C R Package `seatdist`

This paper is accompanied by `seatdist`, an R package which implements a variety of apportionment algorithms and disproportionality measures. The `seatdist` package is developed from the `SciencesPo` (Marcelino 2016) package, but offers more apportionment algorithms and disproportionality measures and corrects some errors.

C.1 Seat Apportionment Methods in `seatdist`

Apportionment algorithms are accessible through a unified interface provided by `seatdist::giveseats()` which takes the following arguments

- `v` — a numeric vector of vote counts;
- `ns` — numeric, the number of seats to allocate;
- `method` — character, name of the method, see Table C.5 for divisor methods and Table C.6 for largest remainder quotas;
- `thresh` — numeric, threshold of exclusion; if in [0,1], treated as a fraction; if in (1, 100), treated as a percent; if larger than 100, treated as a vote count;
- `quota` — character, quota for `method="largest remainders"`; see Table C.6, defaults to `NA`.

For the Largest Remainders method (`method="lr"` or `"largest remainders"`) the Imperiali quota (`quota="im"`) or Reinforced Imperiali (`quota="rei"`) can assign in the first round more seats than available, in which case the function terminates its execution with an error message.

The `seatdist::giveseats()` function returns a named list with items

- `method` — character, name of the apportionment method used;
- `seats` — numeric, vector with seats.

For illustration, 10 seats can be apportioned to parties with 60,000, 28,000, and 12,000 votes under a system with a 5% threshold with the Largest Remainders method and the Hagenbach-Bischoff quota in the following way

```
> seatdist::giveseats(v=c(A=60, B=28, C=12)*1e3, ns=1e1,
                      method="lr", quota="hb", thresh=5e-2)

thresh treated as a fraction
$method
"Largest Remainders with Hagenbach-Bischoff quota"

$seats
A B C
6 3 1
```

Table C.5: Divisor method implemented in `seatdist::giveseats()`. For background on the methods see e.g. Grilli di Cortona et al. (1999).

Method	`method`	Formula	Sequence
D'Hondt	`"dh"`	x	$1, 2, 3, 4, 5, \ldots$
Jefferson	`"je"`	”	”
Hagenbach-Bischoff	`"hb"`	”	”
Adams	`"ad"`	$x - 1$	$0, 1, 2, 3, 4, \ldots$
Smallest Divisors	`"sd"`	”	”
Nohlen	`"no"`	$x + 1$	$2, 3, 4, 5, 6, \ldots$
Imperiali	`"im"`	$(x + 1)/2$	$1, 1.5, 2, 2.5, 3, 3.5, \ldots$
Sainte-Laguë	`"sl"`	$2x - 1$	$1, 3, 5, 7, 9, \ldots$
Webster	`"we"`	”	”
Hungarian Sainte-Laguë	`"hu"`	$2x - 1;\ x > 1$	$1.5, 3, 5, 7, 9, \ldots$
Modified Sainte-Laguë	`"msl"`	$(2x - 1)5/7;\ x > 1$	$1, 2.14, 3.57, 5, 6.43, \ldots$
Danish	`"da"`	$3x - 2$	$1, 4, 7, 10, 13, \ldots$
Huntington-Hill	`"hh"`	$\sqrt{x(x-1)}$	$0, 1.41, 2.45, 3.46, 4.47 \ldots$
Equal Proportions	`"ep"`	”	”

Table C.6: Quotas implemented for the Largest Remainders method (`method="lr"`) in `seatdist::giveseats()`. For background on the methods see e.g. Grilli di Cortona et al. (1999).

Quota	`quota`	Formula
Hare	`"ha"`	$\frac{e}{l}$
Droop	`"dr"`	$\left\lfloor 1 + \frac{e}{l+1} \right\rfloor$
Hagenbach-Bischoff	`"hb"`	$\frac{e}{l+1}$
Imperiali	`"im"`	$\frac{e}{l+2}$
Reinforced Imperiali	`"rei"`	$\frac{e}{l+3}$

C.2 Measures of Disproportionality in `seatdist`

The `seatdist` package computes 24 disproportionality measures (Table C.7) accessible through a unified interface provided by the function `seatdist::disproportionality()`.

