

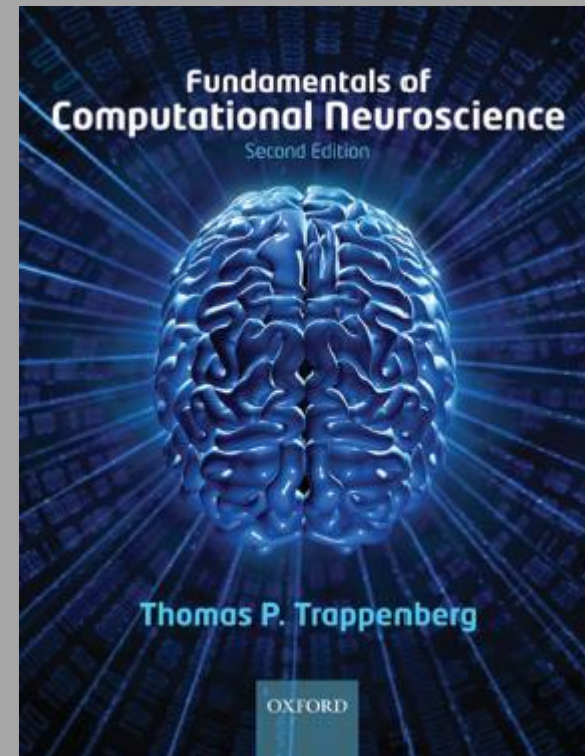
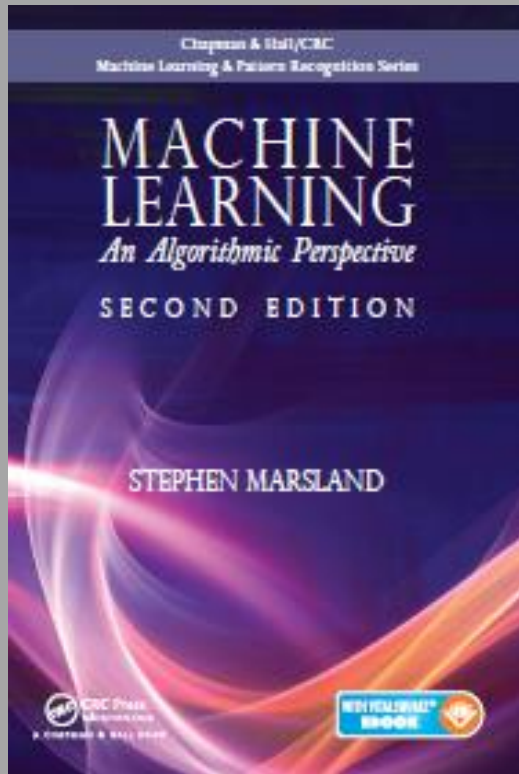


# Redes Neurais Competitivas

Prof. Fabio M Simoes de Souza

Centro de Matemática, Computação e Cognição

# Livros

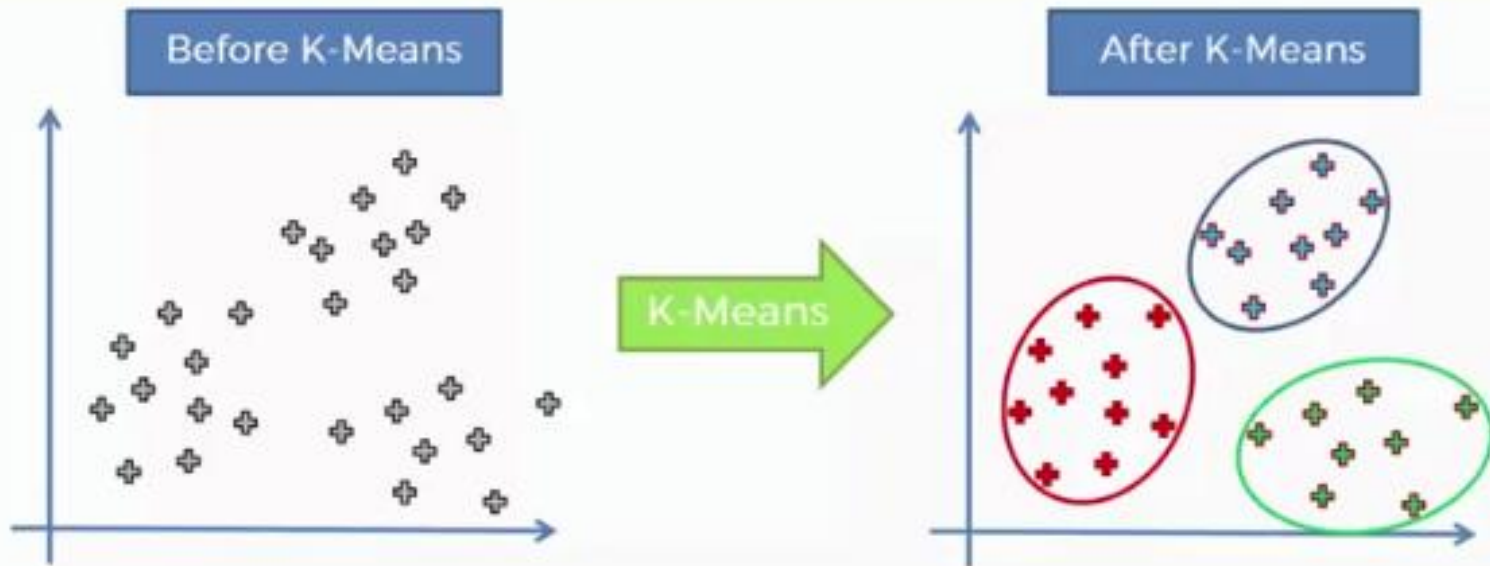


# Aprendizagem Não Supervisionada

- A resposta correta (target) não é conhecida de antemão.
- Como treinar a rede nesse caso, se não existe um sinal de erro?
- Tentar identificar similaridades entre as entradas de maneira a categoriza-las em diferentes conjuntos (clusters).

# K-Means

## What K-Means does for you



# K-Means

STEP 1: Choose the number K of clusters



STEP 2: Select at random K points, the centroids (not necessarily from your dataset)



STEP 3: Assign each data point to the closest centroid → That forms K clusters



STEP 4: Compute and place the new centroid of each cluster



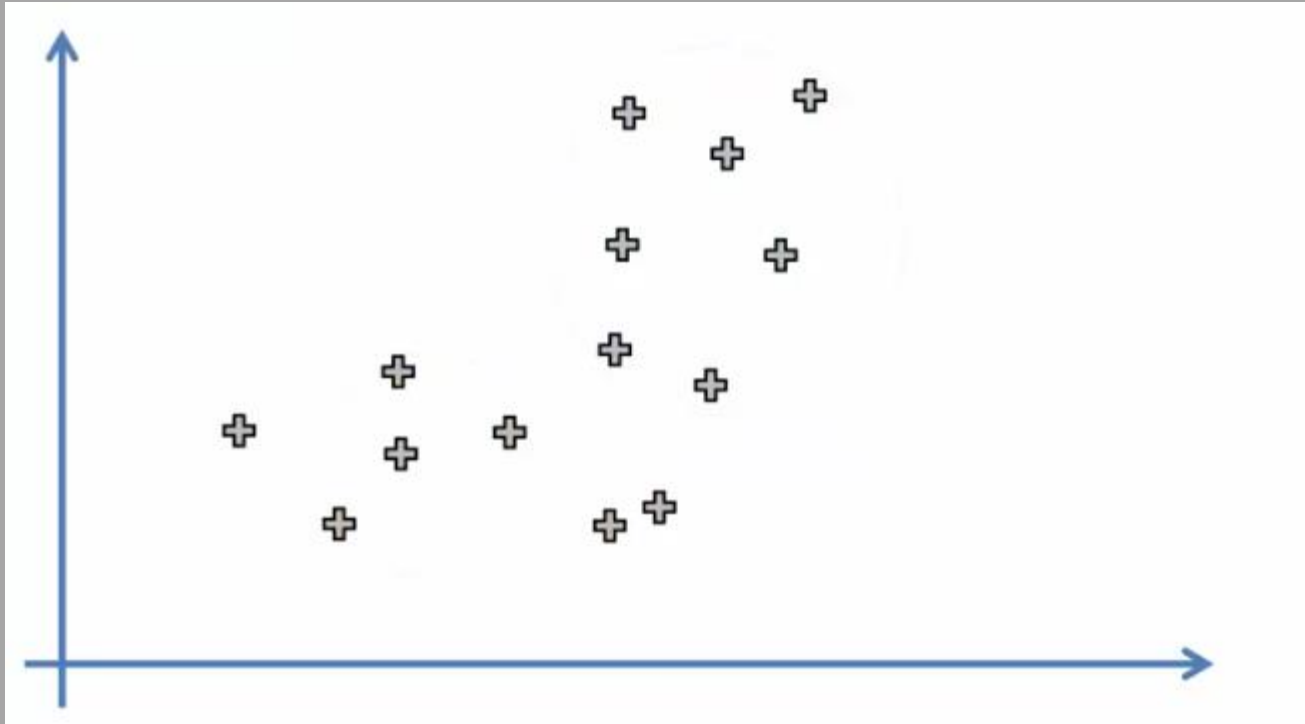
STEP 5: Reassign each data point to the new closest centroid.  
If any reassignment took place, go to STEP 4, otherwise go to FIN.



Your Model is Ready

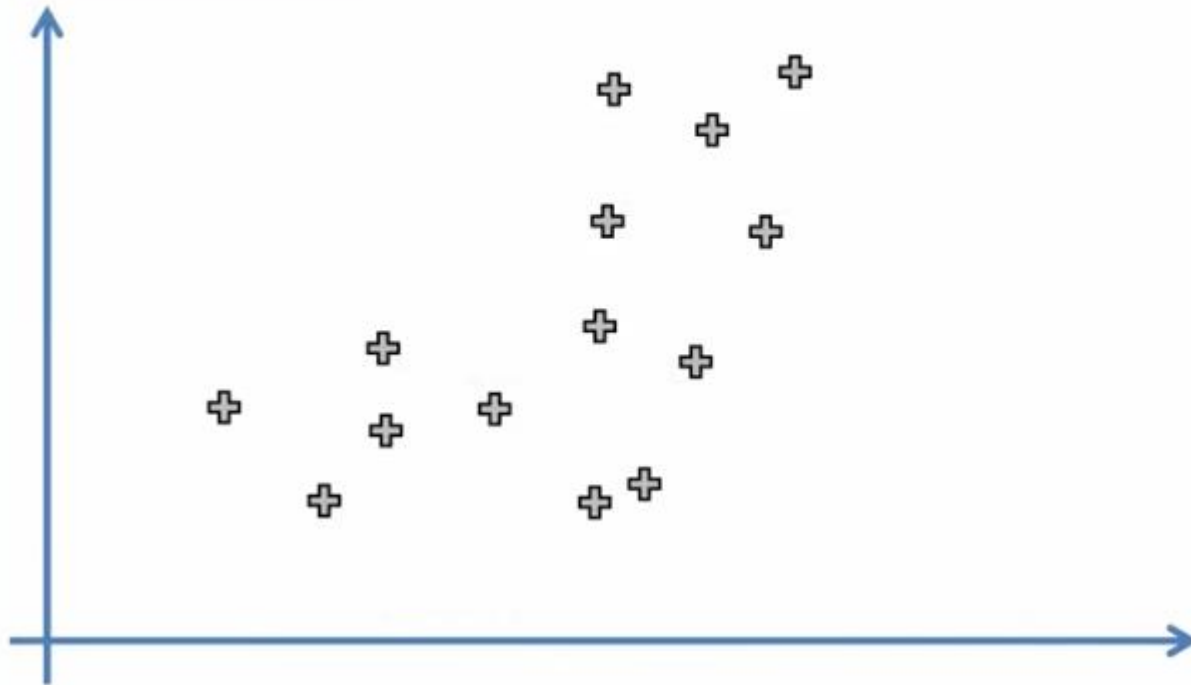
# K-Means

- Classifique esses pontos em clusters



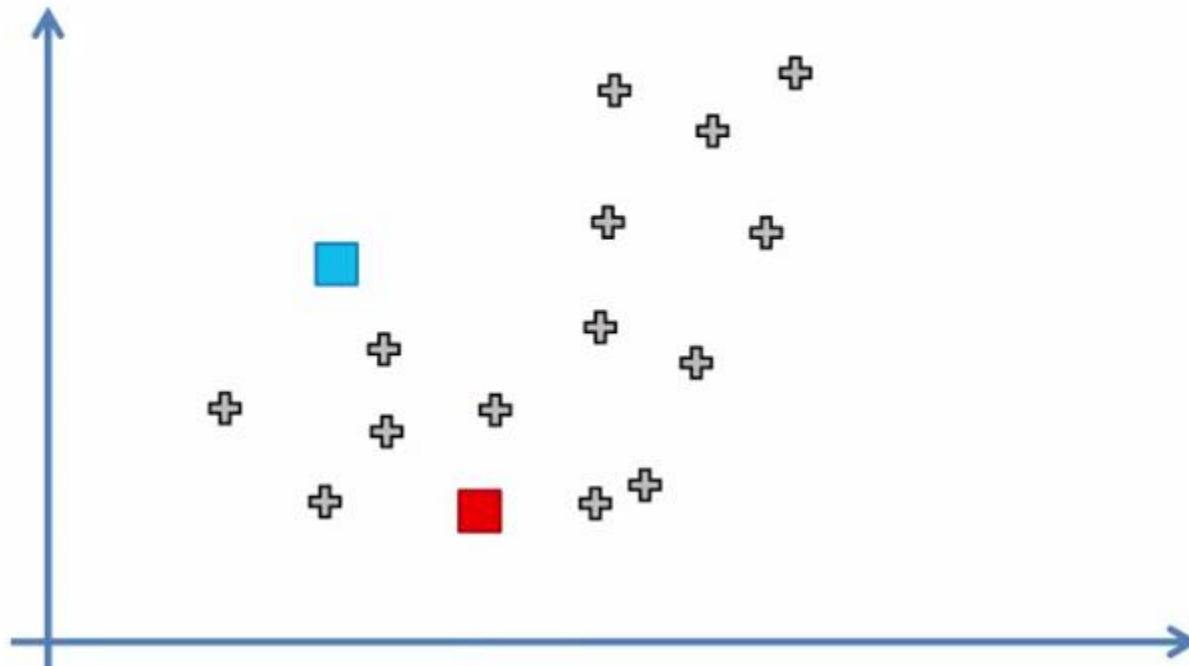
# K-Means

STEP 1: Choose the number K of clusters:  $K = 2$



# K-Means

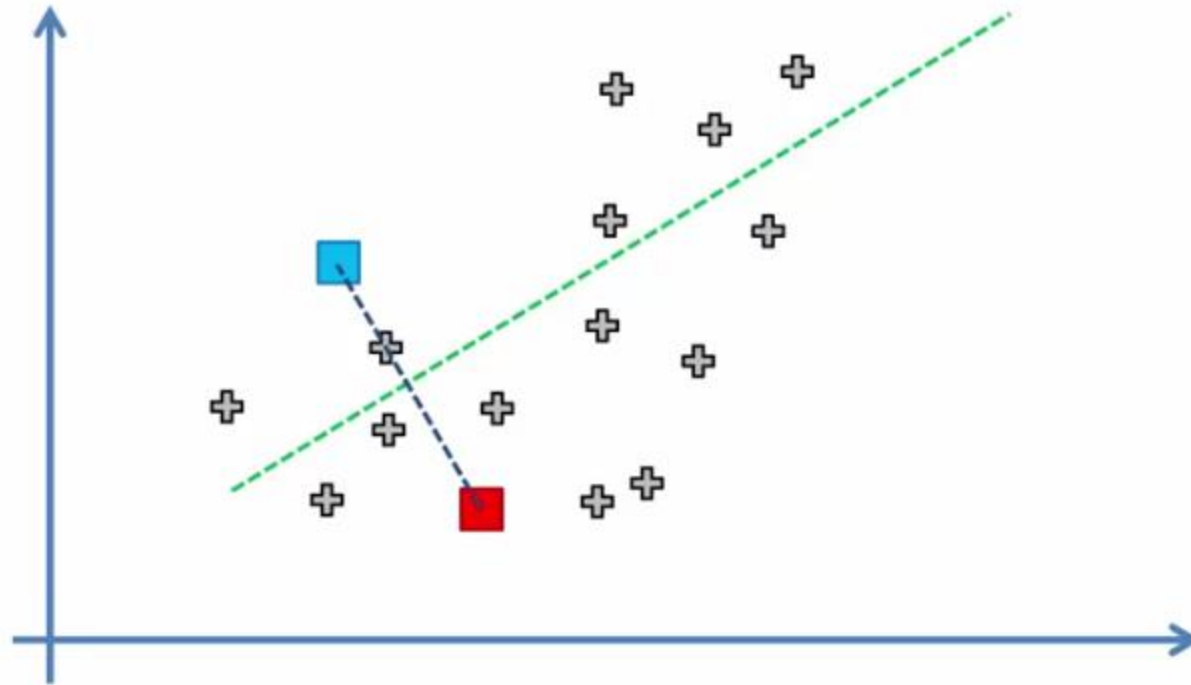
STEP 3: Assign each data point to the closest centroid → That forms K clusters





