Problem Set 3

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Q1

```
suppressMessages(library(xlsx))
suppressMessages(library(tseries))
suppressMessages(library(forecast))
data <- read.xlsx("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/mystery data.xls",sheetName="Shee
data <- subset(data,complete.cases(data$ms1))
```

For ms1

```
adf.test(data$ms1)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$ms1
## Dickey-Fuller = -2.4026, Lag order = 5, p-value = 0.4074
## alternative hypothesis: stationary
```

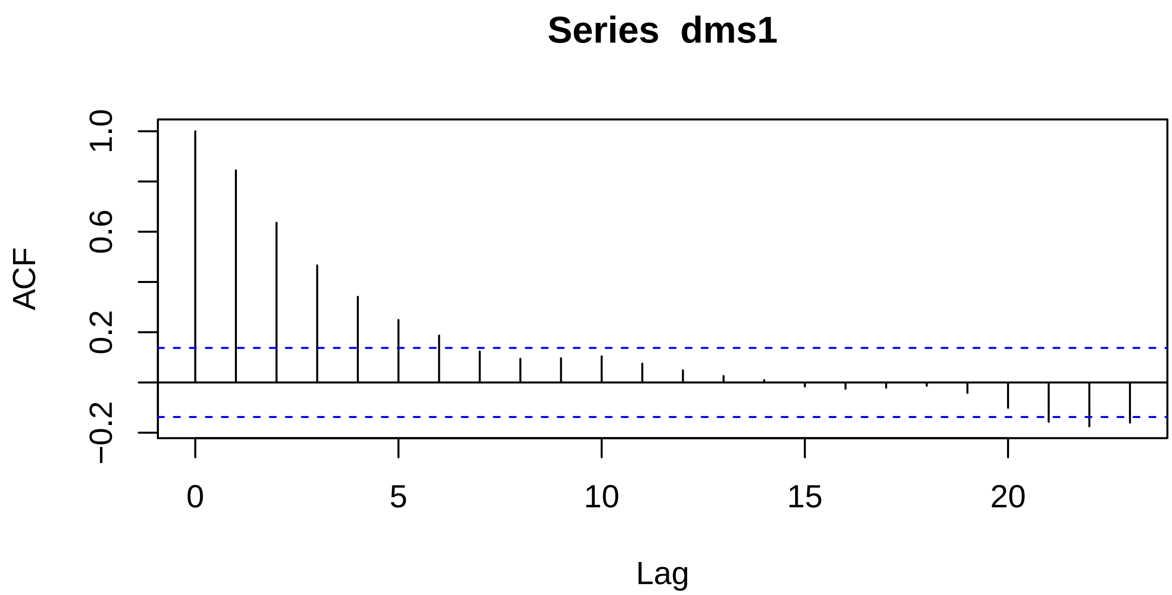
It is non-stationary. We must difference it.

```
dms1 <- diff(data$ms1)
adf.test(dms1)
```

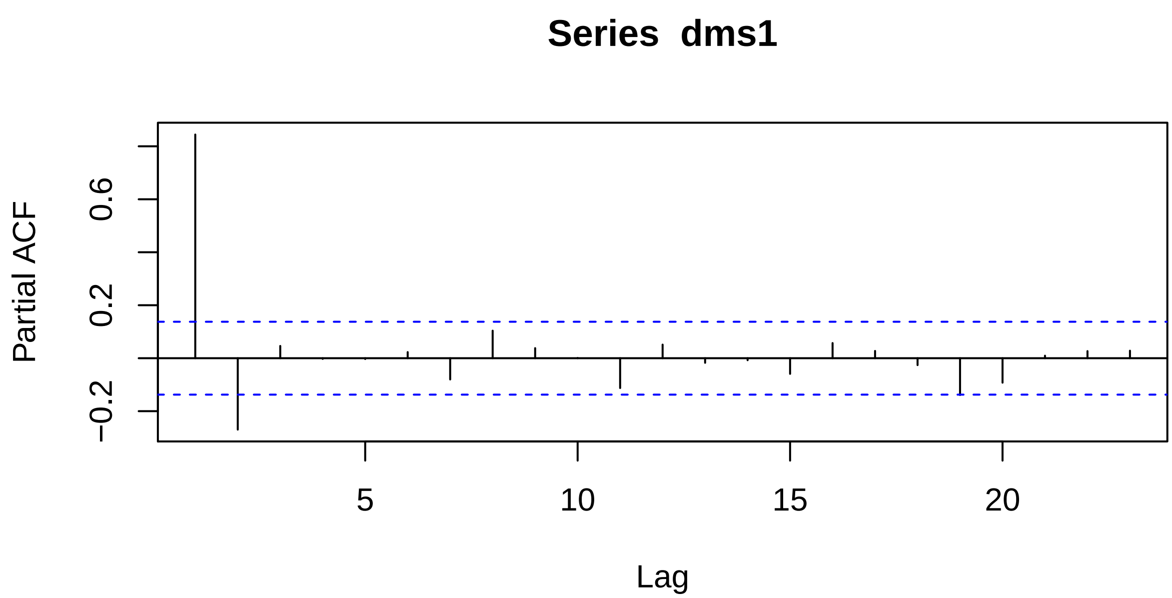
```
##
##  Augmented Dickey-Fuller Test
##
## data:  dms1
## Dickey-Fuller = -3.9496, Lag order = 5, p-value = 0.01279
## alternative hypothesis: stationary
```

The difference series is now stationary.

```
acf(dms1)
```




```r
pacf(dms1)
```



From the graph we think it might be AR(1) or ARMA(1,1).

```r
auto.arima(dms1,ic="aic")
```

```
## Series: dms1
## ARIMA(1,0,1) with zero mean
##
## Coefficients:
```

```
##          ar1     ma1
##       0.7758  0.3415
## s.e.  0.0499  0.0703
##
## sigma^2 estimated as 0.1339:  log likelihood=-83.75
## AIC=173.5   AICc=173.62   BIC=183.44
```

```r
Box.test(auto.arima(dms1)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  auto.arima(dms1)$resid
## X-squared = 0.072656, df = 1, p-value = 0.7875
```

It is ARMA(1,1). And the Q-test shows it is valid.

For ms2

```r
adf.test(data$ms2)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$ms2
## Dickey-Fuller = -2.2302, Lag order = 5, p-value = 0.4796
## alternative hypothesis: stationary
```

It is non-stationary. We must difference it.

