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Replicating Fractal Structures with the Reverse Box Counting Method – An Urban South-east Asian Example

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1 Introduction

Planners, designers, academics, governmental agencies, and citizens are interested in replicating complex spatial patterns for both landscape preservation/conservation and blending new built environments with existing environments. Studying fractal patterns is one general approach to modelling these spatial patterns; however calculating the fractal pattern has been relatively simple and widely employed (DAUPHINÉ 2011, THOMAS et al. 2010a, THOMAS et al. 2010b, MA et al. 2008, DAUPHINÉ 1995, FRANKHAUSER 1994). Fractals can be relatively easily calculated by employing software such as through Fractalyse 2.4 (TheMA 2012). However, replicating the pattern and applying the fractal number in an applied manner has been more elusive.

A French team consisting of geographers, a physicist, and a hydrologist began employing the box counting method to measure and then replicate fractal patterns with the reverse box counting method (the replicating processes for planning and design applications is the relatively new part of the scholarly progression in fractal studies) (DUCHESNE et al. 2002). Recently this box counting approach has been employed to study forest patterns in Michigan for surface mine reclamation (FLEURANT et al. 2009), specific individual tree species patterns in the Upper Peninsula of Michigan (LEHMANN 2009), and to describe and replicate Chinese gardens in Suzhou, China (YUE & BURLEY 2011). These papers review the literature in detail leading up to our study. In addition, software has been compiled by the French team, written in C# to replicate patterns based upon fractal numbers.

2 Study Area and Methodology

In our study we examined the spatial pattern of buildings on Lamma Island (Fig. 1), a part of the Hong Kong island chain, P. R. of China. In Chinese, the island is known as 南丫島, also known as Pok Liu Chau (Chinese 博寮洲) or Pok Liu. We were interested in measuring and establishing a pattern that could be replicated for urban planning and design applications. In China there is interest in making the pattern of new built environments blend with older traditional settings.

To apply the box counting method, one makes a box, and then divides the box into four boxes and continues this reduction method until at least one box is empty. Then the length of the box and the number of filled boxes are recorded. Then the box size is reduced again, the length recorded and the number of filled boxes recorded. The process is stopped when each point is in its own box, the final length is recorded and the final number of boxes with



Fig. 1: A photograph on Lamma Island overlooking the bay and looking to the west

points is counted. The length of the box is transformed in equation 1 and the number of boxes is transformed by equation 2. These pairs of numbers are regressed with $\ln(1/r)$ being the regressor, resulting in the slope of the equation representing the fractal number, equation 3. In our study we mapped the locations of building sites in 2008 (Fig. 2) and visited the island in the summer of 2009 for field corroboration.

$$V1 = \ln(1/r) \quad (1)$$

Where: r = the length of a box

$V1$ = the box length variable for regression

$$V2 = \ln(N) \quad (2)$$

Where: N = the number of boxes with points

$V2$ = the number of boxes variable for regression

$$\ln(N) = \text{slope } \ln(1/r) + \text{intercept} \quad (3)$$

The process is reversed to construct a pattern that is similar to the one measured. Currently there is no mathematical proof demonstrating that the process is reversible. The procedure assumes that the process is reversible. The reverse box-counting method employs the smallest box size used to construct the measured fractal pattern and then a random numbers table is utilized to identify which boxes contain the item of interest and which boxes remain

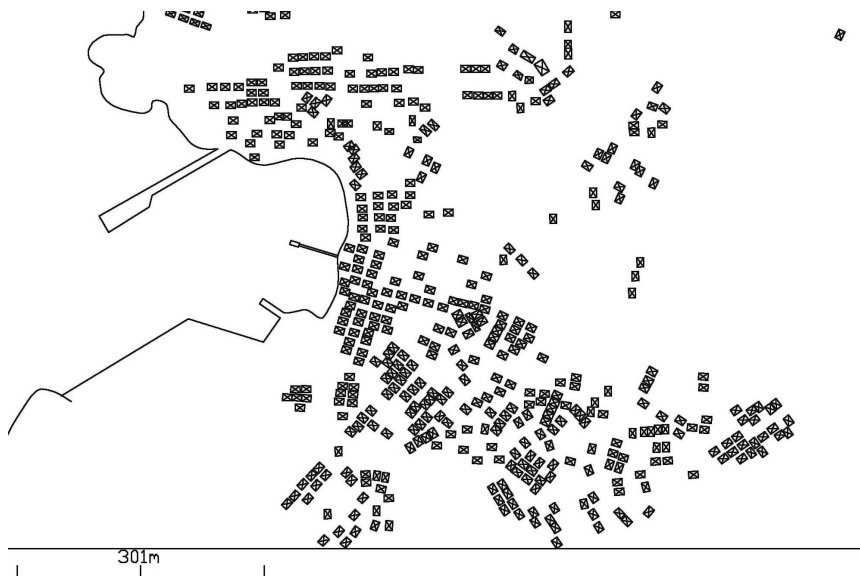


Fig. 2: A map of the building locations on Lamma Island

empty. For example if 95% of the boxes are measured to be empty, then the random numbers tables are set to generate an assignment where approximately 5% of the boxes are filled. FLEURANT et al. (2009) illustrate this approach to produce pattern of trees based upon measurements of tree distributions on xeric environments in the Upper Peninsula of Michigan, USA.

