

Prob 42

let k = key len in bits

D = numeric val of {00 ash.1 hash}

t = len of D in bits

x = offset in bits from right

$N = 2^+ - D$ should be a multiple of 3,
tweak hash if necessary

Desired output

00 01 ff ff .. ff 00 ash.1 hash garbage
15 bits

Achievable with

$$2^{k-15} - 2^{k-x+t} + D2^{k-x} + \text{garbage}$$

where $2^{k-15} - 2^{k-x+t}$ gives 01 ff ff .. ff 00 00 .. 00
k-x+t bits

$D2^{k-x}$ gives 00 ash.1 hash 00 00 .. 00
D having t bits k-x bits

and garbage being less than 2^{k-x+1} and so not impacting D

$$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$

$$\text{let } A = 2^{\frac{k-15}{3}}$$

$$B = \frac{N}{3} 2^{k-x-2(\frac{k-15}{3})} \quad \left. \vphantom{B = \frac{N}{3} 2^{k-x-2(\frac{k-15}{3})}} \right\} \begin{array}{l} \text{induced} \\ \text{from} \\ \text{Finney's} \\ \text{writeup} \end{array}$$

$$= 2^{k-15} - \cancel{2^{2(\frac{k-15}{3})}} N 2^{\cancel{k-x-2(\frac{k-15}{3})}} \quad \left. \vphantom{= 2^{k-15} - \cancel{2^{2(\frac{k-15}{3})}} N 2^{\cancel{k-x-2(\frac{k-15}{3})}}} \right\} \alpha$$

$$+ \cancel{2^{\frac{k-15}{3}}} \frac{N^2}{3} 2^{2k-2x-4(\frac{k-15}{3})} - \frac{N^3}{27} 2^{3k-3x-6(\frac{k-15}{3})} \quad \left. \vphantom{+ \cancel{2^{\frac{k-15}{3}}} \frac{N^2}{3} 2^{2k-2x-4(\frac{k-15}{3})} - \frac{N^3}{27} 2^{3k-3x-6(\frac{k-15}{3})}} \right\} \beta = \text{garbage}$$

$$\begin{aligned} \alpha &= 2^{k-15} - N 2^{k-x} = 2^{k-15} - (2^+ - 1) 2^{k-x} \\ &= 2^{k-15} - 2^{k-x+1} + 1 \cdot 2^{k-x} \quad \checkmark \end{aligned}$$

$$\beta = \frac{N^2}{3} 2^{2k-2x-3(\frac{k-15}{3})} - \frac{N^3}{27} 2^{3k-3x-6(\frac{k-15}{3})} < 2^{k-x+1}$$

$$\begin{array}{l} 2k-2x-k+15 \\ k-2x+15 \end{array}$$

~~therefore so long as~~

$$\begin{array}{l} k-2x+15+2+1 < k-x+1 \\ k-2x+30+3+1 < k-x+1 \end{array} \quad \begin{array}{l} 45+5+1 \\ 45+5+1 \end{array}$$