# Homework#5

# B08902071 塗季芸

1. Answer: (d).

$$X = \begin{bmatrix} 1 & -2 & 4 \\ 1 & 0 & 0 \\ 1 & 2 & 4 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix}$$

$$\begin{cases} (1) & -w_1 + 2w_2 - 4w_3 - b \ge 1 \\ (2) & w_1 + b \ge 1 \\ (3) & -w_1 - 2w_2 - 4w_3 - b \ge 1 \end{cases}$$

(2) 
$$w_1 + b \ge 1$$

$$(3) \quad -w_1 - 2w_2 - 4w_3 - b \ge 1$$

$$(1)\&(2) \quad w_2 - 2w_3 \ge 1$$

$$(2)\&(3) - w_2 - 2w_3 \ge 1$$

Combine the above two results, we have

$$\begin{cases} w_2 \ge 0 \\ w_3 \le -\frac{1}{2} \end{cases}$$

Therefore

$$\frac{1}{2}\mathbf{w}^T\mathbf{w} \ge \frac{1}{2}(w_1^2 + 0 + \frac{1}{4})$$

Also,  $w_1$  only has the constraint of (2), in which b is an arbitrary real number, so

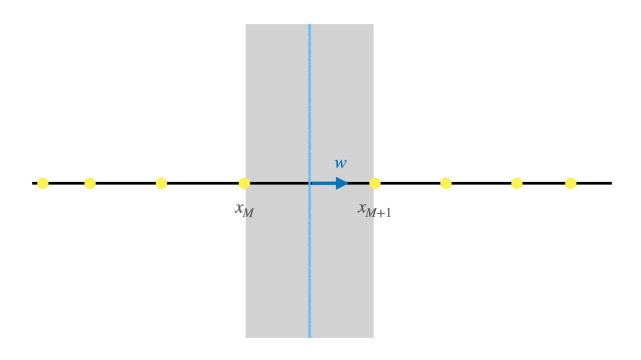
$$w_1^* = 0$$

2. Answer: (b).

$$\text{margin}(b, \mathbf{w}) = \frac{1}{||\mathbf{w}||} = \frac{1}{\sqrt{0 + 0 + \frac{1}{4}}} = 2$$

## 3. Answer: (e).

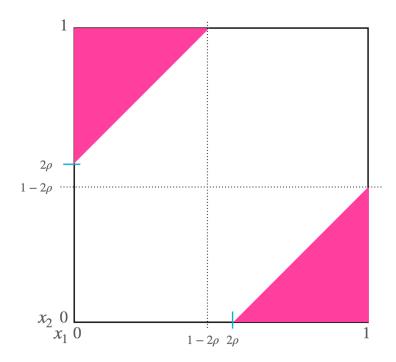
When  $\rho=0$ , the hard-margin SVM is just perceptrons, and we can visualize the optimal perceptron in the figure below:



The largest margin is obviouly  $\frac{1}{2}(x_{M+1}-x_M)$ .

## 4. Answer: (a).

First, consider the probability that  $|x_1 - x_2| \ge 2\rho$ , which can be illustrated in the figure below



The probability of sampling  $x_1, x_2$  from a uniform distribution [0,1] such that  $|x_1 - x_2| \ge 2\rho$  is  $(1 - 2\rho)^2$ .

If  $|x_1 - x_2| \ge 2\rho$ , the perceptron can be between the two points or outside them, so it can shatter the input. While if  $|x_1 - x_2| < 2\rho$ , it can only deal with labels of (+,+) and (-,-). The expected number of dichotomies is then  $4(1-2\rho)^2 + 2(1-(1-2\rho)^2) = 2+2(1-2\rho)^2$ .

