

Running funconstrain tests in package optimx

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Abstract

The **funconstrain** package (<https://github.com/jlmelville/funconstrain>) provides R users with a convenient tool to access the test functions of Moré, Garbow, and Hillstom (1981). This vignette article describes a program to apply these test functions to solvers in the **optimx** package (Nash and Varadhan (2011)).

Background

Numerical optimization of functions of several, namely **n**, parameters is an important computational task. R (R Development Core Team (2008)) is a major platform for scientific and statistical calculations and has provided tools for numerical optimization and nonlinear least squares since its inception. These have been extended via a number of packages. In particular, the author has been heavily involved in this effort, and in collaboration with others has provided the package **optimx** which wraps a number of solvers to allow their invocation by a common calling syntax. Note that *optimization* in R generally means *function minimization*, possibly with bounds (or box) constraints on the function parameters.

It is extremely helpful to users to have examples and tests of function minimization. In many situations it is extremely easy to insert an error into code, so easy-to-apply tests allow for the discovery of such errors. There are a number of collections of test functions with many overlaps and minor differences. A well-established and well-documented set of such functions are those of Moré, Garbow, and Hillstom (1981). These have been translated into R by James Melville in the R package **funconstrain** (<https://github.com/jlmelville/funconstrain>). While initially these provided the function and its gradient given a set of suitable input parameters, the present author added code to compute the Hessian for each test function. This allows Newton-like solvers to be applied. **funconstrain** also provides suggested initial parameter vectors for each of the 35 test functions. However, where there are multiple input possibilities, just one is provided, for example when the test function has a variable number of parameters.

What is then missing is the link between **funconstrain** and the tools in **optimx**, which this article aims to provide.

Function fufn()

Most of the test functions in (**More1981TU?**) are sums of squares of nonlinear functions. While **n** is the number of parameters, we may have a different number of functions squared in the summation. Call this **m**. This may be altered to give different variations of a given function, so **m** must be provided.

Many of the solvers in **optimx** are capable of handling bounds constraints on the **n** parameters. That is parameter **i** must satisfy

```
lower[i] <= prm[i] <= upper[i]
```

where **prm** is the parameter vector and **lower** and **upper** are vectors of numbers providing lower and upper bounds. Methods in **optimx** that can handle masks are listed in the character vector **bdmeth** returned by the function **optimx::ctrldefault(n)**. Note that a number of parameters **n** must nominally be provided to **ctrldefault()** but generally **n** can be specified as 2 to get the default settings for ‘optimx’. At time of writing

```
bdmeth <- c("L-BFGS-B", "nlinb", "lbfgsb3c", "Rcgmin", "Rtnmin", "nvm",
"Rvmmmin", "bobyqa", "nmkb", "hjk", "hjn", "snewtonm", "ncg",
"slsqp", "tnewt", "nlm", "snewtm", "spg")
```

If the upper and lower bound for a parameter are equal, we can say the parameter is **fixed** or **masked**. This may seem to be a silly option, since it essentially reduces the dimensionality of the problem. However, there are many situations where we have evidence that a parameter takes a particular (fixed) value, but know that we may wish to allow optimization over that parameter in later investigations. Masks allow us to avoid having to rewrite the function, gradient and Hessian code. However, only a few optimization solvers handle masks. The function `optimx::ctrldefault()` returns a value `maskmeth` with a list of solvers that do handle the situation where lower and upper bounds coincide. At the time of writing this is specified as

```
maskmeth <- c("Rcgmin", "nvm", "hjn", "ncg", "snewtonm", "nlinb", "L-BFGS-B")
```

With the above in mind, the function `fufn()` was written to access the test functions of `funconstrain`.

