Assignment 1
Phy 426, 2018
J. Klymak

DUE: Fri 2 Feb, 2017, in class (or by email before class) Late penalty 10% per day (days rounded up from 11:31 AM!)

Question 1. Stability of a block

Consider a block of density ρ, with vertical height L, and widths in the other two directions of d floating on top of a fluid of density ρ_o.

1. Demonstrate that the block is stable to tipping if

$$\left(\frac{d}{L}\right)^2 > 6\left(\frac{\rho}{\rho_o}\right)\left(1-\frac{\rho}{\rho_o}\right) \tag{1}$$

   Some hints: Consider the forces in a co-ordinate frame that is rotated by a small perturbation angle θ, where the rotation takes place along the central axis of the block at the water line. The block is still vertical in this reference frame, but the gravity forces no longer point directly down. Further, the center of mass of the water displaced is no longer along the central axis of the block, and hence there is a torque.

2. If the block is unstable, but rotated 90 degrees, will it be stable?

Question 2. Shear Parallel Flow

Consider a flow between two parallel infinite plates a distance H apart. The upper plate is moving with speed U parallel to the other plate, which is motionless. The flow starts at rest and the plate is impulsively moved to speed U. [Assume that the flow is laminar (i.e. no turbulence develops) and that the viscosity of the fluid is given by $\nu/(HU) = 0.1$. Assume that there are no net pressure forces.]

1. What is the steady-state solution of the flow between the two plates (i.e. $u_0(z) \equiv u(z, t = \infty)$)
2. Derive a differential equation for the transient flow ($w(z,t) = u(z,t) - u_0(z)$) and state the spatial and temporal boundary conditions for $w(z,t)$.
3. Determine an appropriate form for the solution to $w(z,t)$ (Hint: The solution to w is separable in time and space. In order to match the initial condition, a discrete Fourier series in z is required.)
4. Using a graphics software, plot the solution at time $t/(H^2/\nu) = 0.01,\ 0.05,\ 0.2,\ 0.5,\ 1.,\ 10.$

Question 3. A hydraulic jack

Consider a hydraulic jack consisting of a wide pipe with cross-sectional area A_1, and a narrow pipe with cross-sectional area A_2, with both pipes connected and filled with a fluid. At rest, the level of the fluids in the two pipes is equal

1. If a mass m is added to the wide pipe on a plunger such that the whole mass is supported by the fluid below. How deep will the plunger plunge relative to the resting depth?
2. Suppose we now exert a force F_2 on the surface of the fluid in the second pipe using a plunger. What will the force F_1 be on the first plunger?
3. Suppose the first plunger rises height h_1 due to the application of this force over time Δt. How much work does the fluid do on the mass?
4. The force exerted on pipe 2 is much less than the force on the mass in pipe 1. Why doesn't this violate the conservation of energy?

Question 4. The nonlinear advective term

We need to use

$$\mathbf{u} \cdot \nabla \mathbf{u} = \omega \times \mathbf{u} + \nabla(\frac{\mathbf{u}^2}{2}) \tag{2}$$

1. Prove that this is true, either by writing out the components or using index notation.