Ex. 26334 (5 digits)

Case 1: 4 digits

$\Rightarrow$ trivial, just compute

$$
\begin{array}{rrcl}
 & XXXX & \Rightarrow & 9\times9\times8\times7 \\
 & XXX & \Rightarrow & 9\times9\times8 \\
 & XX & \Rightarrow & 9\times9 \\
+) & X & \Rightarrow & 9 \\
\hline
 & & & \cdots
\end{array}
$$

Case 2: 5 digits $(X_1 X_2 X_3 X_4 X_5)$

This is the hardest part:

First observe the first digits

1

~ $\Rightarrow$ 2 options

2

However, it is obvious $2\underbrace{XXXX}_{\text{things here is not easy to compute}}$

Suppose we know $f(\underset{\text{first digit}}{X_1})$:

In this case $f(X_1) = 2$ $(1 \sim 2)$

how to compute $f(X_1, X_2)$?

$$f(x_1 x_2) \stackrel{?}{=} f(x_1) \cdot (10-1)$$

Notice numbers like $27x_3x_4x_5$

is > 26334

So we should do some subtraction:

$$f(x_1 x_2) = f(x_1) \cdot (10-1) - \boxed{3}$$

because these

$27x_3x_4x_5$, $28x_3x_4x_5$, $29x_3x_4x_5$

are > 26334

NOTE:

The reason why in code there is `prefix` variable.

Ex. 26334

1000

S + add a digit

2633

Because in $f(x_1 x_2 x_3 x_4)$

2633 doesnt exist!
(it has two 3s)

When this situation appears,
every $x_1x_2x_3x_4$ in $1000 \sim 2633$
contains no repeated digits
is < 2633,
We don't need to substract
numbers > 26334 anymore!
We can just keep multiplying
$(10-i)\cdot(10-i-1)\cdot\cdots\cdots$!