Homework 2

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(10 points) Let φ be a CNF and $C \in \varphi$ a clause in φ and $l \in C$ a literal in C. We say that C is blocked by literal l if for every other clause $D \in \varphi$ which contains $\neg l$ we have that $Res_l(C, D)$ is a tautology (i.e. there is another literal $l_D \in D$ such that $\neg l_D \in C$). Show that φ is equisatisfiable with $\varphi \setminus \{C\}$, i.e. φ is satisfiable if and only if $\varphi \setminus \{C\}$ is satisfiable.

Let α be an assignment over the variables of φ. Then we have the following two options:

1. $\neg l_D \in \alpha$ for at least one of the clauses D: then under the assumption $\neg l_D \in \alpha$ it holds that φ and $\varphi \setminus \{C\}$ are equisatisfiable as C is a tautology under the assumption and removing a tautology has no effect on satisfiability.

2. $l_D \in \alpha$ for each of the clauses D: We know that l is not present in any clause other than C and all clauses that contain $\neg l$ are tautologies under the assumption $\{l_D | \text{for every } D\} \subseteq \alpha$. If α such that $\neg l \in \alpha$ satisfies φ, then $\alpha' = \alpha \triangle \{l, \neg l\}$ also satisfies φ. This learned fact can be formalized into a clause as follows:

   Definition 1 (Learned fact)

   $$\bigwedge \{l_D | \mathit{for\ every\ } D\} \rightarrow \{l\} \equiv \{\neg l_D | \mathit{for\ every\ } D\} \cup \{l\} \equiv A$$

   Then we can easily notice that $A \subseteq C$ and therefore the clause C is a tautology under the assumption $\{l_D | \text{for every } D\} \cup \{l\} \subseteq \alpha$ implied by A from the original assumption; and, similarly to the first option, this means that φ and $\varphi \setminus \{C\}$ are equisatisfiable under the assumption $\{l_D | \text{for every } D\} \subseteq \alpha$.

Observation 1.1 *The fact A in definition 1 can be directly deduced from the setup; and C is weaker than or equivalent to A. And therefore, C is also a fact.*