Propositional Logic and Inference

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CSCI 2824
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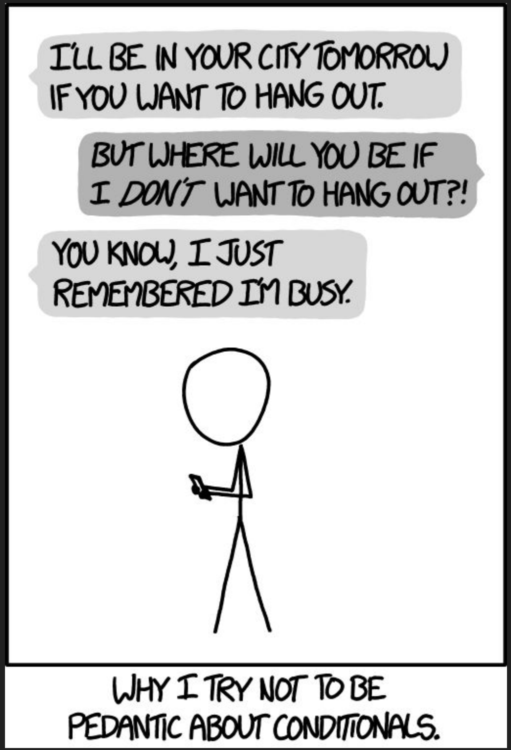
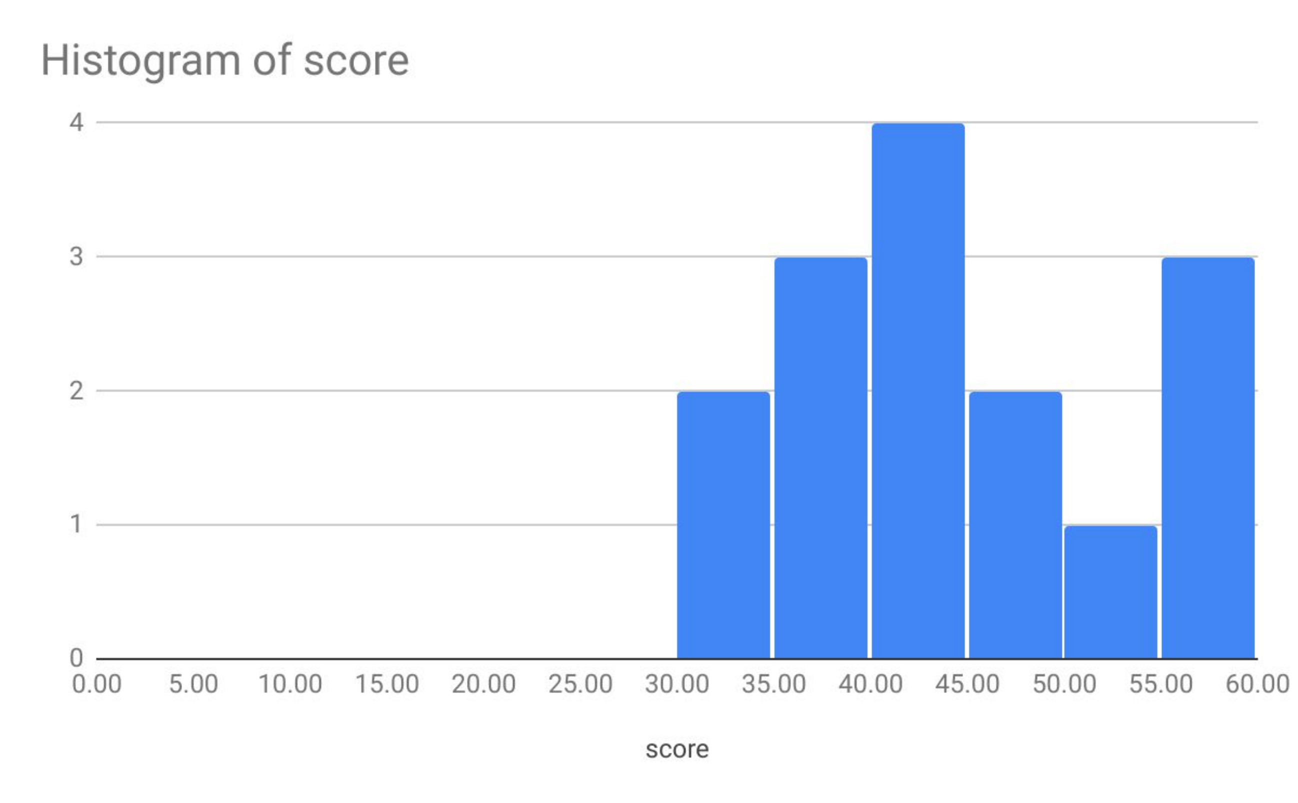


https://xkcd.com/1652/

https://xkcd.com/1856/

Mindset



Histogram of score
4
3
2
1
0
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
55.00
60.00
score

