Works 1: Physics (Work in Progress)

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Preface

A Note from the Author

This set of notes are meant for those taking the Singapore Junior Physics Olympiad(SJPO), and have been tuned to suit the level of the competition itself. It also aids as a guide through the different topics present. I am not held viable for any mistakes in the notes. If there are any mistakes though, please email me at jethrokuan95@gmail.com

1 Preliminary Math

In Physics, Mathematics is indispensable. In order to study physics well, one must familiarise himself with the mathematical concepts which generations of brilliant people have created and concisely compiled into the various textbooks you see sold in bookstores today. Without mathematics, physics would be ambiguous; no precise statement could be made to predict the outcome of the various phenomena. For example, if I were to throw a stone up in the air with an initial speed, without the knowledge of the acceleration due to gravity, and the equation $v^2 = u^2 + 2as$, all I could say was "The stone will move upwards and come to a halt". But with the equation, we are able to "exactly" predict the position at which the stone will come to a halt.

For scientific purposes, I shall adopt similar, widely-used notation so this will not confuse the reader himself. In any case, I will present this by showing some physical phenomena, and attempt at converging the ideas of mathematics and physics together to the best of my ability.

I presume that boring algebra, of which I believe the hardest part in relation to physics is the Vieta's relationship between roots, can be skipped. As a result, it should be okay to move on to the slightly counter–intuitive, but important mathematical tool of calculus. Also note that nothing will be cited because all pictures are drawn using the inbuilt picture environment for LaTeX.

1.1 Fundamental Calculus

Single Variable Calculus is often taught to freshman in university, because they feel that it is an essential course in learning many fields of physics, such as quantum mechanics and electromagnetism. And indeed it is. (I suggest taking a look at the courses 18.01 and 18.02 from the MIT Opencourseware. Personally, I felt that they were pretty helpful in building strong foundation)

Of course, it is first important to understand the meaning, both geometrical and physical, of a derivative. A derivative at a point is basically the equation of the line drawn tangent to that point. We first choose two points on a function, $f(x)$. We want to obtain the gradient of the line passing through the two points. This can be easily done using graph paper.

We then start choosing two points closer and closer to each other, the horizontal (a-axis) distance between the two points is a infinitely small. We denote this infinitely small quantity by Δx. As such, we can obtain the gradient of the line by evaluating.

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{1}$$

and of course this is when Δx nears 0 so a more appropriate mathematical themula would be

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{2}$$

This is the definition of a derivative. Since the two points we chose are so close to each other, it is as if the calculated equation would be the tangent of the original equation. And thus,

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{3}$$

From here onwards, It is easy to derive themulas for different special identities. For example, I want to find the derivative of the equation $f(x) = ax^n$ where a and n are constants.

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^n - ax^n}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{a\left(x^n + \begin{pmatrix} n \\ 1 \end{pmatrix} x^{n-1} (\Delta x) + ...\right) - ax^n}{\Delta x}
\end{aligned}
$$

When Δx approaches 0, the terms behind $\begin{pmatrix} n \\ 1 \end{pmatrix} x^{n-1} (\Delta x)$ will approach zero too. Then this would be equal to

$$
\begin{aligned}
lim_{\Delta x \to 0} \frac{a\left(x^n + \begin{pmatrix} n \\ 1 \end{pmatrix} x^{n-1} (\Delta x) + ...\right) - ax^n}{\Delta x} &= \lim_{\Delta x \to 0} \frac{\left(anx^{n-1}\right)(\Delta x)}{\Delta x} \\
&= anx^{n-1}
\end{aligned}
$$

This is the final result. Of course it is good practice to derive the rest, but I feel that one can do it on his own. The other results are shown as below:

$$
\begin{aligned}
\frac{d}{dx}(\sin x) &= \cos x \\
\frac{d}{dx}(\cos x) &= -\sin x \\
\frac{d}{dx}(e^x) &= e^x \\
\frac{d}{dx} \ln f(x) &= \frac{f'(x)}{f(x)}
\end{aligned}
\tag{4}
$$

Note that other logariforms of different bases can be converted to that of base e, and use the above rule to find its derivative.

There are also certain general rules that can be derived using the definition. These are written in the more commonly adopted notation in physics, which are used only for simplicity.

Let u and v both be differentiable functions.

The product rule is as such:

$$(uv)' = u'v + uv' \tag{5}$$

The quotient rule is as such:

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \tag{6}$$

The more often used rule, for complicated functions (within functions) is known as the **chain rule**. It is as such, in general

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} \tag{7}$$

This covers the basis of single variable calculus. Using these rules, any complicated function most can be differentiated.

Calculus has various applications. Calculus is used to approximate values and relate rates. It can also be used in plotting graphs, finding their minima and maxima. The purpose of that previous section is to provide a foundation for calculus, and I will not be going through the applications, except a few.

The Newton-Rhapson Method is important. The idea of the Newton-Rhapson method is as such: we first start out with a reasonable guess; a nice number reasonably close to the true root, and approximate it with a tangent line. We can calculate the x-intercept easily, and this would often be a better approximation than the previous. This cycle is reiterated. Another often easier type of differentiation is the implicit one. This is nothing more than the special application of the chain rule.

For example to differentiate the function $x^2 + y^2 = 25$ to find the slope of the circle when $x = 3$, one could do so explicitly by shifting the x terms over to the other side, however here i present a simpler method.

$$x^2 + y^2 = 25$$

Differentiating both sides

$$\begin{aligned} 2x + 2yy' &= 0 \\ y' &= -\frac{x}{y} \end{aligned}$$

Substituting the value of x and solving for y, we can easily obtain the slope of the circle at $x = 3$.

An important part of single variable calculus is the Taylor series. The taylor series is the representation of a function as a sum of infinite terms, each additional one correcting and "work" toward theming a polynomial close to the original function. It is very useful for approximation purposes. People often take the first few terms. The themula is as such:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{8}$$

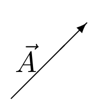
An example of the application of the taylor series is the approximation of the value e^x where we pick $a = 0$

$$1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$$

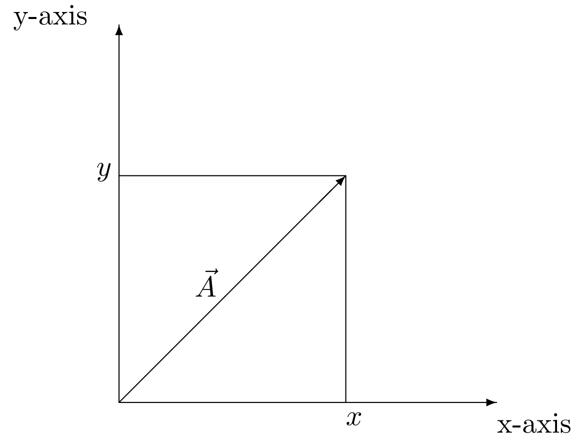
That is about it for single variable calculus, because I do not want to dive into the details. (A single book could be written on single-variable calculus)

1.2 Vectors

Let us move on to some vectors. This is a vector $\vec{A}$:



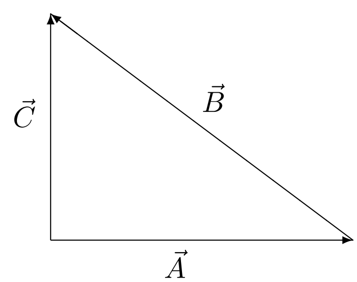
A vector has both magnitude, and direction. A vector can be described by its coordinates.



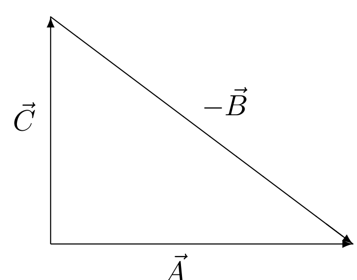
In the above diagram, $\vec{A} = \langle x, y \rangle$

There are some basic rules that are required for computing vector additions, subtractions and products.

Vector addition is simple: $\vec{A} + \vec{B} = \vec{B} + \vec{A} = \vec{C}$



In a similar way, vector subtraction is just the connection between the start point and the end point.



In this similar looking diagram, we see $\vec{A} = \left(-\vec{B}\right) + \vec{C}$

Things get a little tricky when it comes to multiplication. There are two kinds of multiplication for vectors, but they are completely different.

The scalar product, also known as the dot product, produces a scalar quantity. It geometrically represents the the product of the projection of one arrow on another arrow, and the other arrow. It is denoted by a dot, like this: $\cdot$

We then arrive at a formula for calculating a dot product:

Definition 1 (Dot Product)**.**

$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta \tag{9}$$

1.3 Dirac's Bra-ket Notation

No doubt you are now familiar with the world of arrows by now. But in quantum physics the notation often adopted is the bra-ket notation. It is also used in operators such as hamiltonian operators. It is then often important to make the link between the world of arrows, and this bra-ket notation, in linear vector spaces. We start to use this notation, first adopted by Dirac, a physicist, as the first step in weaning the reader away from thinking that vectors are always arrows, which we discussed in the earlier section. We will have to first give a proper definition of a linear vector space:

Definition 2 (Linear Vector Space)**.** A linear vector space denoted by V, is a collection of vectors $\mid A\rangle$, $\mid B\rangle$ and so on called vectors, for which there exists:

(a) A definite rule for theming the vector sum, denoted by $\mid A\rangle + \mid B\rangle$

(b) A definite rule for multiplication by scalars x,y denoted by $x \mid A\rangle$ with the following features:

- The result of these operations is another element of the space, a feature called *closure*: $\mid A\rangle + \mid B\rangle \in V$
- Scalar multiplication is distributive in the vectors: $a\left(|A\rangle + |B\rangle\right) = a\,|A\rangle + a\,|B\rangle$
- Scalar multiplication is also distributive in the scalars: $(a+b)\,|A\rangle = a\,|A\rangle + b\,|A\rangle$
- Addition is commutative: $|A\rangle + |B\rangle = |B\rangle + |A\rangle$
- Addition is associative: $|A\rangle + (|B\rangle + |C\rangle) = (|A\rangle + |B\rangle) + |C\rangle$
- There exists a null vector $|0\rangle$ such obeying $|0\rangle + |A\rangle = |A\rangle$
- For every vector $|A\rangle$ there exists an inverse under addition, $|-A\rangle$, such that $|A\rangle + |-A\rangle = |0\rangle$

Notice, that those axioms come naturally? So there isn't actually a need to remember it. Only remember those that seem to be weird/abnormal.

Definition 3 (Field)**.** The numbers a,b, ... are called the field over which the vector space is defined

If the field consists of real numbers only, it is called the *real vector space.* If they are complex we call it the *complex vector space.*

It is then important to note that a vector itself is neither real nor complex; the adjective only applies to scalars. Also, note that no reference has been made to either magnitude or reference. The point is that while the arrows have these qualities, members of the vector space do not.

The concept of linear dependence is important.

Definition 4. We first consider the linear relation of the them:

$$\sum_{i=1}^{n} a_i\,|i\rangle = |0\rangle \tag{10}$$

Without loss of generality, we can assume none of the elements on the LHS contain a $|0\rangle$.

Then a set of vectors is said to be *linearly independent* if the only linear relation is the trivial one with all $a_i = 0$

This means that as long as the arrows are not parallel to each other, they are linearly independent.

Definition 5 (Dimensions)**.** A vector space has dimensions n if it can contain at most n linearly independent vectors. It will be denoted by $V^n(R)$ if the field is real, and denoted by $V^n(C)$ if the field is complex.

Any vector $|V\rangle$ in an n-dimensional space can be written as a linear combination of the vectors, $|1\rangle, |2\rangle, ..., |n\rangle$

Definition 6 (Basis)**.** A set of n linearly independent vectors in a n-dimensional space is called a *basis*

Therefore, we can write as such:

$$|V\rangle = \sum_{i=1}^{n} v_i |i\rangle \tag{11}$$

where the kets $|i\rangle$ them a basis.

Using this *unique* expansion we can define the addition of kets.

Theorem 1.1 (Additions of Kets)**.** *If*

$$|V\rangle = \sum_i v_i |i\rangle$$

and

$$|W\rangle = \sum_i w_i |i\rangle$$

then

$$|V\rangle + |W\rangle = \sum_i (v_i + w_i) |i\rangle \tag{12}$$

This is equivalent to adding its components.

In a similar way, to multiply a vector by a scalar, it suffices to multiply its components by the scalar, i.e.:

Theorem 1.2 (Multiplication of Kets by a Scalar)**.**

$$a |V\rangle = a \sum_i v_i |i\rangle = \sum_i a v_i |i\rangle$$

Ket Products

The matrix and function examples of kets should have already convinced you that there we can have a vector space with *no* preassigned length or direction for the elements, and as such not look like an arrow.

If you could recall the dot product of a vector (Definition 1)

$$\vec{A} \cdot \vec{B} = |A| |B| \cos\theta \tag{13}$$

The length of the vector **A** using the definitions. We see that this formula for the dot product requires an angle? But in our case, kets might not have "angles" between them. As such, we must adopt the other formula in definition .

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \tag{14}$$

Our goal is to arrive at a similar formula to calculate the product of kets.

We recall again the key features of scalar products between arrowlike vectors.

(a) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

(b) $\vec{A} \cdot \vec{A} \geq 0$ and equal 0 iff $\vec{A} = 0$

(c) $\vec{A} \cdot \left(b\vec{B} + c\vec{C}\right) = b\vec{A} \cdot \vec{B} + c\vec{A} \cdot \vec{C}$

From this, we want to come up with a generalisation which we will call the *inner product* or *scalar product* between two kets $|V\rangle$ and $|W\rangle$. We denote this by the symbol $\langle V|\, W\rangle$.

Definition 7 (Inner product)**.** The inner product of two kets $|V\rangle$ and $|W\rangle$ is denoted by the braket $\langle V|\, W\rangle$

We demand the inner product to follow these axioms:

Theorem 1.3 (Axioms of Inner Products)**.** *The following axioms must hold for inner products:*

(a) $\langle V|\, W\rangle = \langle W|\, V\rangle^*$

(b) $\langle V|\, V\rangle \geq 0$ *and is equal to* 0 *iff* $|V\rangle = 0$

(c) $\langle V|\, a\,|W\rangle + b\,|Z\rangle\,\rangle = a\,\langle V|\, W\rangle + b\,\langle V|\, Z\rangle$

Then similar to arrow vectors, we can say that:

Definition 8 (Orthogonal Kets)**.** Two kets are orthogonal if they are perpendicular, or their inner product vanishes

Definition 9 (Length of a Ket)**.** We shall refer to the length of a ket $|V|$ with the formula $\sqrt{\langle V|\, V\rangle}$

Definition 10 (Orthonormal Basis)**.** A set of basis vectors all of unit norm, which are pairwise orthogonal, are referred to as orthonormal basis

Theorem 1.4 (Formula for Inner Product)**.** *Where*

$$|V\rangle = \sum_i v_i\,|i\rangle$$

and

$$|W\rangle = \sum_j w_j\,|j\rangle$$

$$\langle V|\, W\rangle = \sum_i \sum_j v_i^* w_j\,\langle i|\, j\rangle \tag{15}$$

However, to any further from here, we have to know how to calculate $\langle i|\, j\rangle$ and all we know is that they are linearly independent. However, when we use an orthonormal basis, only some terms survive and we end up with the simple expression, like $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

For a more general case, I shall, at this point, go through the Gram-Schmidt Theorem, as shown in Theorem 1.5.

Theorem 1.5 (Gram-Schmidt)**.** *Given a linearly independent basis we can form linear combinations of the basis vectors to obtain an orthonormal basis*

The reader should attempt to verify the theorem himself.

Assuming the theorem holds, we can assume that the current basis is orthogonal:

$$\langle i|\, j\rangle = \begin{cases} 0 & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases} = \delta_{ij} \tag{16}$$

where δ is the Kronecker delta symbol.

Substituting this back into Equation 15, we get:

$$\langle V|\, W\rangle = \sum_i v_i^* w_i \tag{17}$$

It is this form of the inner product that will be used from now on.

Because the vector is uniquely specified by its components in a given basis, we may, in this basis, write it as a column vector:

$$|V\rangle = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ . \\ . \\ . \\ v_n \end{bmatrix} \text{in this basis} \tag{18}$$

Likewise, we can write $|W\rangle$ as:

$$|W\rangle = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix} \text{in this basis} \tag{19}$$

The inner product $\langle V|\, W\rangle$ can be written as a product of a row and column vector: The transpose conjugate of the column vector representing $|V\rangle$ with the column vector representing $|W\rangle$:

$$\langle V|\, W\rangle = [{v_1}^*, {v_2}^*, \ldots, {v_n}^*] \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix} \tag{20}$$

2 Units and Physical Quantities

Physics is a fundamental science. The study of physics is both challenging and interesting, though occasionally frustrating.

Physics allows you to understand physical phenomena around you, such as mirages etc.

You must first understand that physics is not all about theory. In fact, it is experimental science, and physicists determine formulas based on many assumptions, mostly reasonable, to estimate and predict phenomena.

A theory is first derived, and experimentally shown to hold true. If any evidence shows that this theory does not hold, it is modified, to hold true again. This process repeats until the theory is solid enough to hold for almost all phenomena.

As such, we see that a theory has to be falsifiable. Otherwise this is not science.

2.1 Solving Problems

In their study of Physics, many people complain that they understand the concepts, but are not able to solve problems, or investigate the causes of phenomena. The key lies in *applying* these concepts.

