Replicating species based fractal patterns for reclaiming northern Michigan waste rock piles

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REPLICATING SPECIES BASED FRACTAL PATTERNS FOR RECLAIMING NORTHERN MICHIGAN WASTE ROCK PILES[1]

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Abstract. Landscape planners and designers are interested in replicating natural landscape patterns to reclaim degraded landscapes to blend with existing conditions. One approach that shows promise is the use of fractal geometry to create natural landscape patterns. While the measurement of the actual fractal dimension of an object is difficult, the box-counting method (developed at Agrocampus Ouest, Angers, France) approximates the fractal dimension of an object. This process is illustrated by measuring and replicating a stand of trees in the Upper Peninsula of Michigan and applying the method for a planting plan on a Northern Michigan surface mine. The estimated fractal dimensions for the tree species are calculated: 0.329 for *Tsuga canadensis* Carrière, 0.674 for *Thuja occidentalis* L., 0.607 for *Acer rubrum* L, 0.345 for *Acer saccharum* Marshall, 0.442 for *Pinus strobus* L., and 0.359 for *Picea glauca* (Moench) Voss. and were applied in the design of a revegetation plan.

Additional Key Words: landscape architecture, landscape metrics, landscape ecology, environmental design, GPS, mine reclamation.

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Introduction

Landscape planners, designers, and environmental specialists are concerned with evaluating the spatial composition of landscape features such as composition of vegetation, forms of water bodies, and shape of terrain to unify disturbed landscapes with natural ones. However, natural looking assemblies were difficult to mathematically duplicate. Typical techniques used to replicate natural systems include the gestalt methods and ecological field methods (Fleurant, et al., 2009). The gestalt method was heuristic in nature where one would creatively merge and combine patterns together, until a desired condition was achieved. The ecological field laboratory method used scientific measures such as frequency, density, and size to construct patterns. A new approach has evolved which utilizes fractals to calculate spatial patterns in the landscape. A fractal designates an irregular or fragmented shape that can be divided into parts, each of which is approximately a smaller copy of the entire shape (Foroutan-Pour et al., 1999).

Literature Review

Origin of Fractals

Fractals were originally noticed at the end of the 19th century. Although the term “fractal” was coined later, the Peano curves appeared to be the first example of fractal objects, first explained by Guiseppe Peano. The Peano curves could fill a void through a series of iterations utilizing only a few simple rules (Mandelbrot, 1982).

Fractals have been explored more thoroughly in the latter half of the 20th century most notably by the French mathematician Benoit Mandelbrot. Mandelbrot, while researching “econometry” (mathematics applied to the economy), found that there were no difference in the slopes of curves predicting short-term and long-term market prices. He compiled an extensive description of the curves and created the term fractal (from the Latin word *fractus*, meaning broken) to describe the objects where irregularity separates them from typical Euclidian geometry curves. Upon the discovery of fractals, their use and application has broadened. Mandelbrot (1982) expressed the applications for fractals as follows, “Nature exhibits a high level of complexity in which typical Euclidian geometry classifies as formless, these irregular and fragmented patterns around us can be found using fractal geometry.” This is an explanation of why fractals are used today in such sciences as biology, ecology, and geology.

Fleurant et al.(2009) give a practical definition of the concept of fractals as a "geometrical shape resulting from infinite regular fragmentation of a given form." It is also proper to describe a fractal as a recurrence of the same form on each part of the curve. If one looked closely at any one part of a curve, it would resemble the entire curve itself (Fleurant et al., 2009). Cantor's Dust illustrates this property (Velde et al., 1990). Cantor's Dust is an image which results from Cantor's set, "a collection into a whole, of definite, well distinguished objects of our perception or thought" (Kamke, 1950). Cantor's dust has the geometric property where as the construction iteration process increases towards infinity, the total length L increases towards infinity. Imagine a straight line, then the same line with the middle 1/3rd removed. This process is continued to infinity and eventually the divisions become so small that they are unobservable by the human eye (Fig. 1) (Barnsley, 1988). The rings of Saturn are a real world example of this phenomenon. Saturn's ring was originally thought to be one solid entity, upon closer examination with higher powered telescopes it became clear that the ring was actually comprised of many small rings.

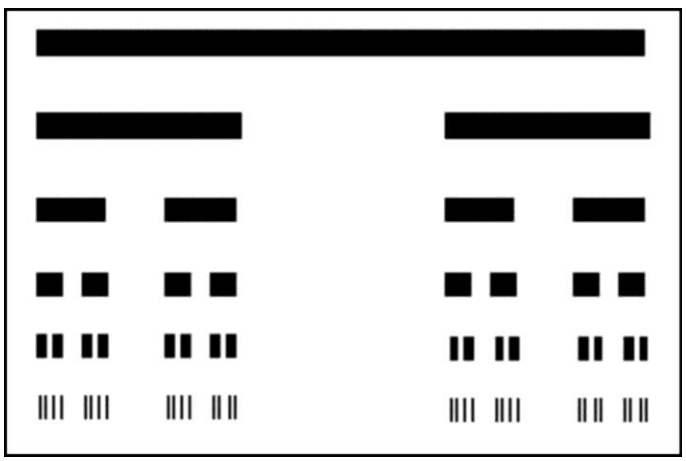

Figure 1. Cantor's Dust fractal object.

