Static and Intertemporal Household Decisions

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November 11, 2021

1 Introduction

Chiappori and Mazzocco (2017) survey the typology of models of household decisions. In the paper, the authors highlight two key features of useful models: testability and identifiability. These features are contrasted with the capability of different models to answer the following types of questions: How do households behave with respect to income transfers and taxes? Does it depend on the bargaining power of household members? Are the household members entitled to different degrees of bargaining power? When describing the models, the general assumption is that the household is formed by two persons (a childless couple) that take decisions over:

- consumption of N commodities: public (Q_k) and private (q_h), and
- allocation of time to leisure, labor supply, and household production.

Each individual has preferences over consumption separated from the spouse, where the general version of an individual's preferences is given by $U_i(Q, q_1, q_2)$, an expression where the consumption of the spouse (part of Q and q_2, if $i = 1$) affects the utility of the individual. This utility form represents an unrestrictive form of altruism.

The decision process in the household depends on:

- The degree of altruism, for example, preferences of the caring type that allow for the separation of an individual's utility in the own consumption of public and private goods and the spouse's consumption of these goods. This is represented by the felicity function:

$$U^i\left(Q, q^1, q^2\right) = W^i\left(u^1\left(Q, q^1\right), u^2\left(Q, q^2\right)\right), \quad (1)$$

  One implication is that this type of preferences doesn't allow to represent direct externalities within the couple. Without altruism, an individual's utility doesn't depend on the consumption or utility of consumption of the individual's spouse, so the felicity function reduces to egotistic preferences:

$$U^i\left(Q,q^1,q^2\right)=W^i\left(u^1\left(Q,q^1\right)\right.$$

- Leisure, which enters into the felicity function as a private good or the spouse's utility function.
- Production of commodities within the household. This feature enters in the model via the budget constraint:

$$p'\left(\sum_{k=1}^{N}X_k+\sum_{h=1}^{n}x_h\right)+\sum_{i=1}^{2}w^i\left(l^i+\sum_{k=1}^{N}D_k^i+\sum_{h=1}^{n}d_h^i\right)=\sum_{i=1}^{2}\left(y^i+w^iT^i\right)=Y, \quad \textbf{(BC)}$$

- A distribution factor, z_k, that doesn't affect preferences or the budget constraint, but affects the bargaining power of the household members.

The survey by Chiappori and Mazzocco discusses only about egotistic preferences and household production where consumption goods are separated from leisure. It is important to also note that the authors take household units as given without reference to their formation or dissolution. Finally, the authors mention that certain aspects of the models are built under the restriction of available data for testing the model predictions. For example, as noted by Chiappori and Mazzocco (p. 4, footnote 2), it is not always possible to measure each household member's consumption to disaggregate household consumption, so the model considers consumption of public and private goods at household level.

Model typology

Type	**Model**	**Preferences and constraints**	**Predicted effects**
Static	Single decision makers		
	Unitary model	Unique utility function independent of prices, income, distribution factors, and household size. "Household preferences" derive from the aggregation of individual preferences (Samuelson's Index, Becker's Rotten Kid Theorem, Transferable Utility).	1. Neutral on which member is the beneficiary of income transfers. 2. Ambiguous with respect to taxation, depending on whether the tax unit is the household or the individual. 3. Not useful to answer about household dissolution.
	Multiple decision makers		
	Noncooperative	Members' decisions derive from taking the others' decision as given.	
	Collective	Assume that household decisions are Pareto efficient. Main alternative to the unitary model.	
Dynamic	Intertemporal models (see section 3, not covered in this notes): intertemporal unitary model, intertemporal collective model with or without commitment to future allocation of resources.		

2 The static unitary model

The static unitary model takes the following form:

$$\max_{(X,x,l^1,l^2,d^1,D^1,d^2,D^2)} U^H\left(Q,q,l^1,l^2\right) \quad \textbf{(UM)}$$

subject to

$$p'\left(\sum_{k=1}^{N}X_k+\sum_{h=1}^{n}x_h\right)+\sum_{i=1}^{2}w^i\left(l^i+\sum_{k=1}^{N}D_k^i+\sum_{h=1}^{n}d_h^i\right)=Y, \quad \textbf{(BC)}$$

$$Q_k=F_k\left(X_k,D_k\right) \quad \textbf{(HP)}$$

and

$$\sum_i q_h^i=f_h\left(x_h,d_h\right) \text{ for all } k \text{ and } h.$$

The hypothesis is that households aggregate individual preferences using an index that is independent of other variables (such as individual income, wealth, wages, human capital, or bargaining power). Three ways of aggregating individual preferences into "household preferences" are used:

1. Samuelson's welfare index: the aggregation of preferences from household members is given by the welfare index W, such that individual utility functions can't be retrieved from W because, for example, consumption and leisure of an individual depends both on the individual's utility over them as well as the household utility from the individual's preference for leisure.

$$\max_{(X,x,l^1,l^2,d^1,D^1,d^2,D^2)} \quad \textbf{(UMW)}$$

$$W\left(u^1\left(Q,q^1,l^1\right),u^2\left(Q,q^2,l^2\right)\right)$$

2. Becker's rotten kid theorem: household welfare depends on the utility of children and altruistic parents, i.e., household welfare depends on household behavior. This behavior is represented as a two-stage game, where children play first, choosing actions that

maximize their welfare over a set of commodities and actions that they could take, Ui (ci, a). These actions are, for example, children's labor supply and contribution to household income to purchase (public) goods. For this reason, children's actions condition available household income, Y=Y(a). Children's actions depend on the transfers that they will receive from their parents. In the second round, parents choose transfers to their children to maximize their own utility. Therefore, all members in the household have their own preferences and the game aggregates them into the "household preferences" that are revealed in data on household consumption and time use. The problem is that to assure the game yields optimal outcomes some conditions should be met: (i) there is only one consumption good, (ii) each child's welfare is a normal good in the parent's utility, and (iii) the parent makes a positive transfer to all children. Bergstrom (1989) shows that the conditions proposed by Becker are not sufficient for the rotten kid theorem to hold, while a necessary and sufficient condition for the rotten kid theorem to be satisfied is that the children's utilities are transferable conditional on the children's actions:

$$u^i\left(c^i, a\right) = A\left(a\right) c^i + B_i\left(a\right).$$

3. Transferable utility (TU): this property requires that the Pareto frontier takes the form of a straight line with slope equal to 1 for all values of prices and income. Therefore, members maximize the sum of individual utilities, and not the weighted sum as in the standard model. As a result, any model that assumes efficient outcomes is reduced to the unitary model.

Another justification for the unitary model is the assumption of income pooling. Total household income is assumed to be the sum of labor and nonlabor income, and particularly nonlabor income is pooled by a linear addition. Then, nonlabor income is not distinguished with specific terms in the budget constraint (the distribution of this type of income doesn't matter to explain household behavior), and the Slutsky matrix should be symmetric and negative semidefinite. When testing these predictions, empirical research has generally rejected this justification of income pooling, suggesting that nonlabor income influences household behavior in other ways than by affecting total household income. In other words, empirical evidence seems to indicate that bargaining power is an important explanatory factor of household decisions. In contrast, the unitary model is a good choice for modeling household behavior if one believes that the intra-household decision power is constant across households and over time.

3 The static noncooperative model

These models are games where the Nash equilibrium is achieved by individuals making independent decisions while taking their spouse's decision as given. The typical situations where these models are appropriate are households with internal conflict (that possibly ends in divorce), and cooperative behaviours that result from trying to avoid the noncooperative

outcome. The survey focuses on two-member households and doesn't include the models that consider household production. The utility function has the same ingredients as before, with the noticeable difference that the consumption of the spouse enters separately,

$$u^i(Q, q^1, q^2, l^1, l^2).$$

Another important feature of the model is the identification of the individual contribution to consumption of public goods: Q = Q1 + Q2. In general, the model takes the following form:

$$\max_{Q^1,q^1,l^1} u^1(Q^1+Q^2, q^1, q^2, l^1, l^2) \quad (5)$$

subject to

$$P'Q^1+p'q^1=Y^1,$$

and

$$\max_{Q^2,q^2,l^2} u^2(Q^1+Q^2, q^1, q^2, l^1, l^2) \quad (6)$$

subject to

$$P'Q^2+p'q^2=Y^2.$$

The outcome of the Nash noncooperative model is generally inefficient, because since individual actions are taken by assuming the spouse's action as given, there's no identification of the utility that the individual's consumption of public goods reports to the spouse. Two feasible solutions are implemented to achieve efficiency: (1) eliminate the consumption of public goods from the model and do not independize the spouse's private consumption and leisure; (2) model household decisions as uniquely dependent on public consumption.

The income pooling hypothesis is derived, for example, in case the couple use their individual income to privately provide a single public good and to purchase a private good. The conclusion is that income pooling would occur only in a range of individual incomes. Additionally, in models with endogenous income, it has been shown that the household adopts one of three types of possible "regimes" in public consumption: the wife is a dictator, the husband is a dictator, or both members share the decision power over the consumption of public goods. In the few studies that tested empirically noncooperative models, the number of children in the household seems to explain the adoption of one of these regimes.

