Welcome to Week #7!

Last Week: Prep for Foundations for Inference

Foundations for Inference - How well can we really know anything?

Overview of next 2 Classes

Hypothesis Testing Framework (Ch. 5-7)

The general outline of the process:

1. Set the hypotheses. ?
   For a single mean this will look like:
   H_0: μ = null value ?
   ? H_A: μ < or > or ≠ null value
2. Check assumptions and conditions
3. Calculate a test statistic ? and a p-value ?
4. Make a decision, and interpret it in context

- If p-value < α, reject H_0,
  ? there is sufficient evidence for [H_A]

- If p-value > α, do not reject H_0,
  there is not sufficient for evidence for [H_A]

Hypothesis Testing Framework (Ch. 5-7)

The general outline of the process:

English

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3. Calculate a test statistic and a p-value
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Provides a rigorous way to determine the answer with a specific level of confidence.

Math

- If p-value < α, reject H_0,
  there is sufficient evidence for [H_A]

- If p-value > α, do not reject H_0,
  there is not sufficient for evidence for [H_A]

English

Hypothesis Testing Framework (Ch. 5-7)

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How do we frame our question into the "null" and "alternative" hypothesis framework? What are these different hypotheses?

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What distributions can we use to explore our sample?

Is our sample large or small?

e.g. if we are asking a question about sample means, do we expect our sample means to be normally distributed?

Use a normal distribution (Ch 5)?
t-distribution (Ch 7)?
Chi-square (Ch 6)?

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Calculate a number using our chosen distribution (e.g. the normal distribution) to see how “weird” a parameter of our sample is.

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  there is sufficient evidence for [H_A]

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Hypothesis Testing Framework (Ch. 5-7)

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Draw a "hard line" to determine if we can reject or we fail to reject the "null hypothesis"

Hypothesis Testing Framework (Ch. 5-7)

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We have actually been doing this mathematically already, you just didn't know!

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Let's look at some examples!

- If p-value < α, reject H_0,
  there is sufficient evidence for [H_A]
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Hypotheses: Definition

In statistics a <u>hypothesis</u> means a very specific thing (slightly different then, for example, a science definition): it is a claim to be tested

<u>H_0, Null Hypothesis</u>: the "default", "standard" or currently accepted claim, the currently accepted value for a parameter. We start this process by assuming this is true.

<u>H_A, Alternative Hypothesis</u>: the "research" hypothesis, or claim we need to test

<u>Possible Outcomes</u>:
(1) We say we "reject the null hypothesis" - i.e. H_A is *more* true than H_0
(2) We say we "fail to reject the null hypothesis"

Note: we *cannot* say that H_A or H_0 is true, only that one is *more likely* to be true than the other.

Examples of stating Hypothesis: Practice #1

It is believed a candy machine makes peanut butter cups that are on average 5g. After maintenance, a worker claims the machine no longer makes the cups at a weight of 5g. What are H_0 and H_A? How do we write them in a statistical format?

The "default" or "previously assumed" claim is the null hypothesis

The alternative hypothesis is the claim to be tested

with math

$$H_0: \mu = 5g$$

$$H_A: \mu \neq 5g$$

population mean

Examples of stating Hypothesis: Practice #2

A company has stated their ping-pong machine makes ping-pongs that are 6mm in diameter. A worker believes the machine no longer makes ping-pongs of this size and samples 100 ping-pongs to perform a hypothesis test with 99% confidence. What are H_0 and H_A?

Think on it for a moment!

Examples of stating Hypothesis: Practice #3

Doctors believe that the average teen sleeps on average no longer than 10 hours per day. A researcher believes that teens on average sleep longer. What are H_0 and H_A?

Examples of stating Hypothesis: Practice #4

The school board claims that at least 55% of students bring an iPhone to school. A teacher believes this number is too high and randomly samples 25 students to test. What are H_0 and H_A?

Examples of stating Hypothesis: Practice #5

A super fan of shopping says that on average buying socks on ebay is cheaper than in person at their local shop. A price comparison study has shown that prices for new socks are on average the same or more expensive on ebay as in their local store. Our shopper wants to setup a statistical test to see if their intuition is right. What are H_0 and H_A?

Summary: Set the hypothesis

The general outline of the process:

1. Set the hypotheses.
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How do we frame our question into the "null" and "alternative" hypothesis framework? What are these different hypotheses?

- If p-value $< \alpha$, reject H_0,
  there is sufficient evidence for [H_A]

- If p-value $> \alpha$, do not reject H_0,
  there is not sufficient for evidence for [H_A]

Summary: Set the hypothesis

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   For a single mean this will look like:
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4. Make a decision, and interpret it in context

tell us something about
what tests we will perform
(more in examples)

- If p-value < α, reject H_0,
  there is sufficient evidence for [H_A]

- If p-value > α, do not reject H_0,
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Summary: Set the hypothesis

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Are we interested in a hypothesis about the population mean (μ)?

Proportion (p)? (Ch. 5)

Difference of 2 means and/or paired data (μ_1 - μ_2)? (Ch. 7)

Difference between observations and theorized results? (Ch. 6)

(more in examples)

Hypothesis Testing: Where we are going

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Picking appropriate distributions and applying - Rest of Ch 5, and 6 & 7

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(a) normal, large sample
(b) normal?, small sample
(c) observations & theory

<u>Test Statistics</u>
(a) Z-score -> P(Z)
(b) T-Score -> P(T)
(c) χ^2 -> P(χ^2)

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<u>Test Statistics</u>

(a) Z-score -> P(Z)

(b) T-Score -> P(T)

(c) χ^2 -> P(χ^2)

Compare Z-score, T-score or χ^2 to our <u>level of significance - α</u> - to see if we can reject the null hypothesis (if the p-value of our test statistic < α)

Anatomy of a test statistic

The general form of a test statistic is

$$\frac{\text{point estimate} - \text{null value}}{\text{SE of point estimate}}$$

Only tricks are:

(1) picking what the point and null values are based on our hypotheses

(2) what the form of the standard errors is based on what our underlying distribution looks like (normal, t-distribution, χ^2)

This construction is based on

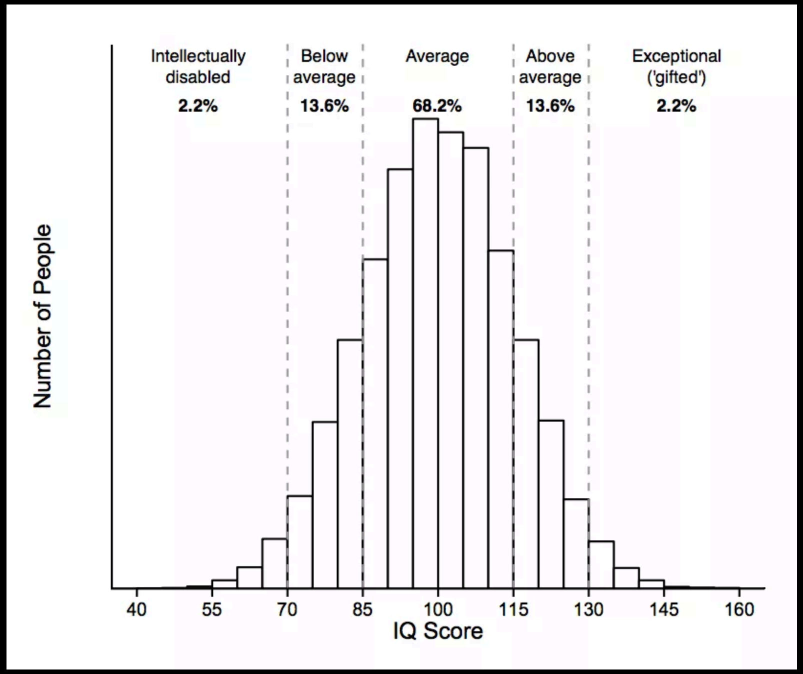
- identifying the difference between a point estimate and an expected value if the null hypothesis was true, and
- standardizing that difference using the standard error of the point estimate.