It is important to understand that there are two different categories of fractals; theoretical fractals, and real fractals. Theoretical fractals, such as the Peano curve and Cantor's dust mentioned prior, exhibit self-similarity and the dimensions can be mathematically calculated to infinity (Mandelbrot, 1982). Real fractals, such as objects found in nature, are not self-similar, and do not continue to infinity. To determine the dimensions of real fractals, one must employ an estimation process such as the box-counting method (Foroutan-Pour et al., 1999).

Fractal Dimensions

In Euclidian geometry, the point has a dimension of 0. Lines and curves have a dimension of 1. Areas have a dimension of 2, such as a triangle or circle. Volumes have a dimension of 4, such as a cylinder or sphere. Fractal objects also have dimensions (Mandelbrot, 1982).

Fractal dimensions have values which cannot be expressed by a simple point or line. Objects such as those found in nature cannot be explained by Euclidian geometry, but can be expressed using fractals. Barnsley (1988) affirmed this idea by stating,

> "Fractal dimensions can be attached to clouds, trees, coastlines, feathers, networks of neurons in the body, dust in the air at an instant in time, the clothes you are wearing, the distribution of frequencies of light reflected by a flower, the colors emitted by the sun, and the wrinkled surface of the sea during a storm."

Fractal dimensions attempt to quantify a subjective feeling which we have about how densely the fractal object fills the space in which it lies. They also provide a means for comparing the complexity of different fractals (Fleurant et al., 2009).

To demonstrate fractal dimensions, consider the Brittany coastline. If one were to calculate a 1 m length of a relatively straight line with a 20 cm ruler, the ruler will be used 5 times, 10 times for a 10 cm ruler, or 20 times with a 5 cm ruler. If one were to measure the same distance along the coastline, the total length will be underestimated due to the irregular pattern of the coast. The smaller the ruler used to measure the coast the more accurate the estimated length. To evaluate this phenomenon mathematically, one can declare that the result is more accurate when using a smaller ruler that fits the curvature of the line. If one can divide the length of the ruler of an infinite small size by "n", one has to use this ruler "n" times more. This property can define the topological dimension of the curve, where Eq. [1] presents the fractal dimension of a line.

$$D_{topological} = \log(n)/\log(n) = 1 \quad \text{[Eq. 1]}$$

Replicating this process again using a surface, one can use a square where the length of the side is L. To measure its area, one can use a smaller square where the length of one side is L/2, then one will need 4 squares, 16 squares using L/4, and so on. If the length of the side of the measuring square is divided by "n", the number of such squares used is multiplied by "n", represented in Eq. [2] for an area.

$D_{topological} = \log(n^2)/\log(n) = 2 \times \log(n)/\log(n) = 2$ [Eq. 2]

Similar results can be obtained for volumes and the topological dimension of a Euclidian geometric object with a fractal dimension of 3 (Fleurant et al., 2009).

In the moderately simple case of self-similar fractal objects (meaning they appear the same no matter which zooming (magnification) factor is used--meaning a closer view or a view from farther away), resulting in a constant iterative factor "k." The fractal dimension is represented in [Eq. 3] for self-similar objects.

$D_{fractal} = \log(n)/\log(k)$ [Eq. 3]

Where: n = the number of subsets counted during the scaling process using a factor 1/k (self-similarity factor).

k = number of iterations

Cantor's Dust illustrates how to calculate the fractal dimension of self-similar fractal objects. Consider a single line with a length of L. If one were to remove the middle 1/3rd of that line, there would be with two lines represented with the remainder where L equals 1/3. One can continue to remove the middle 1/3rd of every line formed by the previous division (the dust presents an infinite number of "lines" with each iteration). This process can be carried on indefinitely. Then, using the same reasoning one can calculate the fractal geometry of Cantor's Dust, [Eq. 4].

$D_{fractal} = \log 2/\log 3 = 0.6309$ [Eq. 4]

Therefore, one can conclude that the fractal dimension of this non-Euclidian curve is not 1 as any of the classic linear geometrical curves. Cantor's Dust has a topological dimension equal to 0 (it is a broken line), but has a fractal dimension of greater than 0, which is not an integer but a real number.

The previous equations [1] through [4] are utilized to calculate the dimension of theoretical fractals. These equations cannot be used to determine the dimension of real fractals due to random elements present in the natural setting (Foroutan-Pour et al., 1999). Instead one must employ a more appropriate method to estimate the fractal dimension. One such method that is commonly utilized is the box-counting method (Foroutan-Pour et al., 1999).