5. Answer: (c).

In this problem, we want to solve

$$\max_{all \ \alpha_n \ge 0} \left( \min_{b, \mathbf{w}} \frac{1}{2} \mathbf{w}^T \mathbf{w} + \sum_{n=1}^N [y_n = +1] \alpha_n (\rho_+ - y_n (\mathbf{w}^T \mathbf{z}_n + b)) + \sum_{n=1}^N [y_n = -1] \alpha_n (\rho_- - y_n (\mathbf{w}^T \mathbf{z}_n + b)) \right)$$

After removing b and  $\mathbf{w}$  from the equation using the same steps as p9-p10 in the lecture 202 slides, the problem becomes

$$\max_{all \ \alpha_n \geq 0, \sum y_n \alpha_n = 0, \mathbf{w} = \sum \alpha_n y_n \mathbf{z}_n} -\frac{1}{2} || \sum_{n=1}^N \alpha_n y_n \mathbf{z}_n ||^2 + \sum_{n=1}^N ([[y_n = +1]] \rho_+ \alpha_n + [[y_n = -1]] \rho_- \alpha_n)$$

which is equivalent to solve

$$\min_{\substack{all \ \alpha_n \geq 0, \sum y_n \alpha_n = 0, \mathbf{w} = \sum \alpha_n y_n \mathbf{z}_n \\ }} \frac{1}{2} || \sum_{n=1}^{N} \alpha_n y_n \mathbf{z}_n ||^2 - \sum_{n=1}^{N} ([[y_n = +1]] \rho_+ \alpha_n + [[y_n = -1]] \rho_- \alpha_n)$$

6. Answer: (c).

First, since

$$\sum_{n=1}^{N} y_n \alpha_n = 0$$

, therefore,

$$\sum_{n:y_n=1} 1 \cdot \alpha_n = -\sum_{n:y_n=-1} -1 \cdot \alpha_n$$

$$\sum_{n:y_n=1} \alpha_n = \sum_{n:y_n=-1} \alpha_n$$

We can rewrite the last item in the dual problem

$$-\sum_{n=1}^{N} ([y_n = +1]] \rho_+ \alpha_n + [y_n = -1]] \rho_- \alpha_n)$$

$$= -\rho_+ \sum_{n:y_n = 1} \alpha_n - \rho_- \sum_{n:y_n = -1} \alpha_n$$

$$= -(\rho_+ + \rho_-) \sum_{n:y_n = 1} \alpha_n$$

$$= \frac{-(\rho_+ + \rho_-)}{2} \sum_{n=1}^{N} \alpha_n$$

Rewrite the dual problem, we have

$$\begin{split} & \min_{\alpha} \frac{1}{2} || \sum_{n=1}^{N} \alpha_{n} y_{n} \mathbf{z}_{n} ||^{2} - \frac{(\rho_{+} + \rho_{-})}{2} \sum_{n=1}^{N} \alpha_{n} \\ & = \frac{(\rho_{+} + \rho_{-})^{2}}{4} \min_{\alpha} \frac{1}{2} || \sum_{n=1}^{N} \left( \frac{2}{\rho_{+} + \rho_{-}} \alpha_{n} \right) y_{n} \mathbf{z}_{n} ||^{2} - \sum_{n=1}^{N} \left( \frac{2}{\rho_{+} + \rho_{-}} \alpha_{n} \right) \\ & = \frac{(\rho_{+} + \rho_{-})^{2}}{4} \left( \frac{1}{2} || \sum_{n=1}^{N} \alpha^{*} y_{n} \mathbf{z}_{n} ||^{2} - \sum_{n=1}^{N} \alpha^{*} \right) \end{split}$$

Let  $\alpha^{*'}$  be the optimal solution of the uneven-margin SVM,

$$\alpha^* = \frac{2}{\rho_+ + \rho_-} \alpha^{*\prime}$$

, so

$$\alpha^{*\prime} = \frac{\rho_+ + \rho_-}{2} \alpha^*$$

7. Answer: (d).

Consider

$$K(\mathbf{x}, \mathbf{x}') = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is symmetric and positive semi-definite.

$$\log_2 K(\mathbf{x}, \mathbf{x'}) = \begin{bmatrix} 0 & -\infty \\ -\infty & 0 \end{bmatrix}$$

Let

$$z = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

We have

$$z^T \log_2 K(\mathbf{x}, \mathbf{x}') z = -\infty < 0$$

Therefore,  $\log_2 K(\mathbf{x},\mathbf{x\prime})$  is not always a valid kernel.

