Problem Set 1

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1 Problem 1

1.1 Part a

If the highest frequency we need is 100 Hz, then we need a Nyquist Frequency greater than 200 Hz in order to properly sample it.

1.2 Part b

If we are sampling a signal at 250 samples/s, then we can sample up to 125 Hz and still have a unique sginal.

2 Problem 2

Given the two equations:

$$\begin{aligned} x_a(t) &= sin(480\pi t) + 3sin(720\pi t) \\ x_b(t) &= 3cos(600\pi t) + 2cos(1800\pi t) \end{aligned} \tag{2.1}$$

2.1 Part a

The Nyquist Sampling frequency will be based off of the higher of the two frequencies. Therefore, in part a the nyquist frequency is:

$$\begin{aligned} x_a(t) &= 3sin(720\pi t) \\ highest\ frequency &= 360\ Hz \\ Nyquist\ frequency &= 720\ samples/second \end{aligned} \tag{2.2}$$

And in equation b:

$$\begin{aligned} x_b(t) &= 2cos(1800\pi t) \\ highest\ frequency &= 900\ Hz \\ Nyquist\ frequency &= 1800\ samples/second \end{aligned} \tag{2.3}$$

2.2 Part b

If the sampled signals are passes through an ideal D/A converter, the reconstructed signals would appear as below:

$$\begin{aligned} x_a(t) &= sin(480\pi t) + 3sin(720\pi t) \\ t &= \frac{n}{Fs} \\ Fs &= 600 \\ x_a(n) &= sin(480\pi \frac{n}{600}) + 3sin(720\pi \frac{n}{600}) \\ x_a(n) &= sin(\frac{4}{5}\pi n) + 3sin(\frac{6}{5}\pi n) \\ x_a(n) &= sin(2\pi n\frac{2}{5}) + 3sin(2\pi n\frac{3}{5}) \\ x_a(n) &= sin(2\pi n\frac{2}{5}) + 3sin(2\pi n(1-\frac{2}{5})) \\ x_a(n) &= sin(2\pi n\frac{2}{5}) - 3sin(2\pi n\frac{2}{5}) \\ Recover\ Signals & \\ y_a(t) &= -2sin(2\pi t\frac{2}{5} * 600) \\ y_a(t) &= -2sin(2\pi 240t)) \end{aligned} \tag{2.4}$$

The two sinewaves appear as one signal due to aliasing in the second part of the signal.

For the Other Signal:

$$\begin{aligned}
x_b(t) &= 3cos(600\pi t) + 2cos(1800\pi t)\\
t &= \frac{n}{Fs}\\
Fs &= 10,000\\
x_b(n) &= 3cos(600\pi \frac{n}{10000}) + 2cos(1800\pi \frac{n}{10000})\\
x_b(n) &= 3cos(\pi n \frac{3}{50}) + 2cos(\pi n \frac{9}{50})\\
&\textit{Recover Signals}\\
y_b(t) &= 3cos(\pi t 600) + 2cos(\pi t 1800)
\end{aligned} \tag{2.5}$$

The two parts of the signal are recovered successfully

3 Part 3

If we have a signal:

$$x(t) = cos(4000\pi t) \tag{3.1}$$

That was sampled to produce

$$x(n) = cos(n\pi/2) \tag{3.2}$$

Then the sampling rate, measured in Hz is:

$$\begin{aligned}
t &= \frac{n}{Fs}\\
cos(4000\pi t) &= cos(n\pi/2)\\
cos(4000\pi \frac{n}{Fs}) &= cos(n\pi/2)\\
4000\pi \frac{n}{Fs} &= \frac{n\pi}{2}\\
\frac{4000}{Fs} &= \frac{1}{2}\\
Fs &= 8000\ Hz
\end{aligned} \tag{3.3}$$