The function takes the following arguments:

`s`	a numeric vector of seat counts or fractions;
`v`	a numeric vector of vote counts or fractions; for `measure = "ortona"` this can alternatively be a vector with seats under the highest possible proportionality
`measure`	character, see Table C.7;
`ignore_zeros`	logical: should parties with 0 votes and 0 seats be ignored?
`k`	numeric, k for the Generalized Gallagher index, defaults to 2;
`eta`	η for the Atkinson index, defaults to 2;
`alpha`	α for the Generalized Entropy index, defaults to 2;
`thresh`	numeric, threshold for the Fragnelli and the Gambarelli & Biella indexes, defaults to `"NULL"`;
`powind`	character, power index for the Fragnelli and the Gambarelli & Biella indexes, defaults to the Shapley-Shubik index, `"shapley shubik"`. no other power indexes implemented yet.

The function returns a named list with the following items

`measure`	character, the measure used;
`distance`	numeric, distance from proportionality.

For illustration, the Gallagher index can be computed for parties with 60,000, 28,000, and 12,000 votes and 6, 3, and 1 seats in the following way:

```
> seatdist::disproportionality(v=c(60,28,12)*1e3,
                               s=c(6,3,1),
                               measure="gallagher")
$measure
[1] "Gallagher"

$distance
[1] 0.02
```

Table C.7: Disproportionality measures in the `seatdist` package. Values for the `measure=` argument in `seatdist::disproportionality()` below index names. For indexes without citations in the table see also Karpov 2008 and Chessa and Fragnelli 2012.

Index	Formula
D'Hondt (Gallagher 1991) `"dhondt"`	$\delta = \max_i \frac{s_i}{v_i}$
Monroe (1994) `"monroe"`	$I_M = \sqrt{\frac{\sum_i (s_i - v_i)^2}{1 + \sum_i v_i^2}}$
Max. Abs. Dev. `"maxdev"`	$I_{MAD} = \max_i \{\|s_i - v_i\|\}$
Rae (1967) `"rae"`	$I_{Rae} = \frac{1}{p} \sum_i \|s_i - v_i\|$
Loosemore & Hanby (1971) `"loosemore hanby"`	$I_{LH} = \frac{1}{2} \sum_i \|s_i - v_i\|$
Grofman `"grofman"`	$I_{Grof} = \frac{1}{e} \sum_i \|s_i - v_i\|;\ e = \frac{1}{\sum_i v_i^2}$
Lijphart `"lijphart"`	$I_L = \frac{\|s_a - v_a\| + \|s_b - v_b\|}{2};\ v_a > v_b > \ldots$
Gallagher (1991) `"gallagher"`	$I_{Gal} = \sqrt{\frac{1}{2} \sum_i (s_i - v_i)^2}$

Table C.8: Table C.7 continued from the previous page

Index	Formula
Generalized Gallagher `"kindex"`	$$I_K = \sqrt[k]{\frac{1}{k}\sum_i (s_i - v_i)^k}$$
Gatev `"gatev"`	$$I_{Gat} = \sqrt{\frac{\sum_i (s_i - v_i)^2}{\sum_i (s_i^2 + v_i^2)}}$$
Ryabtsev `"ryabtsev"`	$$I_{Ryb} = \sqrt{\frac{\sum_i (s_i - v_i)^2}{\sum_i (s_i + v_i)^2}}$$
Szalai (Stewart 2006) `"szalai"`	$$I_{Sz} = \sqrt{\frac{1}{p}\sum_i \left(\frac{s_i - v_i}{s_i + v_i}\right)^2}$$
Weighted Szalai (Stewart 2006) `"weighted szalai"`	$$I_{WSz} = \sqrt{\frac{1}{2}\sum_i \frac{(s_i - v_i)^2}{s_i + v_i}}$$
Aleskerov & Platonov `"aleskerov"`	$$I_{AP} = \frac{\sum_i k_i \frac{s_i}{v_i}}{\sum_i k_i};\; k_i = \mathbb{1}\left(\frac{s_i}{v_i} > 1\right)$$
Gini `"gini"`	The Gini coefficient of inequality
Atkinson `"atkinson"`	$$I_A = 1 - \left[\sum_i v_i \left(\frac{s_i}{v_i}\right)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$$