I shall adopt the use of the acronym and method of problem solving from Young and Freedman, which is **I SEE**. Identify, Set up, Execute, Evaluate.

2.2 Identify

Firstly, we must identify the assumptions we have to make in order to solve the problem. For example, if we were to analyse the motion of a ball, we probably assume that air drag is negligible, or constant against speed for a more accurate result. If we were to consider air resistance, then the equations would become really messy.

A key part of physics is simplification. Physicists believe in simplicity. And it is based on this belief that they have been driven to work on finding a theory of everything, and they have come close, proposing a superstring theory(though still with many loopholes, for example having 11 dimensions instead of 4)

We must first identify that the problem deals with the motion of a particle, and thus classify this as a kinematics problem. We will make the assumption that the ball is a point mass to simplify our question.

3 Kinematics

Kinematics deal with the movement of objects, calculating distances, speeds or velocities. This topic is one of the easiest topics so I will not put much detail in it. More information can be found in other textbooks.

3.1 Unidirectional Motion

This section covers motion in **one direction only**. Many parts of this is also applied in projectile motion, which will be covered later.

3.2 Terminology

- Displacement: Usually denoted by x or s. In this set of notes, I will use s. This also means the net(directional) distance from the start point to the end point.
- Velocity: Denoted by v. Velocity is the **rate of change of position**. We usually denote initial velocity with u and final velocity with v.
- Time: Denoted by t.
- Acceleration: Denoted by a. It is the quantitative description of the rate of change in velocity over time.[1]

3.3 Instantaneous and Average

Instantaneous simply means the value at that specific point in time. it is the gradient of the line tangent to the plotted graph. On the other hand, Average is the total average of the motion throughout its journey. Because velocity and acceleration are both **vector** quantities, the average velocity can be found by this equation, $\frac{s-s_\circ}{t}$ and similarly for acceleration, $\frac{v-u}{t}$.

Theorem 3.1 (Average Velocity)**.** *The average velocity of an object is*

$$v_x = \frac{\Delta s}{\Delta t}$$

*Over some time **t**.*

Theorem 3.2 (Instantaneous Velocity)**.** *The instantaneous velocity of an object is*

$$v_x = \lim_{dt \to 0} \frac{ds}{dt}$$

Theorem 3.3 (Average Acceleration)**.** *The average acceleration of an object is*

$$v_x = \frac{\Delta v}{\Delta t}$$

*Over some time **t**.*

Theorem 3.4 (Instantaneous Acceleration)**.** *The instantaneous acceleration of an object is*

$$v_x = \lim_{dt \to 0} \frac{dv}{dt}$$

[1] College Physics, Young and Geller

3.4 Equations

These equations deal with motion **with constant acceleration**. Those without constant acceleration require integration or differentiation to solve, which are not dealt with at the SJPO level, and thus, not touched on in this manuscript.

Theorem 3.5 (The Five Kinematics Equations)**.**

$$\begin{aligned} v(t) &= u + at \quad \textit{(Gives v if t is known)} \\ s(t) &= s_0 + ut + \frac{1}{2}at^2 \quad \textit{(Gives s if t is known)} \\ v^2 &= u^2 + 2as \quad \textit{(Gives v if s is known)} \end{aligned} \tag{21}$$

3.5 Projectile Motion

This section deals with **bidirectional** motion. The object can moves in a direction up, down, left or right. To solve this kind of questions, we usually resolve the components into their particular x and y components with their own magnitude, and solve the simultaneous equations. Drawing free body diagrams, and using Newton's laws to aid in solving the question is also not uncommon.

3.6 A Projectile

A projectile is any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance(which is usually neglected).

As mentioned earlier, the key to solving questions about projectile motion is that ***we can treat the x and y coordinates separately***. We note that the instantaneous acceleration(see section 3.3). The assumption made in most questions about projectile motion is that the only force acting on the projectile is the gravitational force.

Thus, we conclude that in most questions:

Theorem 3.6 (Projectile motion – information in most questions)**.**

$$a_x = 0 \qquad and \qquad a_y = -g = -9.81ms^{-2}$$

*By definition, the gravitational acceleration is **always positive** and thus, the acceleration has to be defined as* $-g$.

3.7 Equations

Theorem 3.7 (Equations for projectile motion, assuming that $a_x = 0 \quad and \quad a_y = -g$)**.** *Considering the x motion, we substitute a_x into and get:*

$$\begin{aligned} v_x &= u_x \\ s_x &= s_{\circ x} + u_x t = s_{\circ x} + v_x t \end{aligned}$$

For the y motion, we substitute $-g$ for a and obtain:

$$\begin{aligned} v_y &= u_y - gt \\ s_y &= s_{\circ y} + u_y t - \frac{1}{2}gt^2 \end{aligned}$$

Usually it is the simplest to take the initial position (at time t=0) as the origin, in this case, $x_\circ$ and $y_\circ$ are both 0.

Now we attempt to resolve the vectors into their various x and y components. This part requires a little knowledge of trigonometry(mainly TOA CAH SOH).

Anyone with basic knowledge of trigonometry can see that:

Theorem 3.8 (Position and Velocity of a Projectile as functions of time t)**.**

$$
\begin{aligned}
s_x &= (v_\circ \cos\theta)t \\
s_y &= (v_\circ \sin\theta)t - \frac{1}{2}gt^2 \\
v_x &= v_\circ \cos\theta \\
v_y &= v_\circ \sin\theta - gt
\end{aligned}
$$

3.8 Uniform Circular Motion

When a particle moves along a curved path, the direction of its velocity changes. Thus, it *must* have a component of acceleration perpendicular to the path, for its speed to be constant.

When a particle moves in a circle with a constant speed, this motion is also called **Uniform Circular Motion**. Examples of uniform circular motion are:

- A car rounding a curve with constant radius at constant speed
- A satellite moving in a circular orbit

The component of acceleration perpendicular to the path causes the direction of the velocity to change, and is related in a simple way to the speed v of the particle and the radius R of the circle.

First, we note that this is a different problem from the projectile–motion situation in section 3.5 in which the acceleration was always straight down and was constant in both magnitude and acceleration. Here the acceleration is perpendicular to the velocity at each instant; as the direction of the velocity changes, the direction of acceleration also changes. The acceleration vector at each point of the path points toward the center of the circle(path).

The formula for acceleration in uniform circular motion is as follows:

Theorem 3.9 (Acceleration in Uniform Circular Motion)**.** *The acceleration of an object in uniform circular motion is* radical*, meaning that it always points to the center of the circle and is perpendicular to the object's velocity $\vec{v}$. We denote it as $\vec{a}_{rad}$; its magnitude a_{rad} is given by:*

$$a_{rad} = \frac{v^2}{R}$$

Because the acceleration of the object is always directed towards the center, the acceleration is also known as the **centripetal acceleration**, which in Latin means *"seeking the center"*

3.9 Relativity in Kinematics

By the word relativity, I do not mean things related to the Lorentz Transformation, which will be discussed later in this manuscript. I mean what something appears to be relative to an object.

The formula is quite intuitive. Imagine a car moving towards you and you are running towards the car, it would seem like the car is moving faster than it is. This is the velocity of the car relative to you.

If two objects are moving towards each other. the velocity are added to become the relative velocity. If they are moving in the same direction, they are subtracted. This is common sense, and I have put it down in layman terms for easy understanding, and this does not act as “model answers” in any test.

4 Newton's Laws of Motion

4.1 Newton's 3 Laws of Motion

How can a tugboat push a cruise ship that's much heavier than the tug? Why is a long distance needed to stop the ship once it is in motion? These are all questions dealing with **dynamics**, the relationship of motion to the forces associated with it.

All the principles of dynamics can be wrapped up in a neat package containing three statements called **Newton's laws of motion**. They were first clearly stated by Sir Isaac Newton(1642 – 1727). More information can be found about him here.

4.2 Force

The concept of **force** gives us a quantitative description of the interaction between two objects or between an object and its environment.

Types of Forces

(a) When a force involves direct contact between two objects, we call it **contact force**.

(b) When an object rests on a surface, there is always a component of force perpendicular to the surface, called the **normal force**, denoted by $\vec{n}$.

(c) There may also be a component of force parallel to the surface, called **friction or frictional force**, denoted by $\vec{f}$.

(d) When a rope or cord is attached to an object and pulled, the corresponding force acting on the object is referred to as the **tension**, denoted by $\vec{T}$.

(e) A familiar force we'll work with often is the gravitational attraction that the earth exerts on an object. This force is the object's **weight** which can be calculated by taking $mass \times weight$

Measuring force Force is a **vector** quantity; to describe a force, we need to describe the direction in which it acts as well as its magnitude – the quantity that tells us "how much" or "how strongly" the force pushes or pulls. The SI unit of the magnitude of force is **the *newton*** abbreviated N. The official definition is based on the standard kilogram, which will be further explained later at section 4.4.

Resultant of Forces Experiment shows that when two forces $\vec{F_1}$ and $\vec{F_2}$ act at the same time on the same point, the effect is the same as the effect of a single force equal to the vector sum of the two forces. This vector sum is often called the **resultant** of the forces or *net force*. The discovery that forces combine according to vector addition is of the utmost importance, in resolving forces later on in this chapter.

Everyone should already know this, if they have read up on **Scalars and Vectors**, not covered in this manuscript.

4.3 Newton's First Law

The fundamental role of a force is to **change the state of motion of the object on which the force acts**. Newton's first law of motion is as follows:

Definition 11 (Newton's First Law of Motion)**.** Every object continues either at rest or in constant motion in a straight line, unless it is forced to change that state by forces acting on it.

When *no* force acts on the object, then the vector sum of forces on it is zero, and the object will continue in that state of motion. **An object acted on by no net force moves with constant velocity(which could be 0) and thus with zero acceleration.**

Inertia The tendency of an object to remain at rest, or to keep moving once it is set in motion, results from a property called *inertia*.

The quantitative measure of inertia is the **physical quantity called mass**, which we will discuss further in section 4.4

Inertial Frames of Reference In our previous discussions when we mentioned relative velocity, we mentioned "relative to" an object. This is a frame of reference. This concept also plays a important role in Newton's laws of motion.

Newton's law is valid is some frames of reference and not in others. A valid one is called the **inertial frame of reference**.

Although it may seem that there's only one inertial frame of reference in the whole universe, but on the contrary, anything that is moving with the same constant velocity as that frame is also valid, and are all inertial frame of references.

There is no inertial frame of reference that is preferred over all others for formulating Newton's Laws. If one frame s inertial, then another frame moving relative to it at constant velocity is also inertial. Both the state and rest and state of uniform motion can occur when the vector sum of forces acting on the object is zero. Because Newton's first law can be used to describe and define what we mean by an **inertial frame of reference**, it is sometimes called the *law of inertia*.

4.4 Mass and Newton's Second Law

We have learnt that an object acted on by a non-zero net force accelerates. We now want to know the relation of the acceleration to the force, which is what Newton's second law of motion states.

When a force with magnitude F acts on an object with mass m, the magnitude a of the object's acceleration is:

$$a = \frac{F}{m}$$

The Si unit of mass is the **kilogram**. We can use the standard kilogram, and the above equation to define the fundamental SI unit for force, the Newton, N:

Definition 12 (Definition of the Newton)**.** One newton is the amount of force that gives an acceleration of 1 meter per second squared to an object with a mass of 1 kilogram. That is,

$$1N = (1kg)(1ms^{-2})$$

We can use this definition to calibrate the spring balances and other instruments to measure forces.

Now we will move on to Newton's second law of motion, which states that:

Definition 13 (Newton's Second Law of Motion)**.** The vector sum (resultant) of all the forces acting on an object equals the object's mass times its acceleration (the rate of change of its velocity):

$$\sum \vec{F} = m\vec{a}$$

Newton's second law is a fundamental law of nature, the **basic relation between force and motion**. The above equation is one that is vector, and has direction. We can separate it into their various components as such:

Theorem 4.1 (Newton's Second Law Component Form)**.**

$$\sum F_x = ma_x \qquad \textit{and} \qquad \sum F_y = ma_y$$

The definition 13 and equation 4.1 are only valid when the mass m *is constant*. It's easy to think of systems whose mass change, like a leaking tank truck.

Like the first law, Newton's second law only holds true if they are in inertial frames of reference (Refer to section 4.3).

4.5 Mass and Weight

We have previously mentioned that the weight of an object is a force – the force of gravitational attraction of the earth. The terms *mass* and *weight* are often mixed up and misused, interchanged in everyday conversation. The two terms are absolutely different, and it is essential to know the differences.

Mass characterises the *inertial* properties of an object. The greater the mass, the greater the force is required to cause a given acceleration. This is reflected from Newton's Second Law (refer to definition 13)

Weight is the force exerted on an object by the gravitational pull of the earth or some other astronomical body. Everyday experience shows us that large object also have large weight.

Definition 14 (Relation of mass to weight)**.** The weight of an object of mass m then magnitude w is equal to the magnitude of acceleration due to gravity, g times the mass:

$$w = mg$$

Because weight is a force, it has a direction, and we can write it in a vector relation:

$$\vec{w} = m\vec{g}$$

4.6 Newton's Third Law

Again, experiments show that whenever two objects interact, the two forces they exert on each other are equal in magnitude and opposite in direction. This is what Newton's third law states.

Definition 15 (Newton's Third Law)**.** For two interacting objects A and B, the formal statement of Newton's third law is:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Newton's own statement, translated from the Latin of the *Principia*, is:

To every reaction there is always opposed and equal reaction; or, the mutual actions of two objects upon each other are always equal, and directed to contrary parts.

4.7 Application of Newton's Laws

Newton's three laws of motion have already been clearly stated above, but how do we apply them? I shall answer this question in the following section. This whole chapter is about solving problems by applying the three laws.

4.8 Equilibrium of a Particle

We learnt previously that an object is in **equilibrium** when it is at rest or moving with constant velocity in an inertial frame of reference. **When an object is at rest or is moving with constant velocity in an inertial frame of reference, the vector sum of all the forces acting on it must be zero**.

Theorem 4.2 (Necessary condition for equilibrium of an object)**.** *For an object to be in equilibrium, the net force acting on it must be zero.*

$$\sum \vec{F} = 0$$

This condition is sufficient only if the object can be treated as a particle, which we assume in the next principle and throughout the remainder of the chapter.

Similar to the way we treated Newton's Second Law(refer to definition 13) we can separate the forces into their individual components:

Theorem 4.3 (Necessary condition for equilibrium of an object: component form)**.**

$$\sum F_x = 0 \qquad \textit{and} \qquad \sum F_y = 0$$

4.9 Application of Newton's Second Law

We're now ready to discuss problems in **dynamics**. Refer to the definition (13) and the component form (formula 4.1) for a recap of Newton's Second Law.

The steps to solving the question are:

(a) Draw a sketch of the physical situation, and identify the moving object or objects to which you will apply Newton's Second Law.

(b) Draw a free-body diagram for each chosen object, showing all the forces, and depicting their individual magnitudes accurately.

(c) Show coordinate axes explicitly to avoid wrong signs.

(d) Solve for the individual x and y components of the forces and write out equations for each chosen object.

(e) Check if your result makes sense.

4.10 Contact forces and friction

Whenever two objects interact by direct contact, frictional force is produced in the opposite direction.

First, when an object rests or slides on a surface, we can always represent the contact force exerted by the surface on the object in terms of components of force perpendicular and parallel to the surface. We call the perpendicular component the **"normal force"**, denoted by $\vec{n}$ (Normal is a synonym for perpendicular). The component parallel to the surface is the *friction force*, denoted by $\vec{f}$. By definition, $\vec{n}$ and $\vec{f}$ are always perpendicular to each other.

The magnitude f_k of a kinetic-friction force usually increases when the normal force magnitude n increases. Thus, more force is needed to slide a box full of books across the floor than to slide the same box when it is empty. In some cases, the magnitude of the sliding friction force f_k is found to be approximately proportional to the magnitude n of the normal force. In such cases, we call the ratio $\frac{f_k}{n}$ the **coefficient of kinetic friction**, denoted as μ_k.

Definition 16 (Relation between kinetic–friction force and normal force)**.** When the magnitude of the sliding friction force f_k is roughly proportional to the magnitude n of the normal force, the two are related by a constant μ_k called the coefficient of static friction:

$$f_k = \mu_k n$$

Because μ_k is the ratio of two force magnitudes, it has no units.

The numerical value of the coefficient of kinetic–friction for any two surfaces depends on the materials and the surfaces.

Friction forces may also act when there is *no* relative motion between the surfaces of contact. If you try to slide a box of books across the floor, the box may not move at all if you don't push hard enough, because the floor exerts an equal and opposite friction force on the box. This force is called a **static-friction force**.

For a given pair of surfaces, the maximum value of f_s depends on the normal force. In some cases, similar to that of kinetic friction, the maximum value of f_s is approximately *proportional* to n; we call this proportionality factor μ_s, the coefficient of static friction.

Definition 17 (Relation between normal force and maximum static–friction force)**.** When the maximum magnitude of the static-friction force can be represented as proportional to the magnitude of the normal force, the two are related by a constant μ_s, called the coefficient of static friction:

$$f_s \leq \mu_s n$$

4.11 Elastic Forces

Again, experiments show that the amount of compression and stretching of a string is directly proportional to the magnitude of the force exerted on the spring. This proportionality was discovered by Robert Hooke (more information can be found about him here.) This simple equation is known as *Hooke's law.*

Theorem 4.4 (Hooke's Law)**.** *For springs, the spring force F_{spr} is approximately proportional to the distance x by which the spring is stretched or compressed:*

$$F_{spr} = -kx$$

In the above equation, k is the positive proportionality constant called the force constant, or sometimes called the **spring constant**, of the spring.

