Supplementary Material of Manuscript Fast Maximum k-Plex Algorithms Parameterized by Small Degeneracy Gaps

1 Introduction

The supplementary material mainly contains two parts.

- Unfiltered running time of Maple, Maple_{com}, Maple_{hyb}, kPlexS and KpLeX, with different k values.
- Information of graphs in tested benchmarks, i.e. Network-Repo graphs, 10th-DIMACS graphs and 2nd-DIMACS graphs, with different k values. ($\omega_k(G)$, $g_k(G)$ and $cg_k(G)$ included.)

2 Unfiltered Running Time

This section is the running time comparison of five algorithms, Maple, Maple_{com}, Maple_{hyb}, kPlexS and KpLeX. In our experiments, we use second as the time unit and all the results keep two decimals. If the running time of one case exceeds the time limit 1800s, then we will record the running time as "OOT", which is the abbreviation of "out of time". The data of time comparison will be shown by different datasets, which are Network-Repo graphs, 10th-DIMACS graphs and 2nd-DIMACS graphs.

2.1 Network-Repo Graphs

In this subsection, we show the running time comparison of five algorithms on Network-Repo graphs for k=2,5,10,15 and 20. Due to the limit of page size, we divide k values into 3 groups to demonstrate the results. The first group is k=2 and 5, the second group is k=10 and 15 and the last group is k=20.

Table 1: Running time of Maple, Maple $_{com}$, Maple $_{hyb}$, kPlexS and KpLeX for k=2 and 5 on Network-Repo graphs before filtering

			k=2					k=5		
Graph	Maple	$\mathrm{Maple_{com}}$	$\mathrm{Maple_{hyb}}$	kPlexS	KpLeX	Maple	$\mathrm{Maple_{com}}$	$\mathrm{Maple_{hyb}}$	kPlexS	KpLeX
soc-LiveMocha	14.24	5.91	4.40	64.11	18.89	116.89	8.87	10.75	ООТ	ООТ
soc-youtube	0.71	0.84	0.71	0.75	1.39	0.81	0.76	0.82	0.79	57.92
soc-youtube-snap	0.98	1.28	1.06	1.08	1.96	0.92	0.94	1.03	0.96	147.49
soc-lastfm	14.03	3.67	3.29	39.58	8.81	65.33	1.83	3.17	143.43	OOT
$\operatorname{soc-digg}$	580.25	82.76	51.10	OOT	OOT	OOT	135.57	1466.87	OOT	OOT
ia-wiki-Talk	2.31	0.42	0.43	3.84	1.51	2.27	0.31	0.34	4.84	257.87
socfb-Duke 14	4.03	1.99	1.95	54.69	248.48	8.42	1.44	2.75	702.81	OOT
soc-orkut	384.49	935.97	403.93	OOT	OOT	OOT	OOT	OOT	OOT	OOT
sc-ldoor	20.67	146.22	98.52	22.80	OOT	976.69	OOT	1052.74	820.11	OOT
sc-msdoor	11.69	45.25	34.97	13.32	OOT	594.94	841.83	624.05	456.11	OOT
sc-nasasrb	0.56	0.71	0.64	0.57	9.76	2.18	5.73	1.88	3.27	149.48
sc-pkustk11	1.20	4.25	2.17	1.36	0.70	152.09	6.09	3.57	10.79	2.70
soc-flixster	2.51	1.88	1.81	39.09	36.31	2.05	0.94	2.19	190.51	OOT
$\operatorname{sc-pwtk}$	2.20	8.72	3.81	2.31	OOT	3.36	31.99	6.18	4.38	OOT
$scc_reality$	0.77	0.77	0.77	0.08	0.07	14.69	12.39	13.43	18.64	102.76
soc-gowalla	0.34	0.44	0.35	0.32	2.10	0.30	0.28	0.30	0.26	OOT
soc-epinions	0.04	0.04	0.04	0.04	0.07	0.03	0.03	0.03	0.03	OOT
socfb-B-anon	34.93	55.43	38.05	38.22	146.31	46.83	52.18	43.43	42.63	OOT
socfb-A-anon	33.27	41.47	33.52	32.32	30.10	29.35	28.20	29.29	33.63	602.96