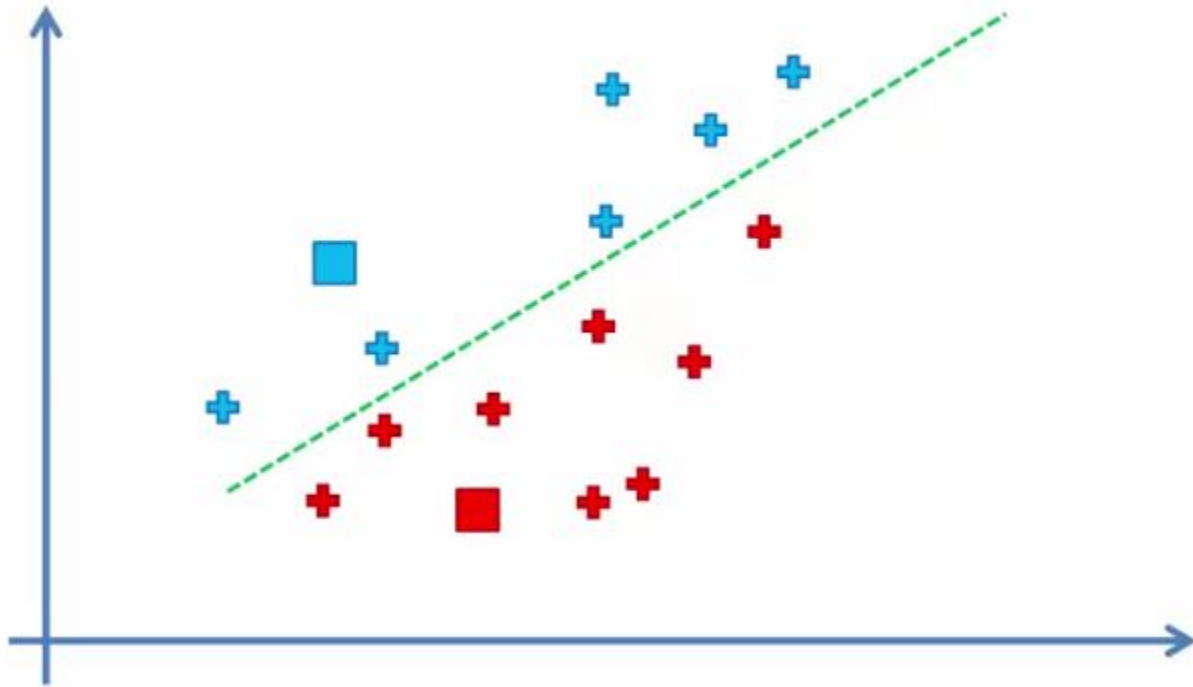
# K-Means

STEP 3: Assign each data point to the closest centroid → That forms K clusters



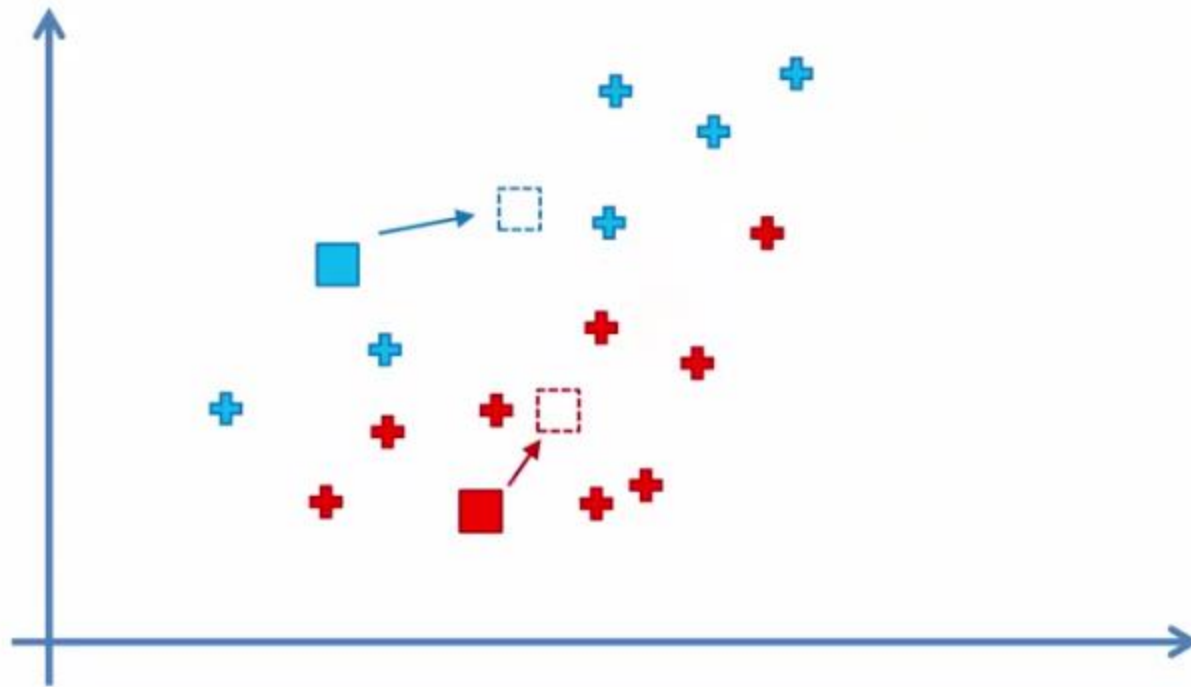
# K-Means

STEP 3: Assign each data point to the closest centroid → That forms K clusters



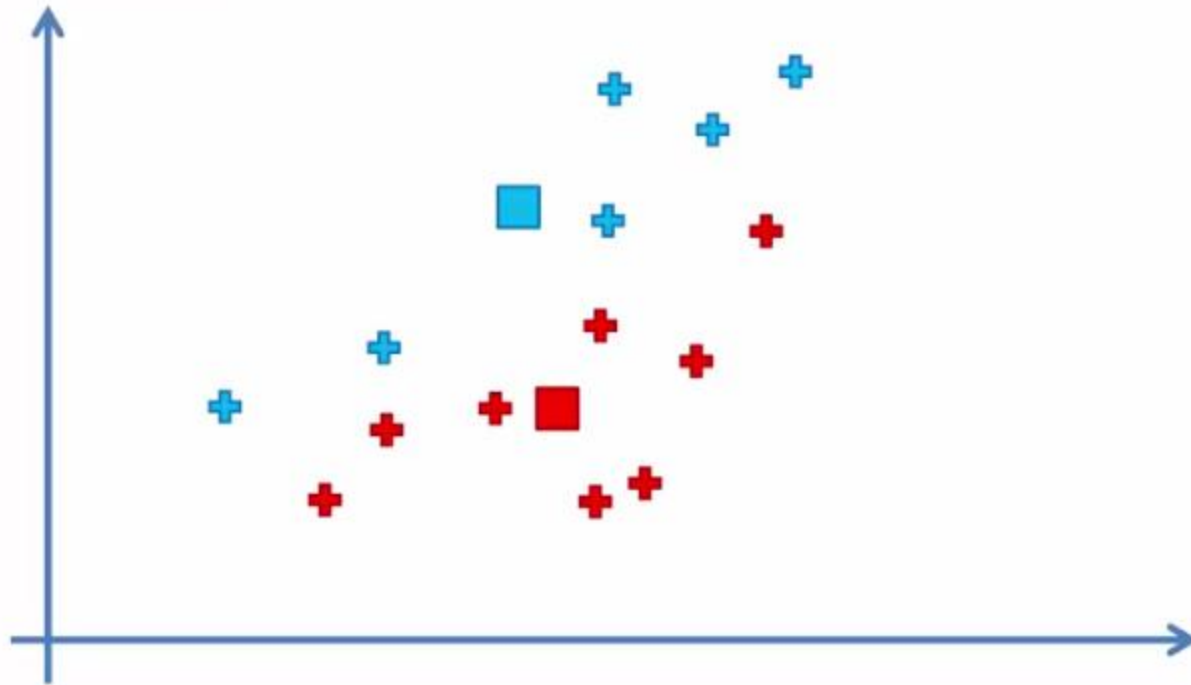
# K-Means

STEP 4: Compute and place the new centroid of each cluster



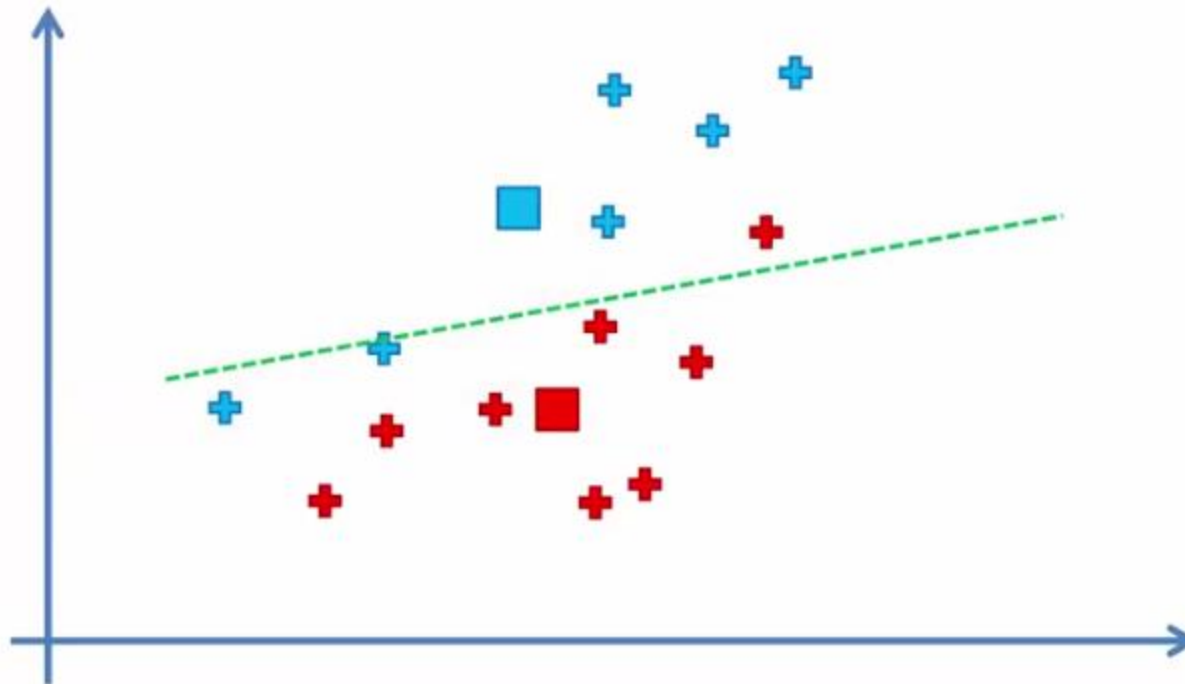
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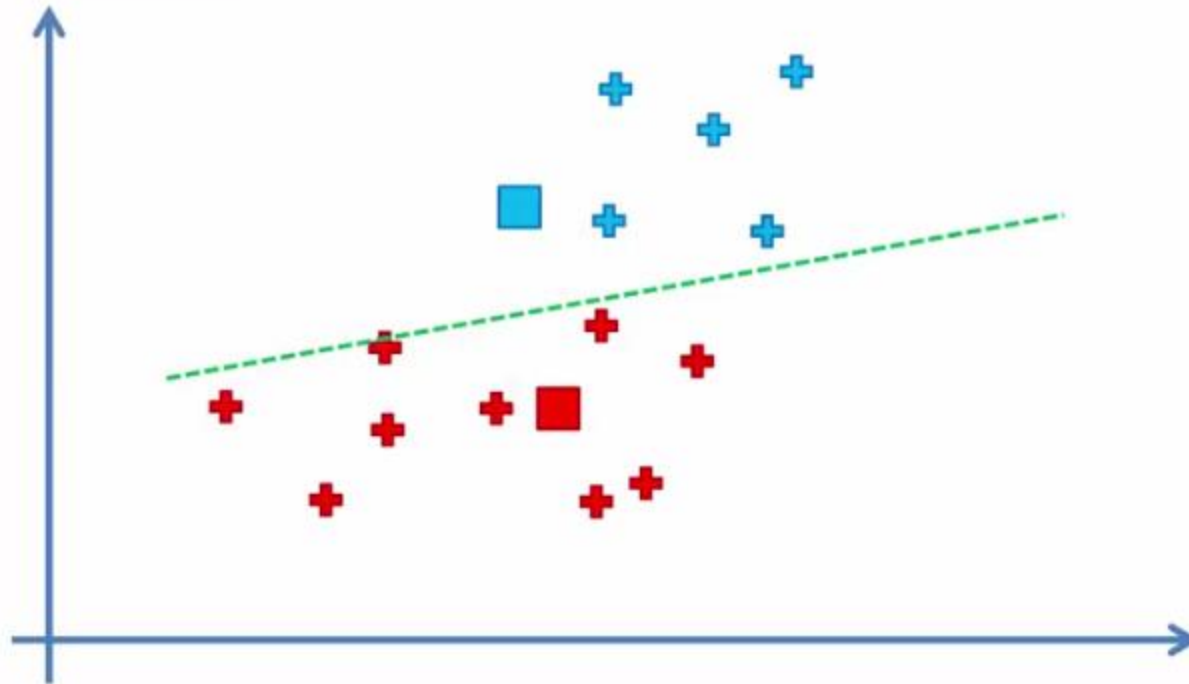
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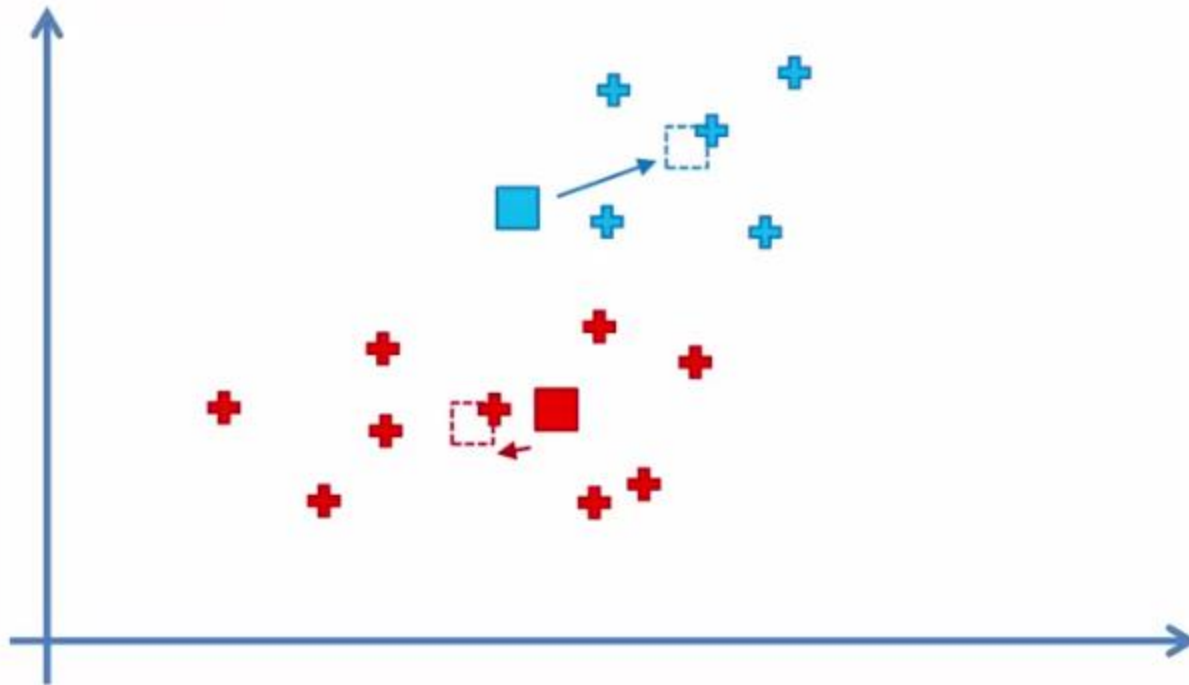
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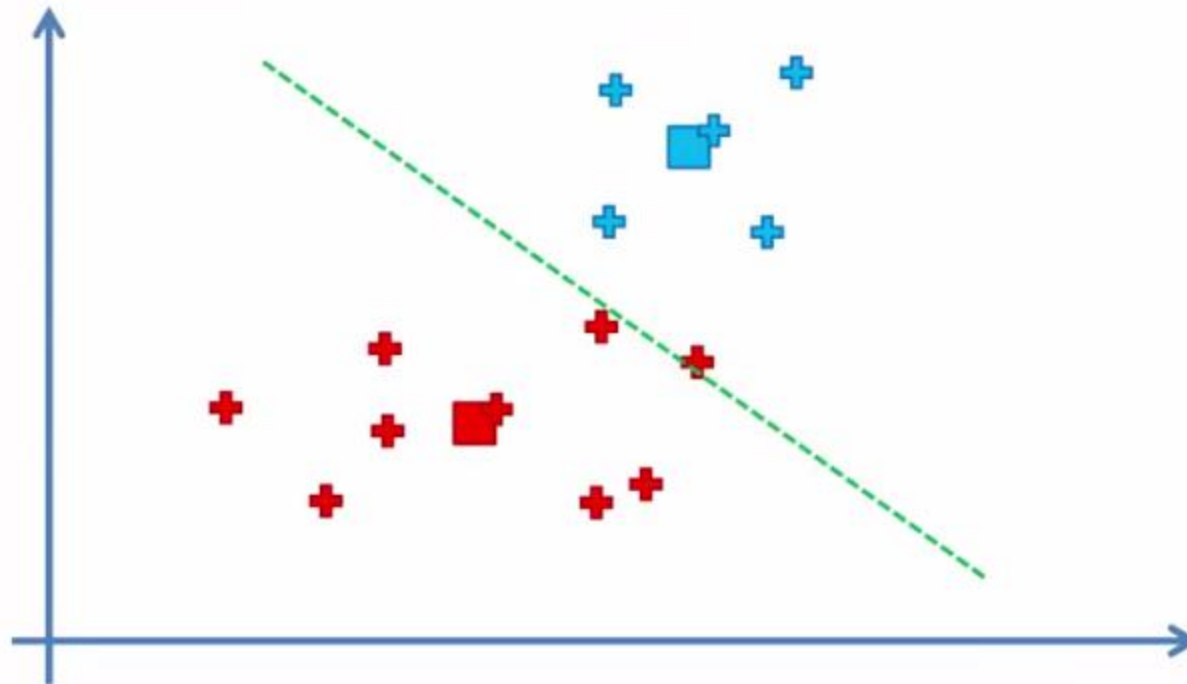
# K-Means

STEP 4: Compute and place the new centroid of each cluster



# K-Means

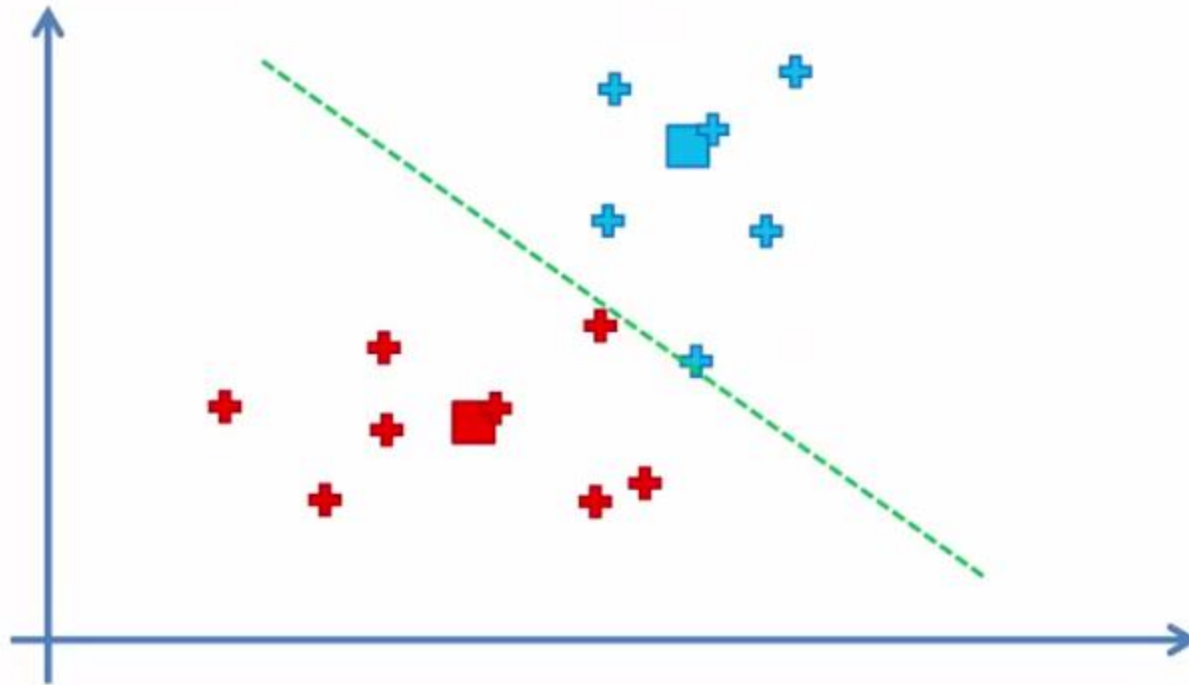
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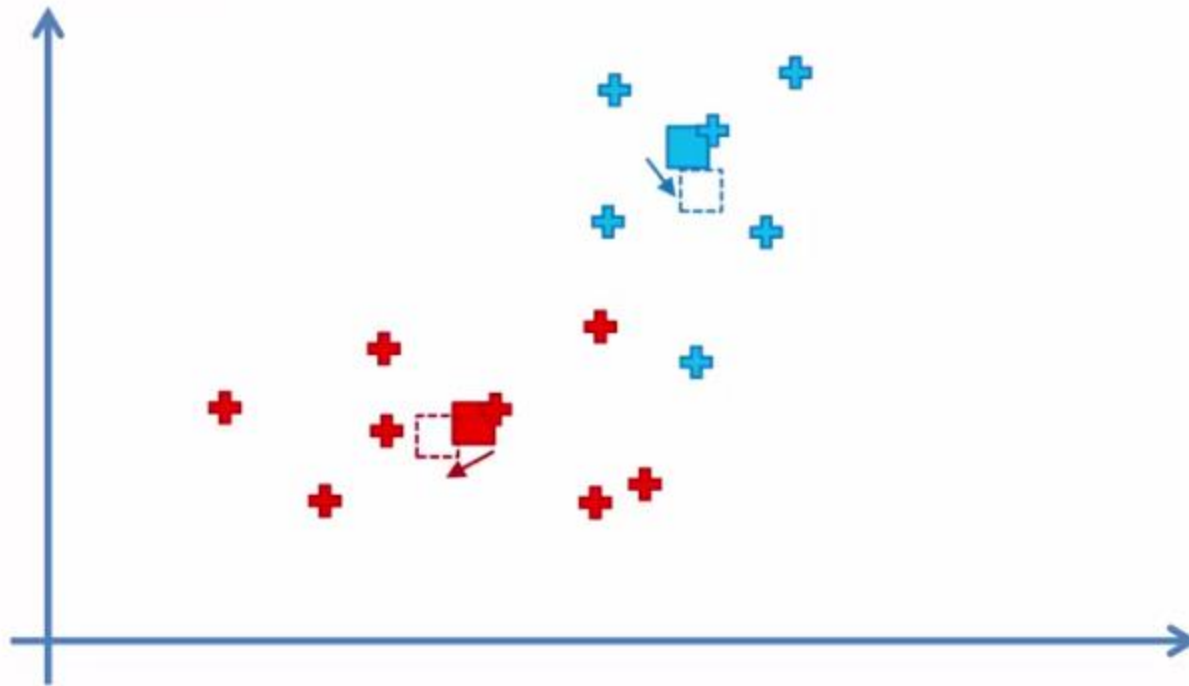
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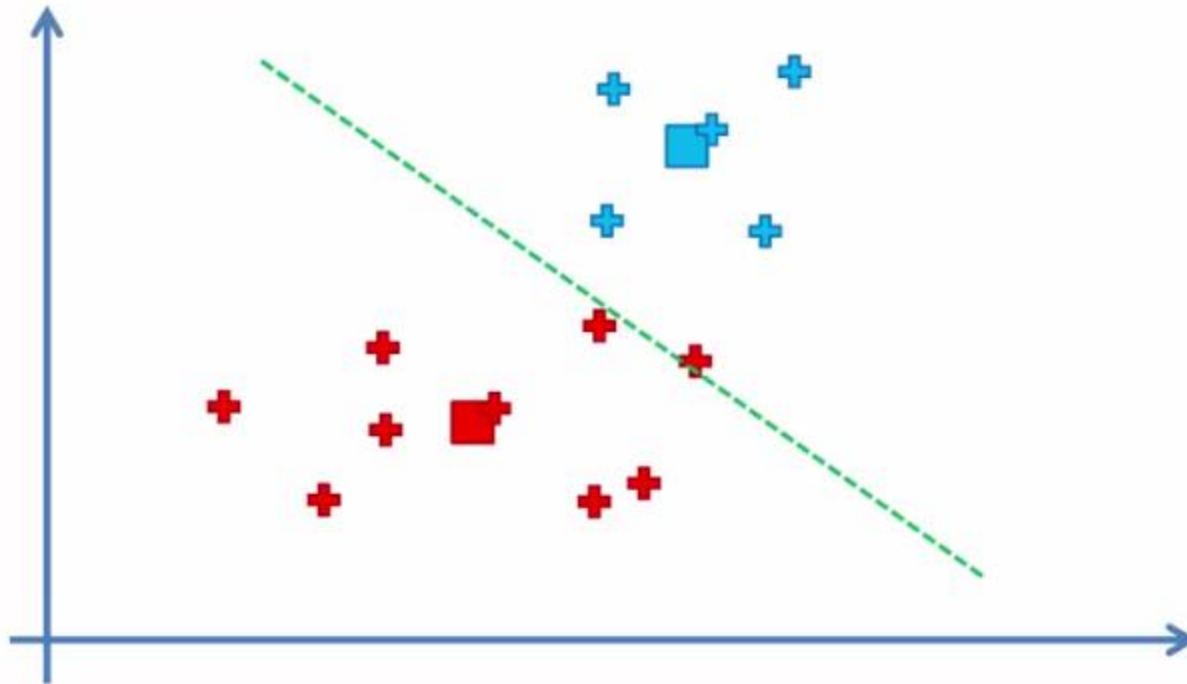
# K-Means

STEP 4: Compute and place the new centroid of each cluster



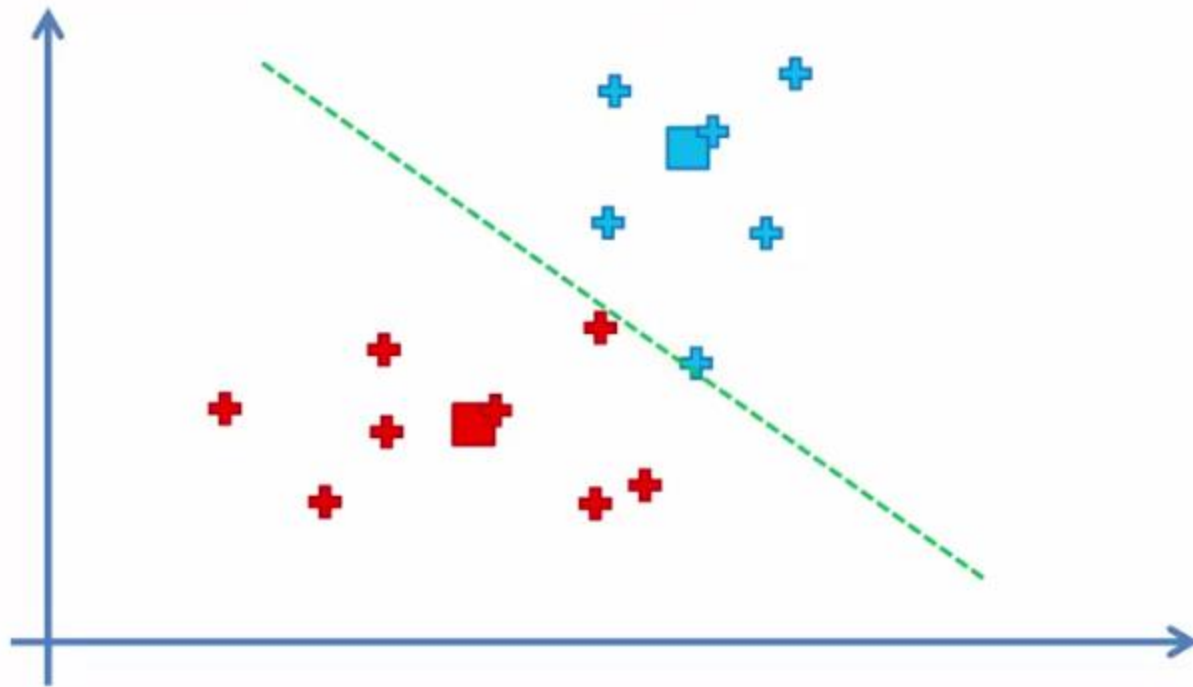
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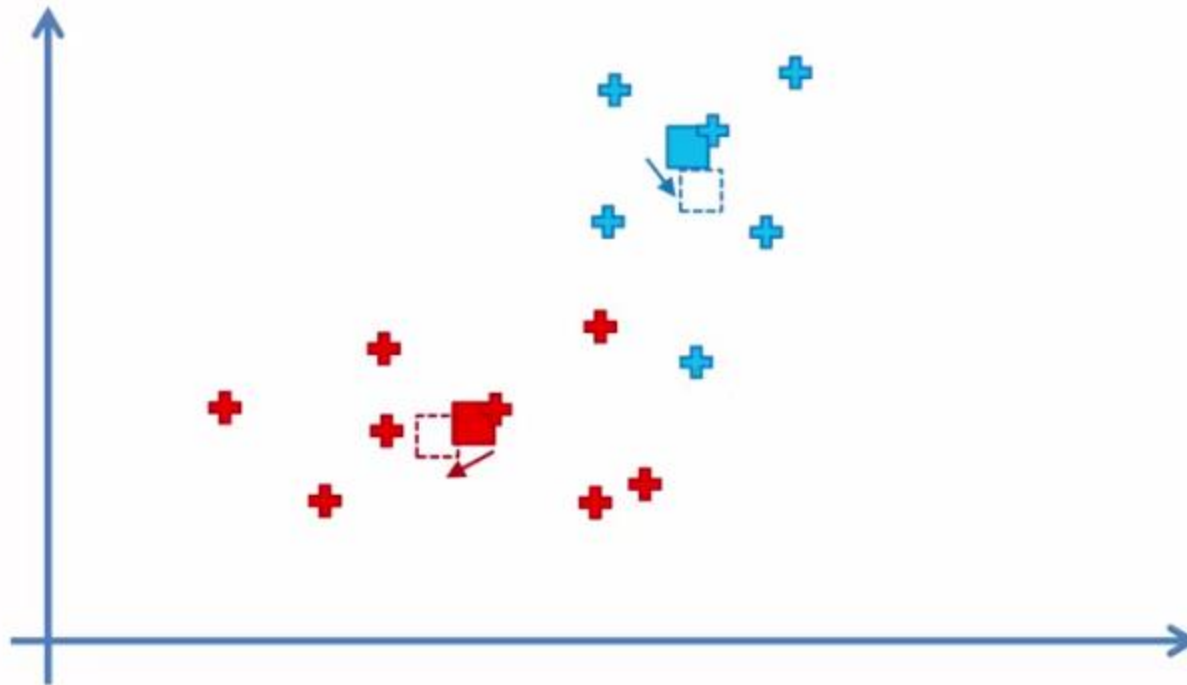
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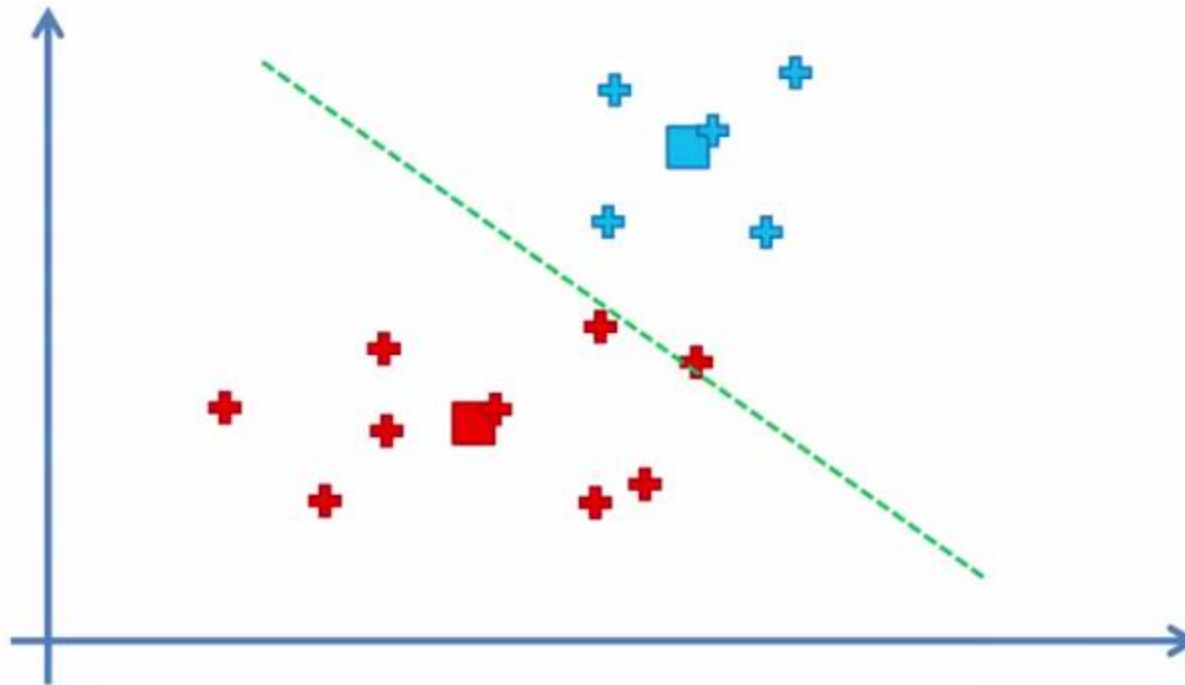
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STEP 4: Compute and place the new centroid of each cluster



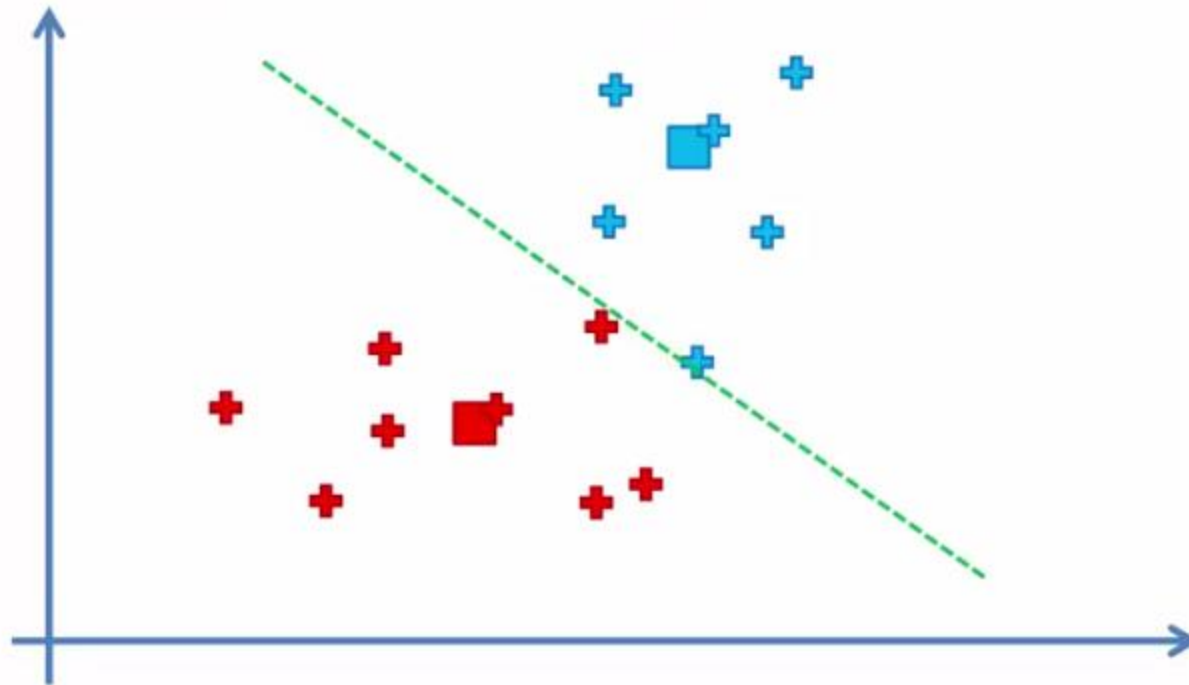
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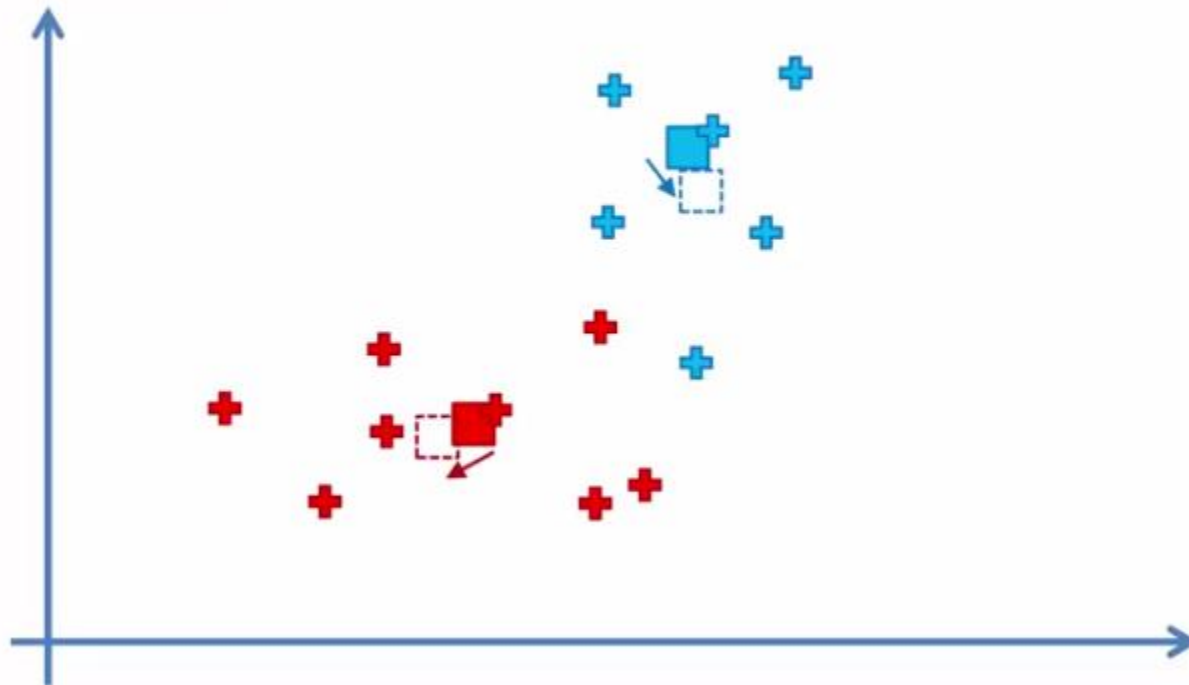
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# K-Means