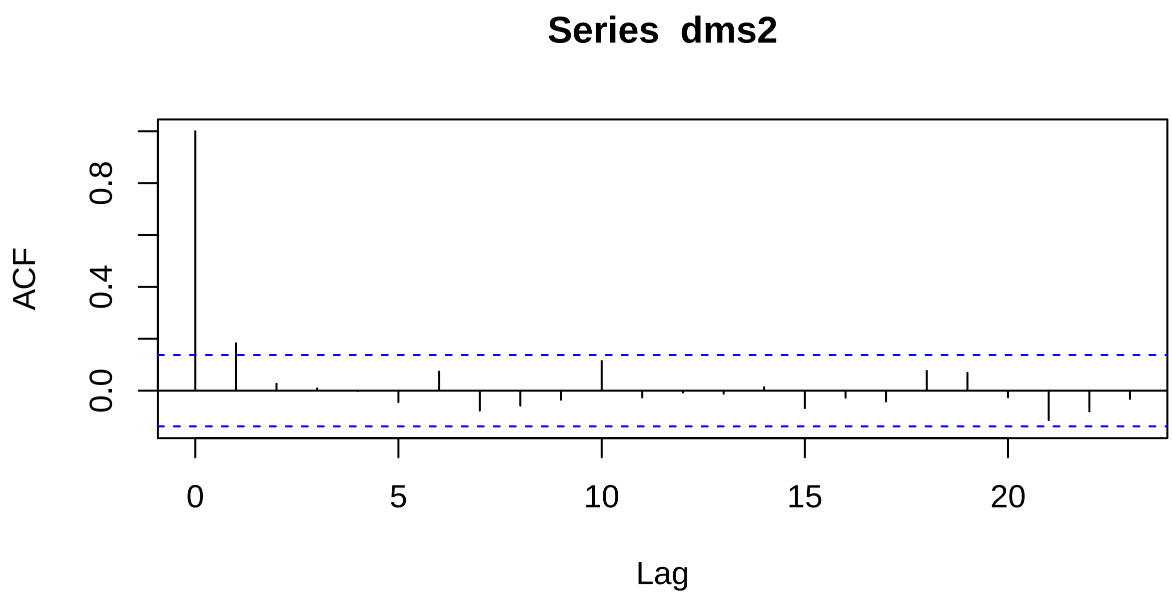
```r
dms2 <- diff(data$ms2)
adf.test(dms2)
```

```
## Warning in adf.test(dms2): p-value smaller than printed p-value
```

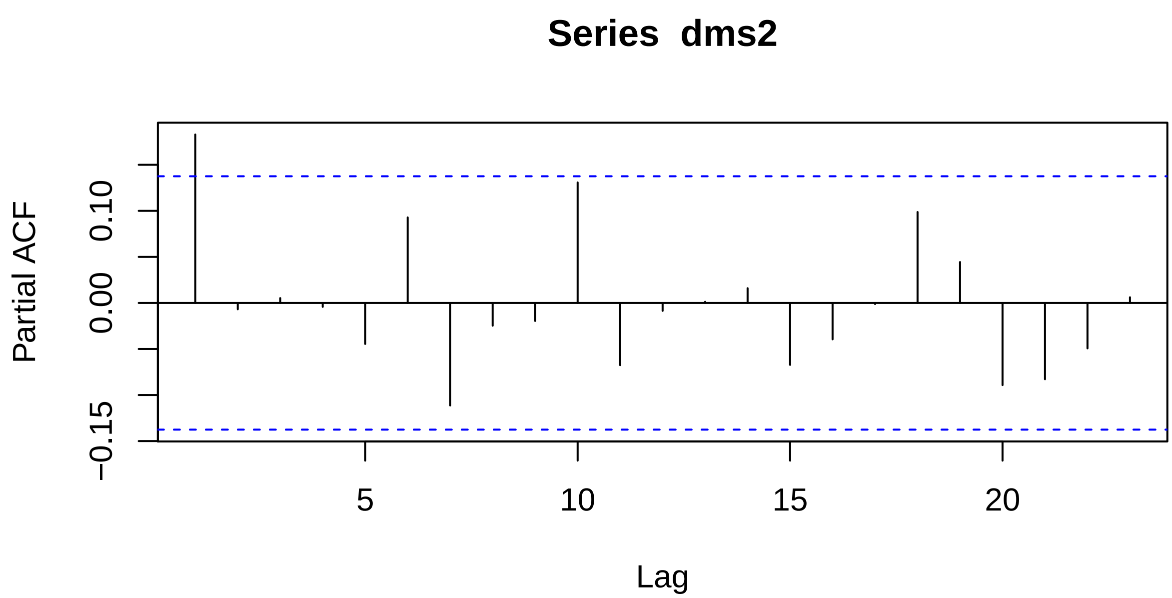
```
##
##  Augmented Dickey-Fuller Test
##
## data:  dms2
## Dickey-Fuller = -4.9243, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary
```

The difference series is now stationary.

```r
acf(dms2)
```




```r
pacf(dms2)
```



From the graph we think it might be AR(1), MA(1) or ARMA(1,1).

```r
auto.arima(dms2,ic="aic",trace = TRUE,allowmean = FALSE)
```

```
##
##  ARIMA(2,0,2) with zero mean     : 173.1417
##  ARIMA(0,0,0) with zero mean     : 174.5879
##  ARIMA(1,0,0) with zero mean     : 169.884
```

```
##  ARIMA(0,0,1) with zero mean     : 169.7023
##  ARIMA(1,0,1) with zero mean     : 171.6919
##  ARIMA(0,0,2) with zero mean     : 171.5648
##  ARIMA(1,0,2) with zero mean     : 171.8239
##
##  Best model: ARIMA(0,0,1) with zero mean
```

```
## Series: dms2
## ARIMA(0,0,1) with zero mean
##
## Coefficients:
##          ma1
##       0.1815
## s.e.  0.0675
##
## sigma^2 estimated as 0.1331:  log likelihood=-82.84
## AIC=169.69   AICc=169.75   BIC=176.32
```

```r
arima(dms2,c(0,0,1),include.mean = FALSE)$aic
```

```
## [1] 169.6897
```

```r
arima(dms2,c(1,0,0),include.mean = FALSE)$aic
```

```
## [1] 169.5314
```

```r
arima(dms2,c(1,0,0),include.mean = FALSE)
```

```
##
## Call:
## arima(x = dms2, order = c(1, 0, 0), include.mean = FALSE)
##
## Coefficients:
##          ar1
##       0.1858
## s.e.  0.0695
##
## sigma^2 estimated as 0.1323:  log likelihood = -82.77,  aic = 169.53
```

```r
Box.test(arima(dms2,c(1,0,0),include.mean = FALSE)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  arima(dms2, c(1, 0, 0), include.mean = FALSE)$resid
## X-squared = 0.00027502, df = 1, p-value = 0.9868
```

There is a really weird bug in R here. When using auto.arima(), it reports that MA(1) is the best model. But when we do arima separately for MA(1) and AR(1), it turns out that AR(1) has a smaller AIC. This might be due to auto.arima() set the number observation to a smaller number to make sure that we use the same nobs to compare the AIC of higher order models. When not using the full sample, the MA(1) has a slightly smaller AIC. We should use the full sample, so the AR(1) is the best model. And the Q-test shows it is valid.