,	04.05	24.00	22.00	20.24	40-4	20.0-	a= a4	25.00	25.50	0.05
soc-pokec	31.35	34.09	32.99	30.24	16.74	28.07	27.34	25.83	25.59	OOT
soc-twitter-follows	0.14	0.20	0.14	0.13	1.11	0.11	0.11	0.12	0.12	ООТ
soc-FourSquare	21.94	34.80	22.31	16.97	969.42	9.13	18.34	3.39	3.88	TOO
socfb-Indiana	2.44	2.78	2.62	2.23	284.86	2.14	2.13	2.11	2.18	TOO
soc-slashdot	0.27	0.16	0.20	1.59	2.05	0.21	0.05	0.17	1.21	ООТ
ia-enron-large	0.09	0.10	0.09	0.09	0.43	0.08	0.09	0.08	0.08	10.91
tech-WHOIS	0.20	0.67	0.20	6.67	296.26	0.44	0.09	0.43	3.25	284.90
ia-email-EU	0.01	0.01	0.01	0.02	0.03	0.01	0.01	0.01	0.01	0.05
socfb-Texas 84	2.89	3.30	2.88	5.36	345.57	1.78	1.88	1.78	2.02	499.56
sc-pkustk13	2.11	2.40	2.36	2.14	655.45	2.03	4.68	2.90	2.04	684.45
$\operatorname{socfb-UF}$	2.76	3.09	2.74	4.67	905.85	1.50	1.51	1.54	1.80	OOT
rt-retweet-crawl	0.44	0.43	0.47	0.42	0.39	0.33	0.37	0.34	0.58	0.45
tech-as-skitter	1.07	1.09	1.10	1.42	OOT	0.87	0.90	0.92	0.90	OOT
soc-livejournal	1.81	1.85	1.71	3.63	4.30	2.54	2.37	2.16	2.35	OOT
socfb-UIllinois	2.48	3.31	2.87	2.38	29.53	2.21	2.61	2.56	2.44	OOT
$scc_fb-messages$	0.10	0.10	0.10	0.01	0.01	0.10	0.10	0.11	0.10	0.01
$scc_twitter-copen$	0.89	2.87	2.18	0.71	17.36	0.88	2.24	1.96	0.76	OOT
socfb-Penn94	1.88	2.00	2.00	1.68	1.35	1.63	1.67	1.65	1.75	1.00
ca-hollywood-2009	0.98	0.99	0.94	1.86	1.49	0.99	0.97	1.20	1.19	1.77
web-uk-2005	0.66	1.77	2.05	0.75	0.77	0.98	2.62	2.77	0.94	1.44
socfb-Stanford3	1.20	1.27	1.27	1.28	896.66	1.19	1.28	1.23	1.37	OOT
socfb-MIT	0.44	0.47	0.46	0.50	15.54	0.40	0.41	0.41	0.40	34.82
soc-delicious	0.14	0.14	0.14	0.15	0.19	0.12	0.11	0.12	0.11	OOT
sc-shipsec1	0.13	0.14	0.14	0.16	0.31	0.14	0.16	0.15	0.15	6.01
socfb-Berkeley13	1.20	1.36	1.23	1.23	11.87	1.11	1.12	1.13	1.17	46.90
ia-infect-hyper	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
socfb-Wisconsin87	1.08	1.18	1.15	1.04	10.38	0.97	0.94	1.02	0.99	0.75
socfb-UCLA	0.88	0.89	0.90	0.83	0.39	0.75	0.79	0.77	0.85	0.43
web-it-2004	0.05	0.06	0.05	0.12	0.14	0.43	1.01	1.02	0.42	0.59
socfb-CMU	0.33	0.34	0.34	0.38	0.49	0.35	0.35	0.35	0.36	4.87
sc-shipsec5	0.23	0.27	0.26	0.25	4.51	0.27	0.38	0.36	0.27	5.79
socfb-OR	0.61	0.58	0.60	0.67	1.54	0.40	0.40	0.40	0.43	1.51
ca-coauthors-dblp	0.11	0.12	0.12	0.37	0.39	0.10	0.11	0.11	0.13	0.46
scc_fb-forum	0.23	0.20	0.31	0.39	0.36	0.58	0.28	0.61	0.56	0.44
scc_retweet	0.06	0.08	0.07	0.05	0.11	0.05	0.07	0.07	0.06	0.06
socfb-UConn	0.38	0.39	0.40	0.39	0.20	0.09	0.10	0.10	0.10	0.06
ca-MathSciNet	0.03	0.03	0.40	0.06	0.08	0.03	0.10	0.10	0.10	0.12
tech-RL-caida	0.06	0.07	0.06	0.19	3.70	0.08	0.09	0.08	0.08	1.07
socfb-UCSB37	0.00	0.07 0.12	0.00 0.12	0.13	0.09	0.08	0.03	0.08	0.08	0.05
web-spam	0.05	0.12	0.12 0.04	0.13	0.03 0.07	0.03	0.00	0.00	0.00	0.03
ca-dblp-2012	0.03	0.02	0.04 0.02	0.03	0.08	0.01 0.02	0.01	0.01	0.01	0.07 0.12
tech-as-caida2007	0.02 0.01	0.02 0.01	0.02	0.01	0.03	0.02 0.01	0.02 0.01	0.02	0.02	0.12 0.01
ca-citeseer	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.04	0.03	0.01
tech-internet-as	0.03	0.03	0.03	0.04 0.01	0.04 0.01	0.03	0.03	0.04	0.03	0.02
ca-AstroPh	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
ca-Astror ii ca-dblp-2010	0.01 0.02	0.01 0.02	0.01	0.01 0.04	0.01 0.04	$0.01 \\ 0.02$	0.01	0.01 0.02	0.01	0.02 0.07
web-arabic-2005	0.02 0.01	0.02 0.01	0.01	0.04 0.03	0.04 0.03	0.02 0.01	0.02 0.01	0.02 0.01	0.01	0.07 0.04
ca-CondMat	0.01	0.01	0.01	0.03	0.03 0.01	0.01 0.00	0.01	0.01	0.01	0.04 OOT
ia-infect-dublin	0.00	0.00	0.00	0.01	0.01	0.00	0.00		0.00	0.01
			0.00					0.00		
web-sk-2005	0.01	0.02		0.02	0.03	0.01	0.02	0.02	0.01	0.05
scc_enron-only	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.02	0.02	0.03
soc-brightkite	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.02	0.01	OOT
ca-HepPh	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
scc_infect-dublin	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.01
web-indochina-2004	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
scc_retweet-crawl	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01
web-webbase-2001	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
ca-GrQc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
scc_rt_lolgop	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
scc_infect-hyper	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
web- $BerkStan$	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00

scc_rt_occupy	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$scc_rt_occupywallstnyc$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$scc_rt_gmanews$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
web-edu	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
tech-routers-rf	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_oman	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$scc_rt_lebanon$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$scc_rt_voteonedirection$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$\operatorname{scc_rt_gop}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
$\operatorname{scc_rt_http}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
scc_rt_israel	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_qatif	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
$\operatorname{scc_rt_tlot}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_obama	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_p2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_uae	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_ksa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_libya	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_tcot	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
scc_rt_dash	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_damascus	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_saudi	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
soc-karate	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
soc-dolphins	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
rt-retweet	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_assad	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$scc_rt_mittromney$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	ООТ
$scc_rt_onedirection$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
scc_rt_bahrain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_barackobama	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_alwefaq	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_justinbieber	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
ia-enron-only	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ca-netscience	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ca-CSphd	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
rt-twitter-copen	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
bio-diseasome	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
web-polblogs	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
bio-yeast	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
bio-celegans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
soc-wiki-Vote	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
web-google	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ia-fb-messages	0.00	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.04
ia-email-univ	0.00	$0.00 \\ 0.00$	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01
inf-power	0.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
ia-reality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ca-Erdos992	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
bio-dmela	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
tech-p2p-gnutella	0.04	0.04	0.04	0.04	0.03	0.01	0.01	0.01	0.01	0.03
rec-amazon	0.01	0.01	0.01	0.01	0.06	0.01	0.01	0.01	0.04	144.84
soc-douban	0.07	0.07	0.07	0.06	0.16	0.06	0.06	0.07	0.06	TOO
inf-roadNet-PA	0.14	0.14	0.15	0.14	0.19	0.07	0.07	0.09	0.15	TOO
inf-roadNet-CA	0.26	0.25	0.26	0.22	0.50	0.13	0.14	0.13	0.27	TOO
inf-road-usa	3.56	3.48	$\frac{3.59}{14.70}$	11.54	8.89	1.91	1.48	1.49	4.25	OOT
socfb-uci-uni	15.04 OOT	14.79	14.70 OOT	21.14 OOT	20.09	10.68	9.53 OOT	8.99 OOT	26.90	25.43 OOT
soc-BlogCatalog	TOO	OOT	OOT	TOO TOO	TOO TOO	TOO	OOT	TOO	TOO	TOO
soc-buzznet soc-flickr	TOO TOO	385.49	415.00		OOT	TOO	1427.91	TOO	TOO	OOT OOT
		OOT	OOT 0.76	OOT		OOT 0.76	OOT	OOT	OOT	
web-wikipedia2009	0.77	1.44	0.76	1.03	7.66	0.76	0.93	0.71	0.84	360.17