These two ideas will help in the construction of an appropriate test statistic for count data.

Examples!

Practice #1

A principle at a certain school claims that the students in his school are above average intelligence. A random sample of 30 students' IQ scores have a mean score of 105. Is there sufficient evidence to support the principle's claims (ignoring the inherent biases in IQ tests)? The mean population IQ is 100 with a standard deviation of 15.



Step 1: Write down Null & Alternative Hypotheses

The "default" or "previously assumed" claim is the null hypothesis

The alternative hypothesis is the claim to be tested

$$H_0: \mu_{students} \leq 100$$

$$H_A: \mu_{students} > 100$$

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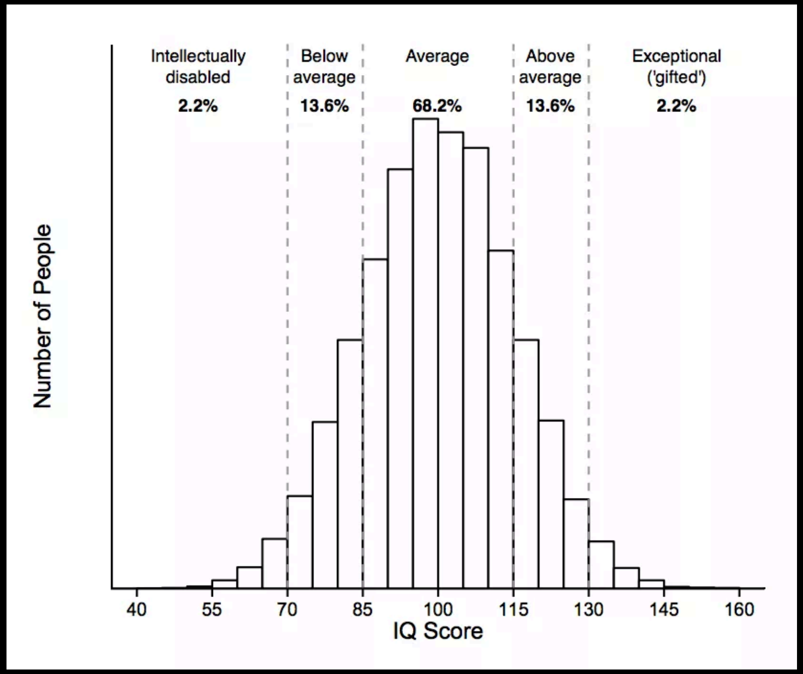

Step 1: Write down Null & Alternative Hypotheses

H_0: $\mu_{students} \leq 100$

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Step 2: Write down assumptions & conditions about the underlying population distribution

Key Insights about how well we know the "average" number representing a sample:

Lets say we want to know the average observation from a random sample taken from a population (usually the case, very rarely can we sample the entirety of the population):

IF the samples are independent (e.g. randomly sampled)
IF the sample size is "large enough" (typically > 30 observations)
IF the underlying population distribution is not strongly skewed

This is admittedly a bit "hand-wavy" and not rigorous (stay tuned for your future stats classes!)

THEN

1. The "average" value of this population mean is the sample mean
2. The error on the measurement of the mean is given by the "standard error": $SE = s/n^{1/2}$

If you are curious, this comes from:

$$\mathrm{Var}\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2}\sum \mathrm{Var}(X_i) = \frac{1}{n^2} \times \sum \sigma^2 = \frac{n}{n^2}\sigma^2 = \frac{\sigma^2}{n}$$

Where "s" is the standard deviation of the sample & n is the number of samples

In practice we have to assume "s" is the standard deviation of the *sample*.

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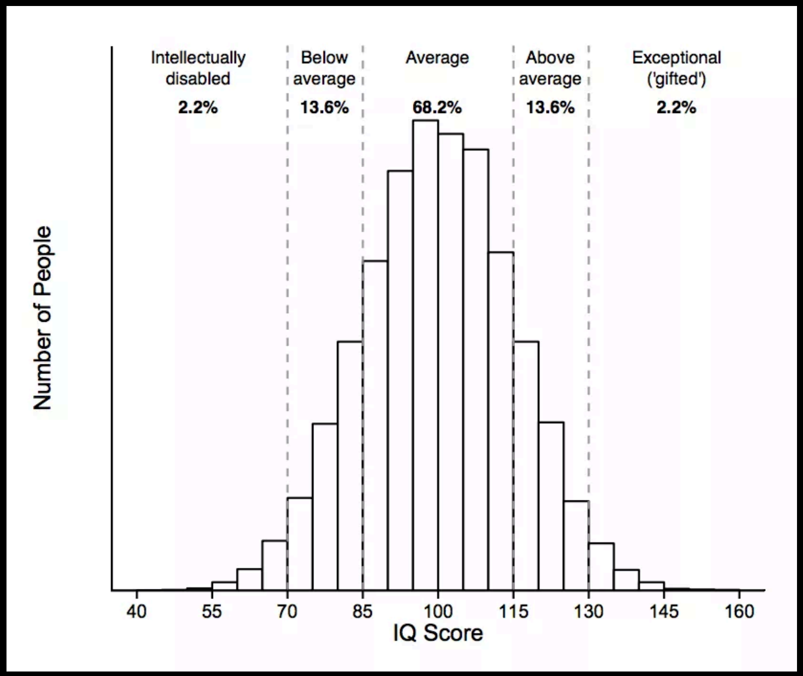

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Step 2: Write down assumptions & conditions about the underlying population distribution

1. # of samples > 30
2. no evidence of strong skew
3. assume independent samples

→ use normal distribution, test statistic will be a Z-score

Practice #1

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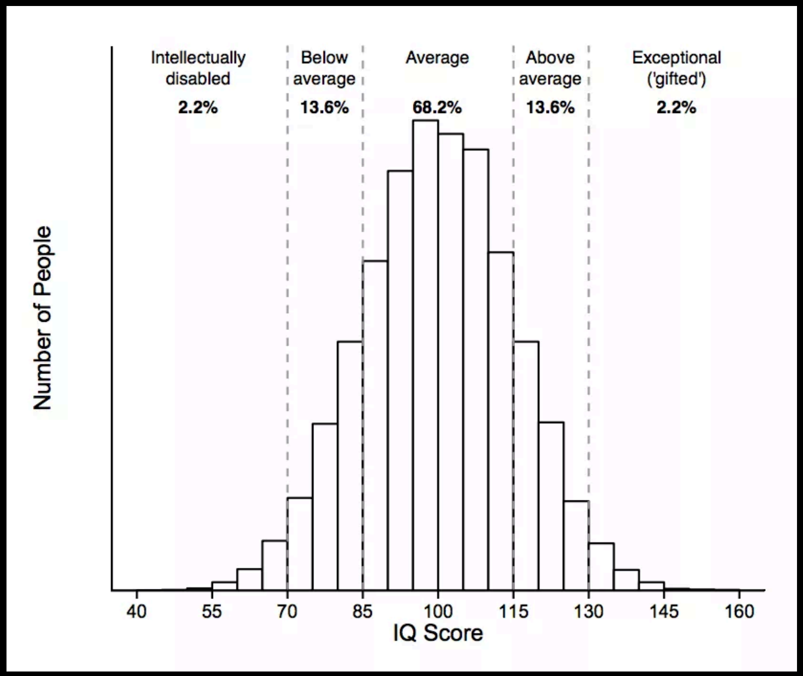

