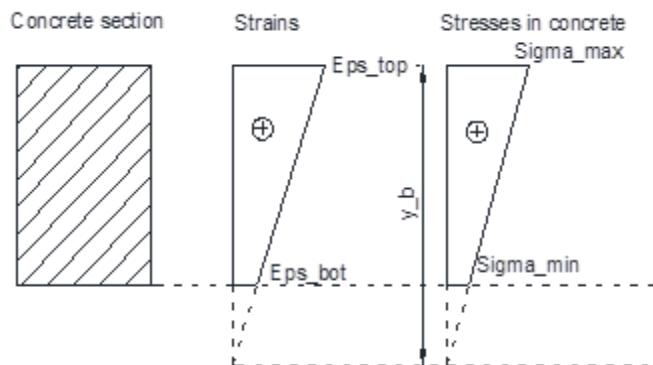
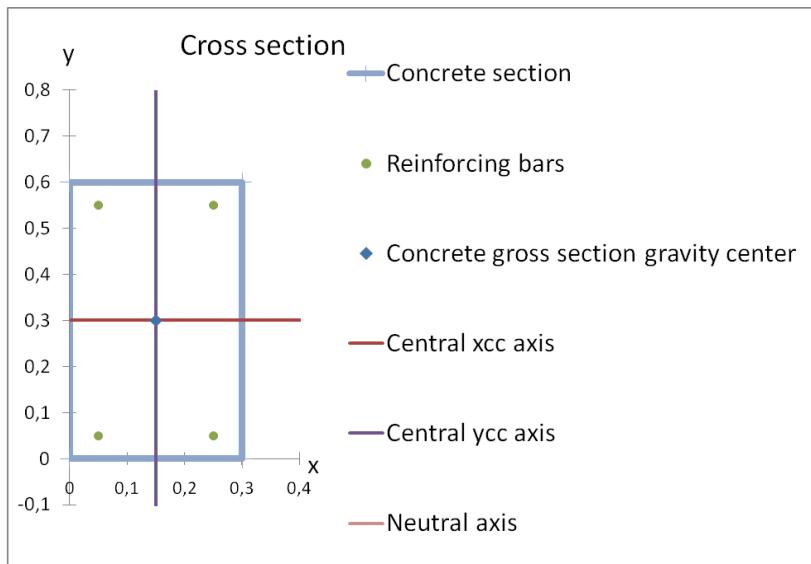
- Results for reinforcement bars:  
 $N_s = 147.03 \text{ e}3$   
 $M_{xs} = -19.79 \text{ e}3$   
 $M_{ys} = 0.00 \text{ e}3$
- Results for concrete section:  
 $A_c = 0.18$   
 $(x_{cg}, y_{cg}) = (0.00000000, -0.07479510)$   
 $N_c = 2051.97 \text{ e}3$   
 $M_{xc} = 153.48 \text{ e}3$   
 $M_{yc} = 0.00 \text{ e}3$

### 13.3.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_x / N = 0.05000 \\ N_f &= \alpha \cdot N = 2199.00 \text{ (OK)} \\ M_{yf} &= \alpha \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section:  $h = 0.6$

$$y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.7$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{cc} = 0.15$$

$$y_{cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If  $|eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses $\sigma = \sigma / 10e6$	force in bar $N = \sigma \cdot \pi \times \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1    -0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1    0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1    0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1    -0,25	4241	1696
						Sum of forces in bars:	147027			-19792    0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

$$Eps_{stop} = \max(eps) = 0.00325$$

$$Eps_{sbot} = \min(eps) = 0.00075$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	$\phi$	strain eps	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon / Eps_{ult} \cdot Area$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,012	0,00075	-0,1    -0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1    0,25	0,000113097	485	121	48

3	0,25	0,55	0,012	0,00325	0,1	0,25	0,000113097		2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1	-0,25	0,000113097		2100	-525	-210
							Sum of forces:		5170	-808	0

Reduction forces in concrete:

$$N_{sc} = 5.17e3$$

$$M_{xsc} = -0.81e3$$

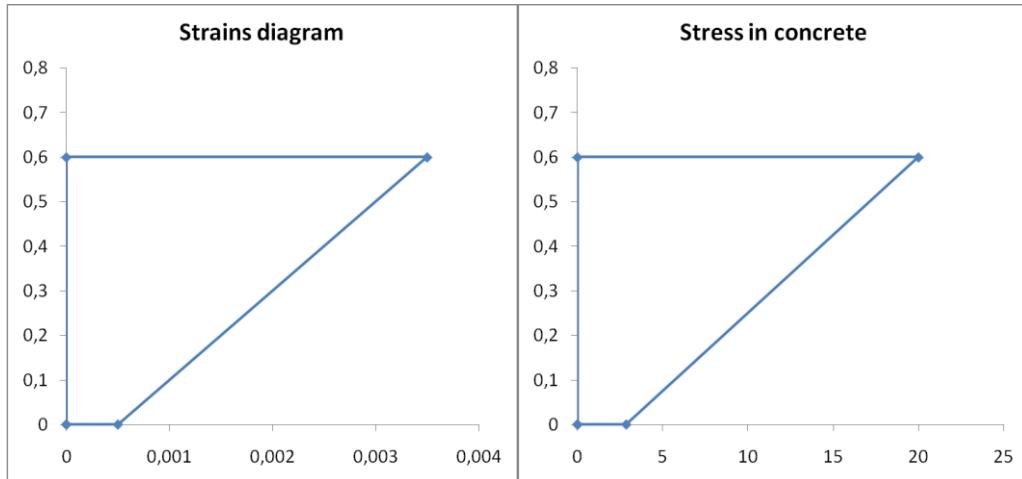
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.00000$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.15625$$

- Results for concrete section:



Height of compressed part of the section:

0.6000

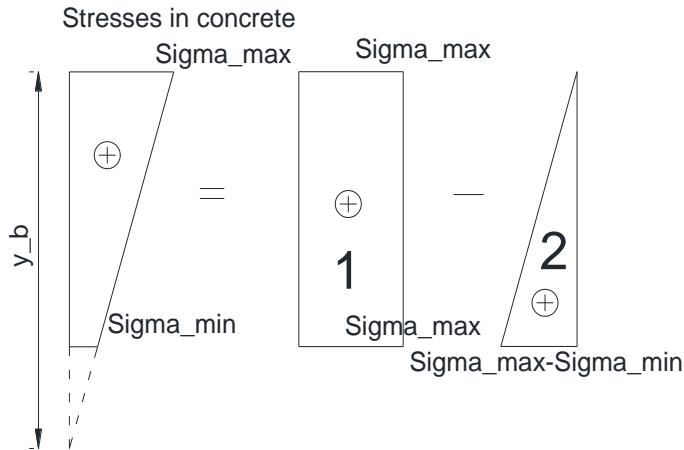
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force  $N_c$  in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of  $N_c$  in Introduction).

$$N_c = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments  $M_{xc}$  and  $M_{yc}$  caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma\_max} = f_{cd} \cdot Eps\_top/Eps\_c1 = 20.000e6$$

$$\text{Sigma\_min} = f_{cd} \cdot Eps\_bot/Eps\_c1 = 2.857e6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma\_max} = 3.600e6$$

$$N_{c2} = A_c \cdot (\text{Sigma\_max} - \text{Sigma\_min}) / 2 = 1.543e6$$

$$N_c = N_{c1} - N_{c2} = 2.057e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -y_{cc} + 0.6/3 = -0.1$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.074795$$

Internal forces in concrete:

$$N_c = N_c - N_{sc} = 2051.97e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -153.48e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N_f = N_c + N_s = 2199.00e3$$

$$M_{xf} = M_{xc} + M_{xs} = -173.27e3$$

$$M_{yf} = M_{yc} + M_{ys} = 0.00e3$$

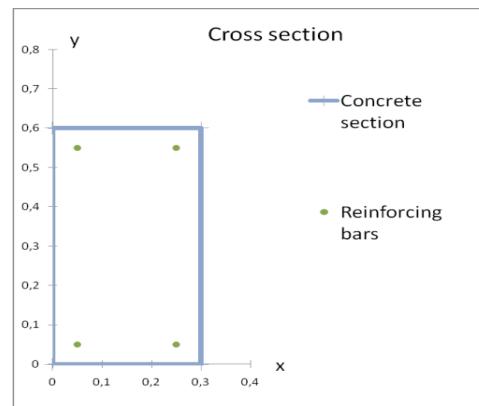
## 13.4 Case 12b

### 13.4.1 Data:

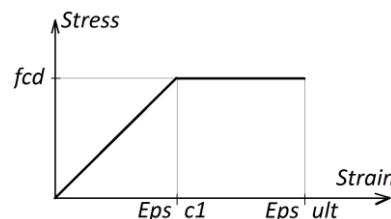
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

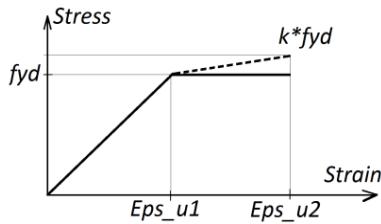
x = 0.05	y = 0.05	$\phi = 0.016$
x = 0.05	y = 0.55	$\phi = 0.016$
x = 0.25	y = 0.55	$\phi = 0.016$
x = 0.25	y = 0.05	$\phi = 0.016$



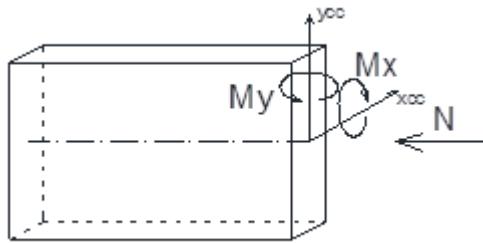
- Concrete parameters:
  - Design strength  $f_{cd} = 30e6$
  - Modulus of elasticity  $E_c = 32e9$
  - Strain-stress model: bilinear
  - Strain ultimate limit  $Eps\_ult = 0.0035$
  - Strain relation change over  $Eps\_c1 = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 205e9$
  - Strain ultimate limit  $Eps\_u2 = 0.1$



- Forces:
  - $N = 114.8397 \text{ e3}$
  - $M_x = -5.634275 \text{ e3}$
  - $M_y = 0.00 \text{ e3}$



### 13.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop$ ,  $Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 13.4.3 Results from Component

- Results for reduced section:  
 $N_f = 4593.59 \text{ e3}$   
 $M_{xf} = -225.37 \text{ e3}$   
 $M_{yf} = 0.00 \text{ e3}$   
 $dist = 0.40000$   
 $Angle = 4.71239 = 270 \text{ deg}$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = 0.000500$   
 $Eps\_stop = 0.003250$   
 $Eps\_sbot = 0.000750$
- Results for reinforcement bars:

$N_s = 222.68 \text{ e3}$   
 $M_{xs} = -24.76 \text{ e3}$   
 $M_{ys} = 0.00 \text{ e3}$

