Homework 6

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We have a conjunctive fragment $e(\varphi)$ of the theory T and we want to describe DP_T procedure which performs an exhaustive theory propagation.

This means we want to find all unassigned atoms (x, y) such that there exists a path in $E_= \cup E_{\neq}$ from x to y with at most one edge in $E_{\neq}$ and we want to distinguish the two cases of whether an edge from $E_{\neq}$ is present on the path: $e(x = y)$ or $\neg e(x = y)$.

First, we describe a simple binary operation $\odot$ for "path lengthening", which takes two connected sub-paths and combines them into larger one distinguishing the cases EQ if a path is a member of $E_=^*$, NEQ if it is a member of $(E_=^* \cdot E_{\neq} \cdot E_=^*)$, and NUL. NUL being a combined class for all other cases.

```
function ⊙(a, b)
    if a == NEQ then
        if b == EQ then return NEQ
        else return NUL                         ▷ b == NUL or b == NEQ
        end if
    else if a == EQ then return b
    else return NUL                             ▷ a == NUL
    end if
end function
```

Then, we describe another binary operation $\oplus$ for "path choosing", which takes two alternative paths from the same source to the same destination and it decides what is the relation of the source and the destination: EQ, NEQ, or NUL, just like in the case of $\odot$.

```
function ⊕(a, b) throws UNSAT
    if a == NEQ then
        if b == EQ then
            throw UNSAT          ▷ Cannot be equal and nonequal at the same time
        else return NEQ                         ▷ b == NUL or b == NEQ
        end if
    else if a == EQ then
        if b == NEQ then
            throw UNSAT          ▷ Cannot be equal and nonequal at the same time
        else return EQ                          ▷ b == NUL or b == EQ
        end if
    else return b
    end if
end function
```

Then we create an adjacency matrix A with values A_{ij} being equal to EQ, NEQ, or NUL, similarly to before, but according to path of length ≤ 1 from x_i to x_j in $E_= \cup E_{\neq}$. And then we compute $B = A^i$ (using $\odot$ for multiplication and $\oplus$ for addition) for minimal i s.t $\forall i' > i : A^i = A^{i'}$, note that this computation can "**throw**" and thus end prematurely, then we return $UNSAT$. Otherwise, we return the conjunction of all unassigned $e(x_i = x_j)$ s.t $B_{ij} = EQ$, and all unassigned $\neg e(x_i = x_j)$ s.t $B_{ij} = NEQ$. (As a side note, we can optimize this by considering only a triangular matrix; and always ends (3^n value space))