Table 2: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=10 and 15 on Network-Repo graphs before filtering

			k=10					k=15		
Graph	Maple	$Maple_{com}$		kPlexS	KpLeX	Maple	$Maple_{com}$		kPlexS	KpLeX
soc-Live $Mocha$	46.25	8.43	11.35	628.76	OOT	3.65	7.33	3.31	3.76	OOT
soc-youtube	0.66	0.86	0.92	0.64	OOT	5.56	5.00	4.03	16.30	OOT
$\operatorname{soc-youtube-snap}$	1.14	1.52	1.01	1.09	TOO	6.43	11.76	6.59	33.11	TOO
soc-lastfm	2.74	1.81	1.61	5.38	OOT	13.14	22.43	14.25	84.67	TOO
$\operatorname{soc-digg}$	1117.74	75.08	1130.93	928.29	OOT	17.03	21.93	19.96	14.85	TOO
ia-wiki-Talk	4.87	3.56	4.39	6.86	OOT	17.46	18.06	13.20	126.63	OOT
socfb-Duke 14	5.42	1.67	3.73	298.91	OOT	32.97	67.06	62.39	OOT	OOT
soc-orkut	359.11	353.60	371.74	OOT	OOT	260.54	287.33	235.00	213.18	OOT
$\operatorname{sc-ldoor}$	11.16	341.32	18.00	11.26	OOT	8.39	121.43	12.28	7.91	OOT
$\operatorname{sc-msdoor}$	5.69	72.46	7.25	5.14	OOT	4.24	77.69	5.70	3.83	OOT
$\operatorname{sc-nasasrb}$	2.07	20.89	3.24	2.22	OOT	12.87	57.01	12.96	148.89	OOT
sc-pkustk11	83.47	68.51	817.22	397.89	431.69	1.41	15.17	2.03	136.90	OOT
soc-flixster	1.78	0.86	1.69	20.99	OOT	0.45	0.58	0.50	1.11	259.17
$\operatorname{sc-pwtk}$	11.52	152.88	14.67	10.85	OOT	23.77	144.32	25.96	116.60	OOT
$scc_reality$	6.89	36.43	41.12	5.83	5.16	75.17	194.93	104.34	58.27	OOT
soc-gowalla	0.24	0.25	0.28	0.26	OOT	0.39	1.02	0.30	0.39	OOT
soc-epinions	0.02	0.02	0.02	0.02	0.03	0.23	0.35	0.21	0.50	5.08
socfb-B-anon	33.13	32.57	29.43	28.19	OOT	31.86	29.20	30.30	24.92	OOT
socfb-A-anon	30.21	28.70	31.14	24.49	OOT	27.64	24.82	26.63	21.67	OOT
soc-pokec	29.97	25.81	28.16		$1,\!451.55$	17.91	22.82	22.32	16.01	OOT
soc-twitter-follows	0.20	0.39	0.13	0.11	TOO	0.13	0.08	0.27	5.60	OOT
soc-FourSquare	5.69	40.23	4.77	4.33	TOO	7.91	44.40	8.19	7.70	OOT
socfb-Indiana	1.70	1.77	1.78	1.56	723.78	1.61	1.60	1.53	1.42	OOT
soc-slashdot	0.06	0.10	0.07	0.13	TOO	1.25	1.32	1.46	9.47	OOT
ia-enron-large	0.10	0.14	0.09	0.08	TOO	0.14	0.33	0.13	0.12	OOT
tech-WHOIS	0.10	0.08	0.11	0.17	552.64	0.30	0.17	0.29	0.40	29.50
ia-email-EU	0.03	0.05	0.03	0.04	0.31	0.18	0.23	0.15	0.18	33.93
socfb-Texas84	1.29	1.33	1.41	1.24	0.58	1.08	1.25	1.18	1.06	10.79
sc-pkustk13	1.80	3.32	2.97	1.85	185.85	3.36	21.24	4.33	2.98	TOO
socfb-UF	0.88	0.95	0.93	0.82	TOO	0.83	0.99	0.84	0.77	20.93
rt-retweet-crawl	0.22	0.22	0.23	0.31	0.30	0.15	0.15	0.17	0.28	0.24
tech-as-skitter	1.07	1.07	1.11	4.35	TOO	1.05	1.08	1.01	1.38	TOO
soc-livejournal	4.02	2.73	3.77	3.52	4.61	2.02	2.65	2.58	3.46	3.77
socfb-UIllinois	2.06	2.40	2.26	2.15	OOT	2.14	2.47	2.21	1.88	ООТ
$scc_fb-messages$	2.40	2.10	2.13	1.98	7.09	0.11	0.12	0.11	0.01	0.01
scc_twitter-copen	0.77	12.99	2.46	0.67	1.12	1.98	1.85	2.46	1.97	29.23
socfb-Penn94	1.54	1.61	1.59	1.45	258.81	1.82	1.81	1.50	1.43	540.98
ca-hollywood-2009	1.06	1.33	1.30	1.73	1.49	1.06	1.44	1.16	1.48	1.53
web-uk-2005	1.31	3.58	3.77	1.38	2.21	1.34	4.34	4.33	1.41	2.21
socfb-Stanford3	1.29	1.19	1.33	1.16	OOT	1.20	1.40	1.17	1.12	TOO
socfb-MIT	0.39	0.41	0.40	0.38	2.00	0.39	0.50	0.40	0.39	OOT
soc-delicious	0.08	0.10	0.09	0.10	0.36	0.06	0.08	0.07	0.09	0.70
sc-shipsec1	0.25	0.47	0.28	0.22	17.38	0.64	1.97	0.96	1.30	OOT
socfb-Berkeley13	1.10	1.11	1.05	0.97	30.37	1.07	1.08	0.96	0.90	0.73
ia-infect-hyper	0.01	0.01	0.01	0.01	0.02	0.10	0.09	0.54	1.08	4.49
socfb-Wisconsin87	0.82	0.84	0.92	0.80	7.63	0.26	0.28	0.26	0.24	0.33
socfb-UCLA	0.82	0.79	0.78	0.77	0.36	0.79	0.92	0.87	0.79	6.56
web-it-2004	0.87	2.00	2.14	0.88	1.50	0.89	2.37	1.96	0.89	1.46
socfb-CMU	0.37	0.35	0.38	0.34	172.35	0.34	0.43	0.35	0.33	OOT
sc-shipsec5	0.61	1.79	0.81	0.59	104.41	0.47	3.16	0.68	0.70	1,373.96
socfb-OR	0.34	0.30	0.34	0.29	0.45	0.26	0.29	0.28	0.26	2.81
ca-coauthors-dblp	0.32	0.57	0.59	0.57	0.57	0.39	0.98	1.00	0.64	0.79
scc_fb-forum	0.02	0.02	0.02	0.00	0.00	0.02	0.02	0.02	0.00	0.00
scc_retweet	0.19	0.25	0.20	0.17	0.17	0.48	0.35	0.39	0.19	0.45
socfb-UConn	0.10	0.10	0.10	0.09	0.09	0.09	0.10	0.09	0.08	0.83

