BFE optimal smoothing

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Choosing the length of the BFE

0.1 Noise source in N-body simulations

1. Noise in the expansion itself.
2. Random distribution of particles in the IC's of the particles. Solution: Bootstrap the computation of the coefficients and it's variance.

0.2 Characterizing noise:

The noise in the BFE due to the discrete nature of the system was quantify in Weinberg 1996. Here we briefly summarize the method. The signal to noise of each coefficient is $S/N = \hat{a}/\sqrt{var(\hat{a})}$

$$var(\hat{a}_{n,l,m}) = m \left[\sum_i^N m_i \Psi^2_{n,l,m}(x_i) - \left(\sum_i^N m_i \Psi_{n,l,m}(x_i) \right)^2 \right] \tag{1}$$

The first term in the right-hand side is the covariance matrix and the second term of Eq.1 are the coefficients $\hat{S}^2_{n,l,m}$ and $\hat{T}^2_{n,l,m}$. m is the particle mass and Ψ_{nlm} is defined as:

$$\Psi_{nlm}(x_i) = (2 - \delta_{m,0})\tilde{A}_{n,l}\Phi_{n,l}(r_i)Y_{l,m}(\theta_i)cos(m\phi_i) \tag{2}$$

The covariance matrix is computed as::

$$\tilde{a}_{nlmn'l'm'} = \hat{a}_{nlm}\hat{a}_{n'l'm'} \tag{3}$$

The smoothening of the coefficients is computed as follows:

$$b_{nlm}(\hat{a}_{nlm}) = \frac{1}{1 + \frac{var(\hat{a}_{nlm})}{\hat{a}^2_{nlm}}} \tag{4}$$

This procedure capture the noise of the particle distribution. However, it does not account of the noise caused the random distribution of particles that are sampling the potential and density. In order to account for this noise, Martin recommended doing $\sqrt{N}$ (N being the number of particles) samples and compute the variance and the coefficients in all of those realizations.

The optimal number of coefficients and it's values is going to be:

$$\hat{a}_{opt} = \frac{1}{\sqrt{N}} \sum_i^{\sqrt{N}} \hat{a}_i \tag{5}$$

$$var(\hat{a}_{opt}) = \frac{1}{\sqrt{N}} \sum_i^{\sqrt{N}} var(\hat{a})_i \tag{6}$$

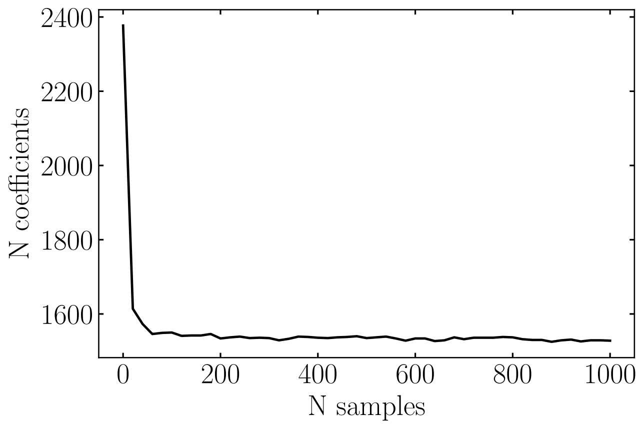

Figure 1: Number of coefficients with S/N>1 as a function of the number of random samples. Convergence is achieve after ∼100 realizations are used. The number of coefficients decrease ∼ 40%.

