Baryo/Leptogenesis

2018. 8. 20. CAU

I. Thermal Universe - Basics

■ Energy content of the current universe

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}, \quad \rho_c = \frac{3H_0^2}{8\pi G_N} = 3M_p^2 H_0^2$$

$$\approx \boxed{10.5\, h^2\ \mathrm{GeV/m^3}}$$

$$M_p = \frac{1}{\sqrt{8\pi G_N}} \approx 2.4\times 10^{18}\ \mathrm{GeV}$$

$$H_0 = 100\,h\ \mathrm{km/s/Mpc}$$

$$h \approx 0.673$$

$$\begin{cases} \Omega_{Atoms} \approx 0.05 \approx \Omega_N \simeq \Omega_B \\ \Omega_{DM} \approx 0.25 \\ \Omega_{DE} \approx 0.70 \end{cases}$$

■ The abundances determined by thermodynamics, particle physics, and ...

■ Distribution ftn of bosonic/fermionic particles

$$f^{eq}(\vec{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \mp 1} \quad \begin{cases} - & b \\ + & f \end{cases}$$

number density $\quad n_i = g_i \int \frac{d^3\vec{p}}{(2\pi)^3} f_i(\vec{p})$

energy density $\quad \rho_i = g_i \int \frac{d^3\vec{p}}{(2\pi)^3} E f_i(\vec{p})$.

	$T \gg m_i$	$T \ll m_i$ ($\lvert\mu\rvert \lll$)
n_i^{eq}	$\frac{\zeta(3)}{\pi^2} g_i T^3 \begin{cases} 1 & b \\ \frac{3}{4} & f \end{cases}$	$g_i \left(\frac{m_i T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_i}{T}}$
ρ_i^{eq}	$\frac{\pi^2}{30} g_i T^4 \begin{cases} 1 & b \\ \frac{7}{8} & f. \end{cases}$	$m_i n_i^{eq}$

Boltzmann approx

■ Total radiation energy density

$$\rho_R = \frac{\pi^2}{30} g_* T^4, \quad g_* = \sum_{i \in b} g_i + \frac{7}{8} \sum_{i \in f} g_i$$

$$T \gg 100\text{ GeV}, \quad g_* = \begin{cases} 106.75 & \text{SM} \\ 227.75 & \text{MSSM} \end{cases}$$

Expansion rate $H = \frac{\dot{a}}{a}$ in radiation era $(H = \frac{1}{2t})$.

$$\rho_R = 3 M_p^2 H^2$$

$$H = \sqrt{\frac{\pi^2}{90} g_*} \frac{T^2}{M_p} \overset{\text{SM}}{\approx} 3.4 \frac{T^2}{M_p}.$$

■ Entropy density s

$$S = \text{constant} = \frac{d((\rho_R + P_R)V)}{dT} = \frac{4}{3} \frac{d(\rho_R V)}{dT} = \frac{4}{3} \frac{\rho_R}{T} V$$

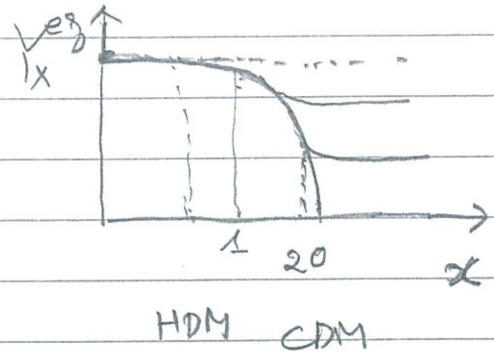
$$s = \frac{S}{V} = \frac{2\pi^2}{45} g_{*S} T^3, \quad g_{*S} = g_* \text{ in most cases}$$

■ For a particle number density n_x that is conserved

$$\frac{n_x V}{s V} = \text{const.} \qquad Y_x = \frac{n_x}{s} = \underline{\text{constant}}.$$

$$\frac{n_X^{eq}}{s} = \begin{cases} \underbrace{\frac{45\zeta(3)}{2\pi^4}}_{\sim 0.28} \frac{g_X}{g_{*s}} \begin{cases} 1 & b \\ \frac{3}{4} & f \end{cases}, & T \gg m_X \\[2ex] \underbrace{\frac{45}{2\pi^2 (2\pi)^{3/2}}}_{0.14} \frac{g_X}{g_{*s}} \underbrace{\left(\frac{m}{T}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}}_{x^{\frac{3}{2}} e^{-x}} , & T \ll m_X \end{cases} \quad \left(x \equiv \frac{m}{T}\right)$$