For ms3

```
adf.test(data$ms3)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$ms3
## Dickey-Fuller = -2.2873, Lag order = 5, p-value = 0.4557
## alternative hypothesis: stationary
```

It is non-stationary. We must difference it.

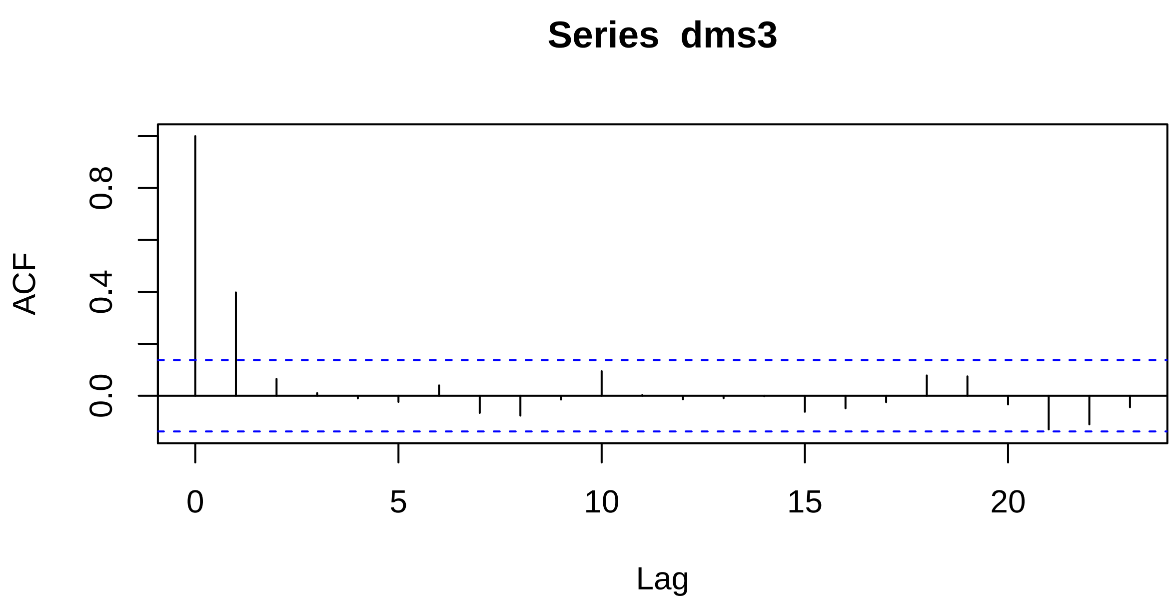
```
dms3 <- diff(data$ms3)
adf.test(dms3)
```

```
## Warning in adf.test(dms3): p-value smaller than printed p-value
```

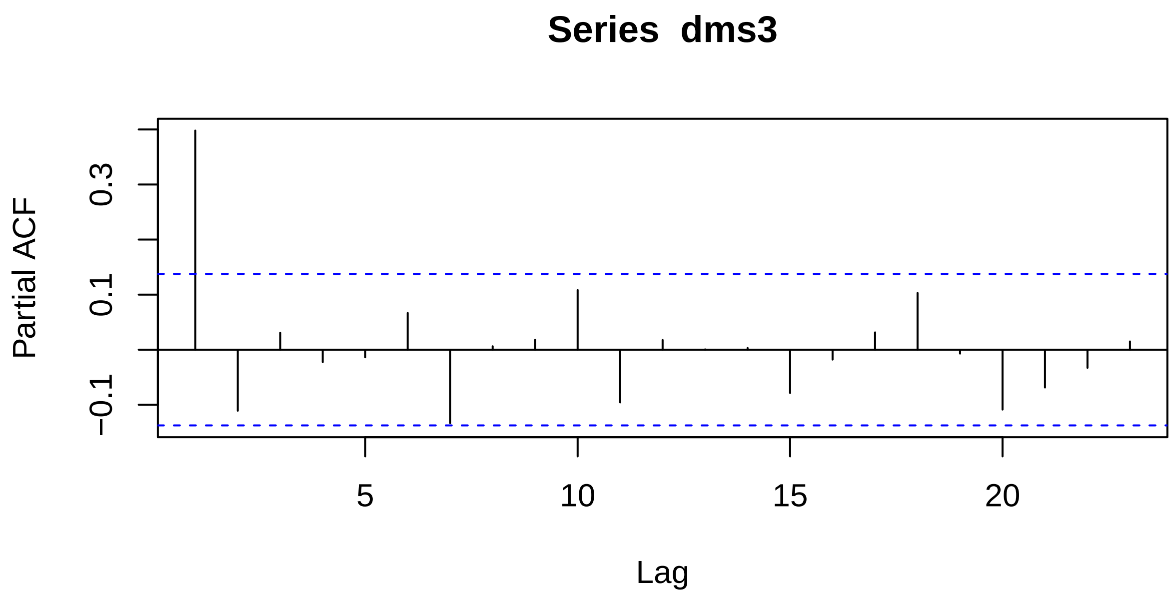
```
##
##  Augmented Dickey-Fuller Test
##
## data:  dms3
## Dickey-Fuller = -4.8258, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary
```

The difference series is now stationary.

```
acf(dms3)
```



```r
pacf(dms3)
```



From the graph we think it might be AR(1), MA(1) or ARMA(1,1).

```r
auto.arima(dms3,ic="aic")
```

```
## Series: dms3
## ARIMA(0,0,1) with zero mean
##
## Coefficients:
##          ma1
##       0.4172
## s.e.  0.0582
##
## sigma^2 estimated as 0.00301:  log likelihood=301.64
## AIC=-599.28   AICc=-599.22   BIC=-592.65
```

```r
Box.test(auto.arima(dms3)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  auto.arima(dms3)$resid
## X-squared = 0.26981, df = 1, p-value = 0.6035
```

It is MA(1). And the Q-test shows it is valid.

For ms4

```r
adf.test(data$ms4)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$ms4
## Dickey-Fuller = -2.3378, Lag order = 5, p-value = 0.4346
## alternative hypothesis: stationary
```

It is non-stationary. We must difference it.