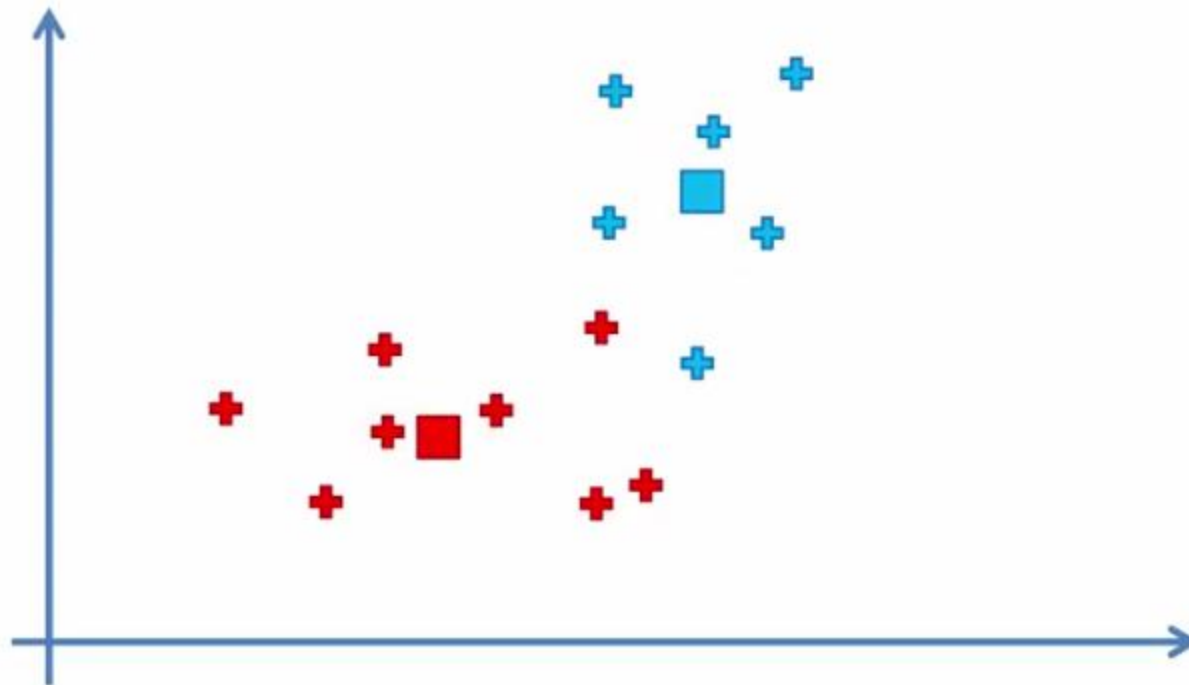
STEP 4: Compute and place the new centroid of each cluster





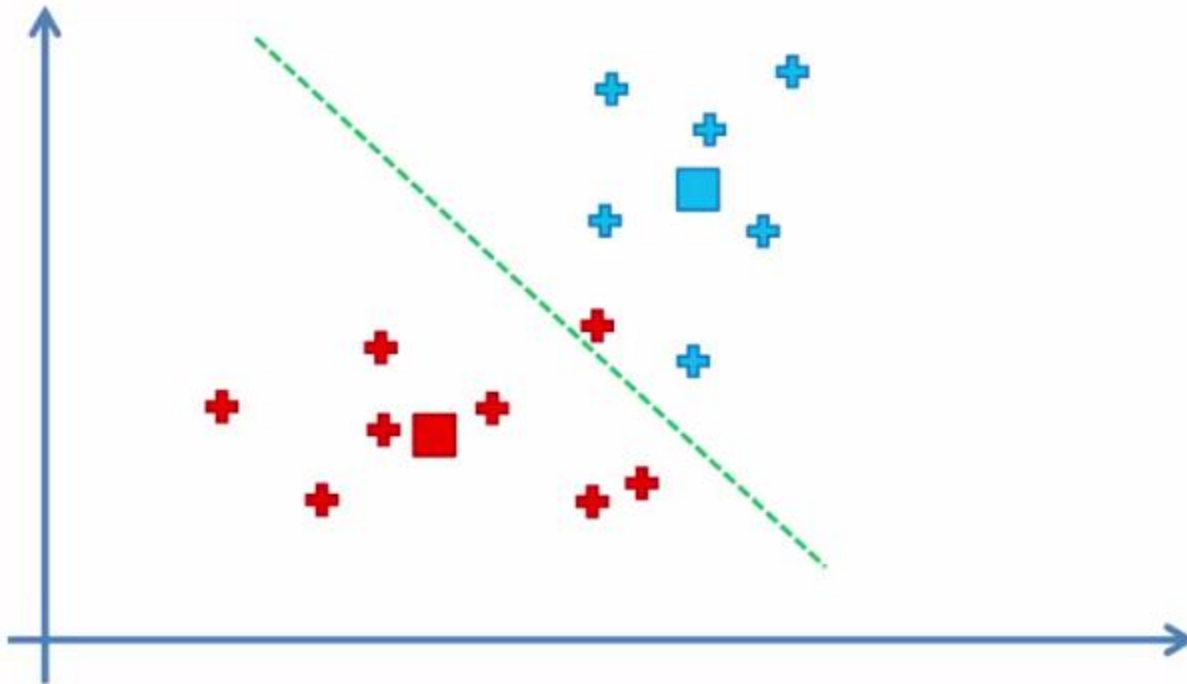
# K-Means

STEP 4: Compute and place the new centroid of each cluster



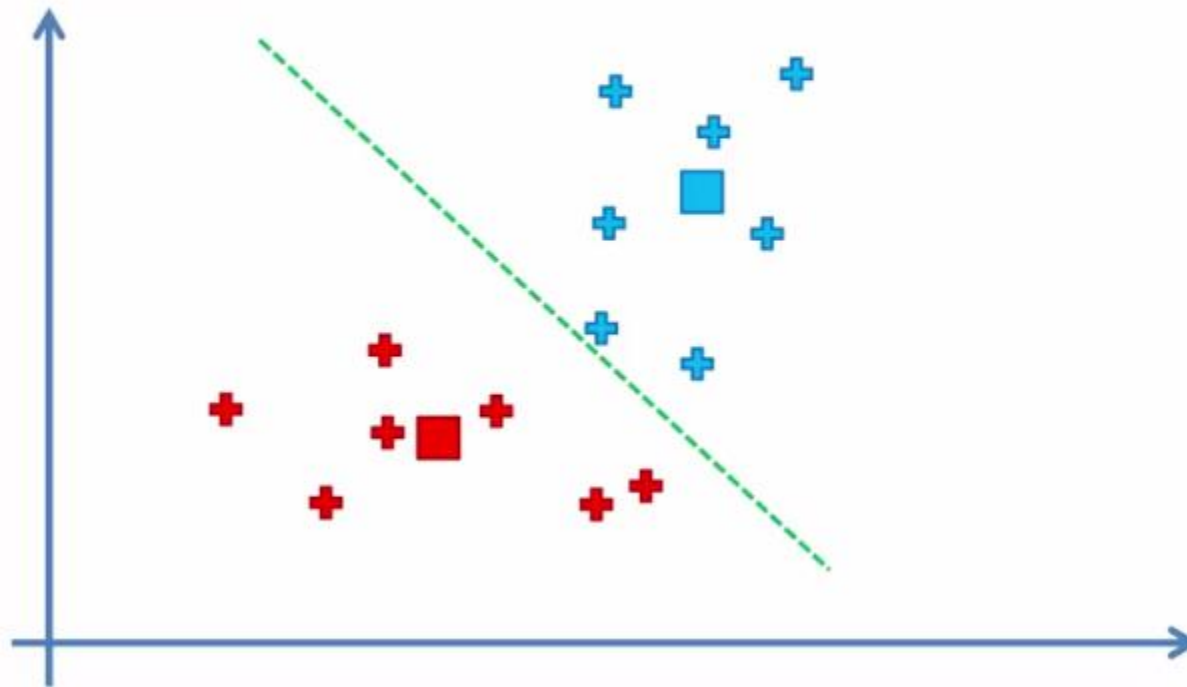
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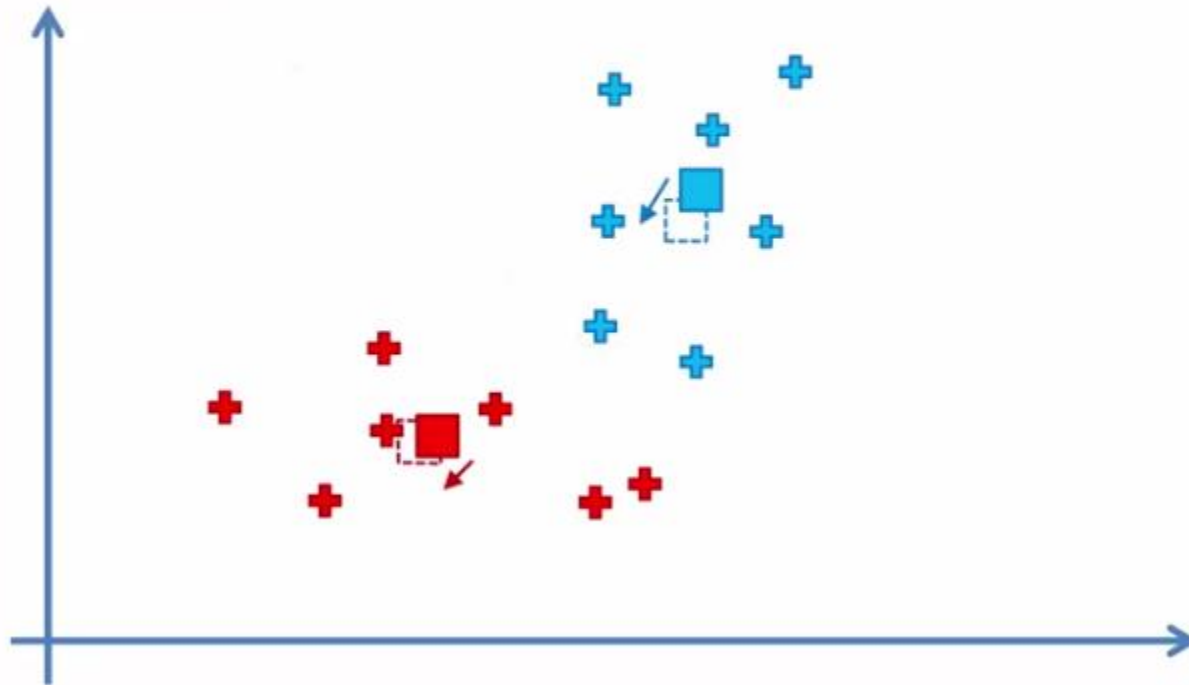
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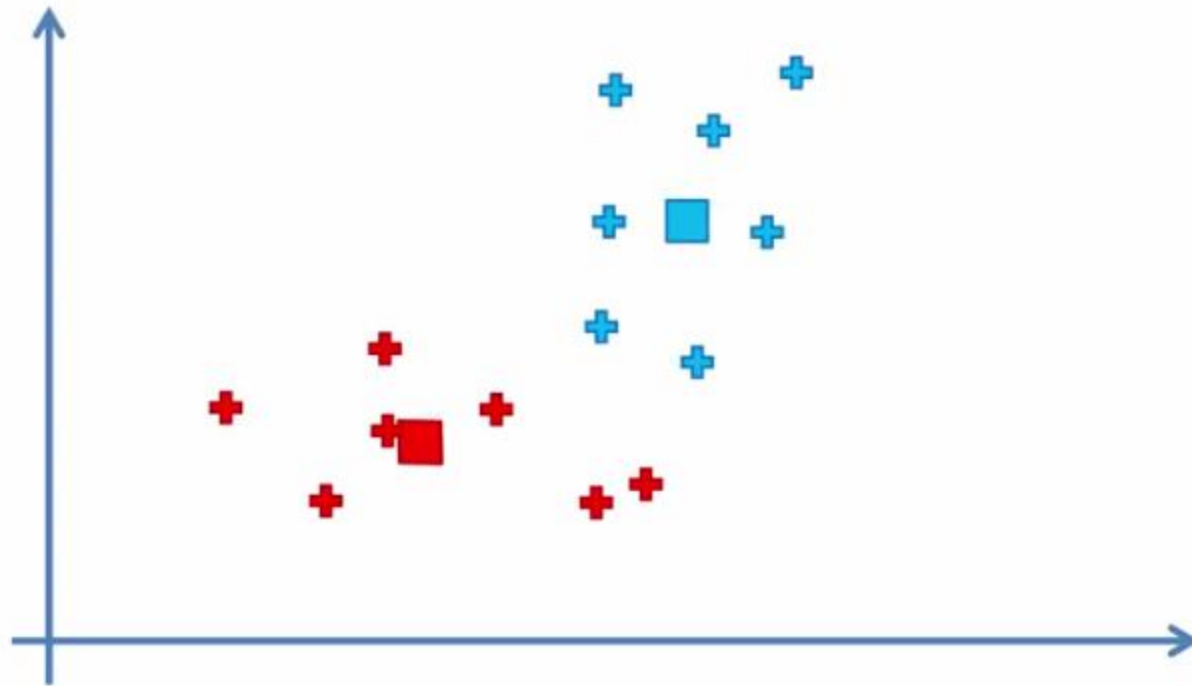
# K-Means

STEP 4: Compute and place the new centroid of each cluster



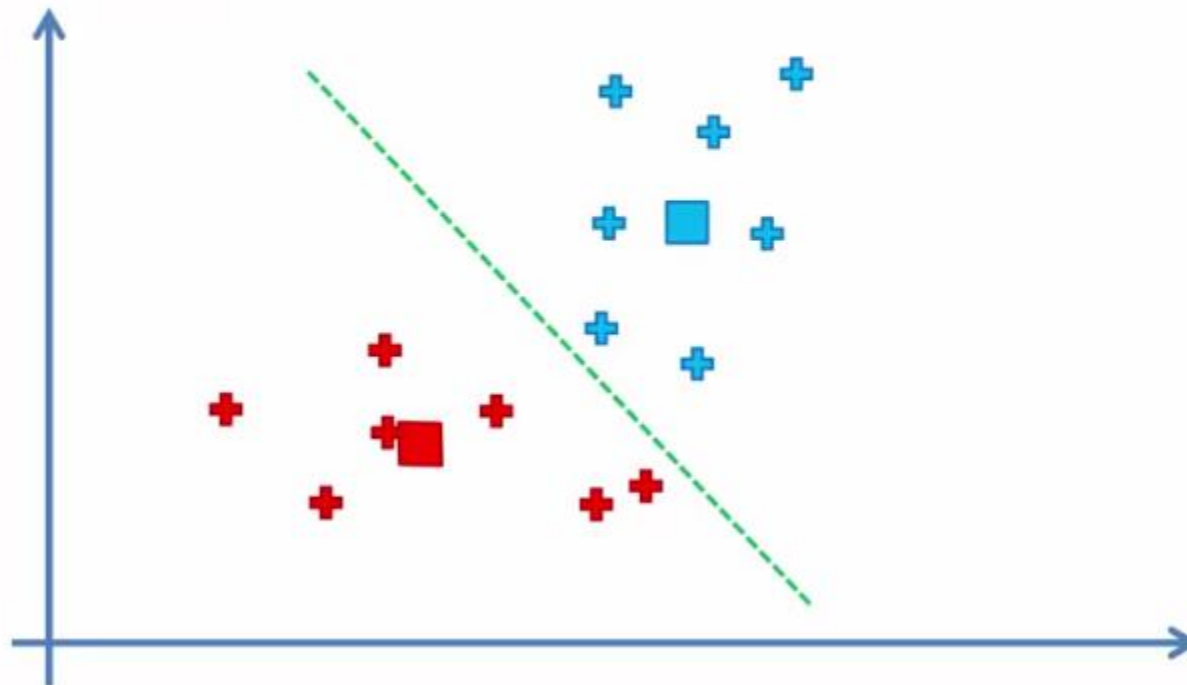
# K-Means

STEP 4: Compute and place the new centroid of each cluster



# K-Means

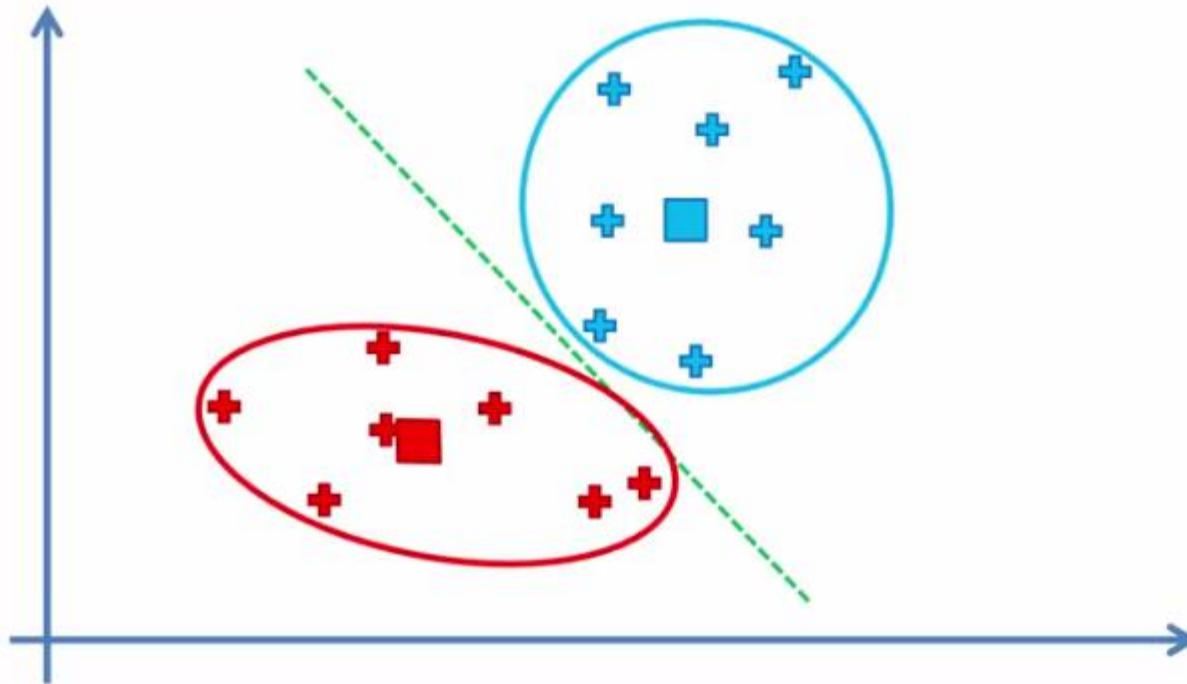
STEP 5: Reassign each data point to the new closest centroid.  
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# K-Means

- **Convergiu!**

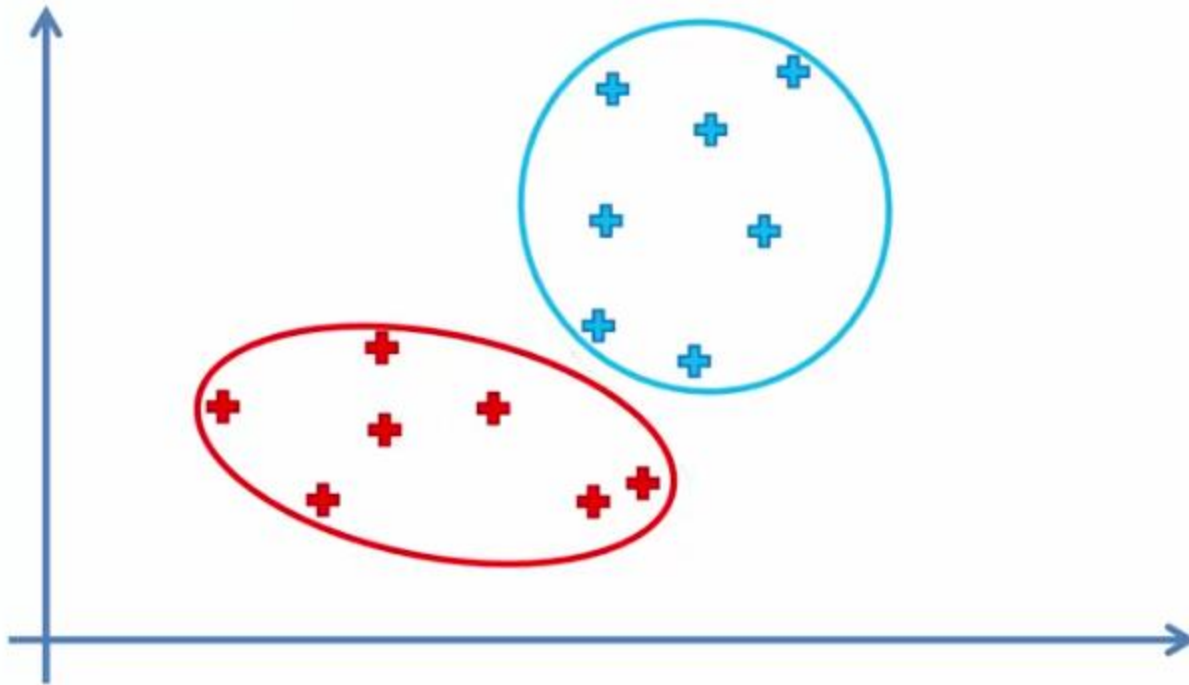
FIN: Your Model Is Ready



# K-Means

- **Convergiu!**

FIN: Your Model Is Ready





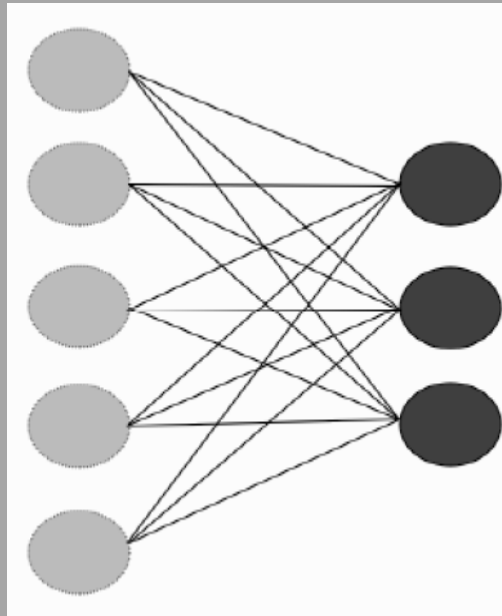
# K-Means

- **Algoritmo pode ser aplicado em dados de  $n$  dimensões!**



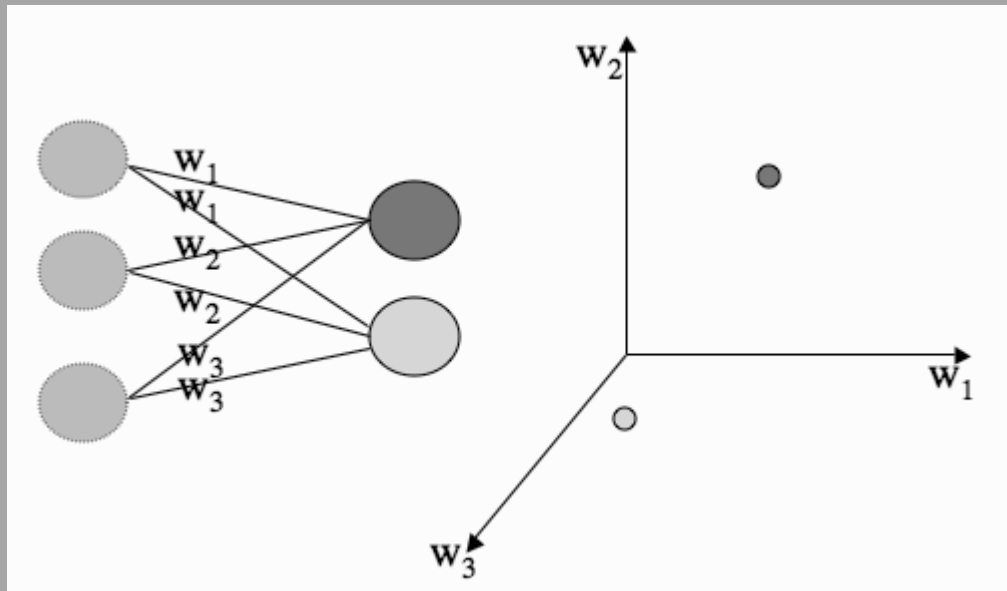
# Rede Neural K-Means

- O treinamento da rede é feito pela movimentação dos neurônios através do ajuste dos pesos
- Uma rede neural de uma única camada consegue implementar o algoritmo K-Means (K neurônios saída)



# Espaço de Pesos

- Posição dos neurônios no espaço de pesos.



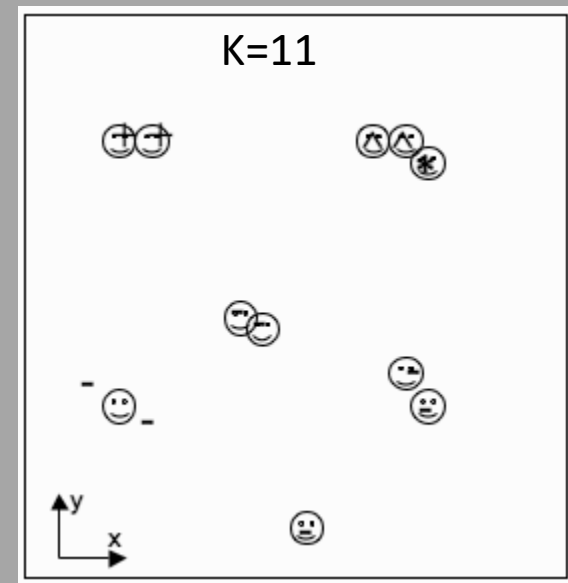
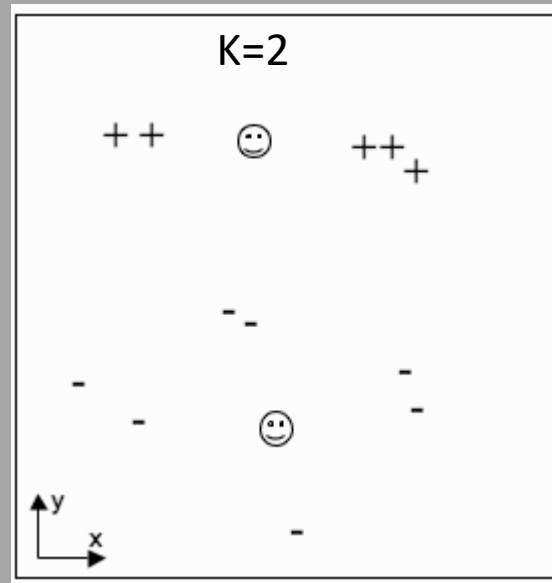
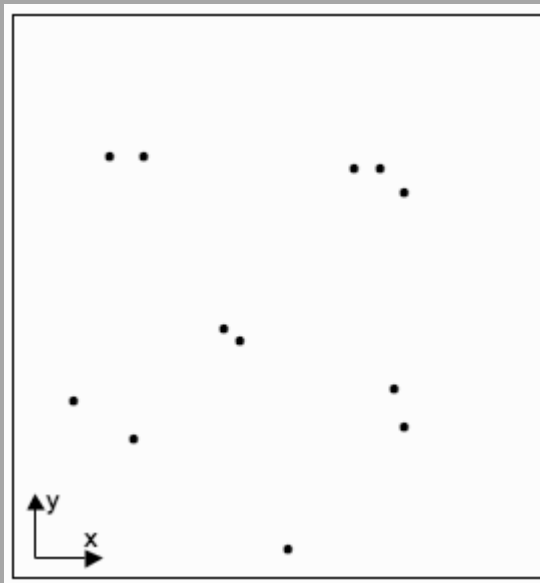
# Espaço de Entrada

- Também é possível calcular a posição dos neurônios no espaço de entrada ( $x_1, x_2, x_3$ ), usando a distância euclidiana.
- Pode-se calcular as distâncias entre os neurônios e entradas para determinar quais neurônios estão mais próximos ou distantes.
- Neurônios próximos representariam um mesmo estímulo.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

# Algoritmo K-Means

- É possível alinhar o espaço de pesos e o espaço de entrada da rede neural de maneira a criar uma representação das entradas na rede.
- Algoritmo K-Means: cria-se K categorias e depois se alinha as categorias (clusters) aos dados de entrada.
- O centro de cada cluster é calculado pela média das distâncias das entradas mais próximas àquele cluster.



# Rede Neural K-Means

- Localiza-se os centros dos clusters no espaço de pesos
- Posiciona-se os neurônios nesses lugares através do treinamento da rede
- A posição de cada neurônio é a sua posição no espaço de pesos.
- Decide-se quais clusters estão mais próximos ao se calcular a distância de cada neurônio em relação à todos os clusters.
- Para cada entrada, determina-se a distância entre o neurônio no espaço de pesos e a entrada.