Freeze-out of a particle X : $\boxed{\Omega_X h^2 \simeq 0.1 \, \frac{x_f}{20} \, \frac{10^{-9}\,\mathrm{GeV}^{-2}}{\langle \sigma v \rangle}}$



$\Gamma = n \langle \sigma v \rangle \ll H$ at x_f

HDM $x_f \ll 1$, $\begin{cases} Y_X \sim 10^{-2} \\ m_X \sim 350\,\mathrm{e} \end{cases}$

CDM $x_f \simeq 20$ $\begin{cases} Y_X \sim 10^{-12} \\ m_X \sim 350\,\mathrm{Ge} \end{cases}$

(*) $\Omega_B = \Omega_N \simeq \boxed{0.05}$, i.e., $\frac{m_N Y_N}{m_{DM} Y_{DM}} \approx \frac{1}{5}$

$= m_N \sim 1\,\mathrm{GeV}$.

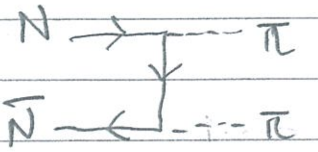
$\Rightarrow Y_N \approx \frac{1}{5} \frac{m_{DM} Y_{DM}}{m_N}$ $\simeq 0.35\,\mathrm{eV}$, $m_N \simeq 1\,\mathrm{GeV}$

$\sim 10^{-10}$

If $N-\bar{N}$ freeze-out by QCD,

$\mathcal{L}_{NN\pi} = g_A \frac{\partial_\mu \pi^a}{f_\pi} \bar{N} \gamma^\mu \gamma_5 \tau^a N$

Classically,
$\langle \sigma v \rangle \approx \pi r_N^2 \sim 3\,f$
$\sim 80\,\mathrm{GeV}^{-2}$



$\langle \sigma v \rangle \sim C^2 \frac{m_N^2}{4\pi f_\pi^4} \sim C^2 10^3\,\mathrm{GeV}^{-2}$

$\Rightarrow \Omega_N \approx \frac{0.1}{h^2} \frac{x_f^N}{20} \frac{10^{-9}}{10^3} \sim \boxed{10^{-13}}$!! $(Y_N \sim 10^{-22})$

■ particle asymmetry

$$\Delta n_X^{eq} = n_X^{eq} - n_{\bar{X}}^{eq} = g_X \int \frac{d^3p}{(2\pi)^3} \left[f_X^{eq}(E,\mu) - f_X^{eq}(E,-\mu) \right]$$

$$\xRightarrow{|\mu| \ll T} \frac{1}{3} g_X T^2 \mu_X \begin{cases} 1 & b \\ \frac{1}{2} & f \end{cases}$$

(*) Baryons must have asymmetry

$$\boxed{Y_{\Delta B} = \frac{\Delta n_B^{eq}}{s} \simeq 0.87 \times 10^{-10}}$$

which survives the $N-\bar{N}$ annihilation by QCD

Is it an initial condition?

Can this be generated dynamically?

↳ Baryogenesis

■ Sakharov condition for generating B(L) asymmetry from a symmetric universe

1) $\not{B}$ ($\not{L}$)

2) $\not{C}$ & $\not{CP}$

3) out of equilibrium.

$$\begin{array}{ccc} (i \xrightarrow{\cancel{P}} f) & \overset{C}{\Longrightarrow} & (\bar{i} \rightarrow \bar{f}) \\ \Downarrow P & & \Downarrow P \\ (i' \rightarrow f') & \overset{C}{\Longrightarrow} & (\bar{i}' \rightarrow \bar{f}') . \end{array}$$

$$\Delta\Gamma = \Gamma(i \to f) + \Gamma(i' \to f') - \Gamma(\bar{i} \to \bar{f}) - \Gamma(\bar{i}' \to \bar{f}') \neq 0 .$$

equilibrium : $\left.\begin{array}{l} i \longleftrightarrow f \\ \bar{i} \longleftrightarrow \bar{f} \end{array}\right\} \Delta\Gamma = 0 .$

SM : 1) B+L violation by quantum anomaly.

2) $\cancel{CP}$ by CKM phase $\delta \Rightarrow \delta_{CP} \approx \lambda^6 \sin\delta \sim 10^{-}$

3) out of equilibrium during EWPT.

1st order PT $\Rightarrow m_h \lesssim 80$ GeV. *

New Physics beyond SM is required!

ex) GUT baryogenesis : (breaking B-L gauge symmetry)

Heavy gauge bosons decay out of equilibrium

generating B-L asymmetry.

II. Spharelon & L-to-B conversion

- Electroweak baryon/lepton number violation

B and L are "anomalous":

J^μ_B and J^μ_L are not consered currents quantum mechanically although they are symmetries of the classical Lagrangian.

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{3g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$\leftarrow SU(2)_L$

RHS is a total derivative

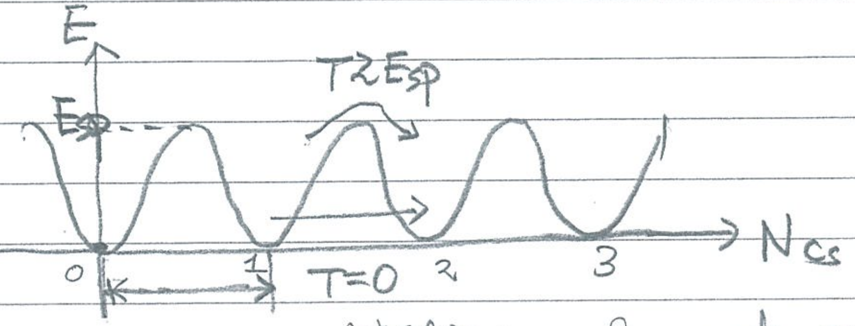
$$F\tilde{F} \propto \partial_\mu K^\mu$$

where $K^\mu = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \left(A^a_\nu \partial_\rho A^a_\sigma + \frac{g}{3} \varepsilon_{abc} A^a_\nu A^b_\rho A^c_\sigma \right)$

and there exists non-trivial field configuration of $A^a_\mu(x)$ with

$$\int_{V_4} \partial_\mu K^\mu = \int_{V_3} K^0 d^3x = \frac{16\pi^2}{g^2} N_{CS}$$

$\leftarrow$ integer



$|0\rangle \longleftrightarrow |q^a q^b q^c l\rangle \varepsilon_{abc}$ for each generation

(SU(2) invariant stat

Different vacua corresponding the topological number N_{cs} ⇒ quantum tunneling between N_{cs} and $N_{cs}+1$

at $T=0$: $e^{-\frac{4\pi}{\alpha_2}} \sim 10^{-164}$

- There is also a field configuration sitting on top of barrier which is a solution of static eqs. of motion with finite energy
  = "sphaleron" : unstable stationary point.

$$A_0^a = 0, \quad A_a^i = \frac{i}{g} \frac{\epsilon_{ijk} x_j \tau_k}{r^2} f(\xi)$$

[rolling solution]

$$\phi^a = \frac{v}{\sqrt{2}} \frac{i \vec{\tau} \cdot \vec{x}}{r} \begin{pmatrix} 0 \\ 1 \end{pmatrix} h(\xi)$$

where $\xi = gvr$ and $f(0) = h(0) = 0$,
and $f(\infty) = h(\infty) = 1$

- The height of the barrier is

$$E_{sp} = \int d^3x \left[\frac{1}{4} F^2 - |D_\mu \phi|^2 + \frac{\lambda}{4} \left(\phi^2 - \frac{v^2}{2} \right)^2 \right]$$

$$\approx \frac{c}{\alpha_2} m_W \approx 10\,\text{TeV}$$

- At finite temperature $T \gtrsim E_{sp}$, the transition is active : $\Gamma_{sp} \sim 250\, \alpha_W^5\, T\, e^{-E_{sp}/T} \gg H \sim \sqrt{g_*} \frac{T^2}{M_P}$

$$\Rightarrow T \lesssim \alpha_W^4 \frac{M_P}{\sqrt{g_*}} \sim 10^{12}\,\text{GeV}$$

providing effective B+L violation at $T \neq 0$

- Initial lepton asymmetry transforms into baryon asymmetry through the sphaleron process.

$\downarrow$ active for $T \approx 10^2 - 10^{12}$ GeV

Naively,

$$L_i \equiv \frac{1}{2}\cancel{(B+L)_i} - \frac{1}{2}(B-L)_i$$

$$\Gamma_{sp}\downarrow \quad \cancel{B+L} \qquad (B-L)_f = (B-L)_i = -2L_i$$

$$B_f = \frac{1}{2}\cancel{(B+L)_f} + \frac{1}{2}(B-L)_f = -L_i$$

- <u>L-to-B conversion</u>

(note) $\Delta n_X = \frac{1}{3} g_X T^2 \mu_X \begin{cases} 1 & b \\ \frac{1}{2} & f \end{cases}$

$$n_B \equiv \sum_{i=1}^{N_f} \frac{1}{3}\left(2\Delta n_{q_i} - \Delta n_{\bar{u}_i} - \Delta n_{\bar{d}_i}\right) = \frac{1}{6}T^2 \sum_i \left(2\mu_{q_i} - \mu_{\bar{u}_i} - \mu_{\bar{d}_i}\right)$$

$$n_L \equiv \sum_{i=1}^{N_f} \left(2\Delta n_{l_i} - \Delta n_{\bar{e}_i}\right) = \frac{1}{6}T^2 \sum_i \left(2\mu_{l_i} - \mu_{\bar{e}_i}\right)$$

