Midterm Study Guide

Here is a list of topics that are fair game for the midterm. Any equivalences you need will be provided for you (no need to memorize tables).

- **Propositional logic/ predicate calculus**
  - connectives: $\wedge, \vee, \neg, \oplus, \Rightarrow, \Leftrightarrow$
  - truth tables
  - necessary/sufficient conditions
  - translating between English and logic and vice versa
  - converse, inverse, contrapositive
  - tautologies/ contradictions
  - logical equivalences (tables below)
  - propositional functions/ quantifiers: $\forall, \exists$
  - rules of inference
    - modus monens
    - modus tollens
    - disjunctive syllogism
    - hypothetical syllogism
    - simplification
    - addition
    - conjunction
    - resolution
  - universal/existential instantiation/generalization
- **Proofs**
  - proof methods for conditionals ($p \Rightarrow q$)
    - direct proof
    - proof by contraposition
    - proof by contradiction
  - definition of $a \mid b$
  - definition of odd/even integers
  - proof by cases
  - existence (proof by construction) and uniqueness
  - strong/weak induction
- **Sets, functions, sequences**
  - common sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
  - set operations $\cup, \cap, \overline{}$
  - roster method/ builder method
  - proving that two sets are equal
  - draw sets (or combinations of sets) in a Venn diagram
  - special sets
    - empty set $\varnothing$

    * power set
  - cardinality
  - cartesian products
  - set identities (table below)
  - domain/ codomain/ range
  - one-to-one/ onto/ bijective
  - special functions
    * composition
    * floor/ ceiling
  - countably infinite/ uncountably infinite
  - arithmetic/ geometric progressions
  - recurrence relations and their solutions

You will have to know formal definitions for the following concepts:

- proposition
- satisfiable
- argument, valid and sound
- function
- injective
- surjective
- sequence

TABLE 1 **Set Identities.**

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution