A Missing Charts

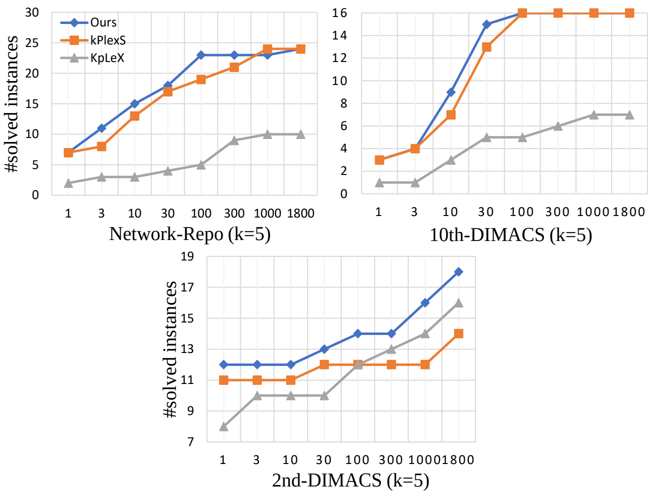

Figure 2: Number of solved instances for Network-Repo, 10th-DIMACS and 2nd-DIMACS graphs with $k = 5$, and time limit 1800 seconds.

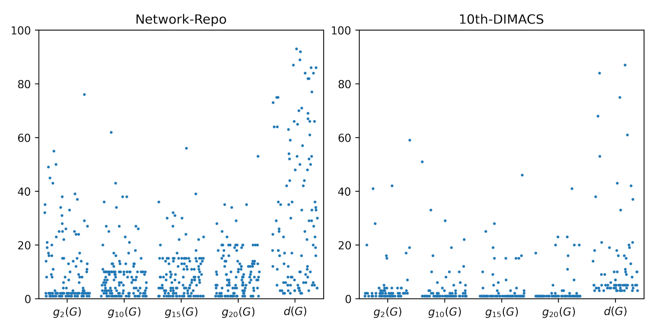

Figure 3: Scatter graph of $g_k(G)$ and $d(G)$ for Network-Repo and 10th-DIMACS graphs. (Y-coordinate is cut off by 100 due to space limit.)

B Proof and Details of Graph Reduction

Proof. Assume that a vertex set $P \subseteq V$ satisfies $|P| = n$, $|E(G[P])| = \lambda$ and $|\Delta_G(P)| < p - nk + n(n-1) - 2\lambda$, but all vertices of P belong to a k-plex P^* of size p. For any vertex $u \in P$, there are at most $k-(n-|N_G(u) \cap P|)$ vertices that are not adjacent to u in $P^* \setminus P$. Hence, there are at most $\sum_{i=1}^{n} k - (n - |N_G(u_i) \cap P|)$ vertices in $P^* \setminus P$ that are not belong to $\Delta_G(P) \cap P^*$. Then,

$$|P^*| - n - |\Delta_G(P) \cap P^*| \leq \sum_{i=1}^{n} k - (n - |N_G(u_i) \cap P|)$$
$$= nk - n^2 + \sum_{i=1}^{n} |N_G(u_i) \cap P| = nk - n^2 + 2\lambda \tag{1}$$

In summary, we have $|\Delta_G(P) \cap P^*| \geq |P^*| - nk + n(n-1) - 2\lambda = p - nk + n(n-1) - 2\lambda$. It is clear that $|\Delta_G(P) \cap P^*| <= |\Delta_G(P)|$, which contradicts the assumption that $|\Delta_G(P)| < p - nk + n(n-1) - 2\lambda$. □

Algorithm 3: Our Graph Reduction

Reduce($G_s = G[\{v_i\} \cup S \cup N_G^+(v_i)], k, p$)
begin
  repeat
    repeat
      if *there is a vertex* $u \in V_s$ *such that* $|\Delta_{G_s}(\{u\})| < p - k$ **then**
        $V_s \leftarrow V_s \setminus \{u\}$
        Remove u and its incident edges
      if *there are two vertices* $u, v \in V_s$ *such that* $u \in N_{G_s}(v)$ *and* $|\Delta_{G_s}(\{u, v\})| < p - 2k$ **then**
        Remove edge (u, v) from G_s
    until *No vertex and edge can be removed*;
    `// Higher Order Reduction.`
    $Hop1 \leftarrow N_{G_s}(v_i)$, $Hop2 \leftarrow N^2_{G_s}(v_i)$
    repeat
      if *there is a vertex* $u \in Hop1$ *such that* $|\Delta_{G_s}(\{v_i, u\})| < p - 2k$ **then**
        $Hop1 \leftarrow Hop1 \setminus \{u\}$
        Remove u and its incident edges
      if *there are two vertices* $u, v \in Hop1$ *such that* $u \in N_{G_s}(v)$ *and* $|\Delta_{G_s}(\{v_i, u, v\})| < p - 3k$ **then**
        Remove edge (p, q) from G_s
    until *No vertex and edge can be removed*;
    repeat
      if *there is a vertex* $u \in Hop2$ *such that* $|\Delta_{G_s}(\{v_i, u\})| < p - 2k + 2$ **then**
        $Hop2 \leftarrow Hop2 \setminus \{u\}$
        Remove u and its incident edges
      if *there are two vertives* $u \in Hop1$ *and* $v \in Hop2$ *such that* $u \in N_{G_s}(v)$ *and* $|\Delta_{G_s}(\{v_i, u, v\})| < p - 3k + 2$ **then**
        Remove edge (u, v) from G_s
      if *there are two vertices* $u, v \in Hop2$ *such that* $u \in N_{G_s}(v)$ *and* $|\Delta_{G_s}(\{v_i, u, v\})| < p - 3k + 4$ **then**
        Remove edge (u, v) from G_s
    until *No vertex and edge can be removed*;
  until *No vertex and edge can be removed*;
  if $\{v_i\} \cup S \nsubseteq V_s$ **then**
    return $G[\emptyset]$
  $n \leftarrow |S| + 1$, $\lambda \leftarrow |E(G_s[\{v_i\} \cup S])|$
  if $|\Delta_G(\{v_i\} \cup S)| < p - nk + n(n-1) - 2\lambda$ **then**
    return $G[\emptyset]$
  return G_s