1 Applying the optimal smoothing of DM halo:

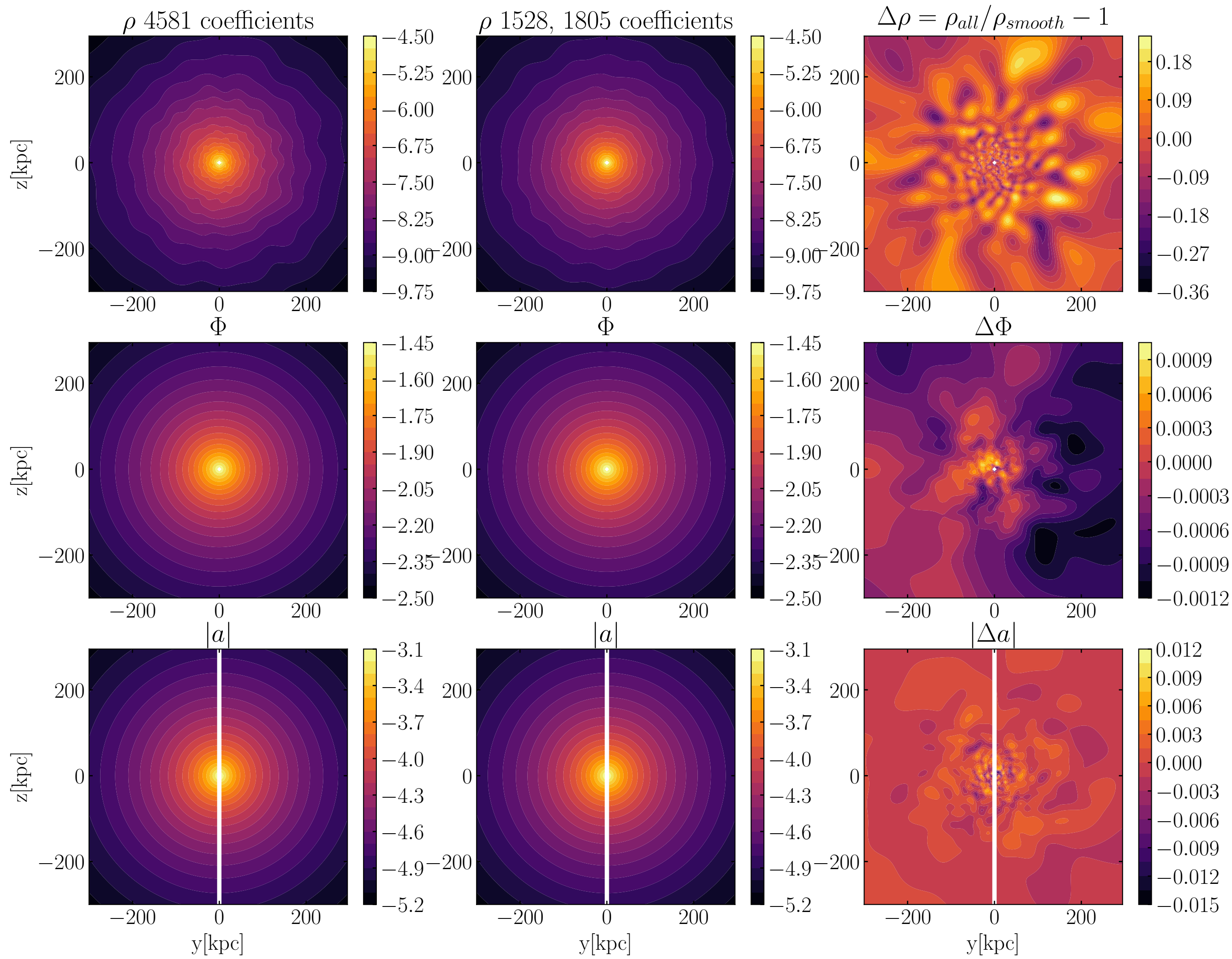

Figure 2: Results of applying the optimal smoothing. Left panels show the results of computing ρ, Φ, and a with all the 4581 coefficients. Middle panels show the results when the optimal smoothing with 1528 and 1805 coefficients that correspond to $S/N > 1$. The optimal smoothing uses 1000 BFE computed in random sampling of the particles. The right panels show the relative difference between the quantities computed using all the coefficients and those with the optimal smoothing. The differences in $\Delta\rho$ are up to 18%, in $\Delta\Phi$ are up to 0.1%, and in Δa up 1.2%.

