Problem 2

Due March 30, 2018

In this problem, we use measurements of the time it takes for a small sphere to fall through a viscous fluid to infer the viscosity of the fluid. The fluid is a silicone oil. A graduated cylinder (photo is shown below) of diameter D was filled with the oil, spheres were dropped from the free surface at the center of the cylinder, and the time t it took for the sphere to fall between two of the markings on the cylinder (usually the 700ml mark and the 300ml mark) was measured. The distance h between the marks was measured with a caliper. The measurement interval was determined to be sufficiently far from the top for the terminal velocity to have been reached.

The measurements were taken in Urbana Illinois on three different days. On each day, the specific gravity S (ratio of fluid density to that of water) was measured with a hydrometer. Before being dropped into the oil, the diameter d of each sphere was measured, and it was weighed to determine its mass m. Spheres of two different nominal diameters and of two different materials (teflon and steel) were used.

The data from the above measurements is available in a spread sheet on canvas, along with this assignment. Also shown in the spread sheet is an estimate of the uncertainty in the measurements of t, S, d and m expressed as the standard deviation σ. The uncertainties in D and h are very small and are not reported.

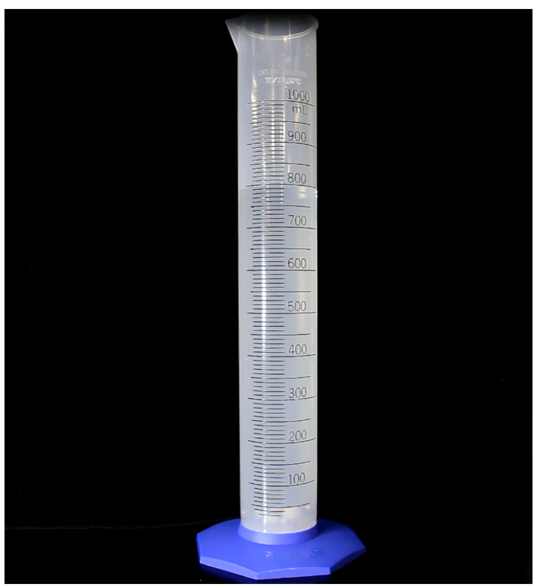

Figure 1: Graduated cylinder in which the spheres were dropped

1. As a preliminary analysis, use the nominal measured values for each sphere drop to compute the viscosity, using the Stokes law model from problem 1 of the first homework. Can you detect any systematic variations in the results? Also estimate the Reynolds numbers to determine whether the Stokes law model is applicable.

2. Formulate a Bayesian inference problem based on the Stokes law model to determine the viscosity of the fluid from the observed drop times, precisely defining your models for all uncertainties, the prior and the likelihood.

3. Determine posterior distributions for the viscosity on each day. What are the mean and standard deviation of these estimates? Can any variation of viscosity from day to day be detected from this data?

4. Recall that Stokes law is valid in the case of a sphere falling in an infinite fluid medium. While the ratio of the sphere diameter to the cylinder diameter is small, there could still be an effect of the cylinder walls. Test whether this is important by using data for the large light (teflon) spheres to infer the viscosity, and then using the inferred distribution of viscosity to predict the fall time of the smaller heavier (steel) spheres. Are these predictions consistent with the observed fall times?

5. Brenner [1] and Brenner & Happel [2] (papers available on canvas with this assignment) developed asymptotic analysis to account for the effects of the walls for a small sphere moving in a highly viscous fluid in a cylinder (as well as other situations). The result relevant for us is for a sphere moving axially in a cylinder and is given by (equations 3.1 and 3.10 in [1]).

$$F_d = \frac{3\pi\mu d V}{1 - [2.1044 - 0.6977\beta^2 + O(\beta^4)]\alpha + O(\alpha^3)}, \tag{1}$$

   where F_d is the drag force on the sphere, V is its axial velocity, $\alpha = d/D$ is the ratio of sphere to cylinder diameters, and $\beta = 2b/D$ is the ratio of the offset b of the center of the sphere from the cylinder axis to the radius of the cylinder. The denominator in (1) is a correction factor to the Stokes law drag.

   Revisit the consistency check in part 4 above using this refined model for the drag. How do the results change?

6. Revise your inference of the viscosity using the refined model in (1). Assess how important an error in dropping the sphere exactly on the axis of the cylinder might be, particularly how precisely must the sphere be located on the axis for the uncertainty due to this error to be unimportant.

References

[1] H. BRENNER, *Effect of finite boundaries on the Stokes resistance of an arbitrary particle*, Journal of Fluid Mechanics, 12 (1962), pp. 35–48.

[2] H. BRENNER AND J. HAPPEL, *Slow viscous flow past a sphere in a cylindrical tube*, Journal of Fluid Mechanics, 4 (1958), pp. 195–213.