① sphaleron in equilibrium : $|0\rangle \leftrightarrow |qqql\rangle$

$$\sum_{SU(2)_L} (B+L) = 0 \Rightarrow \boxed{\sum_i \left(3\mu_{q_i} + \mu_{l_i}\right) = 0}$$

② Yukawa interaction in equilibrium

$$\boxed{\begin{aligned} -\mu_H + \mu_{l_i} + \mu_{\bar{e}_i} &= 0 \\ -\mu_H + \mu_{q_i} + \mu_{\bar{d}_i} &= 0 \\ +\mu_H + \mu_{q_i} + \mu_{\bar{u}_i} &= 0 \end{aligned}}$$

③ charge neutrality : $\sum_X \Delta n_X Y_X = 0$

$$\boxed{\sum_i \left(\mu_{q_i} - 2\mu_{\bar{u}_i} + \mu_{\bar{d}_i} - \mu_{e_i} + \mu_{\bar{e}_i}\right) + 2\mu_H = 0}$$

Assuming "flavor-independent" : $\mu_{q_i} = \mu_q$ etc.
(true for $T \lesssim 10^5$ GeV) ※

6 variables : $\mu_q, \mu_{\bar{u}}, \mu_{\bar{d}}, \mu_e, \mu_{\bar{e}}, \mu_H$.
5 equations : ① + ② + ③
to be solved in terms of $\underline{\mu_e}$

$$B = -\frac{4}{3} N_f \mu_e$$

$$L = \frac{14N_f + 9}{3(2N_f+1)} N_f \mu_e$$

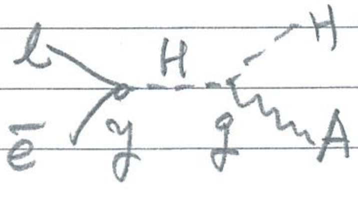
$$B - L = -\frac{22N_f + 13}{3(2N_f+1)} N_f \mu_e$$

$$B = \frac{4(2N_f+1)}{22N_f+13}(B-L) = C(B-L)_i = -CL_i$$

$$\parallel$$

$$C = \frac{28}{79} \text{ for } N_f = 3$$

※ flavor-dependence $\Gamma_{Yuk} > H$



$$4\pi\alpha_e\alpha_2 T > \frac{T^2}{M_P}$$

$$T < \alpha_e \alpha_2 M_P \sim \begin{cases} 10^{12}\text{ GeV} & \tau \\ 10^{10} & \mu \\ 10^5 & e \end{cases}$$

III. Leptogenesis & neutrino masses

* Neutrinos are massive.

ν: neutral $\qquad \nu \neq \nu^c \qquad$ or $\qquad \nu = \nu^c$

Dirac $\qquad\qquad$ Majorana

~~Mass operator in Weyl basis~~

$$\psi_D = \begin{pmatrix} \nu \\ \bar{\nu}^c \end{pmatrix}, \quad m_\nu^D \bar{\psi}_D \psi_D = m_\nu^D (\bar{\psi}_{DR} \psi_{DL} + \bar{\psi}_{DL} \psi_{DR})$$

$$= m_\nu^D (\nu \nu^c + \bar{\nu} \bar{\nu}^c)$$

$$\psi_M = \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix}, \quad m_\nu^M \bar{\psi}_M \psi_M = m_\nu^M (\nu\nu + \bar{\nu}\bar{\nu})$$

(*) The usual Higgs coupling for m_ν^D

"$SU(2) \times U(1)$ singlet"

$$-\mathcal{L} = y_\nu l \cdot H \nu^c + h.c = y_\nu \begin{pmatrix} \nu \\ e \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \nu^c + h.c$$

$$= y_\nu (\nu H^0 \nu^c - e H^+ \nu^c) + h.c$$

$$\Rightarrow m_\nu^D = y_\nu \langle H^0 \rangle \approx 0.1\,\text{eV}.$$

$$y_\nu = \frac{m_\nu^D}{\langle H^0 \rangle} = \frac{0.1\,\text{eV}}{174\,\text{GeV}} \sim \underline{\underline{10^{-12}}}.$$

