Your exam will be shorter than this. These are practice problems of the flavor you can expect. You will be provided with an necessary logical equivalences so no need to memorize anything other than some formal definitions such as cardinality, injective/surjective/bijection, function, proposition, etc., for the exam. Please also see the study guide for topics.

Problem 1
Which of the following are propositions?

(a) Boston is the capital of Massachusetts

(b) Miami is the capital of Florida

(c) 2+3=5

(d) 5+7=10

(e) Go to school

A proposition is a declarative statement that can be assigned some truth value.

(a) Yes

(b) Yes

(c) Yes

(d) Yes

(e) No

Problem 2
For a compound proposition with constituents p, q, r, s, t, u, how many rows would appear in the truth table?

For a compound proposition with n constituent propositions, the number of rows necessary for a truth table is 2^n, so for 6 constituents, we have $2^6 = 64$ rows.

Problem 3
Construct a truth table for each of these compound propositions.

(a) $p \wedge \neg p$

(b) $(p \vee q) \Rightarrow (p \wedge q)$

(c) $(p \Rightarrow q) \oplus (p \Rightarrow \neg q)$

(d) $p \oplus (p \vee q)$

(a)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(b)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \Rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

(c)

p	q	$\neg q$	$p \Rightarrow q$	$p \Rightarrow \neg q$	$(p \Rightarrow q) \oplus (p \Rightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	T	F

(d)

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

Problem 4
Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q and r have the same truth value and is false otherwise.

This proposition is a conjunction of three smaller propositions: $(p \vee \neg q)$, $(q \vee \neg r)$ and $(r \vee \neg p)$, all of which must yield true to satisfy the entire compound proposition. Knowing that the first must be true, we can split the entire problem into two cases: p is true or $\neg q$ is true. If p is true, then by the third conjunct and hypothetical syllogism, we know that r must be true. Combine that fact with the second conjunct, using again hypothetical syllogism, it must be the case that q is true. So if p is true, then q and r must also be true.

Consider now the second case, when $\neg q$ is true. Using a similar line of reasoning, we can conclude that $\neg r$ must be true and $\neg p$ must be true. Therefore, to satisfy this compound proposition, we need p, q, r to be all true or all false.

Problem 5
Translate the following into propositional logic.

(a) To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service.

(b) You are eligible to be the President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country.

(c) You can see the movie only if you are over 18 years old or you have the permission of a parent.

(a) Let p mean "you can use the wireless network," q mean "you pay the daily fee" and r mean "you are a subscriber to the service"

$$p \Rightarrow (q \vee r)$$

(b) Let p mean "you are eligible to be president," q mean "you are at least 35," r mean "you were born in the us," s mean "both parents were citizens when you were born" and t mean "you've lived in the country at least 14 years".

$$p \Rightarrow (q \wedge (r \vee s) \wedge t)$$

(c) Let p mean "you can see the movie," q mean "you are over 18 years old" and r mean "you have the permission of a parent"

$$p \Rightarrow (q \vee r)$$

Problem 6
Knights always tell the truth, Knaves always lie. You run into two people, A and B. A says, "We are both Knights" and B says, "A is a Knave". Can you determine their identities? What if A says "I am a Knave or B is Knight" and B says nothing? Can you determine their identities then?

Let p be the proposition that A is a knight, and let q be the proposition that B is a knight. Then A's statement can be translated to $p \wedge q$ and B's statement can be translated to $\neg p$. Since we can believe them if and only if they are a knight, we can construct a truth table for $p \Leftrightarrow (p \wedge q)$ and $q \Leftrightarrow \neg p$ and see if there is a case when both these compound propositions are true.

p	q	$\neg p$	$p \wedge q$	$p \Leftrightarrow (p \wedge q)$	$q \Leftrightarrow \neg p$
T	T	F	T	T	F
T	F	F	F	F	T
F	**T**	T	F	**T**	**T**
F	F	T	F	T	F

We can see that the only scenario that satisfies both our compound propositions is when p is false and q is true. Thus, A is a knave and B is a knight.