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Step 3.1: Calculate test statistic

$$Z = (\text{point estimate - null value})/SE$$

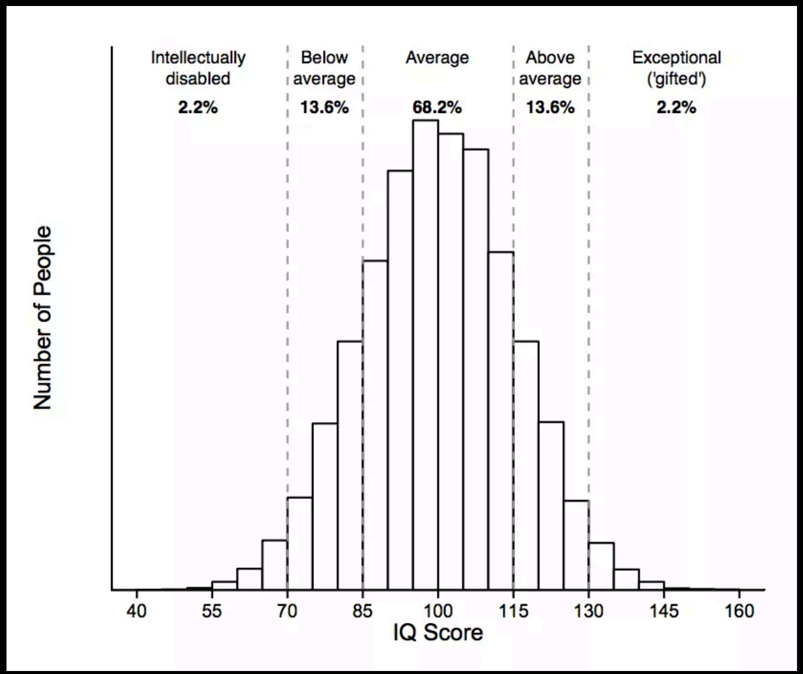
$$SE = SD/n^{1/2}$$

1. point estimate = sample mean = 105
2. null value = population mean = 100
3. SD = 15
4. n = 30

$$Z = (105 - 100)/(\ 15/30^{1/2}\ ) = 1.83$$

Practice #1

A principle at a certain school claims that the students in his school are above average intelligence. A random sample of 30 students' IQ scores have a mean score of 105. Is there sufficient evidence to support the principle's claims (ignoring the inherent biases in IQ tests)? The mean population IQ is 100 with a standard deviation of 15.



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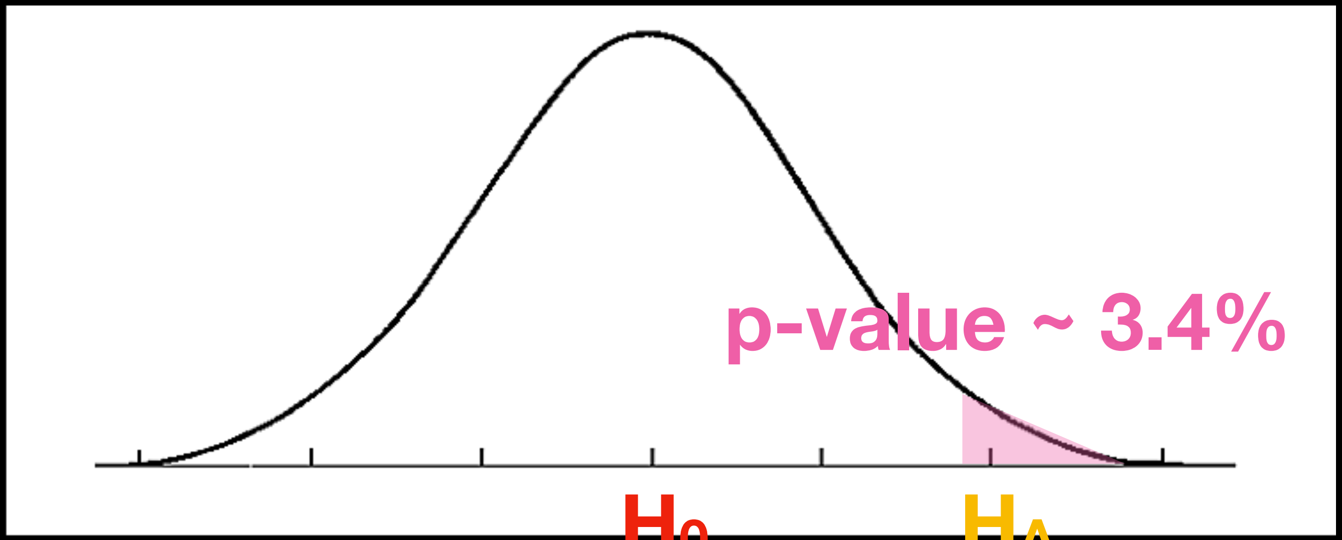
Step 3.1: Calculate test statistic

Z = (point estimate - null value)/SE : $Z = (105 - 100)/(15/30^{1/2}) = 1.83$

Step 3.2: Calculate p-value

p-value = 1-pnorm(1.83)
OR
p-value =
1-pnorm(105, mean=100, sd=15/30**0.5)



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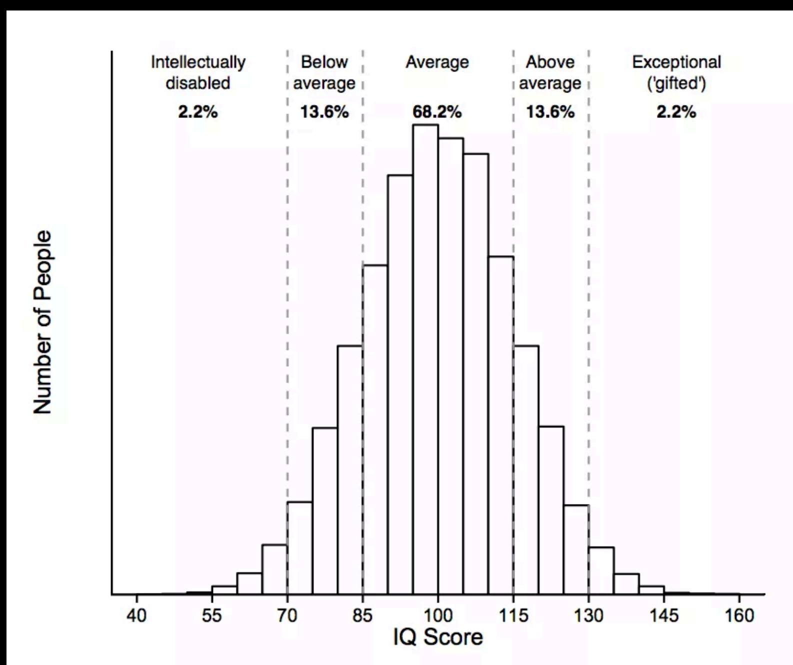

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Step 3.1: Calculate test statistic

Z = (point estimate - null value)/SE : $Z = (105 - 100)/(15/30^{1/2}) = 1.83$

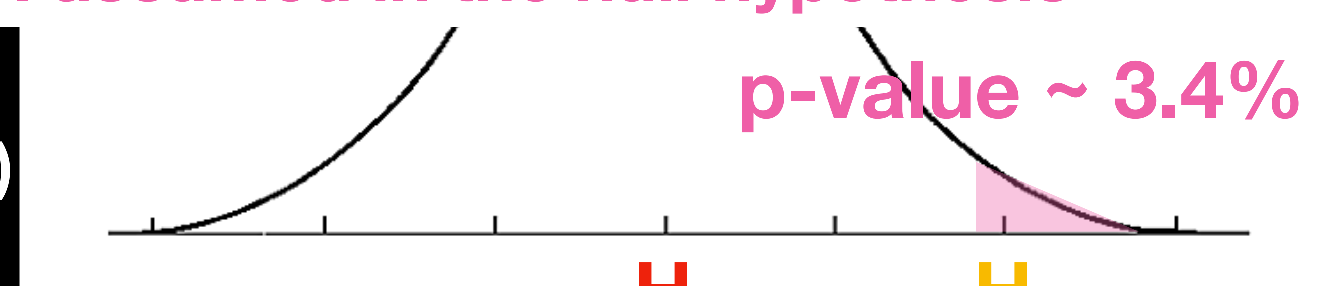
Step 3.2: Calculate p-valu

"how likely is it that the measurement I have is by chance given that the underlying distribution follows what I assumed in the null hypothesis"

p-value = 1-pnorm(1.83)