(*) Seesaw for light Majorana neutrinos

$$-\mathcal{L} = y_\nu l \cdot H \nu^c + \frac{1}{2} M \nu^c \nu^c + h.c.$$

$$\mathcal{M} = \begin{array}{c|cc} & \nu & \nu^c \\ \hline \nu & 0 & m_D \\ \nu^c & m_D & M \end{array} \xrightarrow[\text{Seesaw}]{m_D \ll M} \begin{pmatrix} -\frac{m_D^2}{M} & 0 \\ 0 & M + \frac{m_D^2}{M} \end{pmatrix}$$

$$\Rightarrow m_\nu^M \approx \frac{m_D^2}{M} \approx 0.1\,eV \quad \begin{cases} m_D \sim 100\,GeV \\ M \sim 10^{14}\,GeV \end{cases}$$

$$\begin{cases} m_D \sim 0.1\,MeV \\ M \sim 10^{2}\,GeV \end{cases}.$$

◉ Heavy RHN ($\nu^c = N$) decay can produce Lepton & CP asymmetry.

① $\cancel{L}$: Majorana mass M ($\Delta L = 2$).

② asymmetry : $\Delta\Gamma = \Gamma(N \to \ell H) - \Gamma(N \to \bar{\ell}\bar{H}) \neq 0$

$\cancel{CP}$ in phases of y_ν

③ out of equilibrium : $\Gamma_N < H$.

QFT basics of CP asymmetry

1. $\not{CP}$ effect must arise from loop corrections.

$S = 1 + iT$ $\qquad SS^\dagger = S^\dagger S = 1$

↑ scattering ↑ transition matrix $\qquad$ ↓ unitarity

$$T_{ij} - T^*_{ji} = i\,(TT^\dagger)_{ij}$$

$$|T_{ij}|^2 - |T_{\bar{i}\bar{j}}|^2 \overset{CPT}{=} |T_{ij}|^2 - |T_{ji}|^2$$
$$= |i(TT^\dagger)_{ij} + T^*_{ji}|^2 - |T_{ji}|^2$$
$$= -2\,\mathrm{Im}\left[(TT^\dagger)_{ij} T^*_{ji}\right] + |(TT^\dagger)_{ij}|^2$$

in perturbation of couplings

the difference is at higher order than $|T_{ij}|^2$ ← tree

2. No asymmetry is generated at first order of $H_{\not{B}}$, that is, one must consider at least of second order in $H_{\not{B}}$. (Weinberg theorem)

Nanopoulos – Weinberg 1979 $\qquad (H \equiv H^0 + H_{\not{B}})$

$$A(X \to f) = {}_{out}\langle f | H_{\not{B}} | X \rangle_{in}$$
$$A(\bar{X} \to \bar{f}) = {}_{out}\langle \bar{f} | H_{\not{B}} | \bar{X} \rangle_{in} \overset{CPT}{=} {}_{out}\langle X | H_{\not{B}} | f \rangle_{in}$$
$$= \sum_g {}_{out}\langle X | H_{\not{B}} | g \rangle_{in}\, {}_{out}\langle g | f \rangle_{in}$$

$$= \sum_g A(X\to g)^* S^0_{gf}$$

$$\Gamma_{\bar{B}} \equiv \sum_{\bar{f}\in \bar{B}} \rho_{\bar{f}} |A(\bar{X}\to\bar{f})|^2$$ phase space

$\overset{CPT}{=} \rho_f$

$$= \sum_{g,h} A(X\to g)^* A(X\to h) \underbrace{\sum_{f\in B} \rho_{\bar{f}} S^0_{gf} S^{0*}_{hf}}_{\begin{cases} \rho_g \delta_{gh}, & g\in B \\ 0 & g\notin B \end{cases}}$$

$$= \sum_{g\in B} \rho_g |A(X\to g)|^2$$

$$\equiv \underline{\Gamma_B\ !}$$

3. $\cancel{CP}$ in mixing with $\cancel{B}$ at tree (first order in $H_{\cancel{B}}$).

$$X \xleftrightarrow{CP} \bar{X} \qquad |X_{\substack{1\\2}}\rangle = \frac{1}{\sqrt{2}}(|X\rangle \pm |\bar{X}\rangle) \qquad CP = \pm 1.$$

$\cancel{CP}$ in mixing ($K^0-\bar{K^0}$ system).