Inverse box-counting method: a tool for replicating landscapes

The fractal dimension has not always been easy to calculate but can be estimated using several methods. The box-counting method is one of the simpler and most popular methods to utilize. The box-counting method was developed by Duchesne et al. (2002) and computed by Durandet (2003) in the Landscape Department of the National Institute of Horticulture and Landscape Angers, France, now the Unite de RecherchePaysage; AgroCampusOuest. The natural object is covered with a grid of size r and one counts the number of boxes, N(r) that contain some part of the object. The value of "r" is progressively reduced and N(r) is similarly re-measured. As "r" tends to be very small values (0 in a theoretical way) one finds that the mathematical term in [Eq. 5] becomes the fractal dimension of the object (Fleurant et al., 2009).

$$\log(N(r))/\log(1/r) \qquad \text{[Eq. 5]}$$

The box-counting method is a simple tool to calculate the complexity of a landscape using the value of its fractal dimension. The greater an object's fractal dimension (2 is the maximum value in a plane), the less complex the arrangement of the planting pattern (in terms of scale, structure, alignment, etc.) (Fleurant et al. 2009). By utilizing this method, one is able to control the randomness of plantings or other landscape features with certain parameters: the fractal dimension (D), the average minimum distance between two trees (ϵ_{min}), and the average maximum size of the boxes (ϵ_{max}).

Planning and design applications

There is a belief that fractals may have the ability to re-create complex landscape patterns that are hard to replicate with Euclidian geometry because the landscape is full of fractals: rivers, trees, landscape networks in general (Barnsley, 1988). Fractals are extremely detailed, complex geometric shapes and a measure of their complexity is the fractal dimension (Mandelbrot, 1982). Accordingly, a number of professionals have examined fractals in landscape planning and design including studies by Fernandes et al., (2011); Schwarz, (2010); DiBari (2007); Thomas et al. (2007); Diaz-Delgado et al. (2005); Stamps, (2002); Griffith et al. (2000); Li (2000); Milne (1991); and Palmer (1988). However, for a time, the use of fractals seemed to be looking for a more practical application. For example, in landscapes it has always been relatively simple to describe an existing pattern, but hard to replicate that pattern. Recently, efforts by Wei et al. (2012) and Yue et al. (2012) illustrate practical applications of fractal geometry in urban design and landscape studies through the use of the inverse box-counting method. As a continuation of these promising efforts, this paper presents an approach to replicate vegetation landscape patterns in an applied manner. We were interested in employing the inverse box-counting method for reclaiming surface mine rock piles to depict the natural vegetation patterns based upon a Northern Michigan rock outcrop.

Study Area and Methodology

This study examines the application of fractals in the planting pattern of trees in the Upper Peninsula of Michigan in Dickinson County. The area selected for the study, located in Dickinson County (Fig. 2), was selected on a rocky and dry xeric northern forest (Fig. 3), an environment similar to waste rock piles on a surface mine where the fractal planting plan might be appropriate (Curtis, 1959). Trees equaling 3” dbh (diameter at breast height) or greater were recorded by a GPS (global positioning system) unit. The map of pre-settlement vegetation suggests that the study area was originally a Sugar Maple-Hemlock forest. This forest type was the most predominant upland system in the Upper Peninsula, and also consisted of large numbers of White Pine. Soils associated with this cover type can be steep and rocky, including exposures of basalt and granite bedrock (Albert and Comer, 2008).

According to the Soil Survey of Dickinson County, Michigan prepared by the United States Department of Agriculture, Soil Conservation Service the study area consists of a Pemene-rock outcrop complex (Linsemier, 1989). This complex consists of 35-65% Pemene soil and 15-20% rock outcrop on slopes of 18-35%. Trees to be planted on this complex include (but are not limited to); White Pine, White Spruce, and Sugar Maple.

The location of trees was placed on a map derived from remote sensing field survey. Points were gathered as X, Y data (latitude and longitude) by a remote GPS unit. Points were collected on an entire rock outcrop, and mapped (globally) using ArcGIS 9.3 software (ERSI 2008). The map of points was then projected into UTM’s (universal transverse Mercator) for the placement on a two-dimensional surface. The resulting map was then exported to an AutoCAD 17.2 file (computer aided drafting) (Autodesk 2008).