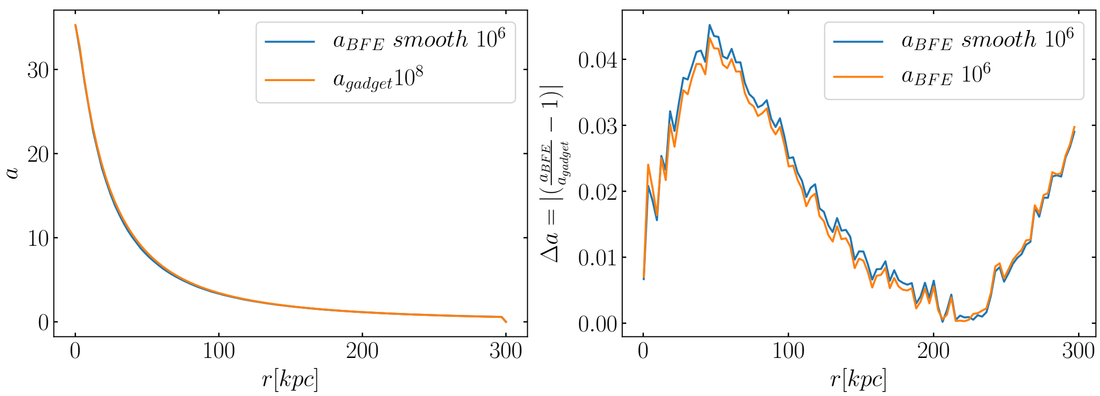

Figure 3: *Right panel:* Accelerations profile computed with optimal smoothing (blue line) and with gadget (orange line). *Left panel:* Δ a between the acceleration computed with gadget and optimal smoothing (blue line), and with no smoothing (orange line). The differences with BFE and gadget are within 4%.

1.1 A BFE for the LMC:

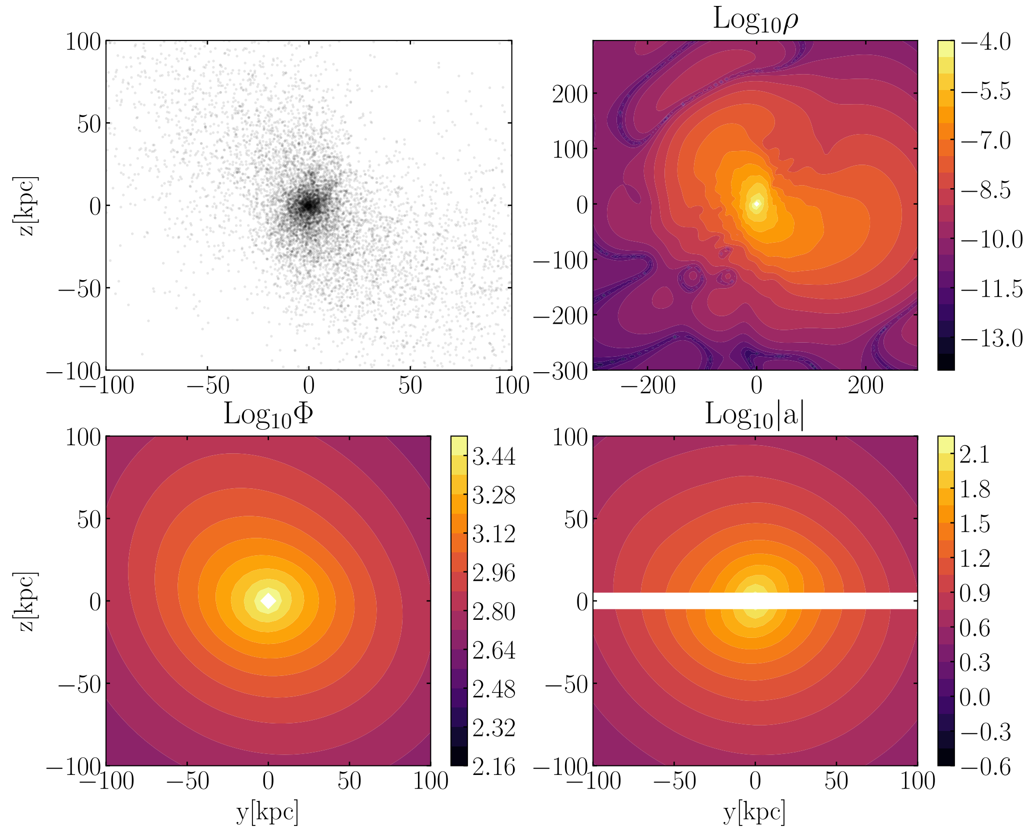

Figure 4: LMC BFE. A BFE with 9472 coefficients ($n_{max} = 40, l_{max}, m_{max} = 20$) was computed for 10^6 random sampled LMC particles from the N-body simulation. The scale length was r_s =10 kpc which corresponds to the scale length of the ICs LMC halo. *Top left:* Distribution of the DM halo particles of the LMC. The density, potential and acceleration computed with the BFE are shown in the Top right, Bottom left, and Bottom right panels respectively.

To choose the length of the BFE for the LMC:

- Compute the optimal coefficients with the optimal smoothing.
- Compare with the total acceleration in the MW's halo including the LMC and cut the expansion based on differences in the acceleration computed with Gadget or with the BFE using all the coefficients.