

# NEH Algorithm

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## ► NEH – Nawaz, Enscore, Ham

- NEH heuristic
- NEH insertion technique

# Flow-Shop Scheduling Problem - FSP

Determining the best sequence of  $n$  jobs to be processed on  $m$  machines in the same order.

Possible No. of sequences =  $(n!)^m$

# Permutation Flow-Shop Scheduling Problem - PFSP

Same job sequence on all machines

Possible No. of sequences =  $(n!)$

For makespan minimisation, PFSP is *NP-complete*, if  $m$  is greater than 2

# NEH Algorithm

## ► *Stage 1:*

The generation of an initial order of jobs with respect to an indicator value.

## ► *Stage 2:*

The iterative insertion of jobs into a partial sequence according to the initial order of stage 1.

# NEH Algorithm for Makespan Minimisation

- *Step 1:* Order the jobs by non-increasing sums of processing times on the machines;
- *Step 2:* Take the first two jobs and schedule them in order to minimise the partial makespan as if there were only these two jobs
- *Step 3:* For  $k = 3$  to  $n$  do Step 4
- *Step 4:* Insert the  $k$ th job at the place, which minimises the partial makespan among the  $k$  possible ones.

Total No. of sequences  
to be enumerated  $= \frac{n(n+1)}{2} - 1$

# Example

4 job x 5 machine flow-shop problem

Process times

		Machines ( $m$ )				
		M1	M2	M3	M4	5
Jobs ( $n$ )	J1	5	9	8	10	1
	J2	9	3	10	1	8
	J3	9	4	5	8	6
	J4	4	8	8	7	2

► Step 1:

$$T_1 = 5 + 9 + 8 + 10 + 1 = 33$$

$$T_2 = 9 + 3 + 10 + 1 + 8 = 31$$

$$T_3 = 9 + 4 + 5 + 8 + 6 = 32$$

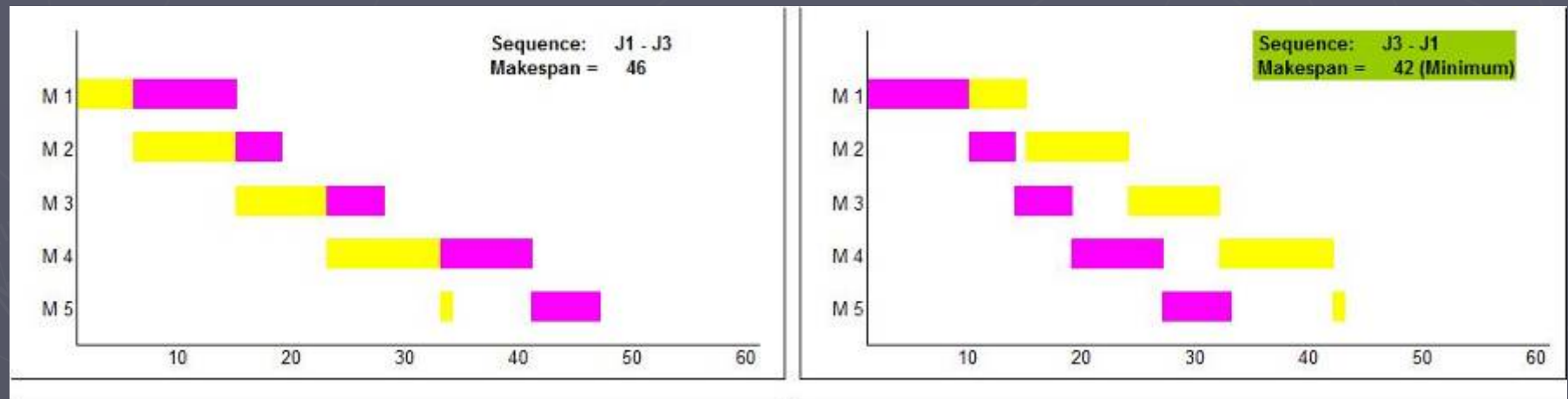
$$T_4 = 5 + 9 + 8 + 10 + 1 = 29$$

► Step 2: J1, J3, J2, J4



**Step 3:** Select J1 and J3, and find the best partial sequence for only these two jobs.

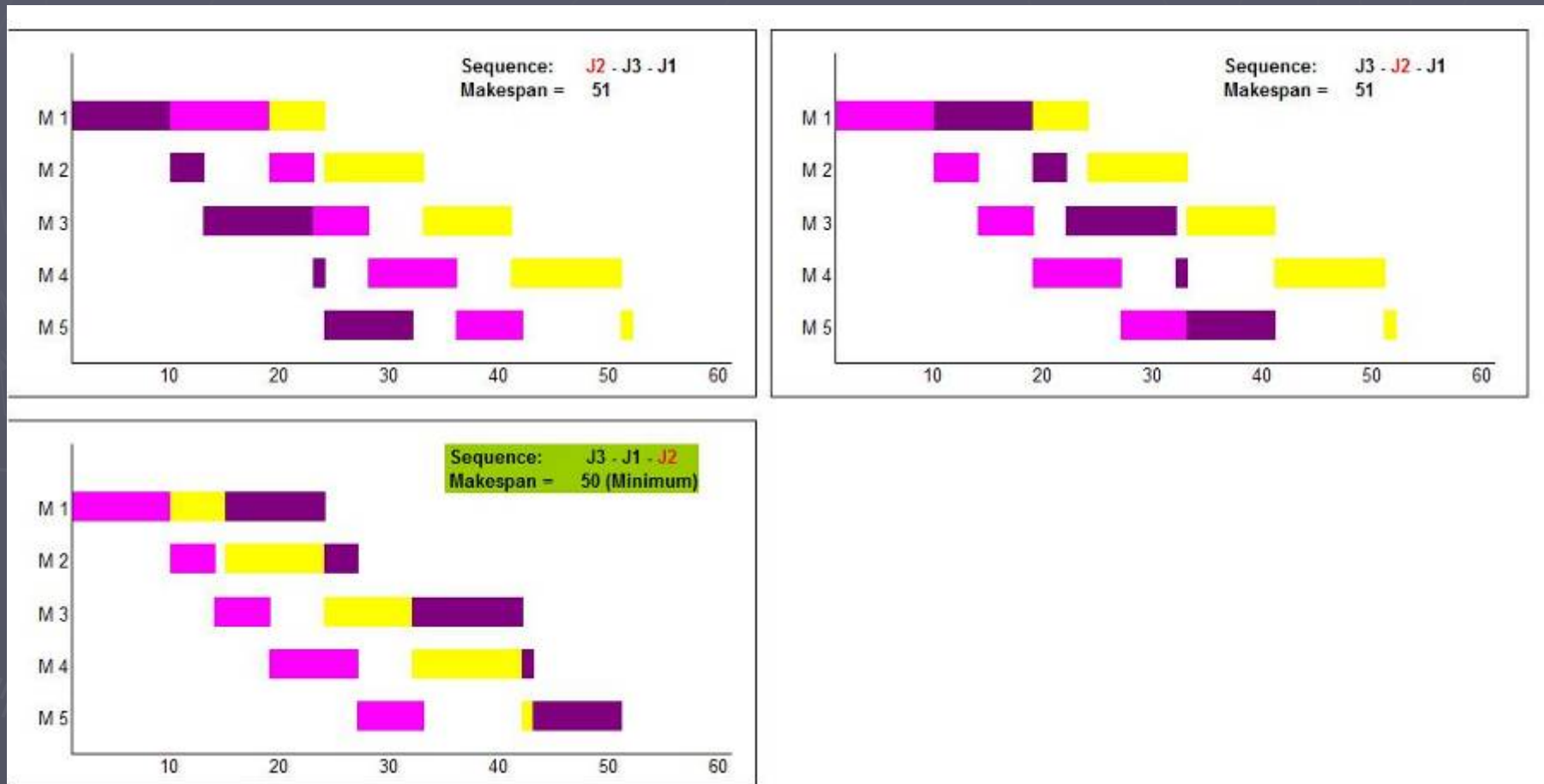
The best sequence is: J1 – J3 with makespan = 42



(In the next step, the relative position of J1 and J3 should always be J3 – J1, i.e. J3 before J1)

