Homework 2

Jiří Klepl

(10 points) Let φ be a CNF and $C \in \varphi$ a clause in φ and $l \in C$ a literal in C. We say that C is blocked by literal l if for every other clause $D \in \varphi$ which contains $\neg l$ we have that $Res_l(C, D)$ is a tautology (i.e. there is another literal $l_D \in D$ such that $\neg l_D \in C$). Show that φ is equisatisfiable with $\varphi \setminus \{C\}$, i.e. φ is satisfiable if and only if $\varphi \setminus \{C\}$ is satisfiable.

We can rewrite φ to $C \wedge \varphi_D \wedge \varphi_r$ where φ_D contains the clauses D and φ_r the clauses of φ that don't contain neither l nor $\neg l$.

It holds that $C \wedge \varphi_D \wedge \varphi_r \equiv (\neg l \to C') \wedge (l \to \varphi'_D) \wedge \varphi_r$ where C' is the clause C after removing the literal l and φ'_D consist of the clauses D after removing the literal $\neg l$.

And then, since neither of C', φ'_D, nor φ_r contain either of the literals l and $\neg l$, it holds that $(\neg l \to C') \wedge (l \to \varphi'_D) \wedge \varphi_r \equiv_{SAT} (C' \vee \varphi'_D) \wedge \varphi_r$.

Then, from distributivity of $\vee$ and $\wedge$, we get $(C' \vee \varphi'_D) \wedge \varphi_r \equiv (\bigwedge\{C' \vee D' | D' \in \varphi'_D\}) \wedge \varphi_r = (\bigwedge\{Res_l(C \vee D) | D \in \varphi_D\}) \wedge \varphi_r$ (the equality follows the definition of C' and φ'_D from C and φ_D).

The assignment declares that $Res_l(C, D)$ is a tautology for every $D \in \varphi_D$, and thus $(\bigwedge\{Res_l(C \vee D) | D \in \varphi_D\}) \wedge \varphi_r \equiv \varphi_r$.

And, since we proved that $\varphi = C \wedge \varphi_D \wedge \varphi_r \equiv_{SAT} \varphi_r$, it follows that $\varphi \equiv_{SAT} \varphi \setminus \{C\} = \varphi_D \wedge \varphi_r$, otherwise φ and φ_r wouldn't be equivalent.