Using a similar strategy, and the same definitions of p and q, we will evaluate the second scenario's compound proposition $p \Leftrightarrow (\neg p \vee q)$

p	q	$\neg p$	$\neg p \vee q$	$p \Leftrightarrow (\neg p \vee q)$
T	**T**	F	T	**T**
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

Here we see the only viable scenario is when p and q are true. Thus, they are both knights.

Problem 7
Use DeMorgan's laws to find the negation of the following statements.

(a) Francois knows Java and Calculus

(b) Rita will move to Oregon or Washington

(a) Francois doesn't know Java or Francois doesn't know Calculus.

(b) Rita will not move to Oregon and Rita will not move to Washington.

Problem 8
Is $[(p \wedge (p \Rightarrow q)] \Rightarrow q$ a tautology?

This is indeed a tautology, as shown by the truth table below.

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$[p \wedge (p \Rightarrow q)] \Rightarrow q$
T	T	T	T	**T**
T	F	F	F	**T**
F	T	T	F	**T**
F	F	T	F	**T**

Problem 9
Let $P(x)$ be the statement "$x = x^3$. If the domain is the integers, what are the truth values of the following?

(a) $P(0)$

(b) $\exists x P(x)$

(c) $\forall x P(x)$

(d) $P(2)$

(a) $0^3 = 0$, **T**

(b) Let $x = 0$, **T**

(c) Let $x = 2$, then $2^3 = 8 \neq 2$, **F**

(d) By (c), **F**

Problem 10
Determine the truth value of each of these statements if the domain consists of all real numbers

(a) $\exists x(x^3 = -1)$

(b) $\exists x(x^4 < x^2)$

(c) $\forall x((-x)^2 = x^2)$

(d) $\forall x(2x > x)$

(a) Let $x = -1$, then $x^3 = -1$, **T**

(b) Let $x = 1/2$, then $x^4 = 1/16 < 1/4$, **T**

(c) **T** since $(-x)^2 = x^2$ and $x^2 = x^2$ is equivalent to **T**.

(d) Let $x = 0$, then $2x = x$, **F**

Problem 11
Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) Something is not in the correct place.

(b) All tools are in the correct place and are in excellent condition

(c) Everything is in the correct place and in excellent condition

(d) Nothing is in the correct place and is in excellent condition

(e) One of your tools is not in the correct place but it is in excellent condition

Let $P(x)$ mean "x is in the right place," $Q(x)$ mean "x is in excellent condition" and $R(x)$ mean "x is a tool" and the domain be all things.

(a) $\exists x \neg P(x)$

(b) $\forall x(R(x) \Rightarrow (P(x) \wedge Q(x))$

(c) $\forall x(P(x) \wedge Q(x))$

(d) $\neg\exists x(P(x) \wedge Q(x))$

(e) $\exists x(\neg P(x) \wedge Q(x))$

Problem 12
Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

(a) $\forall x(x^2 \neq x)$

(b) $\forall x(x^2 \neq 2)$

(c) $\forall x(|x| > 0)$

(a) $x = 1$

(b) $x = \sqrt{2}$

(c) $x = 0$

Problem 13
Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

(a) $\neg T(\text{Jon}, \text{Japanese})$

(b) $\exists x\forall y T(x, y)$

(c) $\forall x\forall z\exists y((x \neq z) \Rightarrow \neg(T(x, y) \wedge T(z, y)))$

(d) $\forall x\forall z\exists y(T(x, y) \Leftrightarrow T(z, y))$

(e) $\exists x\exists z\forall y(T(x, y) \Leftrightarrow T(z, y))$

(a) Jon does not like Japanese food.

(b) There is a person who likes all kinds of cuisine.

(c) For any two (separate) people there is some kind of cuisine such that it's not the case that they both like it.

(d) For any two people there is a cuisine such that the first person likes it if and only if the second person does too.

(e) There exist two people such that the first person likes all cuisine if and only if the second person does too.

Problem 14
What rules of inference are used in the following arguments

(a) "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

(b) "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

(c) "Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials."

(d) "Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum."

(a) Modus Ponens

(b) Modus Tollens

(c) Simplification

(d) Addition

Problem 15
Use a direct proof to show that the sum of two odd integers is even.