$$|\tilde{X}_1\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}(|X_1\rangle + \varepsilon|X_2\rangle) = \frac{(1+\varepsilon)|X\rangle + (1-\varepsilon)|\bar{X}\rangle}{\sqrt{2(1+|\varepsilon|^2)}}$$

$$|\tilde{X}_2\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}(\varepsilon|X_1\rangle + |X_2\rangle) = \frac{(1+\varepsilon)|X\rangle - (1-\varepsilon)|\bar{X}\rangle}{\sqrt{''}}$$

CP-conserving, $\cancel{B}$ decay in X.

$$A(X\to f) = A(\bar{X}\to\bar{f}) = \tfrac{1}{2}(1+\lambda)A_0 \quad \lambda \text{ real}$$
$$A(X\to\bar{f}) = A(\bar{X}\to f) = \tfrac{1}{2}(1-\lambda)A_0 \quad \cancel{B}$$

$$|A(X_{\substack{1\\2}}\to f)|^2 - |A(X_{\substack{1\\2}}\to\bar{f})|^2 = \frac{|A_0|^2}{2(1+|\varepsilon|^2)} 4\lambda\, Re(\varepsilon)$$

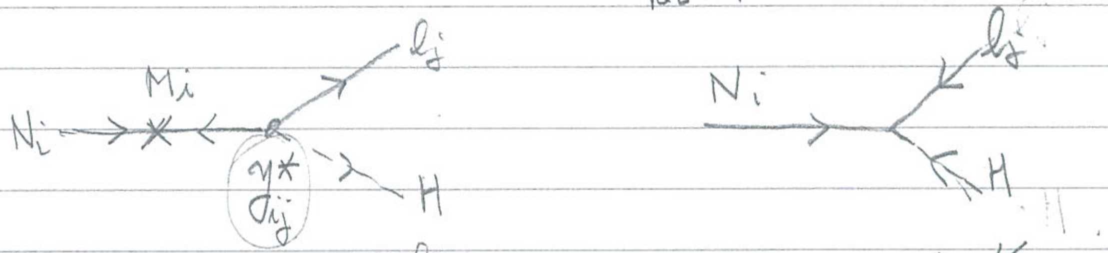
$$\therefore\ \varepsilon_f \equiv \frac{\Gamma_f - \Gamma_{\bar{f}}}{\Gamma_f + \Gamma_{\bar{f}}} = \frac{8\lambda\, Re(\varepsilon)}{(1+|\varepsilon|^2)(1+\lambda^2)} \neq 0$$

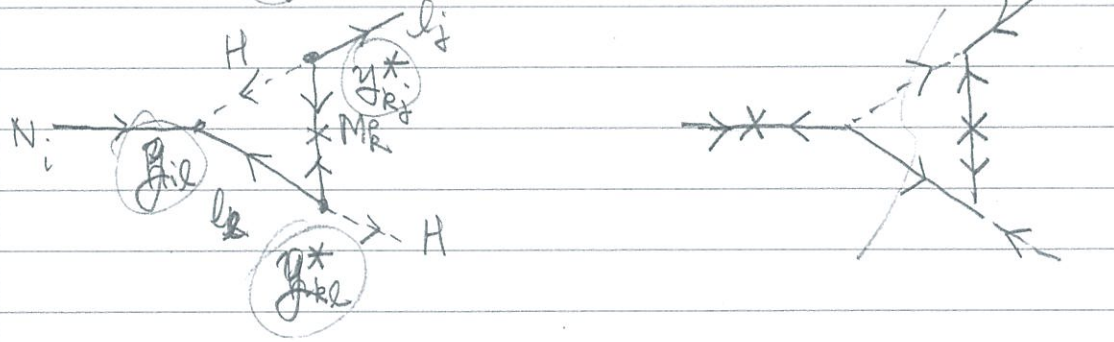
L & CP asymmetry in Leeson

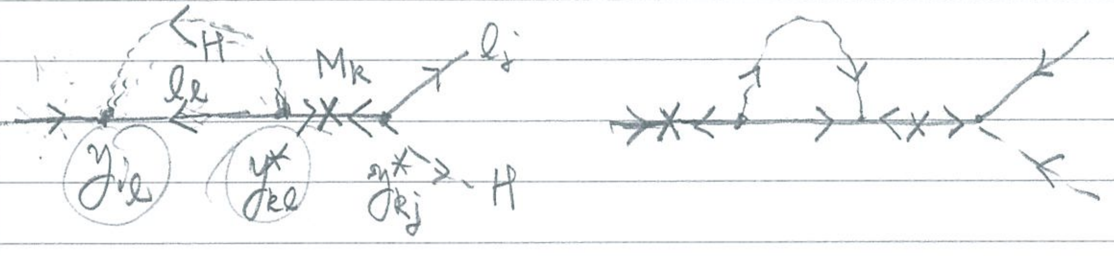
$$-\mathcal{L} = y_{ij} N_i l_j H + \frac{M_i}{2} N_i N_i + h.c.$$

