Baez Exercises: Gauge Transformations

80. Check that if $\phi : \mathbb{R}^4 \to \mathbb{C}^3$ is a solution of

$$(\partial^\mu \partial_\mu + m^2)\phi + \lambda \phi^i \phi_i \phi = 0,$$

so is any $U_1(g)\phi$ for any $g \in SU(2)$

$\phi^i \phi_i$ is the inner product in $\mathbb{C}^3$, and as such is invariant under unitary transformations like $U_1(g)$. Further, since g doesn't depend on x^μ, $\partial_\mu U_1(g) = 0$ and therefore

$$(\partial^\mu \partial_\mu + m^2 + \lambda \phi'^i \phi'_i)\phi' = (\partial^\mu \partial_\mu + m^2 + \lambda \phi^i \phi_i)\phi'$$

$$= U_1(g)(\partial^\mu \partial_\mu + m^2 + \lambda \phi^i \phi_i)\phi = 0.$$

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81. Partition of unity...

82. Show that gauge transformations form a group.

Suppose $f, h \in T$ live in G. This means $[p, v]_\alpha \xrightarrow{f} [p, g_f v]_\alpha$ for $g_f \in G$ and similarly for h and $g_h \in G$.

The product is defined by $fh(p) = f(p)h(p)$ and so we get $fh : [p, v]_\alpha \longrightarrow [p, g_{fh} v]_\alpha$ for $g_{fh} = g_f g_h \in G$. When $h = f^{-1}$ we have a trivial action, so $e = g_{fh} = g_f g_{f^{-1}}$ and thus $g_{f^{-1}} = g_f^{-1}$.