Solutions to Problem Set 2, Econ 587, Summer 2016

1. Dynamic version of Okun's Law

```
      Source |       SS       df       MS              Number of obs =     237
-------------+------------------------------           F(  5,   231) =  100.23
       Model |  396.84544     5  79.3690881            Prob > F      =  0.0000
    Residual | 182.927064   231   .79189205            R-squared     =  0.6845
-------------+------------------------------           Adj R-squared =  0.6777
       Total | 579.772504   236  2.45666315            Root MSE      =  .88988

------------------------------------------------------------------------------
          du |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          du |
         L1. |   .2864088   .0660174     4.34   0.000     .1563355    .4164821
         L2. |  -.1085323   .0579495    -1.87   0.062    -.2227095    .0056448
             |
           g |
         --. |   -.201114    .016118   -12.48   0.000    -.2328711    -.169357
         L1. |  -.0761304   .0208451    -3.65   0.000    -.1172014   -.0350595
         L2. |  -.0586267   .0213618    -2.74   0.007    -.1007156   -.0165379
             |
       _cons |   1.192691   .1260472     9.46   0.000     .9443421     1.44104
------------------------------------------------------------------------------
```

ii. Okun's law lag-length check

```
                     1 lag           2 lag           3 lag
------------------------------------------------------------
N                      236             236             236
aic                  624.4           621.0           621.7
bic                  638.3           641.8           649.4
------------------------------------------------------------
```

AIC is minimized at lag length = 2

ii. Chow break-point test:

```
      Source |       SS       df       MS              Number of obs =     237
-------------+------------------------------           F( 11,   225) =   48.89
       Model |  408.746743    11  37.1587948           Prob > F      =  0.0000
    Residual |  171.025761   225  .760114492           R-squared     =  0.7050
-------------+------------------------------           Adj R-squared =  0.6906
       Total |  579.772504   236  2.45666315           Root MSE      =  .87185

------------------------------------------------------------------------------
          du |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          du |
         L1. |   .2225014     .07354     3.03   0.003     .0775861    .3674167
         L2. |  -.1722662   .0638029    -2.70   0.007    -.2979938   -.0465386
             |
           g |
         --. |  -.2057417   .0170749   -12.05   0.000     -.239389   -.1720945
         L1. |  -.0959639   .0226045    -4.25   0.000    -.1405075   -.0514204
         L2. |  -.0868909   .0233514    -3.72   0.000    -.1329062   -.0408755
             |
        per2 |  -.8618634    .294395    -2.93   0.004    -1.441987   -.2817393
      LduXp2 |   .0584787   .1695321     0.34   0.730    -.2755951    .3925526
     L2duXp2 |   .2482457   .1519251     1.63   0.104    -.0511324    .5476238
        gXp2 |   .0492162    .050782     0.97   0.334    -.0508529    .1492853
       LgXp2 |   .0378015    .058165     0.65   0.516    -.0768164    .1524194
      L2gXp2 |   .0660199   .0599866     1.10   0.272    -.0521875    .1842273
       _cons |    1.55166   .1581464     9.81   0.000     1.240022    1.863297
------------------------------------------------------------------------------

. test per2 LduXp2 L2duXp2 gXp2 LgXp2 L2gXp2 // --> reject null of no break!

 ( 1)  per2 = 0
 ( 2)  LduXp2 = 0
 ( 3)  L2duXp2 = 0
 ( 4)  gXp2 = 0
 ( 5)  LgXp2 = 0
 ( 6)  L2gXp2 = 0

       F(  6,   225) =    2.61
            Prob > F =    0.0183
```

We therefore reject the null hypothesis of no break.

iv. split-sample estimates:

```
* part iv. GDP growth rate consistent with stable unemp (see p. 75 of Knotek paper)
. reg du g if !per2

      Source |       SS       df       MS              Number of obs =     143
-------------+------------------------------           F(  1,   141) =  159.31
       Model |  269.732483     1  269.732483           Prob > F      =  0.0000
    Residual |  238.729627   141  1.69311792           R-squared     =  0.5305
-------------+------------------------------           Adj R-squared =  0.5272
       Total |   508.46211   142  3.58071908           Root MSE      =  1.3012

------------------------------------------------------------------------------
          du |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           g |   -.283147   .0224331   -12.62   0.000    -.3274956   -.2387983
       _cons |   1.180762   .1368002     8.63   0.000     .9103178    1.451207
------------------------------------------------------------------------------

. di -_b[_cons]/_b[g]
4.17

.
. reg du g if per2

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  1,    94) =   45.10
       Model |  21.6175453     1  21.6175453           Prob > F      =  0.0000
    Residual |  45.0607901    94  .479370107           R-squared     =  0.3242
-------------+------------------------------           Adj R-squared =  0.3170
       Total |  66.6783354    95  .701877214           Root MSE      =  .69237

------------------------------------------------------------------------------
          du |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           g |  -.2280816   .0339643    -6.72   0.000    -.2955185   -.1606447
       _cons |   .5857976    .130902     4.48   0.000     .3258887    .8457065
------------------------------------------------------------------------------

. di -_b[_cons]/_b[g]
2.56
```

2. We have $y_t = \varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}$
where $\varepsilon_t \sim WN(0,\sigma^2)$

(i) Theoretical ACF for lags 1 through 4, provide a sketch of the ACF.

