

## Lecture 12: Computability

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Content is borrowed from Susanna Epp's Discrete Mathematics with Applications,  
Rosens's Discrete Mathematics and its Applications,  
Bettina and Thomas Richmond's A Discrete Transition to Advanced Mathematics, and Andrew  
Altomare's notes.

### **Exercise**

Show that the set of all computer programs in a given computer language is countable.

Now, we want to show that there are non-computable functions.

Let  $T$  be the set of all functions from the positive integers to the set  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .

### **Exercise**

Show that  $T$  is uncountable.

### **Exercise**

We're now ready to state that there noncomputable functions. Show that in any computer language there must be a function  $F$  from  $\mathbb{Z}^+$  to  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  with the property that no computer program can be written in the language to take arbitrary values as input and output the corresponding function values.