Proof. Let $a, b \in \mathbb{Z}$ be arbitrary odd numbers. Then by definition, $\exists s, t \in \mathbb{Z}$ such that $a = 2s + 1$ and $b = 2t + 1$. So $a + b = 2s + 1 + 2t + 1 = 2s + 2t + 2 = 2(s + t + 1)$. So their sum must be even. $\square$

Problem 16
Show that the square of an even number is an even number.

Proof. Let $a \in \mathbb{Z}$ be an arbitrary even number. Then by definition $\exists k \in \mathbb{Z}$ such that $a = 2k$. So $a^2 = (2k)^2 = 2(2k^2)$. Therefore a^2 must be even. $\square$

Problem 17
Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

Proof. Assume by way of contradiction that n is a perfect square and $n+2$ is a perfect square. Then $\exists j, k \in \mathbb{Z}$ such that $n = j^2$ and $n + 2 = k^2 \Rightarrow n = k^2 - 2 \Rightarrow j^2 = k^2 - 2 \Rightarrow 2 = k^2 - j^2 = (k - j)(k + j)$. Since 2 is prime, it only has factors of 2 and 1. So we have two cases:

$$\begin{aligned} k - j = 2 \quad &\Leftrightarrow \quad k + j = 1 \quad \text{or} \\ k - j = 1 \quad &\Leftrightarrow \quad k + j = 2 \end{aligned}$$

If $k - j = 2$ and $k + j = 1$ then $k = 2 + j$ and $k = 1 - j \Rightarrow 2 + j = 1 - j \Rightarrow 2j = -1 \Rightarrow j = -1/2$, but then $n = 1/4$.

If instead $k - j = 1$ and $k + j = 2$ then $k = 1 + j$ and $k = 2 - j \Rightarrow 1 + j = 2 - j \Rightarrow 2j = 1 \Rightarrow j = 1/2$. But then we have $n = 1/4$.

Thus if n is a perfect square, then $n + 2$ is not a perfect square. $\square$

Problem 18
Use a proof by contraposition to show that if n is an integer and $n^3 + 5$ is odd, then n is even.

Proof. Suppose that n is odd. Then $\exists k \in \mathbb{Z}$ such that $n = 2k + 1$ and we have

$$n^3 + 5 = (2k + 1)^3 + 5 = (8k^3 + 12k^2 + 6k + 1) + 5 = 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$$

must be even. $\square$

Problem 19
Prove that there is no positive integer n such that $n^2 + n^3 = 100$

Proof. We know that for any positive integers $m, n > 1$, it is true that $m > n \Rightarrow m^2 + m^3 > n^2 + n^3$. Considering that fact, we list the expression $n^2 + n^3$ for the first few positive integers.

$$
\begin{aligned}
1^2 + 1^3 &= 2 < 100 \\
2^2 + 2^3 &= 12 < 100 \\
3^2 + 3^3 &= 36 < 100 \\
4^2 + 4^3 &= 80 < 100 \\
5^2 + 5^3 &= 150 > 100
\end{aligned}
$$

So for every positive integer $n \geq 5$, we have $n^2 + n^3 > 100$, and for any positive integer $n < 5$, $n^2 + n^3 < 100$. $\square$

Problem 20
Prove that $\sqrt[3]{2}$ is irrational using a proof by contradiction.

Proof. Assume by way of contradiction that $\sqrt[3]{2}$ is rational. Then there must exist integers p, q where p and q have no common factors and $q \neq 0$ such that $\sqrt[3]{2} = p/q$. Cube both sides and we have $2 = p^3/q^3$ $\Rightarrow 2q^3 = p^3 \Rightarrow 2 \mid p^3 \Rightarrow 2 \mid p$. So there must exist some integer k such that $p = 2k$. Substituting, we get $2q^3 = (2k)^3$ or $q^3 = 4k^3 = 2(2k^3)$, which means that $2 \mid q^3$ and thus $2 \mid q$. But we assumed that p and q have no common factors. $\square$

Problem 21
Use set builder notation to give a description of each of these sets

(a) $\{0, 3, 6, 9, 12\}$

(b) $\{-3, -2, -1, 0, 1, 2, 3\}$

(c) $\{m, n, o, p\}$

(a) $\{x \mid x \in \mathbb{N}, 3 \mid x, x < 15\}$

(b) $\{x \mid x \in \mathbb{Z}, -3 \leq x \leq 3\}$

(c) $\{\phi \mid \phi \text{ is an English letter}, m \leq \phi \leq p\}$

Problem 22
Determine whether each of these statements is true or false.