- Results for concrete section:  
 $A_c = 0.1800$   
 $(x_{cg}, y_{cg}) = (0.00000000, 0.04589755)$   
 $N_c = 4370.92 \text{ e3}$   
 $M_{xc} = -200.61 \text{ e3}$   
 $M_{yc} = 0.00 \text{ e3}$

### 13.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_x / M_x = 40 \\ N_f &= \alpha \cdot N = 4593.59 \text{ (OK)} \\ M_y &= \alpha \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 4a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

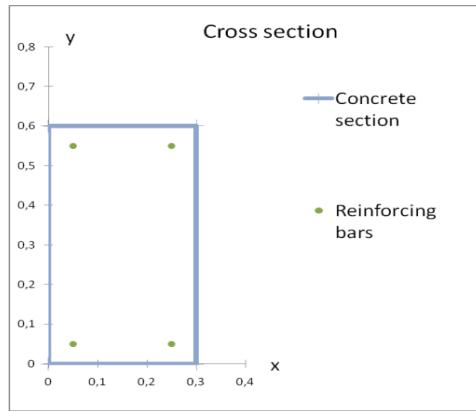
## 13.5 Case 12c

### 13.5.1 Data:

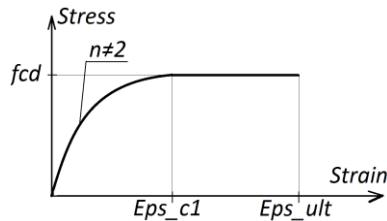
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
  - Reinforcing bars:
 

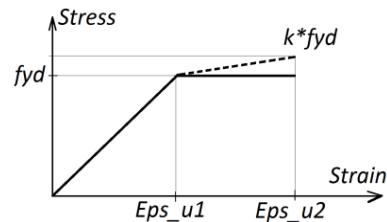
$x = 0.05$	$y = 0.05$	$\phi = 0.016$
$x = 0.05$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.05$	$\phi = 0.016$



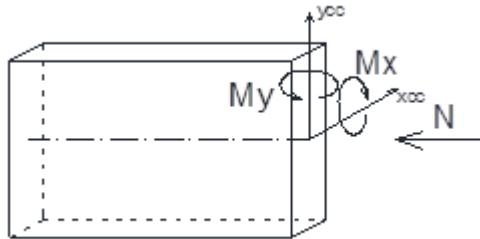
- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: power-rectangular
  - Power:  $n = 1.4$
  - Strain ultimate limit  $\text{Eps\_ult} = 0.0035$
  - Strain relation change over  $\text{Eps\_c1} = 0.0020$



- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.1$



- Forces:
  - $N = 4.41023 \text{ e}3$
  - $M_x = -0.1662045455 \text{ e}3$
  - $M_y = 0.00 \text{ e}3$



### 13.5.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete: ( $x_{cg}$ ,  $y_{cg}$ )
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 13.5.3 Results from Component

- Results for reduced section:  
 $N_f = 4851.25e3$   
 $M_{xf} = -182.83e3$   
 $M_{yf} = 0.00e3$   
 $dist = 0.40000$   
 $Angle = 4.71239 = 270\text{deg}$   
 $\epsilon_{top} = 0.003500$   
 $\epsilon_{bot} = 0.000500$   
 $\epsilon_{stop} = 0.003250$   
 $\epsilon_{sbot} = 0.000750$
- Results for reinforcement bars:  
 $N_s = 221.17e3$   
 $M_{xs} = -25.13e3$   
 $M_{ys} = 0.00e3$
- Results for concrete section:  
 $A_c = 0.1800$   
 $(x_{cg}, y_{cg}) = (0.00000000, 0.03405830)$   
 $N_c = 4630.08e3$   
 $M_{xc} = -157.69e3$   
 $M_{yc} = 0.00e3$

### 13.5.4 Results (manual calculations):

Verification of safety factor:

$$\alpha = M_{xf} / M_x = 1100$$

$$N_f = \alpha \cdot N = 4851.25 \text{ (OK)}$$

$$M_{yf} = \alpha \cdot M_y = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 6a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

## 14 Case 13: Calculation of capacity state in symmetric section for bending moment $M_y$ with axial force

### 14.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $N, 0, M_y$ ) and give state of strain and stress at the failure.

### 14.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 13a)
- A parabolic-rectangular model of concrete stress-strain relation (Case 13b)
- A horizontal branch model of steel
- A bending in  $y$  direction ( $M_y$ )
- An axial force ( $N$ ).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}, y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )

Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}, \epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}, \epsilon_{sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to  $x$  and  $y$  axes.

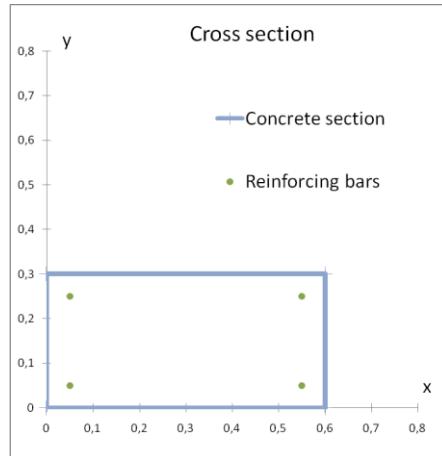
## 14.3 Case 13a

### 14.3.1 Data:

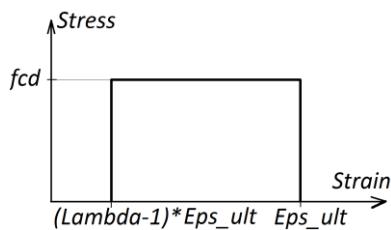
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
  - Reinforcing bars:
 

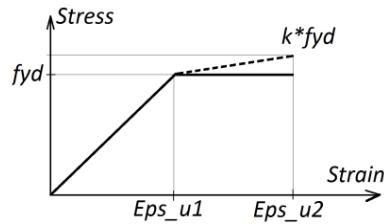
x = 0.05	y = 0.05	$\phi = 0.012$
x = 0.05	y = 0.25	$\phi = 0.012$
x = 0.55	y = 0.25	$\phi = 0.012$
x = 0.55	y = 0.05	$\phi = 0.012$



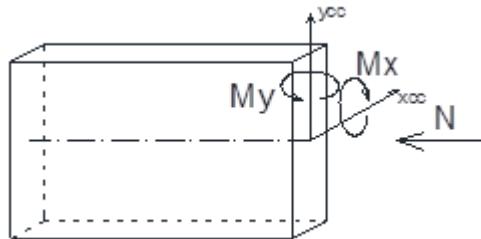
- Concrete parameters:
  - Design strength  $f_{cd} = 20e6$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 30e9$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 500e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Forces:
  - $N = 3331.408571 \text{ e3}$
  - $M_x = 0.00 \text{ e3}$
  - $M_y = -82.84952381 \text{ e3}$



### 14.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop$ ,  $Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 14.3.3 Results from Component

- Results for reduced section:

$N_f = 3497.98 \text{ e3}$   
 $M_{xf} = 0.00 \text{ e3}$   
 $M_{yf} = -86.99 \text{ e3}$   
 $dist = 0.400000$   
 $Angle = 0.00000 = 0.0000\text{deg}$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = 0.000500$   
 $Eps\_stop = 0.003250$   
 $Eps\_sbot = 0.000750$

- Results for reinforcement bars:

$N_s = 147.03 \text{ e3}$   
 $M_{xs} = 0.00 \text{ e3}$   
 $M_{ys} = -19.79 \text{ e3}$

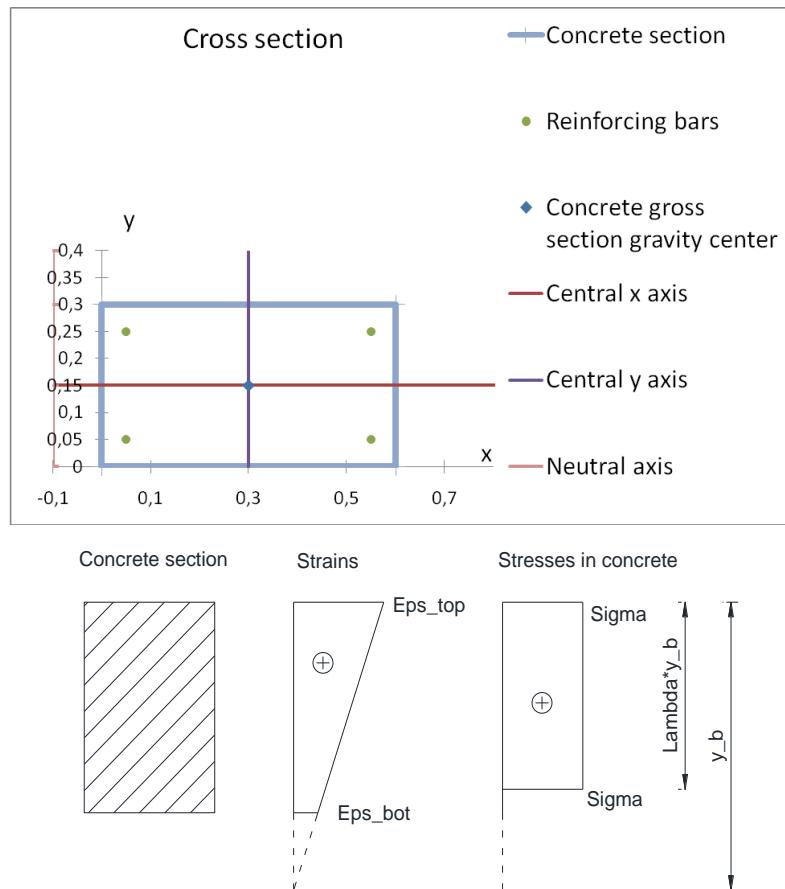
- Results for concrete section:
  $A_c = 0.1680$   
 $(x_{cg}, y_{cg}) = (-0.02005403, 0.00000000)$   
 $N_c = 3350.95 \text{ e}3$   
 $M_{xc} = 0.00 \text{ e}3$   
 $M_{yc} = -67.20 \text{ e}3$