ca-MathSciNet	0.03	0.04	0.05	0.06	0.07	0.05	0.05	0.04	0.09	0.27
tech-RL-caida	0.08	0.11	0.09	0.09	1.44	0.07	0.10	0.09	0.08	0.16
socfb-UCSB37	0.02	0.02	0.02	0.01	0.02	0.08	0.10	0.09	0.08	0.06
web-spam	0.02	0.02	0.02	0.02	0.16	0.02	0.01	0.02	0.03	0.02
ca-dblp-2012	0.02	0.02	0.02	0.07	0.07	0.03	0.05	0.04	0.08	0.07
tech-as-caida2007	0.01	0.01	0.01	0.01	0.02	0.07	0.16	0.07	0.08	0.78
ca-citeseer	0.03	0.03	0.04	0.04	0.04	0.03	0.03	0.03	0.04	0.04
tech-internet-as	0.02	0.03	0.02	0.02	0.10	0.04	0.03	0.03	0.05	0.02
ca-AstroPh	0.01	0.02	0.01	0.01	0.02	0.02	0.04	0.02	0.02	0.04
ca-dblp-2010	0.02	0.02	0.02	0.04	0.03	0.02	0.03	0.02	0.04	0.04
web-arabic-2005	0.02	0.02	0.02	0.04	0.03 0.04	0.01	0.03	0.03	0.04	0.04
ca-CondMat	0.01	0.01	0.01	0.03	0.04 0.01	0.01	0.02	0.02	0.03	0.06
ia-infect-dublin	0.01	0.01	0.01	0.01	0.00	0.01	0.02	0.01	0.02	0.00
web-sk-2005	0.00	0.00	0.00	0.00	0.00	0.00 0.02	0.00	0.00 0.02	0.00	0.00
scc_enron-only	0.02	0.03	0.02 0.00	0.02	0.04 0.02	0.02 0.00	0.03	0.02 0.00	0.02	0.03
soc-brightkite	0.00	0.00	0.00	0.00	0.02 0.05	0.00 0.01	0.00	0.00 0.01	0.00	0.00
ca-HepPh	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.00	0.01
scc_infect-dublin	0.01	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.02
web-indochina-2004	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01
scc_retweet-crawl	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
web-webbase-2001	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.01
ca-GrQc	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
scc_rt_lolgop	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_infect-hyper	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
web- $BerkStan$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
scc_rt_occupy	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_occupywallstnyc	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$scc_rt_gmanews$	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
web-edu	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
tech-routers-rf	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_oman	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$scc_rt_lebanon$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$scc_rt_voteonedirection$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_gop	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_http	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\operatorname{scc_rt_israel}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_qatif	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\operatorname{scc_rt_tlot}$	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	0.00
scc_rt_obama	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_p2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_uae	0.00	0.00	0.00	0.00	ООТ	0.00	0.00	0.00	0.00	0.00
scc_rt_ksa	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	0.00
scc_rt_libya	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_tcot	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\operatorname{scc_rt_dash}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_damascus	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_saudi	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
soc-karate	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
soc-dolphins										
rt-retweet	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
scc_rt_assad	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_mittromney	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
scc_rt_onedirection	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_bahrain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
scc_rt_barackobama	0.00	0.00	0.00	0.00	TOO	0.00	0.00	0.00	0.00	0.00
scc_rt_alwefaq	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
$scc_rt_justinbieber$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ia-enron-only	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	OOT
ca-netscience	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
ca-CSphd	0.00	0.00	0.00	0.00	TOO	0.00	0.00	0.00	0.00	OOT
rt-twitter-copen	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	OOT

bio-diseasome	0.00	0.00	0.00	0.00	ООТ	0.00	0.00	0.00	0.00	ООТ
web-polblogs	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
bio-yeast	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	0.00
bio-celegans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
soc-wiki-Vote	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
web-google	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
ia-fb-messages	0.22	0.01	0.24	10.97	OOT	0.00	0.00	0.00	487.84	TOO
ia-email-univ	0.01	0.00	0.02	0.01	OOT	0.00	0.00	0.00	0.01	TOO
inf-power	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	TOO
ia-reality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
ca-Erdos 992	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	TOO
bio-dmela	0.06	0.02	0.11	0.10	0.86	0.00	0.00	0.00	0.01	TOO
tech-p2p-gnutella	0.01	0.01	0.01	0.18	OOT	0.01	0.01	0.00	0.18	TOO
rec-amazon	0.00	0.00	0.01	0.05	OOT	0.01	0.01	0.01	0.05	OOT
soc-douban	0.06	0.23	0.58	0.12	923.69	0.39	0.09	3.81	4.48	OOT
\inf -roadNet-PA	0.06	0.06	0.06	0.12	OOT	0.07	0.06	0.06	0.15	OOT
\inf -roadNet-CA	0.12	0.10	0.12	0.23	OOT	0.11	0.12	0.12	0.22	OOT
inf-road-usa	1.37	1.49	1.52	12.35	OOT	1.48	1.33	1.48	3.48	OOT
socfb-uci-uni	4.72	4.95	4.24	20.17	19.99	9.48	5.44	4.88	25.37	TOO
soc-BlogCatalog	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
soc-buzznet	OOT	OOT	OOT	OOT	0.00	TOO	OOT	OOT	OOT	OOT
soc-flickr	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
web-wikipedia2009	0.74	TOO	0.36	0.92	OOT	OOT	ООТ	OOT	TOO	OOT

Table 3: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=20 on Network-Repo graphs before filtering

			k=20		
Graph	Maple	$Maple_{com}$	Maple _{hyb}	kPlexS	KpLeX
soc-LiveMocha	591.17	503.83	363.05	OOT	OOT
soc-youtube	652.01	992.56	717.60	OOT	OOT
soc-youtube-snap	OOT	253.62	OOT	OOT	OOT
soc-lastfm	1413.95	173.40	1430.59	OOT	OOT
$\operatorname{soc-digg}$	16.25	21.74	17.30	15.81	OOT
ia-wiki-Talk	OOT	OOT	OOT	OOT	OOT
socfb-Duke14	43.67	8.59	44.43	OOT	OOT
soc-orkut	200.71	245.36	233.96	181.54	OOT
$\operatorname{sc-ldoor}$	13.97	1400.60	25.35	14.11	OOT
$\operatorname{sc-msdoor}$	15.92	336.42	19.94	887.96	OOT
sc-nasasrb	12.34	63.47	12.67	459.33	OOT
sc-pkustk11	14.68	18.48	16.01	407.64	OOT
soc-flixster	0.30	0.31	0.29	0.54	1.41
sc-pwtk	6.30	75.65	86.56	122.78	OOT
$scc_reality$	46.66	OOT	105.09	49.87	OOT
soc-gowalla	7.98	13.95	6.65	64.12	OOT
soc-epinions	10.45	2.61	10.47	57.31	OOT
socfb-B-anon	22.48	30.54	24.21	27.28	OOT
socfb-A-anon	18.53	27.08	20.32	25.05	OOT
soc-pokec	17.52	21.34	15.00	14.99	OOT
soc-twitter-follows	20.55	1.44	44.25	24.93	OOT
soc-FourSquare	7.67	312.91	10.42	8.98	OOT
socfb-Indiana	5.30	11.04	5.49	14.32	446.69
soc-slashdot	0.29	1.44	0.08	0.11	110.08
ia-enron-large	2.05	1.83	1.70	8.21	OOT
tech-WHOIS	0.08	0.36	0.07	0.11	3.08
ia-email-EU	6.23	3.22	4.10	2.26	OOT
socfb-Texas84	0.85	0.94	0.76	0.77	2.99
sc-pkustk 13	4.58	53.81	7.36	4.78	OOT