# Aprendizado Competitivo

- Na camada de saída, os neurônios competem para ativar, de forma que apenas um neurônio por vez é ativado na camada de saída.
- Cada neurônio representa uma categoria (K neurônios)
- Para cada entrada, apenas o neurônio com máxima ativação  $h$  é o vencedor.
- O neurônio que vence é aquele mais próximo da entrada atual.
- Treinamento: move-se o neurônio vencedor para mais perto da entrada atual. Apenas os pesos dos vencedores são alterados
- O vetor de pesos é normalizado entre 0 e 1 para evitar distorções numéricas na competição (pesos muito grandes (10, 1000 etc), sempre iriam ganhar).

$$\Delta w_{ij} = \eta(x_j - w_{ij}),$$

# Algoritmo K-Means

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## The On-Line $k$ -Means Algorithm

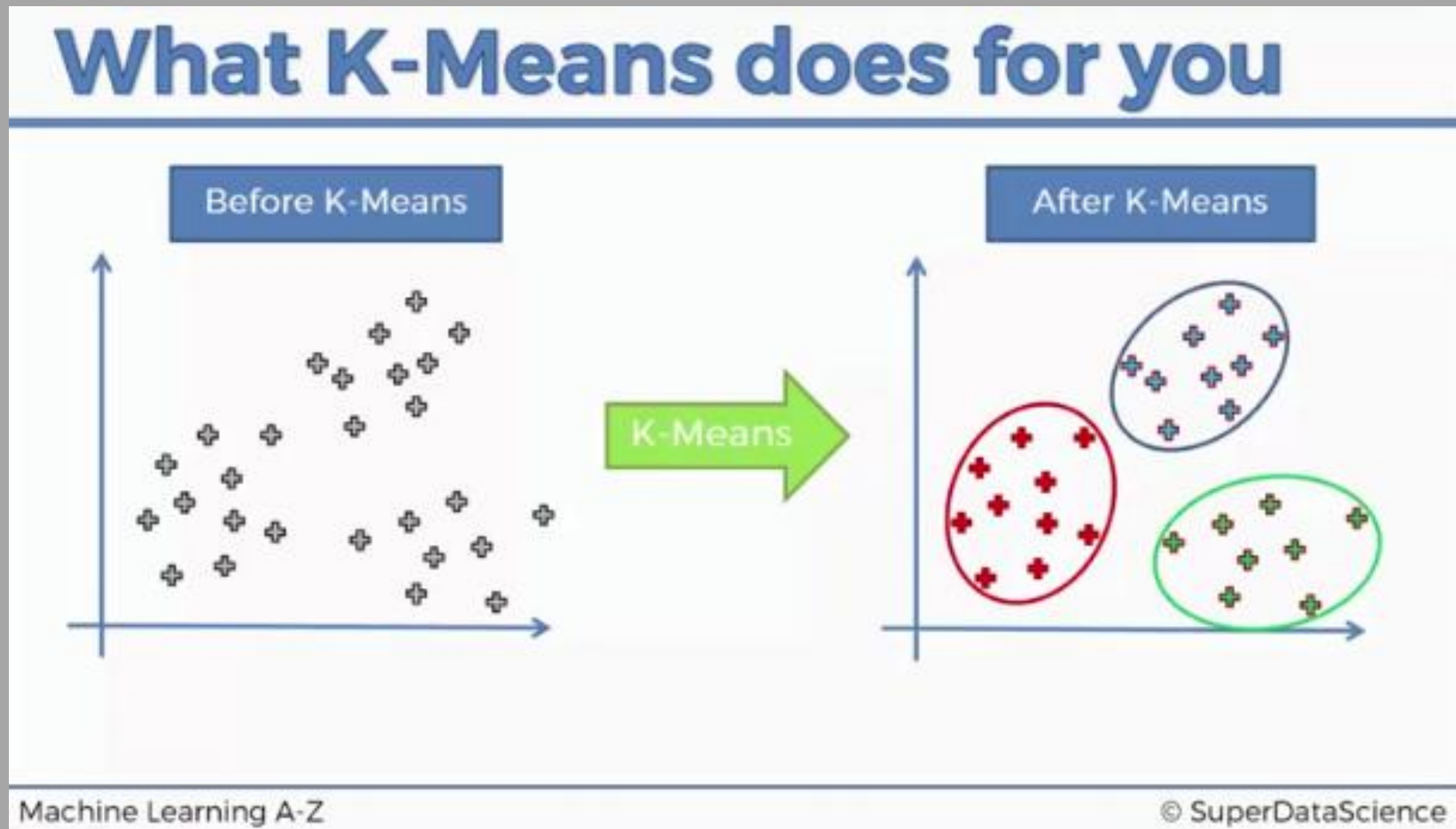
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- **Initialisation**
  - choose a value for  $k$ , which corresponds to the number of output nodes
  - initialise the weights to have small random values
- **Learning**
  - normalise the data so that all the points lie on the unit sphere
  - repeat:
    - \* for each datapoint:
      - compute the activations of all the nodes
      - pick the winner as the node with the highest activation
      - update the weights using Equation (14.7)  $\Delta w_{ij} = \eta(x_j - w_{ij})$ ,
    - \* until number of iterations is above a threshold
- **Usage**
  - for each test point:
    - \* compute the activations of all the nodes
    - \* pick the winner as the node with the highest activation



# Aplicações

- Clustering



# Aplicações

- Aprender informações à respeito da organização dos dados (distribuição não paramétrica)
  - Iris dataset (tipo de flor).
  - 3 Classes: Espécies Iris Setosa, Iris Versicolour, Iris Virginica
- 4 atributos: comprimento e largura da sépala (cm), comprimento e largura da pétala (cm).

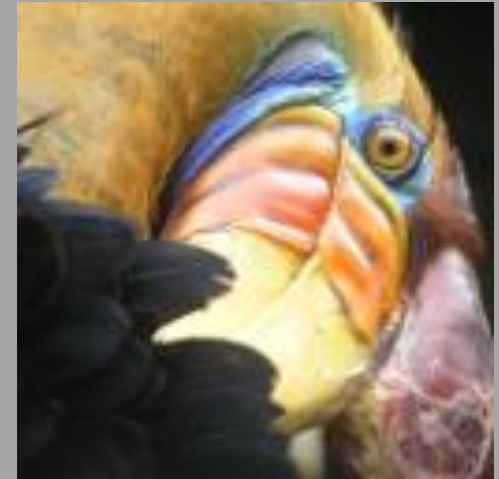


# Aplicações

- `net = kmeansnet.kmeans(3,train)`
- `net.kmeanstrain(train)`
- `cluster = net.kmeansfwd(test)`
- `print cluster`
- `[ 0. 0. 0. 0. 0. 1. 1. 1. 1. 2. 1. 2. 2. 2. 0. 1. 2. 1. 0.`
- `1. 2. 2. 2. 1. 1. 2. 0. 0. 1. 0. 0. 0. 0. 2. 0. 2. 1.]`
- `print iris[3::4,4]`
- `[ 1. 1. 1. 1. 1. 2. 2. 2. 1. 0. 2. 0. 0. 0. 1. 1. 0. 2. 2.`
- `2. 0. 0. 0. 2. 2. 0. 1. 2. 1. 1. 1. 1. 1. 0. 1. 0. 2.]`

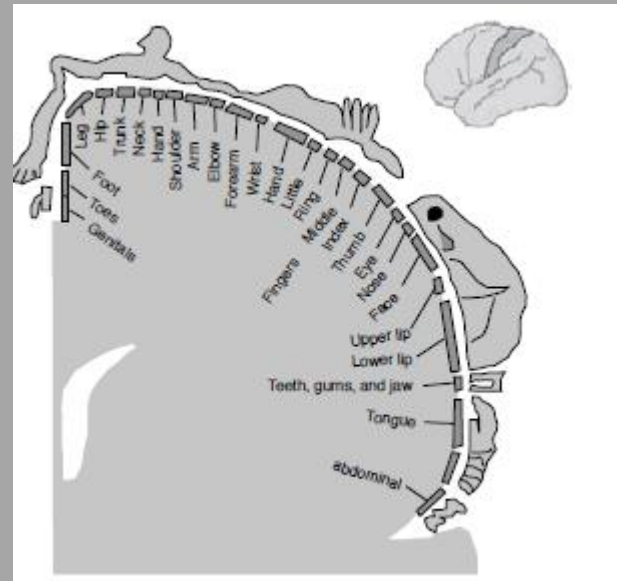
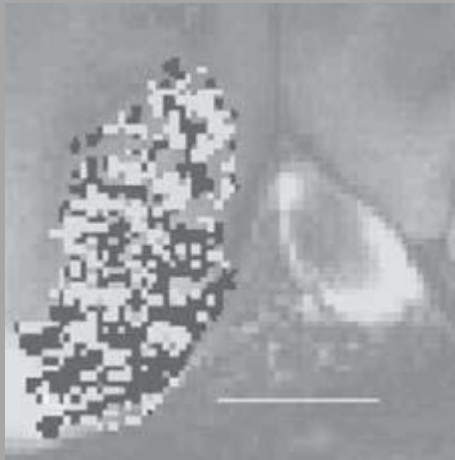
# Aplicações

- Compressão de dados
- Compressão de uma imagem tiff com milhares de cores para uma imagem RGB com apenas 16 cores.
- Usa-se K-means com 16 clusters.
- A imagem original tem as cores dos pixels correspondentes a cada cluster substituídas, gerando a compressão.



# Mapas Corticais

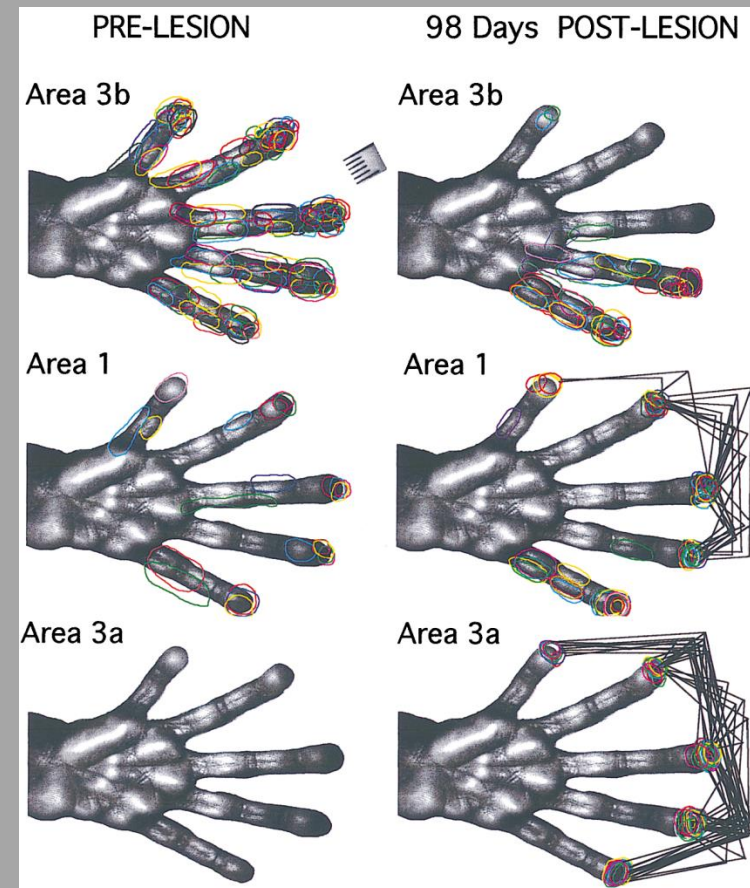
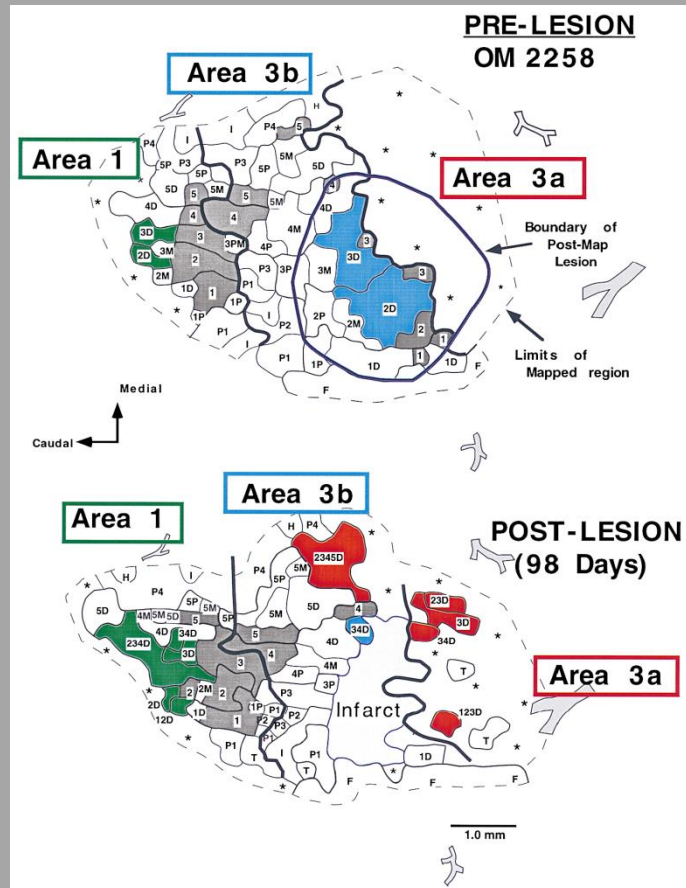
- Organização neocortical
- Mapas de dominância ocular no córtex reveladas por fMRI
- Representação topográfica das áreas sensoriais sensíveis a pressão no córtex somatosensorial
- Atributos de entrada próximos são representados em áreas corticais adjacentes





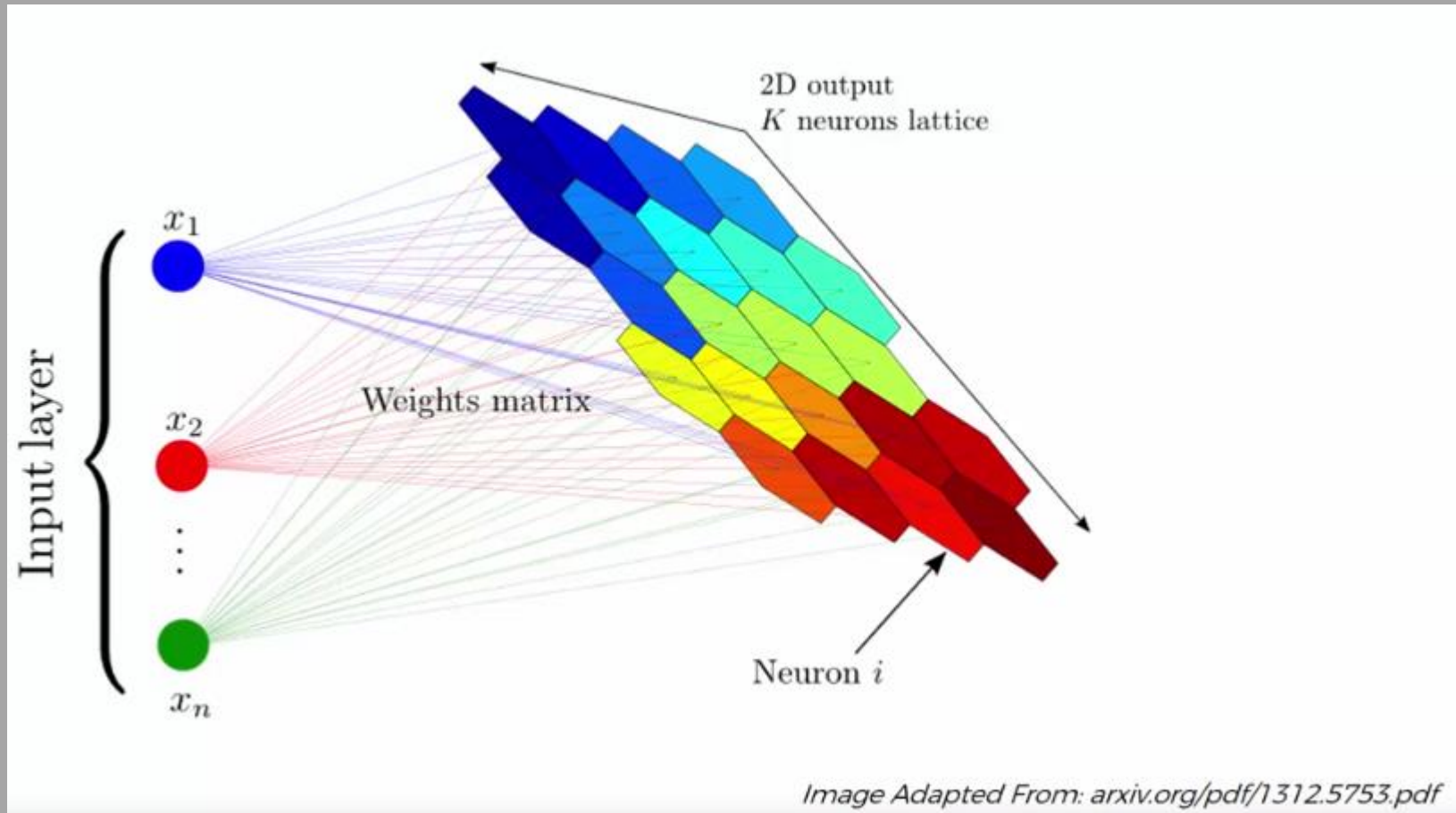
# Lesão Cortical

- Reorganização dos mapas corticais após uma lesão em macacos
- Como essas reorganizações ocorrem?



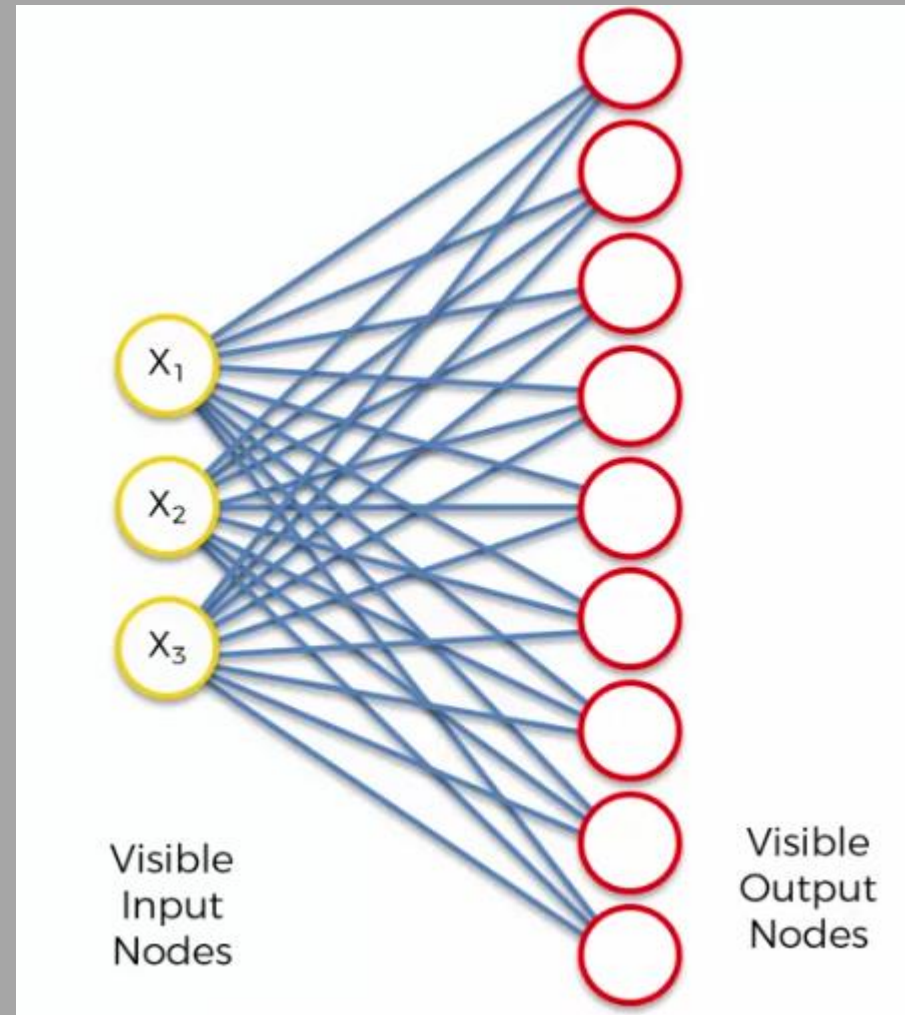
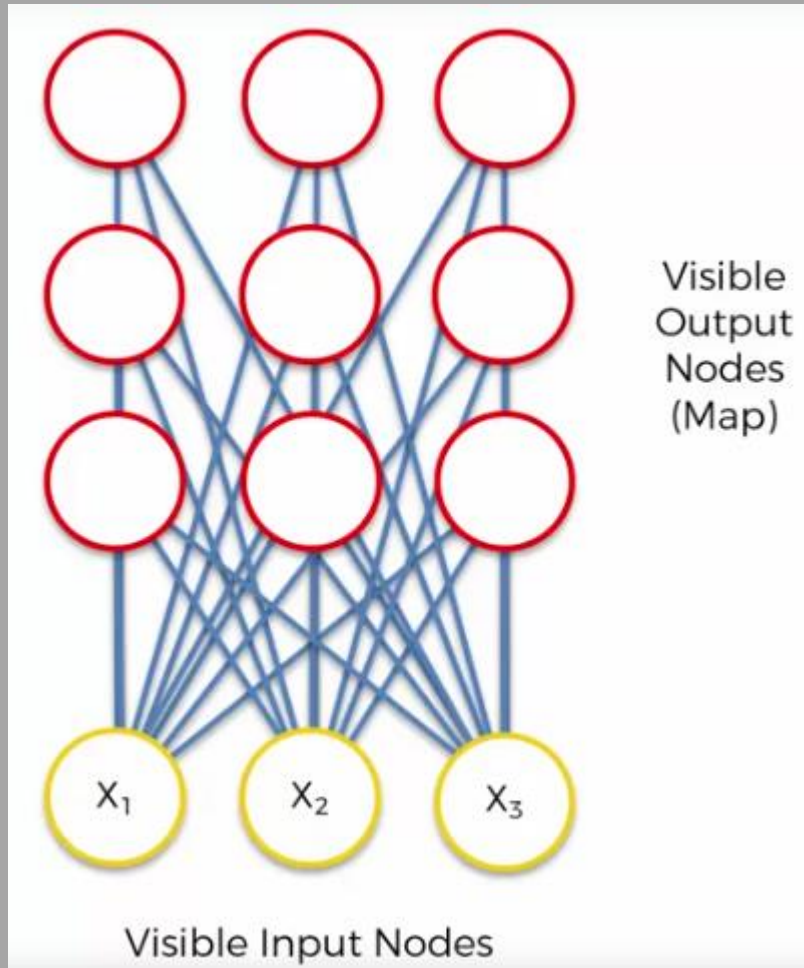
# Mapa Auto-Organizador de Atributos

- Rede do mapa auto-organizado de Kohonen
- Inspirada na representação de mapas sensoriais no córtex cerebral



# Mapa Auto-Organizador de Atributos

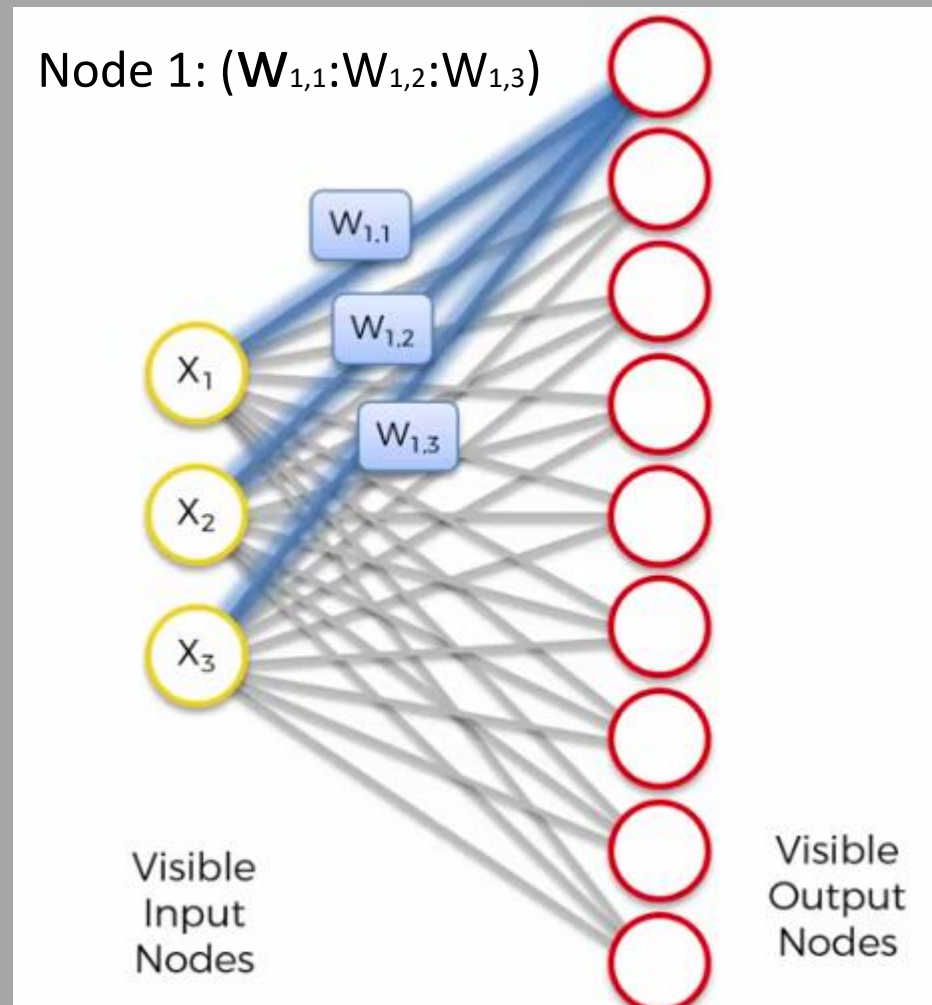
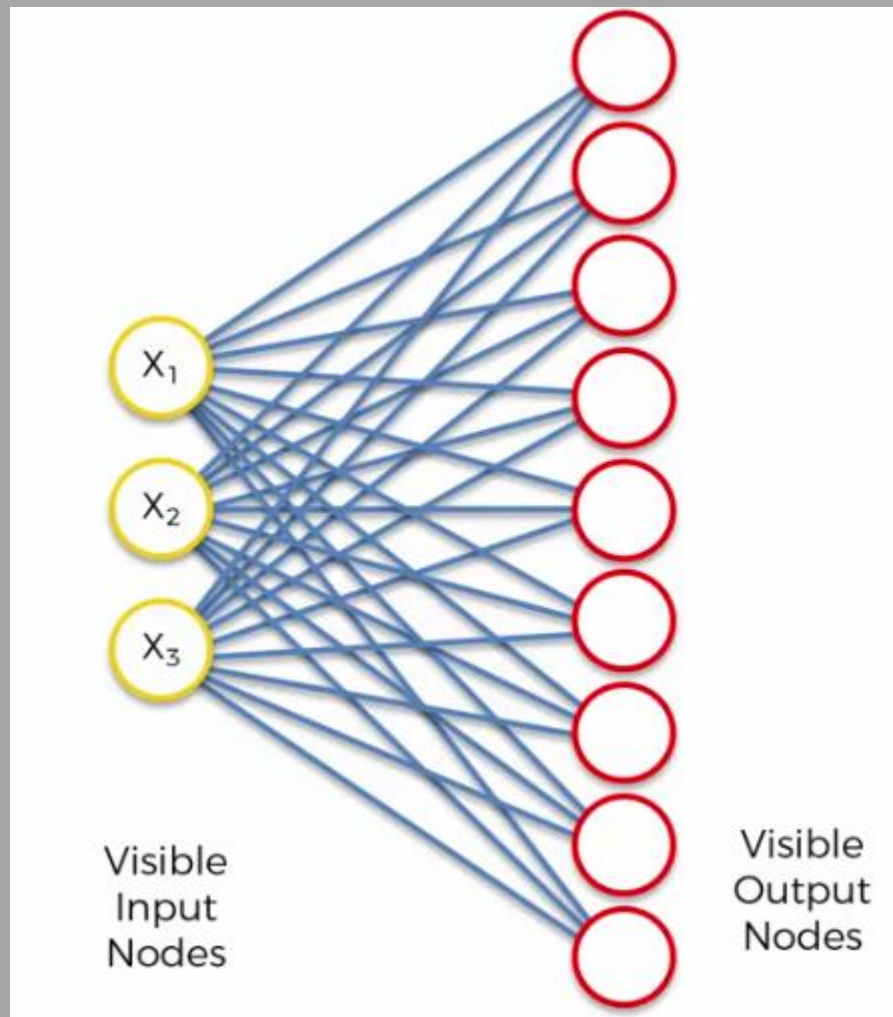
- Mudando a visualização





