Assignment 2
Phy 426, 2018
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DUE: Tue 25 Feb, 2017

Question 1. Conservation of momentum in hydraulically controlled flow

Consider flow over an isolated obstacle in a channel, as in class. Assume that the obstacle is a triangle, with height off a flat channel of $h_m = 10$ m, and that the triangle's ramp has a slope of 1/100. Assume that the incoming two-d flow transport is a fixed 30 $\mathrm{m^2\,s^{-1}}$, that the flow is in steady state, and that the flow is controlled at the crest.

1. What is the thickness of the water column, d_0, far upstream of the obstacle? (OK to use a root finder and give a numeric answer)
2. Knowing the height far upstream you can numerically integrate in x (or calculate the cubic at each point in x) to get the water thickness $d(x)$ as a function of x. Plot the water thickness as a function of x from $x = -1000m$ (upstream) to $x = 0$ m (the crest). Check that $F_m = 1$ in your calculation. For precision, make sure that you have a data point every 10 cm or so; include your code, and the expressions you used to get the interface heights. Make the plot as nice as possible (including the obstacle, helps.
3. Knowing the water thickness as a function of x, use the Momentum Theorem to numerically demonstrate that the momentum balance is satisfied.

Question 2. Standing wave in fjord

Consider a *hydrostatic* wave being forced at the mouth of a rectangular fjord of depth H and length L. The sea-surface height at the mouth of the fjord is prescribed by the sea-surface height in the ocean $\eta(0,t) = \eta_O \cos(\omega t)$, where ω is the tidal frequency.

1. Derive an expression for the the sea-surface height η as a function of x in the fjord assuming that there is no energy dissipation in the fjord. Describe the response of sea-surface height at the head of the fjord as a function of the length of the fjord, L. Also note that there are sometimes nulls in the response in the fjord. Where are they?
2. What is the relationship between $u(x,t)$ and $\eta(x,t)$ in the fjord? What happens to the velocity at the mouth as the fjord length approaches the resonant length?