socfb-UF	0.65	0.73	0.84	0.72	0.40
rt-retweet-crawl	0.10	0.11	0.10	4.53	2.11
tech-as-skitter	1.21	1.20	1.21	1.82	OOT
soc-livejournal	1.38	1.46	1.27	3.60	5.21
socfb-UIllinois	1.98	5.44	2.40	1.88	OOT
$scc_fb-messages$	0.12	0.12	0.11	0.10	0.00
$scc_twitter-copen$	0.10	0.11	0.10	0.09	0.01
socfb-Penn94	1.73	2.29	1.72	1.73	OOT
ca-hollywood-2009	1.31	1.58	1.36	1.03	1.58
web-uk-2005	1.46	6.14	5.46	1.32	2.24
socfb-Stanford3	1.41	2.39	1.26	1.36	OOT
$\operatorname{socfb-MIT}$	0.52	1.49	0.54	1.40	OOT
soc-delicious	1.35	1.58	0.45	0.89	1,033.48
sc-shipsec1	0.48	1.98	1.24	1.28	TOO
socfb-Berkeley13	0.83	1.02	0.97	0.91	5.92
ia-infect-hyper	0.20	0.22	0.26	0.29	7.73
socfb-Wisconsin87	0.22	0.26	0.28	0.22	0.15
socfb-UCLA	0.93	1.16	0.95	0.94	11.02
web-it- 2004	0.91	3.57	2.08	0.88	1.53
socfb-CMU	0.38	0.69	0.46	0.85	TOO
sc-shipsec5	0.30	1.43	0.35	0.36	TOO
socfb-OR	0.27	0.35	0.28	0.26	1,202.98
ca-coauthors-dblp	0.39	1.24	1.10	0.39	0.83
scc_fb -forum	0.01	0.02	0.02	0.02	0.00
scc_retweet	0.04	0.05	0.04	0.04	0.03
socfb-UConn	0.10	0.11	0.12	0.09	5.48
ca-MathSciNet	0.03	0.04	0.03	0.21	33.18
tech-RL-caida	0.04	0.13	0.05	0.07	0.77
socfb-UCSB37	0.08	0.11	0.09	0.08	0.06
web-spam	0.01	0.01	0.01	0.01	0.19
ca-dblp-2012	0.03	0.06	0.05	0.03	0.11
tech-as-caida2007	0.03	0.05	0.03	0.02	0.03
ca-citeseer	0.03	0.05	0.04	0.06	0.07
tech-internet-as	0.02	0.03	0.01	0.03	0.02
ca-AstroPh	0.04	0.20	0.05	0.04	0.12
ca-dblp-2010	0.02	0.03	0.02	0.02	0.06
web-arabic-2005	0.02	0.04	0.02	0.02	0.07
ca-CondMat	0.00	0.01	0.00	0.03	1.22
ia-infect-dublin web-sk-2005	0.00	$0.00 \\ 0.05$	0.00	0.03	0.00
scc_enron-only	$0.02 \\ 0.02$	0.03 0.02	$0.02 \\ 0.02$	$0.02 \\ 0.02$	$0.06 \\ 0.02$
soc-brightkite	0.02 0.01	0.02 0.01	0.02 0.01	0.02 0.01	0.02 0.09
ca-HepPh	0.01	0.01	0.01	0.01	0.03
scc_infect-dublin	0.01	0.02	0.02	0.01	0.01
web-indochina-2004	0.01	0.00	0.01	0.01	0.01
scc_retweet-crawl	0.00	0.01	0.00	0.01	0.01
web-webbase-2001	0.00	0.00	0.00	0.00	0.01
ca-GrQc	0.00	0.01	0.00	0.00	0.01
scc_rt_lolgop	0.00	0.00	0.00	0.00	0.00
scc_infect-hyper	0.00	0.00	0.00	0.00	0.00
web-BerkStan	0.00	0.00	0.00	0.00	0.01
scc_rt_occupy	0.00	0.00	0.00	0.00	0.00
scc_rt_occupywallstnyc	0.00	0.00	0.00	0.00	0.00
scc_rt_gmanews	0.00	0.00	0.00	0.00	0.00
web-edu	0.00	0.00	0.00	0.00	0.01
tech-routers-rf	0.00	0.00	0.00	0.00	0.01
$\operatorname{scc_rt_oman}$	0.00	0.00	0.00	0.00	0.00
$scc_rt_lebanon$	0.00	0.00	0.00	0.00	0.00
$scc_rt_voteonedirection$	0.00	0.00	0.00	0.00	0.00
scc_rt_gop	0.00	0.00	0.00	0.00	0.00
$\operatorname{scc_rt_http}$	0.00	0.00	0.00	0.00	0.00
-					

scc_rt_israel	0.00	0.00	0.00	0.00	0.00
scc_rt_qatif	0.00	0.00	0.00	0.00	0.00
scc_rt_tlot	0.00	0.00	0.00	0.00	0.00
scc_rt_obama	0.00	0.00	0.00	0.00	0.00
scc_rt_p2	0.00	0.00	0.00	0.00	0.00
scc_rt_uae	0.00	0.00	0.00	0.00	0.00
scc_rt_ksa	0.00	0.00	0.00	0.00	0.00
scc_rt_libya	0.00	0.00	0.00	0.00	0.00
scc_rt_tcot	0.00	0.00	0.00	0.00	0.00
scc_rt_dash	0.00	0.00	0.00	0.00	0.00
$scc_rt_damascus$	0.00	0.00	0.00	0.00	0.00
scc_rt_saudi	0.00	0.00	0.00	0.00	0.00
soc-karate	0.00	0.00	0.00	0.00	0.00
soc-dolphins	0.00	0.00	0.00	0.00	0.00
rt-retweet	0.00	0.00	0.00	0.00	0.00
scc_rt_assad	0.00	0.00	0.00	0.00	0.00
$scc_rt_mittromney$	0.00	0.00	0.00	0.00	0.00
$scc_rt_one direction$	0.00	0.00	0.00	0.00	0.00
$scc_rt_bahrain$	0.00	0.00	0.00	0.00	0.00
$scc_rt_barackobama$	0.00	0.00	0.00	0.00	0.00
$scc_rt_alwefaq$	0.00	0.00	0.00	0.00	0.00
$scc_rt_justinbieber$	0.00	0.00	0.00	0.00	0.00
ia-enron-only	0.00	0.00	0.00	0.01	0.00
ca-netscience	0.00	0.00	0.00	0.00	0.00
ca-CSphd	0.00	0.00	0.00	0.00	0.00
rt-twitter-copen	0.00	0.00	0.00	0.00	0.00
bio-diseasome	0.00	0.00	0.00	0.13	0.01
web-polblogs	0.00	0.00	0.00	0.00	0.00
bio-yeast	0.00	0.00	0.00	0.00	0.02
bio-celegans	0.00	0.00	0.00	0.00	0.00
soc-wiki-Vote	0.00	0.00	0.00	0.00	0.00
web-google	0.00	0.00	0.00	0.00	0.00
ia-fb-messages	0.00	0.00	0.00	TOO	0.02
ia-email-univ	0.00	0.00	0.00	0.01	0.01
inf-power	0.00	0.00	0.00	0.00	0.01
ia-reality	0.00	0.00	0.00	0.00	0.00
ca-Erdos 992	0.00	0.00	0.00	0.00	0.01
bio-dmela	0.00	0.00	0.00	0.00	0.00
tech-p2p-gnutella	0.00	0.01	0.01	0.38	10.44
rec-amazon	0.00	0.01	0.01	0.05	66.03
soc-douban	0.01	0.01	0.01	7.93	3.08
\inf -roadNet-PA	0.06	0.07	0.06	0.15	0.32
\inf -roadNet-CA	0.11	0.11	0.12	0.26	OOT
inf-road-usa	1.31	1.88	1.49	13.12	OOT
socfb-uci-uni	4.91	4.93	4.24	24.62	24.59
soc-BlogCatalog	OOT	OOT	OOT	TOO	TOO
soc-buzznet	OOT	OOT	OOT	OOT	TOO
soc-flickr	OOT	OOT	OOT	TOO	TOO
web-wikipedia2009	OOT	OOT	OOT	OOT	OOT

