LaTeXsubmissions are mandatory. Submitting your assignment in another format will result in a **loss of 10 points** on the assignment. The template is here.

Problem 1
[**12pts**] Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{1, \{\varnothing, 2\}\}$. Find each of the following sets:

(a) $A \times (B \cup C)$

(b) $A \times (B \cap C)$

(c) $(A \times B) \cup (A \times C)$

(d) $(A \times B) \cap (A \times C)$

Proof:

Problem 2
[**10pts**] We saw in class that for sets A and B, the cardinality of their union is given by:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Prove the following analogous rule for the union of three sets:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Hint: Start by treating $(B \cup C)$ as one set and then apply the two-set rule given above, along with set identities from lecture. You may use the two-set rule as a given and it can help to draw a Venn diagram.

Proof:

Problem 3
[**20pts**] Let $A = \{x \in \mathbb{Z} \mid x = 5a + 2, a \in \mathbb{Z}\}$, $B = \{y \in \mathbb{Z} \mid y = 10b - 3, b \in \mathbb{Z}\}$, and $C = \{z \in \mathbb{Z} \mid z = 10c + 7, c \in \mathbb{Z}\}$. *Prove or disprove* the following:

(a) $A \subseteq B$

(b) $B \subseteq A$

(c) $B = C$

Proof:

Problem 4
[**20pts**] For the following functions determine whether they are injective, surjective, both or neither.

(a) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = n + 1$

(b) $g : \mathbb{Z} \to \mathbb{Z}$, $g(n) = \lceil \frac{n}{2} \rceil$

(c) $h : \mathbb{R} \to \mathbb{R}$, $h(x) = (x^2 + 1)/(x^2 + 2)$

(d) $j : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $j(m, n) = 3^m 5^n$

Proof:

Problem 5
[10pts] For the following function definitions, provide a domain and codomain that ensures it is actually a function. Remember that the codomain does not have to be precisely the range.

(a) $m(x) = \sqrt{x+1}$

(b) $n(x) = 5$

(c) $p(x) = 1/(x^2 + 2)$

Proof:

Problem 6
[15pts] Consider the following sequence:

$$3, 7, 15, 31, 63, ...$$

(a) Find the next three elements of the sequence

(b) Come up with a recurrence relation for the sequence

(c) Find a closed form for the recurrence relation you found and prove that it is a solution to the recurrence relation.

Proof:

Problem 7
[10pts] Prove that the cardinality of the integers is the same as that of the *even* integers.

Proof:

Problem Reflection
[3pts] Exercise a growth mentality by reflecting on this assignment and your work. Feel free to say whatever you want, but you are required to answer the following. You are graded on whether you complete this, not on what you say.

- How many hours did you spend on this assignment?
- What problem was hardest? Why?
- What problem was easiest? Why?