3 Results

Table 1 presents the values employed to predict the spatial pattern of the structures in the study area. The first empty boxes occurred when the box size was 162.5 meters (Fig 3). When the box size was reduced to 20.3125 meters, each structure was in its own box (Fig. 4). In our study the resulting fractal number was 1.115, with 480 boxes filled across 4,096 boxes. Therefore approximately 11.7% of the boxes are filled.

Box Length m	Filled Boxes	V1	V2
162.5	48	-5.090678002	3.871201011
81.25	139	-4.397530821	4.934473933
40.625	323	-3.704383641	5.777652323
20.3125	480	-3.01123646	6.173786104

Table 1: Values measured and transformed to predict the fractal number of structures in Lamma Island. V1 and V2 are regressed to compute the fractal number with V2 as the dependent variable in the regression.

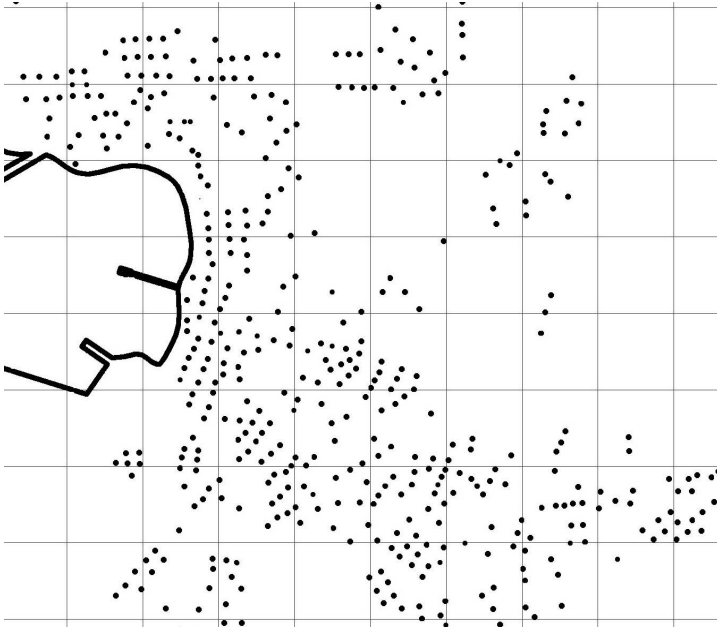


Fig. 3: In this enlargement of a portion of the boxed grid, at least one box is empty and the process of measuring the box length and counting the number of boxes containing at least one point in the box is initiated

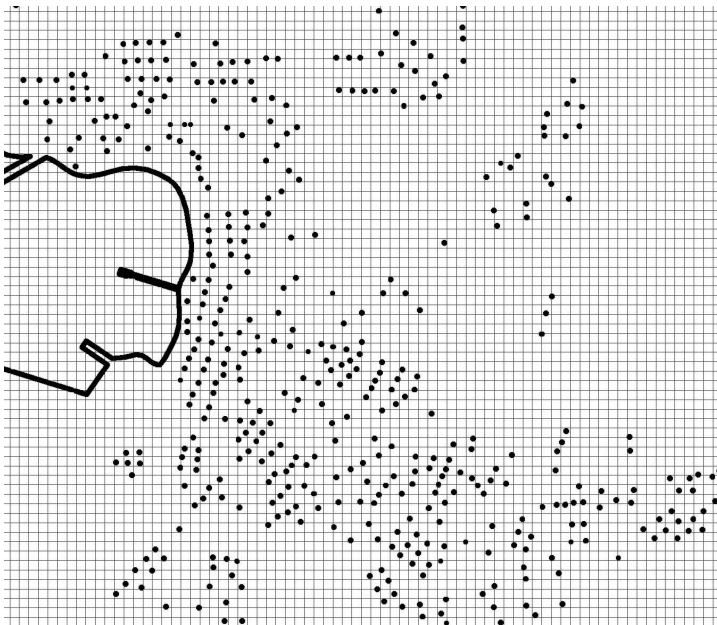


Fig. 4: In this enlargement of a portion of the boxed grid, each point is now in its own box and the process is then stopped

4 Discussion/Conclusion

One can replicate this pattern by randomly filling approximately 480 boxes in a 128 by 128 box grid at a length of 20.3125 meters (see FLEURANT et al. 2009, LEHMANN 2009) for more precise details and examples. The method is simply a tool to assist in emulating a pattern. It does not mean that the pattern must or should be replicated.