8. Answer: (c).

$$\begin{aligned} ||\phi(\mathbf{x}) - \phi(\mathbf{x}\prime)||^2 &= \phi(\mathbf{x})^T \phi(\mathbf{x}) - 2\phi(\mathbf{x})^T \phi(\mathbf{x}\prime) + \phi(\mathbf{x}\prime)^T \phi(\mathbf{x}\prime) \\ &= K(\mathbf{x}, \mathbf{x}\prime) - 2K(\mathbf{x}, \mathbf{x}\prime) + K(\mathbf{x}\prime, \mathbf{x}\prime) \\ &= \exp(0) - 2\exp(-\gamma||\mathbf{x} - \mathbf{x}\prime||^2) + \exp(0) \\ &\leq 2 \end{aligned}$$

When 
$$||\mathbf{x} - \mathbf{x}'|| \to \infty$$
,  $||\phi(\mathbf{x}) - \phi(\mathbf{x}')||^2 = 2$ .

9. Answer: (d).

$$\hat{h}(\mathbf{x}_i) = sign\left(\sum_{n \neq i} y_n \exp(-\gamma ||\mathbf{x}_n - \mathbf{x}_i||^2) + y_i\right)$$

A worst case can happen when all  $y_n, \forall n \neq i$  is different from  $y_i$ . To correctly classify such cases, we want

$$\sum_{n \neq i} \exp(-\gamma ||\mathbf{x}_n - \mathbf{x}_i||^2) < 1$$

Applying the fact that  $||\mathbf{x}_n - \mathbf{x}_i||^2 \ge \epsilon^2$ , we have

$$\sum_{n \neq i} \exp(-\gamma ||\mathbf{x}_n - \mathbf{x}_i||^2) \le \sum_{n \neq i} \exp(-\gamma \epsilon^2) = (N - 1) \exp(-\gamma \epsilon^2)$$

To ensure that

$$(N-1)\exp(-\gamma\epsilon^2) < 1$$

We have

$$\gamma > \frac{\ln(N-1)}{\epsilon^2}$$

10. Answer: (c).

$$\sum_{n=1}^{N} \alpha_{t,n} \phi(\mathbf{x}_n) + y_{n(t)} \phi(\mathbf{x}_{n(t)}) = \sum_{n \neq n(t)} \alpha_{t,n} \phi(\mathbf{x}_n) + (\alpha_{t,n(t)} + y_{n(t)}) \phi(\mathbf{x}_{n(t)})$$

11. Answer: (a).

$$\mathbf{w}_{t}^{T} \phi(\mathbf{x}) = \sum_{n=1}^{N} \alpha_{t,n} \phi(\mathbf{x}_{n})^{T} \phi(\mathbf{x})$$
$$= \sum_{n=1}^{N} \alpha_{t,n} K(\mathbf{x}_{n}, \mathbf{x})$$

12. Answer: (b).

Because every example is a support vector,

$$b = y_n - y_n \xi_n - \mathbf{w}^T \mathbf{z}_n, \ \forall n$$

, and  $\xi_n \geq 0$ . Therefore,

$$\min_{n:y_n < 0} \left( -1 - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) \right) \le -1 + \xi_n - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) 
1 - \xi_n - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) \le \min_{n:y_n > 0} \left( 1 - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) \right) 
\min_{n:y_n < 0} \left( -1 - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) \right) \le b \le \min_{n:y_n > 0} \left( 1 - \sum_{m=1}^{N} y_m \alpha_m K(x_n, x_m) \right)$$

## 13. Answer: (e).

The Lagrange function with Lagrange multipliers  $\alpha_n$  is

$$\mathcal{L}(b, \mathbf{w}, \xi, \alpha) = \frac{1}{2} \mathbf{w}^T \mathbf{w} + C \sum_{n=1}^N \xi_n^2 + \sum_{n=1}^N \alpha_n (1 - \xi_n - y_n (\mathbf{w}^T \phi(\mathbf{x}_n) + b))$$