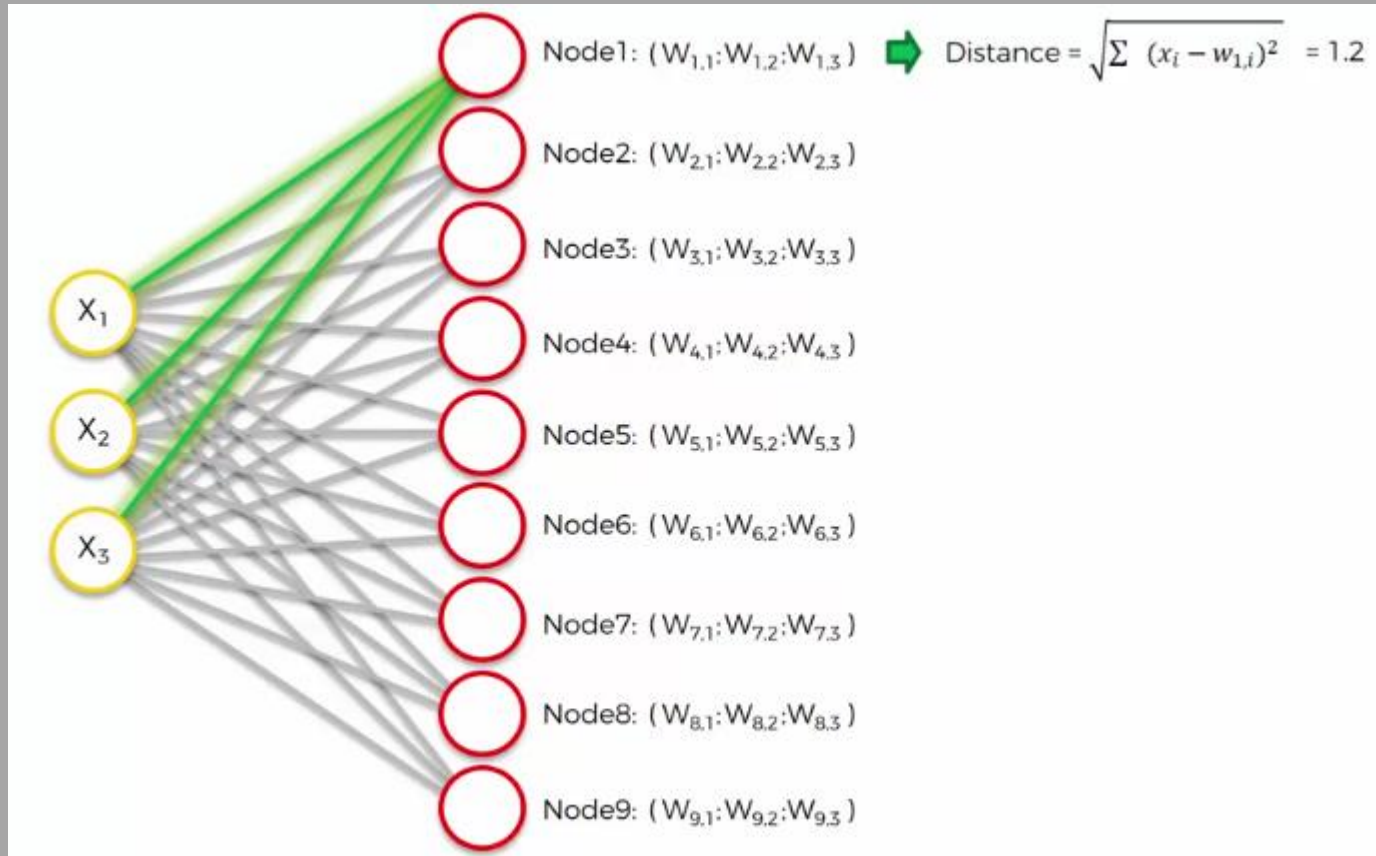
# Mapa Auto-Organizador de Atributos

- Pesos são as coordenadas espaciais do nó no espaço de pesos



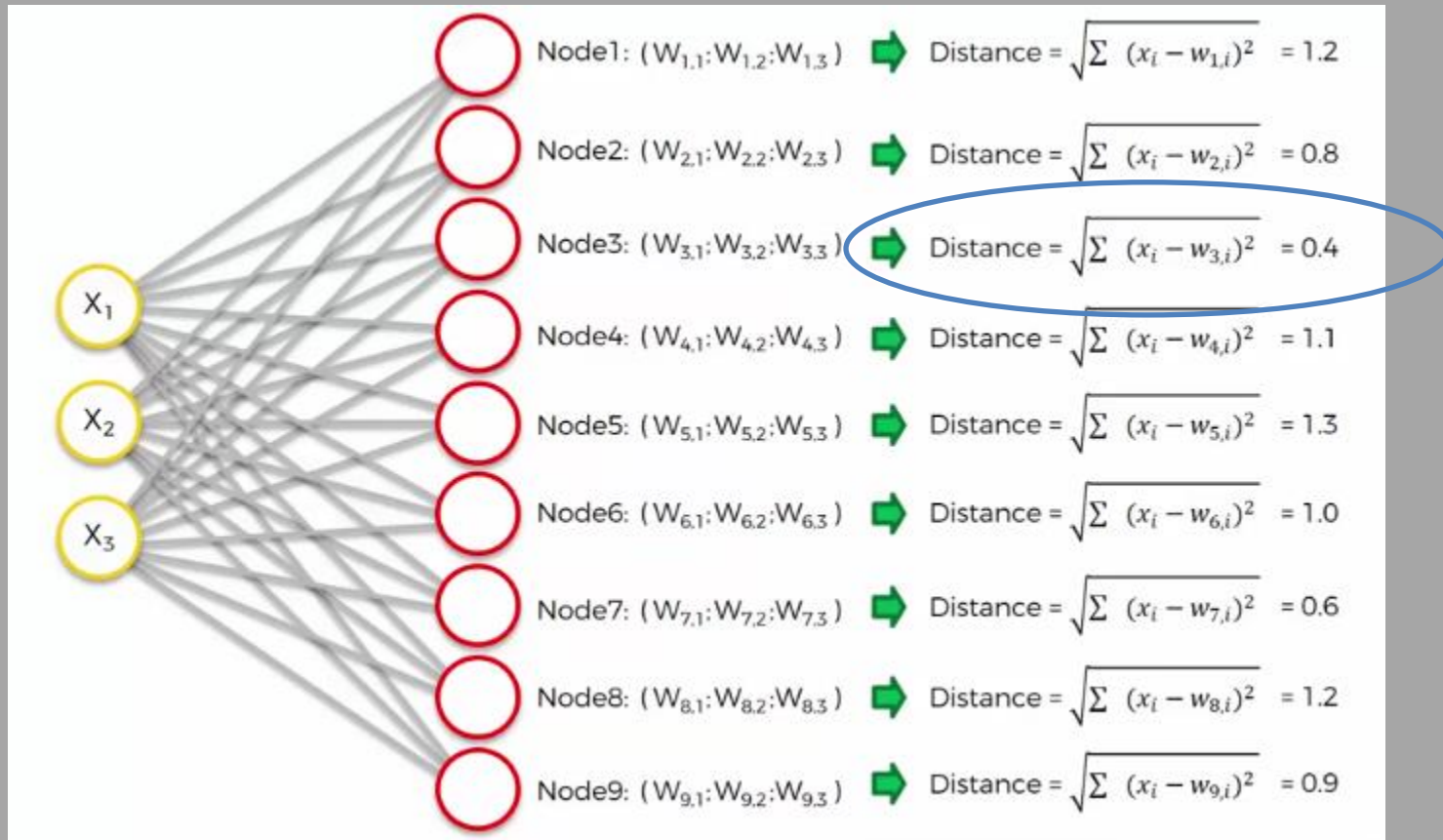
# Mapa Auto-Organizador de Atributos

- Calcule o nó com a menor distância do dado de entrada



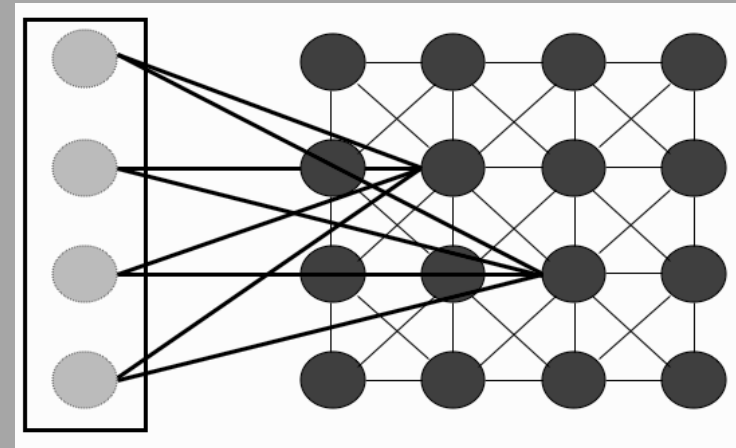
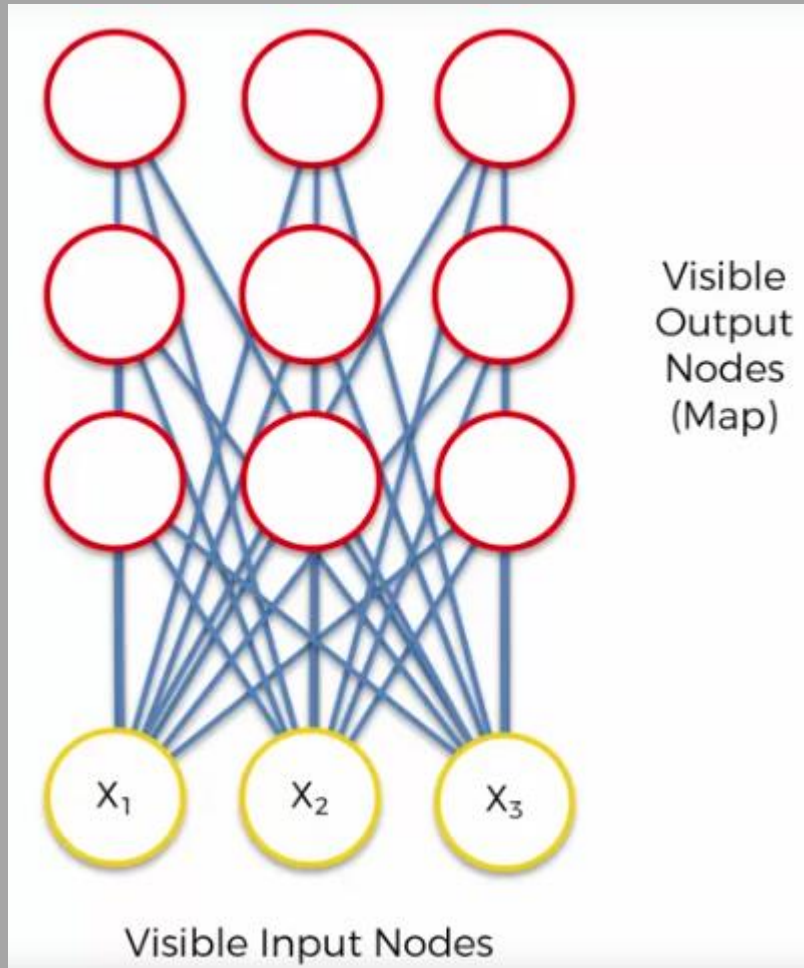
# Mapa Auto-Organizador de Atributos

- Competição: calcule o nó com a menor distância dos dados de entrada



# Mapa Auto-Organizador de Atributos

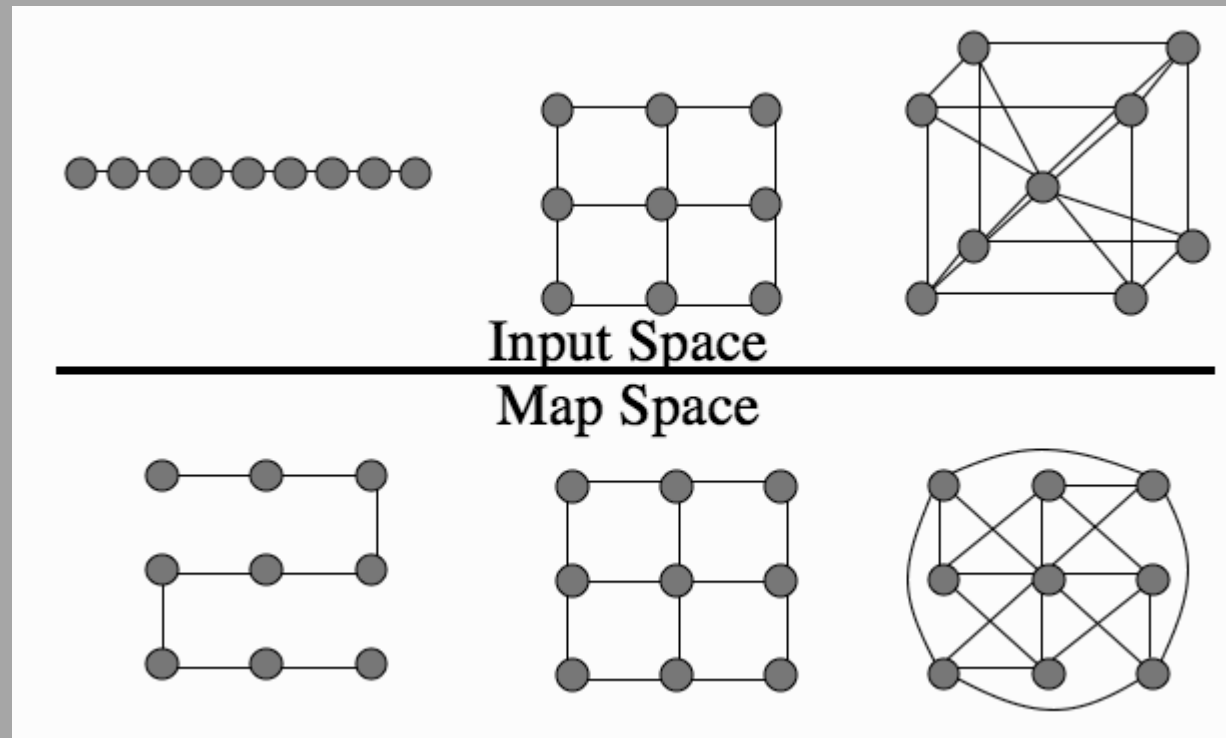
- Exemplos



MACHINE LEARNING: An Algorithmic Perspective

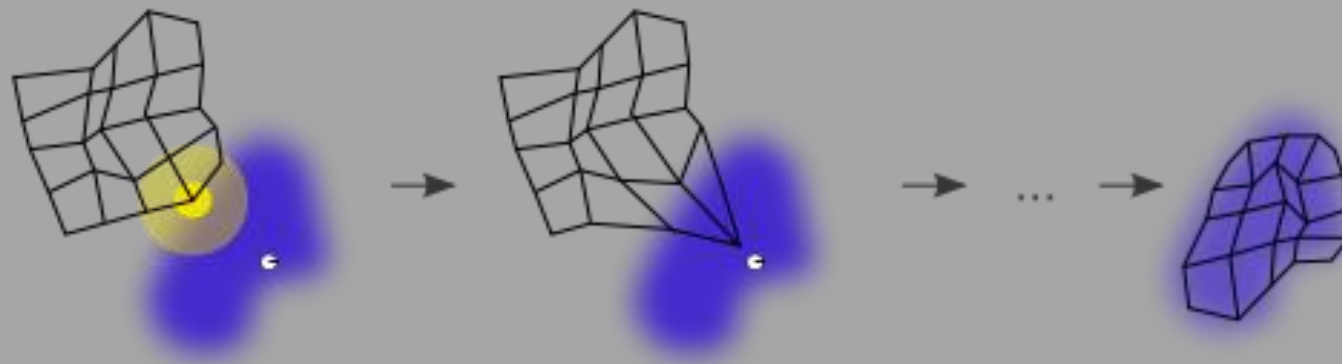
# Mapa Auto-Organizador de Atributos

- Representação do espaço de entrada no espaço do mapa
- Neurônios próximos representam atributos similares
- Aprendizagem não supervisionada



# Como o Mapa Auto-Organizado Aprende

- O mapa se auto-organiza movendo-se na direção dos dados
- Os nós vencedores aumentam o próprio peso e o peso dos outros nós em seu raio de ação, movendo-se na direção dos dados
- Existe competição entre os nós com intersecção de raio de ação
- O raio de ação dos nós vencedores é gradativamente reduzido ao longo do processo



[https://en.wikipedia.org/wiki/Self-organizing\\_map](https://en.wikipedia.org/wiki/Self-organizing_map)

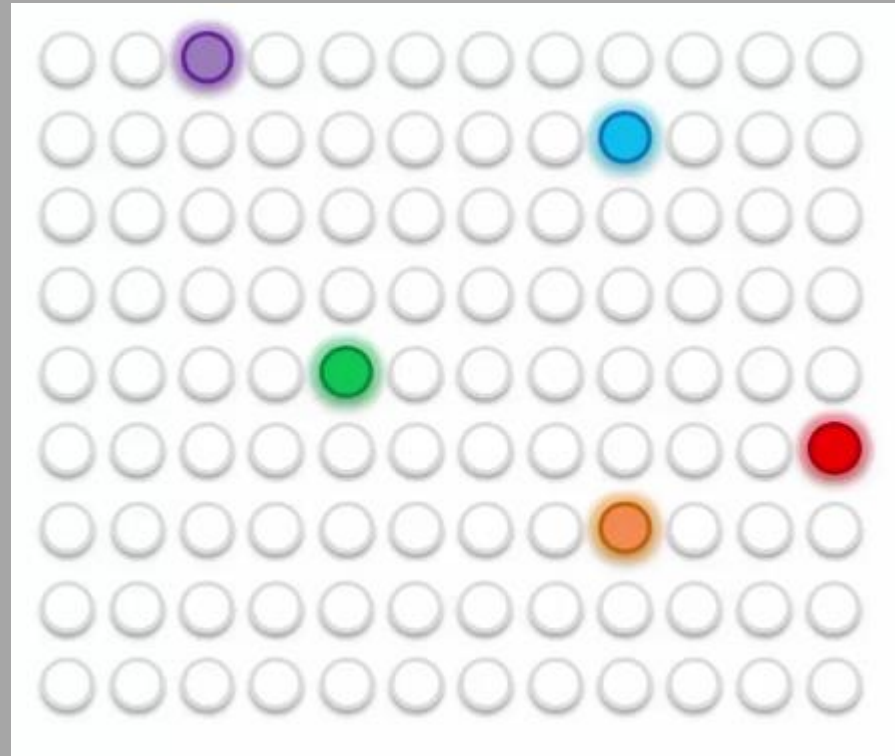
<https://www.superdatascience.com/pages/deep-learning>



# Como o Mapa Auto-Organizado Aprende

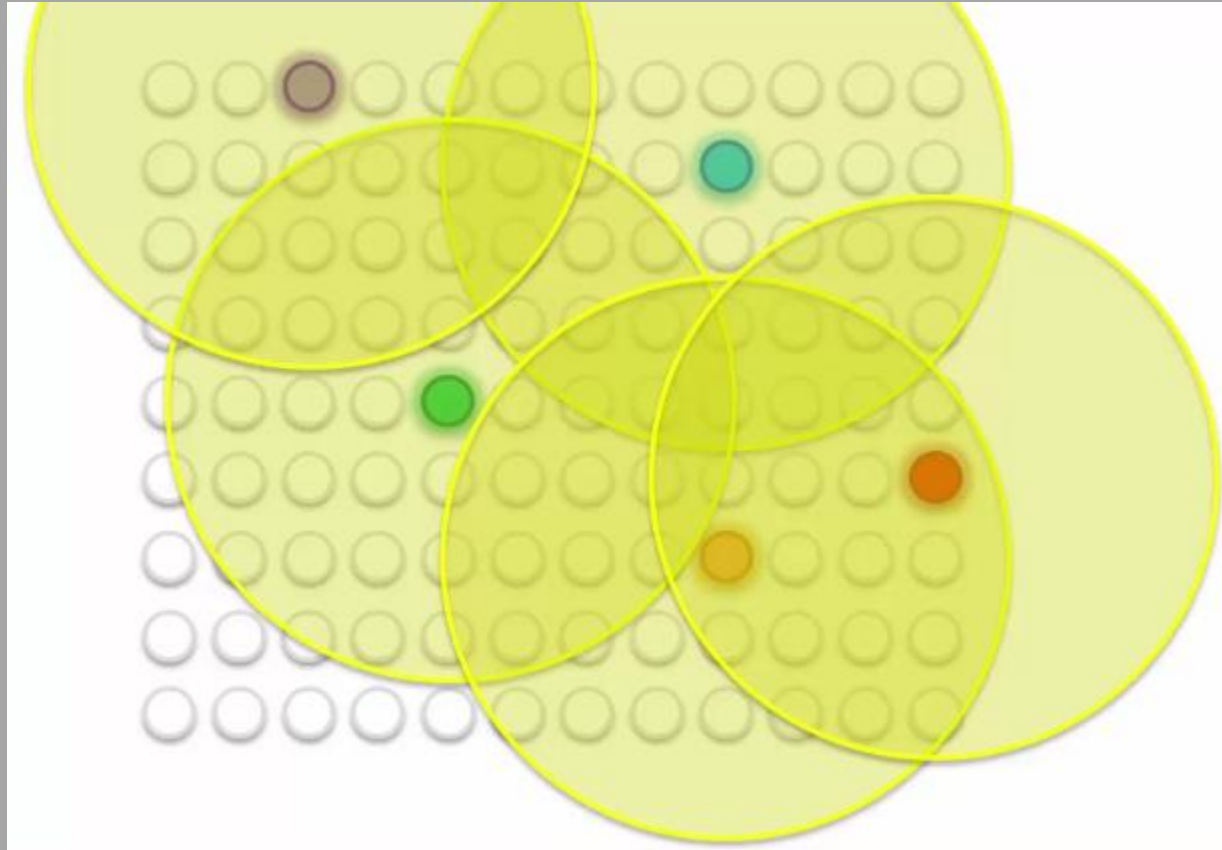
- Auto-organização do mapa

Nós vencedores



# Como o Mapa Auto-Organizado Aprende

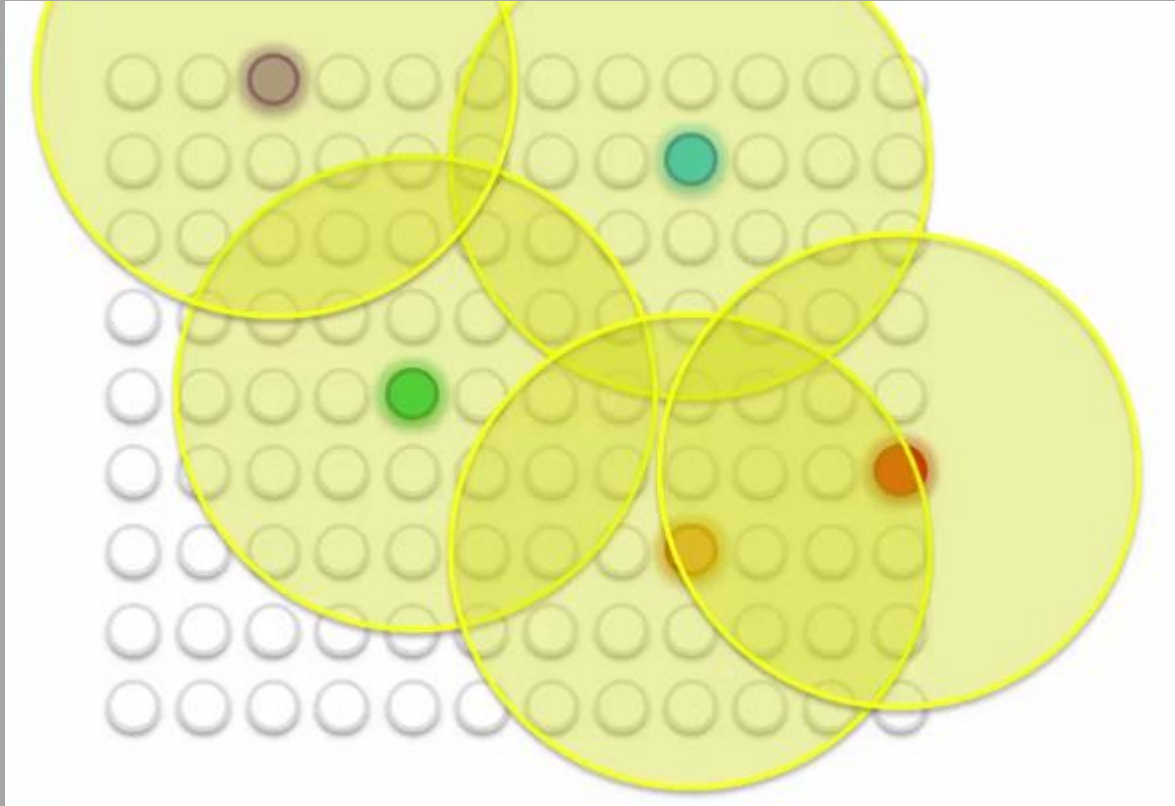
- Raio de ação





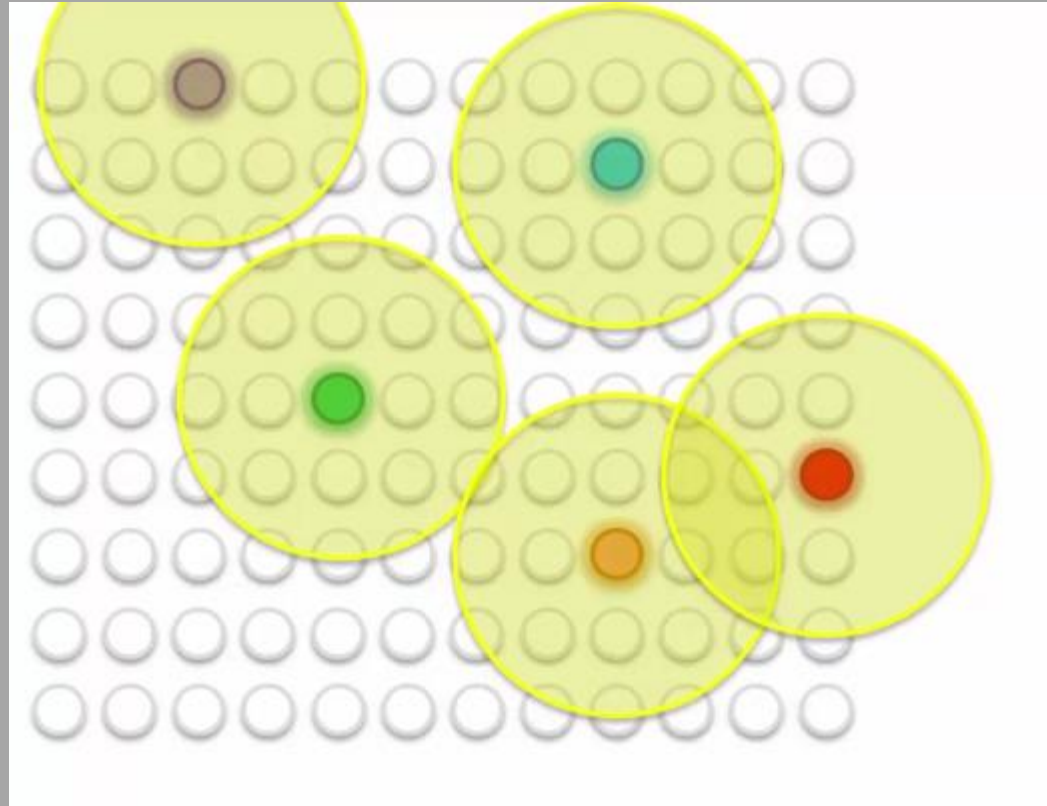
# Como o Mapa Auto-Organizado Aprende

- Raio de ação



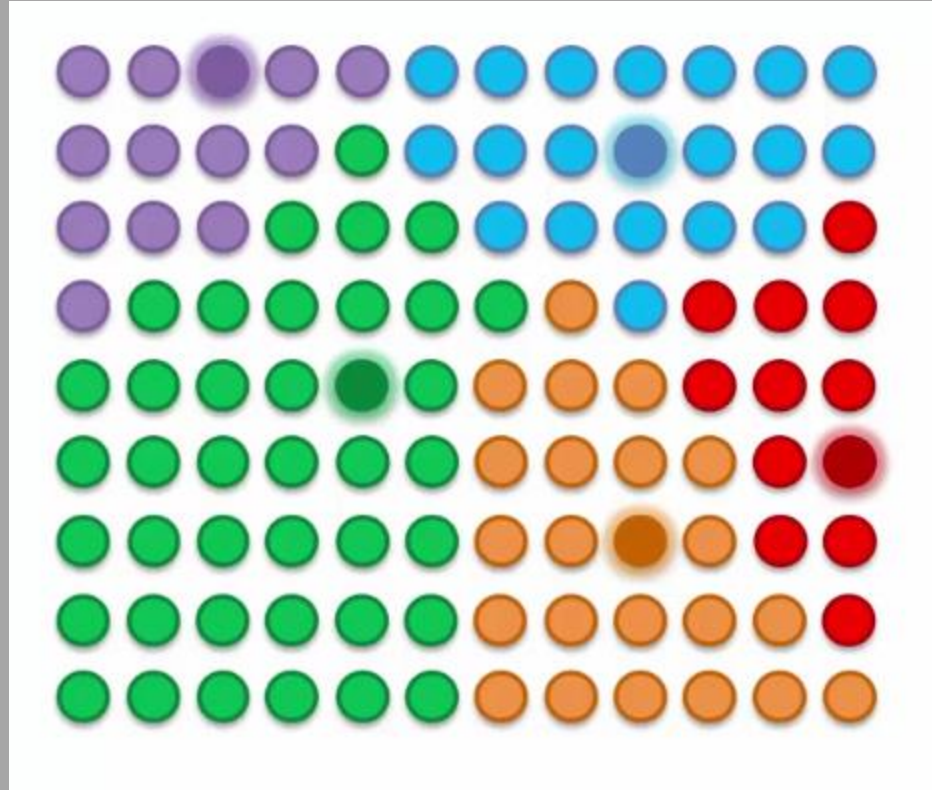
# Como o Mapa Auto-Organizado Aprende

- Raio de ação



# Como o Mapa Auto-Organizado Aprende

- Formação do mapa



# Algoritmo do Mapa Auto-Organizado

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## The Self-Organising Feature Map Algorithm

---

- **Initialisation**

- choose a size (number of neurons) and number of dimensions  $d$  for the map
- either:
  - \* choose random values for the weight vectors so that they are all different OR
  - \* set the weight values to increase in the direction of the first  $d$  principal components of the dataset

- **Learning**

- repeat:
  - \* for each datapoint:
    - select the best-matching neuron  $n_b$  using the minimum Euclidean distance between the weights and the input,

$$n_b = \min_j \|\mathbf{x} - \mathbf{w}_j^T\|. \quad (14.8)$$

- \* update the weight vector of the best-matching node using:

$$\mathbf{w}_j^T \leftarrow \mathbf{w}_j^T + \eta(t)(\mathbf{x} - \mathbf{w}_j^T), \quad (14.9)$$

# Algoritmo do Mapa Auto-Organizado

where  $\eta(t)$  is the learning rate.