4 The static collective model

Collective models distinguish different preferences by member, assuming that the household outcome is efficient: the household behaves as if it were maximizing a weighted sum of the members' utilities subject to a budget constraint and household production constraints (dynamic cooperative models do not necessarily reach the optimum). This second characteristic makes these models axiomatic, while this feature doesn't prevent them to include as special cases most of the static models used to study household behavior.

The reason for efficiency is an agreement between the couple to achieve the optimal by acting cooperatively. Cooperation can be the product of freely taking into account preferences and actions of both spouses, or under the threat of punishment after (repeated games). The main feature that differentiates the collective model from the unitary framework is that household decisions depend on the intra-household decision power, in addition to prices and income.

$$\max_{(x,X,d^1,D^1,d^2,D^2,l^1,l^2)} \quad \textbf{(P)}$$

$$\mu^1 U^1(Q,q^1,l^1)+\mu^2 U^2(Q,q^2,l^2)$$

subject to

$$p'\left(\sum_{k=1}^{N} X_k+\sum_{h=1}^{n} x_h\right)+\sum_{i=1}^{2} w^i\left(l^i+\sum_{k=1}^{N} D_k^i+\sum_{h=1}^{n} d_h^i\right)=Y, \quad \textbf{(BC)}$$

$$Q_k=F_k(X_k,D_k) \quad \textbf{(HP)}$$

and

$$\sum_i q_h^i=f_h(x_h,d_h)$$

for all k and h.

Because the Pareto weights μ_1 an μ_2 generally depend on prices, wages, income, and distribution factors, the model considers an array of factors that explain household decisions and is able to corroborate bargaining power (individual income matters on household decisions before the pooling takes place). In case the decision power is constant, the cooperative model yields the same results as the unitary model.

Collective models are also well suited to implement caring type preferences in Problem (P), instead of egotistic preferences. However, it is proven that the use of egotistic preferences

generate all the results that yield the use of caring type preferences.

5 Empirical part

The empirical test for each model is summarized in table 2 of Chiappori and Mazzocco (2017).

The test for the unitary model that is popular is the income pooling hypothesis. One implication of the unitary model is that individual level of income is irrelevant in the decision making process of the household once the total income level is controlled. In other words, only the total income of the household matters since it enters the budget constrain in the maximization problem. According to Chiappori and Mazzocco (2017), any feature of income allocation that is important in the data but missing from the unitary model will generate a rejection of income pooling. The fact that this only accounts for the resource-allocation of the household makes the test say nothing about the static collective models where intra-household allocation has a significant role. Also, the rejection does not lead to any specific model and also does not only reject the unitary model. Chiappori and Mazzocco (2017) mention that there are numerous literature that rejects income pooling.

There is another test for the unitary model which checks the Slutsky matrix by seeing if whether or not the Slutsky matrix is symmetric and negative semi-definite. There are numerous literature that rejects the symmetric and negative semi-definite Slutsky matrix but the caution is that the test itself has a lot of auxiliary assumptions. Thus, the rejection can be either on the auxiliary assumptions or the hypothesis or both.

For the static collective model, there are three tests presented in Chiappori and Mazzocco (2017) as popular ones. The first one is called the Proportionality condition which uses the fact that the decision factor, which is a variable that affects the decision power, should only affect household decision through the relative decision power. From this assumption, the following equations in figure 1 holds where l is leisure and z_i is any decision factor and μ is the Pareto weights that represents the decision power. Figure 1 means that the ratio of the partial derivatives of leisure or any other good with respect to two decision factor has the same ratio regardless of other variables. It is immediate that the disadvantage of the test is the assumption that the distribution factors affect household decision only through the one-dimensional function μ. Although other limitations, many literature fail to reject the Proportionality condition.

Figure 1: Proportionality condition

$$\frac{\partial \bar{l}^1}{\partial z_j} \Big/ \frac{\partial \bar{l}^2}{\partial z_j} = \frac{\partial l^1}{\partial \mu} \Big/ \frac{\partial l^2}{\partial \mu}. \qquad (18)$$

The next test is based on the Slutsky matrix element $s_{i,j}$ that is obtained by summing the partial derivative of the Marshallian demand function with respect to price and income,

respectively. The part associated with the μ in figure 2 is the effect of the Pareto weights which has an implication of decision power in the household. In the basic test, it is assumed that this decision power is fully characterized by a single factor, the Pareto weights. Thus, the Slutsky matrix is a sum of two parts where the first part is the classic part and the second is a rank one matrix. The generalized version of this test shows the collective approach is not rejected mostly. The last test uses the GARP condition but the difference with the Slutsky matrix plus rank one test is that it is non-parametric.

