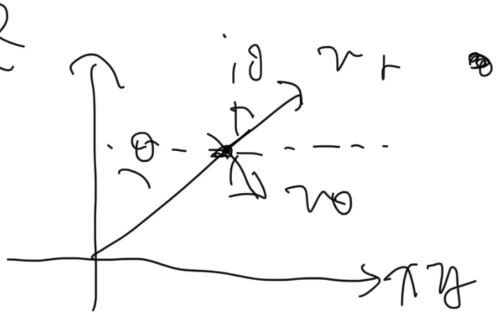
Infall flow of Ulrich 1967

1. Spherial to Cartesian

$(v_r, v_\theta, v_\phi) \longrightarrow (v_x, v_y, v_z)$

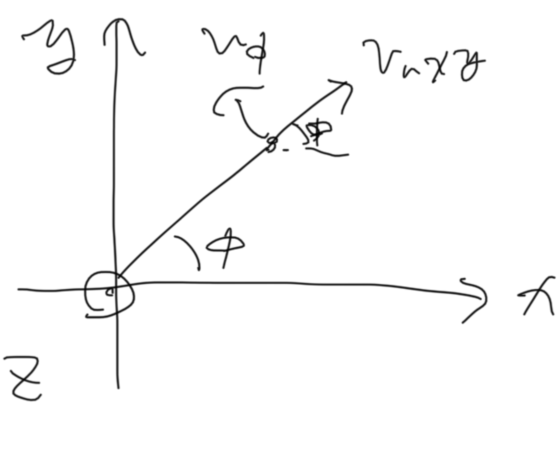
- $v_r, v_\theta \rightarrow v_{rxy}$



- $v_z = v_r \cos\theta - v_\theta \sin\theta$

- $v_{rxy} = v_r \sin\theta + v_\theta \cos\theta$

- $v_{rxy}, v_\phi \rightarrow v_x, v_y$



$$v_x = v_{rxy}\cos\phi - v_\phi \sin\phi$$

$$= v_r \sin\theta\cos\phi + v_\theta\cos\theta\cos\phi - v_\phi\sin\phi$$

$$v_y = v_{rxy}\sin\phi + v_\phi\cos\phi$$

$$= v_r\sin\theta\sin\phi + v_\theta\sin\phi\cos\theta + v_\phi\cos\phi$$

$$\begin{cases} v_x = v_r\sin\theta\cos\phi + v_\theta\cos\theta\cos\phi - v_\phi\sin\phi \\ v_y = v_r\sin\theta\sin\phi + v_\theta\sin\phi\cos\theta + v_\phi\cos\phi \\ v_z = v_r\cos\theta - v_\theta\sin\theta \end{cases}$$

$$v_x^2 + v_y^2 + v_z^2 = v_r^2 + v_\theta^2 + v_\phi^2$$

v_x^2

$$\left(v_r^2\sin^2\theta\cos^2\phi + v_\theta^2\cos^2\theta\cos^2\phi + v_\phi^2\sin^2\phi + 2v_rv_\theta\sin\theta\cos\theta\cos^2\phi - 2v_rv_\phi\sin\theta\cos\phi\sin\phi - 2v_\theta v_\phi\cos\theta\sin\phi\cos\phi\right)$$

v_y^2

$$+\left(v_r^2\sin^2\theta\sin^2\phi + v_\theta^2\sin^2\phi\cos^2\theta + v_\phi^2\cos^2\phi + 2v_rv_\theta\sin\theta\cos\theta\sin^2\phi + 2v_rv_\phi\sin\theta\cos\phi\sin\phi + [illegible]\right)$$

$$+2 v_\theta v_\phi \ldots \sin\theta\cos\theta\,)$$

$$+\underbrace{\left(v_r^2\cos^2\theta + v_\theta^2\sin^2\theta - 2 v_r v_\theta \sin\theta\cos\theta\right)}_{v_z^2}$$

$$= v_r^2\,\underbrace{(\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta)}_{=1}$$

$$+ v_\theta^2\,\underbrace{(\cos^2\theta\cos^2\phi + \sin^2\phi\cos^2\theta + \sin^2\theta)}_{=1}$$

$$+ v_\phi^2\,\underbrace{(\sin^2\phi + \cos^2\phi)}_{=1}$$

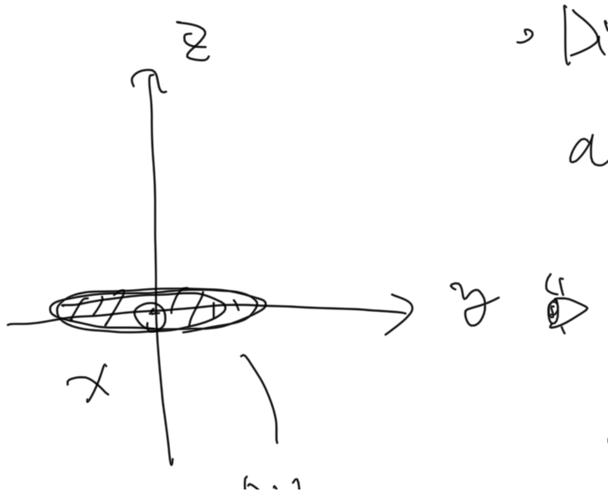
$$+2 \cancel{v_r v_\theta}\,\underbrace{\{\sin\theta\cos\theta(\sin^2\phi + \cos^2\phi) - \sin\theta\cos\theta\}}_{=0}$$