2.2 10th-DIMACS Graphs

In this subsection, we show the running time comparison of five algorithms on 10th-DIMACS graphs for k=2,5,10,15 and 20. Due to the limit of page size, we divide k values into 3 groups to demonstrate the results. The first group is k=2 and 5, the second group is k=10 and 15 and the last group is k=20.

Table 4: Running time of Maple, Maple $_{com}$, Maple $_{hyb}$, kPlexS and KpLeX for k=2 and 5 on 10th-DIMACS graphs before filtering

	k=2	k=5
Graph		

consph 1.72 8.01 2.79 1.75 403.20 24.86 45.09 15.76 56.14 560 connectus 0.30 0.29 0.29 0.24 1.95 0.26 0.26 0.26 0.27 0.26 0.27 0.26 0.27 0.29 0.30 2.2 0.25 0.25
rgg_n_2_24_s0 3.02 3.10 2.98 10.89 18.30 6.17 4.19 4.35 4.14 20 rgg_n_2_23_s0 1.48 1.53 1.46 5.12 8.32 2.67 2.12 2.05 2.52 9 rgg_n_2_22_s0 0.64 0.65 0.70 2.60 3.98 0.93 0.95 0.91 0.90 4 rgg_n_2_21_s0 0.29 0.33 0.32 1.24 1.52 0.40 0.42 0.40 0.42 0.40 0.42 1.80 0.93 0.91 0.90 4 soc-Epinions1 0.81 0.48 0.48 2.40 1.88 0.30 0.27 0.29 0.30 2 bio-pdb1HYS 1.85 1.92 2.01 2.06 9.82 1.85 2.36 OOT 1.87 256 co-papers-dblp 0.11 0.11 0.11 0.41 0.38 0.11 0.12 0.12 0.12 0.12 0.12 0.12
rgg.n_2_23_s0 1.48 1.53 1.46 5.12 8.32 2.67 2.12 2.05 2.52 9. rgg.n_2_22_s0 0.64 0.65 0.70 2.60 3.98 0.93 0.95 0.91 0.90 4. rgg.n_2_21_s0 0.29 0.33 0.32 1.24 1.52 0.40 0.42 0.40 0.42 1. soc-Epinions1 0.81 0.48 0.48 2.40 1.88 0.30 0.27 0.29 0.30 2. bio-pdb1HYS 1.85 1.92 2.01 2.06 9.82 1.85 2.36 OOT 1.87 258 co-papers-dblp 0.11 0.11 0.11 0.41 0.38 0.11 0.12 0.12 0.12 0. rgg.n_2_2_0_s0 0.20 0.19 0.19 0.51 0.50 0.26 0.27 0.26 0.26 0. citationCiteseer 0.25 0.25 0.27 0.28 0.22 0.1
rgg_n_2_22_s0
rgg.n.2_21_s0 0.29 0.33 0.32 1.24 1.52 0.40 0.42 0.40 0.42 1.8 soc-Epinions1 0.81 0.48 0.48 2.40 1.88 0.30 0.27 0.29 0.30 2. bio-pdb1HYS 1.85 1.92 2.01 2.06 9.82 1.85 2.36 OOT 1.87 258 co-papers-dblp 0.11 0.11 0.11 0.41 0.38 0.11 0.12 0.13 0.12 0.13 0.17 0.19 0.17 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
soc-Epinions1 0.81 0.48 0.48 2.40 1.88 0.30 0.27 0.29 0.30 2. bio-pdb1HYS 1.85 1.92 2.01 2.06 9.82 1.85 2.36 OOT 1.87 259 co-papers-dblp 0.11 0.11 0.11 0.41 0.38 0.11 0.12 0.13 0.12 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.17 0.19 0.18<
bio-pdb1HYS 1.85 1.92 2.01 2.06 9.82 1.85 2.36 OOT 1.87 259 co-papers-dblp 0.11 0.11 0.11 0.41 0.38 0.11 0.12 0.12 0.12 0. rgg_n_2_2_0_s0 0.20 0.19 0.19 0.51 0.50 0.26 0.27 0.26 0.26 0. citationCiteseer 0.25 0.25 0.27 0.28 0.22 0.15 0.17 0.19 0.17 0. co-papers-citeseer 0.10 0.11 0.10 0.36 0.35 0.12 0.13 0.12 0.11 0. tech-caidaRouterLevel 0.07 0.07 0.07 0.22 3.76 0.08 0.09 0.09 0.08 1. cnr-2000 0.08 0.08 0.08 0.07 0.09 0.08 0.08 0.09 0.08 0. email-enron 0.10 0.10 0.11 0.44 0.10 <
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rgg.n.2_20.s0 0.20 0.19 0.19 0.51 0.50 0.26 0.27 0.26 0.26 0.26 0.27 0.26 0.26 0.26 0.27 0.26 0.27 0.26 0.26 0.27 0.26 0.26 0.27 0.26 0.26 0.27 0.17 0.17 0.19 0.17 0.17 0.19 0.17 0.17 0.19 0.17 0.10 0.11 0.13 0.12 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.02 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.08 0.09 0.02 0.02
citationCiteseer 0.25 0.25 0.27 0.28 0.22 0.15 0.17 0.19 0.17 0. co-papers-citeseer 0.10 0.11 0.10 0.36 0.35 0.12 0.13 0.12 0.11 0. tech-caidaRouterLevel 0.07 0.07 0.07 0.22 3.76 0.08 0.09 0.09 0.08 1. cnr-2000 0.08 0.08 0.08 0.07 0.09 0.08 0.09 0.08 0. email-enron 0.10 0.10 0.11 0.44 0.10 0.11 0.11 0.10 0. polblogs 0.04 0.03 0.03 0.09 0.14 0.02
co-papers-citeseer 0.10 0.11 0.10 0.36 0.35 0.12 0.13 0.12 0.11 0. tech-caidaRouterLevel 0.07 0.07 0.07 0.22 3.76 0.08 0.09 0.09 0.08 1. cnr-2000 0.08 0.08 0.08 0.07 0.09 0.08 0.09 0.08 0. email-enron 0.10 0.10 0.11 0.44 0.10 0.11 0.11 0.10 0. polblogs 0.04 0.03 0.03 0.09 0.14 0.02 0.02 0.02 0.02 0.02 0. c-62ghs 0.07 0.07 0.07 0.07 0.10 0.06 0.06 0.06 0.07 0. coAuthorsCiteseer 0.03 0.03 0.03 0.04 0.06 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.04 0.04 0.04
tech-caidaRouterLevel 0.07 0.07 0.02 3.76 0.08 0.09 0.09 0.08 1. cnr-2000 0.08 0.08 0.08 0.07 0.09 0.08 0.09 0.08 0. email-enron 0.10 0.10 0.11 0.44 0.10 0.11 0.11 0.11 0.11 0.11 0.10 0. polblogs 0.04 0.03 0.03 0.09 0.14 0.02 0.03 0.03
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polblogs 0.04 0.03 0.03 0.09 0.14 0.02 0.03 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04
c-62ghs 0.07 0.07 0.07 0.10 0.06 0.06 0.06 0.06 0.07 0. coAuthorsCiteseer 0.03 0.03 0.04 0.06 0.03 0.04 0.00
coAuthorsCiteseer 0.03 0.03 0.04 0.06 0.03 0.04 0.00 0.00
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bio-celegansneural $0.00 \ 0.00 \ 0.00 \ 0.00 \ OOT \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00$
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fe-sphere 0.01 0.01 0.01 0.01 4.00 0.01 0.02 0.01 0.01 4.
cti 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.0
wing 0.04 0.05 0.05 0.04 0.03 0.10 0.33 0.17 0.07 52
wing_nodal 0.04 0.05 0.05 0.03 3.55 0.10 0.33 0.17 0.13 3.
delaunay_n14 0.02 0.02 0.02 0.02 5.85 0.03 0.05 0.04 0.03 6.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
t60k 0.02 0.03 0.02 0.02 45.16 0.00 0.00 0.00 0.03 46
delaunay_n15
delaunay_n16 0.08 0.07 0.08 0.07 93.83 0.09 0.45 0.15 0.11 93 inf-luxembourg_osm 0.02 0.02 0.03 0.02 OOT 0.00 0.00 0.00 0.03 115
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
fe-ocean 0.07 0.07 0.07 0.06 0.11 0.01 0.01 0.01 0.08 0.
fe-t10000.0h 0.19 0.22 OOT 0.21 165.56 0.43 5.04 OOT 0.50 167
delaunay_n17 0.15 0.15 0.18 0.14 373.77 0.19 1.63 0.30 0.22 386
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
rgg_n_2_17_s0 0.02 0.02 0.02 0.04 0.03 0.03 0.03 0.06 0.03 0.
fe_rotor 0.33 0.46 1.70 0.32 285.20 0.50 7.09 0.62 0.51 285
144 0.78 1.03 1.06 0.71 632.61 0.79 5.37 0.88 0.77 626
delaunay_n18 0.33 0.31 0.36 0.30 1497.72 0.39 6.21 0.61 0.45 151
rgg_n_2_18_s0 0.04 0.04 0.04 0.10 0.09 0.05 0.05 0.05 0.05 0.
cop20k_A 0.85 2.05 1.02 0.77 359.39 0.82 3.83 1.02 0.87 232
m14b 1.17 2.45 1.73 1.20 1418.07 1.20 19.02 1.37 1.14 138
delaunay_n19
rgg_n_2_19_s0 0.06 0.07 0.07 0.21 0.24 0.09 0.09 0.09 0.10 0.
auto 3.25 4.02 4.15 2.90 OOT 3.61 107.91 4.19 3.72 OO
inf-belgium_osm $0.17 0.17 0.17 0.13 0.16 0.07 0.07 0.07 0.50 00$
delaunay_n20 1.35 1.36 1.37 1.26 OOT 1.40 94.83 2.14 1.79 OO
inf-netherlands_osm 0.25 0.24 0.24 0.22 0.34 0.12 0.11 0.12 0.94 00
delaunay_n21 2.71 2.83 2.78 2.58 OOT 2.34 4.90 3.48 2.04 OO
packing-500x100x100-b050 7.55 8.83 8.00 7.10 OOT 21.24 OOT 31.19 21.12 OO
venturiLevel3 2.07 58.30 2.44 1.90 OOT 0.39 0.37 0.39 2.04 OO