OR

p-value =
1-pnorm(105, mean=100, sd=15/30**0.5)



H_0 H_A

test statistic at 1.83

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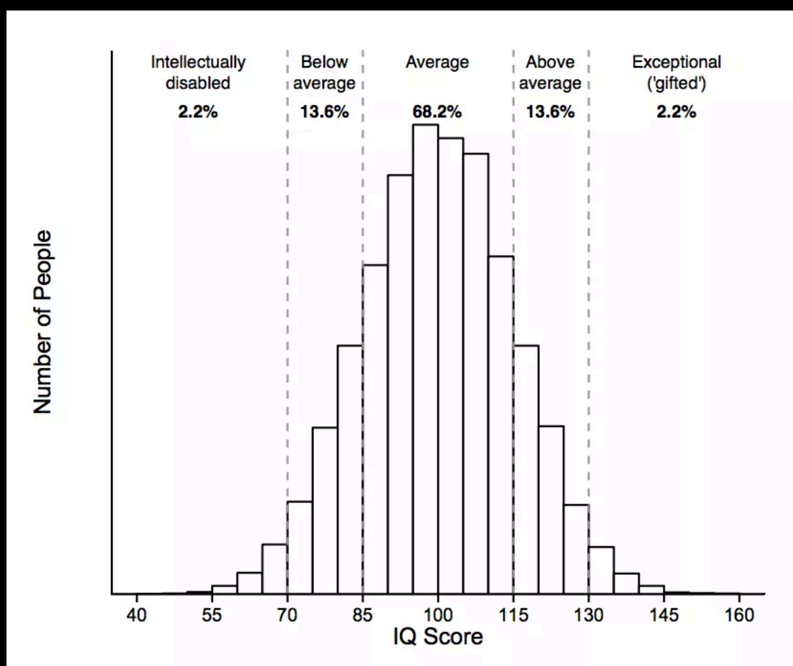

Step 1: Write down Null & Alternative Hypotheses

H_0: $\mu_{students} \leq 100$

H_A: $\mu_{students} > 100$

NOTE: one sided (right handed) test!

Step 2: Write down assumptions & conditions about the underlying population distribution

use normal distribution, test statistic will be a Z-score

Step 3.1: Calculate test statistic

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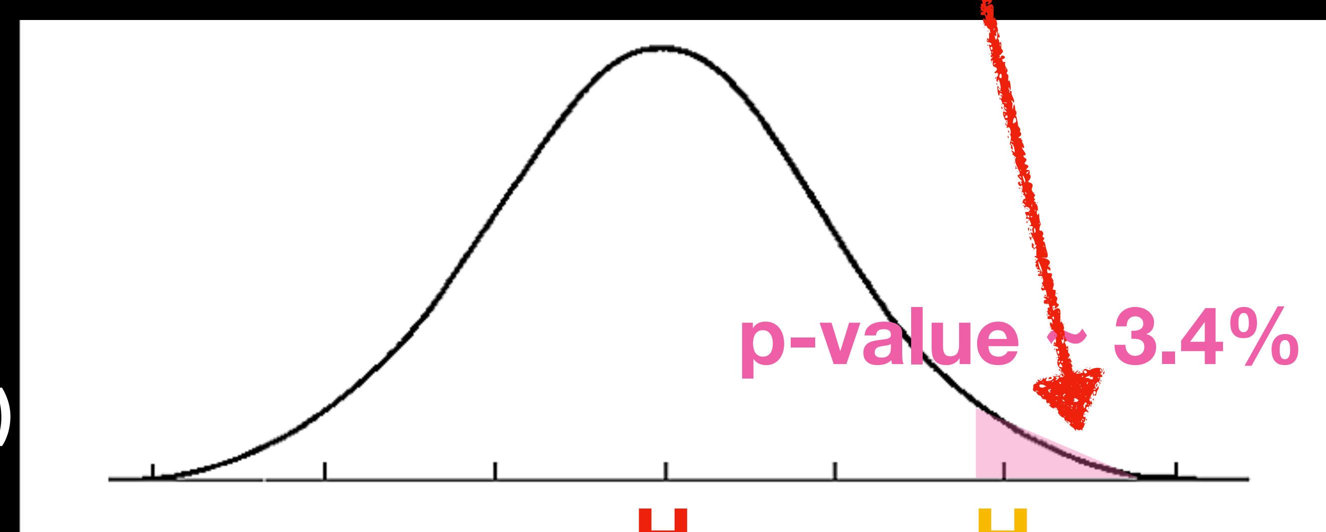

H_0

H_A

test statistic at 1.83

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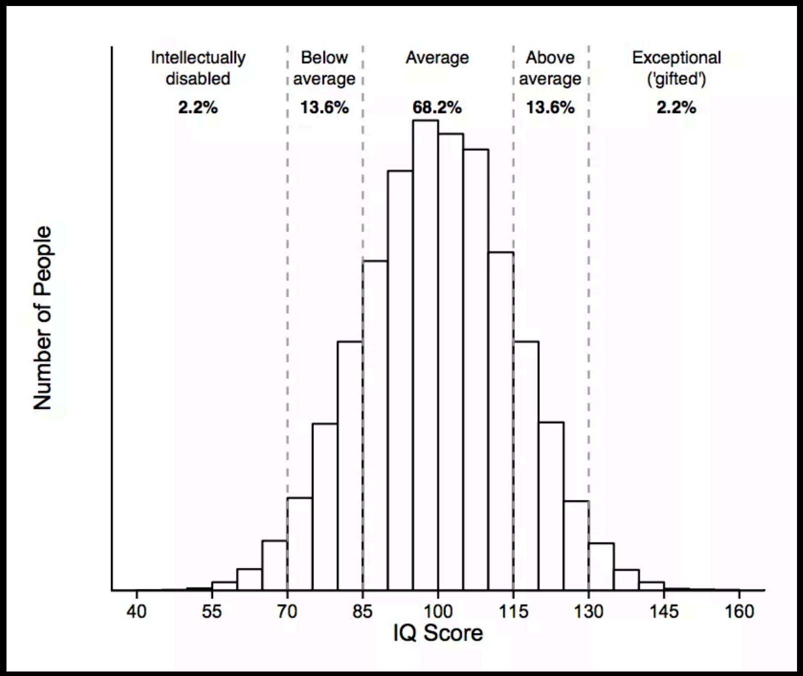