- \* update the weight vector of all other neurons using:

$$\mathbf{w}_j^T \leftarrow \mathbf{w}_j^T + \eta_n(t) h(n_b, t) (\mathbf{x} - \mathbf{w}_j^T), \quad (14.10)$$

where  $\eta_n(t)$  is the learning rate for neighbourhood nodes, and  $h(n_b, t)$  is the neighbourhood function, which decides whether each neuron should be included in the neighbourhood of the winning neuron (so  $h = 1$  for neighbours and  $h = 0$  for non-neighbours)

- \* reduce the learning rates and adjust the neighbourhood function, typically by  $\eta(t+1) = \alpha \eta(t)^{k/k_{\max}}$  where  $0 \leq \alpha \leq 1$  decides how fast the size decreases,  $k$  is the number of iterations the algorithm has been running for, and  $k_{\max}$  is when you want the learning to stop. The same equation is used for both learning rates ( $\eta, \eta_n$ ) and the neighbourhood function  $h(n_b, t)$ .

- until the map stops changing or some maximum number of iterations is exceeded

- Usage

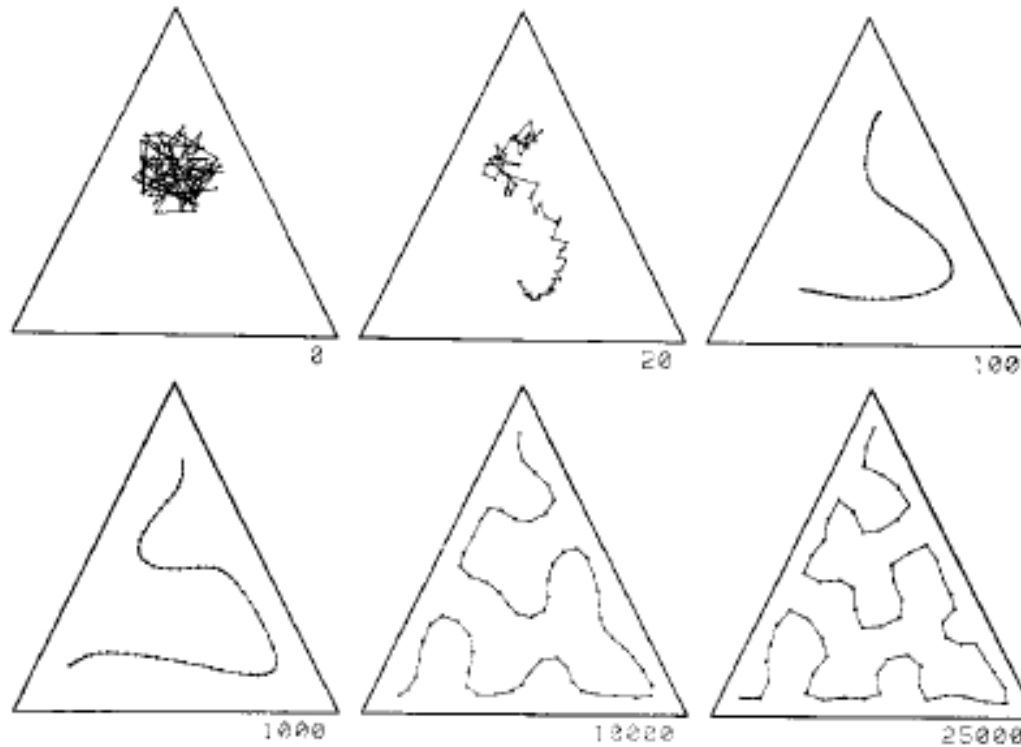
- for each test point:

- \* select the best-matching neuron  $n_b$  using the minimum Euclidean distance between the weights and the input:

$$n_b = \min_j \|\mathbf{x} - \mathbf{w}_j^T\| \quad (14.11)$$

# Self-Organizing Maps (SOM)

1D



**Fig. 4.** Weight vectors during the ordering process, one-dimensional array.

# Self-Organizing Maps (SOM)

2D

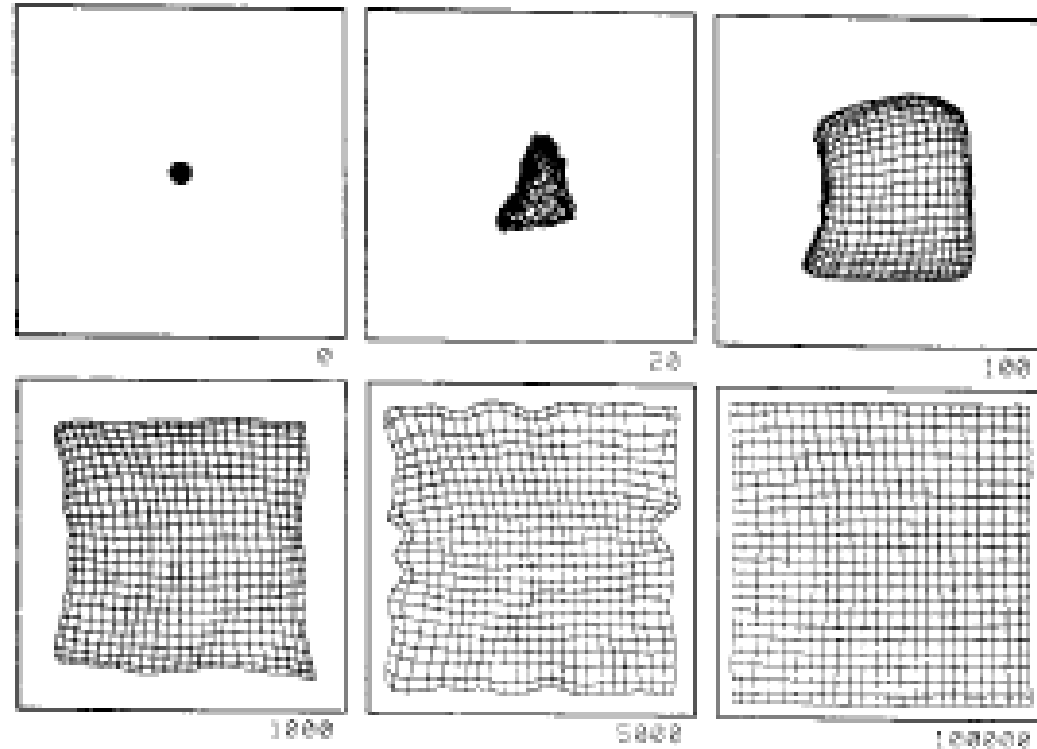
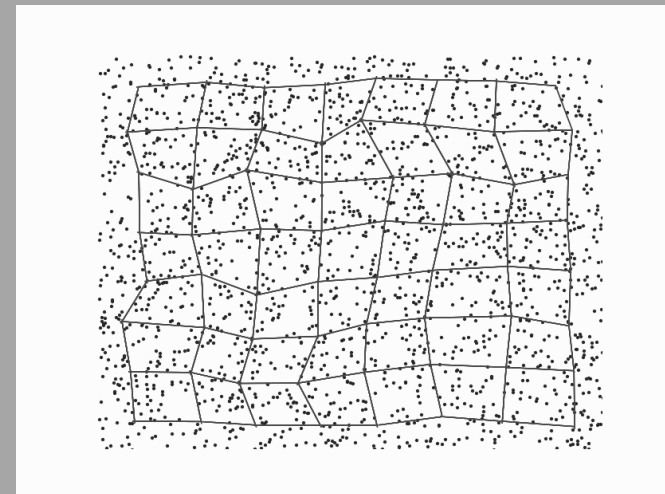
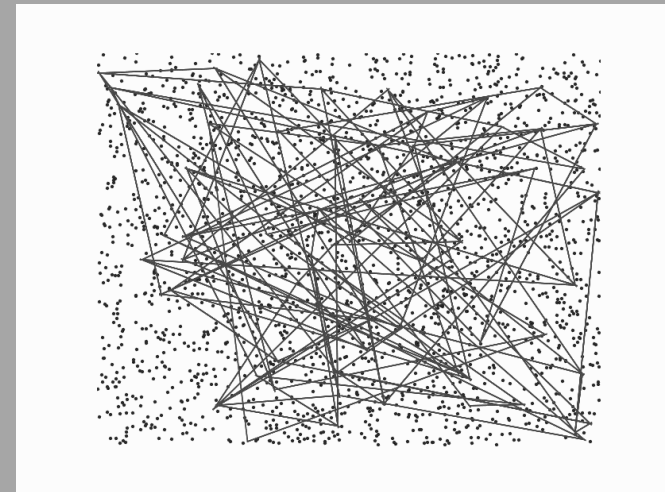


Fig. 3. Weight vectors during the ordering process, two-dimensional array.



# Self-Organizing Maps (SOM)

3D

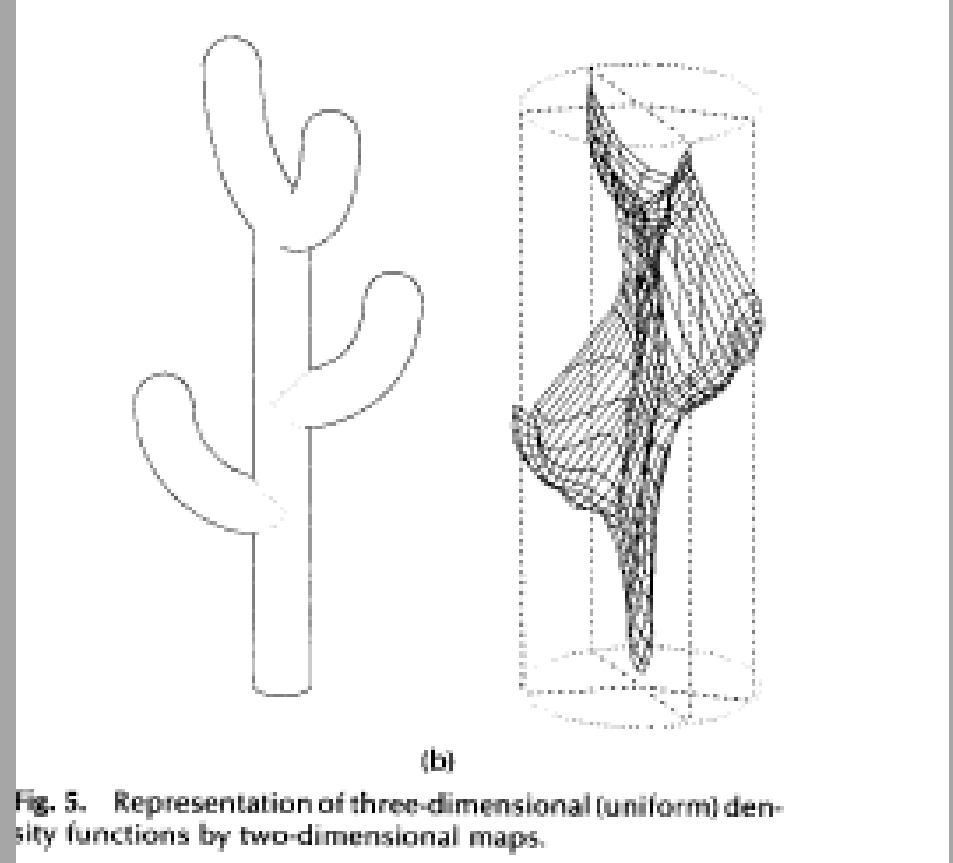
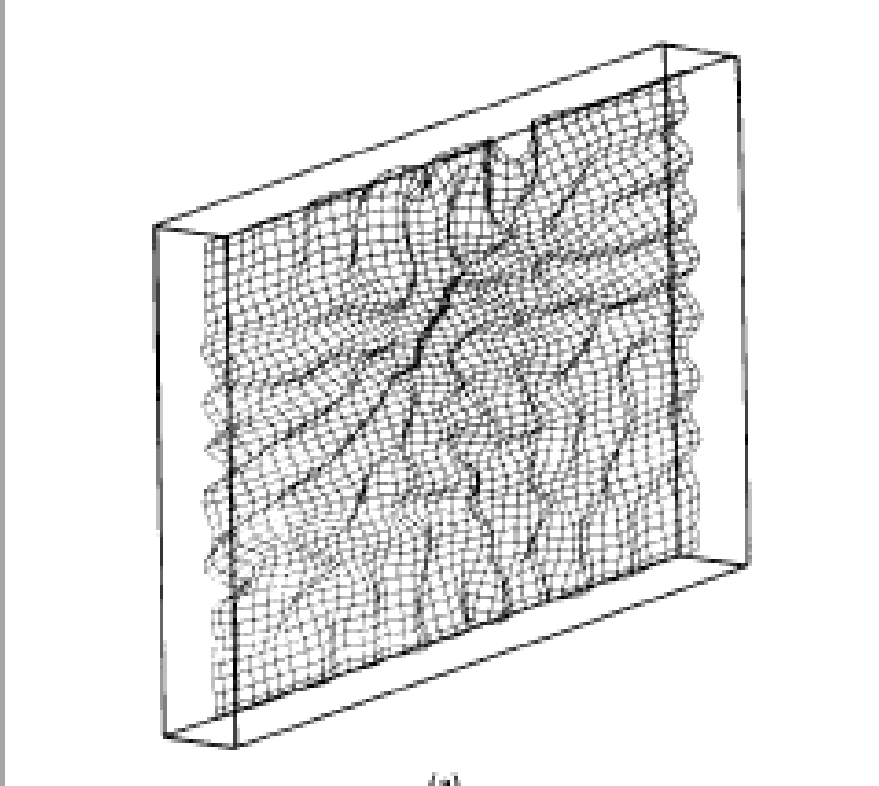
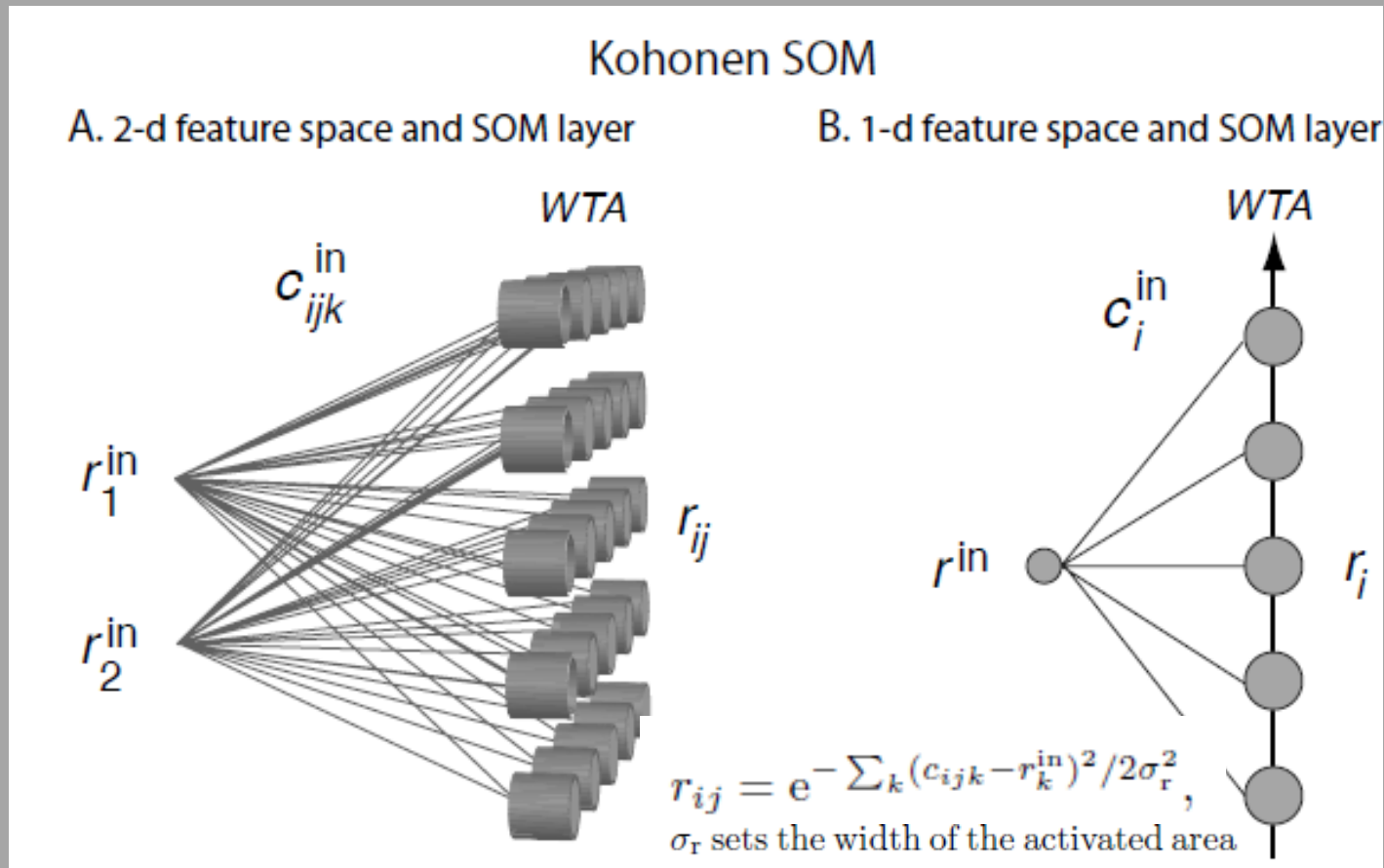


Fig. 5. Representation of three-dimensional (uniform) density functions by two-dimensional maps.



# Self-Organizing Maps (SOM)

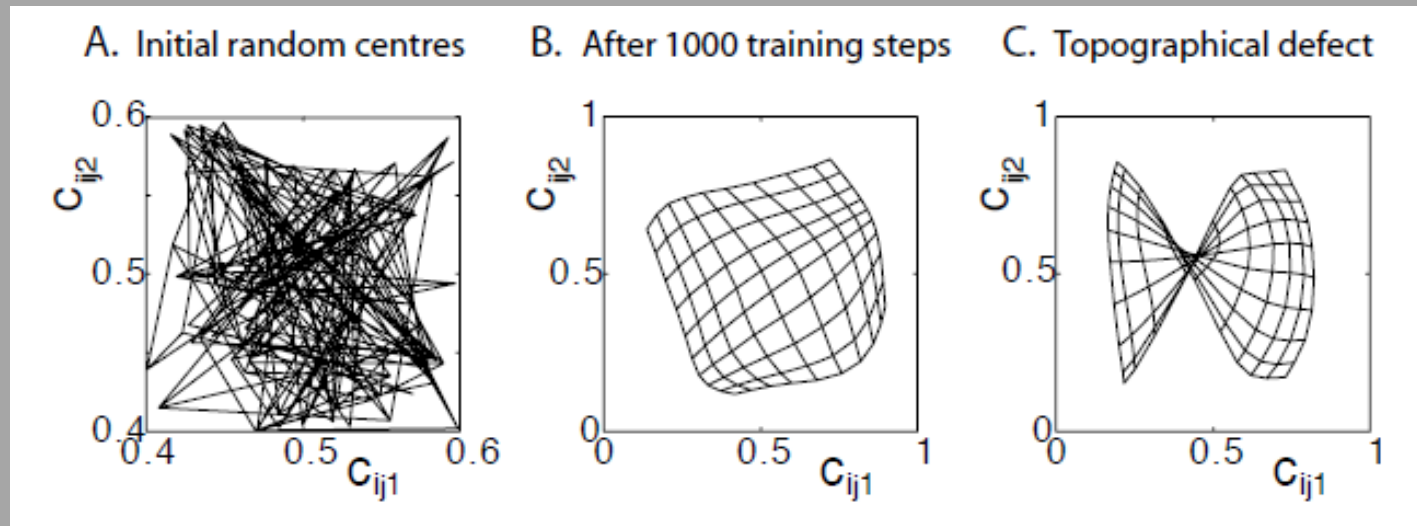
- Mapas auto-organizados
- $r_{ij}$  é a ativação dos neurônios



# Self-Organizing Maps (SOM)

- Regra de aprendizagem
- O vencedor leva tudo: neurônio vencedor aumenta o próprio peso e os pesos dos vizinhos em sua área de influência
- $r_{ij}^*$  é a ativação do neurônio vencedor

$$\Delta c_{ijk} = \epsilon r_{ij}^* (r_i^{\text{in}} - c_{ijk}),$$

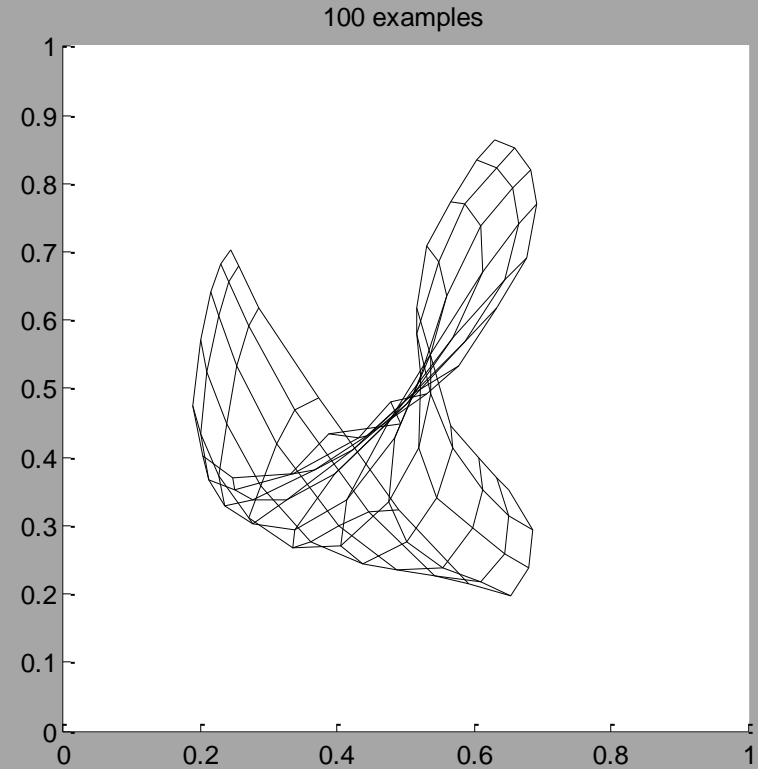
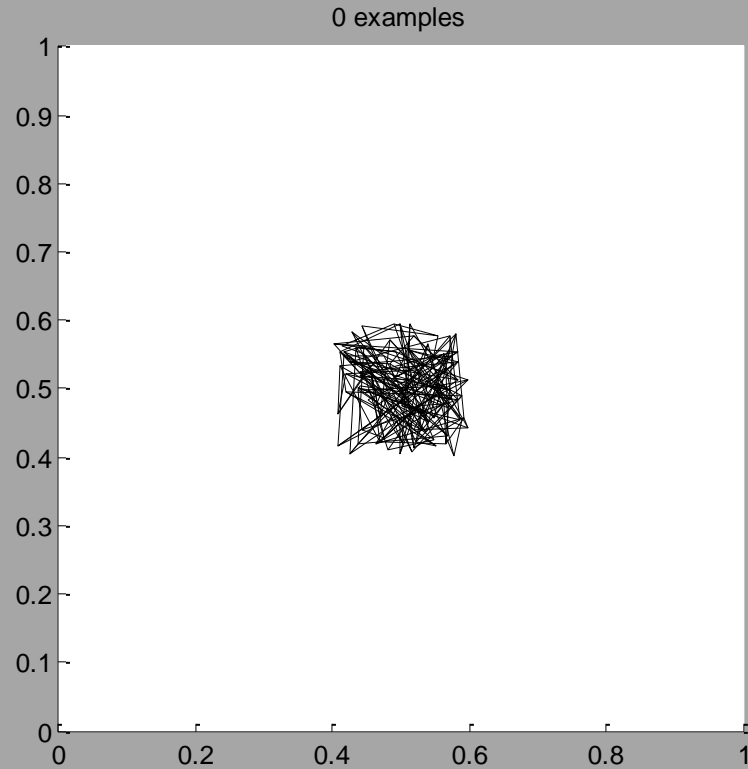


# Self-Organizing Maps (SOM)

```
1  %% Two dimensional self-organizing feature map ala Kohonen
2  clear; nn=10; lambda=0.2; sig=2; sig2=1/(2*sig^2);
3  [X,Y]=meshgrid(1:nn,1:nn); ntrial=0;
4
5  % Initial centres of preferred features:
6  c1=0.5-.1*(2*rand(nn)-1);
7  c2=0.5-.1*(2*rand(nn)-1);
8
9  %% training session
10 while(true)
11     if(mod(ntrial,100)==0) % Plot grid of feature centres
12         clf; hold on; axis square; axis([0 1 0 1]);
13         plot(c1,c2,'k'); plot(c1',c2', 'k');
14         tstring=[int2str(ntrial) ' examples']; title(tstring);
15         waitforbuttonpress;
16     end
17     r_in=[rand;rand];
18     r=exp(-(c1-r_in(1)).^2-(c2-r_in(2)).^2);
19     [rmax,x_winner]=max(max(r)); [rmax,y_winner]=max(max(r'));
20     r=exp(-(X-x_winner).^2+(Y-y_winner).^2)*sig2);
21     c1=c1+lambda*r.*(r_in(1)-c1);
22     c2=c2+lambda*r.*(r_in(2)-c2);
23     ntrial=ntrial+1;
24 end
```

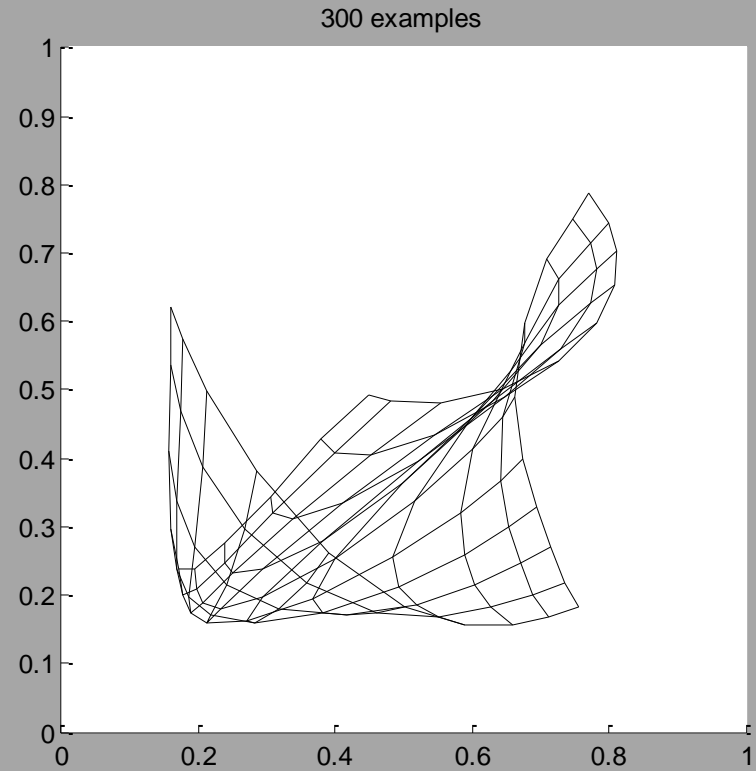
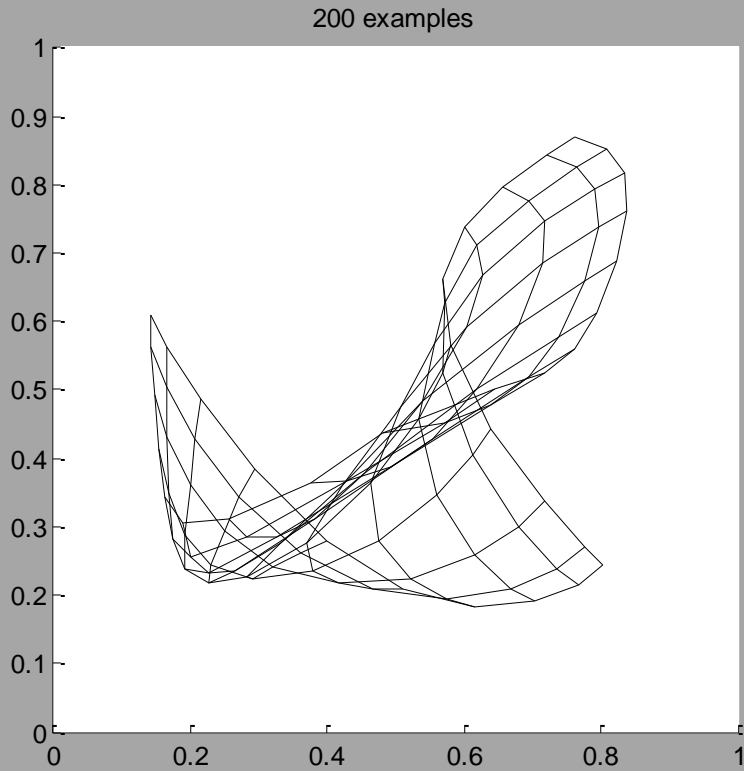
# Self-Organizing Maps (SOM)