$N_i \to l_j H$ $\qquad\qquad$ $N_i \to \bar{l}_j \bar{H}$

$$\varepsilon_{ij} \equiv \frac{\Gamma(N_i \to l_j H) - \Gamma(N_i \to \bar{l}_j \bar{H})}{\Gamma_{tot}}.$$







$$\Delta\Gamma_{ij} \propto \left| y^*_{ij} + \sum_{kl} y_{il} y^*_{kl} y^*_{kj} (I_V + I_S) \right|^2$$

$$- \left| y_{ij} + \sum_{kl} y^*_{il} y_{kl} y_{kj} (I_V + I_S) \right|^2$$

$$= \sum_{kl} \Big[\underbrace{(y_{ij} y_{il} y^*_{kl} y^*_{kj} - c.c)}_{2i\, \mathrm{Im}(y^4)} (I_V + I_S) + c.c \Big]$$

$$= -4\, \mathrm{Im}(y^4)\, \mathrm{Im}(I_V + I_S).$$

$$I_{V,S} = i\,\theta(M - m_l - m_H) + \text{real}.$$

Note) ① $I_{V,S} = i\,\theta(M - m_\ell - m_H) f_{V,S}$ + real ftn.

$\mathrm{Im}(I_{V+S}) \neq 0$ requires $M > m_\ell + m_H$ onshell

contribution from diagrams where an on-shell-cut is present.

② Non-trivial CP phase in y required.

③ No contribution from $k=i$ (one $N \Rightarrow$ No CP)

④ at least, two N and one ℓ is required

$$\boxed{\varepsilon_i = \sum_j \varepsilon_{ij} = -\frac{1}{8\pi}\sum_k \frac{\mathrm{Im}\left[(yy^\dagger)_{ik}\right]^2}{(yy^\dagger)_{11}}\left[f_V\left(\frac{M_k^2}{M_i^2}\right) + f_S\left(\frac{M_k^2}{M_i^2}\right)\right]}$$

$$f_V(x) = \sqrt{x}\,\ln\left(1+\frac{1}{x}\right)$$

$$f_S(x) = \frac{2\sqrt{x}}{x-1}$$

⑤ resonance effect from self-energy diagram when $x \simeq 1$ ($M_k \simeq M_i$).

⑥ Davidson - Ibarra bound.

when $M_1 \ll M_{2,3}$, $f_{V+S}(x) \simeq 3\sqrt{\frac{1}{x}} \simeq 3\frac{M_1}{M_{2,3}}$

$$\varepsilon_1 = -\frac{3}{8\pi}\frac{M_1}{(yy^\dagger)_{11}}\underbrace{\sum_k \frac{\mathrm{Im}\left[(yy^\dagger)_{1k}\right]^2}{M_k}}_{= (yy^\dagger)_{1k}\frac{1}{M_k}(y^* y^T)_{k1}}$$

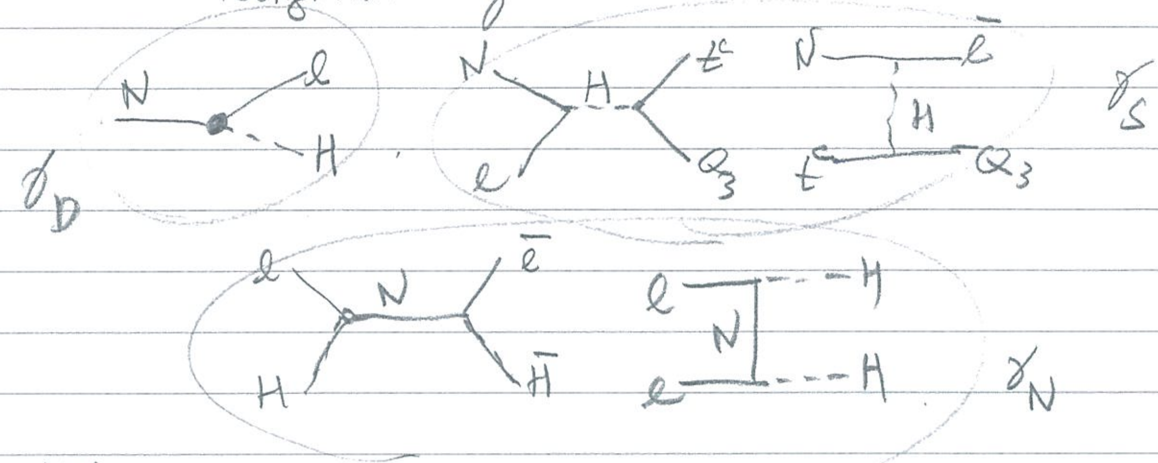
$$\underbrace{y\frac{1}{M_k}y^T}_{= -\frac{M_\nu^*}{v_H^2}}$$