#### 14.3.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_{yf} / M_y = 1.04998 \\ N_f &= 3497.98 \leq \alpha \cdot N = 3497.90 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

width of a section:  $b = 0.6$

$$y_b = -Eps_{top} \cdot b / (Eps_{bot} - Eps_{top}) = 0.7$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{cc} = 0.30$$

$$y_{cc} = 0.15$$

Distance between reinforcing bar and neutral axis:

$$d = y_b + y_{bar}$$

Stress in bar i:

$$\text{If } |\epsilon_i| \leq f_y / E_s \text{ then}$$

$$\sigma_i = \epsilon_i \cdot E_s$$

else

$$\sigma_i = f_y + (|\epsilon_i| - E_s) / (E_s - E_u) \cdot (k - 1) \cdot f_y$$

i	x	y	$\phi$	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot E_{top}$	stresses $\sigma = \epsilon \cdot E_s / 10^6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,012	0,150	0,00325	500	56549	-0,25    -0,10	5655	-14137
2	0,55	0,05	0,012	0,650	0,00075	150	16965	0,25    -0,10	1696	4241
3	0,55	0,25	0,012	0,650	0,00075	150	16965	0,25    0,10	-1696	4241
4	0,05	0,25	0,012	0,150	0,00325	500	56549	-0,25    0,10	-5655	-14137
					Sum of forces in bars:		147027		-19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	$\phi$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars $N_{sc} = f_{cd} \cdot E_{top} / E_{ult} \cdot Area$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,012	-0,25    -0,10	0,000113097	2262	226	-565
2	0,55	0,05	0,012	0,25    -0,10	0,000113097	2262	226	565
3	0,55	0,25	0,012	0,25    0,10	0,000113097	2262	-226	565
4	0,05	0,25	0,012	-0,25    0,10	0,000113097	2262	-226	-565
				Sum of forces:		9048	0	0

Reduction forces in concrete:

$$N_{sc} = 9.05e3$$

$$M_{xsc} = 0.00e3$$

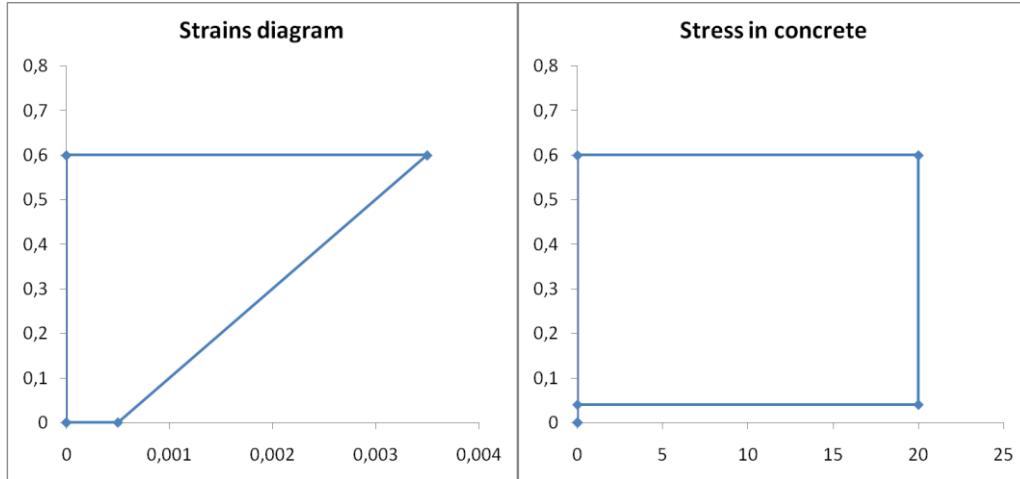
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.00$$

$$y_{scg} = \sum(y_{cc\_i} \cdot Area\_i) / \sum(Area\_i) = 0.00$$

- Results for concrete section:



Height of compressed part of the section:

$$\Lambda \cdot y_b = 0.56$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.56 = 0.168$$

Stress in concrete:

$$\Sigma = Eps\_top / Eps\_ult \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.56 / 2 - x_{cc}) \cdot A_c \cdot \Sigma - x_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = -0.020054$$

$$y_{cg} = ((0.3 / 2 - y_{cc}) \cdot A_c \cdot \Sigma - y_{scg} \cdot N_{sc}) / (A_c \cdot \Sigma + N_{sc}) = 0.000000$$

Internal forces in concrete:

$$N_c = A_c \cdot \Sigma - N_{sc} = 3350.95e3$$

$$M_{xc} = -N_c \cdot y_{cg} = 0.00e3$$

$$M_{yc} = N_c \cdot x_{cg} = -67.20e3$$

- Results for reduced section:

$$N_f = N_c + N_s = 3497.98e3$$

$$M_{xf} = M_{xc} + M_{xs} = 0.00e3$$

$$M_{yf} = M_{yc} + M_{ys} = -86.99e3$$

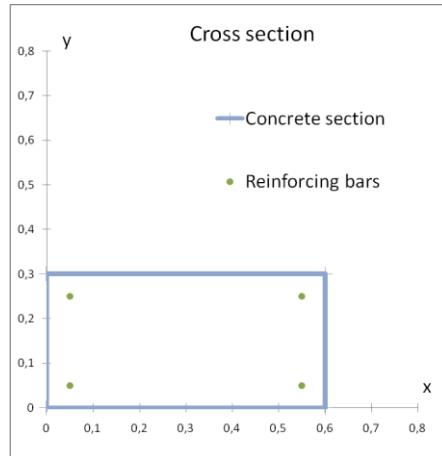
## 14.4 Case 13b

### 14.4.1 Data:

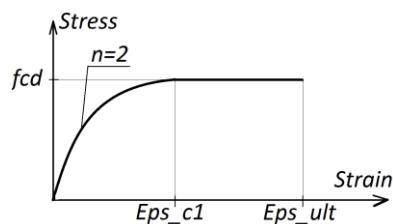
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
  - Reinforcing bars:
 

x = 0.05	y = 0.05	$\phi = 0.016$
x = 0.05	y = 0.25	$\phi = 0.016$
x = 0.55	y = 0.25	$\phi = 0.016$
x = 0.55	y = 0.05	$\phi = 0.016$

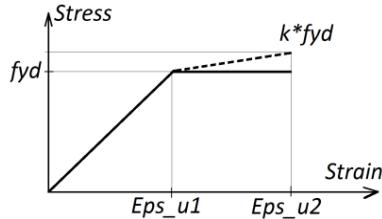


- Concrete parameters:
  - Design strength  $f_{cd} = 30\text{e}6$
  - Modulus of elasticity  $E_c = 32\text{e}9$
  - Strain-stress model: parabolic-rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$
  - Strain relation change over  $\epsilon_{c1} = 0.0020$

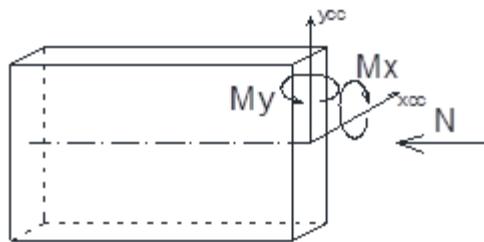


- Steel parameters:
  - Design strength  $f_{yd} = 400\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$

- Strain ultimate limit  $Eps\_u2 = 0.1$



- Forces:
  - $N = 5095.50e3$
  - $M_x = 0.00e3$
  - $M_y = -137.86e3$



#### 14.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f, M_{xf}, M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $Eps\_top, Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_stop, Eps\_sbot$ )
- Results for reinforcement bars:
  - Forces:  $N_s, M_{xs}, M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c, M_{xc}, M_{yc}$

#### 14.4.3 Results from Component

- Results for reduced section:
 

$N_f = 5095.50 e3$   
 $M_{xf} = 0.00 e3$   
 $M_{yf} = -137.86 e3$   
 $dist = 0.400000$   
 $Angle = 0.00000 = 0.0000deg$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = 0.000500$   
 $Eps\_stop = 0.003250$   
 $Eps\_sbot = 0.000750$
- Results for reinforcement bars:
 

$N_s = 221.17 e3$   
 $M_{xs} = 0.00 e3$   
 $M_{ys} = -25.13 e3$

- Results for concrete section:  
 $A_c = 0.1800$   
 $(x_{cg}, y_{cg}) = (-0.02312690, 0.000000000)$   
 $N_c = 4874.33 \text{ e3}$   
 $M_{xc} = 0.00 \text{ e3}$   
 $M_{yc} = -112.73 \text{ e3}$

#### 14.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_{yf} / M_y = 1.000 \\ N_f &= \alpha \cdot N = 5095.50 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_y = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 5a in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

## 15 Case 14: Calculation of capacity state in symmetric section for bidirectional bending with axial force

### 15.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $N$ ,  $M_x$ ,  $M_y$ ) and give state of strain and stress at the failure.

### 15.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ )
- A bending in y direction ( $M_y$ )
- An axial force ( $N$ ).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )
  - Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

### 15.3 Data:

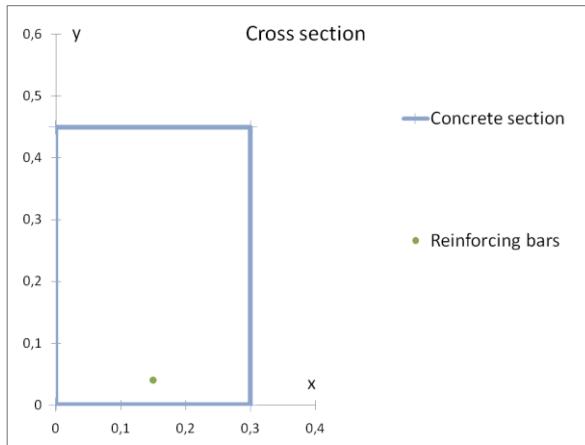
- Geometry:

- Concrete section:

$$\begin{array}{ll} x = 0.0 & y = 0.0 \\ x = 0.0 & y = 0.45 \\ x = 0.3 & y = 0.45 \\ x = 0.3 & y = 0.0 \end{array}$$

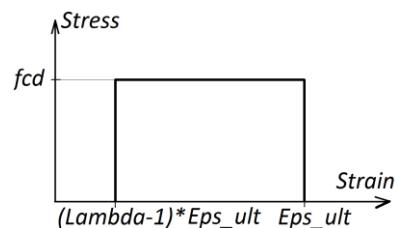
- Reinforcing bars:

$$x = 0.15 \quad y = 0.04 \quad \text{area} = 0.00053 \quad \phi = 0.025977239$$



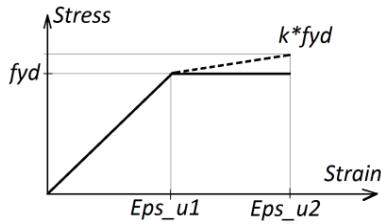
- Concrete parameters:

- Design strength  $f_{cd} = 26.7 \text{ e6} \cdot 0.8 = 21.3 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 35\text{e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$

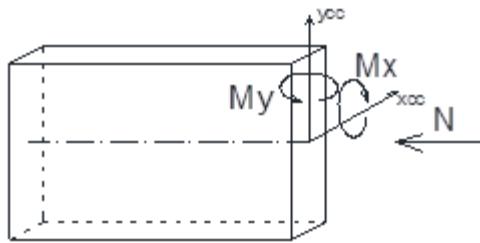


- Steel parameters:

- Design strength  $f_{yd} = 310\text{e6}$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e9}$
  - Strain ultimate limit  $\epsilon_{u2} = 0.075$



- Forces:
  - $N = 200.00 \text{ e3}$
  - $M_x = -N \cdot 0.48 = -96.00 \text{ e3}$
  - $M_y = -N \cdot 0.12 = 24.00 \text{ e3}$



## 15.4 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Force:  $N_c$

## 15.5 Results from Component

- Results for reduced section:
 

$N_f = 199.77 \text{ e3}$   
 $M_{xf} = -95.95 \text{ e3}$   
 $M_{yf} = -24.00 \text{ e3}$   
 Angle =  $5.204135 = 298.174942 \text{ deg}$   
 $dist = -0.120248$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = -0.009152$   
 $Eps\_stop = -0.006659$   
 $Eps\_sbot = -0.006659$
- Results for reinforcement bars:
 

$N_s = -164.41 \text{ e3}$   
 $M_{xs} = -30.41 \text{ e3}$   
 $M_{ys} = 0.00 \text{ e3}$
- Results for concrete section:
 

$A_c = 0.0170$   
 $(x_{cg}, y_{cg}) = (-0.06589487, 0.17995110)$   
 $N_c = 364.018 \text{ e3}$

$$\begin{aligned}M_{xc} &= -65.53 \text{ e3} \\M_{yc} &= -24.00 \text{ e3}\end{aligned}$$

Results from Component are consistent with results from book “*Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2*”, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, example 6.18 on page 352., ISBN 83-7125-136-X. Calculations in the book are performed with smaller precision than in Component which causes some differences in results (about 1%). These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

## 16 Case 15: Calculation of capacity state in asymmetric section for bidirectional bending with axial force

### 16.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $N$ ,  $M_x$ ,  $M_y$ ) and give state of strain and stress at the failure.