$r1 = \text{rand}$ ;  $r2 = \text{rand}$



# Self-Organizing Maps (SOM)

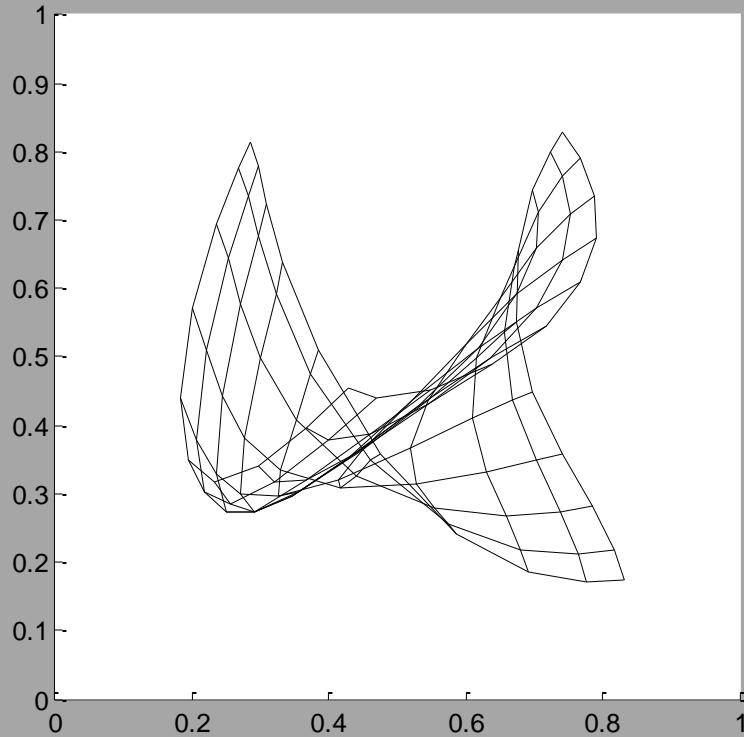
$r1=rand$ ;  $r2=rand$



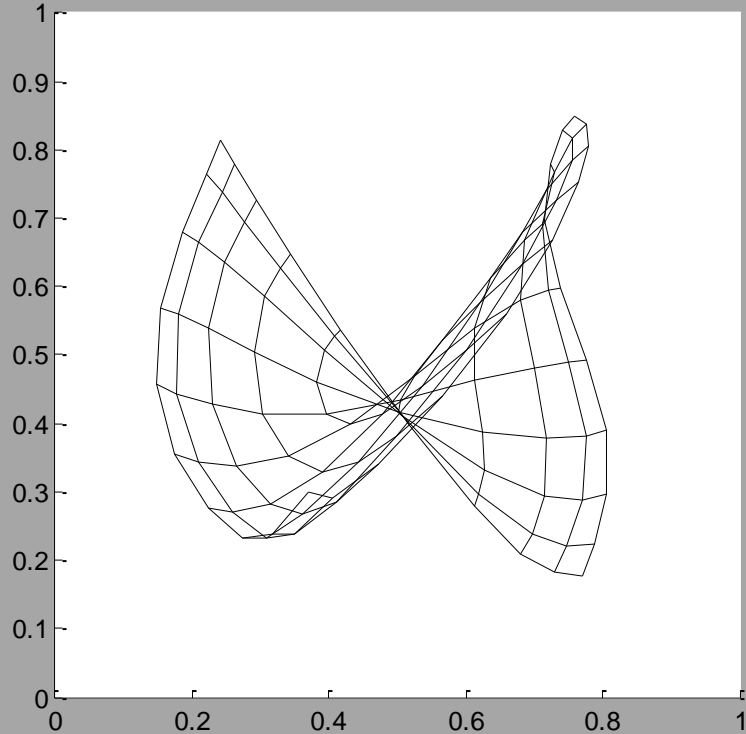
# Self-Organizing Maps (SOM)

$r1=rand$ ;  $r2=rand$

400 examples

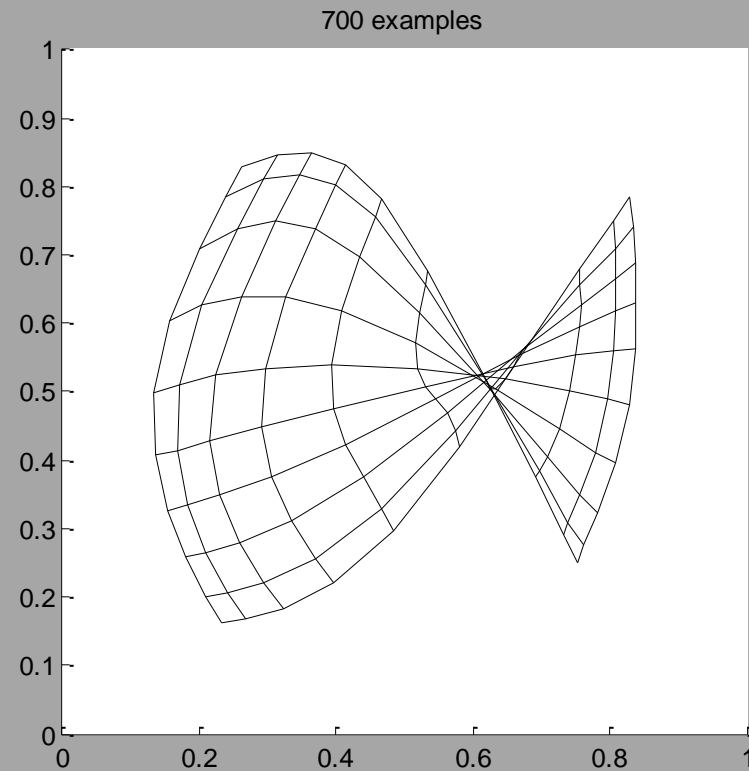
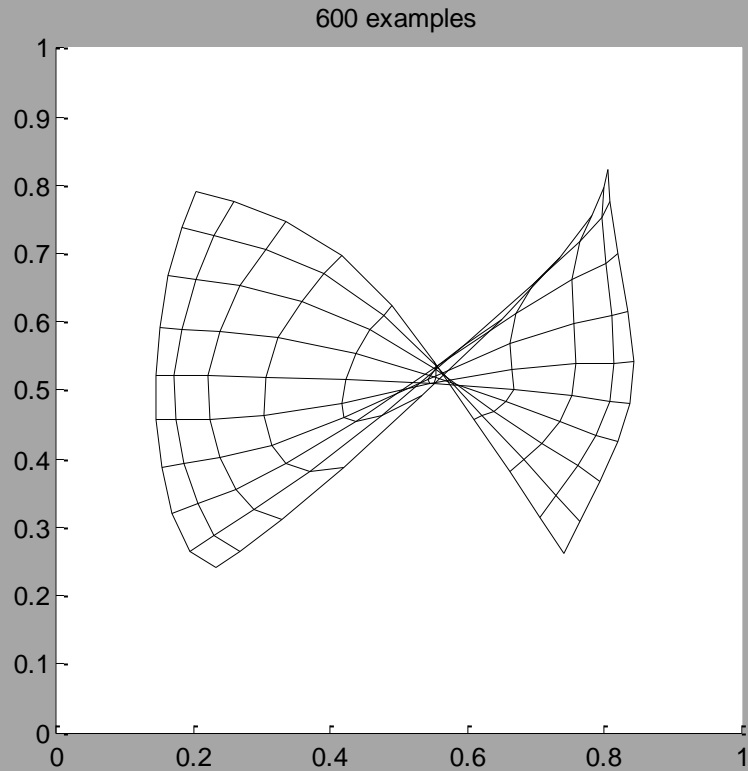


500 examples



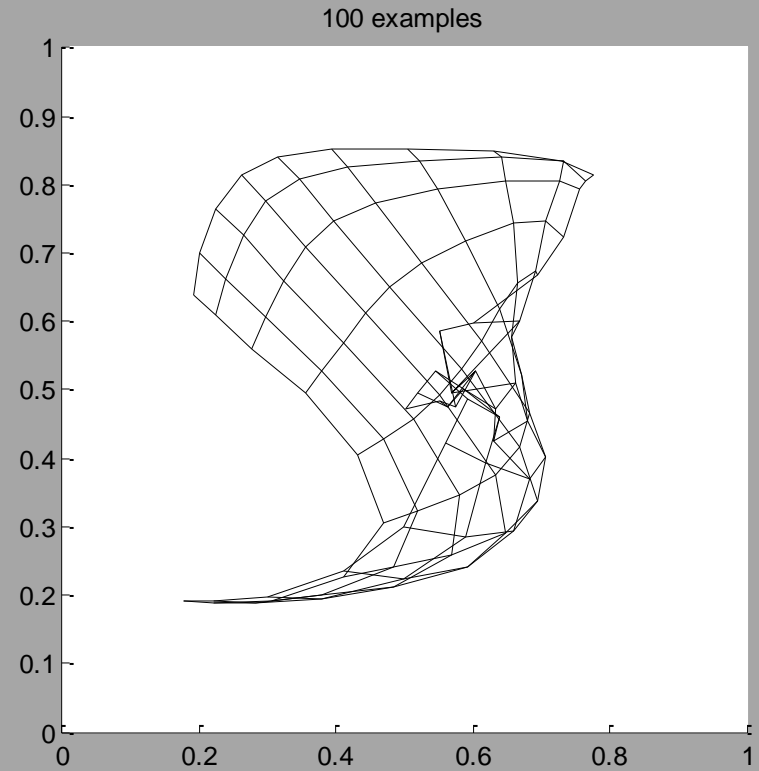
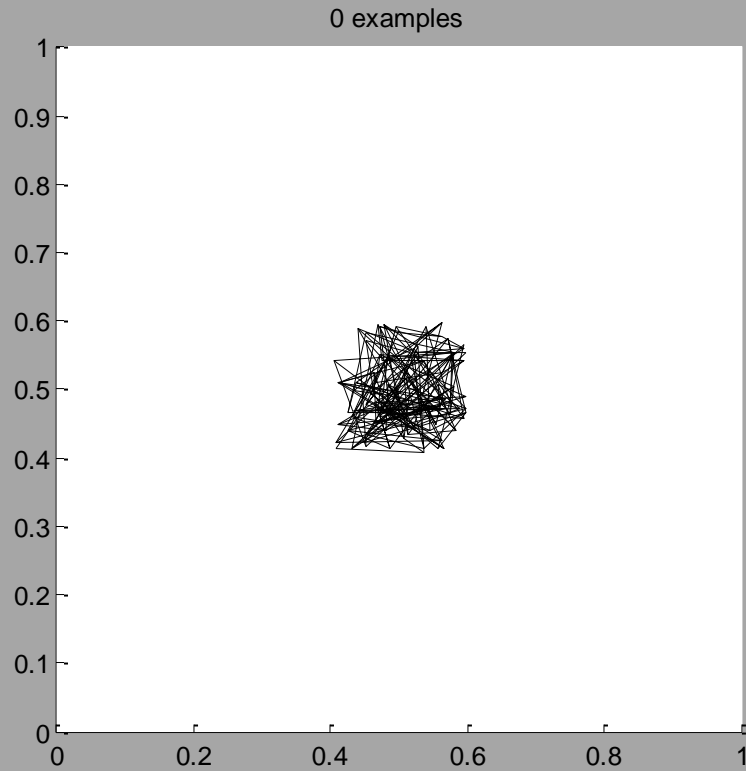
# Self-Organizing Maps (SOM)

$r1=rand$ ;  $r2=rand$



# Self-Organizing Maps (SOM)

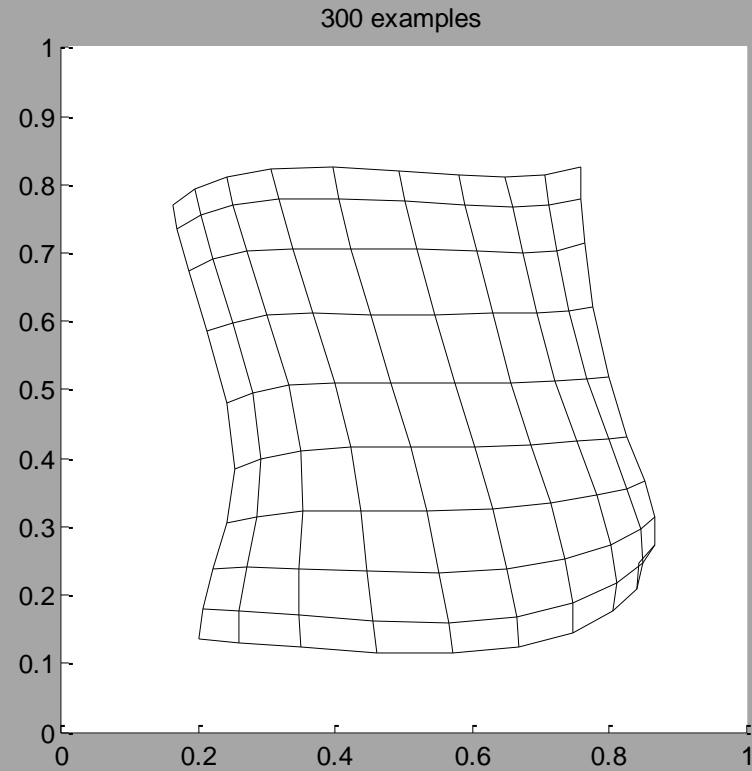
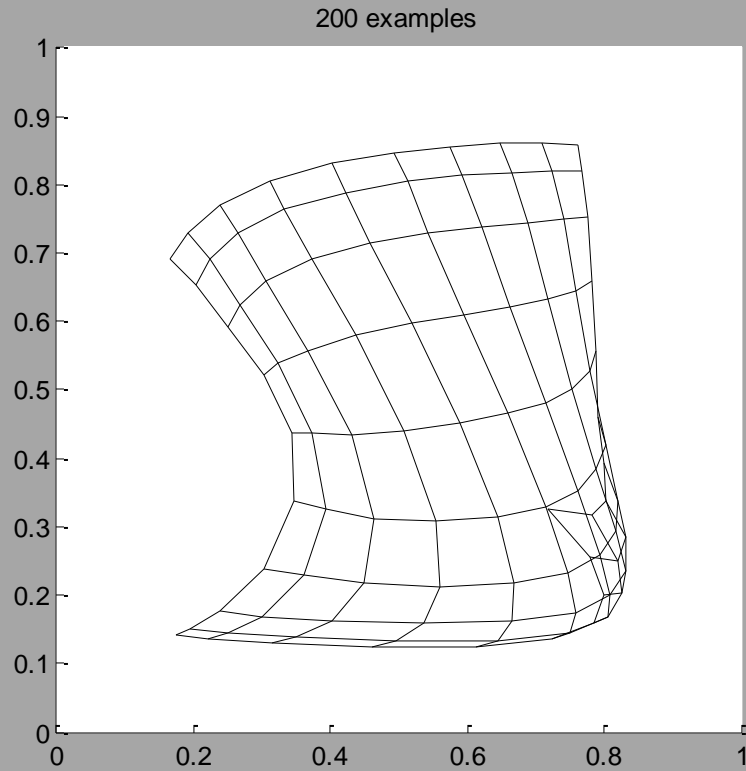
$r1=rand$ ;  $r2=rand$





# Self-Organizing Maps (SOM)

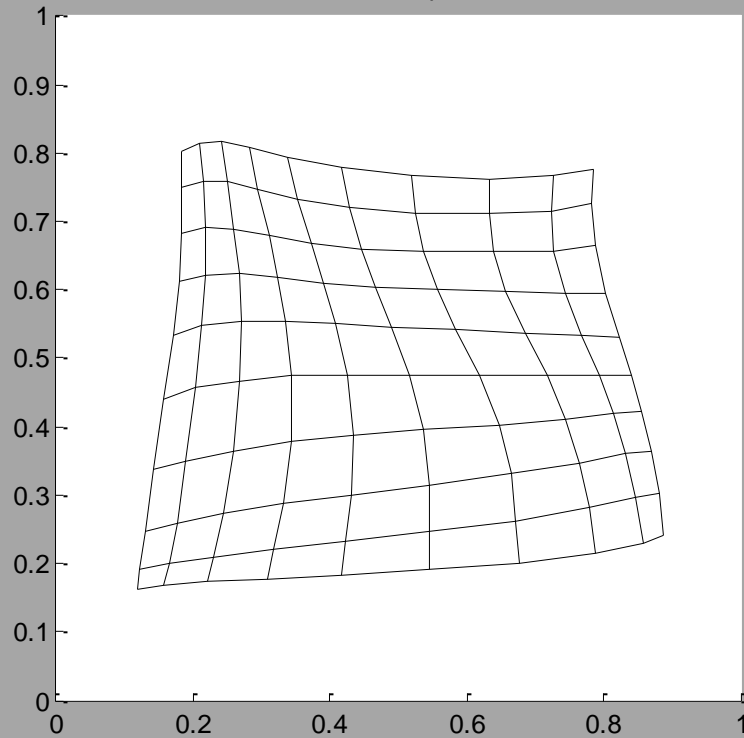
$r1=rand$ ;  $r2=rand$



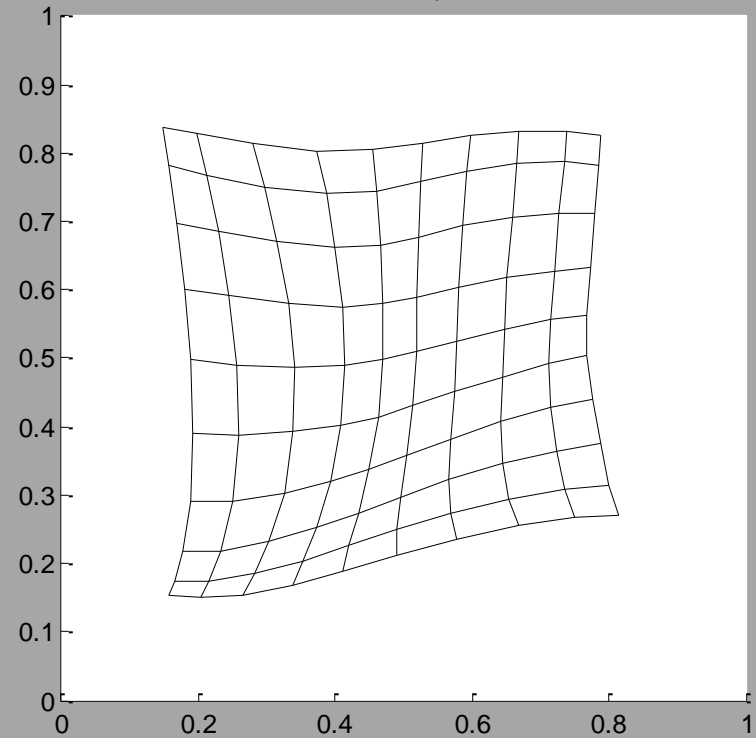
# Self-Organizing Maps (SOM)

$r1 = \text{rand}$ ;  $r2 = \text{rand}$

400 examples

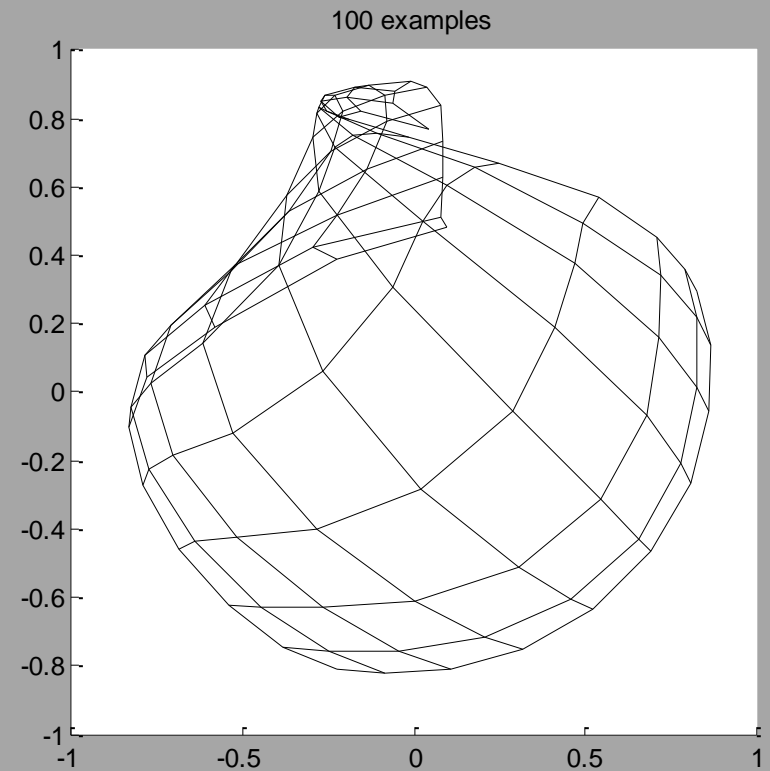
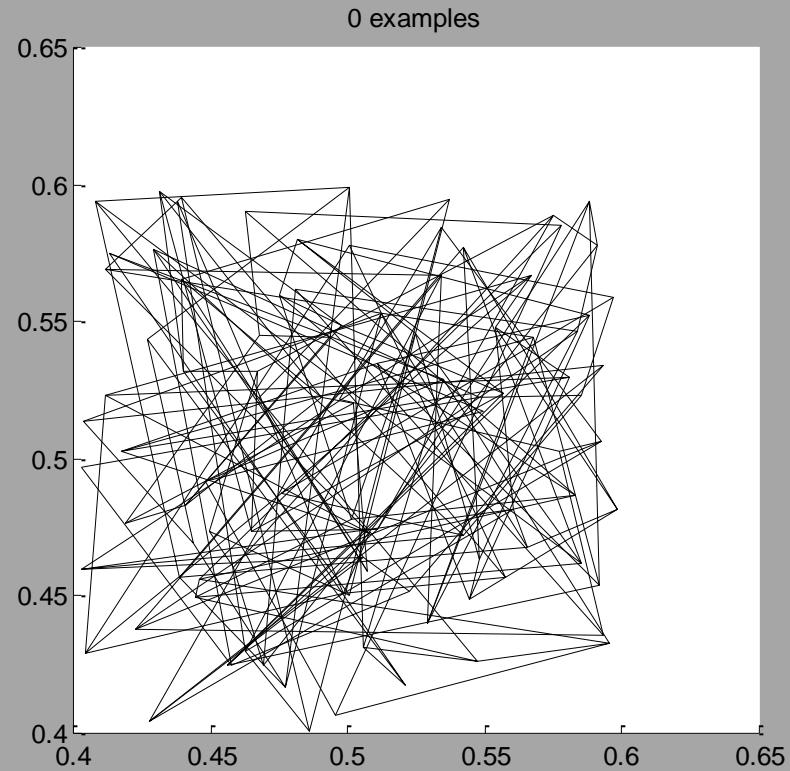


500 examples



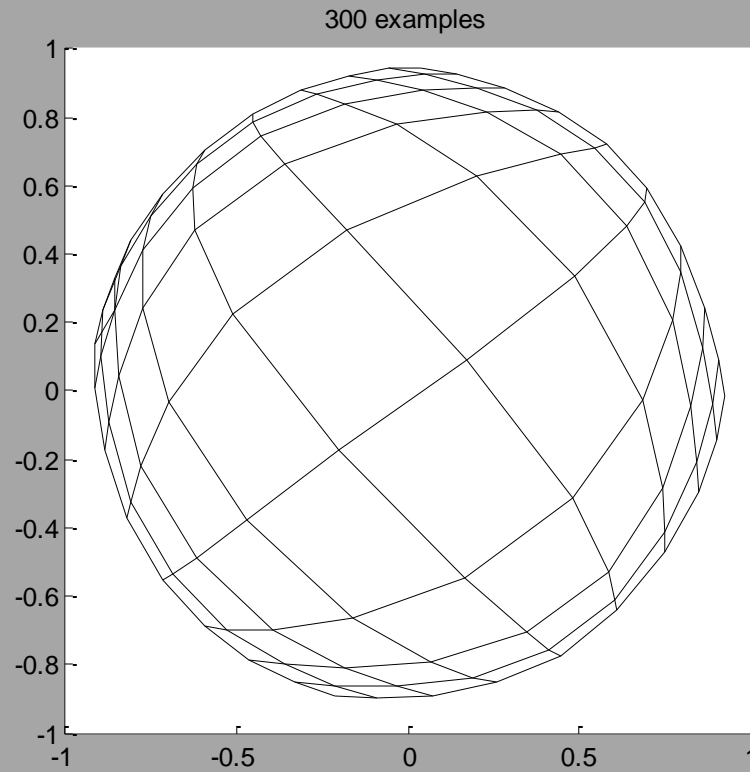
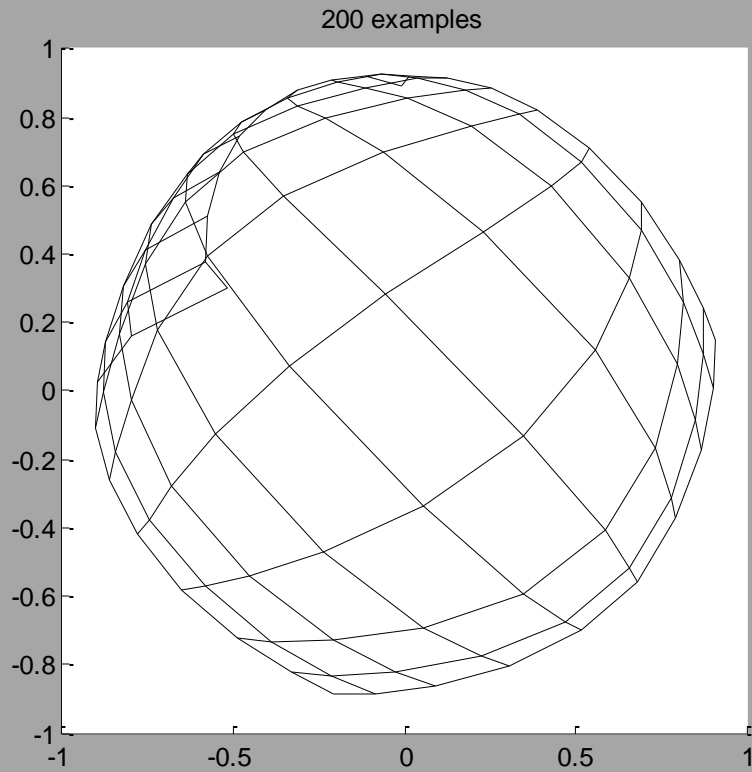
# Self-Organizing Maps (SOM)