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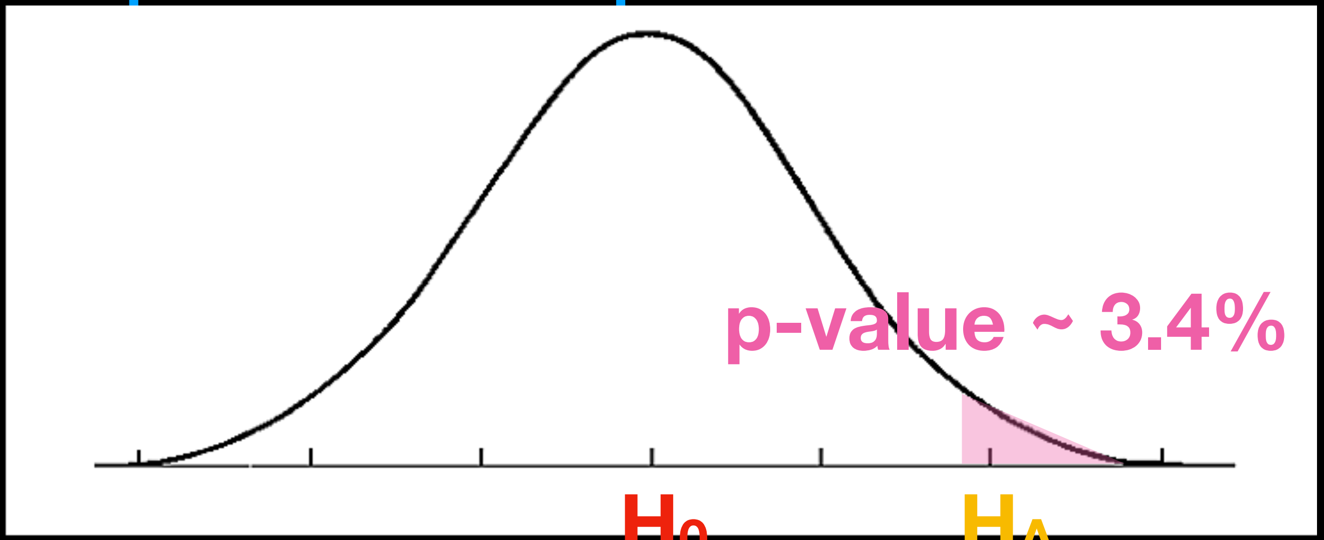


Step 4: Compare p-value to level of significance & conclude

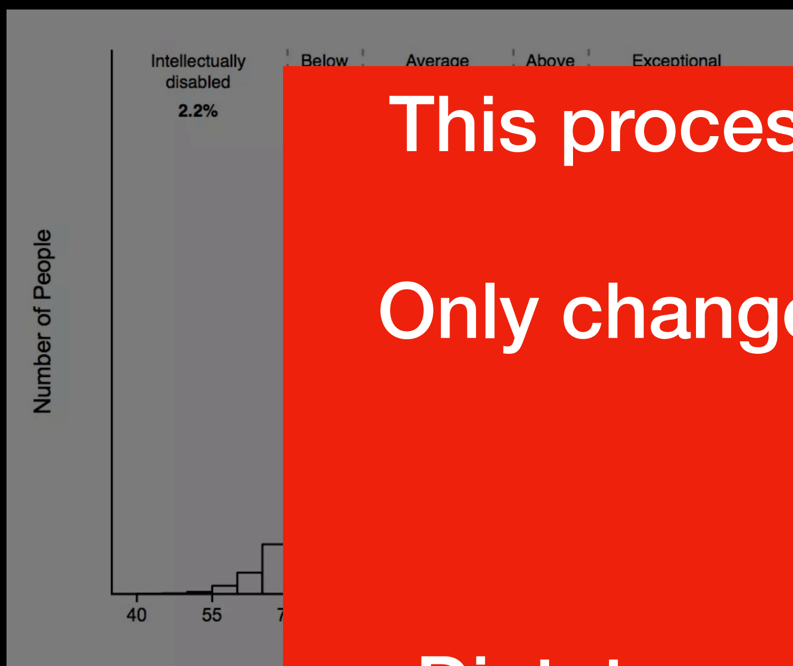
$\alpha = 0.05$ (typical, unless otherwise specified)

$0.034 < 0.05$

so we say we reject the null hypothesis, and there is evidence that the students in the school have above average intelligence

Practice #1

A principle at a certain school claims that the students in his school are above average intelligence. A random sample of 30 students' IQ scores have a mean score of 105. Is there sufficient evidence to support the principle's claims (ignoring the inherent biases in IQ tests)? The mean population IQ is 100 with a standard deviation of 15.



Step 1: Write down Null & Alternative Hypotheses

This processes is essentially all we do in Ch. 5, 6 & 7

Only changes are what distribution we use (in Step 2)

Dictates what test statistic we calculate in Step 3.1
&
What function we use in R to calculate p-value in Step 3.2

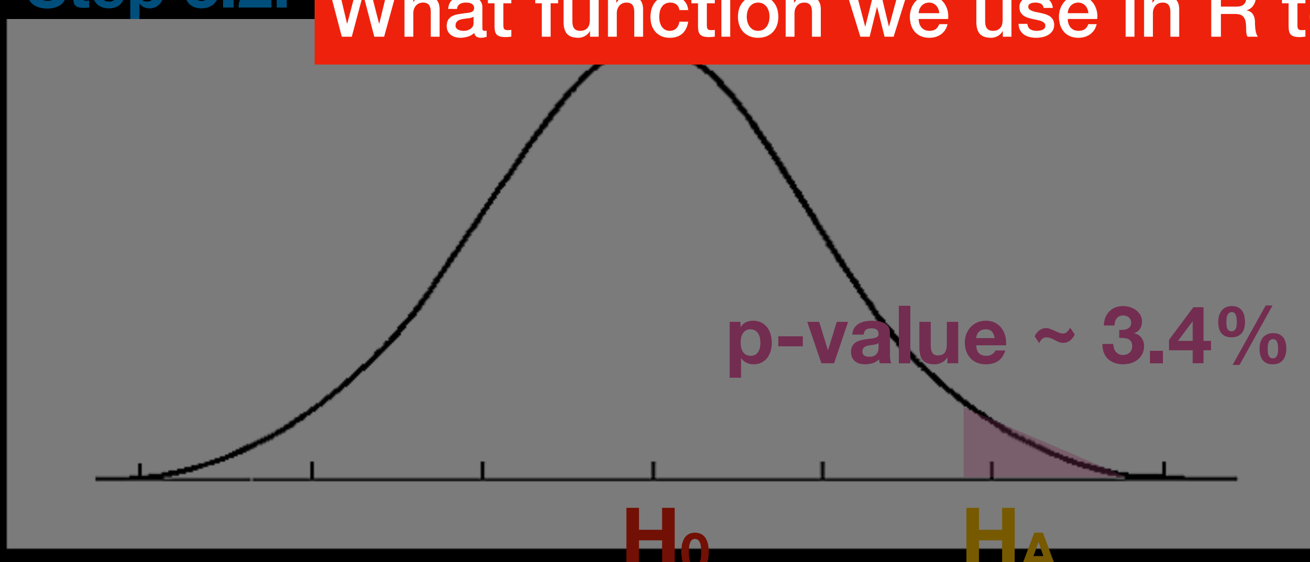
out the

score

) = 1.83

Step 3.2:

vel of



α = 0.05 (typical, unless otherwise specified)

0.034 < 0.05
so we say we reject the null hypothesis, and there is evidence that the students in the school have above average intelligence

Practice #2

The administrator at your local hospital states that on weekends the average wait time for emergency room visits is at most 10 minutes. Based on discussions you have had with friends who have complained on how long they waited to be seen in the ER over a weekend, you dispute the administrator's claim. You decide to test you hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 11 minutes with a standard deviation of 3 minutes. Do you have enough evidence to support your hypothesis that the average ER wait time exceeds 10 minutes? You opt to conduct the test at a 5% level of significance.