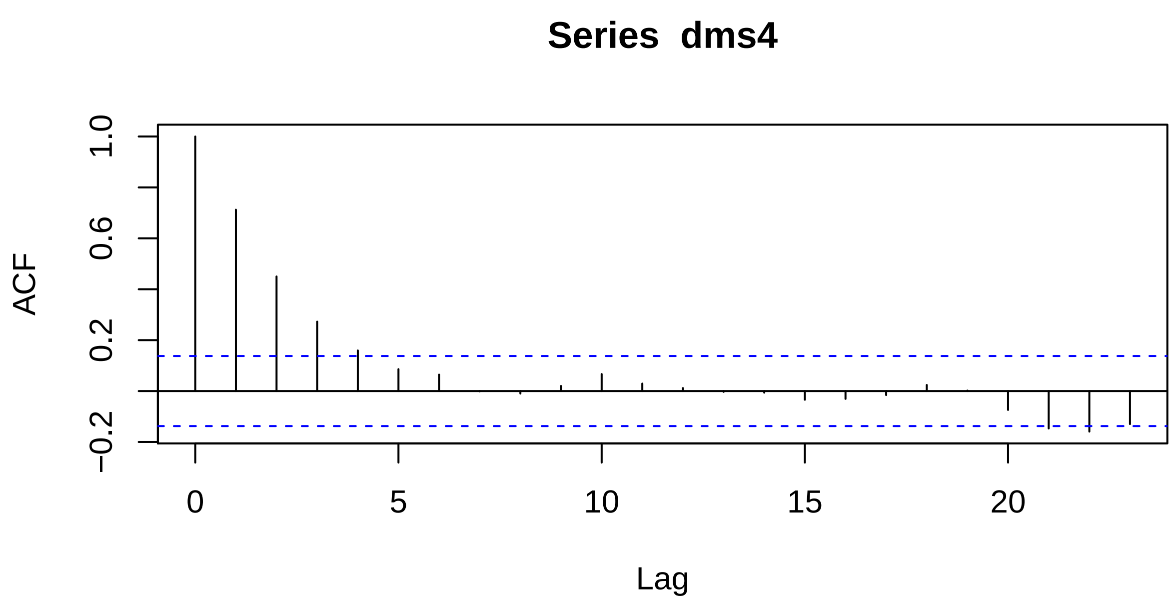
```r
dms4 <- diff(data$ms4)
adf.test(dms4)
```

```
## Warning in adf.test(dms4): p-value smaller than printed p-value
```

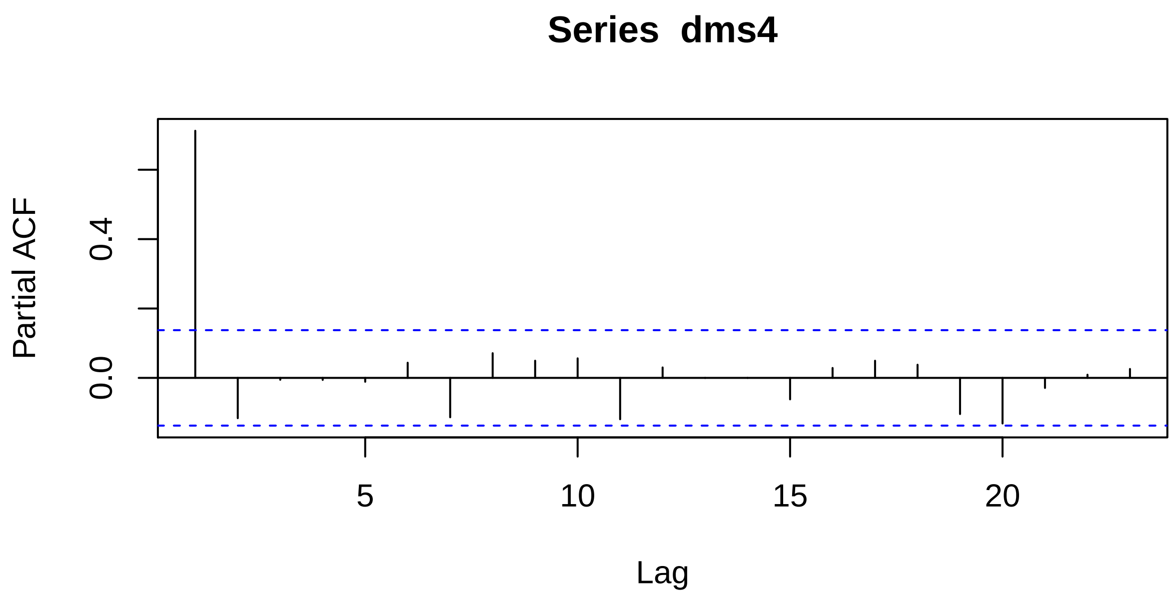
```
##
##  Augmented Dickey-Fuller Test
##
## data:  dms4
## Dickey-Fuller = -4.2876, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary
```

The difference series is now stationary.

```r
acf(dms4)
```



```r
pacf(dms4)
```



From the graph we think it might be AR(1) or AR(2).

```r
auto.arima(dms4,ic="aic")
```

```
## Series: dms4
## ARIMA(2,0,0) with zero mean
##
## Coefficients:
##          ar1      ar2
##       0.8158  -0.1168
## s.e.  0.0699   0.0709
##
## sigma^2 estimated as 0.1339:  log likelihood=-83.32
## AIC=172.65   AICc=172.77   BIC=182.59
```

```r
Box.test(auto.arima(dms4)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  auto.arima(dms4)$resid
## X-squared = 0.0012847, df = 1, p-value = 0.9714
```

It is AR(2). And the Q-test shows it is valid.

1. For a difference stationary series $\{y_t\}$, we can write

$$\phi^*(L)\Delta y_t = \theta(L)\varepsilon_t \quad , \text{and } \phi^*(L) \text{ is invertible}$$

$$\Delta y_t = \phi^*(L)^{-1}\theta(L)\varepsilon_t$$

$$= \psi^*(L)\varepsilon_t$$

Where $\psi^*(L) = \sum_{k=0}^{\infty} \psi_k^* L^k$ with $\psi_0^* = 1$ and $\psi^*(1) \neq 0$

We can write : $y_{t+s} = y_{t-1} + \Delta y_t + \Delta y_{t+1} + \cdots \Delta y_{t+s}$

Impulse response : $\frac{\partial y_{t+s}}{\partial \varepsilon_t} = 0 + \frac{\partial \Delta y_t}{\varepsilon_t} + \frac{\partial \Delta y_{t+1}}{\varepsilon_t} + \cdots + \frac{\partial \Delta y_{t+s}}{\varepsilon_t}$

$$= 1 + \psi_1^* + \psi_2^* + \cdots \psi_s^*$$

Short run impact: $\frac{\partial y_t}{\partial \varepsilon_t} = 1$

long run impact : $\lim_{s\to\infty} \frac{\partial y_{t+s}}{\partial \varepsilon_t} = \sum_{j=1}^{\infty} \psi_j^* = \psi^*(1)$