(a) $0 \in \varnothing$

(b) $\{0\} \subset \varnothing$

(c) $\{0\} \in \{0\}$

(d) $\{\varnothing\} \subseteq \{\varnothing\}$

(e) $\varnothing \in \{0\}$

(f) $\varnothing \subset \{0\}$

(a) **F** because nothing is in the $\emptyset$

(b) **F** because nothing is in the $\emptyset$

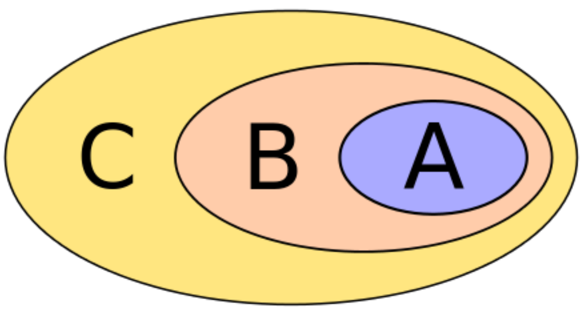
(c) **F** because $\{0\}$ is a set and the set $\{0\}$ is not in $\{0\}$.

(d) **T** because we take the entire thing as a subset

(e) **F** because $\emptyset$ is not an element of $\{0\}$

(f) **T** because the $\emptyset$ is always a subset

Problem 23
Use an Euler diagram to illustrate the relationships $A \subseteq B$ and $B \subseteq C$.



Problem 24
Find two sets A and B such that $A \in B$ and $A \subseteq B$.

Let $A = \{1\}$ and $B = \{1, \{1\}\}$.

Problem 25
What is the cardinality of each of these sets?

(a) $\{a\}$

(b) $\{a, \{a\}\}$

(c) $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$

(a) 1

(b) 2

(c) 3

Problem 26
How many elements does each of these sets have where a and b are distinct elements?

(a) $\mathcal{P}(\{a, b, \{a, b\}\})$

(b) $\mathcal{P}(\mathcal{P}(\{a\}))$

(c) $\mathcal{P}(\mathcal{P}(\{a, b, c\}))$

Let S be a finite set with $|S| = n$ then $|\mathcal{P}(S)| = 2^n$.

(a) $2^3 = 8$

(b) $2^{2^1} = 2^2 = 4$

(c) $2^{2^3} = 2^8 = 256$

Problem 27
Find the sets A and B if $A \setminus B = \{1, 5, 7, 8\}$, $B \setminus A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$

$A = \{1, 3, 6, 5, 7, 8, 9\}$ and $B = \{2, 3, 6, 9, 10\}$ because $\setminus$ is the relative complement.

Problem 28
Prove one of DeMorgan's laws: $\overline{A \cup B} = \overline{A} \cap \overline{B}$. (The bar means complement.)

Proof. Let $x \in \overline{A \cup B}$

$$\begin{aligned} x \in \overline{A \cup B} &\Leftrightarrow \neg(x \in A \cup B) \\ &\Leftrightarrow \neg(x \in A \vee x \in B) \\ &\Leftrightarrow \neg(x \in A) \wedge \neg(x \in B) \\ &\Leftrightarrow (x \in \overline{A}) \wedge (x \in \overline{B}) \\ &\Leftrightarrow x \in \overline{A} \cap \overline{B} \end{aligned}$$

Thus $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ and $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$ $\square$

Problem 29
Determine if $f : \mathbb{Z} \to \mathbb{R}$ is a function if

(a) $f(n) = \pm n$

(b) $f(n) = \sqrt{n^2 + 1}$

(c) $f(n) = 1/(n^2 - 4)$

(a) No; every n (other than 0) is mapped to two elements.

