

Homework 6

Jiří Klepl

We have a conjunctive fragment $e(\varphi)$ of the theory T and we want to describe DP_T procedure which performs an exhaustive theory propagation.

This means we want to find all unassigned atoms (x, y) such that there exists a path in $E_= \cup E_{\neq}$ from x to y with at most one edge in E_{\neq} and we want to distinguish the two cases of whether an edge from E_{\neq} is present on the path: $e(x = y)$ or $\neg e(x = y)$.

First, we describe a simple binary operation \oplus for “path lengthening”, which takes two connected sub-paths and combines them into larger one distinguishing the cases EQ if a path is a member of $E_=$, NEQ if it is a member of $(E_=^* \cdot E_{\neq} \cdot E_=^*)$, and NUL . NUL being a combined class for all other cases.

```

1: function  $\oplus(a, b)$ 
2:   if  $a == NEQ$  then
3:     if  $b == EQ$  then return  $NEQ$ 
4:     else return  $NUL$                                  $\triangleright b == NUL$  or  $b == NEQ$ 
5:   end if
6:   else if  $a == EQ$  then return  $b$ 
7:   else return  $NUL$                                  $\triangleright a == NUL$ 
8:   end if
9: end function

```

Then, we describe another binary operation \odot for “path choosing”, which takes two alternative paths from the same source to the same destination and it decides what is the relation of the source and the destination: EQ , NEQ , or NUL , just like in the case of \oplus .

```

1: function  $\odot(a, b)$  throws  $UNSAT$ 
2:   if  $a == NEQ$  then
3:     if  $b == EQ$  then
4:       throw  $UNSAT$                                  $\triangleright$  Cannot be equal and nonequal at the same time
5:     else return  $NEQ$                                  $\triangleright b == NUL$  or  $b == NEQ$ 
6:     end if
7:   else if  $a == EQ$  then
8:     if  $b == NEQ$  then
9:       throw  $UNSAT$                                  $\triangleright$  Cannot be equal and nonequal at the same time
10:    else return  $EQ$                                  $\triangleright b == NUL$  or  $b == EQ$ 
11:    end if
12:   else return  $b$ 
13:   end if
14: end function

```

Then we create an adjacency matrix A with values A_{ij} being equal to EQ , NEQ , or NUL , similarly to before, but according to path of length ≤ 1 from x_i to x_j in $E_= \cup E_{\neq}$. And then we compute $B = A^i$ (using \odot for multiplication and \oplus for addition) for minimal i s.t $\forall i' > i : A^i = A^{i'}$, note that this computation can “**throw**” and thus end prematurely, then we return $UNSAT$. Otherwise, we return the conjunction of all unassigned $e(x_i = x_j)$ s.t $B_{ij} = EQ$, and all unassigned $\neg e(x_i = x_j)$ s.t $B_{ij} = NEQ$. (As a side note, we can optimize this by considering only a triangular matrix; and always ends (3^n value space))