### 16.2 Summary:

This sample gives as results the internal forces for:

- A asymmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 15a)
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ )
- A bending in y direction ( $M_y$ )
- An axial force ( $N$ ).

In the Case 15b can be seen a comparison of capacity forces for various models of concrete (rectangular, linear, bilinear, parabolic-rectangular and power-rectangular) obtained in Component and in manual calculations.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )
  - Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tension fiber. (Angle)
- Strains
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )

- Extreme strains in steel ( $\text{Eps\_stop}$ ,  $\text{Eps\_sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

## 16.3 Case 15a

### 16.3.1 Data:

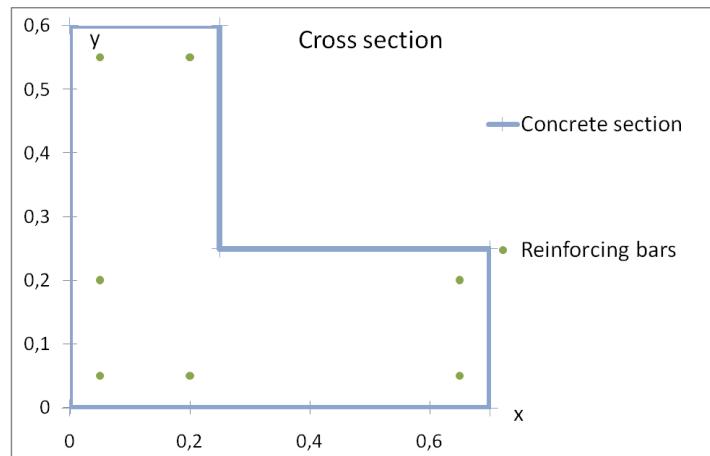
- Geometry:

- Concrete section:

$x = 0.00$	$y = 0.00$
$x = 0.00$	$y = 0.60$
$x = 0.25$	$y = 0.60$
$x = 0.25$	$y = 0.25$
$x = 0.70$	$y = 0.25$
$x = 0.70$	$y = 0.00$

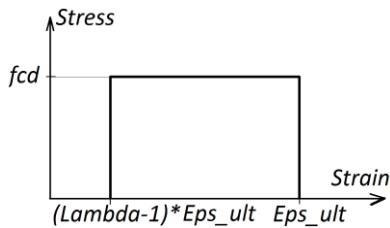
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.020$
$x = 0.05$	$y = 0.55$	$\phi = 0.020$
$x = 0.20$	$y = 0.55$	$\phi = 0.020$
$x = 0.20$	$y = 0.05$	$\phi = 0.020$
$x = 0.65$	$y = 0.05$	$\phi = 0.020$
$x = 0.65$	$y = 0.20$	$\phi = 0.020$
$x = 0.05$	$y = 0.20$	$\phi = 0.020$

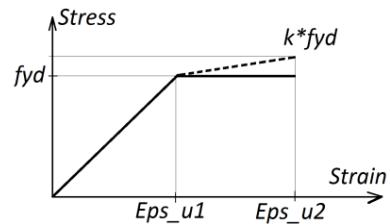


- Concrete parameters:

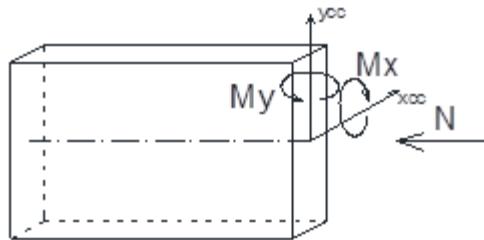
- Design strength  $f_{cd} = 20\text{e}6$
- Effective height reduction factor  $\Lambda = 0.8$
- Modulus of elasticity  $E_c = 35\text{e}9$
- Strain-stress model: rectangular
- Strain ultimate limit  $\text{Eps\_ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 310e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.075$



- Forces:
  - $N = 72.4471 \text{ e3}$
  - $M_x = -28.9825 \text{ e3}$
  - $M_y = 2.5743 \text{ e3}$



### 16.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf}$
  - Neutral axis position: dist, Angle
  - Extreme strains in concrete ( $\text{Eps\_top}$ ,  $\text{Eps\_bot}$ )
  - Extreme strains in steel ( $\text{Eps\_stop}$ ,  $\text{Eps\_sbot}$ )
- Results for reinforcement bars:
  - Forces:  $N_s$ ,  $M_{xs}$ ,  $M_{ys}$
- Results for concrete section:
  - Area of contour under compressive stress:  $A_c$
  - Center of gravity of stresses in the concrete:  $(x_{cg}, y_{cg})$
  - Forces:  $N_c$ ,  $M_{xc}$ ,  $M_{yc}$

### 16.3.3 Results from Component

- Results for reduced section:  
 $N_f = 725.91 \text{ e3}$   
 $M_{xf} = -290.09 \text{ e3}$   
 $M_{yf} = 25.68 \text{ e3}$   
 $\text{Angle} = 4.188995 = 240.011739 \text{ deg}$   
 $\text{dist} = 0.095114$   
 $\text{Eps\_top} = 0.003500$   
 $\text{Eps\_bot} = -0.006888$   
 $\text{Eps\_stop} = 0.002399$   
 $\text{Eps\_sbot} = -0.005788$
- Results for reinforcement bars:  
 $N_s = -108.39 \text{ e3}$   
 $M_{xs} = -101.21 \text{ e3}$   
 $M_{ys} = 31.32 \text{ e3}$
- Results for concrete section:  
 $A_c = 0.0427$   
 $(x_{cg}, y_{cg}) = (-0.00676059, 0.22639273)$   
 $N_c = 834.30 \text{ e3}$   
 $M_{xc} = -188.88 \text{ e3}$   
 $M_{yc} = -5.64 \text{ e3}$

### 16.3.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned} M_{yf} / M_y &= 9.98 \\ M_{xf} / M_x &= 10.01 \\ N_f / N &= 10.02 \\ \text{alfa} &= M_{yf} / M_y \approx M_{xf} / M_x \approx N_f / N (\text{OK}) \end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Position of neutral axis

Concrete gross section gravity center calculations:

i	Corners of concrete section		Area _c	Gravity center of contour corresponding to Area_c		Static moments	
	x	y		x <sub>01</sub>	y <sub>01</sub>	S <sub>cy</sub>	S <sub>cx</sub>
1	0	0	0,000	0,00	0,30	0	0
2	0	0,6	0,150	0,13	0,30	0,01875	0,045
3	0,25	0,6	0,000	0,25	0,43	0	0
4	0,25	0,25	0,113	0,48	0,13	0,05344	0,0141
5	0,7	0,25	0,000	0,70	0,13	0	0
6	0,7	0	0,000	0,35	0,00	0	0
	Total:		0,263			0,0722	0,0591

Where:

$\text{edge}_i = \text{edge between corners: } i \text{ and } i+1$

$\text{Area}_{c\_i} = \text{Area between edge}_i \text{ and axes: } x=x_i, x=x_{(i+1)} \text{ and } y=0$

$$\text{Area}_{c\_i} = 1/2 \cdot (y_{(i+1)} + y(i)) \cdot (x_{(i+1)} - x(i))$$

$x_{01\_i} = x_i + (x_{(i+1)} - x_i) \cdot (2/3 \cdot y_{(i+1)} + 1/3 \cdot y(i)) / (y_i + y_{(i+1)}) \text{ if } (y_i + y_{(i+1)}) \neq 0 \text{ or}$

$$x_{01\_i} = (x_{(i+1)} + x_i) / 2 \text{ if } (y_i + y_{(i+1)}) = 0$$

$y_{01\_i} = y_i / 2 + 0.5 \cdot (x_{01\_i} - x_i) \cdot (y_{(i+1)} - y_i) / (x_{(i+1)} - x_i) \text{ if } x_{(i+1)} - x_i \neq 0 \text{ or}$

$$y_{01\_i} = (y_{(i+1)} + y_i) / 2 \text{ if } x_{(i+1)} - x_i = 0$$

$S_{cy\_i} = \text{Area}_{c\_i} \cdot x_{01\_i}$

$S_{cx\_i} = \text{Area}_{c\_i} \cdot y_{01\_i}$

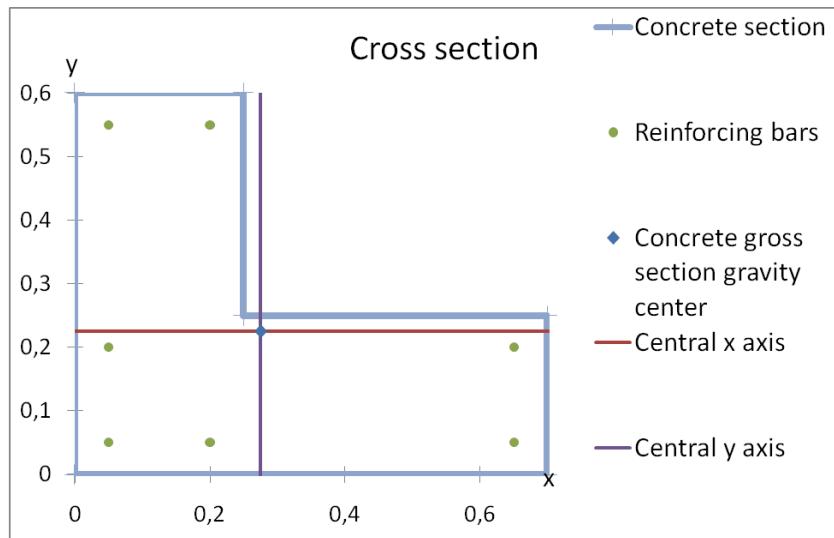
Concrete gross section area:

$$A_c = \sum(\text{Area}_{c\_i}) = 0.263$$

Concrete gross section gravity center

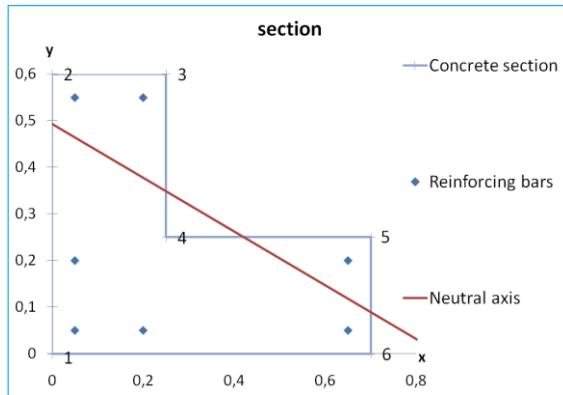
$$x_{Cc} = \sum(S_{cy\_i}) / A_c = 0.275$$

$$y_{Cc} = \sum(S_{cx\_i}) / A_c = 0.225$$



Equation of neutral axis in central axes coordinate system ( $x_{cc}; y_{cc}$ )

$$y_{cc} = -0.577350316 \cdot x_{cc} + 0.110000027 \text{ (e.g. obtained in iteration process)}$$