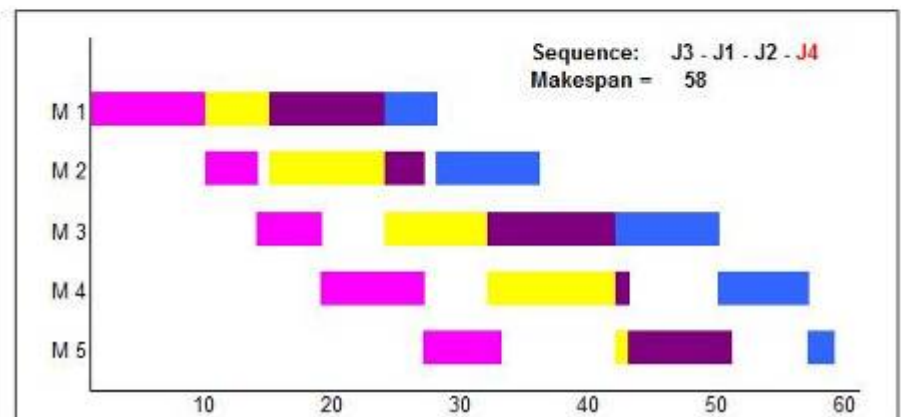
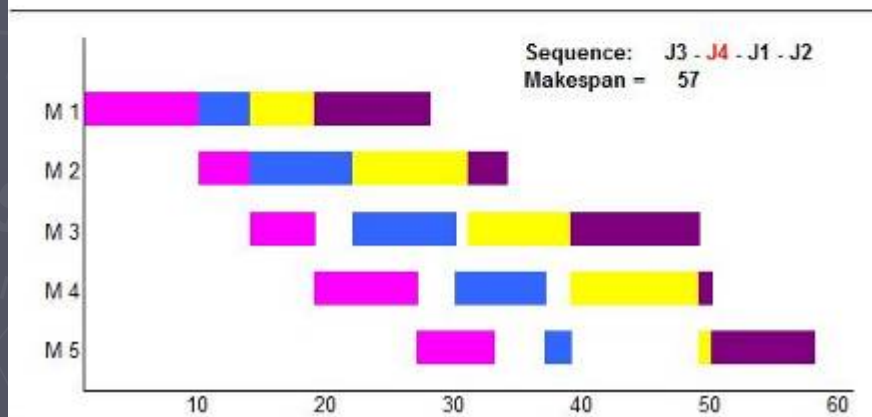
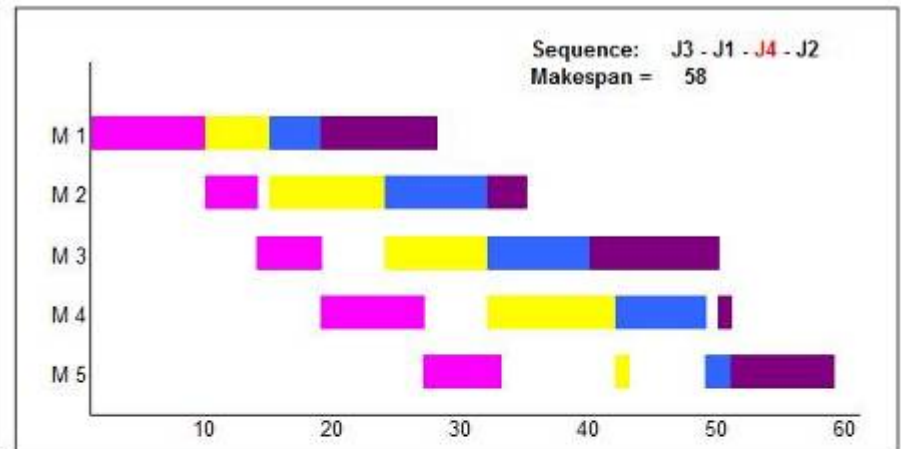
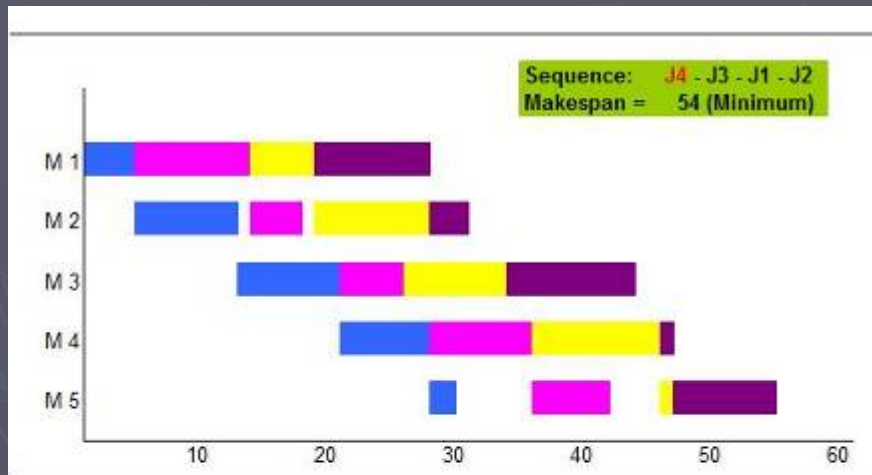
**Step 4:** Select **J2** (job in the 3<sup>rd</sup> position of the list of Step 2) and find the best sequence by inserting **J2** at all three possible positions in the partial sequence J3 - J1 obtained in the previous step.

The best sequence is J3 – J1 – J2 with makespan of 50



**Step 4 a:** Select **J4** (job in the 4th position of the list of Step 2) and find the best sequence by inserting **J4** at all four possible positions in the partial sequence J3 - J1 - J2 obtained in the previous step.

The best sequence is **J4 - J3 - J1 - J2** with makespan of 54



- ▶ NEH algorithm is the most efficient algorithm to minimise makespan for Permutation Flow-Shop Problem (PFSP).
- ▶ Research has shown that, on the average, NEH algorithm returns makespan close to 3% of the optimal for
- ▶  $n = 5, \dots, 500$ , and  $m = 5, \dots, 25$ .

- ▶ NEH algorithm can be used for
  - ▶ - Flow time
  - ▶ - Lateness/Tardiness, etc.
  - ▶ minimisation problems.
- ▶ Almost all metaheuristics are using NEH results as a seed for the next stage improvement.

- ▶ Significance and applications of NEH algorithm can be highlighted by the number of citations reported in the scheduling research literature.
- ▶ SCOPUS database lists 375 citations; at least 70 in 2008 alone.