Step 1: Write down Null & Alternative Hypotheses

Step 2: Write down assumptions & conditions about underlying population distribution

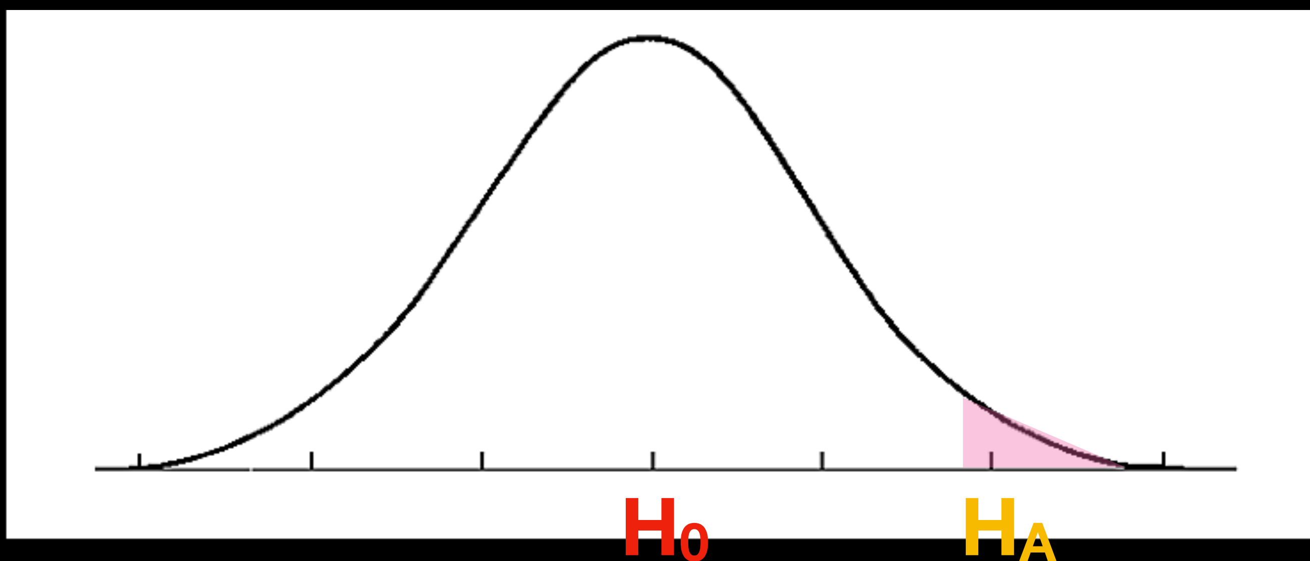
Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

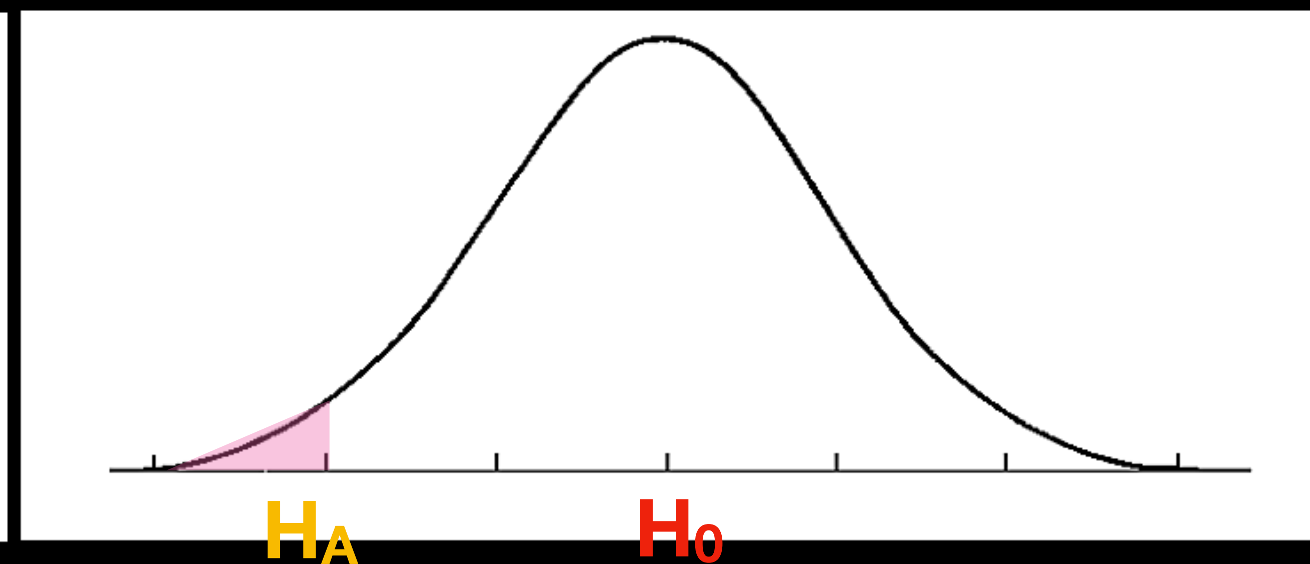
Practice #2 & #2.5

H_0: $\mu_{wait} \leq 10$ min
H_A: $\mu_{wait} > 10$ min



"right tailed test"

H_0: $\mu_{wait} \geq 10$ min
H_A: $\mu_{wait} < 10$ min



"left tailed test"

Practice #2.75

What changes?

The administrator at your local hospital states that on weekends the average wait time for emergency room is <u>exactly</u> 10 minutes. Based on discussions you have had with friends who have mentioned it often does not take 10 minutes to be seen in the ER over a weekend, you dispute the administrator's claim. You decide to test you hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 9 minutes with a standard deviation of 3 minutes. Do you have enough evidence to support your hypothesis that the average ER wait time is <u>not</u> 10 minutes? You opt to conduct the test at a 5% level of significance.

Step 1: Write down Null & Alternative Hypotheses

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

Practice #2.75

What changes?

The administrator at your local hospital states that on weekends the average wait time for emergency room is exactly 10 minutes. Based on discussions you have had with friends who have mentioned it often does not take 10 minutes to be seen in the ER over a weekend, you dispute the administrator's claim. You decide to test you hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 9 minutes with a standard deviation of 3 minutes. Do you have enough evidence to support your hypothesis that the average ER wait time is not 10 minutes? You opt to conduct the test at a 5% level of significance.

Step 1: Write down Null & Alternative Hypotheses

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Think on it for a moment!

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

Practice #2.75

What changes?

The administrator at your local hospital states that on weekends the average wait time for emergency room is <u>exactly</u> 10 minutes. Based on discussions you have had with friends who have mentioned it often does not take 10 minutes to be seen in the ER over a weekend, you dispute the administrator's claim. You decide to test you hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 9 minutes with a standard deviation of 3 minutes. Do you have enough evidence to support your hypothesis that the average ER wait time is <u>not</u> 10 minutes? You opt to conduct the test at a 5% level of significance.

Step 1: Write down Null & Alternative Hypotheses

H_0: $\mu_{wait} = 10$ min

H_A: $\mu_{wait} \neq 10$ min

Step 2: Write down assumptions & conditions about underlying population distribution