General equation of neutral axis in central axes coordinate system ( $x_{cc}$ ;  $y_{cc}$ )

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = -0.202$$

$$B = -0.350$$

$$C = 0.0385$$

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed corner:

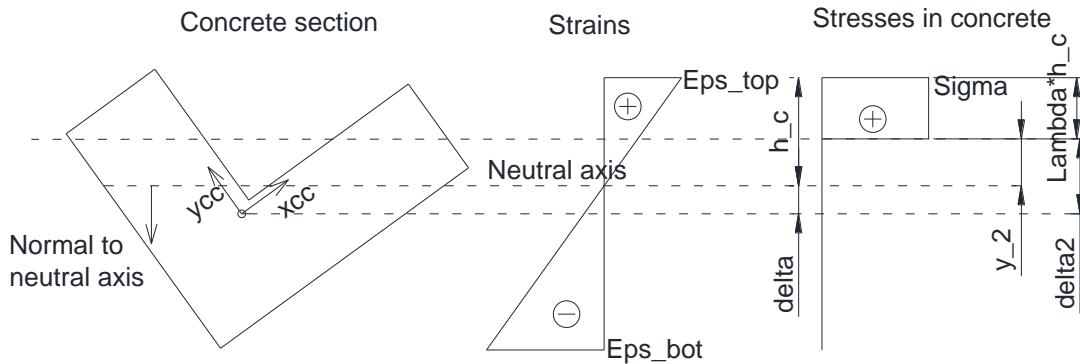
$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	$x_{cc}$	$y_{cc}$		
1	0	0	-0,275	-0,225	tension	0,428
2	0	0,6	-0,275	0,375	compression	-0,092
3	0,25	0,6	-0,025	0,375	compression	-0,217
4	0,25	0,25	-0,025	0,025	tension	0,086
5	0,7	0,25	0,425	0,025	compression	-0,139
6	0,7	0	0,425	-0,225	tension	0,078

Where:

$x_{cc} = X - X_{Cc}$

$y_{cc} = Y - Y_{Cc}$



Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.217$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.428$$

Verification of used equation of neutral axis:

$$Eps_{bot} = -h_t/h_c \cdot Eps_{top} = -0.006903226 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \arctan(-0.577350316) + \pi/2 + \pi = 4.18879 = 240 \text{ deg} = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$Eps_{u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If  $|Eps_i| \leq f_{yd} / E_s$  then

$$\sigma_i = Eps_i / Eps_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|Eps_i| - Eps_{u1}) / (Eps_{u2} - Eps_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	$\phi$	Positions in central axes coordinate system $x_{cc}$ $y_{cc}$	Distance d from neutral axis	strain $eps = d/h_c \cdot Eps_{top}$	stress $\sigma = Eps \cdot 10e6$	force in bar $N_s = \sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,02	-0,23    -0,18	0,359	-0,00580	-310	-97389	-17043	21913
2	0,05	0,55	0,02	-0,23    0,33	-0,074	0,00119	238	74685	-24273	-16804
3	0,2	0,55	0,02	-0,08    0,33	-0,149	0,00240	310	97389	-31652	-7304
4	0,2	0,05	0,02	-0,08    -0,18	0,284	-0,00459	-310	-97389	-17043	7304

5	0,65	0,05	0,02	0,38	-0,18	0,059	-0,00096	-191	-60114	-10520	-22543
6	0,65	0,2	0,02	0,38	-0,03	-0,071	0,00114	228	71535	1788	26826
7	0,05	0,2	0,02	-0,23	-0,03	0,229	-0,00370	-310	-97389	-2435	21913
							Sum of forces in bars:		-108673	101177	31304

$$N_s = -108.67 \text{ e}3$$

$$M_{xs} = -101.18 \text{ e}3$$

$$M_{ys} = 31.30 \text{ e}3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon \leq \text{Eps\_c1} \quad N_{sc} = f_{cd} \cdot \epsilon / \text{Eps\_c1} \cdot \text{Area}$$

$$\text{if } \epsilon > \text{Eps\_c1} \quad N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	φ	strain ε	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force $N_{sc}$ in concrete in place of bars	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot$ $x_{cc}$
1	0,05	0,05	0,02	-0,00580	-0,23	-0,18				
2	0,05	0,55	0,02	0,00119	-0,23	0,33	0,000314159	6283	-2042	-1414
3	0,2	0,55	0,02	0,00240	-0,08	0,33	0,000314159	6283	-2042	-471
4	0,2	0,05	0,02	-0,00459	-0,08	-0,18				
5	0,65	0,05	0,02	-0,00096	0,38	-0,18				
6	0,65	0,2	0,02	0,00114	0,38	-0,03	0,000314159	6283	157	2356
7	0,05	0,2	0,02	-0,00370	-0,23	-0,03				
							0,000942478	18850	-3927	471
					Total:					

Reduction forces in concrete:

$$N_{sc} = 18.850 \text{ e}3$$

$$M_{xsc} = -3.927 \text{ e}3$$

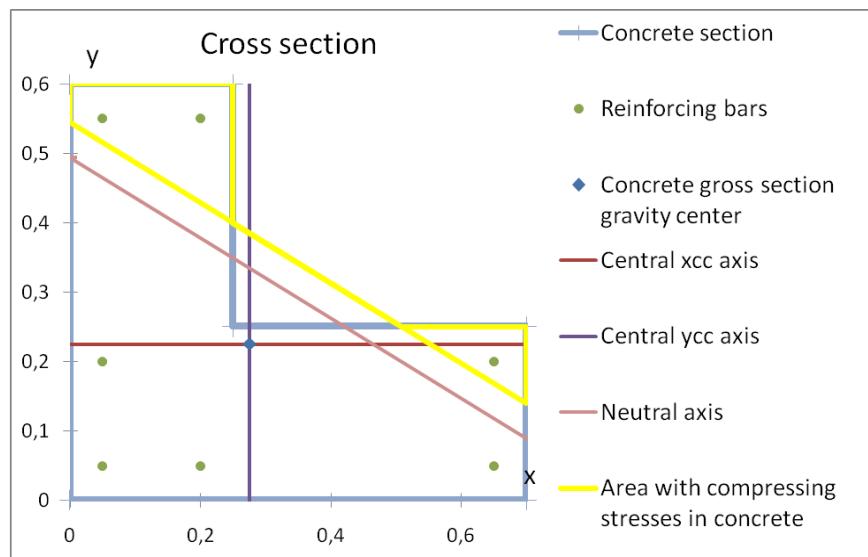
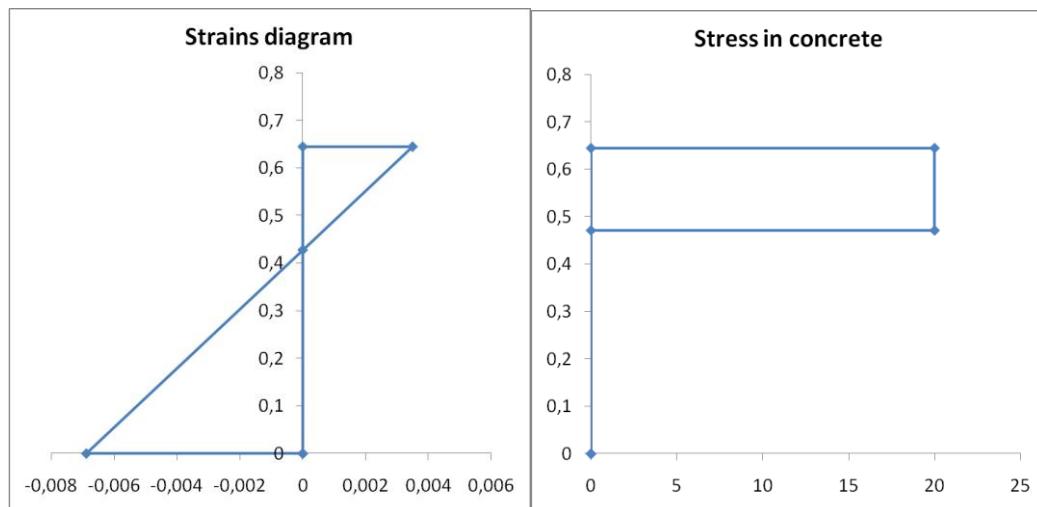
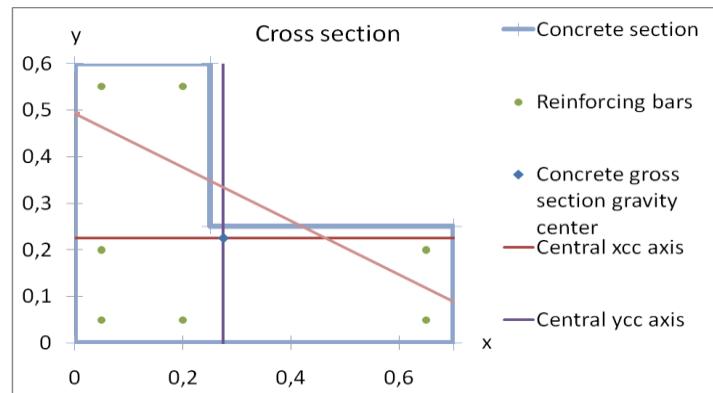
$$M_{ysc} = 0.471 \text{ e}3$$