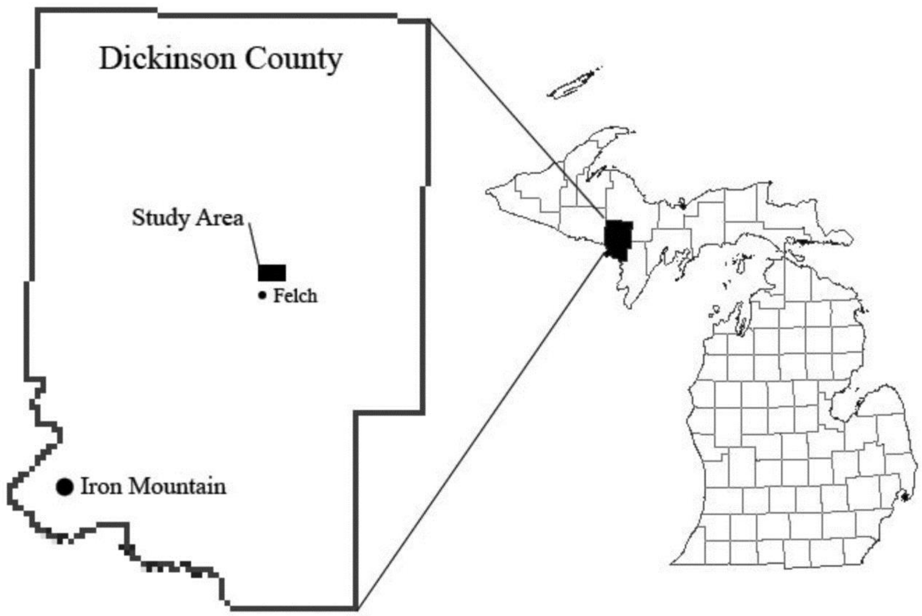

Figure 2. Location of the study area in Michigan.

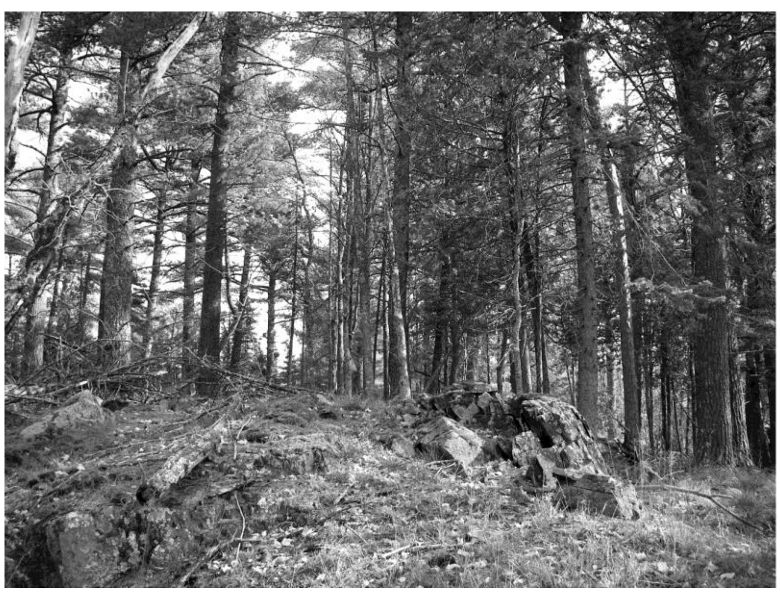

Figure 3. Forest stand at the study site (notice the rocky terrain and exposed bedrock).

Selection criteria for the size of the trial grids were determined by the size of the rock outcrop. The entire outcrop measured approximately 60 m by 80 m. Thus a trial grid of 50 m was selected to encompass the entire site with a series of trials occurring at random placements within the study site. A total of five different trials were completed for each species of tree on the rock outcrop, resulting in 30 (50 by 50 m) trials (see Appendix of Lehmann 2009). Each trial was then subject to the box-counting method.

The box-counting process starts with the pairs of values r and the number of boxes N(r), the starting value of r is 50 m, and the starting value of N(r) is one. Then r is divided in half and the value of r becomes 25 m, while N(r) can range from 1 to 4, depending on the number of boxes which contain trees. The pairs of numbers for the regression analysis includes the first pair where at least one box becomes empty, and continues with successive pairs at smaller sizes until every box contains either one or no trees (Fleurant et al., 2009). In total there were five 50 m by 50 m boxes for every species of tree recorded in the study area with a count of greater than one.

Results

The tree species tallied on site include: Eastern Hemlock (*Tsuga Canadensis* Carrière), Northern White Cedar (*Thuja occidentalis* L.), Red Maple (*Acer rubrum* L.), Sugar Maple (*Acer saccharum* Marshall), White Pine (*Pinus strobus* L.), and White Spruce (*Picea glauca* (Moench) Voss).

Out of the 30 trials, 113 dependent and independent variables for the regression analysis were derived. The regression analysis revealed an adjusted r-square of 0.444, with a significant p-value of 0. The slope of the line expressed in the regression equation is 0.578. This suggests that the fractal dimension is between a point and a line in typology [Eq. 6].

$$Ln(N(r)) = 0.578 Ln(1/r) + 3.107 \quad \text{[Eq. 6]}$$

Where: N(r) = number of boxes with trees

r = length of one side of the box

Eastern hemlock consists of 17 pairs of numbers (Table 1 and Fig. 4). The regression analysis revealed an adjusted r-square of 0.580, with a significant p-value of 0, and a significant t-value of 4.807. The slope of the line expressed in the regression equation is 0.329, suggesting that the fractal dimension is between a point and a line in typology [Eq. 7].

