

$$\min (\max (\text{pair sum}))$$

$$a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$$

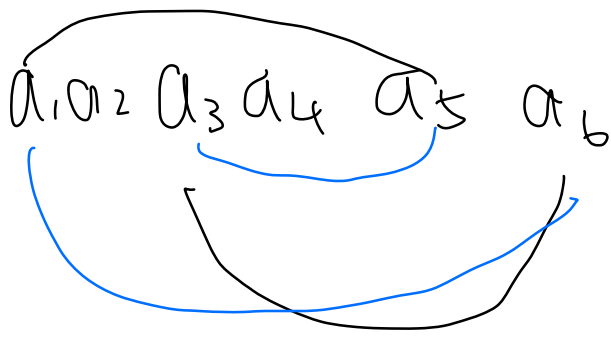
↑

$\frac{n}{2}$ pairs

Prove : Greedy choose (smallest, largest)
will not be worse

suppose:

a_6 is paired with a_i
and a_1 is paired with a_j



Now we swap their pairing

$$\begin{array}{ccc} (a_6, a_i) & \Rightarrow & (a_6, a_i) \\ (a_1, a_j) & & (a_i, a_j) \end{array}$$

Observe =

$$\textcircled{1} \quad a_6 + \underline{a_i} \geq a_6 + \underline{a_1} \quad (\because a_i \geq a_1)$$

$$\textcircled{2} \quad \underline{a_6} + a_i \geq \underline{a_j} + a_i \quad (\because a_6 \geq a_j)$$

$$\textcircled{3} \quad \max(a_6 + a_i, a_1 + a_j) \geq a_6 + a_i \quad (\text{by definition})$$

$$\Rightarrow \textcircled{1}, \textcircled{2}, \textcircled{3}, \max(a_6 + a_i, a_1 + a_j)$$

$$\geq a_6 + a_i \geq \max(a_6 + a_1, a_j + a_i)$$