1 Definitions

We consider the following definitions:

Let $G = (N, A)$ be a directed graph, where G is defined as following:

G= $G_0 \cup G^I \cup G^D \cup G^T$, the graph containing all nodes and arcs of the problem

$G^I = \bigcup_{i=1}^{s} G^i$, the infrastructure graph

$G^D = (N^D, A^D)$, the access graph

$G^T = (N^T, A^T)$, transfer graph

$G^i = (N^i, A^i) \qquad \forall i = 0, \cdots s$ (where s is the number of different technologies, 0 means that there is not bicycle network, s is the best technology for the users but it is the most expensive one)

$N = \bigcup_{i=0}^{s} N^i \cup N^D \cup N^T, \qquad A = \bigcup_{i=0}^{s} A^i \cup A^D \cup A^T$

We named the set of arcs A^I as infrastructure arcs, the set of arcs A^D as access arcs and the set of arc A^T as transfer arcs.

1.1 Parameters

c_a is the variable cost (perceived by the users) on the arc $a \in A$

f_a^i is the fixed cost (construction cost) of the arc $a^i \in A^i$ where $i = 1, \cdots, s$

Usually $c_a^i > c_a^{i+1} \quad \forall i = 0, \cdots, s-1 \quad$ and $f_a^i < f_a^{i+1} \quad \forall i = 1, \cdots, s-1$

Additionally $c_a = \rho \quad \forall a \in A^T$ and $c_a = 0 \quad \forall a \in A^D$.

Parameter ρ is a fix switch cost of changing from a node belonging to a bikeway layer with certain s technology, to another node belonging to a different layer, using a transfer arc $a \in A^T$.

Let K be the set of k demands, $O^k \in N^D$ and $D^k \in N^D$ the set of origin and destination nodes respectively, and $\delta^k > 0$ the value of the demand k.

$$\theta_{nk} = \begin{cases} \delta^k \text{ if } n{=}O^k \\ -\delta^k \text{ if } n{=}D^k \\ 0 \;\; \text{otherwise} \end{cases}$$

Let B be the available budget for constructing arcs $a \in A^I$.

1.2 Decision variables

We define $x_{ak} \in \mathbb{R}^+$ as a variable that indicates the flow through the arc $a \in A$ to meet the demand $k \in K$ (from O^k to D^k).

$$y_a = \begin{cases} 1 \text{ if the arc } a \in A^I \text{ is considered in the solution} \\ 0 \text{ otherwise} \end{cases}$$

1.3 ILP Model

$$min \quad \sum_{k \in K} (\sum_{a \in A \setminus A^T} c_a x_{ak} + \sum_{a \in A^T} \rho x_{ak}) \tag{1}$$

s.t.

$$x_{ak} \leq \delta^k y_a \quad \forall a \in A^I, \forall k \in K \tag{2}$$

$$\sum_{a \in A_n^+} x_{ak} - \sum_{a \in A_n^-} x_{ak} = \theta_{nk} \quad \forall n \in N, \quad \forall k \in K \tag{3}$$

$$\sum_{a \in H(a')} y_a \leq 1 \qquad \forall a' \in A^0, \tag{4}$$

$$\sum_{a \in A^I} f_a y_a \leq B \ , \tag{5}$$

$$x_{ak} \geq 0 \quad \forall a \in A, \forall k \in K \tag{6}$$

$$y_a \in \{0, 1\} \ \forall a \in A^I, \tag{7}$$

Objective function (1) minimizes the total cost perceived by users, and also minimizes the cost of switching between G^i networks.

Constraint (2) states the opening of arc y_a when exists flow in the arc $a \in A^I$ for satisfying the demand k ($x_{ak} \geq 0$).

Constraint (3) is the balance flow equation.

Constraint (4) states the opening at most one arc $a_i \in A^i, \quad \forall i = 1, \cdots, s$. The function $H(a)$ returns the arcs belonging to the different bikeway technologies $(1, \cdots, s)$, corresponding to a certain arc $a \in A^0$

Constraint (5) is the budget restriction, and constraints (6) and (7) are the domain of the decision variables x_{ak} and y_a respectively.