The fufn.R code

```
fufn <- function(fnum=NULL){
  # return list with tfn=function, tgr=gradient given fn number and n
  if (is.null(fnum)) stop("ffn needs a function number fnum")
  if ((fnum < 1) || (fnum > 35)) stop("fnum must be in [1, 35]")
  # cat("entering ffn, fnum=", fnum, "\n")
  # select function
  funnam <- c("rosen", "freud_roth", "powell_bs", "brown_bs", "beale",
    "jenn_samp", "helical", "bard", "gauss", "meyer", "gulf",
    "box_3d", "powell_s", "wood", "kow_osb", "brown_den",
    "osborne_1", "biggs_exp6", "osborne_2", "watson", "ex_rosen",
    "ex_powell", "penalty_1", "penalty_2", "var_dim", "trigon",
    "brown_al", "disc_bv", "disc_ie", "broyden_tri", "broyden_band",
    "linfun_fr", "linfun_r1", "linfun_r1z", "chebyquad")
  # print(str(funnam))
  fname <- funnam[as.integer(fnum)]
  # cat("fname:", fname, "\n")
  while (fnum %in% 1:35) {
    ameth <- optimx::ctrldefault(2)$bdmeth # Choose only bounded methods
    ameth <- ameth[ameth != "lbfgsb3c"] ## ?? Temporarily remove lbfgsb3c
    ameth <- c(ameth, "L-BFGS-B")
    # ?? may want to test allmeth to check that inappropriate methods are captured
    # cat("in while, fnum=", fnum); tmp <- readline("cont.")
    mm <- 0 # in case m value needed
    if (fnum == 1) {
      n <- 2 # fixed
      mm <- 2
      tt <- rosen()
      if (is.function(tt$x0)) {
        xx0 <- tt$x0(n)
      }
      else xx0 <- tt$x0
      lo <- rep((min(xx0)-0.1), n)
      up <- rep((max(xx0)+0.1), n)
      break }

    if (fnum == 2) {
      n <- 2 # fixed
```

```

mm <- 2
tt <- freud_roth()
if (is.function(tt$x0)) {
  xx0<-tt$x0(n)
}
else xx0 <- tt$x0
lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 3) {
  n <- 2 # fixed
  mm <- 2
  tt <- powell_bs()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 4) {
  n <- 2 # fixed
  mm <- 3
  tt <- brown_bs()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
## BAD -- reset 20240323
#   lo <- rep((min(xx0)-0.1), n)
#   up <- rep((max(xx0)+0.1), n)
  lo <- -1e20
  up <- -lo
  break }

if (fnum == 5) {
  n <- 2 # fixed
  mm <- 3
  tt <- beale()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 6) {
  n <- 2 # fixed
  mm <- 10
  tt <- jenn_samp()

```

```

if (is.function(tt$x0)) {
  xx0<-tt$x0(n)
}
else xx0 <- tt$x0
lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 7) {
  n <- 3 # fixed
  tt <- helical()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 8) {
  n <- 3 # fixed
  mm <- 15
  tt <- bard()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 9) {
  n <- 3 # fixed
  mm <- 15
  tt <- gauss()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 10) {
  n <- 3 # fixed
  m <- 16 # ?? how to return
  tt <- meyer()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)

```

```

break }

if (fnum == 11) {
  n <- 3
  mm <- 99
  tt <- gulf()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 12) {
  n <- 3
  mm <- 20
  tt <- box_3d()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 13) {
  n <- 4
  tt <- powell_s()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 14) {
  n <- 4
  tt <- wood()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 15) {
  mm <- 11
  n <- 4
  tt <- kow_osb()
  if (is.function(tt$x0)) {

```

```

    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 16) {
  mm <- 20
  n <- 4
  tt <- brown_den()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 17) {
  mm <- 33
  n <- 5
  tt <- osborne_1()
  ameth<-ameth[-which(ameth=="L-BFGS-B")] # remove L-BFGS-B from this case
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  lo[4] <- 0
  lo[5] <- 0
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 18) {
  mm <- 20
  n <- 6
  tt <- biggs_exp6()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 19) {
  mm <- 65
  n <- 11
  tt <- osborne_2()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }

```

```

else xx0 <- tt$x0
lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 20) {
  n <- 8
  mm <- 31
  tt <- watson()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 21) {
  n <- 10
  tt <- ex_rosen()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 22) {
  n <- 20
  tt <- ex_powell()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 23) {
  n <- 10
  mm <- n + 1
  tt <- penalty_1()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 24) {
  n <- 10

```

```

mm <- n + 1
tt <- penalty_2()
if (is.function(tt$x0)) {
  xx0<-tt$x0(n)
}
else xx0 <- tt$x0
lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 25) {
  n <- 6
  mm <- n + 2
  tt <- var_dim()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 26) {
  n <- 8
  tt <- trigon()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 27) {
  n <- 8
  mm <- n
  tt <- brown_al()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 28) {
  n <- 6
  mm <- n
  tt <- disc_bv()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0

```



```

lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 29) {
  n <- 8
  mm <- n
  tt <- disc_ie()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 30) {
  n <- 8
  mm <- n
  tt <- broyden_tri()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 31) {
  n <- 8
  mm <- n
  tt <- broyden_band()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 32) {
  mm <- 10
  n <- 8
  tt <- linfun_fr()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 33) {

