Your exam will be shorter than this. These are practice problems of the flavor you can expect. You will be provided with an necessary logical equivalences so no need to memorize anything other than some formal definitions such as cardinality, injective/surjective/bijection, function, proposition, etc., for the exam. Please also see the study guide for topics.

Problem 1
Which of the following are propositions?

(a) Boston is the capital of Massachusetts

(b) Miami is the capital of Florida

(c) 2+3=5

(d) 5+7=10

(e) Go to school

Problem 2
For a compound proposition with constituents p, q, r, s, t, u, how many rows would appear in the truth table?

Problem 3
Construct a truth table for each of these compound propositions.

(a) $p \wedge \neg p$

(b) $(p \vee q) \Rightarrow (p \wedge q)$

(c) $(p \Rightarrow q) \oplus (p \Rightarrow \neg q)$

(d) $p \oplus (p \vee q)$

Problem 4
Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q and r have the same truth value and is false otherwise.

Problem 5
Translate the following into propositional logic.

(a) To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service.

(b) You are eligible to be the President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country.

(c) You can see the movie only if you are over 18 years old or you have the permission of a parent.

Problem 6
Knights always tell the truth, Knaves always lie. You run into two people, A and B. A says, "We are both Knights" and B says, "A is a Knave". Can you determine their identities? What if A says "I am a Knave or B is Knight" and B says nothing? Can you determine their identities then?

Problem 7
Use DeMorgan's laws to find the negation of the following statements.

(a) Francois knows Java and Calculus

(b) Rita will move to Oregon or Washington

Problem 8
Is $[(p \land (p \Rightarrow q)] \Rightarrow q$ a tautology?

Problem 9
Let $P(x)$ be the statement "$x = x^3$. If the domain is the integers, what are the truth values of the following?

(a) $P(0)$

(b) $\exists x P(x)$

(c) $\forall x P(x)$

(d) $P(2)$

Problem 10
Determine the truth value of each of these statements if the domain consists of all real numbers

(a) $\exists x(x^3 = -1)$

(b) $\exists x(x^4 < x^2)$

(c) $\forall x((-x)^2 = x^2)$

(d) $\forall x(2x > x)$

Problem 11
Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) Something is not in the correct place.

(b) All tools are in the correct place and are in excellent condition

(c) Everything is in the correct place and in excellent condition

(d) Nothing is in the correct place and is in excellent condition

(e) One of your tools is not in the correct place but it is in excellent condition

Problem 12
Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

(a) $\forall x(x^2 \neq x)$

(b) $\forall x(x^2 \neq 2)$

(c) $\forall x(|x| > 0)$

Problem 13
Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

(a) $\neg T(\text{Jon}, \text{Japanese})$

(b) $\exists x \forall y T(x, y)$

(c) $\forall x \forall z \exists y((x \neq z) \Rightarrow \neg(T(x, y) \wedge T(z, y)))$

(d) $\forall x \forall z \exists y(T(x, y) \Leftrightarrow T(z, y))$

(e) $\exists x \exists z \forall y(T(x, y) \Leftrightarrow T(z, y))$

Problem 14
What rules of inference are used in the following arguments

(a) "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

(b) "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

(c) "Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials."

(d) "Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum."

Problem 15
Use a direct proof to show that the sum of two odd integers is even.

Problem 16
Show that the square of an even number is an even number.

Problem 17
Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

Problem 18
Use a proof by contraposition to show that if n is an integer and $n^3 + 5$ is odd, then n is even.

Problem 19
Prove that there is no positive integer n such that $n^2 + n^3 = 100$

Problem 20
Prove that $\sqrt[3]{2}$ is irrational using a proof by contradiction.

Problem 21
Use set builder notation to give a description of each of these sets

(a) $\{0, 3, 6, 9, 12\}$

(b) $\{-3, -2, -1, 0, 1, 2, 3\}$

(c) $\{m, n, o, p\}$

Problem 22
Determine whether each of these statements is true or false.

(a) $0 \in \varnothing$

(b) $\{0\} \subset \varnothing$

(c) $\{0\} \in \{0\}$

(d) $\{\varnothing\} \subseteq \{\varnothing\}$

(e) $\varnothing \in \{0\}$

(f) $\varnothing \subset \{0\}$

Problem 23
Use an Euler diagram to illustrate the relationships $A \subseteq B$ and $B \subseteq C$.

Problem 24
Find two sets A and B such that $A \in B$ and $A \subseteq B$.

Problem 25
What is the cardinality of each of these sets?

(a) $\{a\}$

(b) $\{a, \{a\}\}$

(c) $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$

Problem 26
How many elements does each of these sets have where a and b are distinct elements?

(a) $\mathcal{P}(\{a, b, \{a, b\}\})$

(b) $\mathcal{P}(\mathcal{P}(\{a\}))$

(c) $\mathcal{P}(\mathcal{P}(\{a, b, c\}))$

Problem 27
Find the sets A and B if $A \setminus B = \{1, 5, 7, 8\}$, $B \setminus A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$

Problem 28
Prove one of DeMorgan's laws: $\overline{A \cup B} = \overline{A} \cap \overline{B}$. (The bar means complement.)

Problem 29
Determine if $f : \mathbb{Z} \to \mathbb{R}$ is a function if

(a) $f(n) = \pm n$

(b) $f(n) = \sqrt{n^2 + 1}$

(c) $f(n) = 1/(n^2 - 4)$

Problem 30
Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$. If not, state whether they are injections or surjections at least.

(a) $f(x) = 2x + 1$

(b) $f(x) = x^2 + 1$

(c) $f(x) = x^3$

(d) $f(x) = (x^2 + 1)/(x^2 + 2)$

Problem 31
If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? If f and $f \circ g$ are onto, does it follow that g is onto?

Problem 32
For each of the following sequences $\{a_n\}$, find a recurrence relation satisfied by the sequence.

(a) $a_n = 2n + 3$

(b) $a_n = n^2$

(c) $a_n = n + (-1)^n$

(d) $a_n = 5^n$

(e) $a_n = n!$

Problem 33
Use induction to prove that $2 \mid (n^2 + n)$ for $n \in \mathbb{Z}^+$.

Problem 34
Use induction to show that

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$$

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Problem 35
Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

(a) What is the statement $P(2)$?

(b) What is the inductive hypothesis?

(c) What do you need to prove in the inductive step?

Problem 36
Use strong induction to show that if you can run one mile or two miles, and if you can always run two more miles once you have run a specified number of miles, then you can run any number of miles.

Problem 37
Let's say I want to use strong induction to prove that I can produce any whole dollar figure greater than 3 using just two dollar bills and five dollar bills. What is my inductive hypothesis and what might I use for my base case(s)?

Problem 38
Prove that any set with an uncountable subset is uncountable.

Problem 39
Prove that if A and B are nonempty finite sets then $|A \times B| = |A| \times |B|$.

Problem 40
Give partitions of $\mathbb{R}$ having one block, two blocks, three blocks, and infinitely many blocks.

Problem 41
State Russell's paradox and its significance to computer science. How do we resolve the paradox?