The Discrete Fourier Transform (DFT)

Overview

In this exercise you will generate a few discrete signals, learn how to write MATLAB functions, and experiment with the DFT.

You may find the `diary` command useful for maintaining a history of your MATLAB session. Other commands you may find useful include `subplot`. As always to get information about a function and its usage, use the MATLAB `help` or `doc` commands, e.g. `help subplot`, `doc filter`, etc.

Basic Signals

1. Generate and plot (using MATLAB'S `stem` command) the sequence

$$x[n] = (0.95)^n \cos(\pi/20\ n)$$

   for $0 \le n \le 63$. Note that MATLAB'S indexing for the first element starts with `1` and not `0`, so you will have to adjust for this in your plot.

2. The following is a simple example of a MATLAB function `framp` which generates an *N*-point ramp *$x[n] = n$ for $1 \le n \le N$*. The input argument is *N* and the function generates the desired values.

```
function x=framp(N)
%  function x=framp(N)
% Generates an N-point ramp sequence
n=1:N;
x=n' ;
```

Write a MATLAB function, `fcosine`, which will generate the values from a finite-length sinusoid $A \cos(2\pi f_0 n + \phi)$ for $n_1 \le n \le n_f$. The function will need a total of five input arguments: A, f_0, ϕ, n_1, and n_f. The function should return a column vector which contains only the desired values of the sinusoid. Test your function by plotting the results with $A = 4$, $f_0 = 1/20$, $\phi = \pi/4$, $n_1 = -20$, and $n_f = 20$.

The DFT

3. Write a MATLAB function `DFT` that returns the N-point DFT of an input signal. Do not use `fft`.
4. Write a MATLAB function `stem_DFT` that perform DFT (or `fft`) on an input signal and plot (i.e. `subplot`) the magnitude and phase angle of the resulting Fourier Transform. You might find the functions `angle` and `abs` useful.
5. Use the function `fcosine` from exercise 2 to generate 2 discrete time signals $x_1[n]$ with $A_1 = 4$, $\boldsymbol{f_1 = 1/20}$, $\phi_1 = \pi/4$, $n_1 = -20$, and $n_f = 20$ and $x_2[n]$ with $A_2 = 4$, $\boldsymbol{f_2 = 1/2}$, $\phi_2 = \pi/4$, $n_1 = -20$, and $n_f = 20$. The choice of f_0 gives radically different results for the DFT. Use `stem_DFT` to compute and compare the DFTs $X_1[m]$ and $X_2[m]$. Comment on your results.

Zero Padding a DFT

6. Let $X_1[m]$ be the 64-point DFT of $x[n]$ in Exercise 1, and let $X_2[m]$ be the 128-point DFT of the 128-point sequence obtained by appending $x[n]$ in Exercise 1 with 64 zeros. Compute $X_1[m]$ and $X_2[m]$, and use `stem_DFT` to plot $|X_1[m]|$ and $|X_2[m]|$. Again, make sure the indexing in your plots is correct. Comment on the relationship, if any, between the values of $X_1[m]$ and $X_2[m]$.

Spectral Analysis

7. Load the file signal1.mat using `load signal1.mat` and look at its spectral component assume a sampling rate Fs = 120 Hz. What are the frequencies present in the signal?
8. Load the file signal2.mat using `load signa2.mat` and look at its spectral component assume a sampling rate Fs = 350 Hz. What are the frequencies present in the signal?
9. Are the frequencies the same in exercises 7 and 8? Given that in both cases the signals were generated using the same equation explain the frequencies encountered. The equation used:

$$x(n) = \cos(2\pi f_0 t + \phi) + 0.5\sin(2\pi 2 f_0 t + \phi)$$
$$f_0 = 50Hz$$
$$\phi = 0$$

Turn in

Turn in a one to two pages memo with a brief description of what you did. Highlight any challenges you had, clever approaches you used, any new insights you gained, etc. Be sure to include your code.