Semi-Nonnegative Matrix Factorization for Motion Segmentation with Missing Data

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1. Introduction

2. Key methods
   - SNMF for motion segmentation problem
   - Segmentation Propagation

Motion Segmentation

Importance

Moving objects segmentation is a precondition for object,action or event recognition.

Complication

Multiple independent motions,transient occlusions among objects,changes in illumination,flexible and/or articulated objects,camera motions.

Challenge

Identifying parts of scenes moving consistently with each other over time.

Problem Definition

- The input to point-based video segmentation is a dense set of points extracted from frames in a video and tracked over time.
- The goal is to group together point tracks corresponding to independently moving objects.
- Grouping Error:two or more points from different objects are grouped together.
- Over-segmentation Error:a single object is split into multiple groups.

SNMF for motion segmentation problem I

Semi-Nonnegative Matrix Factorization(SNMF) based motion segmentation algorithm for object-level segmentation of videos.
Given a video of F frames and P tracked point trajectories,the velocity history matrix X is a $2(F-1) \times P$ matrix:

$$X = \begin{bmatrix} \triangle x_1^1 & \triangle x_2^1 & \cdots & \triangle x_P^1 \\ \triangle y_1^1 & \triangle y_2^1 & \cdots & \triangle y_P^1 \\ \vdots & \vdots & \ddots & \vdots \\ \triangle x_1^{F-1} & \triangle x_2^{F-1} & \cdots & \triangle x_P^{F-1} \\ \triangle y_1^{F-1} & \triangle y_2^{F-1} & \cdots & \triangle y_P^{F-1} \end{bmatrix} \quad (1)$$

where $\triangle x, \triangle y$ are the displacement vectors at coordinate (x, y).The subscripts and superscripts denote the index of the point and the frame number respectively.

SNMF for motion segmentation problem II

Semi-nonnegative matrix factorization(SNMF) factors X into F and G so as to minimize an error function $J(F, G)$:

$$J(F, G) = \min_{F,G} \|(X - FG^T)\|_F, G \geq 0 \tag{2}$$

where X is the velocity history matrix,F is a $2(F-1) \times r$ component matrix representing trajectories,G is an $P \times r$ coefficients matrix.

SNMF models the observed trajectories in X as a sum of components in F,with G providing the relative weights.

The obtained r-column coefficients G serves as the new compact r-dimensional trajectory representation.

SNMF for motion segmentation problem III

Is SNMF a promising framework for the motion segmentation?

The non-negative constraint on coefficients can extract the meaningful "parts" of ensemble data,restricting the linear combination on basis components to be "additive".

In the motion segmentation scenario,the *relative motion* including foreground object motion relative from the camera,the relative motion between different objects or between the sub-parts of specific objects is the natural "parts".

How to handle *missing data*?

Define a $2(F-1) \times P$ indicator matrix W that is $1/0$ at the locations of valid/invalid data.We then seek to minimize:

$$J(F,G) = \min_{F,G} \|W \otimes (X - FG^T)\|_F, G \geq 0 \tag{3}$$

SNMF for motion segmentation problem IV

Initialize F as a random matrix and G as a random positive matrix;

repeat

Updating F by:

$$R = ((W \otimes X)G) \oslash ((W \otimes FG^T)G) \tag{4}$$

$$F = F \otimes R \tag{5}$$

Updating G by:

$$R_1 = ((W^T \otimes X^T)F)^+ + ((W^T \otimes GF^T)F)^- \tag{6}$$

$$R_2 = ((W^T \otimes X^T)F)^- + ((W^T \otimes GF^T)F)^+ \tag{7}$$

$$G = G \otimes R_1 \oslash R_2 \tag{8}$$

until *Converge*;

Algorithm 1: SNMF with missing data

SNMF for motion segmentation problem V

Ambiguity Problem

For any given factorization $X = FG^T$,there exists inherit ambiguity.By normalizing column vectors in F and multiplying back the normalization constant to G,the factorization ambiguity is avoided.

Performance of SNMF

- Produce state-of-the art results on segments of short videos
- Not satisfying when used on longer frame sequences because of frequent local minima.

Segmentation Propagation I

Main Idea

The majority of tracked points last longer than 10 frames,so when SNMF works well on short sequences of data,the segmentation information extracted should be able to be propagated across sliding windows by the majority of interest points tracked through multiple time windows.

1 Do short time SNMF segmentation in every sliding window.$N = n \times t$,N:total number of segmentation,n:number of segmentation per time window,t:number of time window;
2 Construct $N \times N$ affinity matrix by adding up total segmentation co-occurrence within every track;
3 Do Spectral Clustering to make K segmentation groups;
4 Reassign each segmentation with its group labels;
5 Within every track of length l,vote for the most frequent label as track label;

Algorithm 2: Segmentation Propagation