For ms1, $(1-0.7758L)\Delta y_t = (1+0.3415L)\varepsilon_t$

$$\psi^*(L) = \frac{1+0.3415L}{1-0.7758}$$

$$\psi^*(1) = \frac{1+0.3415}{1-0.7758} = 5.9835$$

So short run impact $=1$, long run impact $= 5.9835$

For ms2. $(1-0.1858L)\Delta y_t = \varepsilon_t$

$$\psi^*(L) = \frac{1}{1-0.1858L}$$

$$\psi^*(1) = \frac{1}{1-0.1858} = 1.2282$$

So short run impact $=1$, long run impact $=1.2282$

For ms3. $\Delta y_t = (1+0.4172L)\varepsilon_t$

$$\psi^*(L) = 1+0.4172L$$

$$\psi^*(1) = 1.4172$$

So short run impact $=1$, long run impact $=1.4172$

For ms 4, $(1-0.8158L+0.1168L^2)\Delta y_t = \varepsilon_t$

$$\psi^*(L) = \frac{1}{1-0.8158L+0.1168L^2}$$

$$\psi^*(1) = 3.3223$$

So short run impact $= 1$, long run impact $= 3.3223$

Q2

```r
suppressMessages(library(lmtest))
suppressMessages(library(orcutt))
suppressMessages(library(car))
```

For ms5

Estimate these two models using entire sample

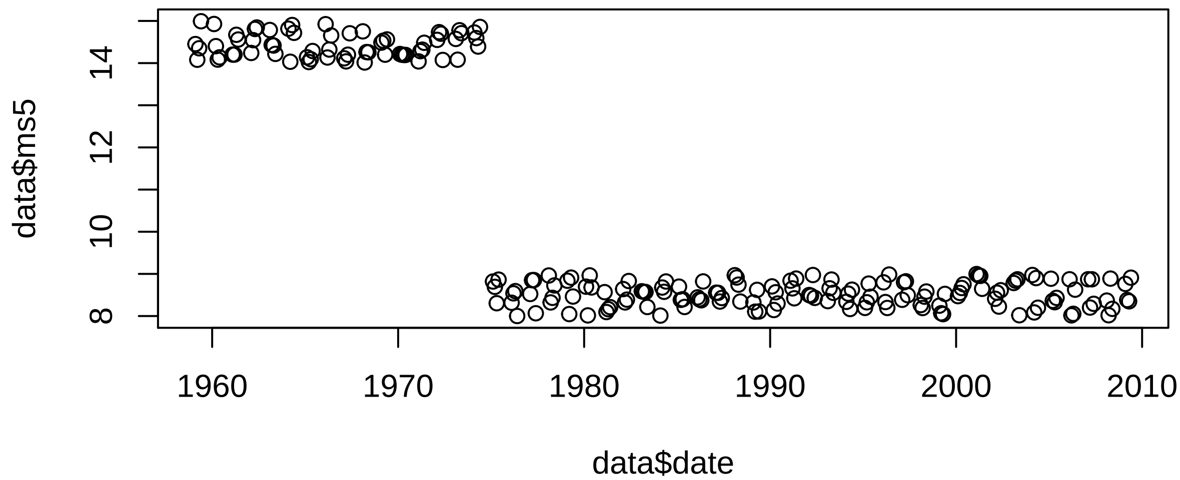
```r
arima(data$ms5)
```

```
##
## Call:
## arima(x = data$ms5)
##
## Coefficients:
##       intercept
##         10.3713
## s.e.     0.1929
##
## sigma^2 estimated as 7.593:  log likelihood = -496.24,  aic = 996.48
```

```r
Box.test(arima(data$ms5)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  arima(data$ms5)$resid
## X-squared = 195.36, df = 1, p-value < 2.2e-16
```

```r
plot(data$date,data$ms5)
```



We can see from the graph that the break point is at about 1974.

Using loops to pinpoint a break point:

```
a <- rep(NA,194)
b <- rep(NA,194)
for(i in 10:194){
  D1 <- c(rep(0,i),rep(1,204-i))
  if(coeftest(arima(data$ms5,xreg = D1))[2,4]<=0.05){
    a[i] <- 1
  }
  if(Box.test(arima(data$ms5,xreg = D1)$resid,type="Ljung-Box")[3]>=0.70){
    b[i] <- 1
  }
}
intersect(which(a==1),which(b==1))
```

```
## [1] 63
```

```
data[intersect(which(a==1),which(b==1)),1]
```

```
## [1] 1974.3
```

The break point is at 1974.3.

```
D1 <- c(rep(0,63),rep(1,204-63))
coeftest(arima(data$ms5,xreg = D1))
```

```
##
## z test of coefficients:
##
```

```
##            Estimate Std. Error z value  Pr(>|z|)
## intercept 14.418423   0.065993 218.484 < 2.2e-16 ***
## D1        -5.855481   0.079379 -73.766 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The model parameter is unstable.

The appropriately modified model:(D1 is a dummy variable that =0 if before 1974.3 and =1 after)

```r
arima(data$ms5,xreg = D1)
```

```
##
## Call:
## arima(x = data$ms5, xreg = D1)
##
## Coefficients:
##       intercept       D1
##         14.4184  -5.8555
## s.e.     0.0660   0.0794
##
## sigma^2 estimated as 0.2744:  log likelihood = -157.55,  aic = 321.1
```

```r
Box.test(arima(data$ms5,xreg = D1)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  arima(data$ms5, xreg = D1)$resid
## X-squared = 0.094195, df = 1, p-value = 0.7589
```

The Q-test shows it is valid.

For ms6 and ms7

Estimate these two models using entire sample

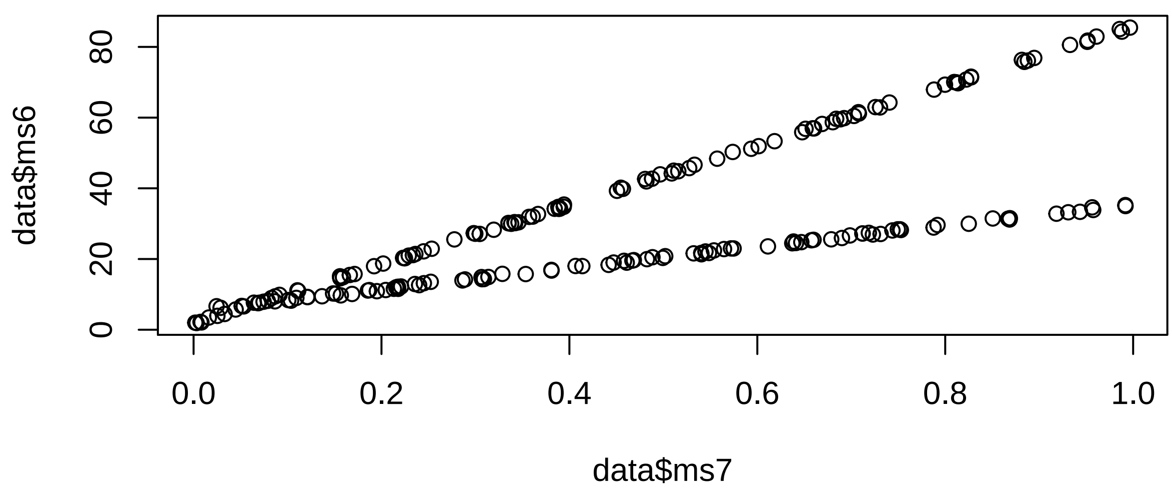
```r
arima(data$ms6,xreg = data$ms7)
```

