

# Homework 2

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(10 points) One of the preprocessing steps can be to eliminate some of the variables using so-called DP-elimination (or DP-resolution). In particular, assume we have a CNF  $\varphi$  and a variable  $x$  which we want to eliminate. Denote

$$\varphi_0 = \{C \in \varphi \mid \neg x \in C\}$$

$$\varphi_1 = \{C \in \varphi \mid x \in C\}$$

$$\varphi_r = \{C \in \varphi \mid C \cap \{x, \neg x\} = \emptyset\}$$

Namely,  $\varphi_0$  consists of the clauses containing negative literal  $\neg x$ ,  $\varphi_1$  consists of the clauses containing positive literal  $x$ ,  $\varphi_r$  contains the rest of the clauses. Let us now define

$$\varphi_{dp} = \{Res(C_0, C_1) \mid C_0 \in \varphi_0, C_1 \in \varphi_1\}$$

where  $Res(C_0, C_1)$  denotes the clauses originating from  $C_0$  and  $C_1$  by resolution. Show that  $\varphi$  is equisatisfiable with  $\varphi_0 = \varphi_r \wedge \varphi_{dp}$ .

First, we define:  $\varphi'_0 = \{C \setminus \{\neg x\} \mid C \in \varphi_0\}$ ,  $\varphi'_1 = \{C \setminus \{x\} \mid C \in \varphi_1\}$

It follows that:  $\varphi_0 \equiv \{x \rightarrow C \mid C \in \varphi'_0\}$ ,  $\varphi_1 \equiv \{\neg x \rightarrow C \mid C \in \varphi'_1\}$  (1)

and from that:  $\varphi_0 \equiv x \rightarrow \{C \mid C \in \varphi'_0\}$ ,  $\varphi_1 \equiv \neg x \rightarrow \{C \mid C \in \varphi'_1\}$

and  $\varphi_0 \wedge \varphi_1 \equiv x \rightarrow \{C \mid C \in \varphi'_0\} \wedge \neg x \rightarrow \{C \mid C \in \varphi'_1\}$  and (since those are the only two occurrences of variable  $x$ ):

$$\varphi_0 \wedge \varphi_1 \equiv \{C \mid C \in \varphi'_0\} \vee \{C \mid C \in \varphi'_1\}$$

From distributivity of  $\wedge$  and  $\vee$  and the previous:  $\varphi_0 \wedge \varphi_1 \equiv \{C_0 \vee C_1 \mid C_0 \in \varphi'_0, C_1 \in \varphi'_1\}$

We can finalize this as follows:

$$\varphi_0 \wedge \varphi_1 \equiv \{C_0 \vee C_1 \mid C_0 \in \varphi'_0, C_1 \in \varphi'_1\} \stackrel{Def}{=} \{Res(\{\neg x\} \cup C_0, \{x\} \cup C_1) \mid C_0 \in \varphi'_0, C_1 \in \varphi'_1\} \stackrel{(1)}{=}$$

$$\{Res(C_0, C_1) \mid C_0 \in \varphi_0, C_1 \in \varphi_1\} = \varphi_{dp}$$