This concludes the chapter, Newton's Law of Motion.

5 Work and Energy

5.1 Introduction

Energy is one of the most important concepts in the world of science. In everyday use, energy is associated with the fuel needed for transportation and heating, with electricity for lights and appliances, and with the foods we consume. These associations, however, don't tell us what energy is, only what it does, and that producing it requires fuel. Our goal in this chapter, therefore, is to develop a better understanding of energy and how to quantify it.

Work has a different meaning in physics than it does in everyday usage. In the physics definition, a programmer does very little work typing away at a computer. A mason, by contrast, may do a lot of work laying concrete blocks. In Physics, work is done **only if an object is moved through some displacement while a force is applied to it**.

Definition 18 (Obtaining Work)**.** The Work W done on an object by a Force of F is given by:

$$W = Fdx$$

where F is the magnitude of the force, dx is the magnitude of the displacement, and $\vec{F}$ and $\vec{x}$ are in the same direction.

The SI unit is **joule(J)** $= newton \circ meter = kg \circ m^2 s^2$

Complications in the definition of work occur when the force exerted on an object is not in the same direction as the displacement. Drawing a vector diagram, we resolve for the component that is parallel, which is $F\cos\theta$. Does, we get a more general definition:

Definition 19 (Obtaining Work – General)**.** The Work W done on an object by a constant force $\vec{F}$ is given by:

$$W = (F\cos\theta)dx$$

where F is the magnitude of the force, dx is the magnitude of the object displacement, and θ is teh angle between the directions of $\vec{F}$ and $d\vec{x}$.

5.2 Kinetic Energy and the Work–energy theorem

Solving problems using Newton's second law(Refer to 4.1) can be difficult if the forces involved are complicated. An alternative is to relate the speed of an object to the net work done on it by external forces.

We know that for a mass m under the action of a constant net force $\vec{F}_{net}$ directed in the same direction, then:

$$W_{net} = F_{net}dx = (ma)dx$$

In the chapter of Kinematics, we have already stated the relationship $v^2 = v_\circ^2 + 2adx$. Then this gives $adx = \frac{v^2 - v_\circ^2}{2}$

Substituting back into the original equation, we get

$$W_{net} = m\left(\frac{v^2 - v_\circ^2}{2}\right)$$

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_\circ^2$$

Then, the net work done on an object equals a change in a quantity of the form $\frac{1}{2}mv^2$

Theorem 5.1 (Calculating Kinetic Energy)**.** *The Kinetic Energy KE of an object of mass m is given by:*

$$KE = \frac{1}{2}mv^2$$

SI Unit = Joule(J)

Then, we can have another formula:

Theorem 5.2 (Calculating Net Work Done)**.** *The net work done on an object W is equal to the change in kinetic energy:*

$$W = KE_f - KE_i = \Delta KE$$

where the change in the kinetic energy is due entirely to the object's change in speed, and is ***independent of mass****.*

5.3 Gravitational Potential energy

An object with kinetic energy can do work on another object, just like moving a hammer can drive a nail through a wall.

Potential energy is a property of a **system**, rather than of a single object. Gravity is a conservative force, and for every conservative force a special expression called a potential energy function can be found.

We apply the definition of work in Equation 19 to get:

$$W_g = Fd\cos\theta = -mg(y_f - y_i)$$

Theorem 5.3 (Gravitational Potential Energy)**.** *The gravitational potential energy of a system consisting of the Earth and an object of mass m near the Earth's surface is given by:*

$$PE = mgh$$

Where g is the gravitational acceleration constant($9.81ms^{-1}$) and h is the vertical height of the Earth's surface.

5.4 Power

The rate at which energy is transferred is important in the design and use of practical devices, such as electrical appliances and engines of all kinds. The issue is particularly interesting for living creatures, since the maximum work per second, or power output, of an animal varies greatly with output duration. Power is defined as such:

Definition 20 (Definition of Power)**.** If an external force is applied to an object and if the work done by this force is in W in the time interval Δt then the **average power** delivered to the object during this interval is the work done divided by the time interval, or:

$$P = \frac{W}{\Delta t}$$

SI unit = Watts, W = J/s.

It is also sometimes useful to rewrite Definition 20 as such:

Theorem 5.4 (Power formula)**.**

$$p = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = Fv$$

This is only true if the force exerted on the object to do work, is constant.

5.5 Conservation of Energy

In a closed system, the total energy is conserved, because energy cannot be created or destroyed.

This means that:

Definition 21 (Conservation of energy)**.** Because the total amount of energy is conserved, the initial kinetic and potential energy adds up to be equal to the final kinetic potential energy, assuming no energy is lost during the process:

$$KE_i + PE_i = KE_f + PE_f$$

There are also other forms of conservations, like conservation of momentum, which will be covered later in the next chapter.

6 Momentum and Impulse

6.1 Momentum

In Physics, momentum has a precise definition. We define momentum as:

Definition 22 (Momentum)**.** The linear momentum $\vec{\rho}$ of an object of mass m with velocity $\vec{v}$ is the product of its mass and velocity:

$$\vec{\rho} = m\vec{v}$$

This means that the momentum is directly proportional to both the mass and velocity of the object. Momentum is a vector quantity and has the same direction as its velocity.

6.2 Impulse

Changing the momentum of an object requires a force. This is, in fact, originally stated in Newton's first law of motion. Then, if a constant net force acts on it, then it is the rate of change of momentum over time, because.

From Newton's Second Law (refer to 4.1)

$$\vec{F} = m\vec{a} = m\frac{dv}{dt} = \frac{dm\vec{v}}{dt} = \frac{\text{Change in momentum}}{\text{Time interval}}$$

This tells us that changing an object's momentum requires the continuous application of a force over a period of time t, leading to the definition of **impulse**:

Definition 23 (Impulse)**.** If a constant force $\vec{F}_{const}$ the impulse $vecI$ delivered to the object over the elapsed time dt is given by:

$$\vec{I} = \vec{F}dt$$

6.3 Conservation of Momentum

When a collision occurs in an isolated system, the total momentum of the system doesn't change with the passage of time. Instead, it remains constant both in magnitude and direction. As such, we call this "theory" the **Conservation of Momentum**.

Definition 24 (Conservation of Momentum)**.** When no net force acts on a system, the total momentum of the system remains constant in time. Thus, in a closed system,

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

6.4 Collisions

We have seen that for any type of collision, the total momentum of the system just before the collision equals the total momentum just after the collision as long as the system may be considered isolated. The total kinetic energy, on the other hand, is generally not conserved in a collision because some of the kinetic energy is converted to internal energy, sound energy, and the work needed to permanently deform the objects involved, such as cars in a car crash. We define **an inelastic collision as a collision in which momentum is conserved, but kinetic energy is not**. The collision of a rubber ball with a hard surface is inelastic, because some of the kinetic energy is lost when the ball is deformed during contact with the surface. When two objects collide and stick together, the collision is called perfectly inelastic. For example, if two pieces of putty collide, **they stick together and move with some common velocity after the collision**. If a meteorite collides head on with the Earth, it becomes buried in the Earth and the collision is considered *perfectly inelastic*. Only in very special circumstances is all the initial kinetic energy lost in a perfectly inelastic collision.

An elastic collision is defined as one in which both momentum and kinetic energy are conserved. Billiard ball collisions and the collisions of air molecules with the walls of a container at ordinary temperatures are highly elastic. Macroscopic collisions such as those between billiard balls are only approximately elastic, because some loss of kinetic energy takes place. For example, in the clicking sound when two balls strike each other. Perfectly elastic collisions do occur, however, between atomic and subatomic particles. Elastic and perfectly inelastic collisions are limiting cases; most actual collisions fall into a range in between them.

In summary,

(a) In an elastic collision, both momentum and kinetic energy are conserved.

(b) In an inelastic collision, momentum is conserved but kinetic energy is not.

(c) In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.

6.5 Perfectly Inelastic Collisions

Because the total momentum of the two-object isolated system before the collision equals the total momentum of the combined-object system after the collision, we can solve for the final velocity using conservation of momentum alone:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Theorem 6.1 (Calculating final velocity in perfectly inelastic collisions)**.** *For two objects each of mass m_1 and m_2 and their initial velocities v_1 and v_2:*

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

It is not necessary to remember this formula, but if it is good for you, then do so as you wish. What you need to understand instead is the concept of conservation of momentum, and the definition of inelastic collisions to obtain the correct figures.

6.6 Elastic Collisions

Now consider two objects that undergo an elastic head-on collision. In this situation, **both the momentum and the kinetic energy of the system of two objects are conserved**. We can write these conditions as:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

and

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

where v is positive if an object moves to the right and negative if it moves to the left.

Problem Solving Strategy for one-dimensional collisions:

(a) **Coordinates**. Choose a coordinate axis that lies along the direction of motion.

(b) **Diagram**. Sketch the problem, representing the two objects as blocks and labeling velocity vectors and masses.

(c) Conservation of Momentum. Write a general expression for the total momentum of the system of two objects before and after the collision, and equate the two. On the next line, fill in the known values.]

(d) Conservation of Energy. If the collision is elastic, write a general expression for the total energy before and after the collision, and equate the two quantities. Fill in the known values. (Skip this step if the collision is not perfectly elastic.)

(e) Solve the equations simultaneously.

6.7 Glancing collisions

In section 6.3 we showed that the total linear momentum of a system is conserved when the system is isolated (that is, when no external forces act on the system). For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the total momentum of the system in each direction is conserved. However, an important subset of collisions takes place in a plane. The game of billiards is a familiar example involving multiple collisions of objects moving on a two-dimensional surface. We restrict our attention to a single two-dimensional collision between two objects that takes place in a plane, and ignore any possible rotation. For such collisions, we obtain two component equations for the conservation of momentum:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

We must use three subscripts in this general equation, to represent, respectively, (1) the object in question, and (2) the initial and final values of the components of velocity. Now, consider a two-dimensional problem in which an object of mass m1 collides with an object of mass m^2 that is initially at rest. After the collision, object 1 moves at an angle θ with respect to the horizontal, and object 2 moves at an angle ϕ with respect to the horizontal. This is called a glancing collision. Applying the law of conservation of momentum in component form, and noting that the initial y-component of momentum is zero, we have x-component:

$$m_1v_{1i} + 0 = m_1v_{1f}cos\theta + m_2v_{2f}cos\phi$$

y component:

$$0 + 0 = m_1v_{1f}sin\theta + m_2v_{2f}sin\phi$$

If the collision is elastic, we can write a third equation, for conservation of energy, in the form

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_1v_{2f}^2$$

If we know the initial velocity v1i and the masses, we are left with four unknowns (v_{1f} , v_{2f} , θ, and ϕ). Because we have only three equations, one of the four remaining quantities must be given in order to determine the motion after the collision from conservation principles alone. If the collision is inelastic, the kinetic energy of the system is not conserved. Then, the above equation does not apply.

Problem Solving Strategy – 2D collisions

(a) **Coordinate Axes**. Use both x- and y-coordinates. It's convenient to have either the x-axis or the y -axis coincide with the direction of one of the initial velocities.

(b) **Diagram**. Sketch the problem, labeling velocity vectors and masses.

(c) **Conservation of Momentum**. Write a separate conservation of momentum equation for each of the x- and y-directions. In each case, the total initial momentum in a given direction equals the total final momentum in that direction.

(d) **Conservation of Energy**. If the collision is elastic, write a general expression for the total energy before and after the collision, and equate the two expressions. Fill in the known values. (Skip this step if the collision is not perfectly elastic.) The energy equation can't be simplified as in the one-dimensional case, so a quadratic expression must be used when the collision is elastic.

(e) Solve the equations simultaneously. There are two equations for inelastic collisions and three for elastic collisions.

7 Rotational Motion

Rotational motion is an important part of everyday life. The rotation of the Earth creates the cycle of day and night, the rotation of wheels enables easy vehicular motion, and modern technology depends on circular motion in a variety of contexts, from the tiny gears in a Swiss watch to the operation of lathes and other machinery. The concepts of angular speed, angular acceleration, and centripetal acceleration are central to understanding the motions of a diverse range of phenomena, from a car moving around a circular race track to clusters of galaxies orbiting a common center.

Rotational motion, when combined with Newton's law of universal gravitation and his laws of motion, can also explain certain facts about space travel and satellite motion, such as where to place a satellite so it will remain fixed in position over the same spot on the Earth. The generalization of gravitational potential energy and energy conservation offers an easy route to such results as planetary escape speed. Finally, we present Kepler's three laws of planetary motion, which formed the foundation of Newton's approach to gravity.

This chapter has striking similarities with that of linear motion(Kinematics). So don't be surprised to see similar equations or formulas, or even definitions.

7.1 Angular Speed and Angular Acceleration

This is a totally new aspect, different from that of linear motion, and deals with motion in a circle. Each of the new concepts have their own analog in rotational motion: *angular acceleration α, angular velocity ω, and angular displacement $d\theta$.*

The **radian**, a unit of angular measure, is essential to the understanding of these concepts. Recall that the distance s around a circle is given by $s = 2\pi r$, where r is the radius of the circle. The radian, can be defined as the arc length s along a circle divided by the radius r:

Definition 25 (Definition of a Radian)**.**

$$\theta = \frac{s}{r}$$

Where s is the circumference of the circle, and r the radius.

One radian is approx. equal to 53°.

Armed with the concept of the radian, we can now discuss angular concepts in physics. The next concept we are going to present is the one on **angular displacement**.

Definition 26 (Definition of Angular Displacement)**.** An object's angular displacement, $d\theta$ is the difference in its final and initial angles.

$$d\theta = \theta_f - \theta_i$$

SI unit: Radian (1 rad).

Note that we use angular variables to describe the rotating disc because each point on the disc undergoes the same angular displacement in any given time interval. Having defined angular displacement, it's natural to define an **angular velocity**:

Definition 27 (Definition of Angular Velocity)**.** The average angular velocity ω_{av} of a rotating rigid object during the time interval dt is defined as the angular displacement $d\theta$ divided by dt:

$$\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{d\theta}{dt}$$

SI unit = rad s^{-1}

Just as changing speed leads to the concept of an acceleration, a change in angular velocity leads to the concept of an **angular acceleration**.

Definition 28 (Angular Acceleration)**.** An object's average angular acceleration α_{av} during the time interval dt is defined as the change in its angular velocity $d\omega$ divided by dt :

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{d\omega}{dt}$$

SI unit: radian per second squared (rad/s^2)

When a rigid object rotates about a fixed axis, as does the bicycle wheel, every portion of the object has the same angular speed and the same angular acceleration. This fact is what makes these variables so useful for describing rotational motion.

7.2 Rotational Motion Under Constant Angular Acceleration

A number of parallels exist between the equations for rotational motion and those for linear motion. These are the similarities among the equations:

Linear Motion with a Constant (Variables: x and v)	Rotational Motion about a Fixed Axis with α Constant (Variables: θ and ω)
$v = v_i + at$	$\omega = \omega_i + \alpha t$
$s = v_i t + \frac{1}{2}at^2$	$d\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2ads$	$\omega^2 = \omega_i^2 = 2\alpha d\theta$

Notice that every term in a given linear equation has a corresponding term in the analogous rotational equation.

7.3 Relations Between Angular and Linear Quantities

Angular variables are closely related to linear variables. From our definition of angular displacement,

$$d\theta = \frac{ds}{r}$$

Dividing both sides of the equation by dt, the time interval during which the rotation occurs, yields

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$$

When dt is very small, the angle $d\theta$ through which the object rotates is also small and the ratio $\frac{d\theta}{dt}$ is close to the instantaneous angular speed ω. On the other side of the equation, similarly, the ratio $\frac{ds}{dt}$ approaches the instantaneous linear speed v for small values of dt. Hence, when dt gets arbitrarily small, the preceding equation is equivalent to:

$$\omega = \frac{v}{r}$$

Theorem 7.1 (Linear and rotational relation)**.** *The formulas for calculating linear quantities from those that are angular, and vice versa, are:*

$$v_t = r\omega$$

$$a_t = r\alpha$$

8 Newtonian Gravitation

Prior to 1686, a great deal of data had been collected on the motions of the Moon and planets, but no one had a clear understanding of the forces affecting them. In that year, Isaac Newton provided the key that unlocked the secrets of the heavens. He knew from the first law that a net force had to be acting on the Moon. If it were not, the Moon would move in a straight-line path rather than in its almost circular orbit around Earth. Newton reasoned that this force arose as a result of an attractive force between Moon and Earth, called the force of gravity, and that it was the same kind of force that attracted objects – such as apples – close to the surface of the Earth.

In 1687, Newton published his work on the law of universal gravitation:

Theorem 8.1 (Force between two objects)**.** *If two particles with masses m_1 and m_2 are separated by a distance r, then a gravitational force acts along a line joining them, with magnitude given by*

$$F = G\frac{m_1 m_2}{r^2}$$

where $G = 6.673 \times 10^{-11} kg^{-1} m^3 s^{-2}$ is a constant of proportionality called the constant of universal gravitation. The gravitational force is always attractive.

8.1 Weight

With this new formula, we can redefine weight, as the total gravitational force exerted on the body by all other bodies in the universe. Fortunately, when the object in consideration is near the Earth, we can neglected the gravitational pull from other celestial bodies, other than Earth.