```
##
## Call:
## arima(x = data$ms6, xreg = data$ms7)
##
## Coefficients:
##       intercept  data$ms7
##          3.0589   58.8719
## s.e.     1.7902    3.3093
##
## sigma^2 estimated as 170:  log likelihood = -785.38,  aic = 1576.76
```

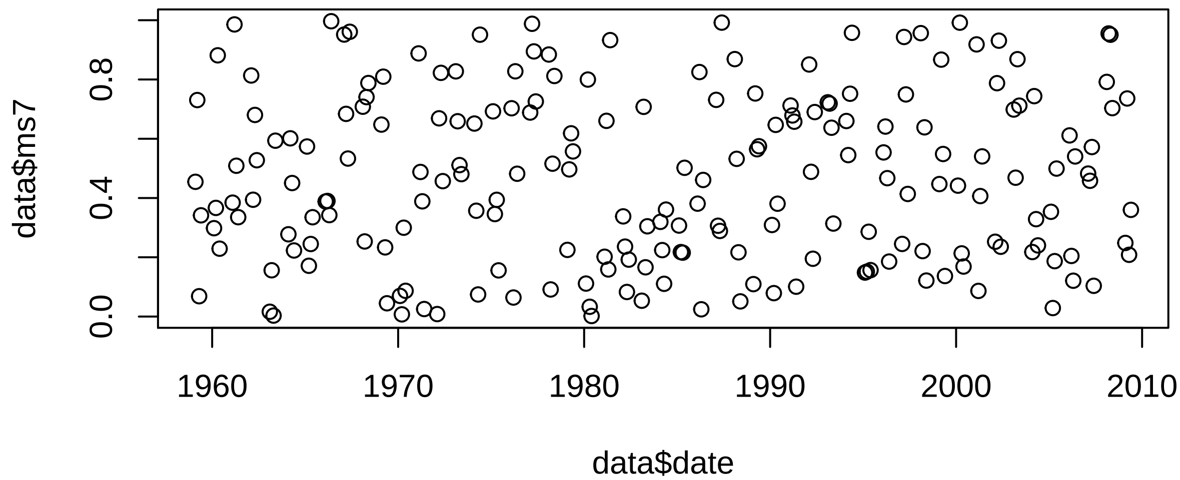
```r
Box.test(arima(data$ms6,xreg = data$ms7)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  arima(data$ms6, xreg = data$ms7)$resid
## X-squared = 90.478, df = 1, p-value < 2.2e-16
```

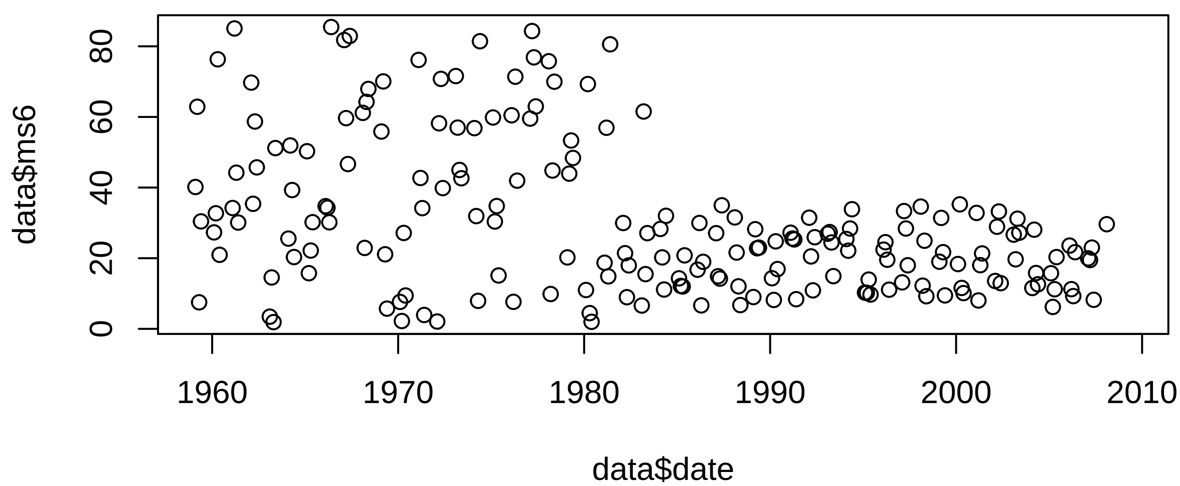
```r
plot(data$ms7,data$ms6)
```




```r
plot(data$date,data$ms7)
```




```
plot(data$date,data$ms6)
```



From the first graph we can see that there is surely two different intercepts and slopes in the relationship of ms7 and ms6. And from the third graph we can see that the break point is at about 1984.

Using loops to pinpoint a break point:

```
a <- rep(NA,194)
b <- rep(NA,194)
for(i in 10:194){
  D2<- c(rep(0,i),rep(1,204-i))
```

```
  if(linearHypothesis(lm(data$ms6 ~ data$ms7+D2*data$ms7),c("D2=0","data$ms7:D2=0"))[2,6]<=0.05){
    a[i] <- 1
  }
  if(Box.test(lm(data$ms6 ~ data$ms7+D2*data$ms7)$resid,type="Ljung-Box")[3]>=0.70){
    b[i] <- 1
  }
}
intersect(which(a==1),which(b==1))
```

```
## [1] 105
```

```
data[intersect(which(a==1),which(b==1)),1]
```

```
## [1] 1985.1
```

The break point is at 1985.1.