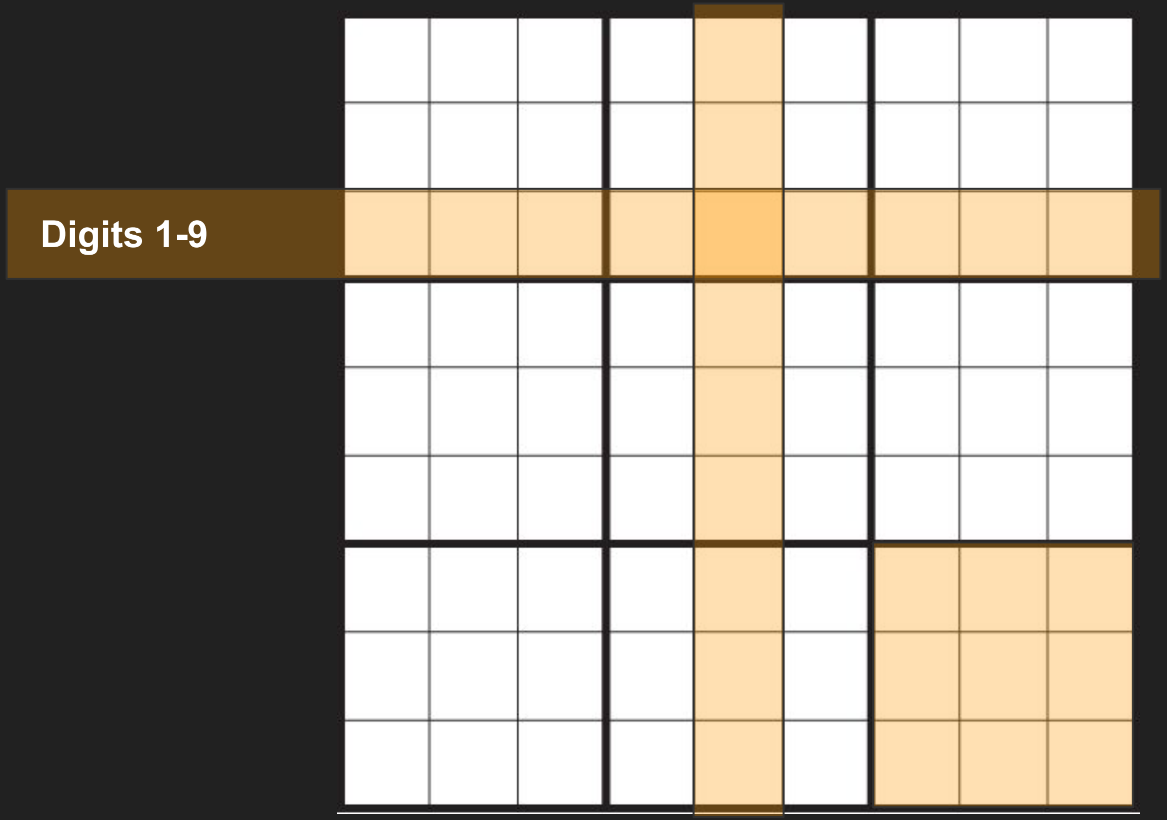
Sudoku as Logic!

Sudoku as Constraint Satisfaction

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

https://en.wikipedia.org/wiki/Sudoku#/media/File:Sudoku_Puzzle_by_L2G-20050714_standardized_layout.svg

Sudoku as Constraint Satisfaction



Sudoku as Constraint Satisfaction

Digits 1-9

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Logical Equivalence

Logical Equivalence

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Epp Book

Are they equivalent?

$\sim(p \wedge q)$ $\quad\quad\quad\quad$ $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

Epp Book

Valid?

$$p \to q \vee \sim r$$
$$q \to p \wedge r$$
$$\therefore p \to r$$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