Theorem 8.2 (Weight of object at Earth's surface)**.**

$$w = mg = F_g = G\frac{M_E m}{{R_E}^2}$$

Then,

Theorem 8.3 (Acceleration of a body at Earth's surface)**.**

$$g = G\frac{M_E}{{R_E}^2}$$

Where R_E is the radius of the Earth, and M_E is the mass of the earth. As stated earlier, G is the gravitational constant.

8.2 Gravitation

We introduced the concept of gravitational potential energy and found that the potential energy associated with an object could be calculated from the equation $PE = mgh$, where h is the height of the object above or below some reference level. This equation, however, is valid only when the object is near Earth's surface. For objects high above Earth's surface, such as a satellite, an alternative must be used, because g varies with distance from the surface.

To find this expression, we consider a body of mass m outside the earth. and first compute the work W_{grav} done by the gravitational force when the body moves directly away from or toward the center of the earth from $r = r_1$ and $r = r_2$.

Then, W_{grav} is given by

$$W_{grav} = \int_{r_1}^{r_2} F_r dr$$

Where F_r is the radial component of the gravitational force $\vec{F}$, the component in the direction outward from the center of the earth.

Because F points directly inward toward the center of the Earth, the force is negative.

$$F_r = -G\frac{M_E m}{r^2}$$

Then,

$$\begin{aligned}
W &= U_1 - U_2 \\
&= -GM_E m \int_{r_1}^{r_2} \frac{1}{r^2} dr \\
&= \frac{-GM_E m}{r_2} - \frac{-GM_E m}{r_1} \\
&= \frac{GM_E m}{r_1} - \frac{GM_E m}{r_2}
\end{aligned}$$

Theorem 8.4 (General Form of Gravitational PE)**.** *The gravitational potential energy associated with an object of mass m at a distance r from the center of Earth is*

$$PE = -G\frac{M_E m}{r}$$

where M_E and R_E are the mass and radius of the Earth respectively, with $r > R_E$

8.3 Escape speeds

If an object is projected upward from Earth's surface with a large enough speed, it can soar off into space and never return. This speed is called Earth's escape speed. (It is also commonly called the escape velocity, but in fact is more properly a speed.) Earth's escape speed can be found by applying conservation of energy. Suppose an object of mass m is projected vertically upward from Earth's surface with an initial speed v_i. The initial mechanical energy (kinetic plus potential energy) of the object, Earth system is given by

$$KE_i + PE_i = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E}$$

We neglect air resistance and assume that the initial speed is just large enough to allow the object to reach infinity with a speed of zero. This value of v_i is the escape speed $v_e sc$. When the object is at an infinite distance from Earth, its kinetic energy is zero, because $v_f = 0$, and the gravitational potential energy is also zero, because 1/r goes to zero as r goes to infinity. Hence the total mechanical energy is zero, and the law of conservation of energy gives

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_e m}{R_E} = 0$$

Thus,

Theorem 8.5 (Finding the escape speed)**.** *The escape speed from earth is given by this equation:*

$$v_esc = \sqrt{\frac{2GM_2}{R_E}}$$

9 Rotational Equilibrium and Rotational Dynamics

In the study of linear motion, objects were treated as point particles without structure. It didn't matter where a force was applied, only whether it was applied or not.

The reality is that the point of application of a force does matter. In football, for example, if the ball carrier is tackled near his midriff, he might carry the tackler several yards before falling. If tackled well below the waistline, however, his center of mass rotates toward the ground, and he can be brought down immediately. Tennis provides another good example. If a tennis ball is struck with a strong horizontal force acting through its center of mass, it may travel a long distance before hitting the ground, far out of bounds. Instead, the same force applied in an upward, glancing stroke will impart topspin to the ball, which can cause it to land in the opponent's court.

9.1 Torque

Forces cause accelerations; *torques* cause angular accelerations. There is a definite relationship, however, between the two concepts.

First, we state the definition of torque:

Definition 29 (Torque)**.** Let $\vec{F}$ be a force acting on an object, and let $\vec{r}$ be a position vector from a chosen point O to the point of application of the force, with perpendicular to $\vec{r}$. The magnitude of the torque $\vec{\tau}$ exerted by the force $\vec{F}$ is given by

$$\tau = rF$$

where r is the length of the position vector and F is the magnitude of the force. SI unit: Newton-meter(Nm)

A more accurate method of calculating torque, is using the cross product. As such note that torque is also a vector.

Theorem 9.1 (Torque: Vector form)**.**

$$\vec{\tau} = \vec{r} \times \vec{F}$$

By convention, **counterclockwise is taken to be the positive direction**, **clockwise the negative direction**. When an applied force causes an object to rotate counterclockwise, the torque on the object is positive. When the force causes the object to rotate clockwise, the torque on the object is negative. When two or more torques act on an object at rest, the torques are added. If the net torque isn't zero, the object starts rotating at an ever-increasing rate. If the net torque is zero, the object's rate of rotation doesn't change. These considerations lead to the rotational analog of the first law: **the rate of rotation of an object doesn't change, unless the object is acted on by a net torque.**

The applied force isn't always perpendicular to the position vector . Thus, we need a more general form of the definition of torque, which applies for all angles:

Definition 30 (General definition of torque)**.** Let $\vec{F}$ be a force acting on an object, and let $\vec{r}$ be a position vector from a chosen point O to the point of application of the force. The magnitude of the torque $\vec{\tau}$ exerted by the force is

$$\tau = rF\sin\theta$$

where r is the length of the position vector, F the magnitude of the force, and θ the angle between $\vec{r}$ and $\vec{F}$. SI unit: Newton-meter (Nm)

Torque is a vector perpendicular to the plane determined by the position and force vectors, as illustrated in Figure 8.4. The direction can be determined by the *right-hand rule*:

(a) Point the fingers of your right hand in the direction of $\vec{r}$.

(b) Curl your fingers toward the direction of vector $\vec{F}$.

(c) Your thumb then points approximately in the direction of the torque.

9.2 Torque and the two conditions for equilibrium

Theorem 9.2 (The two conditions for equilibrium). *An object in mechanical equilibrium must satisfy the following two conditions:*

(a) The net external force must be zero: $\sum \vec{F} = 0$ *(translational)*

(b) The net external torque must be zero: $\sum \vec{\tau} = 0$ *(rotational)*

9.3 Center of Gravity

To compute the torque on a rigid body due to the force of gravity, the body's entire weight can be thought of as concentrated at a single point. The problem then reduces to finding the location of that point. If the body is homogeneous (its mass is distributed evenly) and symmetric, it's usually possible to guess the location of that point.

Consider an object of arbitrary shape lying in the xy-plane. The object is divided into a large number of very small particles of weight $m_1 g$, $m_2 g$, $m_3 g$, ... having coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, ... If the object is free to rotate around the origin, each particle contributes a torque about the origin that is equal to its weight multiplied by its lever arm. For example, the torque due to the weight $m_1 g$ is $m_1 g x_1$, and so forth.

We wish to locate the point of application of the single force of magnitude $w = F_g = Mg$ (the total weight of the object), where the effect on the rotation of the object is the same as that of the individual particles. This point is called the object's center of gravity. Equating the torque exerted by w at the center of gravity to the sum of the torques acting on the individual particles gives

$$(m_1 g + m_2 g + m_3 g + ...) = m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + ...$$

Thus,

$$x_{cg} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + ...}{m_1 + m_2 + m_3 + ...} = \frac{\sum m_i x_i}{\sum m_i}$$

where x_{cg} is the x-coordinate of the center of gravity. Similarly, the y-coordinate and z-coordinate of the center of gravity of the system can be found from

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i}$$

and

$$z_{cg} = \frac{\sum m_i z_i}{\sum m_i}$$

9.4 Relationship between torque and angular acceleration

When a rigid object is subject to a net torque, it undergoes an angular acceleration that is directly proportional to the net torque. This result, which is analogous to Newton's second law, is derived as follows.

$$F_t = ma_t$$

Multiply both sides of the equation by r:

$$F_t r = mra_t$$

Substituting the equation relating tangential and angular acceleration, the above expression gives

$$F_t r = mr^2 a$$

Because torque $\tau = F_t r$, we get

$$\tau = mr^2 a$$

9.5 Rotational Kinetic Energy

We previously defined the kinetic energy of a particle moving through space with a speed v as the quantity $\frac{1}{2}mv^2$.

Let there be a point of mass m_i in a rigid body object. The mass of the rigid body would be the sum of all such point masses.

In addition to that, we know

$$K_i = \frac{1}{2}m_i v_i^2$$

Because we are concerned with the rotational energy, we substitute $v = \omega r$, and obtain

$$K_i = \frac{1}{2}m_i r_i^2 \omega_i^2$$

The total rotational energy would be

$$K_R = \sum_i K_i = \sum_i \frac{1}{2}m_i r_i^2 \omega_i^2$$

ω is constant in a rigid body object, so:

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

We call the quantity, $\left(\sum_i m_i r_i^2\right)$ the **moment of inertia** of the rigid object.

Therefore, somewhat analogously, an object rotating about some axis with an angular speed ω has rotational energy given by $\frac{1}{2}I\omega^2$

Theorem 9.3 (Rotational Kinetic Energy)**.** *Given the inertia I, and angular speed ω, then the rotational kinetic energy is given by:*

$$KE_r = \frac{1}{2}I\omega^2$$

9.6 Calculating Moments of Inertia

The moment of inertia of a system can be obtained by dividing the rigid body into small elements of mass Δm_i.

We can then take the limit where by Δm_i approaches zero, then the sum becomes an integral over the volume of the object.

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

But note $\rho = \frac{\text{mass}}{\text{volume}}$ so $dm = \rho\ dV$

The equation can then be rewritten as:

$$I = \int \rho r^2 dV$$

9.7 Energy Considerations in Rotational Motion

Consider a rigid body pivoted at point O. and there is a point P a distance r away from the point O. Suppose a single force $\vec{F}$ is applied at P, And the work done on the object by $\vec{F}$ to rotate through an infinitesimal distance $ds = r\ d\theta$ is then:

$$dW = \vec{F} \cdot \vec{ds} = (F \sin\phi)\, r\ d\theta$$

Where ϕ is the angle of applied force. We can rewrite work done for the infinitesimal rotation as

$$dW = \tau d\theta$$

Consequently,

$$P = \frac{dW}{dt} = \tau\omega$$

Many other formulas can be derived, but will just be listed, in comparison to the linear equations, for a juxtapose.

Linear Motion with a Constant (Variables: x and v)	Rotational Motion about a Fixed Axis with α Constant (Variables: θ and ω)
$v = dx/dt$	$\omega = d\theta/dt$
$a = dv/dt$	$\alpha = d\omega/dt$
$\sum F = ma$	$\sum \tau_{ext} = I\alpha$
$KE = \frac{1}{2}mv^2$	$K_R = \frac{1}{2}I\omega^2$
$P = Fv$	$P = \tau\omega$
$W = \int_{\theta_i}^{\theta f} \tau d\theta$	$\int_{x_i}^{x_f} F_x dx$
$p = mv$	$L = I\omega$
$\sum \tau = dL/dt$	$\sum F = dp/dt$

9.8 Angular Momentum

Let there be an object of mass m rotates in a circular path of radius r, acted on by a net force, F_{net}. The resulting net torque on the object increases its angular speed from the value $\omega_\circ$ to the value ω in a time interval dt. Therefore, we can write:

$$\sum \tau = I\alpha = I\frac{d\omega}{dt} = I\left(\frac{\omega - \omega_\circ}{dt}\right) = \frac{I\omega - I\omega_\circ}{dt}$$

If we define the product

$$L = I\omega$$

as the **angular momentum** of the object, then we can write:

$$\sum \tau = \frac{\text{angular momentum}}{\text{time interval}} = \frac{dL}{dt}$$

Note that **the net torque acting on an object is equal to the rate of change of the object's angular momentum**.

Theorem 9.4 (Conservation of Angular Momentum). *Let L_s and L_f be the angular momenta of a system at two different times, and suppose there is no net external torque, so $\sum \tau = 0$, then:*

$$L_s = L_f$$

Angular momentum is said to be conserved.

10 Solids and Liquids

There are four known states of matter: solids, liquids, gases, and plasmas. In the universe at large, plasmas's systems of charged particles interacting electromagnetically's are the most common. In our environment on Earth, solids, liquids, and gases predominate.

10.1 States of Matter

Matter is normally classified as being in one of three states: **solid, liquid, or gas**. Often this classification system is extended to include a fourth state of matter, called a **plasma**.

Solids can be classified as either crystalline or amorphous. In a crystalline solid the atoms have an ordered structure. For example, in the sodium chloride crystal (common table salt), sodium and chlorine atoms occupy alternate corners of a cube. In an **amorphous solid**, such as glass, the atoms are arranged almost randomly.

For any given substance, the liquid state exists at a higher temperature than the solid state. The intermolecular forces in a liquid aren't strong enough to keep the molecules in fixed positions, and they wander through the liquid in random fashion. Solids and liquids both have the property that when an attempt is made to compress them, strong repulsive atomic forces act internally to resist the compression.

In the gaseous state, molecules are in constant random motion and exert only weak forces on each other. The average distance between the molecules of a gas is quite large compared with the size of the molecules. Occasionally the molecules collide with each other, but most of the time they move as nearly free, noninteracting particles. As a result, unlike solids and liquids, gases can be easily compressed. We'll say more about gases in subsequent chapters.

When a gas is heated to high temperature, many of the electrons surrounding each atom are freed from the nucleus. The resulting system is a collection of free, electrically charged particles – negatively charged electrons and positively charged ions. Such a highly ionized state of matter containing equal amounts of positive and negative charges is called a **plasma**. Unlike a neutral gas, the long-range electric and magnetic forces allow the constituents of a plasma to interact with each other. Plasmas are found inside stars and in accretion disks around black holes, for example, and are far more common than the solid, liquid, and gaseous states because there are far more stars around than any other form of celestial matter, except possibly **dark matter**.

10.2 Deforming Solids

While a solid may be thought of as having a definite shape and volume, it's possible to change its shape and volume by applying external forces. A sufficiently large force will permanently deform or break an object, but otherwise, when the external forces are removed, the object tends to return to its original shape and size. This is called **elastic behavior**.

For sufficiently small stresses, stress is proportional to strain, with the constant of proportionality depending on the material being deformed and on the nature of the deformation. We call this proportionality constant the elastic modulus:

Definition 31 (Elastic Modulus)**.**

$$\text{stress} = \text{elastic modulus} \times \text{strain}$$

The elastic modulus is analogous to a spring constant. It can be taken as the stiffness of a material: A material having a large elastic modulus is very stiff and difficult to deform. There are three relationships having the form of Definition 10.2, corresponding to tensile, shear, and bulk deformation, and all of them satisfy an equation similar to Hooke's law for springs:

Theorem 10.1 (Hooke's Law)**.**

$$F = kdx$$

10.3 Young's Modulus: Elasticity in Length

The word tensile has the same root as the word tension. The SI unit of stress is the newton per square meter (N/m^2), called the pascal (Pa):

Definition 32 (Pascal)**.**

$$1Pa = 1N/m^2$$

The tensile strain in this case is defined as the ratio of the change in length dL to the original length $L_\circ$ and is therefore a dimensionless quantity. Using Definition 10.2, we can write an equation relating tensile stress to tensile strain:

Theorem 10.2 (Calculating Tensile Stress)**.**

$$\frac{F}{A} = Y\frac{dL}{L_\circ}$$

In this equation, Y is the constant of proportionality, called **Young's modulus**. It could be solved for F and put in the form $\mathbf{F} = \mathbf{kdL}$, where $k = YA/L_\circ$, making it look just like Hooke's law.

A material having a large Young's modulus is difficult to stretch or compress. This quantity is typically used to characterize a rod or wire stressed under either tension or compression. Because strain is a dimensionless quantity, Y is in pascals.

10.4 Shear Modulus: Elasticity in Shape

Another type of deformation is when an object is subjected to a force $\vec{F}$ *parallel* to one of its faces while the opposite is held fixed by a second force, usually friction.

This kind of friction is usually called **shear stress**

We define this shear stress as:

Definition 33 (Shear Stress)**.** $\frac{F}{A}$, the ratio of the magnitude of the parallel force to the area A being sheared. The shear strain is the ratio $\frac{dx}{h}$, where dx is the horizontal distance the sheared face moves and h is the height of the object.

Theorem 10.3 (Calculating Shear Stress)**.**

$$\frac{F}{A} = S\frac{dx}{h}$$

Under this kind of stress, there is no volume deformation.

10.5 Bulk Modulus: Volume Elasticity

Suppose that the external forces acting on an object are all perpendicular to the surface on which the force acts and are distributed uniformly over the surface of the object. This occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape.

Definition 34 (Bulk Modulus)**.** The volume stress dP is defined as the ratio of the magnitude of the change in the applied force dF to the surface A

Theorem 10.4 (Calculating Bulk Modulus)**.**

$$dP = -B\frac{dV}{V}$$

10.6 Density and Pressure

Equal masses of aluminium and gold have different volume distance.