$$Ln(N(r)) = 0.329 Ln(1/r) + 1.936 \quad \text{[Eq. 7]}$$

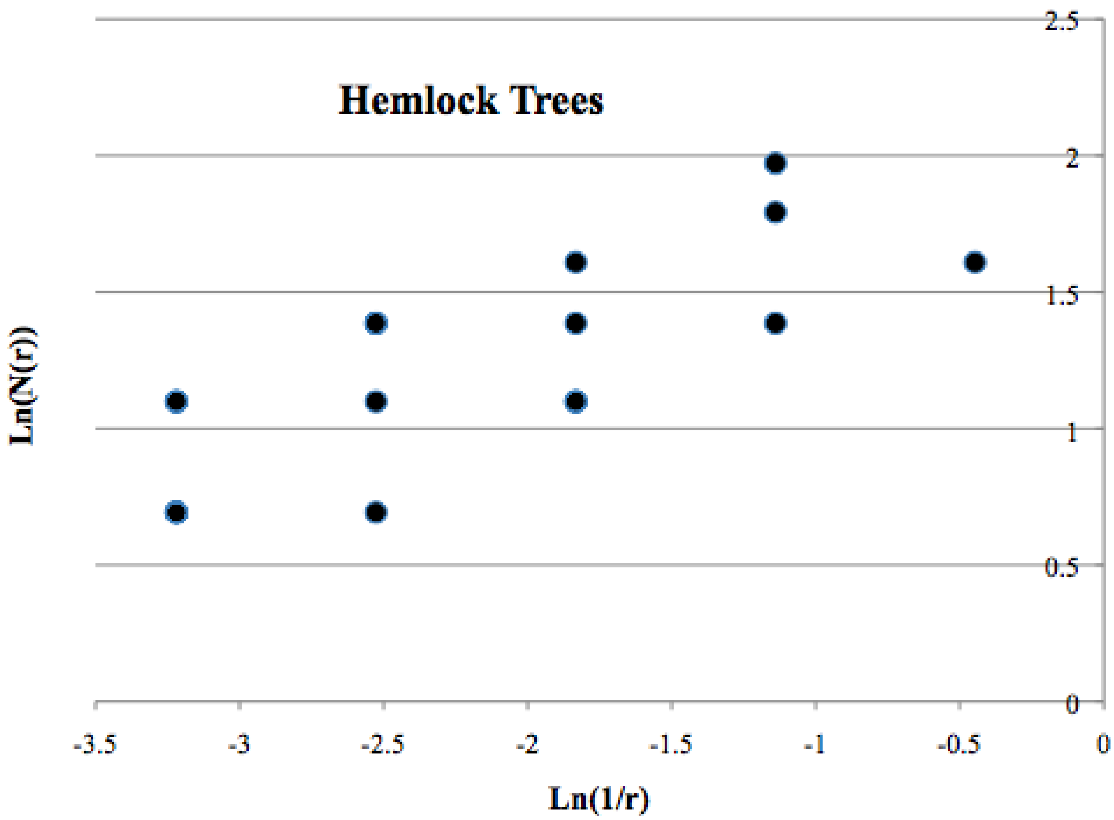

Figure 4. Scatter plot of results from Table 1.

Northern white cedar consists of 17 pairs of numbers (Table 2). The regression analysis revealed an adjusted r-square of 0.845, with a significant p-value of 0, and a significant t-value of 9.382. The slope of the line expressed in the regression equation is 0.674, suggesting that the fractal dimension is between a point and a line in typology [Eq. 8].

$$Ln(N(r)) = 0.674Ln(1/r) + 3.582 \qquad \text{[Eq. 8]}$$

Table 1. Dependent and independent variables for Eastern Hemlock regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
Eastern Hemlock 1	-3.219	0.693
	-2.526	1.386
	-1.833	1.609
	-1.139	1.792
Eastern Hemlock 2	-3.219	1.099
	-2.526	1.386
	-1.833	1.386
	-1.139	1.792
Eastern Hemlock 3	-3.219	0.693
	-2.526	0.693
	-1.833	1.099
Eastern Hemlock 4	-3.219	0.693
	-2.526	1.099
	-1.833	1.099
	-1.139	1.386
	-0.447	1.609
Eastern Hemlock 5	-3.219	1.099

Table 2. Dependent and independent variables for Northern White Cedar regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
Northern White Cedar 1	-3.219	1.099
	-2.526	2.079
	-1.833	2.639
	-1.139	2.833
Northern White Cedar 2	-2.526	2.079
	-1.833	2.565
	-1.139	2.708
Northern White Cedar 3	-2.526	2.197
	-1.833	2.398
	-1.139	2.773
Northern White Cedar 4	-3.219	1.099
	-2.526	1.792
	-1.833	2.303
	-1.139	2.565
Northern White Cedar 5	-2.526	1.946
	-1.833	2.303
	-1.139	2.639

Red maple consists of 20 pairs of numbers (Table 3). The regression analysis revealed an adjusted r-square of 0.768, with a significant p-value of 0, and a significant t-value of 7.994. The slope of the line expressed in the regression equation is 0.607, suggesting that the fractal dimension is between a point and a line in typology [Eq. 9]).