23	96	75	5	35	88	87	89	84	63
84	94	67	85	71	66	29	93	70	31
2	77	52	71	92	17	92	93	42	65
89	38	7	39	99	89	97	30	46	23
63	61	3	100	37	63	85	93	89	57
41	1	8	78	38	82	55	73	9	9
6	50	97	15	1	21	50	88	22	28
66	93	16	85	53	25	69	32	32	4
12	73	13	95	30	65	51	55	31	79
98	52	5	75	40	95	98	57	39	46

Table 2: 100 random numbers from a random number table

To replicate the pattern, suppose one had a 100 box grid area with 20.3125 m lengths for each box. Then approximately 12 of the boxes would need to have locations for structures. Table 2 presents 100 random numbers with each number assigned to a box. Since 12% of the boxes need to be filled, the numbers equal to or less than 12 represent locations where the structures are located and numbers greater than 12 are empty. In the first column, the numbers 2, 6 and 12 are equal to or less than 12 and represent the location of structures in rows 3, 7, and 9. Figure 5 presents the location of structures based upon Table 2. This figure is strictly a pattern that represents the fractal approximation of 1.115. Notice that there are 13 locations, not 12. In addition, this pattern is quite a different basis for structure location than typical environmental concerns, such as not being in a floodway (physical) or on a sacred site (cultural). Figure 5 is simply a guide suggesting form. It is not the complete answer or solution to any landscape plan.

The method is supposed to be independent upon the size of box implemented to initiate the measurements. YUE, WEI & BURLEY (unpublished, in review) chose a different size box to initiate the process, including more structures for Lamma Island and calculated a fractal number of 1.158 with the smallest box size of 18.75 m and 556 filled boxes. This replication should give some insight into the variance, reliability, and robust nature of the reverse box-counting process. In other words, the exact box size is not necessarily essential, neither is the exact fractal number, but rather, it is a neighborhood of similar ranges that appear consistent. These results are the first report indication that the reverse box-counting method has some related similarity.

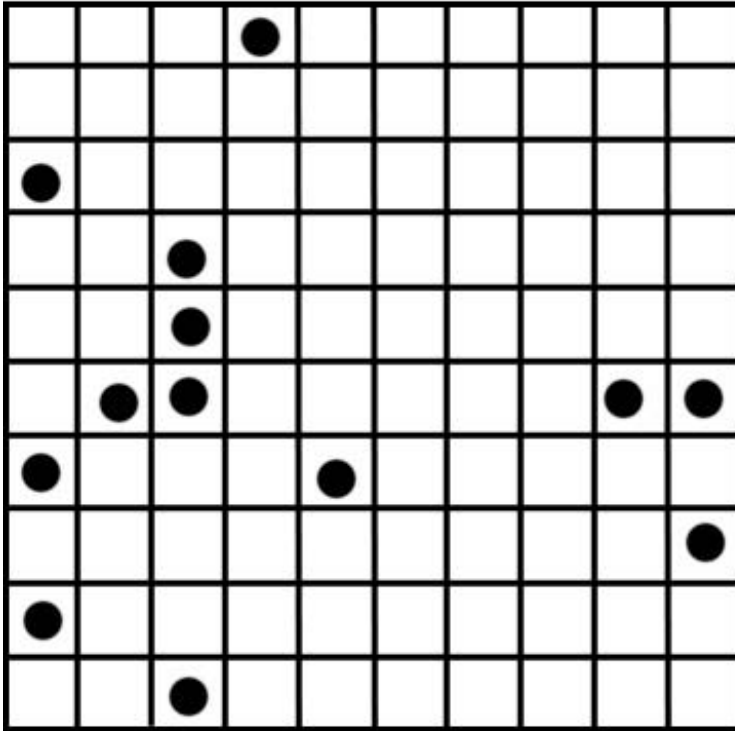


Fig. 5: A replicated fractal pattern from based upon measurements on Lamma Island

YUE, WEI & BURLEY (unpublished, in review) have also examined the reverse box-counting method in conjunction with other spatial measures such as with GIS and logistic regression to explain the physical properties of spatial elements. At Lamma Island, we noticed the development did not occur on hill-tops (Fig. 1). Thus the pattern of housing on Lamma Island could be possibly refined with spatial models that combine fractal methods and other physical spatial variables. We believe there are numerous opportunities to combine spatial methods such as auto-correlation measures, logistic regression, and fractal patterns to explain more of the variance in spatial phenomena.

We would encourage other investigators to employ such methods to study the replication properties of their urban and natural areas. We believe that there is much to explore concerning the properties and creation of spatial patterns.

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