Table C.9: Table C.7 continued from the previous page

Index	Formula
Generalized Entropy "gen entropy"	$I_{GE} = \frac{1}{\alpha^2 - \alpha}\left[\sum_i v_i \left(\frac{s_i}{v_i}\right)^{\alpha} - 1\right]$
Sainte-Laguë (1910) "sainte lague"	$I_{SL} = \sum_i \frac{(s_i - v_i)^2}{v_i}$
Cox & Shugart "cox shugart"	$I_{CS} = \frac{\sum_i (s_i - \bar{s})(v_i - \bar{v})}{\sum_i (v_i - \bar{v})^2}$
Farina (Kestelman 2005) "farina"	$I_{Far} = \arccos\left[\frac{\sum_i s_i v_i}{\sqrt{\sum_i s_i^2 \sum_i v_i^2}}\right]\frac{10}{9}$
Ortona "ortona"	$I_O = \frac{\sum_i \lvert s_i - v_i\rvert}{\sum_i \lvert u_i - v_i\rvert};\ u_i = \mathbb{1}(s_i = \max_i s_i)$
Fragnelli "fragnelli"	$I_{Frag} = \frac{1}{2}\sum_i \lvert\varphi_i(s) - \varphi_i(v)\rvert;$
Gambarelli & Biella "gambarelli biella"	$I_{GB} = \max_i\{\lvert s_i - v_i\rvert,\ \lvert\varphi_i(s) - \varphi_i(v)\rvert\}$
Cosine Dissimilarity "cosine"	$I_{CD} = 1 - \frac{\sum_i s_i v_i}{\sqrt{\sum_i s_i^2}\sqrt{\sum_i v_i^2}}$
Mixture D'Hondt "mixture"	$\pi^*_{DH} = 1 - \frac{1}{\max_i s_i / v_i}$

References

Chessa, Michela, and Vito Fragnelli. 2012. "A note on 'Measurement of disproportionality in proportional representation systems'". *Mathematical and Computer Modelling* 55 (3): 1655–1660.

Eurostat. 2017. "Nomenclature of territorial units for statistics".

Gallagher, Michael. 1991. "Proportionality, disproportionality and electoral systems". *Electoral Studies* 10 (1): 33–51.

Grilli di Cortona, Pietro, et al. 1999. *Evaluation and Optimization of Electoral Systems*. SIAM.

Jeworutzki, Sebastian. 2016. *cartogram: Create Cartograms with R*. R package version 0.0.2. `https://CRAN.R-project.org/package=cartogram`.

Karpov, Alexander. 2008. "Measurement of disproportionality in proportional representation systems". *Mathematical and Computer Modelling* 48 (9): 1421–1438.

Kestelman, Philip. 2005. "Apportionment and proportionality: A measured view". *Voting Matters* 20:12–22.

Loosemore, John, and Victor J Hanby. 1971. "The theoretical limits of maximum distortion: some analytic expressions for electoral systems". *British Journal of Political Science* 1 (4): 467–477.

Marcelino, Daniel. 2016. *SciencesPo: A tool set for analyzing political behavior data*. R package version 1.4.1. `http://CRAN.R-project.org/package=SciencesPo`.

Monroe, Burt L. 1994. "Disproportionality and malapportionment: Measuring electoral inequity". *Electoral Studies* 13 (2): 132–149.

Rae, Douglas W. 1967. *The Political Consequences of Electoral Laws*. New Haven: Yale University Press.

Sainte-Laguë, André. 1910. "La représentation proportionnelle et la méthode des moindres carrés". In *Annales scientifiques de l'École Normale Supérieure*, 27:529–542.

Stewart, Jay, et al. 2006. "Assessing alternative dissimilarity indexes for comparing activity profiles". *Electronic International Journal of Time Use Research* 3 (1): 49–59.