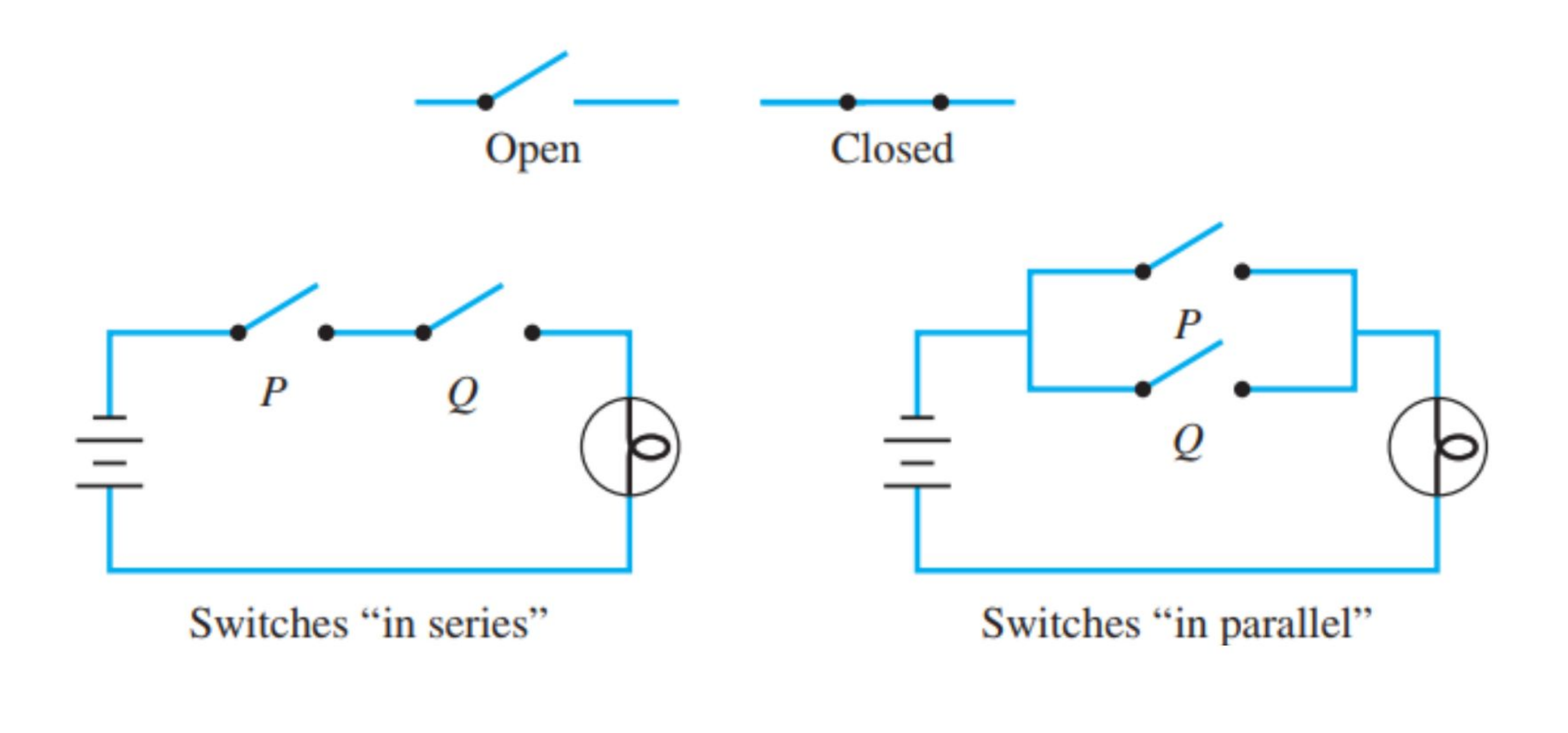
This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

■

Inference

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

Digital Circuits



Circuits and Logic?

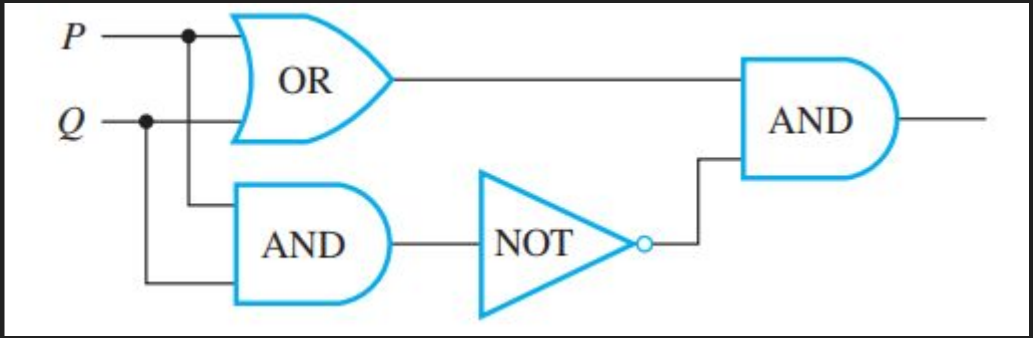
(a) Switches in Series

Switches		Light Bulb
P	*Q*	State
closed	closed	on
closed	open	off
open	closed	off
open	open	off

(b) Switches in Parallel

Switches		Light Bulb
P	*Q*	State
closed	closed	on
closed	open	on
open	closed	on
open	open	off

What is the formula?



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What did you like? What did you not like?