LaTeXsubmissions are mandatory. Submitting your assignment in another format will result in a **loss of 10 points** on the assignment. The template is here.

Problem 1 *worth 15 points*
Two different factories both produce a certain automobile part. The probability that a component from the first factory is defective is 2%, and the probability that a component from the second factory is defective is 5%. In a supply of 180 of the parts, 100 were obtained from the first factory and 80 from the second factory.

(a) What is the probability that a part chosen at random is from the first factory?

(b) What is the probability that a part chosen at random is defective?

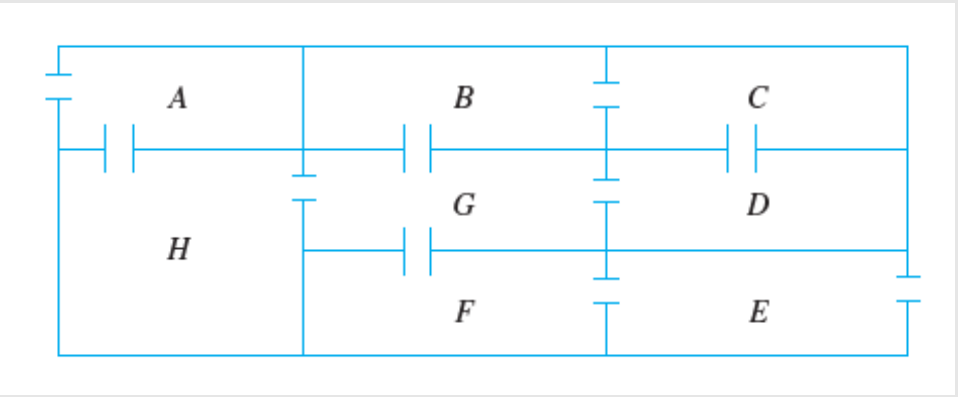
(c) If the chosen part is defective, what is the probability that it came from the first factory?

Problem 2 *worth 10 points*
A software company uses two quality assurance (QA) checkers X and Y to check an application for bugs. X misses 12% of the bugs and Y misses 15%. Assume that the QA checkers work independently.

(a) What is the probability that a randomly chosen bug will be missed by both QA checkers?

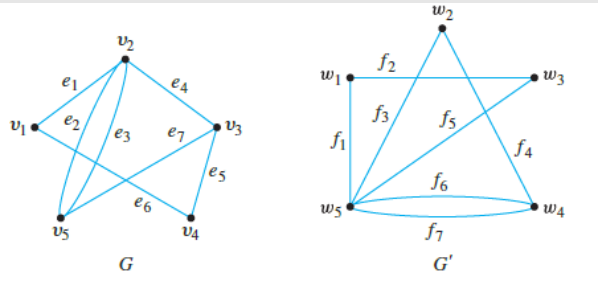
(b) If the program contains 1,000 bugs, what number can be expected to be missed?

Problem 3 *worth 6 points*
The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done? What kind of path is this? Be sure to use the language of graph theory in your explanation.



Problem 4 *worth 7 points*
Show that at a party with at least two people, there are at least two mutual acquaintances or at least two mutual strangers.

Problem 5 *worth 7 points*
Determine if G and G' are isomorphic. If they are, give functions $g : V(G) \to V(G')$ and $h : E(G) \to E(G')$ that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.



Problem 6 *worth 10 points*
Draw all nonisomorphic graphs with four vertices and no more than two edges.

Problem 7 *worth 14 points*
For each, either draw a graph with the given specifications or explain why no such graph exists

(a) Graph, circuit-free, seven vertices, four edges

(b) Tree, twelve vertices, fifteen edges

(c) Graph, six vertices, five edges, not a tree

(d) Tree, five vertices, total degree 10

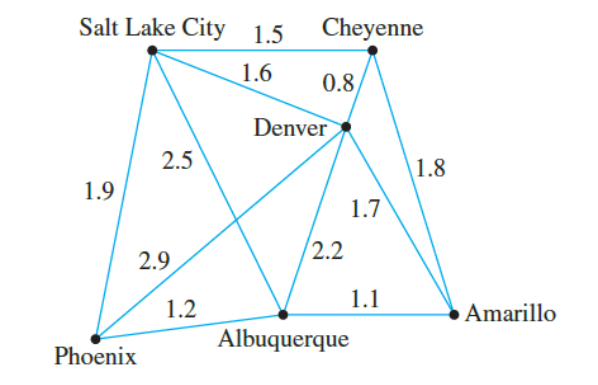
(e) Graph, connected, ten vertices, nine edges, has a circuit

(f) Simple graph, connected, six vertices, six edges

(g) Tree, ten vertices, total degree 24

Problem 8 *worth 10 points*
Prove that if G is a graph with spanning tree T and e is an edge of G that is not in T, then the graph obtained by adding e to T contains one and only one set of edges that forms a circuit.

Problem 9 *worth 7 points*
A pipeline is to be built that will link six cities. The cost (in hundreds of millions of dollars) of constructing each potential link depends on distance and terrain and is shown in the weighted graph below. Find a system of pipelines to connect all the cities and yet minimize the total cost.



Problem 10 *worth 10 points*
Suppose a graph G and a graph G' are combined to create a graph H by connecting each vertex of G to each vertex of G' and otherwise all vertices and edges remaining unchanged. Prove that $\chi(G) + \chi(G') = \chi(H)$ where $\chi(\cdot)$ is the chromatic number, i.e. the minimal number of colors needed to color each vertex so that no two adjacent vertices have the same color.

Problem R *worth 4 points*
Exercise a growth mentality by reflecting on this assignment and your work. Feel free to say whatever you want, but you are required to answer the following. You are graded on whether you complete this, not on what you say.

- How many hours did you spend on this assignment?
- What problem was hardest? Why?
- What problem was easiest? Why?
- What are you most concerned about for the final exam?

Proof. **1 point for each question they answered** □