Assume normality, independence, #samples > 30

Step 3.1: Calculate test statistic

Z = (point estimate - null value)/SE : $Z = (9 - 10)/(3/40^{1/2}) = -2.11$

Step 3.2: Calculate p-value

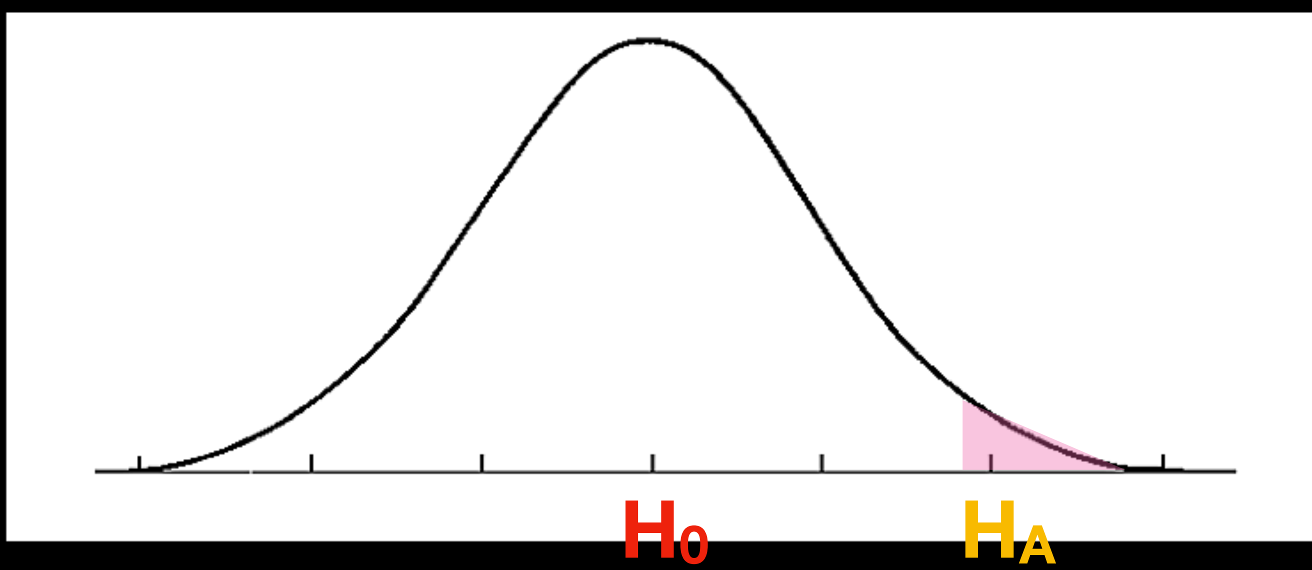
p-value = 2 X pnorm(-2.11) = 2 X 0.018 = 0.035

Step 4: Compare p-value to level of significance & conclude

0.035 < 0.05 so we can reject the null hypothesis and say that it is likely the wait time is NOT exactly 10 minutes on average

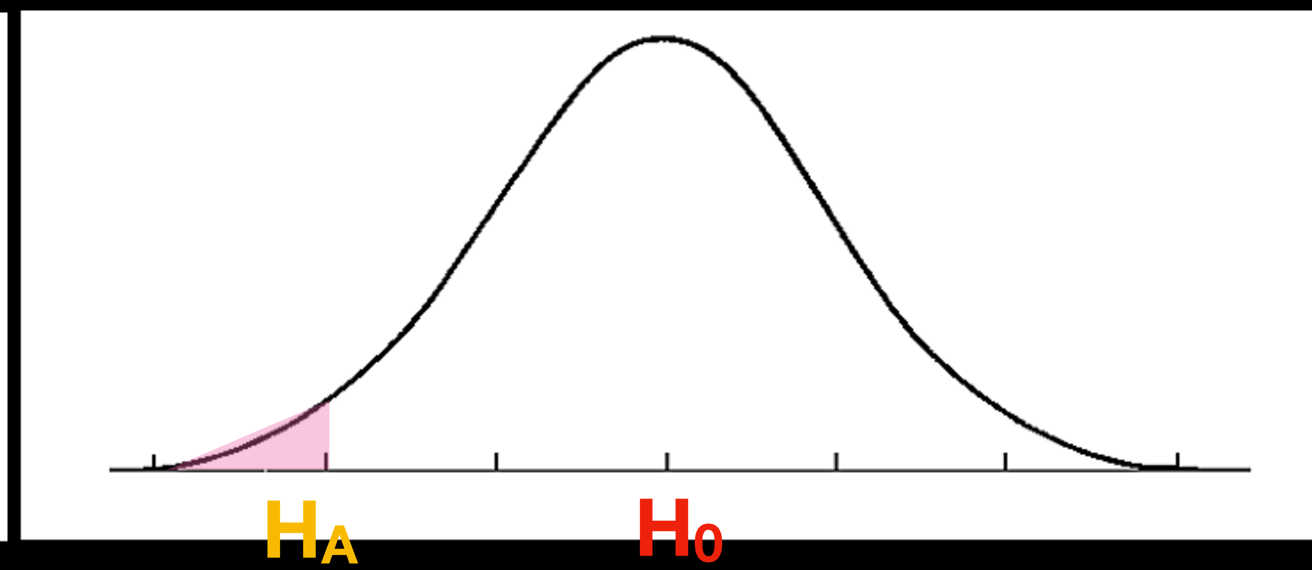
Practice #2 & #2.5 & #2.75

H_0: $\mu_{wait} \leq 10$ min
H_A: $\mu_{wait} > 10$ min



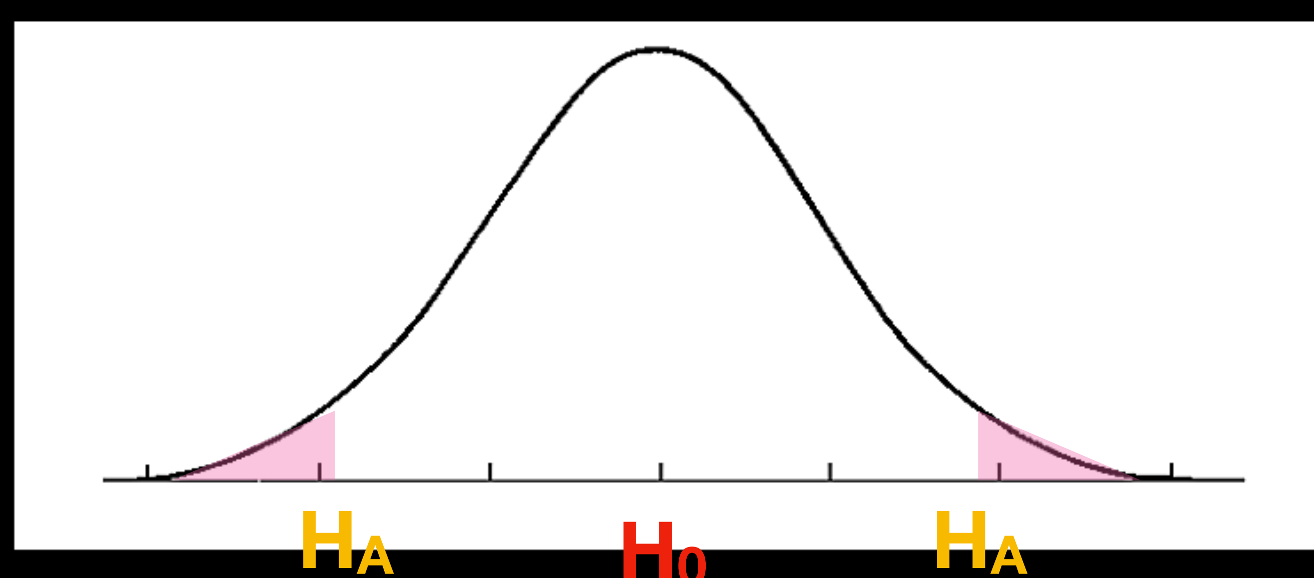
"right tailed test"

H_0: $\mu_{wait} \geq 10$ min
H_A: $\mu_{wait} < 10$ min



"left tailed test"

H_0: $\mu_{wait} = 10$ min
H_A: $\mu_{wait} \neq 10$ min



"two tailed test"

Practice #3

Is there a difference in serum uric acid levels between populations with and without Down's syndrome? A dataset from individuals without Down's syndrome has a sample mean of x_1 = 4.5 and standard deviation SD_1 = 1 for a sample of 35 individuals. The dataset from individuals with Down's syndrome has a sample mean of x_2 = 3.5 and standard deviation SD_2 = 1.5 for a sample of 45 individuals.

Step 1: Write down Null & Alternative Hypotheses

What do these look like?