```

```

mm <- 10
n <- 8
tt <- linfofun_r1()
if (is.function(tt$x0)) {
  xx0<-tt$x0(n)
}
else xx0 <- tt$x0
lo <- rep((min(xx0)-0.1), n)
up <- rep((max(xx0)+0.1), n)
break }

if (fnum == 34) {
  mm <- 10
  n <- 8
  tt <- linfofun_r1z()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }

if (fnum == 35) {
  n <- 8
  m <- n
  tt <- chebyquad()
  if (is.function(tt$x0)) {
    xx0<-tt$x0(n)
  }
  else xx0 <- tt$x0
  lo <- rep((min(xx0)-0.1), n)
  up <- rep((max(xx0)+0.1), n)
  break }
}
# NOTE: bounds are experimental only
mask <- rep(1L, n) # masks set to "free" (not masked)
val <- list(npar = n, ffn=tt$fn, fgr=tt$gr, x0=xx0, lo=lo, up=up,
           mask=mask, fname=fname, ameth=ameth)
# cat("val:"); print(val); tmp<-readline('exit ffn')
val
} # end fufn

```

Calling fufn()

While we can write our own driver for `fufn()`, I wanted to make the task extremely easy. Thus the script `fufnrun.R` is provided. This is set up to use a simple text file to specify which test functions are to be applied to which solvers. Moreover, a “sink” function name can be specified to save the text output of the run.

Test specification file `RFO.txt`

Let us consider an example.

```

testsink240408A.txt
1, 6:8, 35
c("L-BFGS-B", "lbfgsb3c", "lbfgs")
FALSE

```

The lines of the above file provide the following information:

- the first line is the name of the text file to use to save the output
 - line 2 says that test functions 1, 6, 7, 8, and 35 are to be used. Note that we can use the colon “:” when giving a contiguous range of function numbers. These numbers – by referring back to the vector `funnam` at the top of function `fufn()` – specify functions “rosen”, “jenn-samp”, “helical”, “bard” and “chebyquad”. Using the function numbers. Appendix A lists the numbers and corresponding names.
 - line 3 gives an R character vector of the solver methods to be applied.
- ‘ line 4 is TRUE if the experimental bounds constraints are to be applied.

A driver program for `fufn()`

The following driver program will run the tests specified by `RFO.txt`:

```

# fufnrun.R -- J C Nash 2024-4-8
## ?? fixing kkt
# RFO.txt is input file
source("./fufn.R") # ensure fufn() loaded
library(funconstrain) # get the functions
library(optimx)
mycon<-file("RFO.txt", open="r", blocking = TRUE)
sfname<-readLines(mycon, n=1)
if (length(sfname) == 0) {
  cat("no sink file\n")
} else {
  cat("opening sink file ",sfname,"\n")
  sink(sfname, split=TRUE)
} # open sink file

## opening sink file testsink240408A.txt
cat("sink file name=",sfname,"\n")

## sink file name= testsink240408A.txt
lin2 <- readLines(mycon, n=1)
cat("probs =",lin2,"\n")

## probs = 1, 6:8, 35
if (length(lin2) == 0) stop("Unexpected null probs")
txt<-paste("probc<-c(",lin2,")","")
tryparse<-eval(parse(text=txt))
# ?? should we check it worked?
cat("Problem numbers:\n"); print(probc)

## Problem numbers:
## [1] 1 6 7 8 35

# check loop
for (iprob in probc){ # loop over problems

```

```

    if ( (iprob < 1) || (iprob > 35) ) {
        stop('Problem number out of range. Stopping.')
    }
} # end check loop
meths <- readLines(mycon, n=1)
if (length(meths) == 0) stop("Unexpected null meths")
cat("Methods:\n")

## Methods:
cat(meths, "\n")

## c("L-BFGS-B", "lbfgsb3c", "lbfgs")
methvec<-paste("methc<-c(",meths,")","")
tryparse<-eval(parse(text=methvec))
# cat("methods in list form:"); print(methc)
tbounds<-readLines(mycon, n=1)
have.bounds<-FALSE
if (tbounds == "TRUE") have.bounds<-TRUE
cat("have.bounds:",have.bounds, "\n")

## have.bounds: FALSE
close(mycon)
for (iprob in probc){ # loop over problems
    tfun <- fufn(fnum=iprob)
    # print(tfun)
    cat("Problem:", tfun$fname, "\n")
    x0 <- tfun$x0
    if (have.bounds){
        lo <- tfun$lo
        up <- tfun$up
    }
    else {
        lo <- -Inf
        up <- Inf
    }
    tfn <- tfun$fffn
    attr(tfn, "fname") <- tfun$fname
    tgr <- tfun$ffgr
    the <- tfun$ffhe
    nx0<-length(x0)
    # cat("about to call opm\n")
    if (have.bounds) {
        t21 <-opm(x0, tfn, tgr, hess=the, lower=lo, upper=up, method=methc,
            contro=list(trace=0))
    } else {
        t21 <-opm(x0, tfn, tgr, hess=the, method=methc, contro=list(trace=0))
    }
    print(summary(t21, order=value, par.select=1:min(nx0,5)))
    cat("END :", tfun$fname, "\n\n")
}

## Problem: rosen
## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :

```

```

## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

##           p1 s1           p2 s2           value fevals gevals hevals conv kkt1
## L-BFGS-B 0.9999997 0.9999995 2.267577e-13 47 47 0 0 TRUE
## lbfgs 1.0000006 1.0000012 3.545445e-13 45 45 0 0 TRUE
## lbfgsb3c 1.0000275 1.0000560 8.551377e-10 74 74 0 0 TRUE
##           kkt2 xtime
## L-BFGS-B TRUE 0.004
## lbfgs TRUE 0.004
## lbfgsb3c TRUE 0.052
## END : rosen
##
## Problem: jenn_samp
## Error in optim(par = par, fn = efn, gr = egr, method = method, hessian = FALSE, :
## non-finite value supplied by optim

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

##           p1 s1           p2 s2           value fevals gevals hevals conv
## lbfgs 0.2578252 0.2578252 1.243622e+02 67 67 0 -1001
## lbfgsb3c 0.2578252 0.2578252 1.243622e+02 45 45 0 0
## L-BFGS-B NA NA 8.988466e+307 24 24 0 9999
##           kkt1 kkt2 xtime
## lbfgs TRUE TRUE 0.001
## lbfgsb3c TRUE TRUE 0.001
## L-BFGS-B NA NA 0.001
## END : jenn_samp
##
## Problem: helical

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

##           p1 s1           p2 s2           p3 s3           value fevals
## lbfgs 1.0000000 -2.066737e-08 -3.335635e-08 1.517549e-15 34
## L-BFGS-B 0.9999999 -8.281311e-07 -1.286731e-06 2.203966e-12 30
## lbfgsb3c 0.9999990 5.628310e-06 8.770209e-06 1.855335e-10 53
##           gevals hevals conv kkt1 kkt2 xtime
## lbfgs 34 0 0 TRUE TRUE 0.001
## L-BFGS-B 30 0 0 TRUE TRUE 0.000

```

```

## lbfgsb3c      53      0      0 TRUE TRUE 0.001
## END : helical
##
## Problem: bard

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

##
##          p1 s1      p2 s2      p3 s3      value fevals gevals hevals
## L-BFGS-B 0.08241058    1.133036    2.343695    0.008214877      24      24      0
## lbfgs     0.08240992    1.133030    2.343698    0.008214877      24      24      0
## lbfgsb3c 0.08232066    1.130012    2.346610    0.008214943      46      46      0
##          conv kkt1 kkt2 xtime
## L-BFGS-B    0 TRUE TRUE 0.001
## lbfgs       0 TRUE TRUE 0.001
## lbfgsb3c    0 TRUE TRUE 0.002
## END : bard
##
## Problem: chebyquad

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

## Warning in kktchk(ans$par, fn, wgr, hess = NULL, upper = NULL, lower = NULL, :
## kktchk: pHes not symmetric -- symmetrizing

##
##          p1 s1      p2 s2      p3 s3      p4 s4      p5 s5
## L-BFGS-B 0.04315278    0.1930908    0.2663287    0.4999999    0.5000001
## lbfgs     0.04315297    0.1930910    0.2663289    0.4999998    0.5000002
## lbfgsb3c 0.04315505    0.1930877    0.2663283    0.4999442    0.5000558
##          value fevals gevals hevals conv kkt1 kkt2 xtime
## L-BFGS-B 0.003516874     28     28      0      0 TRUE TRUE 0.001
## lbfgs     0.003516874     25     25      0      0 TRUE TRUE 0.001
## lbfgsb3c 0.003516880     46     46      0      0 TRUE TRUE 0.002
## END : chebyquad

```

```

sink()

```

Appendix A: function numbers and names

```

1      rosen
2      freud_roth
3      powell_bs
4      brown_bs
5      beale
6      jenn_samp
7      helical

```

```
8      bard
9      gauss
10     meyer
11     gulf
12     box_3d
13     powell_s
14     wood
15     kow_osb
16     brown_den
17     osborne_1
18     biggs_exp6
19     osborne_2
20     watson
21     ex_rosen
22     ex_powell
23     penalty_1
24     penalty_2
25     var_dim
26     trigon
27     brown_al
28     disc_bv
29     disc_ie
30     broyden_tri
31     broyden_band
32     linfofr
33     linfofr1
34     linfofr1z
35     chebyquad
```

References

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- R Development Core Team. 2008. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. <http://www.R-project.org>.