C R Package `seatdist`

This paper is accompanied by `seatdist`, an R package which implements a variety of apportionment algorithms and disproportionality measures. The `seatdist` package is developed from the `SciencesPo` (Marcelino 2016) package, but offers more apportionment algorithms and disproportionality measures and corrects some errors.

C.1 Seat Apportionment Methods in `seatdist`

Apportionment algorithms are accessible through a unified interface provided by `seatdist::giveseats()` which takes the following arguments

`v`	a numeric vector of vote counts;
`ns`	numeric, the number of seats to allocate;
`method`	character, name of the method, see Table C.5 for divisor methods and Table C.6 for largest remainder quotas;
`thresh`	numeric, threshold of exclusion; if in [0,1], treated as a fraction; if in (1, 100), treated as a percent; if larger than 100, treated as a vote count;
`quota`	character, quota for `method="largest remainders"`; see Table C.6, defaults to `NA`;
`divs`	numeric, divisors for `method="custom"`, must be non-negative.

For the Largest Remainders method (`method="lr"` or `"largest remainders"`) the Imperiali quota (`quota="im"`) or Reinforced Imperiali (`quota="rei"`) can assign in the first round more seats than available, in which case the function terminates its execution with an error message.

The `seatdist::giveseats()` function returns a named list with items

`method`	character, name of the apportionment method used;
`seats`	numeric, vector with seats.

For illustration, 10 seats can be apportioned to parties with 60,000, 28,000, and 12,000 votes under a system with a 5% threshold with the Largest Remainders method and the Hagenbach-Bischoff quota in the following way

```
> seatdist::giveseats(v=c(A=60, B=28, C=12)*1e3, ns=1e1,
                      method="lr", quota="hb", thresh=5e-2)

thresh treated as a fraction
$method
"Largest Remainders with Hagenbach-Bischoff quota"

$seats
A B C
6 3 1
```

Table C.5: Divisor method implemented in `seatdist::giveseats()`. For background on the methods see e.g. Grilli di Cortona et al. (1999), Agnew (2008), Ichimori (2010), and Wada (2016).

Method	`method`	x^{th} Divisor	Sequence
Plurality	`"pl"`	1	$1, 1, 1, 1, 1, \ldots$
D'Hondt	`"dh"`	x	$1, 2, 3, 4, 5, \ldots$
Jefferson	`"je"`	"	"
Hagenbach-Bischoff	`"hb"`	"	"
Adams	`"ad"`	$x - 1$	$0, 1, 2, 3, 4, \ldots$
Smallest Divisors	`"sd"`	"	"
Nohlen	`"no"`	$x + 1$	$2, 3, 4, 5, 6, \ldots$
Imperiali	`"im"`	$(x + 1)/2$	$1, 1.5, 2, 2.5, 3, 3.5, \ldots$
Sainte-Laguë	`"sl"`	$2x - 1$	$1, 3, 5, 7, 9, \ldots$
Webster	`"we"`	"	"
Swedish Sainte-Laguë	`"sw"`	$2x - 1;\ x > 1$	$1.2, 3, 5, 7, 9, \ldots$
Nepalese Sainte-Laguë	`"ne"`	$2x - 1;\ x > 1$	$1.4, 3, 5, 7, 9, \ldots$
Norwegian Sainte-Laguë	`"nor"`	"	"
Hungarian Sainte-Laguë	`"hu"`	$2x - 1;\ x > 1$	$1.5, 3, 5, 7, 9, \ldots$
Modified Sainte-Laguë	`"msl"`	$(2x - 1)5/7;\ x > 1$	$1, 2.14, 3.57, 5, 6.43, \ldots$
Danish	`"da"`	$3x - 2$	$1, 4, 7, 10, 13, \ldots$
Huntington-Hill	`"hh"`	$\sqrt{x(x - 1)}$	$0, 1.41, 2.45, 3.46, 4.47, \ldots$
Equal Proportions	`"ep"`	"	"
Dean	`"de"`	$x(x - 1)/(x - 0.5)$	$0, 1.33, 2.4, 3.43, 4.44 \ldots$
Theil-Schrage	`"ts"`	$\frac{1}{\ln x - \ln(x-1)}$	$0, 1.44, 2.47, 3.48, 4.48, \ldots$
Agnew	`"ag"`	$\frac{1}{e}\left(\frac{x^x}{(x-1)^{(x-1)}}\right)$	$0.37, 1.47, 2.48, 3.49, 4.49, \ldots$
Ichimori 1/3	`"ich"`	$\sqrt{x^2 + x + 1/3}$	$1.73, 2.65, 3.61, 4.58, 5.57, \ldots$
Custom	`"custom"`		user-supplied in argument `divs`

Table C.6: Quotas implemented for the Largest Remainders method (`method="lr"`) in `seatdist::giveseats()`. For background on the methods see e.g. Grilli di Cortona et al. (1999).