$$Ln(N(r)) = 0.607 Ln(1/r) + 3.689 \quad \text{[Eq. 9]}$$

Sugar maple consists of 18 pairs of numbers (Table 4). The Regression analysis revealed an adjusted r-square of 0.681, with a significant p-value of 0, and a significant t-value of 6.106. The slope of the line expressed in the regression equation is 0.345, suggesting that the fractal dimension is between a point and a line in typology [Eq. 10].

$$Ln(N(r)) = 0.345 Ln(1/r) + 2.168 \quad \text{[Eq. 10]}$$

Table 3. Dependent and independent variables for Red Maple regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
Red Maple 1	-2.526	2.197
	-1.833	2.773
	-1.139	3.091
	-0.447	3.296
Red Maple 2	-2.526	2.485
	-1.833	2.890
	-1.139	3.258
	-0.447	3.367
Red Maple 3	-2.526	2.079
	-1.833	2.565
	-1.139	2.944
Red Maple 4	-2.526	2.398
	-1.833	2.944
	-1.139	3.219
	-0.447	3.367
Red Maple 5	-3.219	1.099
	-2.526	1.792
	-1.833	2.565
	-1.139	2.773
	-0.447	2.944

Table 4. Dependent and independent variables for Sugar Maple regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
Sugar Maple 1	-3.219	1.099
	-2.526	1.609
	-1.833	1.792
	-1.139	1.792
	-0.447	1.946
Sugar Maple 2	-3.219	1.099
	-2.526	1.386
	-1.833	1.792
	-1.139	1.946
Sugar Maple 3	-3.219	1.099
Sugar Maple 4	-3.219	1.099
	-2.526	1.386
	-1.833	1.386
	-1.139	1.792
Sugar Maple 5	-3.219	0.693
	-2.526	1.099
	-1.833	1.099
	-1.139	1.609

Table 5. Dependent and independent variables for White Pine regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
White Pine 1	-2.526	2.197
	-1.833	2.485
	-1.139	2.565
	-0.447	2.639
White Pine 2	-2.526	2.303
	-1.833	2.708
	-1.139	3.091
	-0.447	3.135
White Pine 3	-2.526	2.485
	-1.833	3.135
	-1.139	3.135
	-0.447	3.219
White Pine 4	-2.526	2.485
	-1.833	2.833
	-1.139	2.890
	-0.447	2.944
White Pine 5	-3.219	1.099
	-2.526	2.079
	-1.833	2.485
	-1.139	2.565

White pine consists of 20 pairs of numbers (Table 5). The regression analysis revealed an adjusted r-square of 0.554, with a significant p-value of 0, and a significant t-value of 4.962. The slope of the line expressed in the regression equation is 0.442, suggesting that the fractal dimension is between a point and a line in typology [Eq. 11].

$$Ln(N(r)) = 0.442Ln(1/r) + 3.342 \qquad [Eq. 11]$$

White spruce consists of 21 pairs of numbers (Table 6). The regression analysis revealed an adjusted r-square of 0.387, with a significant p-value of 0.002, and a significant t-value of 3.689. The slope of the line expressed in the regression equation is 0.359, suggesting that the fractal dimension is between a point and a line in typology [Eq. 12].

$$Ln(N(r)) = 0.359Ln(1/r) + 2.387 \qquad [Eq. 12]$$

Table 6. Dependent and independent variables for White Spruce regression analysis.

Species Plot	Ln(1/r)	Ln(N(r))
White Spruce 1	-3.219	1.099
	-2.526	1.946
	-1.833	2.303
	-1.139	2.303
	-0.447	2.398
White Spruce 2	-3.219	1.099
	-2.526	1.609
	-1.833	1.792
	-1.139	1.946
White Spruce 3	-3.219	1.099
	-2.526	1.099
	-1.833	1.099
	-1.139	1.386
	-0.447	1.609
White Spruce 4	-2.526	1.946
	-1.833	2.079
	-1.139	2.303
White Spruce 5	-3.219	0.693
	-2.526	1.792
	-1.833	1.792
	-1.139	1.946

Application and Discussion

To apply the inverse box-counting method to the reclaimed landscape one would follow these procedures:

1. Divide the landscape to be planted in 50 m grids.
2. Divide each 50 m grid into grids with sides equal to 1.563 m (the size of the smallest boxes).
3. Use a random number generator to fill the grid with numbers from 1to 1024 for each tree species. Fill any box which contains a number that is less than or equal to the mean number of trees for each trial. Next, count each box that contains a tree and make sure the total falls within one standard deviation of the expected mean number of trees affiliated with the grid (see Lehmann 2009). The resulting pattern represents that particular species planting plan. Repeat these processes for each species of tree recorded and then combine the grids of all species onto one grid of the same size for the overall planting plan (Fig. 5). The number of trees per grid can be increased proportionally if the mortality rate of the trees is known.

The results of this approach are illustrated in Fig. 4. This process generated seven different fractal results, one for each of the tree species examined and one for all species combined and can be combined into a map for the placement of vegetation.