$$+2 \cancel{v_r v_\phi}\,\underbrace{\{-\sin\phi\cos\phi\sin\theta + \sin\phi\cos\phi\sin\theta\}}_{=0}$$

$$+2 \cancel{v_\theta v_\phi}\,\underbrace{(-\sin\phi\cos^2\theta + \sin\phi\sin^2\theta)}_{=0}$$

$$= v_r^2 + v_\theta^2 + v_\phi^2$$

2. Rotation



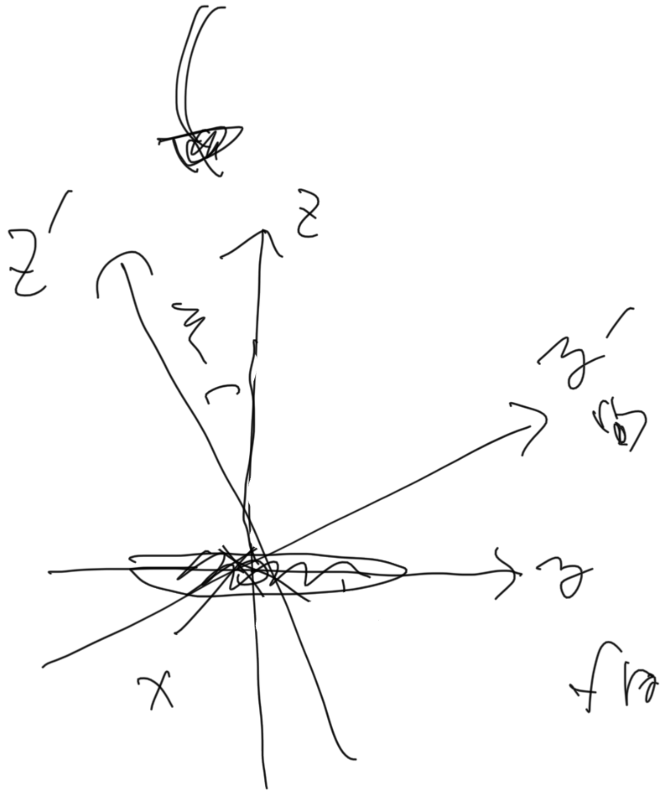
- Disk is initially aligned to xz-plane

- Take y-axis

Disk

(z'-axis)

as the line-of-sight



Rotate coordinates

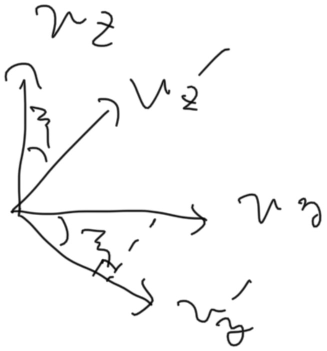
AO code model

from different angle.

· Take anti-clockwise positive

· Coordinate rotation with angle ξ

$$\begin{cases} x' = x \\ y' = y\cos\xi + z\sin\xi \\ z' = -y\sin\xi + z\cos\xi \end{cases}$$



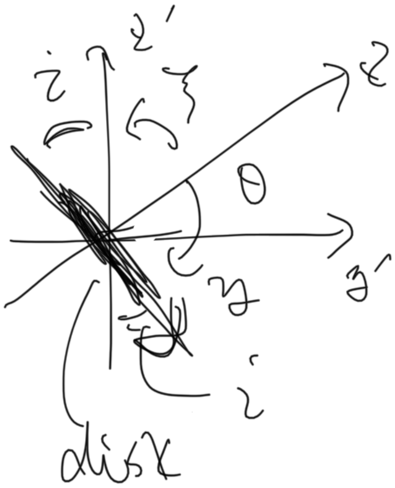
Same for v

$$\begin{cases} v_x' = v_x \\ v_y' = v_y\cos\xi + v_z\sin\xi \\ v_z' = -v_y\sin\xi + v_z\cos\xi \end{cases}$$

By taking y'-axis as the line-of-sight

$v_{LOS} = v_{y'}$

∘ Inclination i → rotation angle ξ



$i = 0°$ face-on

or $i = 90°$ edge-on

$i = 0°$ so that

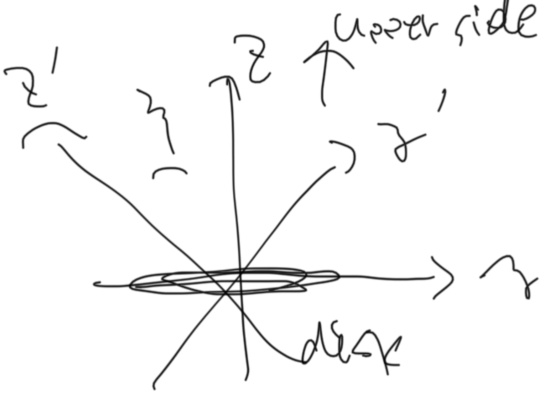
Define Disk upper side faces to us.

$i \leftrightarrow \theta$

$i = 90° - \theta$, $\theta = 90° - i$

To look disk upper side at $i = 0°$

i.e., $\theta = 90°$



Rotation must be taken to negative θ

direction.

$$\therefore \xi = -(90^\circ - i)$$

Again, take anti-clockwise positive.

$$\begin{cases} v_x' = v_x, \\ v_y' = v_y \cos\xi + v_z \sin\xi, \\ v_z' = -v_y \sin\xi + v_z \cos\xi, \\ \xi = -(90^\circ - i), \end{cases}$$

where v_z' is the line-of-sight velocity V_{los}, i is the inclination angle, and ξ is the rotational angle.