Problem Set 3 Macroeconometrics, Spring 2016
Solutions

Problem 1.

[I will put the Beveridge Nelson decomposition part in a separate document]

MS1:
DF test indicates a UR, so I took difference.
ACF and PACF of D.ms1 suggested AR(2). Possibly AR(1) or AR(1,1). I tested four models and AR(1,1) and AR(2) fit best.

	(1) D.ms1	(2) D.ms1	(3) D.ms1	(4) D.ms1
ms1				
_cons	-0.0154	-0.0149	-0.0134	-0.00756
	(0.137)	(0.144)	(0.153)	(0.187)
ARMA				
L.ar	1.114***	0.935***	0.776***	0.858***
	(0.0731)	(0.246)	(0.0553)	(0.0405)
L2.ar	-0.301***	-0.147		
	(0.0720)	(0.218)		
L.ma		0.197	0.342***	
		(0.238)	(0.0755)	
sigma				
_cons	0.364***	0.364***	0.364***	0.382***
	(0.0259)	(0.0258)	(0.0256)	(0.0260)
N	203	203	203	203
aic	175.6	177.0	175.5	192.4
bic	188.9	193.5	188.7	202.3

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

MS2:

Unit root. So I took difference

The ACF and PACF suggested AR(1) to me. I estimated three models and AR(1) and MA(1) fit best:

	(1)	(2)	(3)
	D.ms2	D.ms2	D.ms2
ms2			
_cons	-0.00224	-0.00225	-0.00231
	(0.0314)	(0.0313)	(0.0303)
ARMA			
L.ar	0.186*	0.161	
	(0.0742)	(0.407)	
L.ma		0.0257	0.181*
		(0.404)	(0.0712)
sigma			
_cons	0.364***	0.364***	0.364***
	(0.0257)	(0.0257)	(0.0257)
N	203	203	203
aic	171.5	173.5	171.7
bic	181.5	186.8	181.6

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

MS3:

Another unit root process according to DF. So I differenced it.

ACF and PACF may indicate AR(1)? Tested three models and MA(1) fit best

	(1)	(2)	(3)
	D.ms3	D.ms3	D.ms3
ms3			
_cons	-0.000415	-0.000495	-0.000458
	(0.00650)	(0.00546)	(0.00596)
ARMA			
L.ar	0.406***		0.177
	(0.0707)		(0.179)
L.ma		0.417***	0.276
		(0.0637)	(0.167)
sigma			
_cons	0.0549***	0.0547***	0.0546***
	(0.00389)	(0.00387)	(0.00386)
N	203	203	203
aic	-595.9	-597.3	-596.5
bic	-586.0	-587.3	-583.3

MS4:

Unit root process according to DF test. For the difference, ACF and PACF indicated AR(2) or ARMA(1,1).

	(1)	(2)	(3)	(4)
	D.ms4	D.ms4	D.ms4	D.ms4
ms4				
_cons	-0.0113	-0.0186	-0.0125	-0.0127
	(0.0952)	(0.0462)	(0.0862)	(0.0844)
ARMA				
L.ar	0.732***		0.659***	0.816***
	(0.0532)		(0.0788)	(0.0747)
L2.ar				-0.117
				(0.0737)
L.ma		0.604***	0.156	
		(0.0544)	(0.0970)	
sigma				
_cons	0.366***	0.409***	0.364***	0.364***
	(0.0259)	(0.0276)	(0.0257)	(0.0258)
N	203	203	203	203
aic	175.3	219.9	174.7	174.6
bic	185.3	229.9	187.9	187.9

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Problem 2.

For ms5, I found a break at 1975q1. I ran the Chow for each period and found the largest F stat at that period. Then I confirmed there was stability within the pre-1975 and post 1975 periods. (It's also pretty obvious if you look at a plot of the data!)

```
----------------------------
                      (1)   
                      ms5   
----------------------------
per2               -5.907***
                 (0.0428)   

_cons               14.43***
                 (0.0355)   
----------------------------
N                     204   
aic                 66.83   
bic                 73.47   
----------------------------
Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

For ms6, I found a break at 1985q1. I did this using the multiple Chow test, but again, it is pretty clear from looking at a graph of the data there is a break.