(b) Yes; every integer gets mapped to one real number.

(c) No; for $n = 2$ the function is undefined.

Problem 30
Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$. If not, state whether they are injections or surjections at least.

(a) $f(x) = 2x + 1$

(b) $f(x) = x^2 + 1$

(c) $f(x) = x^3$

(d) $f(x) = (x^2 + 1)/(x^2 + 2)$

(a) Yes; $(y - 1)/2$ is defined for all real numbers y and if $2a + 1 = 2b + 1$ then $a = b$.

(b) No; $f(x) > 0$ for all x (not onto). Also $f(-x) = f(x)$ (not 1-1)

(c) Yes $\sqrt[3]{y}$ is defined for all real numbers y and if $a^3 = b^3$ then $a = b$

(d) No; $f(x) > 0$ for all x (not onto). Also $f(-x) = f(x)$ for all x (not 1-1)

Problem 31
If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? If f and $f \circ g$ are onto, does it follow that g is onto?

Proof. Let $f : B \to C$ and $g : A \to B$ be functions such that f is 1-1 and $f \circ g$ is 1-1. Then we know that $\forall c, d \in B$, $f(c) = f(d) \Leftrightarrow c = d$ and $\forall a, b \in A$, $(f \circ g)(a) = (f \circ g)(b) \Leftrightarrow a = b$. Since f is 1-1 and $(f \circ g)(a) = f(g(a))$, we can say that $f(g(a)) = f(g(b)) \Leftrightarrow g(a) = g(b)$. Therefore $g(a) = g(b) \Leftrightarrow a = b$ $\square$

Problem 32
For each of the following sequences $\{a_n\}$, find a recurrence relation satisfied by the sequence.

(a) $a_n = 2n + 3$

(b) $a_n = n^2$

(c) $a_n = n + (-1)^n$

(d) $a_n = 5^n$

(e) $a_n = n!$

(a) $a_0 = 3$, $a_n = a_{n-1} + 2$

(b) $a_0 = 0$, $a_n = a_{n-1} + 2(n - 1) + 1$

(c) $a_0 = 1$, $a_1 = 0$, $a_n = a_{n-2} + 2$

(d) $a_0 = 1$, $a_n = 5a_{n-1}$

(e) $a_0 = 1$, $a_n = n \cdot a_{n-1}$

Problem 33
Use induction to prove that $2 \mid (n^2 + n)$ for $n \in \mathbb{Z}^+$.

Proof. **Base case:** Let $n = 1$. Then $n^2 + n = 1^2 + 1 = 2$ and $2 \mid 2$ ✓

Inductive step: Suppose that for some $k \in \mathbb{Z}^+$ $2 \mid (k^2 + k)$.

Consider now the expression $(k + 1)^2 + (k + 1) = k^2 + 2k + 1 + k + 1 = (k^2 + k) + 2k + 2$. We know that $2 \mid (k^2+k)$ by assumption, so $\exists a \in \mathbb{Z}$ such that $k^2+k = 2a$. Substituting, we get $(k+1)^2+(k+1) = 2a+2k+2 = 2(a+k+1)$ must be even. $\square$

Problem 34
Use induction to show that

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$$

.

Proof. **Base case:** Let $n = 0$. $0(1) = 0(1)(2)/3 = 0$ ✓

Inductive step: Suppose that for some $k \in \mathbb{N}$ $1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) = k(k+1)(k+2)/3$

Add the $(k+1)^{th}$ term of the sum to both sides of this equality.

$$\begin{aligned} 1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) + (k+1)(k+2) &= k(k+1)(k+2)/3 + (k+1)(k+2) \\ &= k(k+1)(k+2)/3 + 3(k+1)(k+2)/3 \\ &= (k+1)(k+2)(k+3)/3 \end{aligned}$$

□

Problem 35
Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

(a) What is the statement $P(2)$?

(b) What is the inductive hypothesis?

(c) What do you need to prove in the inductive step?