This is due to the difference, the concept of density

Definition 35 (Density)**.** The density ρ is equal to the mass divided by its volume

$$\rho = \frac{m}{V}$$

Another important aspect of this subject is the specific gravity, which is defined as follows:

Definition 36 (Specific Gravity)**.** The specific gravity of a substance is the ratio of its density to the density of water at $4°C$, which is $1.0 \times 10^3 kgm^{-3}$

We also define pressure here, just in case (:

Definition 37 (Pressure)**.** If F is the magnitude of a force exerted perpendicular to a given surface of area A, then the pressure P is equal to the Force divided by the Area,

$$P = \frac{F_{perpendicular}}{A}$$

SI unit: Pascal (Pa)

The formula for calculating the pressure under the water can be given by:

Theorem 10.5 (Calculating Pressure at different depths)**.**

$$P = \rho gh$$

Where P is the pressure, ρ is the density of the fluid, g is the gravitational acceleration, and h is the depth under water.

Because pressure in a fluid depends on depth, and on the value of P_o, any increase in pressure at the surface must be transmitted to every point in the fluid.

French scientist Pascal took this understanding, and came up with a principle:

Definition 38 (Pasca"s Principle)**.** A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid

This principle is largely applied in hydraulic presses. Because fluid pressure is always equal, then let there be two pressure at different points. $P_1 = P_2$ at all times. Then it follows that $F_1/A_1 = F_2/A_2$, then the magntiude of $\vec{F_2}$ is larger than $\vec{F_1}$ by a factor of A_2/A_1. This is idea used in lifting heavy items.

10.7 Buoyant Forces and Archimedes' Principle

A fundamental principle affecting objects submerged in fluids was discovered by the Greek Mathematician Archimedes. Archimedes' principles can be stated as follows:

Definition 39 (Archimedes Principle)**.** Any object completely or partially submerged in a fluid is buoyed up by a force with magnitude **equal to the weight** of the fluid displaced by the object

Water provides partial support to any object placed in it. This force is called the **buoyant force**.

We can actually derive this from the first equation, on deriving pressure from . Because horizontal forces cancel, and in the vertical direction P_2A acts upwards on the bottom of the block of fluid and P_1A and the gravity force on the fluid, Mg, act downwards, giving

$$B = P_2A - P_1A = Mg$$

The buoyancy force can be identified as a difference in pressure equal in magnitude to the weight of the displaced fluid.

Using the definition of density, we get the equation:

$$B = \rho_{fluid} V_{fluid} g$$

where ρ_{fluid} is the density of the liquid and V_{fluid} is the **volume of fluid displaced**

It is also good to note the different case breakdowns in the analysis of buoyant force:

(a) Fully submerged objects

When an object is completely submerged in the fluid, the the volume of water displaced is the volume of the object itself, and thus, the Buoyant force experienced, $B = \rho_{fluid} V_{obj} g$ where V_{obj} is the volume of the object.

As a result, if the density of the object is less than the density of the fluid, the net force exerted on the object is positive(upward) and the object accelerates upward.

If the density of the object is greater than the density of the fluid, it will have a negative net force, and accelerate downwards

(b) Floating objects

We assume that the object is in static equilibrium floating in a fluid.

The upward buoyant force is balanced by the downward force of gravity acting on the object. If V_{fluid} is the volume of liquid displaced by the object, then the magnitude of the buoyant force is given by $B = \rho_{fluid} V_{fluid} g$. Because the weight of the object is $w = mg = \rho_{obj} V_{obj} g$, and because w = B, it follows that:

$$\frac{\rho_{obj}}{\rho_{fluid}} = \frac{V_{fluid}}{V_{obj}}$$

Note that buoyant fore of the air is neglected, but is insignificant, since the density of air is low at sea level.

10.8 Fluids in Motion

When a fluid is in motion, its flow can be characterized in one of the two ways. The flow is said to be **streamline**, or **laminar**

In discussions of fluid flow, the term **viscosity** is used often, to describe the degree of internal friction of the fluid.

The most important part of this section is the **Law of Continuity** and I guess I shall go straight to the point.

10.9 Law of Continuity

This law is usually used and applied in tubes containing fluid.

For the case of an incompressible liquid, which is often assumed to be so, the equation that is always followed is:

Definition 40 (Law of Continuity)**.**

$$A_1v_1 = A_2v_2$$

Where A represents the cross-sectional area of the tube, and v representing the velocity of the fluid

In other words, the condition Av is always a constant throughout a closed tube.

Following this discovery, Scientist Bernoulli also discovered an amazing finding. He managed to relate the pressure of a flujid to its speed and elevation. He then called this equation the **Bernoulli's equation**.

10.10 Bernoulli's Equation

As stated earlier, this equation relates the pressure of a fluid to the speed and elevation.

I shall go through the derivation of this equation with you. Before this, I must make some assumptions. As before, we assume that the fluid is incompressible, non viscous and flows in a steady state manner.

Consider the flow of a liquid in a non-uniform tube, and consider the time period dt. The force on the lower end of the tube is P_1A_1 (recall that $P = \frac{F}{A}$ by definition), where P_1 is the pressure at the lower end.

Then the work done on the lower end of the fluid by the fluid behind it is:

$$W_1 = F_1dx_1 = P_1V$$

where V is the volume of the initial region of the fluid. In a similar manner, the work done on the upper region at the time dt is

$$W_2 = -P_2A_2dx_2 = -P_2V$$

This is volume of the liquid is the same because by the previous equation, the law of continuity(Refer to 10.9)

The net work done by these forces in the time dt is

$$W_{fluid} = P_1V - P_2V$$

Part of this work goes into changing the fluid's kinetic energy, and part goes into changing the gravitational potential energy of the fluid–earth system.

If m is the mass of the fluid passing through the pipe in the time interval dt, then the change in kinetic energy of the fluid is

$$dKE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The change in the gravitational potential energy is

$$dPE = mgy_2 - mgy_1$$

Because the net work done by the fluid on the segment of fluid shown changes the KE and GPE only, then:

$$W_{fluid} = dKE + dGPE$$

$$P_1V - P_2V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by V and recall that $\rho = \frac{m}{V}$, then we get

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}v_1^2$$

Rearranging, we get:

$$P_1 + frac12\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho b_2^2 + \rho gy_2$$

This is **Bernoulli's equation**, often expressed as

Theorem 10.6 (Bernoulli's equation)**.**

$$P + \frac{1}{2}\rho v^2 + \rho gy = constant$$

The proper definition of the equation is:

Definition 41 (Bernoulli's equation)**.** Bernoulli's equation states that the sum of the pressure P, the kinetic energy per unit volume, $\frac{1}{2}\rho v^2$, and the potential energy per unit volume, ρgy has the same value at all points along a streamline

10.11 Surface Tension

This section will be ignored for the moment, since it is not in syllabus.

This concludes our chapter on Solids and Fluids.

11 Thermal Physics

Thermal physics is the study of temperature, heat, and how they affect matter. Quantitative descriptions of thermal phenomena require careful definitions of the concepts of temperature, heat, and internal energy. Heat leads to changes in internal energy and thus to changes in temperature, which cause the expansion or contraction of matter.

Gases are critical in the harnessing of thermal energy to do work. Within normal temperature ranges, a gas acts like a large collection of non-interacting point particles, called an ideal gas. Such gases can be studied on either a macroscopic or microscopic scale. On the macroscopic scale, the pressure, volume, temperature, and number of particles associated with a gas can be related in a single equation known as the ideal gas law. On the microscopic scale, a model called the kinetic theory of gases pictures the components of a gas as small particles. This model will enable us to understand how processes on the atomic scale affect macroscopic properties like pressure, temperature, and internal energy.

11.1 Temperature and the 0^{th} Law of Thermodynamics

When placed in contact with each other, two objects at different initial temperatures will eventually reach a common intermediate temperature. If a cup of hot coffee is cooled with an ice cube, for example, the ice rises in temperature and eventually melts while the temperature of the coffee decreases. Understanding the concept of temperature requires understanding thermal contact and thermal equilibrium. Two objects are in thermal contact if energy can be exchanged between them. Two objects are in thermal equilibrium if they are in thermal contact and there is no net exchange of energy. The exchange of energy between two objects because of differences in their temperatures is called heat. Using these ideas, we can develop a formal definition of temperature. Consider two objects A and B that are not in thermal contact with each other, and a third object C that acts as a thermometer's a device calibrated to measure the temperature of an object. We wish to determine whether A and B would be in thermal equilibrium if they were placed in thermal contact. The thermometer (object C) is first placed in thermal contact with A until thermal equilibrium is reached, whereupon the reading of the thermometer is recorded. The thermometer is then placed in thermal contact with B, and its reading is again recorded at equilibrium. If the two readings are the same, then A and B are in thermal equilibrium with each other. If A and B are placed in thermal contact with each other, there is no net transfer of energy between them. We can summarize these results in a statement known as the **zeroth law of thermodynamics (the law of equilibrium)**:

Definition 42 (Zeroth Law of Thermodynamics)**.** If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other.

11.2 Ideal Gases

The properties of gases are important in a number of processes. It is important to note that these gases are asuumed to have these certain properties, and are thus considered ideal gases.

The equation of state can be very complicated, but is found experimentally to be relatively simple if the gas is maintained at low pressure.

A gas usually consists of a very large number of particles, so it is convenient to express the amount of gas in a given volume in terms of the number of moles, n. One mole of gas contains a fixed number of particles, and this number is the *Avogadro's Constant*

Definition 43 (Avogadro's Constant)**.**

$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

Ideal gases follow the ideal gas law, which summarizes three scientist's experimental findings. They have found that if a cylinder contains an ideal gas, and the cylinder does not leak, then the number of moles of gas remained constant. If a gas is kept at constant temperature, then the pressure is inversely proportional to its volume(Boyle's Law) When the pressure of a gas is kept constant, the volume of the gas is directly proportional to the temperature (Charles' law). When the volume is kept constant, the pressure is directly proportional to the temperature(Gay-Lussac's Law). These different laws were all summarized to form the ideal gas law, stated below.

11.3 The Ideal Gas Law

Definition 44 (The Ideal Gas Law)**.**

$$PV = nRT$$

R is the constant for a specific gas that must be determined from experiments. As the pressure approaches zero, R becomes the same value for all gases: $R = 8.31 J/molK$

As previously stated, the number of molecules contained in one mole of any gas is the Avogadro's number, $N_A = 6.02 \times 10^{23}$ particles/mole, so

$$n = \frac{N}{N_A}$$

where n is the number of moles and N is the number of molecules in the gas. With the above equation, we can rewrite the ideal gas law in terms of the total number of molecules as

$$PV = nRT = \frac{N}{N_A} RT$$

or

$$PV = N k_B T$$

where

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-}23 J/K$$

is **Boltzmann's constant** this reformation is useful to relate the temperature of a gas to the average kinetic energy of particles in a gas.

11.4 The Kinetic Theory of Gases

Using the previous model of an ideal gas, we will describe the **kinetic theory of gases**. With this theory we can interpret the pressure and temperature of an ideal gas in terms of microscopic variables. The kinetic theory of gases model makes the following assumptions:

(a) **The number of molecules in the gas is large, and the average separation between them is large compared with their dimensions.** The fact that the number of molecules is large allows us to analyse their behaviour statistically.

(b) **The molecules obey Newton's laws of motion, but as a whole they move randomly.** By "randomly" it is meant that any molecule can move in any direction of equal probability.

(c) **The molecules interact only though short-range forces during elastic collisions.** This assumption is consistent with the ideal gas model.

(d) **The molecules make elastic collisions with the walls**

(e) **All molecules in the gas are identical**

11.5 Molecular Model for the Pressure of an Ideal Gas

This would be our first application of the kinetic theory. We first assume N molecules in a container of volume V. We use m to represent the mass of one molecule. The container is a cube with edges of length d.

After colliding elastically with a wall. The molecule with originally a momentum of +mv changes to -mv, assuming that it was initially moving in the positive x direction.

$$dp_x = mv_x - (-mv_x) = 2mv_x$$

Then if F_1 is the magnitude of the average force exerted by a molecule of the wall in the time dt, then applying Newton's second law to the wall gives:

$$F_1 = \frac{2mv_x}{dt} = \frac{2mv_x}{dt}$$

In order for the molecule to make two collisions with the same wall, it must travel a distance 2d along the x-direction in a time dt. Therefore the time interval between two collisions with the same wall is $dt = \frac{2d}{v_x}$ and the force imparted to the wall by a single molecule is

$$F_1 = \frac{2mv_x}{dt} = \frac{2mv_x}{2d/v_x} = \frac{mv_x^2}{d}$$

The total force F exerted by all the molecules on the wall is found by adding the forces exerted by the individual molecules:

$$F_{total} = \frac{m}{d}(v_{1x}^2 + v_{2x}^2 + \cdots)$$

In this equation, v_{1x} is the x-component of velocity of molecule 1. Note that the average value of the square of the velocity in the x-direction is

$$\bar{v_x^2} = \frac{v_{1x}^2 + v_{2x}^2 + \cdots}{N}$$

where $\bar{v_x^2}$ is the average value of v_x^2. The total force on the wall can then be written as:

$$F = \frac{Nm}{d}\bar{v_x^2}$$

Now we focus on one molecule in the container traveling in some arbituary direction with velocity $\vec{v}$ and having components, v_x, v_y and v_z. In this case, we must express the total force on the wall in terms of the speed of the moelcules rather than just a single component. The Pythagorean theorem relates the square of the speed to the square of these components according to the expression $v^2 = v_x^2 + v_y^2 + v_z^2$. Hence, the average value of v^2 for all the molecules in the container is related to the average values $\bar{v_x^2}$, $\bar{v_y^2}$ and $\bar{v_z^2}$ according to the expression $\bar{v^2} = \bar{v_x^2} + \bar{v_y^2} + \bar{v_z^2}$. Because the motion is completely random, the average values of $\bar{v_x^2}$, $\bar{v_y^2}$ and $\bar{v_z^2}$ are equal to each other. Using this fact and the earler equation for $\bar{v_x^2}$, we find that

$$\bar{v_x^2} = \frac{1}{3}\bar{v^2}$$

The total force on the wall, then is

$$F = \frac{N}{3}\left(\frac{m\bar{v^2}}{d}\right)$$

This equation allows us to find the total pressure exerted on the wall by dividing the force by the area:

$$\frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\left(\frac{N}{d^3}m\bar{v^2}\right) = \frac{1}{3}\left(\frac{N}{V}\right)m\bar{v^2}$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\bar{v^2}\right)$$

The above equation says that **the pressure is proportional to the number of molecules per unit volume and to the average translational kinetic energy of a molecule, $\frac{1}{2}\mathbf{m\bar{v^2}}$.**

11.6 Molecular Interpretation of Temperature

Having related the pressure of a gas to the average kinetic energy of the molecules, we can now relate temperature to a description of the gas.

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\bar{v^2}\right)$$

Comparing this equation with the equation of state for an ideal gas stated earlier, ($PV = Nk_BT$), we note that the left-hand sides are identical. Then we get:

$$T = \frac{2}{3k_B}\left(\frac{1}{2}m\bar{v^2}\right)$$

This means that **the temperature of a gas is a direct measure of the average molecular kinetic energy of a gas**

Rearranging the above equation, we get:

$$\frac{1}{2}m\bar{v^2} = \frac{3}{2}Nk_BT$$

So the average translational kinetic energy per molecule is $\frac{3}{2}Nk_BT$. The total translational energy of N molecules of gas is simply N times the average energy per molecule,

$$KE_{total} = N\left(\frac{1}{2}m\bar{v^2}\right) = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

where we earlier used $k_B = \frac{R}{N_A}$ for Boltzmann's constant and and $n = \frac{N}{N_A}$ for the number of moles in a gas. From this result, we see that **the total translational kinetic energy of a system of molecules is proportional to the absolute temperature of the system.**

$$U = \frac{3}{2}nRT \text{ (monoatomic gas)}$$

For diatomic and polyatomic molecules, additional possibilities of energy storage are available in the vibration and rotation of the molecule.

The square root of $\bar{v^2}$ is called the **root–mean–square** (rms) speed of the molecules. We get

$$v_{rms} = \sqrt{\bar{v^2}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

where M is the molar mass in *kilograms per mole* if R is given in SI units.

12 Energy in Thermal Processes

In this chapter, we will discuss temperature changes(energy transfer) in thermal processes.

Experiments showed to a certain extent that the conservation of energy rule seemed to apply only for certain kinds of mechanical systems. Experiments conducted showed that the sum of the gravitational energy and kinetic energy was not constant. Some of it were converted to heat energy, rising the temperature of the object. We will deal with this scenarios in this chapter.

12.1 Heat and Internal Energy

There is a difference between the terms internal energy, and heat. In fact, the differences are major, so major that the terms internal energy and heat cannot be used interchangeably.

Heat is the transfer of energy between a system and the environment, because of a temperature difference between them.

Internal energy U is the energy associated with the microscopic components of a system's the atoms and molecules of the system. The internal energy includes kinetic and potential energy associated with the random translational, rotational, and vibrational motion of the particles that make up the system, and any potential energy bonding the particles together.

The calorie had been defined as **the amount of energy required to raise the temperature of water by** $1°C$

$$1cal = 4.186J$$

Scientists agreed that since heat was a direct measure of the transfer of energy, As such, the SI unit should be Joule.

12.2 Specific Heat

Previously, the historical definition of the calorie was the amount of energy necessary to raise the temperature of a specific substance – water – by 1 degree. This amount was $4.186J$. Rasing the temperature of different substances by one degree would require different amount of energy.