In the Upper Peninsula of Michigan, a typical mine site contains waste rock, with environmental conditions similar to xeric forest sites in the region. The planting method can be completed with seedlings being planted by hand or machine, as long as the tree is planted in the correct designated box. Berger (2008) and Burley (2001) provided additional background concerning reclamation planning and design.

The composition of trees in the study are similar to those specified by Curtis, dominant trees, however vary from typical northern xeric forest. This is not a rare condition as stated by Curtis (1959):

"Vegetation is a chaotic mixture of communities, each composed of a random assortment of species, each independently adapted to a particular set of external environmental factors. Rather there is a certain pattern to the vegetation, with more or less similar groups of species re-occurring from place to place."

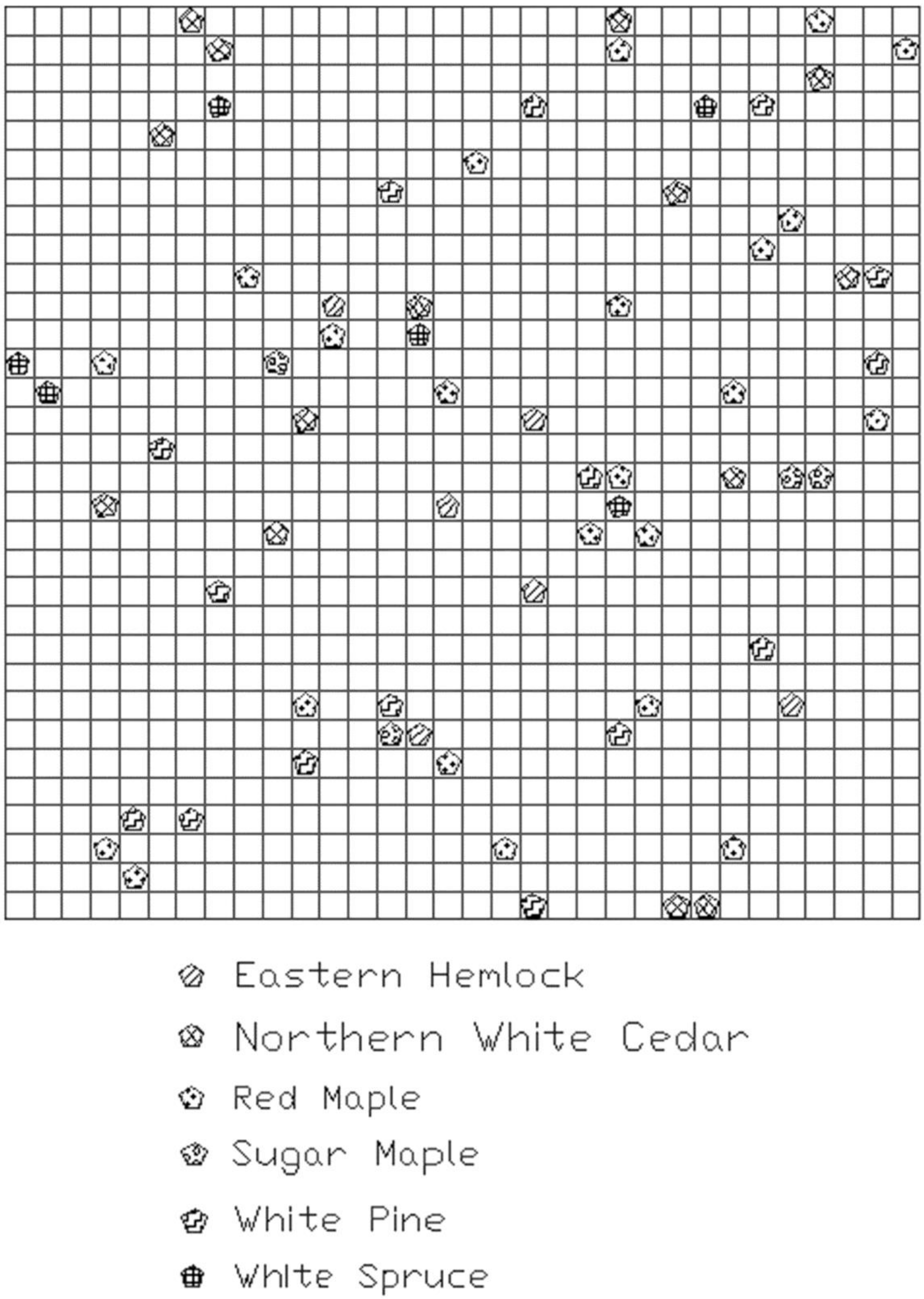

Figure 5. Fractal based planting plan. Each grid cell is 1.563 m.

This explanation from Curtis (1959) can also be attributed to cover change over time. According to Albert and Comer (2008), the existing tree species composition is different from the pre-settlement vegetation according to an interpretation of the 1816-1856 general land office surveys (2008).