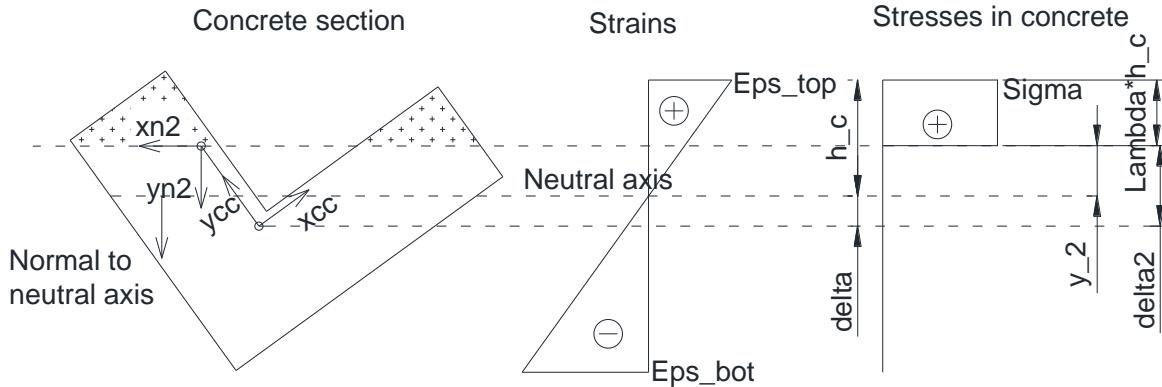
Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.0250$$

$$y_{scg} = \text{sum}(y_{cc\_i} \cdot \text{Area}_i) / \text{sum}(\text{Area}_i) = 0.2083$$

- Results for concrete section:





$$\Sigma = f_{cd} = 20.000 \text{ e6}$$

Distance between gravity center of contour and neutral axis:

$$\delta = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

for \$x\_{cc} = y\_{cc} = 0\$ and coefficients A, B, C as defined in paragraph: Position of neutral axis, above.  
 $\delta = 0.09526$

Distance between the gravity center of contour of the section and area with compressive stresses in concrete:

$$\delta_2 = \delta + (1 - \Lambda) \cdot h_c = 0.138862$$

To calculate a sum of stresses over the contour with compressive stresses in concrete we define a new coordinate system \$x\_{n2}\$, \$y\_{n2}\$ (see picture above), where \$x\_{n2}\$ is an axis parallel to neutral axis, in distance \$\delta\_2\$ from the gravity center of contour of the section, and it crosses axis \$y\_{n2}\$ in point \$(x\_{n2c}; y\_{n2c})\$ given in central axis coordinate system.

Equation of neutral axis in central axes coordinate system:

$$y_{cc} = -0.577350316 \cdot x_{cc} + 0.110000027$$

Equation of \$x\_{n2}\$ axis in central axes coordinate system

$$y_{nc} = -0.577350316 \cdot x_{nc} + \delta_2 / \cos(\text{Angle}) = -0.577350316 \cdot x_{nc} + 0.16011327$$

For \$x\_{n2c} = 0\$ and \$y\_{n2c} = 0.16011327\$

Now we find coordinates of contour under stress solid in \$(x\_{cc}, y\_{cc})\$ coordinates system.

i	Contour x y		Contour in central axes coordinate system \$x_{cc}\$ \$y_{cc}\$	Equation of line containing edge_i, in central axes coordinate system	Intersection point of line containing edge_i and \$x_{n2}\$ axis \$x_{cc}\$ \$y_{cc}\$	Is intersection on the contour?		
1	0	0	-0,275	-0,225	\$x_{cc} = -0,275\$	-0,275	0,3189	Yes
2	0	0,6	-0,275	0,375	\$y_{cc} = 0,375\$	-0,3722	0,375	No
3	0,25	0,6	-0,025	0,375	\$x_{cc} = -0,025\$	-0,025	0,1745	Yes
4	0,25	0,25	-0,025	0,025	\$y_{cc} = 0,025\$	0,2340	0,025	Yes
5	0,7	0,25	0,425	0,025	\$x_{cc} = 0,425\$	0,425	-0,0853	Yes
6	0,7	0	0,425	-0,225	\$y_{cc} = -0,225\$	0,6670	-0,225	No
7	0	0	-0,275	-0,225	\$x_{cc} = -0,275\$	-0,275	0,3189	Yes

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

edge\_i = edge between corners: i and i+1

Now we calculate area and gravity center of contour under compressive stresses in concrete in  $(x_{cc}, y_{cc})$  coordinates system:

i	Contour with compressive stresses in concrete		Contour with compressive stresses in concrete		Area_c	Gravity center of contour corresponding to Area_c		Static moments	
	x_cc	y_cc	x	y		x_01	y_01	S_cy	S_cx
1	-0,275	0,375	0,000	0,600	0,1500	0,125	0,3	0,019	0,045
2	-0,025	0,375	0,250	0,600	0,0000	0,25	0,6	0	0
3	-0,025	0,175	0,250	0,400	0,0841	0,370	0,165	0,031	0,014
4	0,234	0,025	0,509	0,250	0,0477	0,605	0,125	0,029	0,006
5	0,425	0,025	0,700	0,250	0,0000	0,7	0,25	0	0
6	0,425	-0,085	0,700	0,140	-0,2393	0,281	0,191	-0,067	-0,046
7	-0,275	0,319	0,000	0,544	0,0000	0	0,544	0	0
8	-0,275	0,375	0,000	0,000	0,0000	0,125	0,3	0	0
			Total	0,0426				0,0115	0,0192

Where:

edge\_i = edge between corners: i and i+1

Area\_c\_i = Area between edge\_i and axes:  $x=x_i$ ,  $x=x_{(i+1)}$  and  $y=0$

$$\text{Area}_c_i = 1/2 \cdot (y_{(i+1)} + y(i)) \cdot (x_{(i+1)} - x(i))$$

$$x_{01\_i} = x_{(i)} + (x_{(i+1)} - x_{(i)}) \cdot (2/3 \cdot y_{(i+1)} + 1/3 \cdot y_{(i)}) / (y_{(i)} + y_{(i+1)}) \text{ if } (y_{(i)} + y_{(i+1)}) \neq 0 \text{ or}$$

$$x_{01\_i} = (x_{(i+1)} + x_{(i)}) / 2 \text{ if } (y_{(i)} + y_{(i+1)}) = 0$$

$$y_{01\_i} = y_{(i)} / 2 + 0.5 \cdot (x_{01\_i} - x_{(i)}) \cdot (y_{(i+1)} - y_{(i)}) / (x_{(i+1)} - x_{(i)}) \text{ if } x_{(i+1)} - x_{(i)} \neq 0 \text{ or}$$

$$y_{01\_i} = (y_{(i+1)} + y_{(i)}) / 2 \text{ if } x_{(i+1)} - x_{(i)} = 0$$

$$S_{cy\_i} = \text{Area}_c_i \cdot x_{01\_i} - y_{cc}$$

$$S_{cx\_i} = \text{Area}_c_i \cdot y_{01\_i} - x_{cc}$$

Compressed concrete area:

$$A_c = \sum(\text{Area}_c_i) = 0,0426$$

Gravity center of compressed concrete contour (gross) in central axes coordinate system

$$x_c = \sum(S_{cy\_i}) / A_c - x_{Cc} = -0,0060$$

$$y_c = \sum(S_{cx\_i}) / A_c - y_{Cc} = -0,2260$$

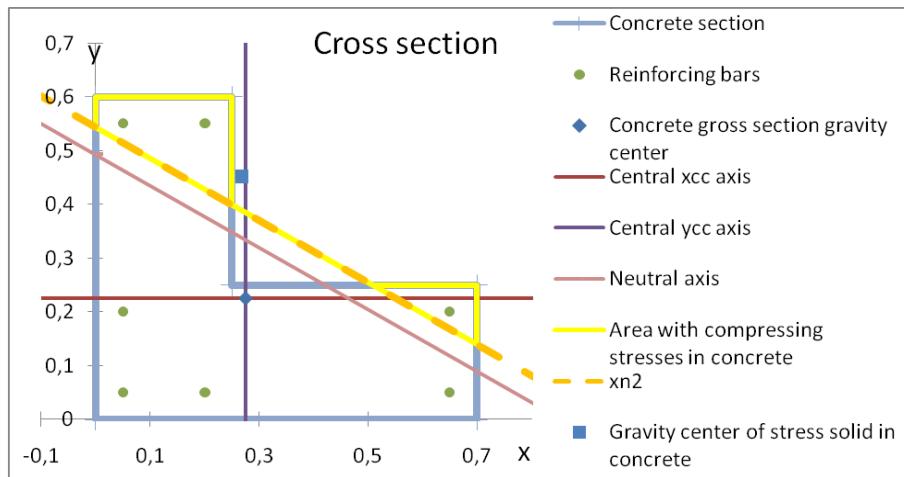
Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \Sigma = 0,851993 \text{ e6}$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_c \cdot N_c - x_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = -0,006674$$

$$y_{cg} = (y_c \cdot N_c - y_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0,226430$$



Internal forces in concrete:

$$\begin{aligned} N_c &= N_c' - N_{sc} = 833.14 \text{ e3} \\ M_{xc} &= -N_c \cdot y_{cg} = -188.65 \text{ e3} \\ M_{yc} &= N_c \cdot x_{cg} = -5.56 \text{ e3} \end{aligned}$$

- Results for reduced section:

$$\begin{aligned} N_f &= N_c + N_s = 724.47 \text{ e3} \\ M_{xf} &= M_{xc} + M_{xs} = -289.83 \text{ e3} \\ M_{vf} &= M_{yc} + M_{ys} = 25.74 \text{ e3} \end{aligned}$$

Small differences between results from Component and from manual calculations are below 1.5% and are due to precision of iterations.

## 16.4 Case 15b

Comparison of capacity forces for various models of concrete (obtained in Component / in manual calculations) for example as in Case 15a:

stress-strain model of concrete behavior:	rectangular	linear	bilinear	parabolic-rectangular	power-rectangular
parameters of concrete	as in Case 15a $f_{cd} = 20\text{e}6$	$Eps\_ult= 0,0035$ $f_{cd} = 20\text{e}6/0.8= 25\text{e}6$	$Eps\_ult= 0,0035$ $Eps\_c1= 0,0020$ $f_{cd} = 20\text{e}6/0.8= 25\text{e}6$	$Eps\_ult= 0,0035$ $Eps\_c1= 0,0020$ $f_{cd} = 20\text{e}6/0.8= 25\text{e}6$	$Eps\_ult= 0,0035$ $Eps\_c1= 0,0020$ $n = 1.5$ $f_{cd} = 20\text{e}6/0.8= 25\text{e}6$
$N_f / \text{e3}$	725.91 / 724.47	658.70 / 657.37	737.18 / 736.31	760.14 / 759.21	751.24 / 750.30
$M_{xf} / \text{e3}$	-290.09 / -289.83	-263.20 / -262.98	-294.66 / -294.56	-303.83 / -303.72	-300.26 / -300.16
$M_{vf} / \text{e3}$	25.68 / 25.74	23.29 / 23.35	26.24 / 26.17	27.06 / 26.98	26.74 / 26.66

## 17 Case 16: Calculation of capacity state in symmetric section for fixed axial force

### 17.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $N$ ) and give state of strain and stress at the failure for fixed axial force  $N$  and  $M_x$  or  $M_y = 0$ .