Figure 2: Sluskty equation

$$s_{i,j}=\frac{\partial q_i}{\partial p}+\frac{\partial q_i}{\partial Y}q_i^*+\frac{\partial q_i}{\partial \mu}\left(\frac{\partial \mu}{\partial p}+\frac{\partial \mu}{\partial Y}\right)q_i^*.$$

The inter-temporal models can be tested with the Euler equation of the dynamic problem, which is called as excess sensitivity test. For the unitary model, the decision is fully characterized by the Euler equation given the current information set. Thus, according to Chiappori and Mazzocco (2017), the difference between the current marginal utility of consumption and next period expected marginal utility of consumption should be independent of variables that are known to the household at the time of the decision. This hypothesis is regularly rejected.

The difference with the intertemporal collective models with unitary is that variables that have an effect on the decision power can be accounted for the difference between the current and expected future marginal utility of consumption. The paper gives an interesting and intuitive example. Say we have two households where husbands have same risk aversion and wives have same risk aversion but the wives are more risk averse than the husbands. Assume that the wife of the first household has more decision power than the other wife because the first household wife has more education. All the other conditions are identical. The first household will be more risk averse and that will appear in the Euler equation and will lead to a different consumption path. Now the difference in consumption path will be explained by education and other education correlated factors such as wage. The reduced form result is therefore that education and wage will have an effect on the household Euler equation. Therefore, the intertemporal unitary model can be rejected if we can find a variable that has a significant effect on the Euler equation with the explanation associated with decision power.

The problem is that this test does not distinguish between the environment difference associated with commitment. Limited-commitment intertemporal collective models (LIC) happens when the decision to stay as a couple can be changed each period, meaning that the people in the model cannot fully commit. On the other hand, Full-commitment intertemporal collective model happens when the decision power is decided at the formation of household and kept constant. Thus, the decision power in the FIC model only affect household decision at the formation of the household but in the LIC, this happens each period. Back to the survey example, if the second household wife decides to get more education and this decision was not expected at the time of marriage, after getting married, the FIC model will assume

that there is no change in the decision power even though the wife might attain the same level of education as the first household wife while the LIC will take the difference into account. This implies that economists can test if the consumption path of a given household changes over time in response to variation that are known at the time the Euler equation were computed (current variables) but were unexpected before that period. The FIC model is said to be rejected in the US.

Further on in section 4, Chiappori and Mazzocco (2017) gives a survey of literature where non-unitary models are favored over unitary models after empirical tests. Moreover, the authors introduce literature about the identification results, meaning how much can we retrieve from data.

Figure 3: Testable implications

TABLE 2

Relationship between the Three-stage Formulation and Different Testable Implications

	Static models		
	Unitary	**Nash bargaining**	**Collective**
Stage 3:		Proportionality	Proportionality
Intra-household allocation stage		Slutsky symmetry + rank 1 Efficiency with production Collective GARP	Slutsky symmetry + rank 1 Efficiency with production Collective GARP
Stage 2:	Income		
Resource allocation stage	Pooling		
Stage 1:			
Intertemporal stage			
	Intertemporal models		
	Unitary	Full efficiency	No commitment
Stage 3:		Proportionality	
Intra-household allocation stage		Slutsky symmetry + rank 1 Efficiency with production Collective GARP	
Stage 2:	Income		
Resource allocation stage	Pooling		
Stage 1:	Euler test	Euler test	Euler test
Intertemporal stage	(Distribution factors have no effect)	(Only cross-sectional variation in distribution factors has effect)	(Both cross-sectional and longitudinal variation have effect)

6 Policy

Chiappori and Mazzocco (2017) gives a survey of what different model can say about different policies. They consider three policies which are cash transfers, joint versus individual taxation and laws related to household dissolution. Regardless of the policy of interest, unitary models cannot explain the intra-household decision power change coming from the policy change. Meanwhile, LIC and FIC can account for the decision power change but the FIC only considers the decision power at the formation of the household while LIC can account for the change over time.

When analyzing cash transfer and joint versus individual taxation, unitary models can explain the effect of the policies on the budget constraint but fail to account for the household decision power change from the change of policy. For instance, if the research is focusing on the effect of Progressa, which gives cash to the wife, on the decision power inside the household, the model should adopt LIC or FIC. Laws regarding dissolution of household is usually not suitable to be analyzed with the unitary model according to Chiappori and Mazzocco (2017) except for the restrictive case of transferable utility. Thus, LIC and FIC should be a better model while Chiappori and Mazzocco (2017) gives suggestive evidence that LIC might be emprically more supported.

References

Chiappori, Pierre-Andre and Maurizio Mazzocco (2017). "Static and Intertemporal Household Decisions". In: *Journal of Economic Literature* 55.3, pp. 985–1045. DOI: 10.1257/jel.20150715. URL: https://www.aeaweb.org/articles?id=10.1257/jel.20150715.