```
----------------------------
                      (1)   
                      ms6   
----------------------------
ms7                 84.05***
                 (0.0967)   

per2                3.750***
                 (0.0792)   

ms7Xper2           -54.03***
                  (0.146)   

_cons               1.739***
                 (0.0519)   
----------------------------
N                     197   
aic                 68.99   
bic                 82.12   
----------------------------
Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

Problem 3.

I found a break in the parameters in 1981q3. I found this testing whether any of the gammas changed for various years. The largest F stat was when I specified 1981q3 as the break. I also tested for a change in the slope of the Phillips curve, i.e., the sum of the gammas.

The same method and test can also indicate there is a break in 1974. With that break, there was also a change in the gammas and in the sum of the gammas.

After identifying a potential break, it is a good idea to test for stability within each. Indeed, to the extent that the gammas changed and the sum of the gammas changed there is evidence that there were breaks in 1970, 1974, and 1981.

The results below show estimates over the pre-1981 and post-1981 periods, and F tests of the equality of the gammas, and the equality of the sum of the gammas. Over the first period, the sum of the gammas is .049. Over the second period it is .034. So the slope became less steep, that is, the relationship between the GDP gap and inflation lessened.

```
--------------------------------------------
                      (1)             (2)
                       pi              pi
--------------------------------------------
L.pi                0.570***        0.395***
                  (0.109)         (0.0838)

L2.pi              0.0925         -0.0917
                  (0.125)         (0.0841)

L3.pi             0.00942          0.385***
                  (0.139)         (0.0778)

L4.pi              -0.131          -0.138
                  (0.122)         (0.0741)

L.gap               0.117         -0.0872
                  (0.179)          (0.130)

L2.gap             -0.382           0.294
                  (0.269)          (0.201)

L3.gap              0.353          0.0859
                  (0.261)          (0.194)

L4.gap            -0.0391          -0.308*
                  (0.181)          (0.118)

_cons               9.512**         0.595***
                  (2.915)          (0.160)
--------------------------------------------
N                      93             140
aic                 360.0           347.3
bic                 382.7           373.8
--------------------------------------------
Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

. test ([mod1_mean]L1.gap = [mod2_mean]L1.gap) ([mod1_mean]L2.gap = [mod2_mean]L2.gap)///
>       ([mod1_mean]L3.gap = [mod2_mean]L3.gap) ([mod1_mean]L4.gap = [mod2_mean]L4.gap)

 ( 1)  [mod1_mean]L.gap - [mod2_mean]L.gap = 0
 ( 2)  [mod1_mean]L2.gap - [mod2_mean]L2.gap = 0
```

```
 ( 3)  [mod1_mean]L3.gap - [mod2_mean]L3.gap = 0
 ( 4)  [mod1_mean]L4.gap - [mod2_mean]L4.gap = 0

           chi2(  4) =   23.81
         Prob > chi2 =    0.0001

.
. test [mod1_mean]L1.gap + [mod1_mean]L2.gap + [mod1_mean]L3.gap + [mod1_mean]L4.gap =///
>      [mod2_mean]L1.gap + [mod2_mean]L2.gap + [mod2_mean]L3.gap + [mod2_mean]L4.gap

 ( 1)  [mod1_mean]L.gap + [mod1_mean]L2.gap + [mod1_mean]L3.gap + [mod1_mean]L4.gap -
       [mod2_mean]L.gap - [mod2_mean]L2.gap - [mod2_mean]L3.gap - [mod2_mean]L4.gap = 0

           chi2(  1) =   17.76
         Prob > chi2 =    0.0000
```

Problem 4.

