Take-home Final
Phy 426, 2019
J. Klymak

DUE: Fri 26 Apr, 2019, 17:00 Exam to be completed independently. Open book, open notes are fine. Show all work, define any constants you need that I don't provide, check your units, etc. Except as noted, the density of the fluid is ρ, gravity is g, the kinematic viscosity ν, and the fluid can be assumed Bousinesque and incompressible.

Please try to make it readable. I will deduct up to 10% for illegible chicken scratches, so please take the time to recopy your work.

The value of each question is indicated in square brackets, the total is out of 22.

Question 1. Laminar flow out a pipe

A barrel with a viscous oil in it has a thin pipe with radius of 0.01 m leading out of it 1 m below the surface of the oil. Assume that the flow is laminar and viscous, with viscosity $\nu = 10^{-4}$ m^2 s^{-1}. Assume that surface tension is not important (though in such a thin pipe it might be).

1. [4] If the pipe is 4-m long and lays flat what is the flow rate out the end of the pipe (in m^{3}s^{-1})? (HINT: from the point of view of deriving the velocity in the pipe, assume that the pipe is infinite - i.e. don't worry about the end-effects.)

2. [2] Given the flow speed at the centre line of the pipe as a velocity scales, what is a horizontal length scale over which the boundary layer would become fully developed? Is the assumption suggested above OK? Is the assumption that the flow is laminar likely OK?

3. [5] Show that the rate of pressure work done by the fluid in the barrel on the fluid in the hose is equal to the rate of turbulent dissipation in the pipe.

Question 2. Waves reflecting from a step

Consider shallow water waves in a narrow channel where the water depth (at rest) shoals from 10 m to 3 m at a step. Assume no viscous losses. Measurements of along-channel velocity and water depth are taken 50m from the step in the deep part of the channel. Both signals are found to have a period of 8 s, and the peak of the positive sea surface height is found to occur 1 s before the peak of positive down-channel velocity. The sea surface height *in the shallow water* is measured to have an amplitude of 0.1 m.

1. [8] The signal at $x = -50$ m consists of a wave moving towards the step, and one partially reflected from the step. Give the information above, derive the amplitudes of these two waves, expressing your answer as two equations in two unknowns. Note that the incoming and reflected waves are in phase at $x = 0$.

2. [3] Numerically solve for the amplitude of the incoming are reflected waves given the information above. (I used `scipy.optimize.fmin`).