Definition 45 (Specific Heat)**.** If a quantity of energy Q is transferred to a substance m, changing its temperature from T_i to T_f, the **specific heat** of the substance, denoted by c, is defined by

$$c = \frac{Q}{m\left(T_f - T_i\right)}$$

From the definition of specific heat, we can easily see that the energy Q needed to raise the temperature of a system of mass m by temperature Δt is:

Theorem 12.1 (Energy Required to Raise Temperature)**.**

$$Q = mc\Delta T$$

A method to determine the specific heat of a substance is **calorimetry**. This process involves heating up a substance, and then putting it into cold water. Assuming that the system is isolated, it is possible to calculate the amount of energy invested into changing the temperature of the substance, and thus use this information to calculate the specific heat.

12.3 Latent Heat and Phase Change

We use the term **phase** to describe a specific state of matter, such as solid, liquid and gas. The compound H_2O exists as ice in the solid phase, water in the liquid phase, and steam in the gaseous state. A transistion from one phase to another is called *phase transition*.

We know that it takes energy to change from one state to another. This is obvious because at the phase transistion, though there is no temperature change, energy is constantly invested in this process. As such this process consumes energy. Physicists aim to quantify this amount. They call the amount of energy required per unit mass for the phase transistion, **heat of fusion** or **latent heat of fusion**, and this is denoted by L_f.

By the definition of the value L_f, the amount of energy Q required, then, to change the state of a substance of mass m, is then

Theorem 12.2 (Energy Required for Phase Transistion)**.**

$$Q = mL_f$$

12.4 Mechanisms for Heat Transfer

We have talked about conductors and insulators, materials that permit or prevent heat transfer between bodies. In this section, we will investigate the mechanisms in which heat is transferred, and the rate of which it is transferred.

The three mechanisms for heat transfer are:

(a) Conduction

Occurs within a body or between two bodies in contact

(b) Convection

Depends on the motion of mass from one region of space to another (such as hot air rises)

(c) Radiation

Heat transfer by electromagnetic radiation

We will now go through each single mechanism more in depth.

12.5 Conduction

On the atomic level, atoms in the hotter regions have higher kinetic energy, than those atoms at the cooler end. As such these atoms jostle and transfer some of their energy to the neighbours. These neighbours continue the transfer of energy to their cooler neighbouring atoms. This happens until energy will no longer be transferred (i.e. they are in thermal equilibrium). This process is called **conduction**.

When a quantity dQ is transferred over in a time interval of dt, the rate of heat flow is $\frac{dQ}{dt}$. This is also known more commonly by scientists as **heat current**, often denoted by H. Experiments show that heat current is proportional to the cross–sectional area A, to the temperature difference $T_H - T_C$ and is inversely proportional to the length of the rod L.

From these experiments, we can obtain a relationship between heat current and all the above parameters. We now introduce a constant, the thermal conductivity of the material, denoted by k.

Theorem 12.3 (Thermal Conductivity of a Material)**.**

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L} = kA\left(T_{grad}\right)$$

As seen from above, we introduced a new term, T_{grad}. This is the **temperature gradient**.

Engineers often use the term **thermal resistance** for buildings. The thermal resistance R of a slab of material with area A is defined so that the heat current H through the slab is:

Definition 46 (Thermal Resistance)**.**

$$H = \frac{A\left(T_H - T_C\right)}{R}$$

Comparing to the previous equation, we can obtain a relationship between thermal resistance and thermal conductivity of a material.

$$R = \frac{L}{k}$$

where L is the thickness of the slab.

12.6 Convection

Convection is the transfer of heat by mass motion of the fluid from one region of space to another. Familiar examples would be hot air that rises. Conective heating is a very complicated process, and physicists have not found any equation to relate the relevant parameters.

However, these are some experimental facts:

(a) The heat current due to convection is inversely proportional to surface area.

(b) Viscosity of fluids shows natural convection near a stationary surface, giving a surface film that on a vertical surface typically has an insulating value of 0.7. Forced convection decreases the thickness of this film, increasing the rate of heat transfer.

(c) The heat current due to convection is found to be approximately proportional to $\frac{5}{4}$ power of the temperature difference between the surface and the main body of the fluid.

12.7 Radiation

Radiation is the transfer of heat through electromagnetic waves, such as visible light, infrared and ultraviolet radiation. This mechanism of heat transfer is unique, in that since electromagnetic waves are transverse, it was possible for this heat transfer to occur even if it was vacuum between the bodies.

It is common misunderstanding, that since nothing can be seen at low temperatures. However, every body emits energy. At lower temperatures, the heat is radiated normally in the form of infrared waves. As the temperature rises, the wavelengths shift to shorter values, so at high temperatures the radiation contains enough visible light to appear "white–hot".

The rate of energy radiation from a surface is proportional to the surface area A. The rate increases very rapidly with temperature, depending on the fourth power of the absolute (Kelvin) temperature. The rate also depends on the nature of the surface; this dependence is described by a quantity e called the emissivity. A dimensionless number between 0 and 1, it represents the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends some–what on temperature. Thus the heat current H = dQ/dt due to radiation from a surface area A with emissivity e at absolute temperature T can be expressed as

Theorem 12.4 (Heat current in radiation)**.**

$$H = Ae\sigma T^4$$

Where σ is a fundamental physical constant, the Stefan–Boltzmann constant, This relationship is called the **Stefan–Boltzmann law**. The current best numerical value for σ is

$$\sigma = 5.670400(40) \times 10^{-8} W/m^2 \cdot K^4$$

While a body at absolute temperature T is radiating, its surroundings at temperature T. are also radiating, and the body absorbs some of this radiation. If it is in thermal equilibrium with its surroundings, $T = T_S$ and the rates of radiation and absorption must be equal. For this to be true, the rate of absorption must be given in general by $H = Ae\sigma T_s^4$. Then the net rate of radiation from a body at temperature T with surroundings at temperature T, is

$$H_{net} = Ae\sigma T^4 - Ae\sigma T_s^4 = Ae\sigma \left(T^4 - T_s^4\right)$$

In this equation a positive value of H means a net heat flow out of the body. The above equation shows that for radiation, as for conduction and convection, the heat current depends on the temperature difference between two bodies.

13 The Laws of Thermodynamics

In this section, we further expand on the first law of thermo dynamics discussed earlier on.

The second law of thermodynamics constrains the first law by establishing which processes allowed by the first law actually occur. For example, the second law tells us that energy never flows by heat spontaneously from a cold object to a hot object. One important application of this law is in the study of heat engines (such as the internal combustion engine) and the principles that limit their efficiency.

13.1 Work in Thermodynamic Processes

Energy can be transferred to a system by heat and by work done on the system. In most cases of interest treated here, the system is a volume of ideal gas, which is important for understanding engines. We need a method of obtaining the work done as a function of pressure and change and volume, and this formula is relatively simple.

We let there be a system of pressure P, and a piston having pushed down a distance of Δx. Then, $W = -F\Delta x = -PA\Delta x$. We also know that the change in volume is $\Delta V = A\Delta x$, then we get

Theorem 13.1 (Expressing Work Done)**.** *The **work done W on a gas** at constant pressure is given by*

$$W = P\Delta V$$

where P is the pressure throughout the gas and ΔV is the change in volume of the gas during the process.

13.2 Paths Between Thermodynamic States

We have seen thermodynamic processes which involve change in volume and change in pressure. It is useful to plot a pressure against volume graph, which allows us to see the amount of work done easily.

The work done throughout the entire process, can be calculated by finding the area under the graph, or rather enclosed by the graph, where pressure is the y-axis, while volume is the x-axis.

13.3 First Law of Thermodynamics

The **first law of thermodynamics** is a law regarding energy conversion. It relates internal energy to energy transfers during heat gain and work done. In fact, the previous two are the only methods of change in internal energy.

As such, by conservation of energy, we get:

Definition 47 (First Law of Thermodynamics)**.** If a system undergoes a change from an initial state to a final state, where Q is the energy transferred to the system by heat and W is the work done on the system, then the change in internal energy can be obtained from the following equation:

$$\Delta U = U_f - U_i = \Delta Q + \Delta W$$

It is important to note that the quantity Q is positive when energy is **TRANSFERRED INTO** the system, and negative when energy is **TRANSFERRED OUT** of the system. Similarly, as stated earlier, work done on the system is positive, and work done by the system is negative.

Sometimes, it is also important to realise that in an isolated system, the change in internal energy is 0. as such, the energy conversions from heat to work done and vice versa are due to each other.

Recall the equation describing the amount of internal energy of an ideal gas:

$$U = \frac{3}{2}nRT$$

This expression is only valid for a monoatomic gas, meaning the gas is only made up of single atoms. Then the change in internal energy of the gas is:

$$\Delta U = \frac{3}{2}nR\Delta T$$

13.4 Special Processes in Thermodynamics

There are four basic types of thermal processes, which will be studied in this section.

13.5 Isobaric Processes

From the name, it is easy to tell that isobaric processes are thermodynamic processes in which the pressure remains constant. (You would know this if you studied Geography). An expanding gas does work on the environment,

$$W_{env} = P\Delta V$$

$$Q = \Delta U - W = \Delta U + P\Delta V$$

$$Q = \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nRT = nC_p\Delta T$$

where C_p is the molar heat capacity for constant pressure, $C_p = \frac{5}{2}R$.

13.6 Adiabatic Processes

An **adiabatic process** is defined as one whereby there is no heat transfer out or into the system (i.e. $\Delta Q = 0$). This can be attained by surrounding the system with a thermally insulating material. Applying the first law of thermodynamics, we get:

$$\Delta U = -\Delta W$$

This equation can be easily interpreted from the different scenarios. When the system does work on the surroundings, the total internal energy would decrease ($W = 0$).

13.7 Isochoric Processes

An **isochoric process** is a process in which volume remains constant. We go back to the ideal gas law:

$$PV = nRT$$

$$V = nR\left(\frac{T}{P}\right)$$

This means that the value $\frac{T}{P}$ is a constant: Temperature change in the process is equivalent to pressure change.

13.8 Isothermal Processes

An **isothermal process** is one in which temperature remains constant. Again, we go back to the ideal gas law:

$$PV = nRT$$
$$T = \frac{PV}{nR}$$

This means that PV remains constant throughout the process. As such, we get that the pressure is inversely proportional to the volume of the gas.

Recall that temperature is a direct measure of the amount of kinetic energy of the gas. If temperature is constant, then the internal energy of the system is a constant: $\Delta U = 0$

As a result we get:

$$\Delta Q = \Delta W$$

13.9 Heat Engines

Heat has been, since the dawn of time, a very potent source of energy. Heat can be derived from the sun's radiation, the earth's geothermal energy etc. As such, it is important to understand how to take heat from a source and convert it into mechanical energy to do work. This machinery is called a **heat engine**. In a heat engine, a quantity of matter undergoes addition or subtraction of heat. This matter is called the **working substance**.

The simplest machines undergo **cyclic processes**. These processes eventually leave the working substance in the same state as originally started. This means that its initial and final internal energy is the same, i.e. $\Delta U = 0$.

As such, the first law of thermodynamics requires that:

$$\Delta U = 0 = \Delta Q - \Delta W \text{and thus, } \Delta Q = \Delta W$$

When analysing heat engines, it is helpful to think of two objects with which the working substance of the engine can interact. One of these, called the *hot reservoir*, can give the working substance large amounts of energy at a constant temperature T_H without losing much of its own heat. The other is called the *cool reservoir*, which on the other hand can absorb large amounts of heat at a constant lower temperature T_C.

We denote the quantities of heat transferred from the hot reservoir and the cold reservoir Q_H and Q_C respectively. The image on the right shows the transfer of heat in a process.

When an engine repeats the same cycle over and over, Q_H and Q_C represents the amount of energy absorbed and rejected respectively. The net heat Q absorbed per cycle is:

$$Q = Q_H + Q_C = |Q_H| - |Q_C|$$

The net work done is also:

$$W = Q = |Q_H| - |Q_C|$$

Ideally, we would like to convert all the heat into work, i.e. $|Q_C| = 0$, but experiments have shown that such a thermal engine is not possible to build, and that $|Q_C|$ can never be equal to zero. As such, we created a term to define the thermal efficiency of the engine, denoted by e.

Definition 48 (Thermal Efficiency of a Heat Engine)**.**

$$e = \frac{W}{Q_H}$$

The thermal efficiency e represents the fraction of Q_H that is actually converted into work.

From here, we also arrive at our second law of thermodynamics:

Theorem 13.2 (Second Law of Thermodynamics)**.** *It is impossible to have the sole result the transfer of heat from a cooler to hotter body*

The above statement is also known as the "refrigerator" statement.

13.10 Carnot Cycle

In 1824, french engineer Sadi Carnot created a hypothetical engine that has the maximum possible efficiency consistently to the second law of thermodynamics. This engine is called the **carnot engine**.

Conversion of work to heat is a irreversible process; the purpose of a heat engine is a partial reversal of this process; the conversion of heat to work with as high an efficiency as possible. As such, for maximum efficiency, it is necessary to avoid all irreversible processes.

Heat flow through a finite temperature drop is a irreversible process. Therefore, during heat transfer in the Carnot cycle there must be *no* finite temperature difference. When the engine takes heat from the hot reservoir at temperature T_H , the working substance of the engine must also be at T_H; otherwise, irreversible heat flow would occur. Similarly, when the engine discards heat to the cold reservoir at T_C, the engine itself must be at T_C. That is, every process that involves heat transfer must be isothermal at either T_H or T_C.

Conversely, in any process in which the temperature of the working substance of the engine is intermediate between T_H and T_C, there must be no heat transfer between the engine and either reservoir because such heat transfer could not be reversible. Therefore any process in which the temperature T of the working substance changes must be adiabatic.

The carnot cycle consists of two adiabatic and isothermal processes each, all of which are reversible:

(a) The process $A \to B$ is an isothermal expansion at temperature Th in which the gas is placed in thermal contact with a hot reservoir (a large oven, for example) at temperature T_h. During the process, the gas absorbs energy Q_h from the reservoir and does work $W_{A\to B}$ in raising the piston.

(b) In the process $B \to C$, the base of the cylinder is replaced by a thermally non–conducting wall and the gas expands adiabatically, so no energy enters or leaves the system by heat. During the process, the temperature falls from T_H to T_C.

(c) In the process $C \to D$, the gas is placed in thermal contact with a cold reservoir at temperature T_C and is compressed isothermally at temperature T_C. During this time, the gas expels energy Q_C to the cold reservoir and the work done on the gas is $W_{C\to D}$.

(d) In the final process, $D \to A$, the base of the cylinder is again replaced by a thermally nonconducting wall and the gas is compressed adiabatically. The temperature of the gas increases to T_H, and the work done on the gas is $W_{D\to A}$.

13.11 Entropy

Entropy is a state variable, denoted by S, which is related to the second law of thermodynamics.

We begin by stating the definition of entropy:

Definition 49 (Entropy)**.** Let Q_r be the energy absorbed or expelled during a reversible, constant temperature process between two equilibrium states. Then the change in entropy during any constant temperature process connecting the two equilibrium states is defined as:

$$\Delta S = \frac{Q_r}{T}$$

SI units: joules/Kelvin (J/K)

The concept of entropy had become widely-accepted because it's significance was enhanced when it was found that the entropy of the Universe increases in all natural processes. This is another fancy way of stating the second law of thermodynamics.

13.12 Entropy and Disorder

A large element of chance is inherent in natural processes. The spacing between trees in a natural forest, for example, is random; if you discovered a forest where all the trees were equally spaced, you would conclude that it had been planted. Likewise, leaves fall to the ground with random arrangements. It would be highly unlikely to find the leaves laid out in perfectly straight rows. We can express the results of such observations by saying that a disorderly arrangement is much more probable than an orderly one if the laws of nature are allowed to act without interference.

Entropy originally found its place in thermodynamics, but its importance grew tremendously as the field of statistical mechanics developed. This analytical approach employs an alternate interpretation of entropy. In statistical mechanics, the behavior of a substance is described by the statistical behavior of the atoms and molecules contained in it. One of the main conclusions of the statistical mechanical approach is that isolated systems tend toward greater disorder, and entropy is a measure of that disorder.

In light of this new view of entropy, Boltzmann found another method for calculating entropy through use of the relation

$$S = k_B \ln W$$

where $k_B = 1.38 \times 10^{23} J/K$ is Boltzmanns constant and W is a number proportional to the probability that the system has a particular configuration. The second law of thermodynamics is really a statement of what is most probable rather than of what must be.

14 Vibrations and Waves

Periodic motion, from masses on springs to vibrations of atoms, is one of the most important kinds of physical behavior.

14.1 Hooke's Law

One of the simplest types of vibrational motion is that of an object attached to a spring. If the spring is stretched or compressed a small distance x from its unstretched or equilibrium position and then released, it exerts a force on the object.

It obeys the equation

$$F_s = -kx$$

k is a positive constant called the **spring constant**. It is important to note that the force exerted by the spring is always directed opposite the displacement of the object. Because the spring force always acts toward the equilibrium position, it is sometimes called a restoring force. A restoring force always pushes or pulls the object toward the equilibrium position.

Suppose the object is initially pulled a distance A to the right and released from rest. The force exerted by the spring on the object pulls it back toward the equilibrium position. As the object moves toward $x = 0$ the magnitude of the force de- creases (because x decreases) and reaches zero at $x = 0$. However, the object gains speed as it moves toward the equilibrium position, reaching its maximum speed when $x = 0$. The momentum gained by the object causes it to overshoot the equilibrium position and compress the spring. As the object moves to the left of the equilibrium position (negative x-values), the spring force acts on it to the right, steadily increasing in strength, and the speed of the object decreases. The object finally comes briefly to rest at $x = -A$ before accelerating back towards $x = 0$ and ultimately returning to the original position at $x = A$. The process is then repeated, and the object continues to oscillate back and forth over the same path. This type of motion is called simple harmonic motion. Simple harmonic motion occurs when the net force along the direction of motion obeys Hookes law– when the net force is proportional to the displacement from the equilibrium point and is always directed toward the equilibrium point.