It is clear that pi Granger-causes unemp and unemp Granger-causes pi. Granger-causation is not really causation, but an intertemporal correlation.

	pi		unemp
L.pi	0.605*** (0.0653)	L.pi	0.0511*** (0.0131)
L2.pi	0.166* (0.0756)	L2.pi	-0.00934 (0.0151)
L3.pi	0.267*** (0.0750)	L3.pi	-0.00535 (0.0150)
L4.pi	-0.0780 (0.0674)	L4.pi	-0.0155 (0.0135)
L.unemp	-1.364*** (0.318)	L.unemp	1.587*** (0.0636)
L2.unemp	1.740** (0.607)	L2.unemp	-0.594*** (0.122)
L3.unemp	-0.680 (0.606)	L3.unemp	-0.0611 (0.121)
L4.unemp	0.192 (0.316)	L4.unemp	0.0342 (0.0633)
_cons	0.835* (0.356)	_cons	0.124 (0.0712)
N			233
aic			799.4
bic			861.5

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

```
. vargranger

  Granger causality Wald tests
 +------------------------------------------------------------------+
 |          Equation           Excluded |   chi2     df Prob > chi2 |
 |--------------------------------------+---------------------------|
 |                pi              unemp |  24.081     4    0.000    |
 |                pi                ALL |  24.081     4    0.000    |
 |--------------------------------------+---------------------------|
 |             unemp                 pi |  25.878     4    0.000    |
 |             unemp                ALL |  25.878     4    0.000    |
 +------------------------------------------------------------------+
```

Problem 5.

I wrote this as a system of four equations

```
s2*(1+t1^2+t2^2+t3^2) = 1
  (t1+t2*t1+t3*t2)*s2 = .35
        s2*(t3*t1+t2) = .15
                t3*s2 = .1
```

I used R to solve the system of nonlinear equations. I got:

```
s2 = 0.8718088
t1 = 0.3408908
t2 = 0.1329545
t3 = 0.1147040
```

I modified the model to include a dummy variable equal to 1 after 1976.2:

Dependent Variable: MS5
Method: Least Squares
Date: 04/03/08 Time: 15:23
Sample: 1959Q1 2008Q4
Included observations: 200

	Coefficient	Std. Error	t-Statistic	Prob.
C	1.040865	0.114123	9.120520	0.0000
DUM	0.889633	0.141011	6.308945	0.0000

R-squared	0.167377	Mean dependent var	1.623574
Adjusted R-squared	0.163172	S.D. dependent var	1.036290
S.E. of regression	0.947980	Akaike info criterion	2.740983
Sum squared resid	177.9360	Schwarz criterion	2.773967
Log likelihood	-272.0983	Hannan-Quinn criter.	2.754331
F-statistic	39.80278	Durbin-Watson stat	2.138293
Prob(F-statistic)	0.000000		

MS6:

Dependent Variable: MS6
Method: Least Squares
Date: 04/03/08 Time: 15:25
Sample: 1959Q1 2008Q4
Included observations: 200

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.868845	0.079332	10.95197	0.0000
MS7	0.684835	0.078434	8.731295	0.0000

R-squared	0.277993	Mean dependent var	0.801699
Adjusted R-squared	0.274346	S.D. dependent var	1.310841
S.E. of regression	1.116644	Akaike info criterion	3.068483
Sum squared resid	246.8851	Schwarz criterion	3.101466
Log likelihood	-304.8483	Hannan-Quinn criter.	3.081831
F-statistic	76.23550	Durbin-Watson stat	1.951641
Prob(F-statistic)	0.000000		

Based on the Quandt-Andrews test, I detected a break in the slope coefficient at 1984:2. The modified model is as follows:

Dependent Variable: MS6
Method: Least Squares
Date: 04/03/08 Time: 15:30
Sample: 1959Q1 2008Q4
Included observations: 200

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.953505	0.068163	13.98859	0.0000
MS7	0.107405	0.093792	1.145144	0.2535
MS7*DUM	1.174931	0.134149	8.758379	0.0000