(a) $P(2)$ is the statement $2! < 2^2$

(b) The inductive hypothesis is: Assume that for some $k \in \mathbb{Z}_{>1}$ $k! < k^k$

(c) We need to show that our assumption implies $(k+1)! < (k+1)^{k+1}$

Problem 36
Use strong induction to show that if you can run one mile or two miles, and if you can always run two more miles once you have run a specified number of miles, then you can run any number of miles.

Proof. The base cases are given (I can run one mile or two miles). Assume by way of (strong) induction, that for some $k > 0$, I can run any amount of miles between 0 and k. Then, by assumption I can definitely run $k-1$ miles. And it is given that I can always run two more miles once I have run a specified number of miles, so I can also run $(k-1)+2$ miles, or $k+1$ miles. □

Problem 37
Let's say I want to use strong induction to prove that I can produce any whole dollar figure greater than 3 using just two dollar bills and five dollar bills. What is my inductive hypothesis and what might I use for my base case(s)?

The inductive hypothesis is to assume for some $k > 3$, we can make k dollars using just two dollar bills and five dollar bills. The base cases would be to prove that we can make four dollars and five dollars.

Problem 38
Prove that any set with an uncountable subset is uncountable.

Proof. Let's prove the contrapositive, i.e. any set that is countable has a countable subset. We get this from a stronger statement that every subset of a countable set is countable:

Let A be a particular but arbitrarily chosen countable set and let B be any subset of A. Either B is finite or it is infinite. If it is finite, then B is countable by definition. Suppose B is infinite. Since A is countable its elements can be represented as a sequence $a_1, a_2, a_3, \ldots$. We now define a function $g : \mathbb{Z}^+ \longrightarrow B$ as follows:

1. Search sequentially through $a_1, a_2, a_3, \ldots$ until an element of B is found. Call that element $g(1)$.
2. For each integer $k \geq 2$ suppose $g(k-1)$ has been defined, i.e. $g(k-1) = a_i$ for some a_i in the sequence. Starting with a_{i+1} search sequentially through $a_{i+1}, a_{i+2}, a_{i+3}, \ldots$ trying to find an element of B. We must find one eventually because B is infinite and we have only defined finitely many elements in g so far. Call the one that that is found $g(k)$.

Thus, g is defined for each positive integer. Since all $a_1, a_2, a_3, \ldots$ is distinct then g is injective. Further, the searches for elements of B are sequential. Thus, every element of A is reached during the search and all elements of B is somewhere in the sequence. So, every element of B is eventually found and made the image of some integer. So, g is surjective. Thus, we have a bijection between $\mathbb{Z}^+$ and B proving it is countably infinite. □

Problem 39
Prove that if A and B are nonempty finite sets then $|A \times B| = |A| \times |B|$.

Consider $a \in A$. It can be paired off with any of the elements of B, i.e. there are $|B|$ pairings for it. Further, there are $|A|$ possible a to consider and thus $|A \times B| = |A| \times |B|$.

Problem 40
Give partitions of $\mathbb{R}$ having one block, two blocks, three blocks, and infinitely many blocks.

- **One block**: $\mathbb{R}$
- **Two blocks**: $\{x \in \mathbb{R} | x \leq 0\}$ and $\{x \in \mathbb{R} | x > 0\}$
- **Three blocks**:$\{x \in \mathbb{R} | x \notin \mathbb{Q}\}$, $\{x \in \mathbb{R} | x \in \mathbb{Q}$ and $x \leq 0\}$ and $\{x \in \mathbb{R} | x \in \mathbb{Q}$ and $x > 0\}$
- **Infinitely many blocks**: $\{\{x\} | x \in \mathbb{R}\}$.

Problem 41
State Russell's paradox and its significance to computer science. How do we resolve the paradox?

Russell's paradox states that the naive interpretation of set theory with sets just being collections of objects leads to problems with definitions when we allow a set to contain itself. It leads to a contradiction where $P \wedge \neg P$ has to be true since a set can be both "normal" and "abnormal." One way to resolve this paradox is to use a carefully axiomatized system like ZFC. Other research is exploring hypersets and other cool resolutions.