CSCI-GA.1170-001/002 Fundamental Algorithms	September 4, 2019

Solutions to Problem 1 of Homework 1 (10 Points)

Name: Jingshuai Jiang (jj2903) *Due: 5 pm on Thursday, September 12*

Collaborators: NetID1, NetID2

For each of the following pairs of functions $f(n)$ and $g(n)$, state whether f is $O(g)$; whether f is $o(g)$; whether f is $\Theta(g)$; whether f is $\Omega(g)$; and whether f is $\omega(g)$. (More than one of these can be true for a single pair!)

(a) $f(n) = n!; \quad g(n) = (n+1)!$.

Solution: INSERT YOUR SOLUTION HERE ☐

(b) $f(n) = n^{\log_2 c}; \quad g(n) = c^{\log_2 n}$.

Solution: INSERT YOUR SOLUTION HERE ☐

(c) $f(n) = \log(n^5 + 5n^4 + 4n^3 + 3n^2 + 2n + 1); \quad g(n) = \log(n^{10} + n^8 + n^6 + n^4 + n^2 + 100$.

Solution: INSERT YOUR SOLUTION HERE ☐

(d) $f(n) = \frac{3^n}{n + n\log n}; \quad g(n) = 9^{\sqrt{n}}$.

Solution: INSERT YOUR SOLUTION HERE ☐

(e) $f(n) = 2^{\sqrt{\log_2 n}}; \quad g(n) = n^{1/3}$.

Solution: INSERT YOUR SOLUTION HERE ☐

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Solutions to Problem 2 of Homework 1 (12 Points)	
Name: Jingshuai Jiang (jj2903)	*Due: 5 pm on Thursday, September 12*
Collaborators: NetID1, NetID2	

(a) (8 Points) For each of the following functions $f(n)$, find a "canonical" function[1] $g(n)$ such that $f(n) = \Theta(g(n))$. For example, $3200n + 2n^2 \log^{24}(n^2) = \Theta(n^2 \log^{24} n)$. Briefly justify your answers (and I mean *briefly*).

$$3^n + 7n^{73}, \quad 3^{n+\log_3 n}, \quad \log(n^{34}+5), \quad 2^{2n}, \quad \sqrt{n^5 + 30n^4}, \quad \frac{n^4 - 5n}{55555}, \quad \log^2 n + 33, \quad n^2 \log^3 n + n^3 \log^2 n$$

Solution: INSERT YOUR SOLUTION HERE $\square$

(b) (4 Points) Based on your answers in part (a), sort the resulting "canonical" (not original)[2] functions in asymptotically increasing order.

Solution: INSERT YOUR SOLUTION HERE $\square$

[1]I.e., function of the form $a^n n^b \log^c n$ for constants $a \geq 1$ and $b, c \geq 0$.
[2]Thsi way even if you got part (a) wrong, you can still have correct solution to part (b).

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Solutions to Problem 3 of Homework 1 (16 points)

Name: Jingshuai Jiang (jj2903) *Due: 5 pm on Thursday, September 12*

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Let $A[1, \ldots, n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion* of A.

(a) (2 points) List all inversions of the array $\langle 8, 5, 2, 7, 9 \rangle$.

Solution: INSERT YOUR SOLUTION HERE $\square$

(b) (3 points) Which arrays with distinct elements from the set $\{1, 2, \ldots, n\}$ have the smallest and the largest number of inversions and why? State the expressions *exactly* in terms of n.

Solution: INSERT YOUR SOLUTION HERE $\square$

(c) (5 points) What is the relationship between the running time of Insertion-Sort and the number of inversions I in the input array? *Justify your answer.*

Solution: INSERT YOUR SOLUTION HERE $\square$

(d) (3 points) For any $0 < a < 1/2$, construct an array for which insertion sort has a run time of $an^2 + \Theta(n)$.

Solution: INSERT YOUR SOLUTION HERE $\square$

(e) (3 points) Let $A[1, \ldots, n]$ be a random permutation of $\{1, 2, \ldots, n\}$. What is the expected number of inversions of A. What can you conclude about the average case running time of Insertion-Sort (where the average is over all arrays A of size n)?

Hint: Recall the linearity of expectation, i.e., for any real a, b, c and any random variables X, Y,

$$E(aX + bY + c) = aE(X) + bE(Y) + c\ .$$

Solution: INSERT YOUR SOLUTION HERE $\square$