```
D2<- c(rep(0,105),rep(1,204-105))
linearHypothesis(lm(data$ms6 ~ data$ms7+D2*data$ms7),c("D2=0","data$ms7:D2=0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## D2 = 0
## data$ms7:D2 = 0
##
## Model 1: restricted model
## Model 2: data$ms6 ~ data$ms7 + D2 * data$ms7
##
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    195 33481
## 2    193   189  2     33292 16996 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The model parameters are unstable.

The appropriately modified model:(D2 is a dummy variable that =0 if before 1985.1 and =1 after)

```
lm(data$ms6 ~ data$ms7+D2*data$ms7)
```

```
##
## Call:
## lm(formula = data$ms6 ~ data$ms7 + D2 * data$ms7)
##
## Coefficients:
## (Intercept)     data$ms7           D2  data$ms7:D2
##       1.515       84.270        3.984      -54.255
```

```r
Box.test(lm(data$ms6 ~ data$ms7+D2*data$ms7)$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  lm(data$ms6 ~ data$ms7 + D2 * data$ms7)$resid
## X-squared = 0.078017, df = 1, p-value = 0.78
```

The Q-test shows it is valid.

Q3

```r
suppressMessages(library(car))
Inflation <- read.csv("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/CPILFESL.csv")
GDPC1 <- read.csv("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/GDPC1.csv")
GDPPOT <- read.csv("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/GDPPOT.csv")
GAP <- 100*(log(GDPC1[,2])-log(GDPPOT[,2]))
L.GAP <- c(NA,GAP[-208])
L2.GAP <- c(NA,L.GAP[-208])
L3.GAP <- c(NA,L2.GAP[-208])
L4.GAP <- c(NA,L3.GAP[-208])
```

Full sample estimates:

```r
arima(Inflation[,2],order = c(4,0,0),xreg =cbind(L.GAP,L2.GAP,L3.GAP,L4.GAP))
```

```
##
## Call:
## arima(x = Inflation[, 2], order = c(4, 0, 0), xreg = cbind(L.GAP, L2.GAP, L3.GAP,
##     L4.GAP))
##
## Coefficients:
##          ar1     ar2     ar3      ar4  intercept   L.GAP   L2.GAP  L3.GAP
##       0.6373  0.0946  0.2823  -0.1019     3.9969  0.2159  -0.0844  0.1607
## s.e.  0.0700  0.0809  0.0874   0.0752     0.9322  0.1185   0.1233  0.1297
##       L4.GAP
##       0.0092
## s.e.  0.1161
##
## sigma^2 estimated as 1.528:  log likelihood = -333.58,  aic = 687.16
```

```r
armax <- arima(Inflation[,2],order = c(4,0,0),xreg =cbind(L.GAP,L2.GAP,L3.GAP,L4.GAP))
Box.test(armax$resid,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  armax$resid
## X-squared = 0.034094, df = 1, p-value = 0.8535
```

The Q-test shows it is valid.

Using loops to pinpoint a break point:

```
a <- rep(NA,207)
b <- rep(NA,207)
for(i in 10:197){
  D <- c(rep(0,i),rep(1,208-i))
  DLGAP <- D*L.GAP
  DL2GAP <- D*L2.GAP
  DL3GAP <- D*L3.GAP
  DL4GAP <- D*L4.GAP
  armax.test <- arima(Inflation[,2],order = c(4,0,0),
                      xreg =cbind(D,L.GAP,L2.GAP,L3.GAP,L4.GAP,DLGAP,DL2GAP,DL3GAP,DL4GAP))
  if(linearHypothesis(armax.test,c("DLGAP=0","DL2GAP=0","DL3GAP=0","DL4GAP=0"))[2,3]<=0.05){
    a[i] <- 1
  }
  if(Box.test(armax.test$resid,type="Ljung-Box")[3]>=0.70){
    b[i] <- 1
  }
}
intersect(which(a==1),which(b==1))
```

```
## [1] 68
```

```
Inflation[intersect(which(a==1),which(b==1)),1]
```

```
## [1] 1974-10-01
## 208 Levels: 1958-01-01 1958-04-01 1958-07-01 1958-10-01 ... 2009-10-01
```

The break point is at 1974.4.

Test for the structural break:

```
D <- c(rep(0,68),rep(1,208-68))
DLGAP <- D*L.GAP
DL2GAP <- D*L2.GAP
DL3GAP <- D*L3.GAP
DL4GAP <- D*L4.GAP
armax.test <- arima(Inflation[,2],order = c(4,0,0),
                    xreg =cbind(D,L.GAP,L2.GAP,L3.GAP,L4.GAP,DLGAP,DL2GAP,DL3GAP,DL4GAP))
coeftest(armax.test)
```

```
##
## z test of coefficients:
##
##             Estimate Std. Error z value  Pr(>|z|)
## ar1         0.570112   0.070621  8.0728 6.871e-16 ***
## ar2         0.174583   0.080566  2.1670 0.0302382 *
## ar3         0.381346   0.083116  4.5881 4.473e-06 ***
## ar4        -0.175280   0.075497 -2.3217 0.0202495 *
## intercept   6.010650   1.683038  3.5713 0.0003552 ***
## D          -2.975950   1.334613 -2.2298 0.0257593 *
```

```
## L.GAP     -0.112417   0.167325 -0.6718 0.5016828
## L2.GAP    -0.210196   0.178833 -1.1754 0.2398426
## L3.GAP     0.317484   0.178762  1.7760 0.0757310 .
## L4.GAP     0.011553   0.170481  0.0678 0.9459727
## DLGAP      0.540361   0.218725  2.4705 0.0134923 *
## DL2GAP     0.060166   0.246491  0.2441 0.8071626
## DL3GAP    -0.225798   0.250224 -0.9024 0.3668539
## DL4GAP    -0.048616   0.223512 -0.2175 0.8278104
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
linearHypothesis(armax.test,c("DLGAP=0","DL2GAP=0","DL3GAP=0","DL4GAP=0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## DLGAP = 0
## DL2GAP = 0
## DL3GAP = 0
## DL4GAP = 0
##
## Model 1: restricted model
## Model 2: armax.test
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  4 9.9949    0.04051 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We can see that $gama_1$ is significant here, and all the $gama$'s are jointly significant.

Test for the slope change:

```r
linearHypothesis(armax.test,c("DLGAP+DL2GAP+DL3GAP+DL4GAP=0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## DLGAP  + DL2GAP  + DL3GAP  + DL4GAP = 0
##
## Model 1: restricted model
## Model 2: armax.test
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  1 1.7955     0.1803
```

There is no change of slope.

Q4

```r
suppressMessages(library(vars))
suppressMessages(library(lmtest))
Inflation <- read.csv("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/CPILFESL.csv")
UNRATE <- read.csv("D:/Dropbox/16summer/Macroeconometrics/Problem Set3/UNRATE.csv")
```

VAR model:

```r
y <- cbind(Inflation[,2],UNRATE[,2])
colnames(y)[2] <- "UNRATE"
colnames(y)[1] <- "Inflation"
VAR(y,p=5,type="const")
```

```
##
## VAR Estimation Results:
## =======================
##
## Estimated coefficients for equation Inflation:
## ==============================================
## Call:
## Inflation = Inflation.l1 + UNRATE.l1 + Inflation.l2 + UNRATE.l2 + Inflation.l3 + UNRATE.l3 + Inflati
##
## Inflation.l1    UNRATE.l1 Inflation.l2    UNRATE.l2 Inflation.l3
##   0.60767298  -1.38190265   0.14504144   1.73932093   0.25862045
##    UNRATE.l3 Inflation.l4    UNRATE.l4 Inflation.l5    UNRATE.l5
##  -0.66864562  -0.12920884   0.08711882   0.09957515   0.07966871
##        const
##   0.93710322
##
##
## Estimated coefficients for equation UNRATE:
## ===========================================
## Call:
## UNRATE = Inflation.l1 + UNRATE.l1 + Inflation.l2 + UNRATE.l2 + Inflation.l3 + UNRATE.l3 + Inflation.l
##
## Inflation.l1    UNRATE.l1 Inflation.l2    UNRATE.l2 Inflation.l3
##  0.054759088  1.578637717 -0.004575879 -0.558478994 -0.015191488
##    UNRATE.l3 Inflation.l4    UNRATE.l4 Inflation.l5    UNRATE.l5
##  0.001199227 -0.022622346 -0.156394781  0.009168466  0.096769193
##        const
##  0.150367319
```

Granger Causality:

```r
grangertest(Inflation[,2],UNRATE[,2],order = 5)
```

```
## Granger causality test
##
## Model 1: UNRATE[, 2] ~ Lags(UNRATE[, 2], 1:5) + Lags(Inflation[, 2], 1:5)
## Model 2: UNRATE[, 2] ~ Lags(UNRATE[, 2], 1:5)
##   Res.Df Df      F    Pr(>F)
## 1    192
```

```
## 2    197 -5 4.6414 0.0005062 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
grangertest(UNRATE[,2],Inflation[,2],order = 5)
```

```
## Granger causality test
##
## Model 1: Inflation[, 2] ~ Lags(Inflation[, 2], 1:5) + Lags(UNRATE[, 2], 1:5)
## Model 2: Inflation[, 2] ~ Lags(Inflation[, 2], 1:5)
##   Res.Df Df      F   Pr(>F)
## 1    192
## 2    197 -5 4.2453 0.001103 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The result shows that unemployment rate Granger causes inflation, and inflation also Granger causes unemployment rate.

There are some pitfalls. The Granger causality refers only to the effects of past values of one series on the current value of another series. Hence, Granger causality actually measures whether current and past values of one series help to forecast future values of another series. It is a kind of intertemporal correlation, but a very weak form of "causality".

5. $\gamma_0 = Var(X_t) = E(\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3})^2$

$= E(\varepsilon_t^2 + \theta_1^2\varepsilon_{t-1}^2 + \theta_2^2\varepsilon_{t-2}^2 + \theta_3^2\varepsilon_{t-3}^2)$

$= (1+\theta_1^2+\theta_2^2+\theta_3^2)\sigma_\varepsilon^2$

$\gamma_1 = E X_t X_{t-1} = E(\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\theta_3\varepsilon_{t-3})(\varepsilon_{t-1}+\theta_1\varepsilon_{t-2}+\theta_2\varepsilon_{t-3}+\theta_3\varepsilon_{t-4})$

$= E(\theta_1\varepsilon_{t-1}^2 + \theta_1\theta_2\varepsilon_{t-2}^2 + \theta_2\theta_3\varepsilon_{t-3}^2)$

$= (\theta_1+\theta_1\theta_2+\theta_2\theta_3)\sigma_\varepsilon^2$

$\gamma_2 = E X_t X_{t-2} = E(\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\theta_3\varepsilon_{t-3})(\varepsilon_{t-2}+\theta_1\varepsilon_{t-3}+\theta_2\varepsilon_{t-4}+\theta_3\varepsilon_{t-5})$

$= E(\theta_2\varepsilon_{t-2}^2+\theta_1\theta_3\varepsilon_{t-3}^2)$

$= (\theta_2+\theta_1\theta_3)\sigma_\varepsilon^2$

$\gamma_3 = E X_t X_{t-3} = E(\theta_3\varepsilon_{t-3}^2) = \theta_3\sigma_\varepsilon^2$

$\rho_1 = \dfrac{\theta_1+\theta_1\theta_2+\theta_2\theta_3}{1+\theta_1^2+\theta_2^2+\theta_3^2} = 0.35$

$\rho_2 = \dfrac{\theta_2+\theta_1\theta_3}{1+\theta_1^2+\theta_2^2+\theta_3^2} = 0.15$

$\rho_3 = \dfrac{\theta_3}{1+\theta_1^2+\theta_2^2+\theta_3^2} = 0.1$

Q5

```
suppressMessages(library(BB))
fun <- function(x) {
  f <- numeric(length(x))
  f[1] <-  x[1]+x[1]*x[2]+x[2]*x[3]-0.35*(1+x[1]^2+x[2]^2+x[3]^2)
  f[2] <-  x[2]+x[1]*x[3]-0.15*(1+x[1]^2+x[2]^2+x[3]^2)
  f[3] <-  x[3]-0.1*(1+x[1]^2+x[2]^2+x[3]^2)
  f
}
startx <- c(0.5,0.5,0.5)
result = dfsane(startx,fun,control=list(maxit=2500,trace = TRUE))
```

```
## Iteration:  0  ||F(x0)||:  0.4055603
## iteration:  10  ||F(xn)|| =   2.184985e-07
```

```
theta = result$par
sigma2 = 1/(1+t(theta)%*%theta)
theta
```

```
## [1] 0.3408908 0.1329545 0.1147040
```

```
sigma2
```

```
##           [,1]
## [1,] 0.8718089
```

So θ_1=0.3408908; θ_2=0.1329545; θ_3=0.1147040; σ_ε^2=0.8718089

Let's check whether our results can get the right ACF's

```
(theta[1]+theta[1]*theta[2]+theta[2]*theta[3])*sigma2
```

```
##      [,1]
## [1,] 0.35
```

```
(theta[2]+theta[1]*theta[3])*sigma2
```

```
##      [,1]
## [1,] 0.15
```

```
(theta[3])*sigma2
```

```
##            [,1]
## [1,] 0.09999999
```

They are right.