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

Practice #4

An e-commerce research company claims that 60% or more graduate students have bought merchandise on-line. A consumer group is suspicious of the claim and thinks that the proportion is lower than 60%. A random sample of 80 graduate students show that only 22 students have ever done so. Is there enough evidence to show that the true proportion is lower than 60%? Conduct the test at a 0.05 level of significance.

Step 1: Write down Null & Alternative Hypotheses

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

Practice #4

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Step 1: Write down Null & Alternative Hypotheses

Think on it!

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude

Hypothesis Testing Framework (Ch. 4-6)

The general outline of the process:

1. Set the hypotheses.
   For a single mean this will look like:
   H_0: p = null value
   H_A: p < or > or ≠ null value
2. Check assumptions and conditions
3. Calculate a test statistic and a p-value
4. Make a decision, and interpret it in context

- If p-value < α, reject H_0,
  there is sufficient evidence for [H_A]
- If p-value > α, do not reject H_0,
  there is not sufficient for evidence for [H_A]

But things can go wrong!

Decision errors

Hypothesis tests are not flawless.

In the court system innocent people are sometimes wrongly convicted, and the guilty sometimes walk free.
Similarly, we can make a wrong decision in statistical hypothesis tests as well.

The difference is that we have the tools necessary to quantify how often we make errors in statistics.

Decision errors (cont.)

There are two competing hypotheses: the null and the alternative. In a hypothesis test, we make a decision about which might be true, but our choice might be incorrect.

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

A Type 1 Error is rejecting the null hypothesis when H_0 is true.
A Type 2 Error is failing to reject the null hypothesis when H_A is true.

We (almost) never know if H_0 or H_A is true, but we need to consider all possibilities.

Hypothesis Test as a trial

If we again think of a hypothesis test as a criminal trial then it makes sense to frame the verdict in terms of the null and alternative hypotheses:

H_0: Defendant is innocent
H_A: Defendant is guilty

Which type of error is being committed in the following circumstances?

Declaring the defendant innocent when they are actually guilty

Declaring the defendant guilty when they are actually innocent

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

Hypothesis Test as a trial

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H_0: Defendant is innocent
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Which type of error is being committed in the following circumstances?

Declaring the defendant innocent when they are actually guilty

Type 2 error

Declaring the defendant guilty when they are actually innocent

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

Type 1 error rate

As a general rule we reject H_0 when the p-value is less than 0.05, i.e. we use a significance level of 0.05, $\alpha = 0.05$.

This means that, for those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times.

In other words, when using a 5% significance level there is about 5% chance of making a Type 1 error if the null hypothesis is true.

$P(\text{Type 1 error} \mid H_0 \text{ true}) = \alpha$

This is why we prefer small values of α -- increasing α increases the Type 1 error rate.

Choosing a significance level

Choosing a significance level for a test is important in many contexts, and the traditional level is 0.05. However, it is often helpful to adjust the significance level based on the application.

We may select a level that is smaller or larger than 0.05 depending on the consequences of any conclusions reached from the test.

If making a Type 1 Error is dangerous or especially costly, we should choose a small significance level (e.g. 0.01). Under this scenario we want to be very cautious about rejecting the null hypothesis, so we demand very strong evidence favoring H_A before we would reject H_0.

If a Type 2 Error is relatively more dangerous or much more costly than a Type 1 Error, then we should choose a higher significance level (e.g. 0.10). Here we want to be cautious about failing to reject H_0 when the null is actually false.

Practice #5

The average IQ of the adult population is at least 100. A researcher believes the average IQ of adults is lower. A random sample of 5 adults are tested and scored:

69, 79, 89, 99, 109 (SD = 15.81)

Is there enough evidence to suggest the average IQ of adults is lower based on this sample?

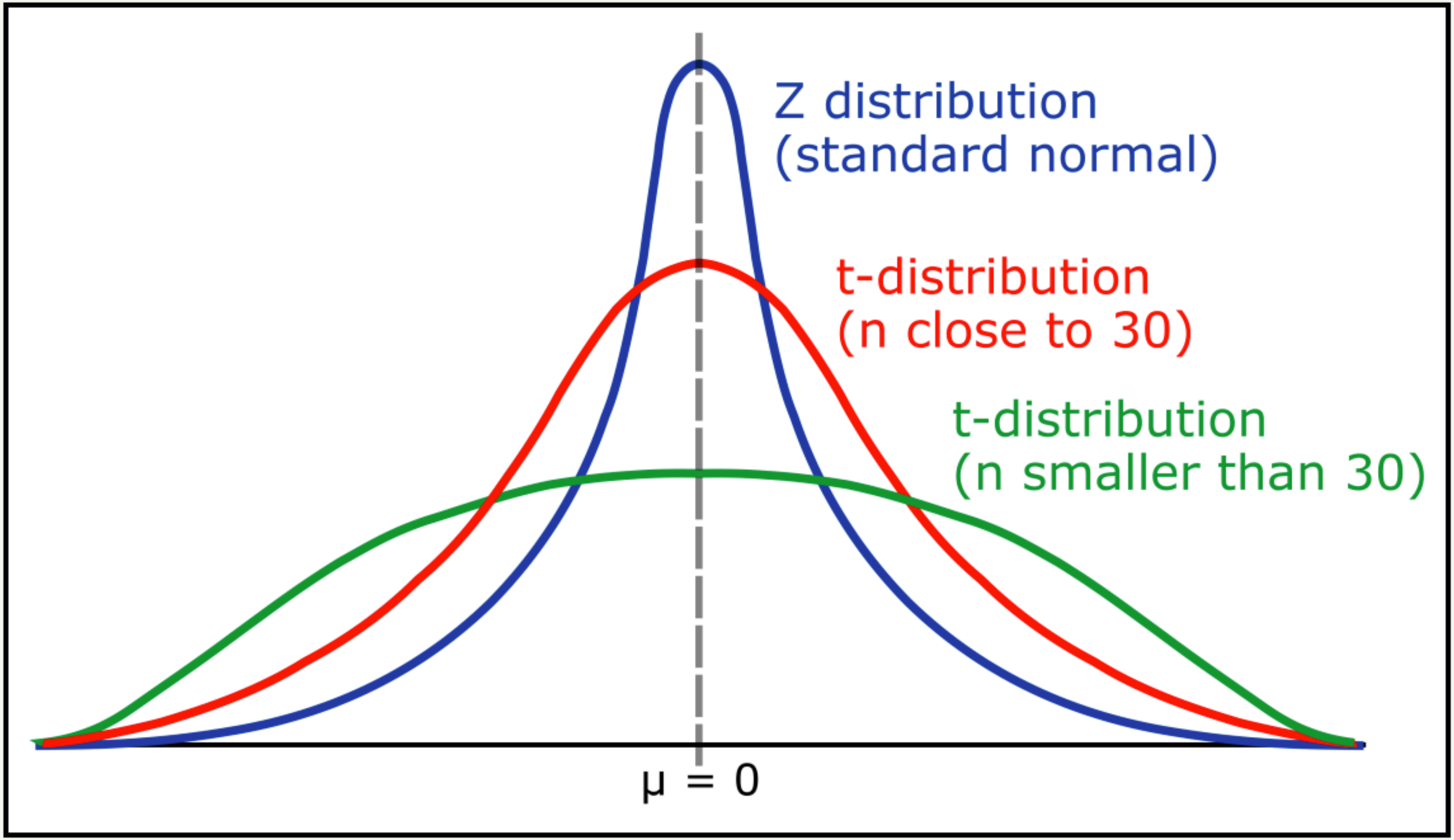
Step 1: Write down Null & Alternative Hypotheses

Step 2: Write down assumptions & conditions about underlying population distribution

Step 3.1: Calculate test statistic

Step 3.2: Calculate p-value

Step 4: Compare p-value to level of significance & conclude


Z distribution
(standard normal)
t-distribution
(n close to 30)
t-distribution
(n smaller than 30)
μ = 0