To solve

$$\max_{\alpha_n \ge 0} \left( \min \mathcal{L}(b, \mathbf{w}, \xi, \alpha) \right) \tag{1}$$

we have

$$\frac{\partial \mathcal{L}}{\partial \xi_n} = 0 = 2C\xi_n - \alpha_n$$

therefore,

$$\mathcal{L}(b, \mathbf{w}, \xi, \alpha) = \frac{1}{2} \mathbf{w}^T \mathbf{w} - \frac{1}{4C} \sum_{n=1}^{N} \alpha_n^2 + \sum_{n=1}^{N} \alpha_n (1 - y_n(\mathbf{w}^T \phi(\mathbf{x}_n) + b))$$

Then, apply the same routine that taught in class,

$$\frac{\partial \mathcal{L}}{\partial b} = 0$$
$$\frac{\partial \mathcal{L}}{\partial w_i} = 0$$

Solving (1) is equal to solving

$$\min_{\alpha} \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_n \alpha_m y_n y_m K(\mathbf{x}_n, \mathbf{x}_m) + \frac{1}{4C} \sum_{n=1}^{N} \alpha_n^2 - \sum_{n=1}^{N} \alpha_n$$

$$= \min_{\alpha} \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_n \alpha_m y_n y_m \left( K(\mathbf{x}_n, \mathbf{x}_m) + \frac{1}{2C} \llbracket n = m \rrbracket \right) - \sum_{n=1}^{N} \alpha_n$$

14. Answer: (e).

From the condition we already set when solving Q13, we have

$$\frac{\partial \mathcal{L}}{\partial \xi_n} = 0 = 2C\xi_n - \alpha_n$$

which means

$$\xi^* = \frac{\alpha^*}{2C}$$

# Experiments with Soft-Margin SVM

# Preparation

• build training data according to the classification rule.

15. Answer: (d).

```
b08902071@linux14 [/tmp2/b08902071/mlt-hw5/libsvm] svm-train -s 0 -t 0 -c 10 satimage.scale.3 .....*..*........*

optimization finished, #iter = 22874

nu = 0.108695

obj = -4784.881503, rho = 3.623003

nSV = 500, nBSV = 466

Total nSV = 500

b08902071@linux14 [/tmp2/b08902071/mlt-hw5/libsvm]
```

```
#include <bits/stdc++.h>
2 using namespace std;
3 double sum[40];
5 int main()
6 {
    ios_base::sync_with_stdio(false); cin.tie(0);
    freopen("satimage.scale.3.model", "r", stdin);
8
    string s;
9
    for(int i=0;i<8;i++) getline(cin,s);</pre>
10
    while (getline(cin,s)){
11
      string t;
      stringstream ss(s);
13
14
      ss>>t;
      sum[0]=stod(t);
15
      while(ss>>t){
16
        istringstream ist(t);
        string tmp;
18
        int id;
19
        double x;
20
         getline(ist,tmp,':');
         id=stoi(tmp);
22
         getline(ist,tmp);
        x=stod(tmp);
24
         sum[id] += sum[0] *x;
25
26
    }
27
    double ans=0.0;
28
    for(int i=1;i<40;i++) ans+=pow(sum[i],2);</pre>
    cout << sqrt(ans) << "\n";</pre>
30
31 }
```

#### 16. Answer: (b).

### 17. Answer: (c).

```
b089022718\inux14 [/\textsup2/b08902871/\slt-\ms/\libsvm] svm-train -s 0 -t 1 -d 2 -g 1 -r 1 -c 10 satimage.scale.1

optimization finished, #lter = 15704

un = 0.008583

abj = 228.264815, rho = 0.006089

nby = 145, nby = 10

Total nby = 145

b0890267218\inux14 [/\textsup2/b08902871/\slt-\ms/\libsvm] svm-train -s 0 -t 1 -d 2 -g 1 -r 1 -c 10 satimage.scale.2

optimization finished, #lter = 2908

nby = 37, 85V = 10

Total nby = 25.47625

nby = 37, 85V = 0

Total nby = 87

Fortal nby = 87

B0890267216\inux14 [/\textsup2/b08902871/\slt-\ms/\libsvm] svm-train -s 0 -t 1 -d 2 -g 1 -r 1 -c 10 satimage.scale.3

optimization finished, #lter = 69995

n = 0.0259

solution finished, #lter = 73743

n = 0.13255

optimization finished, #lter = 41834

n = 0.013669

optimization finished, #lter = 41834

optimization finished, #lter = 41834

optimization finished, #lter = 41834

optimization finished, #lter = 61835

optimization finished, #lter = 61835

optimization finished, #lter
```