### 17.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ ) (Case 16a)
- A bending in y direction ( $M_y$ ) (Case 16b)
- A fixed axial force ( $N$ ).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )

Where  $\alpha$  is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

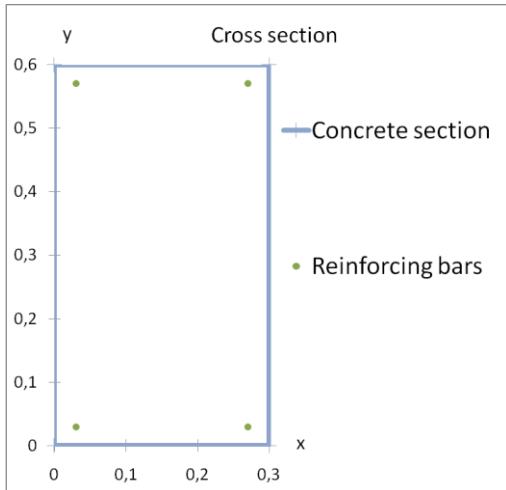
## 17.3 Case 16a

### 17.3.1 Data:

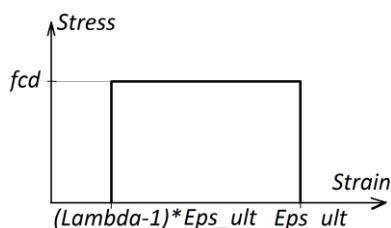
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

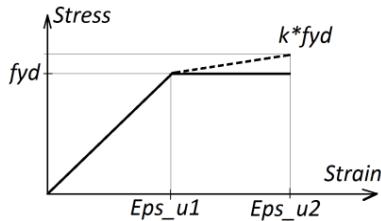
x = 0.03	y = 0.03	$\phi = 0.04$
x = 0.03	y = 0.57	$\phi = 0.04$
x = 0.27	y = 0.57	$\phi = 0.04$
x = 0.27	y = 0.03	$\phi = 0.04$



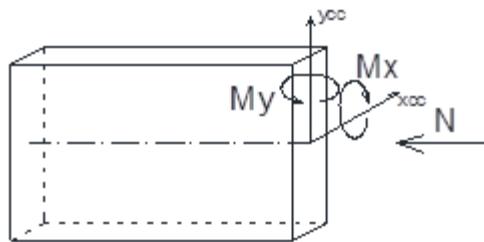
- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 310 \text{ e6}$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200 \text{ e9}$
  - Strain ultimate limit  $\epsilon_{u2} = 0.025$



- Forces:
  - $N = 678.00e3$
  - $M_x$
  - $M_y$



### 17.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f = 678.00e3$ ,  $M_{xf}$ ,  $M_{yf}$
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_top$ ,  $Eps\_bot$ )

### 17.3.3 Results from Component

- Results for reduced section:
 

$N_f = 677.98 e3$   
 $M_{xf} = -574.80 e3$   
 $M_{yf} = 0.00 e3$   
 $Eps\_top = 0.003500$   
 $Eps\_bot = -0.008467$   
 $Eps\_stop = 0.002902$   
 $Eps\_sbot = -0.007869$

Results from Component are consistent with a graph from book “Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 360., ISBN 83-7125-136-X.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

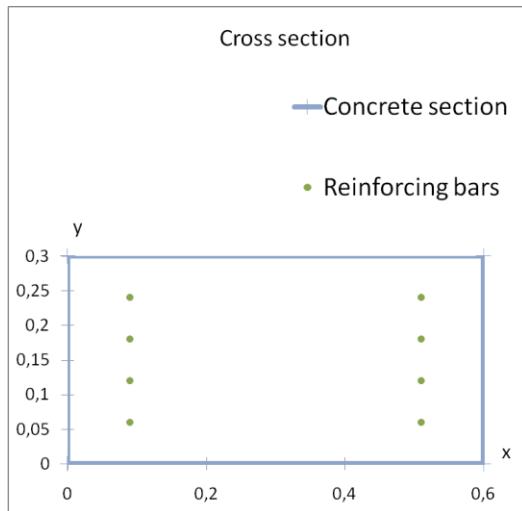
## 17.4 Case 16b

### 17.4.1 Data:

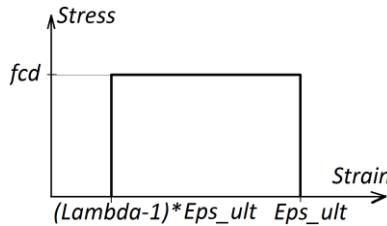
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
  - Reinforcing bars:
 

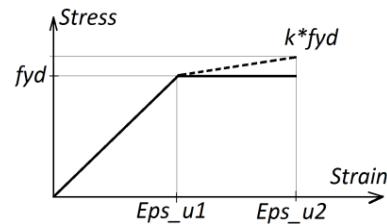
x = 0.09	y = 0.06	$\phi = 0.036$
x = 0.09	y = 0.12	$\phi = 0.036$
x = 0.09	y = 0.18	$\phi = 0.036$
x = 0.09	y = 0.24	$\phi = 0.036$
x = 0.51	y = 0.06	$\phi = 0.036$
x = 0.51	y = 0.12	$\phi = 0.036$
x = 0.51	y = 0.18	$\phi = 0.036$
x = 0.51	y = 0.24	$\phi = 0.036$



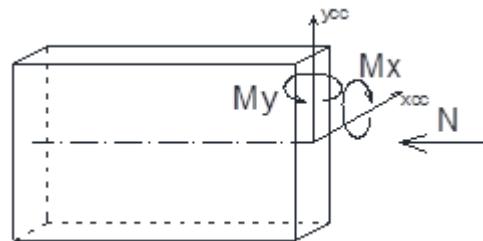
- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 420e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $E_{s,u2} = 0.025$



- Forces:
  - $N = 1700.00e3$
  - $M_x$
  - $M_y$



#### 17.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f = 1700.00e3$ ,  $M_{xf}$ ,  $M_{yf}$
  - Extreme strains in concrete ( $E_{s,top}$ ,  $E_{s,bot}$ )
  - Extreme strains in steel ( $E_{s,top}$ ,  $E_{s,bot}$ )

#### 17.4.3 Results from Component

- Results for reduced section:
  - $N_f = 1700.07 e3$
  - $M_{xf} = 0.00 e3$
  - $M_{yf} = 859.56 e3$
  - $E_{s,top} = 0.003500$
  - $E_{s,bot} = -0.002581$

Eps\_stop = 0.002588  
Eps\_sbot = -0.001669

Results from Component are consistent with a graph from book “*Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2*”, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

## 18 Case 17: Calculation of capacity state in symmetric section for fixed moment

### 18.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ( $M_x$  or  $M_y$ ) and give state of strain and stress at the failure for fixed moment ( $M_x$  or  $M_y$ ) and another moment= 0.

### 18.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ ) (Case 17a) with fixed moment.
- A bending in y direction ( $M_y$ ) (Case 17b) with fixed moment.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )  
Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

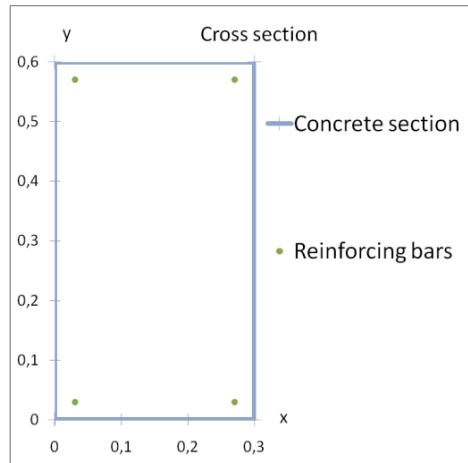
## 18.3 Case 17a

### 18.3.1 Data:

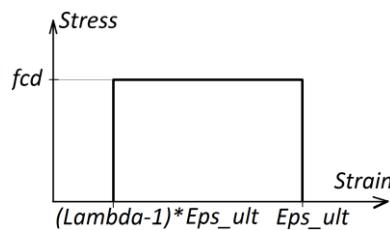
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

x = 0.03	y = 0.03	$\phi = 0.04$
x = 0.03	y = 0.57	$\phi = 0.04$
x = 0.27	y = 0.57	$\phi = 0.04$
x = 0.27	y = 0.03	$\phi = 0.04$

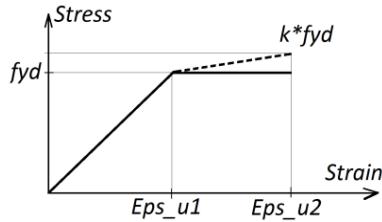


- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$

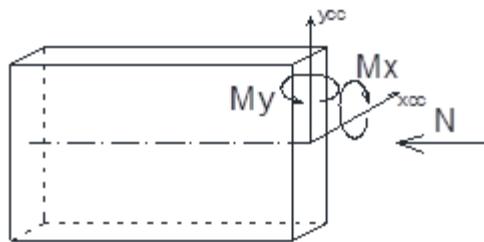


- Steel parameters:
  - Design strength  $f_{yd} = 310 \text{ e6}$
  - Hardening factor  $k = 1.0$

- Modulus of elasticity  $E_s = 200e9$
- Strain ultimate limit  $\epsilon_{u2} = 0.025$



- Forces:
  - $N$
  - $M_x = 184.90e3$
  - $M_y$



### 18.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f, M_{xf} = 184.90 e3, M_{yf}$
  - Extreme strains in concrete ( $\epsilon_{top}, \epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{top}, \epsilon_{bot}$ )

### 18.3.3 Results from Component

- Results for reduced section:
  - $N_f = 3876.03 e3$
  - $M_{xf} = 184.90 e3$
  - $M_{yf} = 0.00 e3$
  - $\epsilon_{top} = 0.003500$
  - $\epsilon_{bot} = 0.000486$
  - $\epsilon_{stop} = 0.003349$
  - $\epsilon_{sbot} = 0.000637$

Results from Component are consistent with a graph from book "Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2", Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 1%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

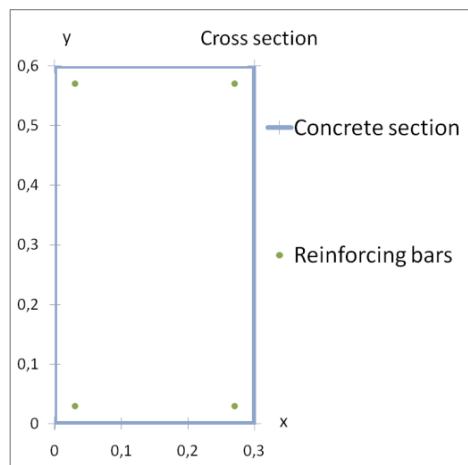
## 18.4 Case 17b

### 18.4.1 Data:

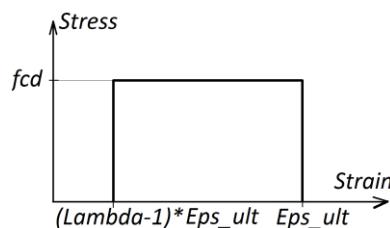
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
  - Reinforcing bars:
 

x = 0.03	y = 0.03	$\phi = 0.032$
x = 0.03	y = 0.57	$\phi = 0.032$
x = 0.27	y = 0.57	$\phi = 0.032$
x = 0.27	y = 0.03	$\phi = 0.032$