The following three concepts are important in discussing any kind of periodic motion:

(a) The **amplitude** A is the maximum distance of the object from its equilibrium position. In the absence of friction, an object in simple harmonic motion oscillates between the positions $x = -A$ and $x = A$.

(b) The **period** T is the time it takes the object to move through one complete cycle of motion, from $x = A$ to $x = -A$ and back to $x = A$.

(c) The **frequency** f is the number of complete cycles or vibrations per unit of time, and is the reciprocal of the period ($f = \frac{1}{T}$).

15 Electrical Energy and Capacitance

Here we learn that the potential energy concept is also useful in the study of electricity. Because the Coulomb force is conservative, we can define an electric potential energy corresponding to that force. In addition, we define an electric potential as the potential energy per unit charge corresponding to the electric field.

15.1 Electric Potential Difference and Electric Potential

Electric potential energy and electric potential are closely related concepts. The electric potential turns out to be just the electric potential energy per unit charge. This is similar to the relationship between electric force and the electric field, which is the electric force per unit charge.

15.2 Work and Electrical Potential Energy

Theorem 15.1 (Electric Potential Energy)**.** *The change in the electric potential energy, dPE, of a system consisting of an object of charge q moving through a displacement dx in a constant electric field is given by*

$$dPE = -W_{AB} = -qE_x dx$$

where Ex is the x-component of the electric field and $dx = x_f + x_i$ is the displacement of the charge along the x-axis. SI Unit: joule (J)

15.3 Electric Potential

It is useful to define an electric potential difference dV related to the potential energy by $dPE = qdV$:

Theorem 15.2 (Electric Potential)**.** *The electric potential difference dV between points A and B is the change in electric potential energy as charge q moves from A and B, divided by the charge q:*

$$dV = V_B - V_A = \frac{dPE}{q}$$

16 Electric Charge and Electric Field Calculations

In this chapter, we proceed with the study of charges, and the fields they produce. Fields might not appear all new to you. This is because we have also dealt with the gravitation field theory earlier.

16.1 Electric Charge

So what exactly is an electric charge? How do we know that it exists in the first place?

Some simple experiments demonstrate the existence of electric forces. For example, after rubbing a balloon on your hair on a dry day, we will find that the balloon would attract pieces of paper. The balloon is said to have been charged, and this electric force is actually the electric force of attraction.

When such materials exhibit this behaviour, they are said to be *electrified* or to have become **electrically charged**. One can easily electrify himself by rubbing his/her shoes against a cloth rug.

Using this series of experiments, **Benjamin Franklin** found that there were two kinds of electric charges, which were given the names **positive** and **negative**. Electrons have been identified to possess **negative** charge, while protons possess **positive** charge. **Charges of the same sign repel one another, while charge sof the opposite sign attract one another**.

One observation of much importance is the electrical charge is conserved in an isolated system. That is, when one object is rubbed against another, charge is not created in the process. Instead **charge is transferred** from one object to another, and result in both objects having an imbalance of charge.

Later on, Robert Millikan discovered that electric charge was a quantised. electric charge, q, was always in integer multiples of the fundamental amount of charge e. Therefore, we can write $q = \pm Ne$ where N is some integer.

16.2 Induction

It is important and convenient that we are able to classify materials according to their ability of electrons to move around the material. This is the classification of conductors, insulators, and the "in-betweens".

> Electrical conductors are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material; electrical insulators are materials in which all electrons are bound to atoms and cannot move freely through the material.

17 Electrical Energy and Capacitance

Here we learn that the potential energy concept is also useful in the study of electricity. Because the Coulomb force is conservative, we can define an electric potential energy corresponding to that force. In addition, we define an electric potential as the potential energy per unit charge corresponding to the electric field.

17.1 Electric Potential Difference and Electric Potential

Electric potential energy and electric potential are closely related concepts. The electric potential turns out to be just the electric potential energy per unit charge. This is similar to the relationship between electric force and the electric field, which is the electric force per unit charge.

17.2 Work and Electrical Potential Energy

Theorem 17.1 (Electric Potential Energy)**.** *The change in the electric potential energy, dPE, of a system consisting of an object of charge q moving through a displacement dx in a constant electric field is given by*

$$dPE = -W_{AB} = -qE_x dx$$

where Ex is the x-component of the electric field and $dx = x_f + x_i$ is the displacement of the charge along the x-axis. SI Unit: joule (J)

17.3 Electric Potential

It is useful to define an electric potential difference dV related to the potential energy by $dPE = qdV$:

Theorem 17.2 (Electric Potential)**.** *The electric potential difference dV between points A and B is the change in electric potential energy as charge q moves from A and B, divided by the charge q:*

$$dV = V_B - V_A = \frac{dPE}{q}$$

18 Current and Resistance

18.1 Electric Current I

Whenever a charge is flowing, an electric current said is to exist. The electric current, I, is defined as the **net amount of charge passing a point per unit time**. Hence, the *magnitude of the current tells us the rate of the new flow of charge.*

The current is the rate at which charge flows through this surface. If dQ is the amount of charge that passes through the surface in a time interval dt, the **average electric current** I_{ave} is equal to the charge that passes through A per unit time.

Definition 50 (Average Current)**.**

$$I = \frac{dQ}{dt}$$

Where Q is charge and t is time.

It is sometimes, even very often, useful to describe current in terms of current density. Current density is the amount of "current" per unit area. This quantity is rather important in the derivations for the magnetic field chapter. We denote current density by J.

Definition 51 (Current Density)**.**

$$J = \frac{I}{A}$$

Where I is current, and A is area.

19 Quantum Physics

This branch of Physics describe the behavior and energy and matter at the atomic and subatomic scale. [2]

19.1 Photoelectric Experiment

It was Albert Einstein who conducted this experiment. If you are interested in his works, you can find more information about him here. I shall continue explaining the Photoelectric experiment.

The experiment's purpose was to **study the emission of electrons from a metal surface which is irradiated with light and experimentally determine the value of Planck's constant (Refer to section 19.1) "h" by making use of the spectral dependency of the photoelectric effect**.

Max Planck was a brilliant physicist, who was considered to be the **founder of the quantum theory**. Planck was awarded the Nobel Prize in Physics in 1918. In 1894 Planck turned his attention to the problem of black-body radiation. He had been commissioned by electric companies to create maximum light from lightbulbs with minimum energy. Many argue that Einstein, who discovered the photoelectric effect should be given credit for quantum physics because Planck did not understand in a deep sense that he was "introducing the quantum" as a real physical entity. But it was true that Planck was the one who first proposed the idea and deserved credit too.

19.2 Classical Wave Theory

The classical wave theory states that:

- Photoelectrons emitted for radiation are in all wavelengths
- Maximum $K.E.$ of photoelectron depends on light intensity, and is independent of frequency
- Measurable time lag between emission of electrons(photons) $\Longrightarrow$ Need to gain enough energy

The photoelectric effect **does not** agree with this theory. A new theory had to be created to explain the effect.

19.3 Quantum theory

The idea suggests that:

(a) electromagnetic energy is particulate in nature $\Longrightarrow$ transmitted through quanta[3]

(b) The packets of energy are called photons, which travel in **only one direction**.

(c) The amount of energy E contained in each quantum is directly proportional to the frequency:

Theorem 19.1 (Einstein's Photoelecric equation – Basic)**.**

$$E = hf$$

E = energy contained, h = Planck's constant $= 6.63 \times 10^{-34} Js$, *and f = frequency of radiation*

[2] Quantum mechanics - Wikipedia, the free encyclopedia

[3] A fundamental particle, building block of protons and neutrons, as well as all other hadrons and mesons

19.4 Einstein's Photoelectric equation

This section requires thorough knowledge in the photoelectric experiment (Refer to 19.1 For a photoelectron to be emitted, sufficient energy is needed. Every material or metal has got a **characteristic work function,** ϕ. This is the **minimum amount of energy needed to liberate an electron from its surface.**

If the photoelectron is emitted without any further interactions with other atoms, its kinetic energy would be at maximum. Thus, Einstein proposed this equation:

Theorem 19.2 (Einstein's Photoelectric Equation – work function)**.**

$$hf = \phi + KE_{max}$$

Since from section , $KE_{max} = \frac{1}{2}mv_{max}^2$*, we get:*

$$hf = \phi + \frac{1}{2}mv_{max}^2$$

The two equations above are known as **Einstein's Photoelectric equation**.

The maximum KE is measured by measuring the **stopping potential** V_s, of the collector that is made negative with respect to the emitter. This represents the minimum negative potential required to stop even the most energetic electron from reaching the collector(there will be no current in the ammeter).

Thus, the stopping potential and maximum kinetic energy are related by the following equation:

$$\frac{1}{2}mv^2 = eV_s$$

Thus, the photoelectric equation can also be rewritten as $hf = \phi + eV_s$.

19.5 The Wave-Particle Duality

Light can behave as a particle(from the photoelectric effect, refer to 19.1) when it interacts with matter or as a wave(reflection, refraction and interference(Young's Double Split Experiment)). This shows the wave particle duality of electromagnetic energy.

De Broglie suggested that matter might also exhibit this duality and have wave properties.

He suggested that for a particle of momentum $p = mv$, which exhibits wave behavior, it will have an associated wavelength, λ, given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where:
h = Planck's constant $= 6.63 \times 10^{-34} Js$, p = momentum, m = mass, v = velocity

This is also known as the **de Broglie principle**.

19.6 Electron Diffraction

When a beam of electron falls on a thin layer of graphite atoms, the electrons disperse and their pattern is captured on a coated fluorescent screen. The pattern is identical to that obtained due to the interference of electromagnetic radiation. This is known as electron diffraction phenomenon.

19.7 Bohr's Atomic Model

Several models of atom suggest that it consist of a nucleus with electrons orbiting around the nucleus. Maxwell's theory of electromagnetic radiation meant that electrons would emit radiation and lose energy while orbiting.

Niel Bohr had a different model. His model stated that:

- electrons within atoms could exist in stable energy states or levels without emitting radiation
- electrons occupy the lowest energy level(known as the ground state) so that the energy of the atom is at the lowest and is the most usable
- electrons would absorb energy in quanta of certain definite amounts when bombarded by atoms or other electrons
- electron will jump to a higher level on gaining its quantum of energy, thus being in an **excited state(excitation)**. If quantum of energy is sufficiently high, the electron might escape from **atom surface(ionisation)**.

19.8 Line Spectra

When a gas is energised sufficiently by being heated or having an electric current passed through it, it emits visible light. Using a spectrometer, an emission spectrum is observed.

Gases like neon and hydrogen give a spectrum that consists of a few colours only. A line spectrum is then observed. The term "line spectra" indicates that only certain frequencies are present. These lines are evidence of "quantised" energy levels in an atom. **A line spectra is the characteristic of each element.**

When a gas is energised, the electron jumps to a higher energy level according to the amount of energy received. The atom would not be stable. The electron would then fall back to its original energy level, and in the process emit radiation for the amount of energy.

19.9 Energy levels and Line Spectrum of Hydrogen

Hydrogen gas is filled inside a tube and bombarded with electrons. The resulting spectrum is observed using a spectrometer.

The energy level diagram and the corresponding line spectrum is shown below:

(a) All the energy levels have negative energy values. Energy of an electron at rest outside atom is taken to be 0.

(b) The energy value of a given level may be expressed in units of an eV (electronvolt). $1eV = 1.6 \times 10^{-19}J$.

(c) The frequency of the photon or radiation emitted when a electron falls from energy level E_2 to $E_1$is given by:

$$hf = E_2 - E_1$$

19.10 Features of line spectra

Consider the Balmer series ($E_n \rightarrow E_2$) for the hydrogen atom:

(a) There is an infinite number of spectrum lines in the Balmer series

(b) Spacing between adjacent lines increase with increasing wavelength(or decrease with increasing frequency)

(c) The series limit of $365.6nm$ is the shortest possible wavelength emitted, corresponding to the transition ($E_\infty \rightarrow E_2$)

(d) Generally, spectral lines corresponding to longer wavelengths are more intense because these transitions are more common.

19.11 Emission Line Spectra

When a gas is heated or bombarded by electrons, the electrons in the gas atoms are excited to higher energy level. They will only remain there momentarily before emitting a photon and moving to lower energy state. This causes a series of lines in a spectrum, which is called the emission line spectrum. It consists of a series of separate bright line of definite wavelength on a dark background.

19.12 Blackbody Radiation

The Stefan–Boltzmann Equation

Every object at a temperature greater than 0K emits electromagnetic radiation consisting of a mixture of wavelengths. The rate of energy at which energy is radiated is given by the Stefan-Boltzmann equation:

Theorem 19.3 (The Stefan–Boltzmann Equation)**.**

$$P = e\sigma AT^4$$

Where P = *total power radiated in all wavelengths,*
T = *absolute(Kelvin) temperature of the object,*
σ = *Stefan's constant* $= 5.67 \times 10^{-8} Wm^{-2}K^{-4}$
A = *Surface area of radiating object*
e = *emissivity*

The emissivity e:

- is characteristic of the surface of the radiating surface
- takes on a value **between 0 and 1**, i.e. $0 \leq \sigma \leq 1$
  - Very black and rough surfaces have a value closer to 1.
  - White and smooth surfaces have a value closer to 0.
- Is somewhat dependent on the temperature of the body

19.13 Uncertainty Principles

Uncertainty Principles are well-popularised iconic principles of quantum theory due to its major departure from the determinism of classical physics. The program of quantum mechanics is very different – the way physical observables are understood and calculated involves a *probabilistic* interpretation of many physical phenomena around us. Thus, uncertainty principles – such as those involving position–momentum, energy–time, often become the first striking features of quantum physics.

Imagine that we have to measure the position and momentum of a particle at the same time. Classically, the only limitation to the precision at which both its position and momentum can be measured is due to technology and the method of measurement. However, it turns out that in quantum mechanics, this fuzziness in spatial position and momentum is *fundamental.* Werner Heisenberg derived this notion in 1927, now popularly known as the **Heisenberg's uncertainty principle** which states:

If a measurement of position is made with precision dx and a simultaneous measurement of momentum in the x–direction, is made with precision dp then the product of the two uncertainties can never be smaller than $\frac{h}{2}$ i.e.

Theorem 19.4 (Heisenberg's Uncertainty Principle — position–momentum)**.**

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

This uncertainty principle deals with position and momentum. Now I shall introduce another uncertainty principle, the energy-time uncertainty principle. From the name of the principle it is obvious that the latter deals with energy and time. The formula for the equation is:

Theorem 19.5 (Heisenberg's Uncertainty Principle – energy–time)**.**

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Now the usual context in which one uses the energy-time uncertainty principle is that of electromagnetic radiation involved in atomic processes. Suppose we wish to measure the energy E emitted/absorbed during the time interval dt, the finite time of the radiation process limits the accuracy with which we can determine the frequency – and hence the energy E of the waves. Thus, the energy-time uncertainty principle is a statement about the uncertainty in an energy measurement taken within the time interval dt.

20 Matter

In this topic, I will be going through three main points:

(a) Properties of nuclei

(b) Nuclear Reactions

(c) Radioactivity

20.1 Properties of Nuclei

Rutherford's α-particle Scattering Experiment

In 1911, Ernest Rutherford and his students Hans Geiger and Ernest Marsden used a beam of positively charged alpha particles to fire on a thin gold foil, trying to break open an atom to identify any particle that emerged.

(Note: An α-particle is a Helium nucleus stripped of its electrons, hence having a positive charge of +2e, where e is the fundamental charge)

Results:

- Most of the particles went straight through or were scattered through very small angles
- BUT a very small fraction(less than 1%) of the particles were scattered through very large angles, some of which were close to 180^0.

20.2 Atomic Structure

(a) Every atom has a central, positively charged nucleus that is very small but negative.

(b) The nucleus is made up of positive protons and neutral neutrons.

(c) Electrons revolve around the nucleus.

(d) There is a vast amount of empty space within an atom.

	Proton	Neutron	Electron
Mass(kg)	$1.673 \times 10^{-27} kg$	$1.675 \times 10^{-27} kg$	$9.109 \times 10^{-31} kg$
Charge(C)	$+1.602 \times 10^{-19} C$	0	$-1.602 \times 10^{-19} C$
Mass (u)	$1.007276u$	$1.008665u$	$0.0005486u$

u is the atomic mass unit, defined to be $\frac{1}{12}$ of the mass of a ^{12}C atom and is equivalent to $1.6605402 \times 10^{-27} kg$. At the atomic and subatomic level, u is used as the standard unit for masses instead of kg.

Use Avogadro's number to show that $u = 1.66 \times 10^{-27}$.

20.3 Properties of Nuclei

- The nucleus contains protons and neutrons – collectively referred to as nucleons
- The nucleus is very small as compared to the size of the atom
  - Nuclear diameters are around $10^{-15}m$
  - Atomic diameters are around $10^{10}m$
- Over 99.9% of the mass of an atom is in its nucleus
- All nuclei(except for hydrogen nucleus) are composed of two types of particles, protons and neutrons. In representing nuclei, scientists use symbols in the general form of ${}^{A}_{Z}X$, where **X** is the chemical symbol for the element and:

  A - mass number The number of nucleons (neutrons + protons) in the nucleus. Also known as nucleon number.