### 18. Answer: (d)(e).

```
b88902071elinuxi4 [/tmp2/b88902071/mlt-hm5/libxwm] svm-train -s 0 -t 2 -d 2 -g 10 -c 0.01 -g satimage.scale.t > satimage.scale.t. 6 b89802071elinuxi4 [/tmp2/b88902071/mlt-hm5/libxwm] svm-train -s 0 -t 2 -d 2 -g 10 -c 0.1 -g satimage.scale.t. > satimage.scale.t.6.out Accuracy = 76.5% (1536/2060) (classification) b89802071elinuxi4 [/tmp2/b88902071/mlt-hm5/libxwm] svm-train -s 0 -t 2 -d 2 -g 10 -c 0.1 -g satimage.scale.6. should satimage.scale.t.6.out b89802071elinuxi4 [/tmp2/b88902071/mlt-hm5/libxwm] svm-train -s 0 -t 2 -d 2 -g 10 -c 0.1 -g satimage.scale.6. should s
```

## 19. Answer: (b).

```
b8898/20718tinux14 [/tmp2/m88902071/mlt-hm5/libxvm] svm-train -s 0 -t 2 -d 2 -g 0.1 -c 0.1 -g satimage.scale.6 b889020718tinux14 [/tmp2/m88902071/mlt-hm5/libxvm] svm-predict satimage.scale.6.soxel satimage.scale.6 soxel satimage.scale.6 soxel satimage.scale.5 soxel satimage.scale.5 soxel satimage.scale.5 soxel satimage.scale.6 soxel satimage.scale.5 soxel soxel satimage.scale.6 soxel soxel satimage.scale.6 soxel soxel satimage.scale.6 soxel soxel
```

#### 20. Answer: (b).

```
1 from symutil import *
2 import random
4 y, x = svm_read_problem('../satimage.scale.6')
5 L = len(y)
6 \text{ numlist} = [0] *L
7 for i in range(L):
      numlist[i]=i
y_val, x_val = [0]*200, [0]*200
y_{train}, x_{train} = [0]*(L-200), [0]*(L-200)
compare = [0]*5
12 for rounds in range (1000):
      print(f'round={rounds}, max_index={compare.index(max(compare))}')
13
      ACC = [0]*5
14
      cnt_val=0
      cnt_train=0
      S=random.sample(numlist,200)
      for i in range(L):
18
          #print(f'{cnt_val}, {cnt_train}')
19
          if(i in S):
20
               y_val[cnt_val]=y[i]
21
               x_val[cnt_val]=x[i]
22
               cnt_val+=1
23
          else:
               y_train[cnt_train]=y[i]
25
               x_train[cnt_train]=x[i]
26
               cnt_train+=1
27
      m = svm_train(y_train, x_train, '-s 0 -t 2 -c 0.1 -g 0.1 -q')
28
      p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
29
      ACC[0], MSE, SCC = evaluations(y_val, p_label)
30
      m = svm_train(y_train, x_train, '-s 0 -t 2 -c 0.1 -g 1 -q')
      p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
      ACC[1], MSE, SCC = evaluations(y_val, p_label)
33
      m = svm_train(y_train, x_train, '-s 0 -t 2 -c 0.1 -g 10 -q')
34
      p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
35
      ACC[2], MSE, SCC = evaluations(y_val, p_label)
36
      m = svm_train(y_train, x_train, '-s 0 -t 2 -c 0.1 -g 100 -q')
37
      p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
38
      ACC[3], MSE, SCC = evaluations(y_val, p_label)
39
40
      m = svm_train(y_train, x_train, '-s 0 -t 2 -c 0.1 -g 1000 -q')
      p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
41
      ACC[4], MSE, SCC = evaluations(y_val, p_label)
42
      #compare ACC
43
      compare[ACC.index(max(ACC))]+=1
44
print(compare.index(max(compare)))
```