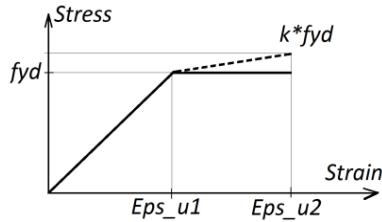


- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$

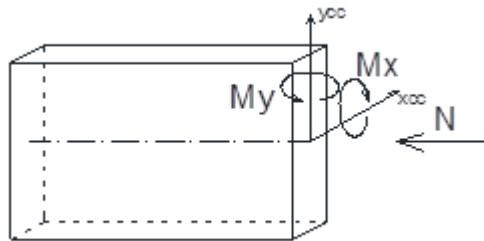


- Steel parameters:
  - Design strength  $f_{yd} = 310\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$

- Strain ultimate limit  $Eps\_u2 = 0.025$



- Forces:
  - $N$
  - $M_x$
  - $M_y = 138.67e3$



#### 18.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f$ ,  $M_{xf}$ ,  $M_{yf} = 138.67e3$
  - Extreme strains in concrete ( $Eps\_top$ ,  $Eps\_bot$ )
  - Extreme strains in steel ( $Eps\_top$ ,  $Eps\_bot$ )

#### 18.4.3 Results from Component

- Results for reduced section:
  - $N_f = 2826.85 e3$
  - $M_{xf} = 0.00 e3$
  - $M_{yf} = 138.67 e3$
  - $Eps\_top = 0.003500$
  - $Eps\_bot = -0.000204$
  - $Eps\_stop = 0.003130$
  - $Eps\_sbot = 0.000167$

Results from Component are consistent with a graph from book "Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2", Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 363., ISBN 83-7125-136-X. Differences between results are smaller than 2%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

## 19 Case 18: Calculation of capacity state in symmetric section for fixed negative (tensioning) axial force

### 19.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (negative N) and give state of strain and stress at the failure for fixed axial force N and  $M_x$  or  $M_y = 0$ .

### 19.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction ( $M_x$ ) (Case 18a)
- A bending in y direction ( $M_y$ ) (Case 18b)
- A fixed negative axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
  - Axial force ( $N_s$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xs}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{ys}$ )
- Results for concrete section:
  - Area of contour under compressive stress ( $A_c$ )
  - Center of gravity of stresses in the concrete ( $x_{cg}$ ,  $y_{cg}$ )
  - Axial force ( $N_c$ )
  - Moment with respect to  $x_{cc}$  axis ( $M_{xc}$ )
  - Moment with respect to  $y_{cc}$  axis ( $M_{yc}$ )
- Results for reduced section:
  - Axial force at the failure related to acting forces ( $N_f = \alpha \cdot N$ )
  - Moment at the failure related to acting forces, with respect to  $x_{cc}$  axis ( $M_{xf} = \alpha \cdot M_x$ )
  - Moment at the failure related to acting forces, with respect to  $y_{cc}$  axis ( $M_{yf} = \alpha \cdot M_y$ )

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
  - Distance between the axis and a gravity center of concrete gross section (dist)
  - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{stop}$ ,  $\epsilon_{sbot}$ )

Where  $x_{cc}$  and  $y_{cc}$  are central axes of concrete gross section parallel to x and y axes.

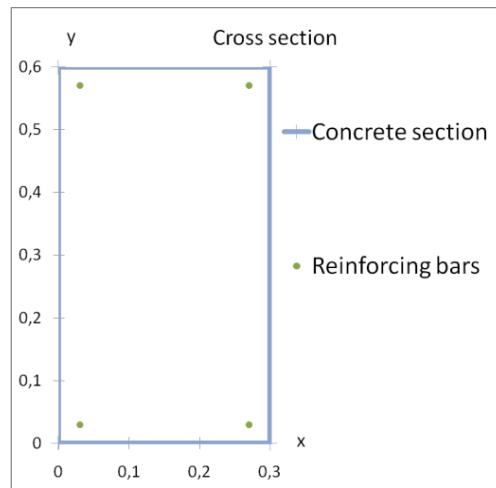
## 19.3 Case 18a

### 19.3.1 Data:

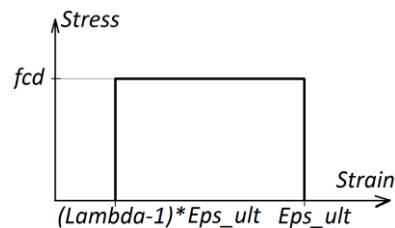
- Geometry:
  - Concrete section:
 

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
- Reinforcing bars:
 

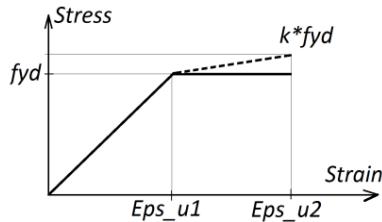
$x = 0.03$	$y = 0.03$	$\phi = 0.04$
$x = 0.03$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.03$	$\phi = 0.04$



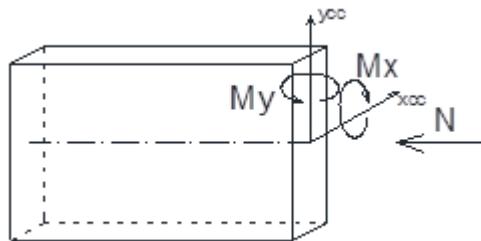
- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 310\text{e}6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200\text{e}9$
  - Strain ultimate limit  $\epsilon_{u2} = 0.010$



- Forces:
  - $N = -493.06 \text{ e}3$
  - $M_x$
  - $M_y$



### 19.3.2 Search for:

- Results for reduced section:
  - Forces:  $N_f = -493.06 \text{ e}3$ ,  $M_{xf}$ ,  $M_{yf}$
  - Extreme strains in concrete ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )
  - Extreme strains in steel ( $\epsilon_{top}$ ,  $\epsilon_{bot}$ )

### 19.3.3 Results from Component

- Results for reduced section:
 

$N_f = -493.05 \text{ e}3$   
 $M_{xf} = -288.16 \text{ e}3$   
 $M_{yf} = 0.00 \text{ e}3$   
 $\epsilon_{top} = 0.001044$   
 $\epsilon_{bot} = -0.010581$   
 $\epsilon_{stop} = 0.000463$   
 $\epsilon_{sbot} = -0.010000$

Results from Component are consistent with a graph from book "Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2", Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 3%. These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.



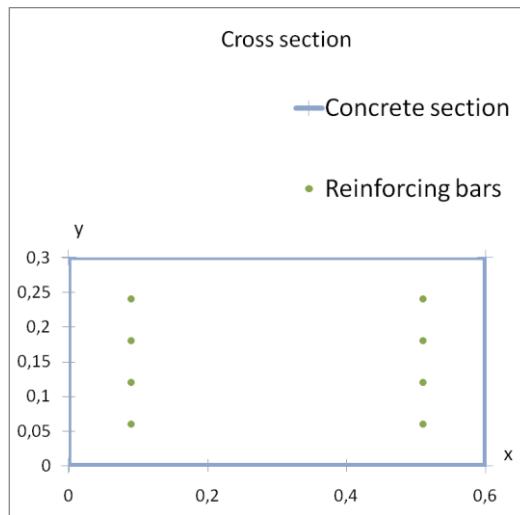
## 19.4 Case 18b

### 19.4.1 Data:

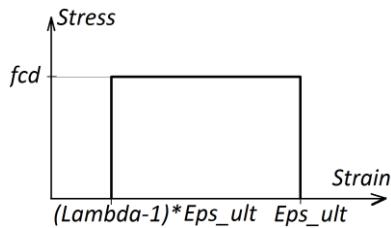
- Geometry:
  - Concrete section:
 

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
  - Reinforcing bars:
 

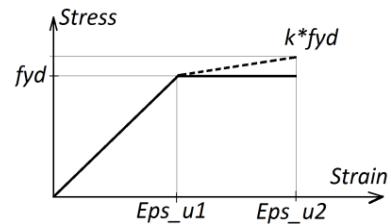
x = 0.09	y = 0.06	$\phi = 0.036$
x = 0.09	y = 0.12	$\phi = 0.036$
x = 0.09	y = 0.18	$\phi = 0.036$
x = 0.09	y = 0.24	$\phi = 0.036$
x = 0.51	y = 0.06	$\phi = 0.036$
x = 0.51	y = 0.12	$\phi = 0.036$
x = 0.51	y = 0.18	$\phi = 0.036$
x = 0.51	y = 0.24	$\phi = 0.036$



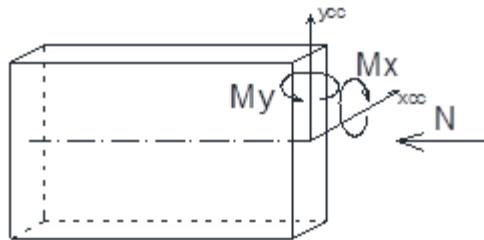
- Concrete parameters:
  - Design strength  $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
  - Effective height reduction factor  $\Lambda = 0.8$
  - Modulus of elasticity  $E_c = 32 \text{ e9}$
  - Strain-stress model: rectangular
  - Strain ultimate limit  $\epsilon_{ult} = 0.0035$



- Steel parameters:
  - Design strength  $f_{yd} = 420e6$
  - Hardening factor  $k = 1.0$
  - Modulus of elasticity  $E_s = 200e9$
  - Strain ultimate limit  $\text{Eps\_u2} = 0.010$



- Forces:
  - $N = -862.85 e3$
  - $M_x$
  - $M_y$



#### 19.4.2 Search for:

- Results for reduced section:
  - Forces:  $N_f = -862.85 e3$ ,  $M_{xf}$ ,  $M_{yf}$
  - Extreme strains in concrete ( $\text{Eps\_top}$ ,  $\text{Eps\_bot}$ )
  - Extreme strains in steel ( $\text{Eps\_top}$ ,  $\text{Eps\_bot}$ )

#### 19.4.3 Results from Component

- Results for reduced section:
 

$N_f = -862.80 e3$   
 $M_{xf} = 0.00 e3$   
 $M_{yf} = 554.30 e3$   
 $\text{Eps\_top} = 0.002908$   
 $\text{Eps\_bot} = -0.012278$   
 $\text{Eps\_stop} = 0.000630$

Eps\_sbot = -0.010000

Results from Component are consistent with a graph from book “*Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2*”, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.