# Homework 6

**Jiří Klepl**

We have a conjunctive fragment $e(\varphi)$ of the theory $T$ and we want to describe $DP_T$ procedure which performs an exhaustive theory propagation.

This means we want to find all unassigned atoms $(x, y)$ such that there exists a path in $E_= \cup E_{\neq}$ from $x$ to $y$ with at most one edge in $E_{\neq}$ and we want to distinguish the two cases of whether an edge from $E_{\neq}$ is present on the path: $e(x = y)$ or $\neg e(x = y)$.

First, we describe a simple binary operation $\odot$ for "path lengthening", which takes two connected sub-paths and combines them into larger one distinguishing the cases $EQ$ if a path is a member of $E_=^*$, $NEQ$ if it is a member of $(E_=^* \cdot E_{\neq} \cdot E_=^*)$, and $NUL$. $NUL$ being a combined class for all other cases.

**function** $\odot(a, b)$<br>
 **if** $a == NEQ$ **then**<br>
  **if** $b == EQ$ **then return** $NEQ$<br>
  **else return** $NUL$   $\triangleright$ $b == NUL$ or $b == NEQ$<br>
  **end if**<br>
 **else if** $a == EQ$ **then return** $b$<br>
 **else return** $NUL$   $\triangleright$ $a == NUL$<br>
 **end if**<br>
**end function**

Then, we describe another binary operation $\oplus$ for "path choosing", which takes two alternative paths from the same source to the same destination and it decides what is the relation of the source and the destination: $EQ$, $NEQ$, or $NUL$, just like in the case of $\odot$.

**function** $\oplus(a, b)$ **throws** $UNSAT$<br>
 **if** $a == NEQ$ **then**<br>
  **if** $b == EQ$ **then**<br>
   **throw** $UNSAT$   $\triangleright$ Cannot be equal and nonequal at the same time<br>
  **else return** $NEQ$   $\triangleright$ $b == NUL$ or $b == NEQ$<br>
  **end if**<br>
 **else if** $a == EQ$ **then**<br>
  **if** $b == NEQ$ **then**<br>
   **throw** $UNSAT$   $\triangleright$ Cannot be equal and nonequal at the same time<br>
  **else return** $EQ$   $\triangleright$ $b == NUL$ or $b == EQ$<br>
  **end if**<br>
 **else return** $b$<br>
 **end if**<br>
**end function**

Then we create an adjacency matrix $A$ with values $A_{ij}$ being equal to $EQ$, $NEQ$, or $NUL$, similarly to before, but according to path of length $\leq 1$ from $x_i$ to $x_j$ in $E_= \cup E_{\neq}$. And then we compute $B = A^i$ (using $\odot$ for multiplication and $\oplus$ for addition) for minimal $i$ s.t $\forall i' > i : A^i = A^{i'}$, note that this computation can "**throw**" and thus end prematurely, then we return $UNSAT$. Otherwise, we return the conjunction of all unassigned $e(x_i = x_j)$ s.t $B_{ij} = EQ$, and all unassigned $\neg e(x_i = x_j)$ s.t $B_{ij} = NEQ$. (As a side note, we can optimize this by considering only a triangular matrix; always ends ($\leq 3^{n^2}$ states; $i \leq n$))