Propositional Logic and Inference

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https://xkcd.com/1652/

https://xkcd.com/1856/
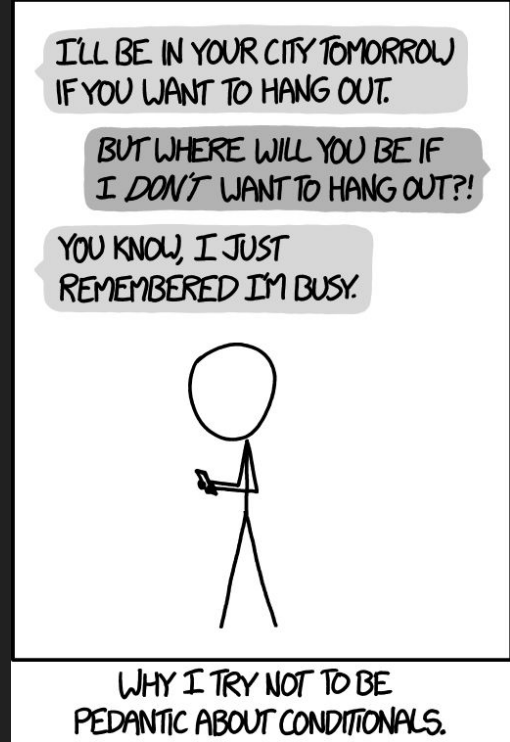
Logical Equivalence

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Epp Book

Are they equivalent?

$$\sim(p \wedge q) \qquad \sim p \wedge \sim q$$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

Valid?

$$p \rightarrow q \vee \sim r$$
$$q \rightarrow p \wedge r$$
$$\therefore p \rightarrow r$$

			premises		conclusion

| p	q	r	$\sim r$	$q \lor \sim r$	$p \land r$	$p \to q \lor \sim r$	$q \to p \land r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

Inference

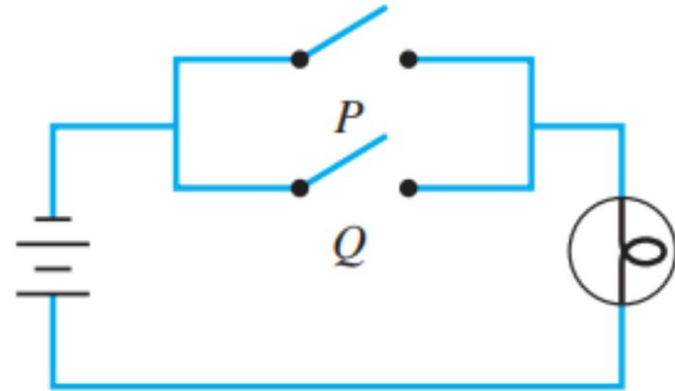| Modus Ponens	$p \rightarrow q$ p $\therefore q$	Elimination	**a.** $p \vee q$ $\sim q$ $\therefore p$	**b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Transitivity	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	
Generalization	**a.** p **b.** q $\therefore p \vee q$ $\therefore p \vee q$	Proof by Division into Cases	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$	
Specialization	**a.** $p \wedge q$ **b.** $p \wedge q$ $\therefore p$ $\therefore q$			
Conjunction	p q $\therefore p \wedge q$	Contradiction Rule	$\sim p \rightarrow c$ $\therefore p$	

Digital Circuits
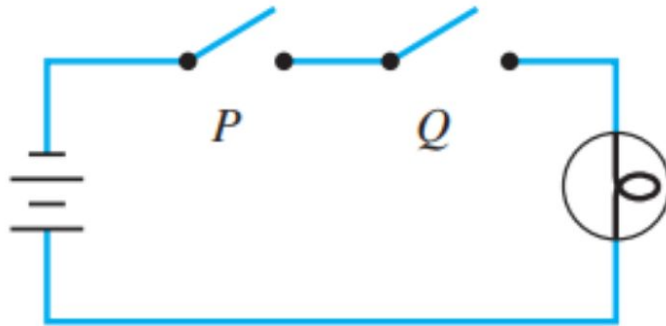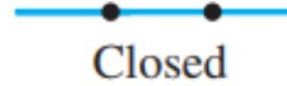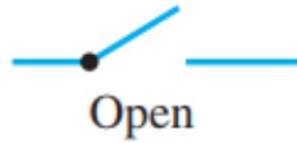
Open   Closed

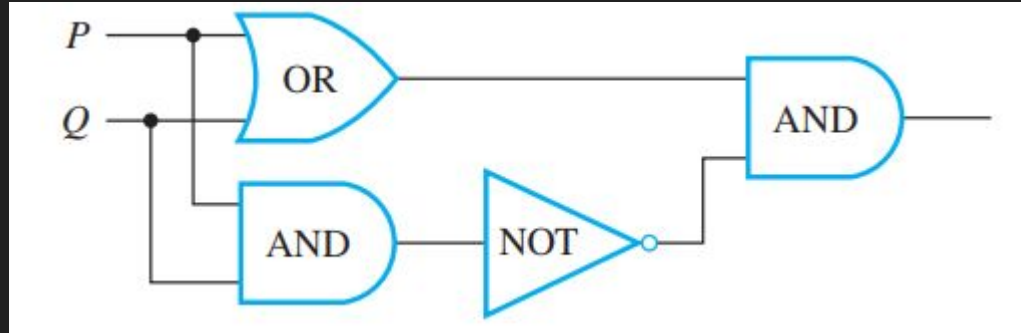Switches "in series"   Switches "in parallel"

Circuits and Logic?

(a) Switches in Series		
Switches		**Light Bulb**
P	*Q*	State
closed	closed	on
closed	open	off
open	closed	off
open	open	off

(b) Switches in Parallel		
Switches		**Light Bulb**
P	*Q*	State
closed	closed	on
closed	open	on
open	closed	on
open	open	off

What is the formula?

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What did you like? What did you not like?