$$= \frac{3}{8\pi}\frac{M_1 m_{\nu_3}}{v_H^2}\frac{\mathrm{Im}\left[y M_\nu^* y^T\right]_{11}}{(yy^\dagger)_{11}}$$

$$\leq \frac{3}{8\pi}\frac{M_1 m_{\nu_3}}{v_H^2} \simeq 2\times 10^{-6}\left(\frac{M_1}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{m_{\nu_3}}{0.05\,\mathrm{eV}}\right)$$

Asymmetry generation out of equilibrium

- Boltzmann equation -



$$\frac{dY_N}{dz} = -zK\,(Y_N - Y_N^{eq})(\gamma_D + 2\gamma_{S_s} + 4\gamma_{S_t})$$

$$\frac{dY_{\Delta l}}{dz} = zK\gamma_D\left[\varepsilon(Y_N - Y_N^{eq}) - \frac{Y_N^{eq}}{2Y_l^{eq}}Y_{\Delta L}\right]$$

$$- \frac{Y_{\Delta l}}{Y_l^{eq}}\left(2\tilde{\gamma}_N + 2\gamma_{S_t} + 2\gamma_{S_s}\frac{Y_N}{Y_N^{eq}}\right)$$

$$K \equiv \frac{\Gamma_N}{H(T=M)}, \quad z = \frac{M_1}{T}, \quad \begin{cases} \Gamma_N = \frac{1}{8\pi}(yy^\dagger)_{11}M \\ H_M \approx \sqrt{g_*}\frac{M^2}{m_{pl}} \end{cases}$$

$$Y_{\Delta B} = \frac{28}{79}Y_{\Delta L}, \quad Y_{\Delta L} = \frac{n_N^{eq}}{s}\,\varepsilon\cdot\kappa$$

Sphaleron conversion in SM

"later"

$= \frac{1}{2}\cancel{(B+L)} \to 0$

$- \frac{1}{2}(B-L)$

?

QFT factor

efficiency factor depending on K

solving B.E.

$$\frac{\frac{3}{4}\frac{\zeta(3)}{\pi^2}g_N T^3}{\frac{2\pi^2}{45}g_{*s}T^3} = 0.21\frac{g_N}{g_{*s}}$$

$$\Rightarrow \boxed{Y_{\Delta B} \simeq \frac{0.15}{g_*}\cdot\varepsilon\cdot\kappa} \quad \frac{0.15}{g_*} \sim 10^{-3}$$

■ Efficiency factor $\boxed{\kappa}$

i) $K \ll 1$. lepogenesis occurs at $T \gg M$. $(z \ll 1)$
$|y|^2$ too small to populate N sufficiently
$\Rightarrow \kappa \ll 1$.

ii) $K \lesssim 1$, leptogenesis occurs at $T \gtrsim M$ $(z \lesssim 1)$
$\Rightarrow$ maximum efficiency $\kappa \sim 0.1$.

iii) $K \gg 1$, Inverse decay $(\ell H \to N)$ efficient enough for $T < M$ $(z > 1)$ $\Rightarrow$ reduce $Y_{\Delta L}$.
leptogenesis occurs when Γ_{ID} freeze-out at $z_f \ll 1$.

$$1 = zK \left.\frac{Y_N^{eq}}{Y_\ell^{eq}}\right|_{z_f} \Rightarrow K z_f^{\frac{5}{2}} e^{-z_f} \simeq 1.$$

$$\Rightarrow z_f \approx \ln K$$

$$\Rightarrow \kappa \sim \frac{1}{z_f K}$$

(B+L)
(B-L)

$$Y_{\Delta B} \simeq \frac{0.15}{g_{*s}} \varepsilon \cdot \kappa \sim 10^{-10}$$

g_{*s}: 10^3, κ: 0.1

$\boxed{\varepsilon \gtrsim 10^{-6}}$

$\downarrow$ DI bound.

$\boxed{M_1 \gtrsim 10^{10}\ \text{GeV}}$