Quota	`quota`	Formula
Hare	`"ha"`	$\frac{e}{l}$
Droop	`"dr"`	$\left\lfloor 1 + \frac{e}{l+1} \right\rfloor$
Hagenbach-Bischoff	`"hb"`	$\frac{e}{l+1}$
Imperiali	`"im"`	$\frac{e}{l+2}$
Reinforced Imperiali	`"rei"`	$\frac{e}{l+3}$

C.2 Measures of Disproportionality in `seatdist`

The `seatdist` package computes 24 disproportionality measures (Table C.7) accessible through a unified interface provided by the function `seatdist::disproportionality()`.

The function takes the following arguments:

`s`	a numeric vector of seat counts or fractions;
`v`	a numeric vector of vote counts or fractions; for `measure = "ortona"` this can alternatively be a vector with seats under the highest possible proportionality
`measure`	character, see Table C.7;
`ignore_zeros`	logical: should parties with 0 votes and 0 seats be ignored?
`k`	numeric, k for the Generalized Gallagher index, defaults to 2;
`eta`	η for the Atkinson index, defaults to 2;
`alpha`	α for the Generalized Entropy index, defaults to 2;
`thresh`	numeric, threshold for the Fragnelli and the Gambarelli & Biella indexes, defaults to `"NULL"`;
`powind`	character, power index for the Fragnelli and the Gambarelli & Biella indexes, defaults to the Shapley-Shubik index, `"shapley shubik"`. no other power indexes implemented yet.

The function returns a named list with the following items

`measure`	character, the measure used;
`value`	numeric, value.

For illustration, the Gallagher index can be computed for parties with 60,000, 28,000, and 12,000 votes and 6, 3, and 1 seats in the following way:

```
> seatdist::disproportionality(v=c(60,28,12)*1e3,
                                 s=c(6,3,1),
                                 measure="gallagher")
$measure
[1] "Gallagher"

$value
[1] 0.02
```

Table C.7: Disproportionality measures in the `seatdist` package. Values for the `measure=` argument in `seatdist::disproportionality()` below index names. For indexes without citations in the table see also Karpov 2008 and Chessa and Fragnelli 2012.

Index	Formula
D'Hondt (Gallagher 1991) `"dhondt"`	$\delta = \max_i \frac{s_i}{v_i}$
Monroe (1994) `"monroe"`	$I_M = \sqrt{\frac{\sum_i (s_i - v_i)^2}{1 + \sum_i v_i^2}}$
Max. Abs. Dev. `"maxdev"`	$I_{MAD} = \max_i \{\lvert s_i - v_i \rvert\}$
Rae (1967) `"rae"`	$I_{Rae} = \frac{1}{p} \sum_i \lvert s_i - v_i \rvert$
Loosemore & Hanby (1971) `"loosemore hanby"`	$I_{LH} = \frac{1}{2} \sum_i \lvert s_i - v_i \rvert$
Grofman `"grofman"`	$I_{Grof} = \frac{1}{e} \sum_i \lvert s_i - v_i \rvert; \; e = \frac{1}{\sum_i v_i^2}$
Lijphart `"lijphart"`	$I_L = \frac{\lvert s_a - v_a \rvert + \lvert s_b - v_b \rvert}{2}; \; v_a > v_b > \ldots$
Gallagher (1991) `"gallagher"`	$I_{Gal} = \sqrt{\frac{1}{2} \sum_i (s_i - v_i)^2}$