The autocorrelation is the ratio of the autocovariance function to the variance. Therefore, we first derive the variance of y_t which is the same as the autocovariance at lag 0:

$$\text{var}(y_t) = \gamma(0) = E[\varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}]^2 = E[\varepsilon_t^2 + .09\varepsilon_{t-1}^2 + .0289\varepsilon_{t-2}^2 - .102\varepsilon_{t-1}\varepsilon_{t-2} - .6\varepsilon_t\varepsilon_{t-1} + .34\varepsilon_t\varepsilon_{t-2}]$$

Because ε_t is zero mean white noise, it follows that:

$$E[\varepsilon_t]^2 = \sigma^2 \text{ and } E[\varepsilon_t\varepsilon_{t-i}] = 0 \text{ for all } i \neq 0.$$

Therefore:

$$\text{var}(y_t) = \sigma^2 \times (1 + .09 + .289) = 1.379\sigma^2$$

Next we derive the autocovariance at lags 1 through 4:

Lag 1:
$\gamma(1) = E[\varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}] \times E[\varepsilon_{t-1} + -.3\varepsilon_{t-2} + .17\varepsilon_{t-3}]$
Ignoring the cross products which disappear (equal zero):
$\gamma(1) = E[-.3\varepsilon_{t-1}^2 - .051\varepsilon_{t-2}^2] = -.351\sigma^2$

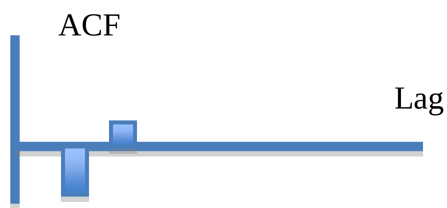
Lag 2:
$\gamma(2) = E[\varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}] \times E[\varepsilon_{t-2} - .3\varepsilon_{t-3} + .17\varepsilon_{t-4}] = .17\sigma^2$

Lag 3:
$\gamma(3) = E[\varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}] \times E[\varepsilon_{t-3} - .3\varepsilon_{t-4} + .17\varepsilon_{t-5}] = 0$

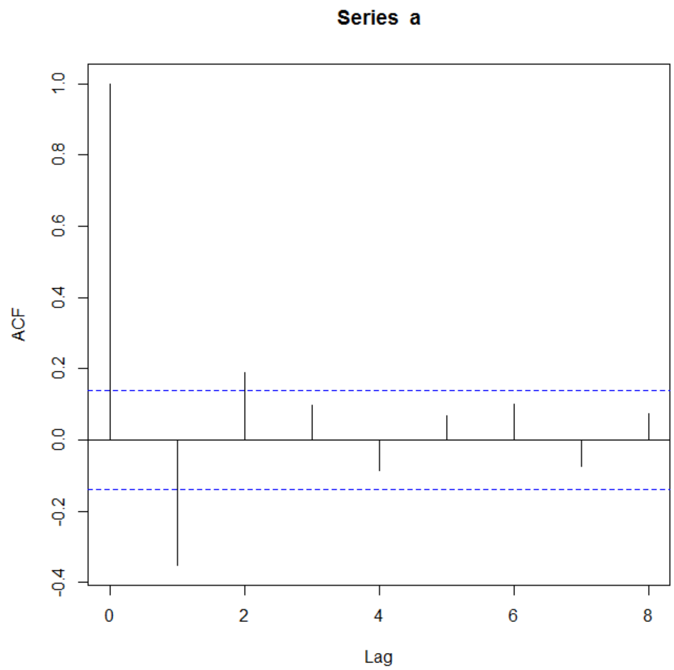
Lag 4:
$$\gamma(4) = E[\varepsilon_t - .3\varepsilon_{t-1} + .17\varepsilon_{t-2}] \times E[\varepsilon_{t-4} - .3\varepsilon_{t-5} + .17\varepsilon_{t-6}] = 0$$

The ACF (autocorrelation function):
$$\rho(i) = \frac{\gamma(i)}{\gamma(0)}$$
$$\rho(1) = \frac{\gamma(1)}{\gamma(0)} = \frac{-.351\sigma^2}{1.379\sigma^2} = -0.25$$
$$\rho(2) = \frac{\gamma(2)}{\gamma(0)} = \frac{.17\sigma^2}{1.379\sigma^2} = 0.12$$
$$\rho(i) = \frac{\gamma(i)}{\gamma(0)} = 0 \quad \text{for } i \geq 3$$

sketch:



(ii) Code in R:
```
a <- arima.sim(n = 200, list(ma = c(-.3,.17)))
acf(a, lag.max=8)
```



It roughly corresponds: the first lag is negative and significant as predicted. The second lag is positive as predicted. All of the remaining lags are insignificant

iii. Code in R:

```
arima(a,order=c(0,0,2))
```

Output in R:

```
Coefficients:
          ma1     ma2  intercept
      -0.3405  0.2800     0.1075
s.e.   0.0679  0.0686     0.0700
```

3. We have $y_t = .28 y_{t-1} + \varepsilon_t$

where $\varepsilon_t \sim WN(0,1.15)$

Recall that an AR(1) can be re-written as an MA(∞):

$$y_t = \sum_{i=0}^{\infty} .28^i \varepsilon_{t-i}$$

$$\text{var}(y_t) = \gamma(0) = E[\sum_{i=0}^{\infty} .28^i \varepsilon_{t-i}]^2$$

Since all of the cross-products cancel out:

$$\text{var}(y_t) = \gamma(0) = E[\sum_{i=0}^{\infty} .28^i \varepsilon_{t-i}]^2 = 1.15 \times [1 + .28^2 + .28^4 + .28^6 + \cdots] = \frac{1.15}{1 - .28^2} = 1.25$$

4. This is an exercise in inspecting ACFs, PACFs, and the estimation results from multiple candidate specifications. Also it had you (unwittingly) model a couple unit root series as ARMA. See my Stata output file for details.

GDP growth: AIC chooses ARMA(2,1), BIC chooses ARMA(1,1).

	(1)	(2)	(3)	(4)	(5)
	g	g	g	g	g
g					
_cons	2.923***	2.977***	2.976***	2.974***	2.977***
	(0.394)	(0.498)	(0.513)	(0.508)	(0.550)
ARMA					
L.ma	0.380***	-0.193	0.439	0.448	-0.370
	(0.0807)	(0.257)	(0.389)	(0.351)	(0.201)
L2.ma	0.309**	0.147	-0.0206		
	(0.0953)	(0.126)	(0.284)		
L.ar		0.579*	-0.0511	-0.0553	0.756***
		(0.225)	(0.376)	(0.325)	(0.142)
L2.ar			0.451	0.435***	
			(0.335)	(0.131)	
sigma					
_cons	2.250***	2.224***	2.203***	2.203***	2.235***
	(0.121)	(0.149)	(0.145)	(0.137)	(0.144)
N	104	104	104	104	104
aic	472.1	471.8	471.9	469.9	470.8
bic	482.7	485.0	487.7	483.1	481.4

T Bill Rate: [We now know that this is probably a unit root time series so we should difference it first. But when the homework was due, we hadn’t gotten that far yet.]Without differencing, the model I thought the ARMA(2,2) fit pretty well.

	(1) trate	(2) trate	(3) trate	(4) trate
trate				
_cons	3.868*	3.901	3.847***	3.855***
	(1.789)	(2.464)	(0.544)	(0.546)
ARMA				
L.ar	1.469***	0.995***	1.956***	1.951***
	(0.0429)	(0.0115)	(0.0303)	(0.0318)
L2.ar	-0.474***		-0.961***	-0.957***
	(0.0450)		(0.0303)	(0.0321)
L.ma		0.450***	-0.566***	-0.578***
		(0.0482)	(0.0599)	(0.0596)
L2.ma			-0.229***	-0.298***
			(0.0570)	(0.0703)
L3.ma				0.109
				(0.0583)
sigma				
_cons	0.191***	0.194***	0.182***	0.181***
	(0.00606)	(0.00634)	(0.00580)	(0.00607)
N	232	232	232	232
aic	-96.49	-88.22	-113.5	-114.6
bic	-82.71	-74.44	-92.78	-90.49

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Labor productivity: [Another unit root!] AIC chose ARMA(1,2).

	(1) lp	(2) lp	(3) lp
lp			
_cons	76.14***	79.77***	79.56***
	(1.335)	(20.68)	(20.72)
ARMA			
L.ma	1.616***	0.273**	
	(0.171)	(0.0857)	
L2.ma	1.000***	0.240**	
	(0.213)	(0.0929)	
L.ar		0.999***	0.999***
		(0.00807)	(0.00625)
sigma			
_cons	3.753	0.619***	0.679***
	(.)	(0.0421)	(0.0474)
N	103	103	103
aic	579.8	210.7	225.4
bic	587.7	223.9	233.3

5. The statistic I started with was the sum of squared residuals (SSR). The SSR however is (weakly) decreasing in the number of lags included. So more lags will never lead to an increase in the SSR. Therefore, I adjusted the information criterion by adding a penalty of .15 for each lag (why .15? I figured out what would make 8 lags have the min!)

Here is a plot of SSR and SSR + .15*(number of lags) which is minimized at lag 8 :