Assignment 3
Phy 426, 2018
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Question 1. Numerical Potential Flow

The text describes an iterative procedure for solving for solving for potential flow. Consider a channel, 50 m wide, z=0 in the centre, with a 2-D transport of 1 $m^2 s^{-1}$ in steady state. Two half cylinders are in the flow along the lower boundary, with their centers at (0.,-25) and radius 20 m, and (40,-25), radius 14 m.

For all of these, please turn in your code. I did this in Matlab and Python.

1. Solve, numerically, for the flow and plot 50 evenly-spaced streamlines (in streamline space). (HINTS: The code for this can be very short in a matrix language like matlab or numpy in Python. You only need one for-loop to do the iterations on ψ. You need to set the lower boundary value of $\psi = 0$ $m^2 s^{-1}$, and the upper boundary to $\psi = 1$ $m^2 s^{-1}$.)

2. Construct a solution with two doublets and calculate the streamlines from that, and *quantitatively* compare with your fully numerical solution. What is the major difference between the two solutions, and how would you make your numerical solution more like the analytical? (HINTS, in class we did the analytical solution using cylindrical co-ordinates, but that is awkward here, where there are two doublets. Instead, just form the complex flow potential $w(z)$ and contour the imaginary part. Also note that the values of ψ are arbitrary to a constant, so you may want to set the lowest streamline's value to zero to compare with your numerical answer.)

Question 2. Zhukhovski Airfoil

Using the Zhukhovski Transform, numerically demonstrate the lift law for an airfoil.

1. For suitable choice of an offset in the ζ-plane, transform a circle into an airfoil in the z-plane. Note that for full marks it should look like an airfoil!

2. Show the streamlines for a flow with mean speed U and circulation $-\Gamma$, where you chose the appropriate values for those constants around the sphere and airfoil. Note that you should chose Γ to satisfy the Kutta condition. (HINT: you want to define your cylinder in the ζ-plane, but you want to plot your streamlines in the z-plane where the cylinder is an airfoil. So on an even grid in x and y, for each value of $z = x + iy$ determine the value of ζ, and then determine the value of $\psi(z)$ from its value $\psi(\zeta(x, y))$.)

3. Repeat the above for two other angles of attack α, where you chose α.

4. Calculate and plot the pressure field (up to an arbitrary constant) around the circle and airfoil. ($|\mathbf{u}(z)|$=$|dw/dz| \approx |dw/dx|$ which you can calculate from first difference from your grid (or analytically for the cylinder) Then you need to evaluate on the edge of the body.).

5. Calculate the lift from the pressure field for all three values of the angle of attack. Compare to the Zhukovski lift law.