

L^AT_EX submissions are mandatory. Submitting your assignment in another format will result in a **loss of 10 points** on the assignment. The template is here.

Problem 1 *worth ? points*

Answer the following with justification:

- (a) If p is a prime number and a is a positive integer, how many distinct positive divisors does p^a have?
- (b) If p and q are distinct prime numbers and a and b are positive integers, how many distinct positive divisors does $p^a q^b$ have? Justify your answer.
- (c) If p_1, p_2, \dots, p_m are distinct prime numbers and a_1, a_2, \dots, a_m are positive integers, how many distinct positive divisors does $p_1^{a_1} p_2^{a_2} \dots p_m^{a_m}$ have? Justify your answer.
- (d) What is the smallest positive integer with exactly 42 distinct positive divisors? Justify your answer.

Problem 2 *worth ? points*

Suppose A is a set with m elements and B is a set with n elements.

- (a) How many relations are there from A to B ? Justify.
- (b) How many functions are there from A to B ? Justify?
- (c) Finally, what fraction of the relations from A to B are functions? What is the exact answer when $|A| = 60$ and $|B| = 60$, since 60 is approximately how many students are in CSCI 2824 this summer. Are you likely to encounter a function if you pick a relation at random when m and n are bigger than a few elements?

Problem 3 *worth ? points*

Six people attend the theater together and sit in a row with exactly 6 seats.

- (a) How many ways can they be seated together in the row?
- (b) Suppose one of the six is a doctor who must sit on the aisle in case she is called to an emergency. How many ways can the people be seated together in the row with the doctor in an aisle seat?
- (c) Suppose the six people consist of three married couples and each couple wants to sit together. How many ways can the six be seated in the row?

Problem 4 *worth ? points*

Suppose you are playing a game that requires rolls of two 4-sided dice, which have sides numbered 1, 2, 3, and 4.

- (a) If both dice are fair, then what is the probability distribution over the set of possible outcomes for the sum of the two dice when rolled?
- (b) Now suppose that one die is weighted such that a 3 is twice as likely to be rolled as any single number of that die, and 1, 2, and 4 are all equally likely to be rolled? What is the probability distribution for outcomes of a single roll of just that one loaded die?
- (c) What is the probability that the total of the fair and unfair dice is 6, when both are rolled?
- (d) Suppose you roll both dice but one falls on the floor and rolls under the couch so you can't see it, but the other die has rolled a 3. The unfair die is made of a strange material that makes it twice

as likely as the fair die to fall off the table. Without seeing the die that is under the couch, what is the probability that the dice total is 6?

- (e) Suppose the fair and unfair 4-sided dice are in a bag with two regular and fair 6-sided dice. If you pick a die at random from the bag and roll it, what is the probability of rolling a 3?

Problem 5 *worth ? points*

When each of 702, 787, and 855 is divided by the positive integer m , the remainder is always the positive integer r . When each of 412, 722, and 815 is divided by the positive integer n , the remainder is always the positive integer $s \neq r$. Find $m + n + r + s$.

Problem 6 *worth ? points*

Prove that every prime number except 2 and 3 has the form $6q + 1$ or $6q + 5$ for some integer q .

Problem 7 *worth ? points*

Find the largest possible value of k for which 3^{11} is expressible as the sum of k consecutive positive integers. Prove that your answer is correct.

Problem 8 *worth ? points*

Read [this New York Times post](#). Summarize their goal and approach in a few sentences. Be sure to comment on where probability comes in.

Problem Reflection *worth 3 points*

Exercise a growth mentality by reflecting on this assignment and your work. Feel free to say whatever you want, but you are required to answer the following. You are graded on whether you complete this, not on what you say.

- How many hours did you spend on this assignment?
- What problem was hardest? Why?
- What problem was easiest? Why?