Results of the data collection process reveal a consistent vegetation type to those described by Curtis (1959), Linsemier (1989), and Albert and Comer (2008). There were a number of trees not identified by these sources, however changes to composition and introduction of new species by humans can attribute to these changes. It is also important to remember that each area has its own unique set of environmental conditions which can affect the composition of vegetation within a given cover type (Curtis, 1959). One constant that holds true throughout these investigations is soil conditions; rocky and steep terrain with exposed bedrock. It can be concluded that most tree species identified by these investigations will be appropriate for reforestation of surface mine reclamation projects within the Upper Peninsula of Michigan.

Statistical analysis of the fractal dimensions of each species, and the combined analysis reveal that all species have similar patterns. The total fractal dimension of all species revealed a slope of 0.587. The fractal dimension of each species ranged from 0.329 to 0.674, revealing that each species has the same Euclidian dimension of 0, but their own distinctive fractal dimension. The intercept value of all species was 3.107. The intercept value of each species ranged from 1.936 to 3.582, revealing that each species indeed has their own pattern and the overall species composition falls within the parameters of these patterns. The investigation of the fractal dimension of each species reveals numbers which are similar to that of Cantor's Dust. This result suggests that these fractal patterns may be expressed at different scales (100 m by 100 m, 1 mile by 1 mile, etc.). Further research is needed to determine if it is possible to apply these findings to areas larger than 50 m by 50 m.

This fractal landscape pattern replication approach can be applied to other spatial objects such as hills and lakes in the general study area. The pattern of peaks and lakes for the Porcupine Mountains and the Huron Mountains in Michigan, north of the study area were examined during 2009. Table 7 illustrates the results.

Table 7. Properties of peaks and lakes in the Porcupine Mountains and Huron Mountains.

	Total Length	Box Length	Total Boxes	Filled Boxes	Fractal
Porcupine Mountain Peaks	8.046.72 m	252.46 m	1,024	69	0.039
Porcupine Mountains Lakes	8,046.72 m	1,005.84 m	64	5	0.320
Huron Mountains Peaks	10,058.4 m	78.58 m	16,384	154	0.038
Huron Mountains Lakes	10,058.4 m	314.32 m	1,024	13	0.156

The patterns can be replicated in the same manner as the tree patterns were replicated by (Fleurant, et al., 2009). Instead of having tree species, the classes of the pattern are by elevation. For example, in Table 7, for the Huron Mountain peaks, there are expected to be 154 boxes filled from a total of 16,384 box, with a box length of 78.58 m in a pattern approaching fractal dimension 0.038. Table 8 presents the classes of heights for the 154 filled boxes. For the Huron Mountain peak class of 201.17 m (660 ft) to 251.46 m (825 ft) eight of the 154 boxes would contains peaks of this class. Besides vegetation, hill peaks, and lake positions, it may be possible to recreate other landscape features. We encourage investigators to explore and describe landscape features in their study areas.

Table 8. The height class of peaks in the Huron Mountains.

Height class	Filled Boxes
201.17 m to 251.46 m	8
251.46 m to 301.75 m	23
301.75 m to 352.04 m	29
352.04 m to 402.34 m	57
402.34 m to 452.63 m	30
452.63 m to 502.92 m	7

A limitation of this study is the scale of investigation. Our investigation used 50 by 50 m square grids, however, most reclamation projects are larger than this. To be able to apply these findings at a larger scale is an area of further investigation one may choose to explore. The box-counting method expresses this scale-based limitation in the upper and lower limits of the regression line. As the regression line continues past the boundaries (the defined line and limits of the regression pairs) of the box-counting method, the line is skewed. The upper limit of the regression line flattens (Duchesne et al., 2002 and Dauphiné, 2001) while the lower limit of the regression line is abnormally steep. To scale the results of this study without the proper mathematical function would yield an unreliable result. Another limitation of the box-counting method also relates to scale, specifically the maximum size of the grid. The fractal dimension estimate is highly correlated to the size of the largest box (Kenkel and Walker, 1996). Site limitations which caused this experiment to utilize 50 m grids may have ultimately affected the fractal dimension estimated. The estimated fractal dimension of all species was relatively low when compared to the previous investigations of Fleurant et al. (2009) and does not meet the standards for a set of unaligned points in a two dimensional plane. According to Kenkel and Walker (1996) the fractal dimension of a two dimensional point pattern should be less than 2.

Another limitation of this study is the site of application. This study focused solely on the vegetation of a Northern Michigan rock outcrop. To apply these findings anywhere but a Northern Michigan surface mine, one would have to conduct their own survey of a natural area they wished to replicate. This investigation determined the fractal pattern of trees; while this is not a limitation, further research is needed to determine if this process can be used for other landscape features such as topography, or water networks.

In conclusion, it is determined that the box-counting method can be used to estimate the fractal dimension of an individual species of tree within a vegetation stand. The inverse box-counting method can then be applied to re-create the fractal patterns found in the landscape. Successfully replicating natural tree stands is important to many disciplines outside of mine reclamation. This method can be applied for many projects including reforestation after forest fire (or other natural disaster), restoration, or any project which attempts to blend in with the surrounding vegetative community.

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