Take-home Final
Phy 426, 2018
J. Klymak

DUE: Fri 13 Apr, 2017, 17:00 Exam to be completed independently. Open book, open notes are fine. Show all work, define any constants you need that I don't provide, check your units, etc. Except as noted, the density of the fluid is ρ, gravity is g, the kinematic viscosity ν, and the fluid can be assumed Bousinesque and incompressible.

Please try to make it readable. I will deduct up to 10% for illegible chicken scratches, so please take the time to recopy your work.

The value of each question is indicated in square brackets, the total is out of 75.

Question 1. Laminar boundary layer under a shallow-water wave

Consider a shallow-water wave over a flat-bottom sea-floor of depth H. Assume the wave is specified by the sea-surface displacement $\eta(x,t) = \eta_0 \cos(kx - \omega t)$, where ω is the frequency of the wave (rad/s) and k is the wavenumber (rad/m). Assume that the flow remains laminar throughout the water column, has a viscosity ν, and obeys a no-slip boundary condition at the sea floor.

1. [4] Describe an appropriate Reynolds number for the flow, and show for a 0.1-m amplitude wave in 10-m of water with a 16-s period the Reynolds number is very large.
2. [4] Derive a scaling for the thickness of the bottom boundary layer δ and state how large it might be given the parameters above.
3. [12] Derive an expression for the velocity above the sea-floor at $x = 0$, $u(0, z, t)$. (HINT: think about the scaling here - from the very thin boundary layer, the flow above looks infinite in both the vertical and the horizontal). Sketch the solution at a couple of times.
4. [12] What is the rate of energy dissipation due to the boundary layer viscosity, averaged over a wave period? What is an estimate of the decay time-scale of the wave (i.e. *do not* solve the full decaying time-dependent equation; assume the decay timescale is much greater than the period of the waves, so that you can assume the wave forcing is steady for the calculation).

Question 2. Lossy standing waves

Consider the quasi-steady response in a rectangular basin H deep, W wide, with a vertical wall at one end ("the head") to forcing at a tidal frequency ω. Assume that ω is small enough that the waves are shallow-water waves.

1. [4] A distance L from "the head" of the channel, the pressure is measured with a gauge to vary as $p = p_o \cos \omega t$. Assuming no energy losses in the basin, what is the water height as a function of x, and t?
2. [4] Again, assume that the flow loses no energy, what would the functional dependence of $u(x,t)$ be?

3. [12] Suppose we measure u at $x = L$ from the head, and it is found to be given by $u = u_o \sin(\omega t + \phi)$. what is the average rate of energy loss inside the embayment in terms of p_o, u_o and ϕ?

4. [7] If $\phi << 1$, $p_o << \rho g H$, and $L << \frac{2\pi\sqrt{gH}}{\omega}$, approximate the energy loss just in terms of p_o and ϕ.

Question 3. Flow around a Headland

Consider a headland protruding into a rectangular channel of free-stream depth H. The headland is approximated by a circle with radius R. Assume that R is much less than the width of the channel, and that the free-stream speed far from the headland is U along the channel. The flow separates from the headland at the tip.

1. [4] What is the fastest speed along the headland?

2. [4] Assume that the pressure in the separation bubble behind the tip is the same as at the tip. What is the total drag on the flow exerted by the headland?

3. [4] Assuming that the separation vortex is a Rankine vortex with a radius similar to the headland (i.e. approximately R) and outer speed given by the speed of the flow past the headland, derive an expression for the sea-surface height in the eddy?

4. [4] Suppose the mean flow suddenly turns off. What direction, and how fast will the headland eddy start to move?