Table C.8: Table C.7 continued from the previous page

Index	Formula
Generalized Gallagher `"kindex"`	$$I_K = \sqrt[k]{\frac{1}{k}\sum_i (s_i - v_i)^k}$$
Gatev `"gatev"`	$$I_{Gat} = \sqrt{\frac{\sum_i (s_i - v_i)^2}{\sum_i (s_i^2 + v_i^2)}}$$
Ryabtsev `"ryabtsev"`	$$I_{Ryb} = \sqrt{\frac{\sum_i (s_i - v_i)^2}{\sum_i (s_i + v_i)^2}}$$
Szalai (Stewart 2006) `"szalai"`	$$I_{Sz} = \sqrt{\frac{1}{p}\sum_i \left(\frac{s_i - v_i}{s_i + v_i}\right)^2}$$
Weighted Szalai (Stewart 2006) `"weighted szalai"`	$$I_{WSz} = \sqrt{\frac{1}{2}\sum_i \frac{(s_i - v_i)^2}{s_i + v_i}}$$
Aleskerov & Platonov `"aleskerov"`	$$I_{AP} = \frac{\sum_i k_i \frac{s_i}{v_i}}{\sum_i k_i};\ k_i = \mathbb{1}\left(\frac{s_i}{v_i} > 1\right)$$
Gini `"gini"`	The Gini coefficient of inequality
Atkinson `"atkinson"`	$$I_A = 1 - \left[\sum_i v_i \left(\frac{s_i}{v_i}\right)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$$

Table C.9: Table C.7 continued from the previous page

Index	Formula
Generalized Entropy `"gen entropy"`	$I_{GE} = \frac{1}{\alpha^2 - \alpha} \left[\sum_i v_i \left(\frac{s_i}{v_i} \right)^\alpha - 1 \right]$
Sainte-Laguë (1910) `"sainte lague"`	$I_{SL} = \sum_i \frac{(s_i - v_i)^2}{v_i}$
Cox & Shugart `"cox shugart"`	$I_{CS} = \frac{\sum_i (s_i - \bar{s})(v_i - \bar{v})}{\sum_i (v_i - \bar{v})^2}$
Farina (Kestelman 2005) `"farina"`	$I_{Far} = \arccos \left[\frac{\sum_i s_i v_i}{\sqrt{\sum_i s_i^2 \sum_i v_i^2}} \right] \frac{10}{9}$
Ortona `"ortona"`	$I_O = \frac{\sum_i \lvert s_i - v_i \rvert}{\sum_i \lvert u_i - v_i \rvert};\ u_i = \mathbb{1}(s_i = \max_i s_i)$
Fragnelli `"fragnelli"`	$I_{Frag} = \frac{1}{2} \sum_i \lvert \varphi_i(s) - \varphi_i(v) \rvert;$
Gambarelli & Biella `"gambarelli biella"`	$I_{GB} = \max_i \{ \lvert s_i - v_i \rvert,\ \lvert \varphi_i(s) - \varphi_i(v) \rvert \}$
Cosine Dissimilarity `"cosine"`	$I_{CD} = 1 - \frac{\sum_i s_i v_i}{\sqrt{\sum_i s_i^2}\sqrt{\sum_i v_i^2}}$
Mixture D'Hondt a.k.a. Lebeda's RR `"mixture"`	$\pi^*_{DH} = 1 - \frac{1}{\max_i s_i / v_i}$

Table C.10: Table C.7 continued from the previous page

Index	Formula
Lebeda's ARR `"arr"`	$ARR = \frac{1}{p}\left(1 - \frac{1}{\max_i s_i/v_i}\right)$
Lebeda's SRR `"arr"`	$SRR = \sqrt{\sum_i \left(v_i - \frac{s_i}{\max_i s_i/v_i}\right)^2}$
Lebeda's WDRR `"wdrr"`	$WDRR = \frac{1}{3}\left(\left(\sum_i \lvert v_i - s_i\rvert\right) + \left(1 - \frac{1}{\max_i s_i/v_i}\right)\right)$
Kullback-Leibler Surprise `"surprise"`	$KL = \sum_{s_i>0} s_i \ln \frac{s_i}{v_i}$
Likelihood Ratio Statistic `"lrstat"`	$G = 2\sum_i v_i \ln \frac{v_i}{s_i}$
Chi Squared `"chisq"`	$\chi^2 = \sum_{s_i>0} \frac{(v_i - s_i)^2}{s_i}$
Hellinger Distance `"hellinger"`	$HD = \frac{1}{\sqrt{2}}\sqrt{\sum_i \left(\sqrt{s_i} - \sqrt{v_i}\right)^2}$

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