R-squared	0.480341	Mean dependent var	0.801699
Adjusted R-squared	0.475065	S.D. dependent var	1.310841
S.E. of regression	0.949736	Akaike info criterion	2.749620
Sum squared resid	177.6936	Schwarz criterion	2.799095
Log likelihood	-271.9620	Hannan-Quinn criter.	2.769642
F-statistic	91.04743	Durbin-Watson stat	2.139533
Prob(F-statistic)	0.000000		

3. Estimate the following Phillips curve model:

$$\pi_t = \alpha + \sum_{i=1}^{4} \beta_i \pi_{t-i} + \sum_{i=1}^{4} \gamma_i U_{t-i} + \varepsilon_t$$

Full-sample estimates:

Dependent Variable: PIE
Method: Least Squares
Date: 04/03/08 Time: 15:42
Sample (adjusted): 1958Q2 2006Q2
Included observations: 193 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.228137	0.510856	-0.446578	0.6557
PIE(-1)	0.627170	0.074561	8.411449	0.0000
PIE(-2)	0.078953	0.087043	0.907065	0.3656
PIE(-3)	0.240883	0.087776	2.744296	0.0067
PIE(-4)	-0.114359	0.075945	-1.505810	0.1338

U(-1)	0.466687	0.431214	1.082264	0.2806
U(-2)	-0.198574	0.802586	-0.247418	0.8049
U(-3)	0.106611	0.801801	0.132964	0.8944
U(-4)	-0.215274	0.427501	-0.503563	0.6152

R-squared	0.774309	Mean dependent var	4.149562
Adjusted R-squared	0.764496	S.D. dependent var	2.874371
S.E. of regression	1.394895	Akaike info criterion	3.549025
Sum squared resid	358.0148	Schwarz criterion	3.701172
Log likelihood	-333.4810	Hannan-Quinn criter.	3.610640
F-statistic	78.90928	Durbin-Watson stat	1.997092
Prob(F-statistic)	0.000000		

The recursive residuals and CUSUM tests are suggestive of a break but the Quandt-Andrews test does not indicate a significant break. It is possible that there is an outlier which is causing the CUSUM and recursive residuals to show instability in the early 1980s.

4. Estimate the bivariate VAR:

Pairwise Granger Causality Tests
Date: 04/03/08 Time: 15:45
Sample: 1957Q1 2008Q4
Lags: 4

Null Hypothesis:	Obs	F-Statistic	Prob.
U does not Granger Cause PIE	192	1.57473	0.1829
PIE does not Granger Cause U		2.79994	0.0274

The above Granger test results indicate that Pie "causes" U but U does not "cause" pie. Of course there could be (and probably is) omitted variables which means that some third missing variable could be causing both Pie and U. It would be more accurate to call this the Granger predictability test. Given past values of pie, past values of u help to predict pie. But given past values of u, past values of pie do not help to predict u.

$x_t = \alpha + B(L)x_{t-1} + \varepsilon_t$

$B(L)$ is a 4th-order polynomial in the lag operator and $x_t = \begin{bmatrix} \pi_t \\ U_t \end{bmatrix}$ where π_t is the consumer price index, excluding food and energy prices and U_t is the civilian unemployment rate.

- Test for Granger-Causality
- report and interpret your results
- what are the potential pitfalls associated with interpreting your results as "causal?"

5. The Yule-Walker equations for an MA(2):

$$\gamma(0) = \sigma^2(1+\theta_1^2+\theta_2^2)$$
$$\gamma(1) = \sigma^2(\theta_1+\theta_1\theta_2)$$
$$\gamma(2) = \sigma^2\theta_2$$

we can directly calculate the covaraince $\gamma(0), \gamma(1), \gamma(2)$ from the data and then use the Y-W equations to solve for the three unknows: $\sigma^2, \theta_1, \theta_2$. In this particular example, the nonlinear equations produce imaginary solutions.