  Z - atomic number The number of protons in the nucleus. It is also referred to as the proton number. It defines the chemical characteristics of an atom and the place of the element in the periodic table.

  N - neutron number It can be obtained by taking $A - Z$.

20.4 Isotopes

Definition 52 (Isotopes)**.** Isotopes are atoms that have the same number of protons but different number of neutrons

Atoms that have the same number of protons will also have the same number of electrons. Since the number of electrons determines the chemical properties of an element, isotopes therefore exhibit the same chemical properties.

Some examples of isotopes:

1. ${}^{1}_{1}H$ – Hydrogen	2. ${}^{12}_{6}C$ – Carbon–12	3. ${}^{35}_{12}Cl$ – Chlorine–35
${}^{2}_{1}H$ – Deuterium	${}^{14}_{6}C$ – Carbon–14	${}^{37}_{12}Cl$ – Chlorine–37
${}^{3}_{1}H$ – Tritium		

A **radioisotope** is an isotope of an element that is **radioactive**.

A nuclide is any particular atomic nucleus with a **specific** atomic number Z and mass number A. E.g. ${}^{12}_{6}C$ and ${}^{56}_{26}Fe$. Isotopes and nuclides having the same atomic number.

20.5 Nuclear Stability (The Nuclear Force)

Why does a nucleus remain intact and not burst apart?

There must be an even stronger force (which we call “nuclear force”) and attractive in nature at this distance ($10^{-15}m$). ”Nuclear force” Is the strongest force in nature and has the following characteristics:

(a) It is independent of electric charge

(b) It is a very short range attractive force. This means that it is very strong when nucleon–nucleon distance is approximately $10^{-15}m$, and decreases to zero at large distances($> 10^{-15}m$)

For a nuclide to be stable, there must be sufficient neutrons to "glue" the protons together.

Every proton repels every other proton, i.e. a proton is repelled by all other protons via electric force. But they are attracted by neighbouring nucleons via a strong nuclear force. Hence, for heavier nuclides, more neutrons are needed to separate the protons further apart. As the atomic number increases, the line of stability deviates upwards from the line $N = Z$. This can be understood by recognising that, as the number of protons increases, the strength of the Coulomb force increases, which tend to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable because neutron experience only the attractive nuclear force.

20.6 Einstein's Mass–Energy Equivalence

In 1905, Einstein showed from his theory of relativity that mass and energy are equivalent.

As a result of Einstein's theory, the separate conservation principles of mass and energy can be unified as the principle of mass-energy. Mass can be 'created' or 'destroyed' but when this happens, an equivalent amount of energy simultaneously vanishes or comes into being.

Theorem 20.1 (Einstein's Mass–energy Equivalence)**.** *The energy E produced by a charge of mass m is given by the mass-energy equivalence relation:*

$$E = mc^2$$

where c is the speed of light $= 299792458\ ms^{-1}$*, E is in Joules and m is in kg.*

The relation shows that mass is a form of energy. A small mass is equivalent to an enormous amount of energy.

20.7 The Mass Defect

When individual protons and neutrons come together to form the nucleus, there is a **decrease in mass**. i.e. The mass of the nucleus is smaller than the sum of masses of the individual nucleons. This decrease in mass is also known as the **mass defect**.

Definition 53 (Mass Defect)**.** The mass defect of a nucleus is defined as the difference between the mass of the separated nucleons and the combined mass of the nucleus

For example, to calculate the mass defect of a $^{40}_{20}Ca$ nucleus of mass $39.95159u$: [*mass of a proton* $= 1.00728u$ *; mass of a neutron* $= 1.00867u$]

$$\begin{gathered}\text{Mass of protons} = 20 \times 1.00728\\ \text{Mass of neutrons} = 20 \times 1.00867\\ \text{Total Mass} = 40.3190u\\ \text{Mass Defect } dM = 40.3190u - 39.95159u = 0.36741u\end{gathered}$$

In general, to calculate the mass defect dM for a **nucleus** that has **Z** protons and **N** neutrons,

Theorem 20.2 (Calculating Mass Defect - Nucleus). $dM = Zm_p + Nm_n - M_n$
Where: m_p *= mass of a proton,* m_n *= mass of a neutron, and* M_n *= mass of the nucleus*

As it is usually the masses of **neutral atoms** that are given, we can write the above equation as

Theorem 20.3 (Calculating Mass Defect - Neutral Atoms). $dM = Zm_p + Nm_n + Zm_e - M_n$
Where: m_e *= mass of electron, and* M_n *= mass of neutral atom*

(Note: the second equation acutally gives the mass defect of the entire atom, and not only the nucleus.)

20.8 The Binding Energy

The mass of a nucleus is **always less** than the sum of the mass of its nucleons.

$$\begin{aligned} dM &= (Zm_p + nm_n) - m_{nucleus} \\ &= (Zm_p + Nm_n) - (M_A - Zm_e) \\ &= ZM_H + Nm_n - M_A \end{aligned}$$

where Z is the proton number, N is the **neutron number**, i.e. the number of neutrons in the nucleus (Numerically $A - Z$), m_p the mass of proton, m_n the mass of neutron, m_e the mass of electron, $m_{nucleus}$ the actual mass of the nucleus, M_n the mass of **hydrogen atom** and M_A the atomic mass. Previously, we learnt that nucleons inside the nucleus are held tightly together. Therefore, ENERGY is required to separate the nucleus into its constituent protons and neutrons.

Definition 54 (Binding Energy). The **binding energy of a nucleus** is the work which must be done on the nucleus to separate it completely into its constituent nucleons.

Applying Einstein's mass-energy equivalence(Refer to *Formula 20.1*), this shows that the binding energy of any nucleus is

Theorem 20.4 (Calculating Binding Energy). *Binding Energy (B.E.)* $= (dm)c^2$
Where dm *= mass defect and* c *= the speed of light*

Hence, the **binding energy of a nucleus** is the energy equivalent to the mass defect when nucleons bind together to form an atomic nucleus.

Combining with the fact that the mass of a nucleus is ***always less*** than the sum of the masses of the nucleons, the binding energy of a nucleus is also the amount of energy released when nucleons band together to form an atomic nucleus.

At the atomic level,

(a) the unit for mass is atomic mass unit(u), in place of kg.

(b) the unit for energy is mega electron-volt, MeV, in place of J.

SI units at subatomic levels	
Mass	$1u = 1.6605402 \times 10^{-27} kg$
Energy	1 **MeV** $= 1.60217733 \times 10^{-13} J$

Thus,

$$\begin{aligned} 1u &= 1.6605402 \times 10^{27} \times \frac{(2.99792458 \times 10^8)^2}{1.60217733 \times 10^{-23}} MeV \\ &= 931.494 MeV \end{aligned}$$

Calculation of Binding Energy

Approach 1: B.E. $= (dm)c^2$
(find dm in kg, use m_p, m_n and M_A expressed in **kg**, to obtain B.E. in J)

Approach 2: B.E. $= (Zm_p + Nm_n - M_A) \times 931.494MeV$
(use m_p, m_n and M_A expressed in **u** to compute dm to find B.E. in **MeV**)

20.9 Binding Energy per Nucleon and Nuclear Stability

The **binding energy(B.E.) per nucleon** of a nucleus is the **binding energy divided by the total number of nucleons**.

Definition 55 (Binding energy per nucleon)**.** B.E. per nucleon $= \frac{B.E._{total}}{Nucleons}$

This is also a measure of how stable the nucleus is – the larger the binding energy per nucleon, the more stable.

The greater the binding energy per nucleon, the greater the work that must be done to remove the nucleon from the nucleus. Hence, a larger B.E. per nucleon indicates a stable nucleus.

The important features of the curve are:

- Except for the lighter nuclei, the **average binding energy per nucleon** is about 8 MeV.
- The peak, i.e. the maximum B.E. per nucleon occurs at around mass number A = 50, and corresponds to the most stable nuclei. From the graph, it can be seen that the **iron nucleus** $^{56}_{26}Fe$ is located close to the peak with a B.E. per nucleon value of approximately 8.8MeV. It is one of the most stable nuclides that exist.
- Nuclei with very low or very high mass numbers have lesser binding energy per nucleon and are less stable because the less B.E. per nucleon, the easier it is to separate the nucleus into its constituent nucleons.
- Nuclei with low mass numbers may undergo **nuclear fusion**, where light nuclei are joined together under certain conditions so that the final product may have a greater binding energy per nucleon.
- Nuclei with high mass numbers may undergo **nuclear fission** – the nucleus may split into two daughter nuclei with the **release of neutrons**. The daughter nuclei will possess **a greater binding energy per nucleon.**

$$\boxed{\text{Energy released in reactions} = (m_{reactants} - m_{products})c^2 = (BE_{products} - BE_{reactants})}$$

20.10 Nuclear Reactions

A nuclear reaction involves the rearrangement of nuclear constituents. **In all nuclear processes, the following quantities are conserved:**

- nucleon number
- proton number(charge)

- mass-energy
- momentum

Nuclear reactions can be represented by an equation in which the total nucleon number and proton number balance on each side.

Example: $\,^{1}_{0}n +^{14}_{7} N \rightarrow^{14}_{6} C +^{1}_{1} p$

Nuclear reactions can also be represented by a symbolic expression of the form

Example: $\,^{14}_{7}N(n,p)^{14}_{6}C$

Nuclear reactions can be classified into two types: *spontaneous radioactive decay* and *induced nuclear reactions.*

20.11 Induced nuclear reactions

Induced nuclear reactions occur when *a nucleus changes as a result of being strucked by a particle.*

For example:

- Bombardment of nitrogen atoms N with α–particles *(Rutherford, 1919, refer to section 20.1)*

$$^{14}_{7}N +^{4}_{2} He \rightarrow^{17}_{8} O +^{1}_{1} H$$

- Bombardment of beryllium atoms by α–particles

$$^{9}_{4}Be +^{4}_{2} He \rightarrow^{12}_{6} C +^{1}_{0} n$$

Note that in every reaction, the proton and nucleon number is conserved, i.e. equal on both sides of the reactions

If the products have greater mass than the reactants(i.e. nucleus & incident particle) before the reaction, than the incident particle must supply enough energy to make up for the increases in mass of the products to allow a reaction to take place

subsectionNuclear fission Nuclear fission is the **disintegration of a heavy nucleus** into **two lighter nuclei of approximately equal masses**

Since the B.E. per nucleon of the daughters nuclei are higher than the parent's nucleus, energy is released.

For example:

- When Uranium–235 is bombarded by slow neutrons

$$^{235}_{92}U +^{1}_{0} n \rightarrow^{142}_{56} Ba +^{91}_{36} Kr + 3(^{1}_{0}n) + energy$$

- The energy released by the fission of a single uranium–236 atom is about 200 MeV.
- Most of this energy will appear in the form of the kinetic energies of the fission fragments and the neutrons, and in the form of the γ–radiation produced
- Neutrons emitted would strike other Uranium–235 nuclei
  - Collision of the neutrons produced in one fission reaction with other nuclei can give rise to a *nuclear chain reaction*

20.12 Nuclear Fusion

Nuclear fusion is the **combining of two light nuclei to produce a heavier nucleus**

A large amount of energy is released during the process because the average binding energy per nucleon of the product has a greater binding energy per nucleon than the two light nuclei before fusion.

Fusion is a difficult process to achieve because of the strong electrical repulsion between the nuclei when they are close to each other. At extremely high temperatures (in excess of 10^8 K) the nuclei have enough energy to overcome the repulsion.

An example is the fusion of two deuterium nuclei to produce helium–3:

$$^2_1H +^2_1 H \rightarrow^3_2 He +^1_0 n$$

Reactions of this type(conversion of hydrogen to helium) are the source of the Sun's energy.

Energy released by the fusion of two nuclei is very much less than that which results from fission. (However, fusion offers the possibility of energy from almost unlimited fuels, with the key advantage of non–radioactive waste.)

20.13 Radioactivity

20.14 Nature of Radioactivity

Definition 56 (Radioactive decay)**.** Radioactive decay is the **spontaneous disintegration** of the nucleus of an atom which results in the **emission of particles** and/or **atom**

- A radioactive nucleus consists of an unstable assembly of protons and neutrons which becomes more stable by emitting an α, β or γ photon.
- **Spontaneous Processes**
  - Radioactive decay occurs *spontaneously.* The process cannot be sped up or slowed down by physical means such as changes in pressure or temperature.
  - The decay of a radioactive atom is *not affected* by any chemical condition or the chemical compound it exists in and is independent of physical conditions such as temperature, pressure and most importantly the decay of other atoms.
- **Random Processes**
  - Radiation is emitted at random. By random, we mean that it is impossible to predict which nucleus and when any particular nucleus will disintegrate.

20.15 Types of radiation

Property	α–particles	β–particles	γ–particles
Nature	Helium–4 nucleus	Electrons	Electromagnetic waves of short wavelength
Charge	+2e	-e	0
Mass	$6.6464835 \times 10^{-27}$kg	$9.109387 \times 10^{-31} kg$	0
Deflection by E and B fields	Deflected by strong fields	Deflected by weak fields	Undeflected
Energy	Constant for a given source	From zero up to a maximum, depending on source	Depends on frequency
Speed	around $10^7 ms^{-1}$	around $10^8 ms^{-1}$	$3.0 \times 10^8 ms^{-1}$
Range in Air	few cm	few metres	Follows the inverse square law
Ionising Power	Strong, producing 10^3 to 10^4 ions per mm of path	Less strong	Weak
Penetrating Power	$10^{-2} mm$ Aluminum	5mm Aluminum	100mm lead

- Alpha Decay
  - Alpha decays can be represented by the following equation, where **P** represents the parent nuclide and **D** the daughter nuclide)

$$^{A}_{Z}P \rightarrow ^{A-4}_{Z-2} D + ^{4}_{2} He$$

  - Examples of alpha decays are:

$$^{238}_{92}U \rightarrow ^{234}_{90} D + ^{4}_{2} He + \gamma$$

$$^{230}_{90}Th \rightarrow ^{226}_{88} D + ^{4}_{2} He + \gamma$$

- Beta Decay
  - In beta decay the general equation is as follows:

$$^{A}_{Z}P \rightarrow ^{A}_{Z+1} D + ^{0}_{-1} e$$

  - Examples of beta radiation are:

$$^{234}_{90}Th \rightarrow ^{234}_{91} Pa + ^{0}_{-1} e + \gamma$$

$$^{234}_{91}Pa \rightarrow ^{234}_{92} U + ^{0}_{-1} e + \gamma$$

- Gamma Decay
  - Gamma decay represents the ***emission of energy*** from a nucleus which is returning to its ground state.
  - Excited nucleus $\rightarrow$ more stable nucleus $+\gamma$

***Note: atomic and mass numbers are all conserved during all types of decay**

20.16 Deflection of α, β and γ in a strong magnetic field

Above shows the paths of α, β and γ–particles/rays in a strong magnetic field.

20.17 Activity, Half-life and Decay constant

Definition 57 (Radioactivity of a substance)**.** The activity of a radioactive substance is defined as the average number of atoms disintegrating per unit time

- The **Activity A** or rate of decay of the parent nuclei is given by

  Theorem 20.5 (Radioactivity Equation 1)**.**

  $$A = -\frac{dN}{dt}$$

  *Where **N** is the number of nuclei, and **t** is the time. An activity of one decay per second is one Becquerel (1 Bq)*

- As radioactive decay is a random process, it follows the laws of statistics. The more radioactivity nuclei there are, the greater the activity

- The activity **A** is directly proportional to the number of parent nuclei **N** at that instant

  Theorem 20.6 (Radioactivity Equation 2)**.**

  $$A \propto N$$
  $$A = -\frac{dN}{dt} = \lambda N$$

  Where the constant of proportionality, λ, is called the decay constant, and it is the property of the particular radioactive nuclei.

  Definition 58 (Decay constant)**.** The decay constant of a nucleus is defined as its probability of decay per unit time

  Rationale: $\lambda = \frac{A}{N} = \frac{-\frac{dN}{dt}}{N} = \frac{-\frac{dN}{N}}{dt} = \frac{\text{probability of decay}}{\text{time interval}}$

- Integrating, we get:

  Theorem 20.7 (Radioactivity Equation 3)**.**

  $$\ln N = \ln N_\circ = -\lambda t$$

  Where $N_\circ$ is the initial number of radioactive nuclides and N the number of nuclides remaining after a time t.

- This gives:

  $$N = N_\circ e^{-\lambda t}$$

Hence, it can be seen that the number of radioactive nuclides remaining after the time **t** decreases exponentially with time. This is the solution to the decay law.

- The above equation shows how the number of parent nuclei varies with time. It is an exponential decay.
- The statistical uncertainty in a count of **N** is equal to $\sqrt{N}$. E.g. (100 ± 10) counts in 1s gives uncertainty of 10% while (10000 ± 100) counts in 100s gives uncertainty of 1%.
- Therefore, $A = \lambda N = \lambda N_\circ e^{-\lambda t} = A_\circ e^{-\lambda t}$, where $A_\circ$ is the initial activity of the sample.

20.18 Half-life

Definition 59 (Half-life)**.** Half-life is defined as the time taken for half the original number of radioactive nuclei to decay

Substituting $N = \frac{1}{2}N_\circ$ and $t = t_{1/2}$ we get:

$$\frac{1}{2}N_\circ = N_\circ e^{-\lambda t_{1/2}}$$

$$2 = e^{\lambda t_{1/2}}$$

Theorem 20.8 (Half-life formula)**.** $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$