$$r1=\sin; r2=\cos$$



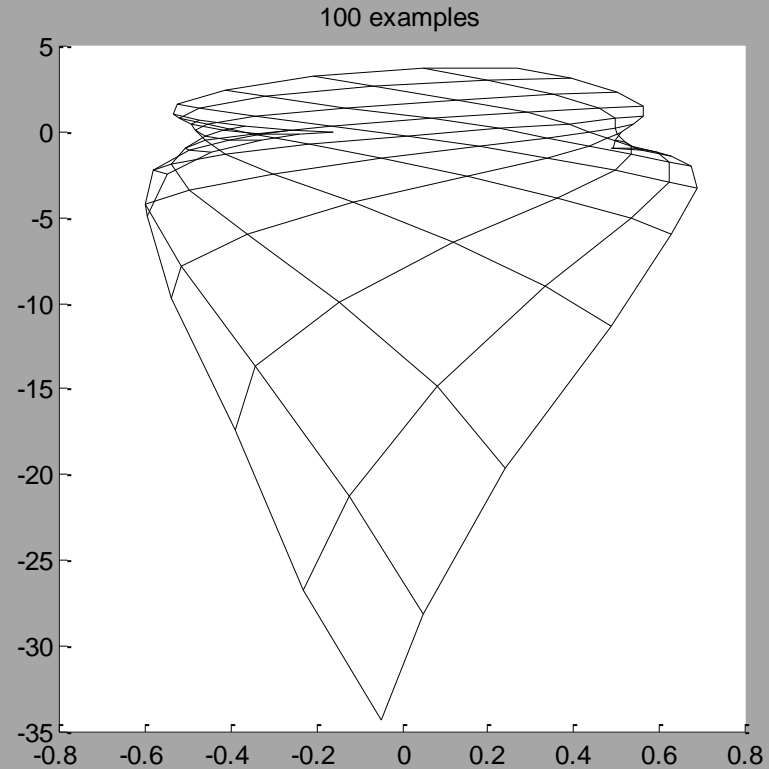
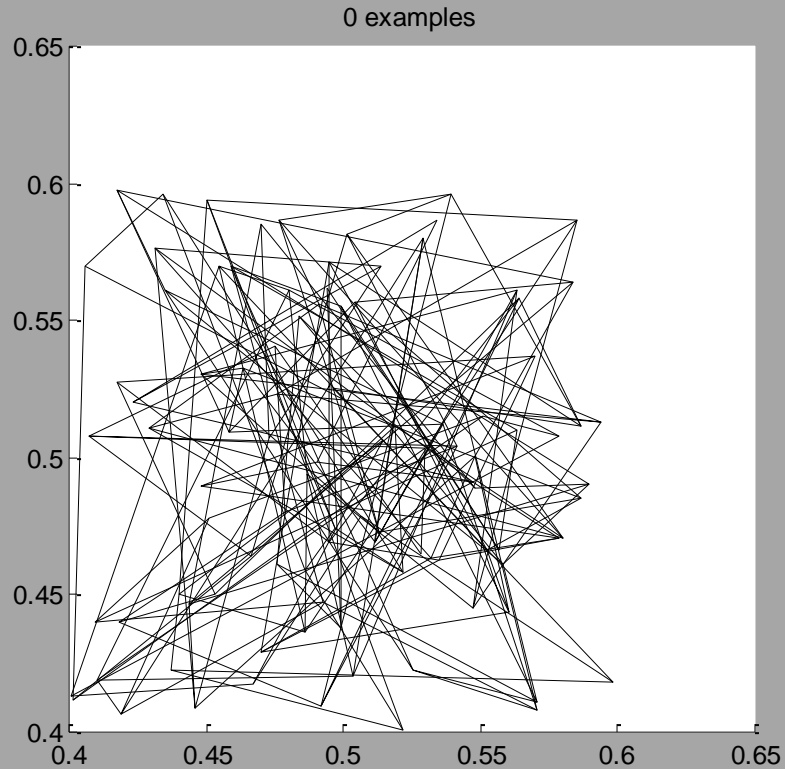
# Self-Organizing Maps (SOM)

$$r_1 = \sin; r_2 = \cos$$



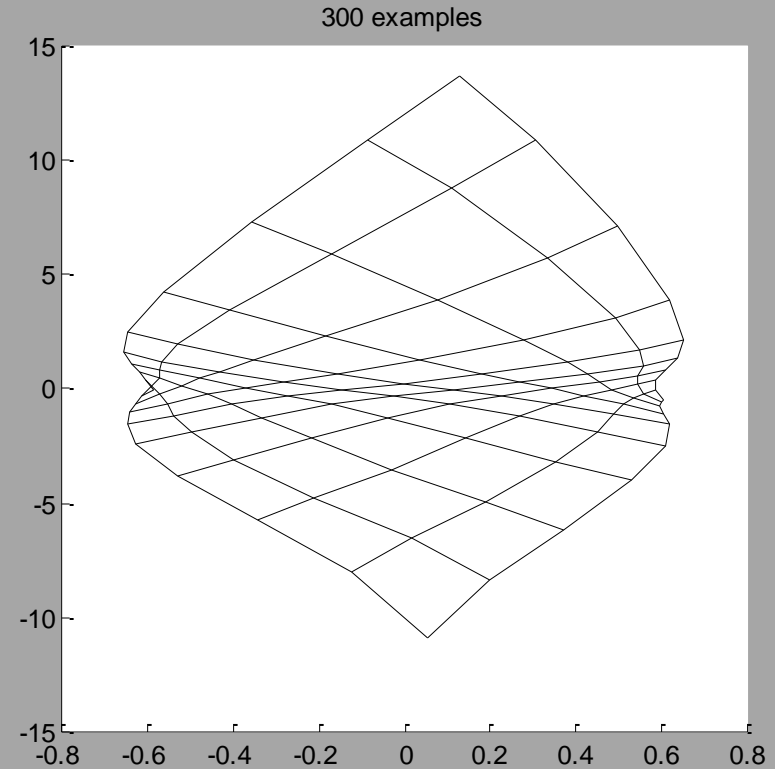
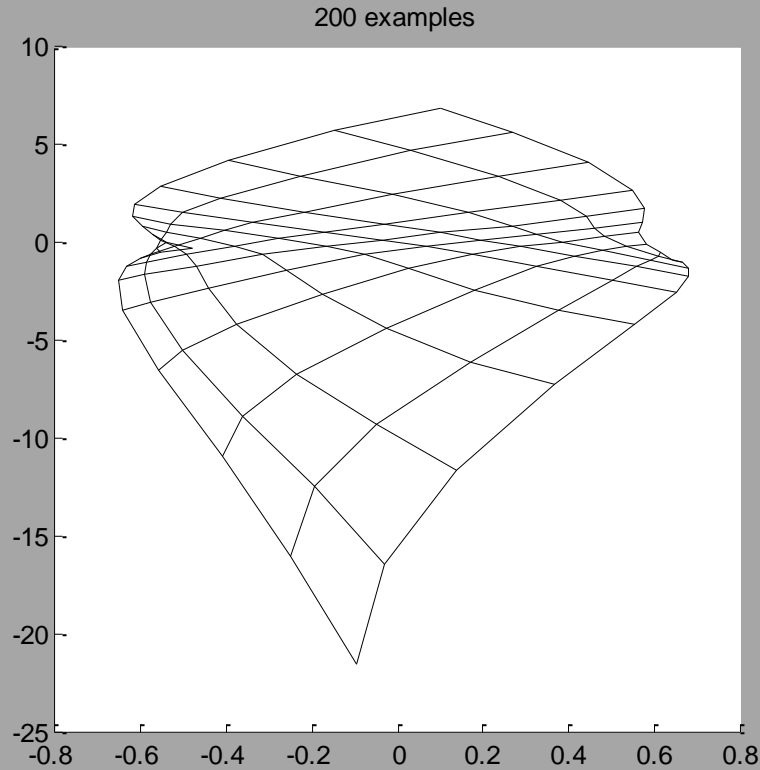
# Self-Organizing Maps (SOM)

$r1=\sin$ ;  $r2=\tan$



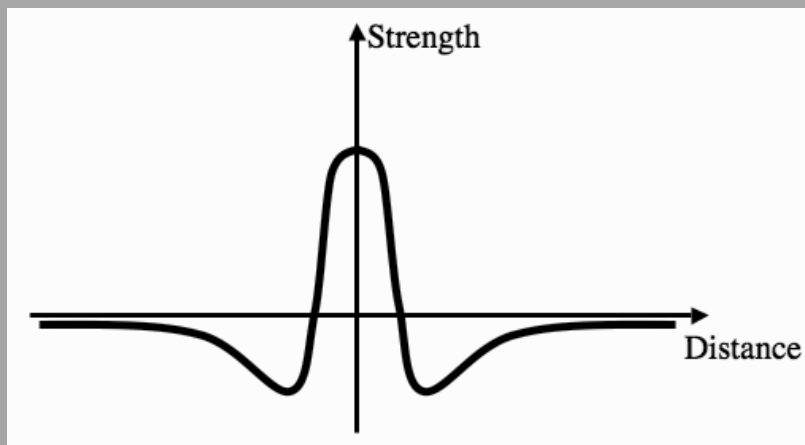
# Self-Organizing Maps (SOM)

$r1=\sin$ ;  $r2=\tan$



# Mapa Auto-Organizador de Atributos

- Auto-organização: interação é local, gerando uma organização global (como na inteligência de abelhas e formigas)
- Neurônios vencedores atraem outros neurônios próximos no espaço de pesos.
- Neurônios vencedores repelem neurônios distantes através do uso de pesos negativos.
- Neurônios muito distantes são ignorados (representam outros atributos)
- Função Chapéu Mexicano



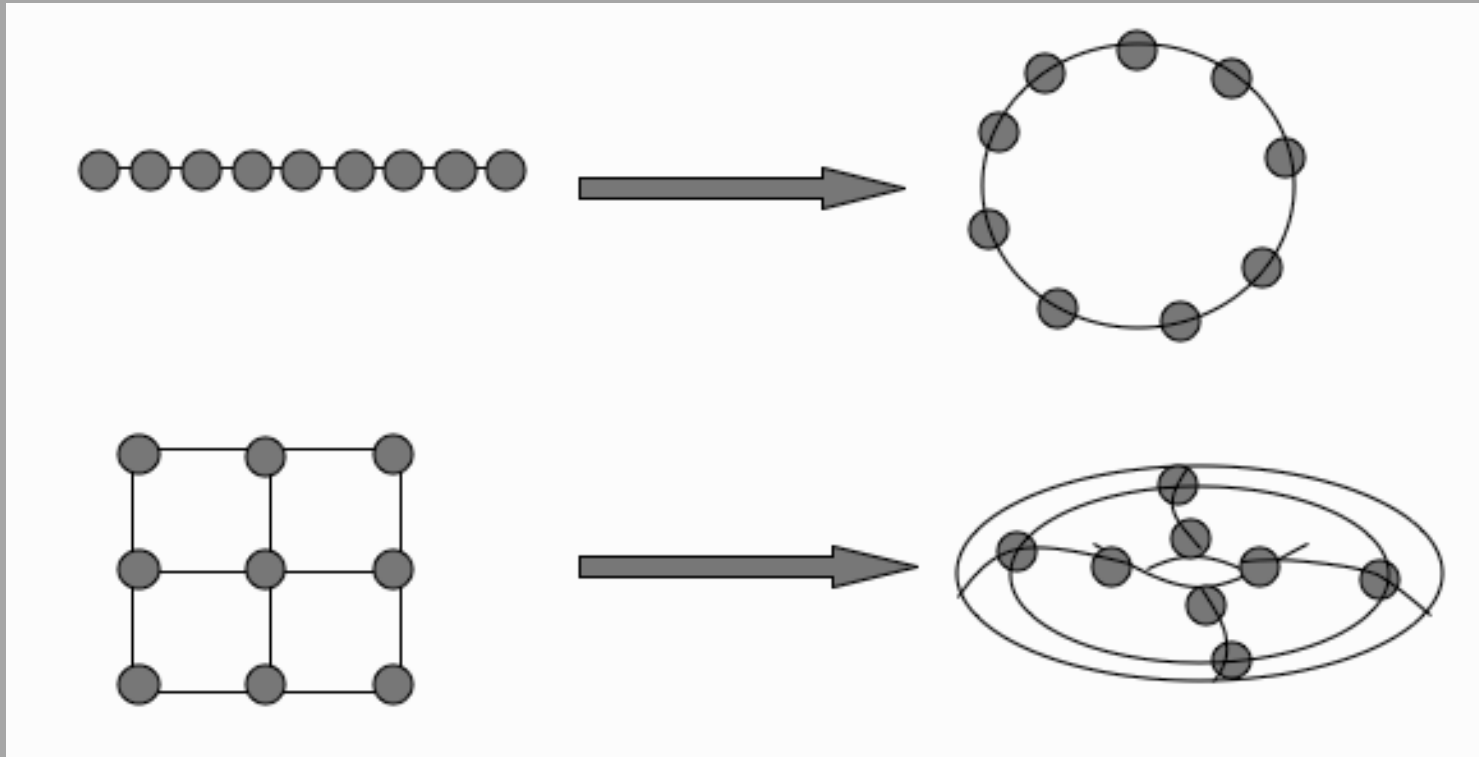
# Mapa Auto-Organizador de Atributos

- Mapas auto-organizados detém a topologia dos dados de entrada (como uma máscara)
- Mapas auto-organizados revelam correlações que não são facilmente identificados
- Mapas auto-organizados classificam os dados sem supervisão
- Não precisa de target-vector e backpropagation para treinamento da rede
- Não existem conexões laterais entre os nós do mapa



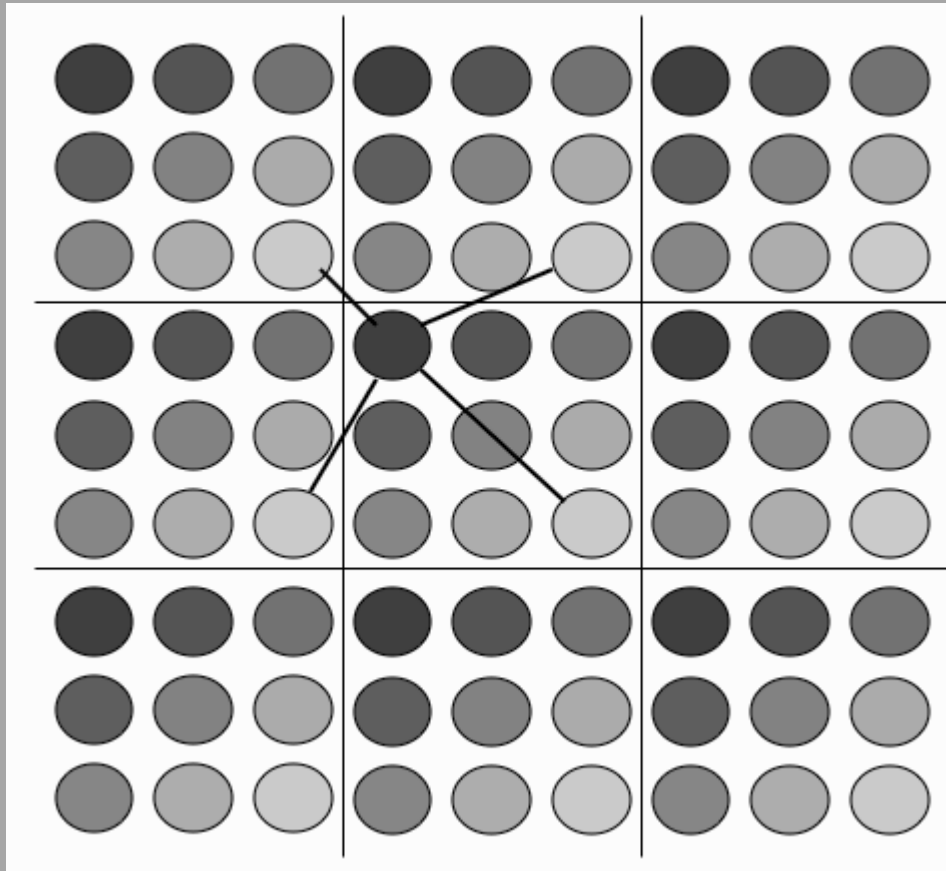
# Mapa Auto-Organizador de Atributos

- Bordas circulares (anel e torus) são utilizadas para minimizar efeitos de borda.



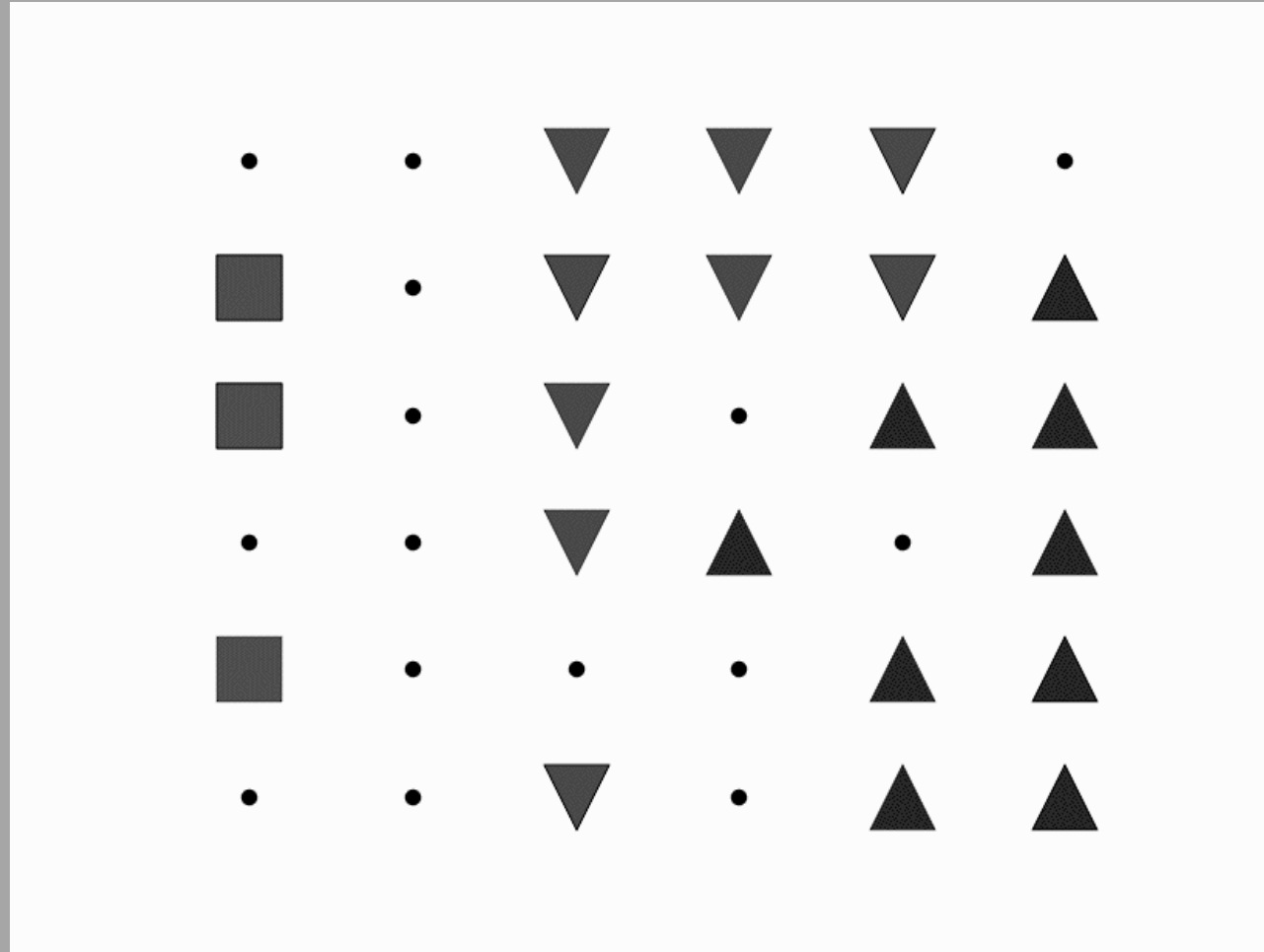
# Mapa Auto-Organizador de Atributos

- É computacionalmente eficiente considerar múltiplas cópias de um mesmo mapa colocados ao redor do mapa original



# Exemplos de Aplicações

- Taxonomia: Organização do Banco de Dados Iris

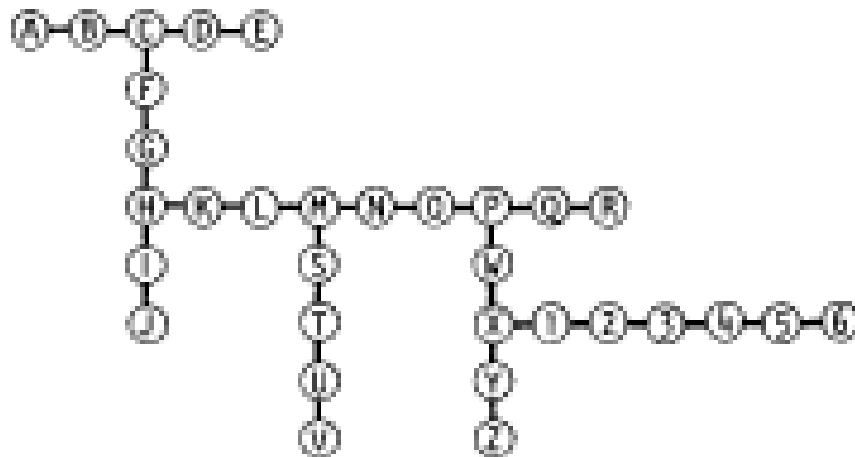


# Exemplos de Aplicações

- Taxonomia: cluster hierárquico

Table 1 Input Data Matrix

Attribute	Item																									
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
$a_1$	1	2	3	4	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
$a_2$	0	0	0	0	0	1	2	3	4	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
$a_3$	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	3	3	3	3	6	6	6	6
$a_4$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	1	2	3	4
$a_5$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

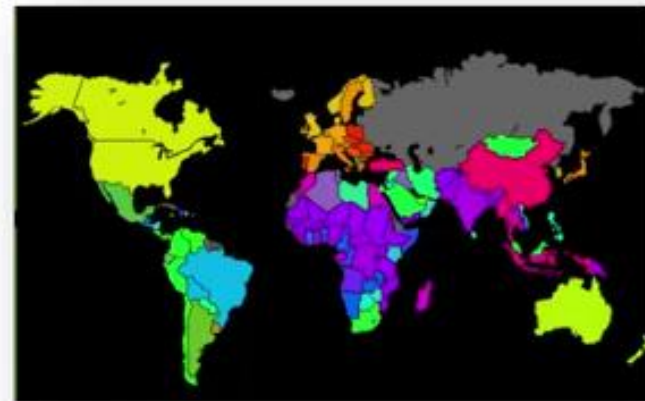
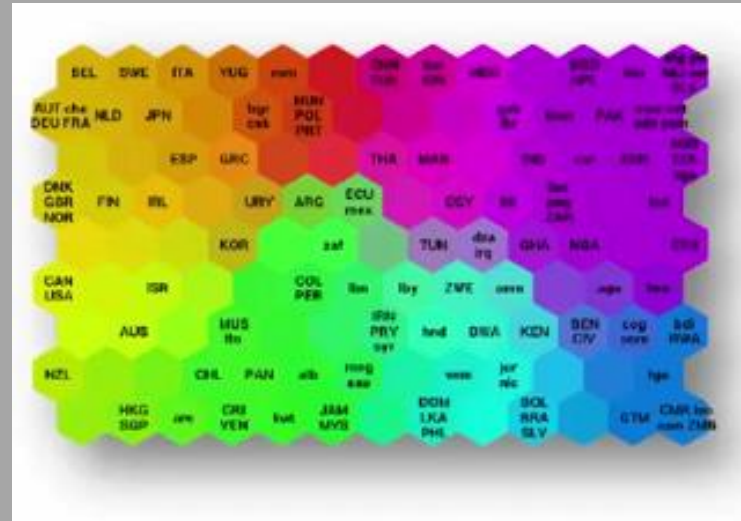


	B	C	D	E	*	Q	R	*	Y	Z
A	*	*	*	*	*	P	*	*	X	*
	*	F	*	N	O	*	W	*	*	1
	*	G	*	M	*	*	*	*	2	*
	H	K	L	*	T	U	*	3	*	*
	*	I	*	*	*	*	*	*	4	*
	*	J	*	S	*	*	V	*	5	6

# Organização de Atributos

- Nível de desenvolvimento dos países

	A	B	C	D	E
1	Country	Country C	Health Ex	Education E	Inflation
2	Aruba	ABW	9.418971	5.92467022	-2.13637
3	Afghanistan	AFG	4.371774		-8.28308
4	Angola	AGO	5.791339		13.73145
5	Albania	ALB	6.75969		2.280502
6	Andorra	AND	4.57058	3.1638701	
7	Arab World	ARB	4.049924		3.524814
8	United Arab Emirates	ARE	7.634758		
9	Argentina	ARG	4.545323	4.88997984	6.282774
10	Armenia	ARM		3.84079003	3.406767
11	American Samoa	ASM	4.862062		
12	Antigua and Barbuda	ATG	9.046056	2.55447006	-0.55016
13	Australia	AUS	11.19444	5.09262991	1.820112
14	Austria	AUT	5.85024	5.7674098	0.506313
15	Azerbaijan	AZE	6.964187	3.22430992	1.401056
16	Burundi	BDI	10.39434	6.3197999	10.98147
17	Belgium	BEL	4.46431	6.41535997	-0.05315
18	Benin	BEN	7.405431	4.22204018	2.15683



<http://www.ai-junkie.com/ann/som/som5.html>

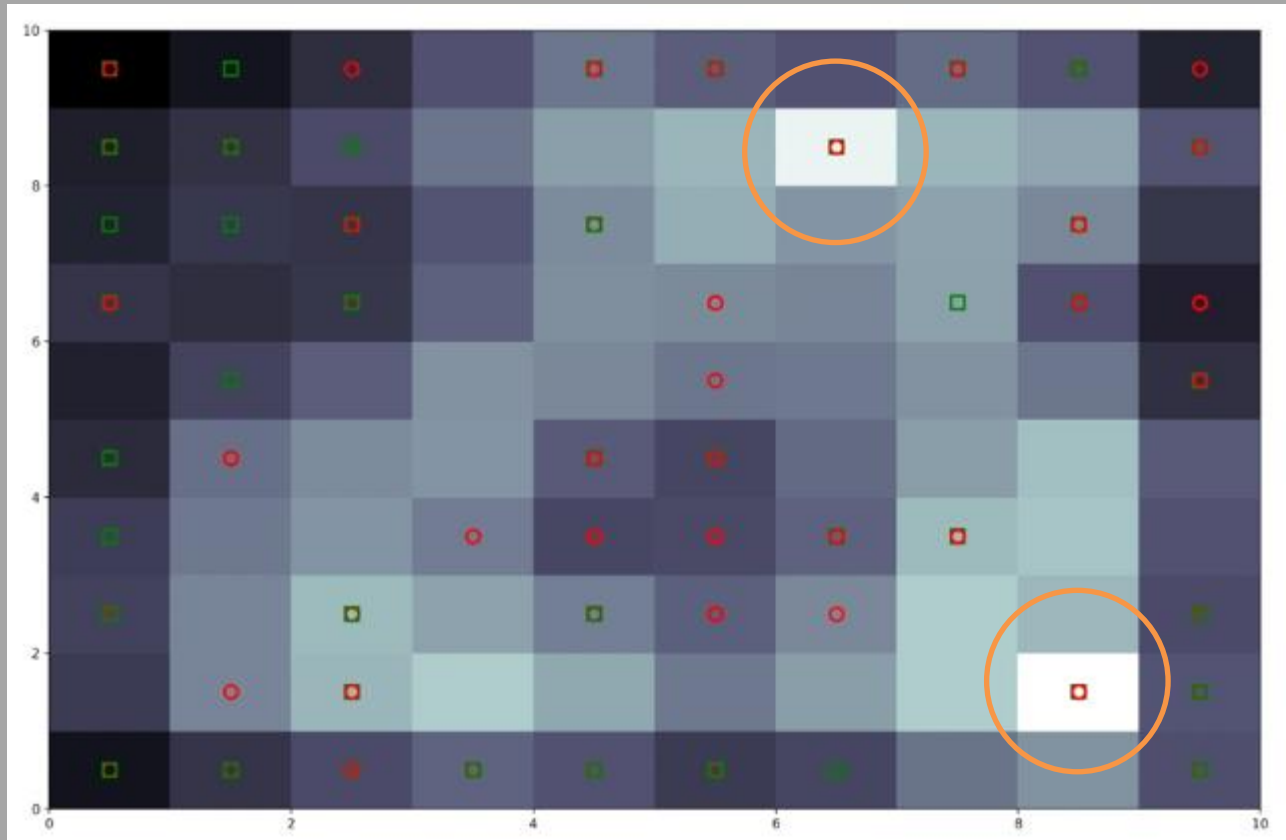
<http://www.cis.hut.fi/>

<https://www.superdatascience.com/pages/deep-learning>

# Detecção de Fraude Cartão Crédito

- Casos são raros
- Quantidade de dados disponível muito baixa para aplicar aprendizado supervisionado
- Solução possível através de clustering dos dados por mapas auto-organizados
- Dataset:  
[http://archive.ics.uci.edu/ml/datasets/statlog+\(australian+credit+approval\)](http://archive.ics.uci.edu/ml/datasets/statlog+(australian+credit+approval))
- As fraudes são os outliers (os pontos fora da curva, aqueles que não seguem as regras)

# Detecção de Fraude Cartão Crédito



Até a Próxima Aula!