hugetrace-00000	4.84	4.69	4.05	3.34	ООТ	0.22	0.22	0.21	4.28	ООТ
delaunay_n22	5.57	5.89	5.89	5.19	OOT	3.14	1146.97	3.59	3.51	OOT
hugetric-00000	1.17	1.18	1.24	0.93	OOT	0.27	0.28	0.28	6.40	OOT
channel- $500x100x100$ -b050	15.47	13.08	13.41	12.56	198.51	35.37	OOT	61.64	35.40	OOT
adaptive	0.95	0.97	0.93	0.74	OOT	0.51	0.50	0.57	11.12	1.64
$\inf - italy_osm$	0.78	0.73	0.75	0.54	0.90	0.30	0.30	0.30	2.62	OOT
hugetric-00010	1.56	1.54	1.52	1.44	OOT	0.37	0.37	0.41	9.52	OOT
inf -great-britain_osm	0.97	1.04	0.93	0.76	OOT	0.43	0.37	0.37	3.71	1.90
hugetric-00020	9.87	10.77	10.47	8.61	OOT	0.43	0.40	0.41	10.19	OOT
\inf -asia_osm	1.47	1.48	1.40	1.10	2.32	0.59	0.55	0.55	1.35	OOT
$delaunay_n23$	11.62	13.48	12.38	11.75	OOT	7.29	777.01	8.84	7.69	OOT
\inf -germany_osm	1.76	1.68	1.73	1.55	2.33	0.63	0.59	0.61	6.77	OOT
hugetrace-00010	2.68	2.58	2.52	3.02	OOT	0.68	0.68	0.67	14.41	OOT
\inf -road_central	2.67	2.65	2.61	2.99	6.54	1.13	1.15	1.09	10.37	TOO
hugetrace-00020	5.62	4.26	3.96	3.32	OOT	0.95	0.95	0.97	22.36	TOO
$delaunay_n24$	27.69	27.01	24.88	23.81	OOT	13.11	OOT	18.06	13.20	TOO
hugebubbles-00020	5.62	6.57	5.80	5.06	OOT	1.40	1.32	1.35	33.85	TOO
$inf-road_usa$	3.57	3.50	3.44	11.27	8.00	1.53	1.48	1.50	4.42	OOT
$inf-europe_osm$	6.77	6.84	9.82	5.80	10.19	2.62	2.69	2.66	28.71	12.79
$kron_g500-logn16$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	TOO
$kron_g500-logn17$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	TOO
$kron_g500-logn18$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	TOO
$kron_g500-logn19$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	TOO
$kron_g500$ - $logn20$	OOT	OOT	OOT	TOO	OOT	TOO	TOO	OOT	OOT	OOT
$kron_g500-logn21$	OOT	TOO	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT

Table 5: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=10 and 15 on 10th-DIMACS graphs before filtering

			k=10					k=15		
Graph	Maple	$Maple_{com}$	$\mathrm{Maple_{hyb}}$	kPlexS	KpLeX	Maple	$Maple_{com}$	$\mathrm{Maple_{hyb}}$	kPlexS	KpLeX
consph	9.03	78.42	29.56	19.27	OOT	19.71	94.43	21.85	437.79	OOT
connectus	1.07	0.80	1.23	0.82	OOT	72.46	1440.75	666.50	77.09	OOT
$rgg_n_2_24_s0$	8.05	8.19	8.39	13.78	27.52	5.13	45.94	5.14	28.13	OOT
$rgg_n_2_3s0$	4.46	16.75	4.59	5.69	10.07	3.24	8.16	3.22	30.35	OOT
$rgg_n_2_22s0$	3.23	74.02	3.73	3.95	OOT	1.12	2.18	1.14	13.99	OOT
$rgg_n_2_1s0$	2.17	59.86	2.82	2.55	OOT	0.49	0.54	0.44	6.35	OOT
soc-Epinions1	0.34	0.38	0.34	0.60	20.13	0.27	0.29	0.26	0.26	4.87
bio-pdb1HYS	1.75	3.40	2.80	1.87	447.45	1.93	3.53	3.62	1.85	OOT
co-papers-dblp	0.29	0.57	0.54	0.57	0.56	0.40	0.83	0.77	0.65	0.72
$rgg_n_2_20_s0$	0.38	0.42	0.37	0.52	1.73	0.22	0.22	0.18	0.47	0.50
citationCiteseer	0.16	0.25	0.19	0.20	19.87	0.05	0.05	0.05	0.34	210.25
co-papers-citeseer	0.12	0.12	0.11	0.32	0.30	0.12	0.13	0.13	0.31	0.31
tech-caida $RouterLevel$	0.08	0.11	0.09	0.10	2.02	0.07	0.09	0.09	0.08	0.16
cnr-2000	0.09	0.09	0.09	0.07	0.08	0.09	0.11	0.10	0.07	0.09
email-enron	0.09	0.10	0.10	0.09	0.21	0.11	0.10	0.11	0.09	0.17
$\operatorname{polblogs}$	0.02	0.04	0.02	0.03	0.27	0.01	0.01	0.01	0.01	0.02
c-62ghs	0.05	0.05	0.05	0.05	0.06	0.04	0.04	0.04	0.04	0.05
coAuthorsCiteseer	0.03	0.03	0.03	0.04	0.05	0.03	0.03	0.03	0.05	0.05
c-66b	0.03	0.03	0.03	0.03	0.05	0.02	0.02	0.02	0.02	0.04
che sape ake	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
$delaunay_n10$	0.00	0.00	0.00	0.00	OOT	0.00	0.00	0.00	0.00	OOT
bio-celegansneural	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.06	OOT
delaunay_n11	0.00	0.00	0.00	0.01	OOT	0.00	0.00	0.00	0.00	OOT
$delaunay_n12$	0.00	0.00	0.00	0.01	OOT	0.00	0.00	0.00	0.01	OOT
fe-4elt2	0.00	0.00	0.00	0.01	OOT	0.00	0.00	0.00	0.01	OOT
$delaunay_n13$	0.00	0.00	0.00	0.02	OOT	0.00	0.00	0.00	0.01	OOT
fe-sphere	0.00	0.00	0.00	0.01	TOO	0.00	0.00	0.00	0.02	ООТ

cti	0.00	0.00	0.00	0.03	OOT	0.00	0.00	0.00	0.03	OOT
wing	0.00	0.00	0.00	0.06	OOT	0.00	0.00	0.00	0.05	OOT
$\operatorname{wing_nodal}$	0.00	0.00	0.00	0.21	OOT	0.00	0.00	0.00	0.20	OOT
$delaunay_n14$	0.00	0.00	0.00	0.03	OOT	0.00	0.00	0.00	0.02	OOT
cs4	0.00	0.00	0.00	0.02	OOT	0.00	0.00	0.00	0.02	OOT
t60k	0.00	0.00	0.00	0.02	52.57	0.00	0.00	0.00	0.03	52.61
$delaunay_n15$	0.00	0.00	0.00	0.05	OOT	0.00	0.00	0.00	0.04	OOT
fe-body	0.00	0.00	0.00	0.04	OOT	0.00	0.00	0.00	0.05	OOT
$rgg_n_2_{15}s0$	0.01	0.01	0.01	0.01	27.52	0.00	0.00	0.00	0.04	0.01
delaunay_n16	0.00	0.01	0.01	0.09	OOT	0.00	0.00	0.00	0.07	OOT
inf-luxembourg_osm	0.00	0.00	0.00	0.03	OOT	0.00	0.00	0.00	0.03	131.11
$rgg_n_2_{16}$ s0	0.02	0.06	0.02	0.04	8.77	0.01	0.01	0.01	0.11	116.57
fe-ocean	0.01	0.01	0.01	0.21	OOT	0.01	0.01	0.01	0.14	OOT
fe-t10000.0h	0.01	0.01	OOT	0.62	OOT	0.01	0.01	OOT	0.31	OOT
$delaunay_n17$	0.01	0.01	0.01	0.18	OOT	0.01	0.01	0.01	0.15	OOT
$598 m \mathring{a}$	0.02	0.02	0.02	1.81	OOT	0.02	0.02	0.02	1.77	OOT
$rgg_n_2_17_s0$	0.07	0.26	0.06	0.06	10.07	0.01	0.02	0.02	0.24	473.21
fe_rotor	0.02	0.02	0.02	1.14	ООТ	0.01	0.01	0.01	1.22	ООТ
144	0.52	0.77	0.66	2.16	OOT	0.03	0.03	0.03	2.17	OOT
delaunay_n18	0.02	0.02	0.02	0.33	OOT	0.02	0.02	0.02	0.30	0.04
rgg_n_2_18_s0	0.06	0.07	0.06	0.09	0.09	0.04	0.04	0.04	0.09	553.59
cop20k_A	0.91	7.21	1.08	0.85	358.69	3.25	7.40	7.17	4.72	OOT
m14b	1.54	7.08	2.90	2.37	1445.14	0.05	0.05	0.05	4.57	OOT
delaunay_n19	0.04	0.04	0.04	0.50	OOT	0.04	0.04	0.04	0.62	OOT
rgg_n_2_19_s0	0.27	1.84	0.29	0.31	1.40	0.08	0.09	0.08	1.28	OOT
auto	0.68	1.04	0.79	5.25	OOT	0.11	0.03	0.10	7.10	OOT
inf-belgium_osm	0.00	0.06	0.06	0.51	OOT	0.11 0.07	0.06	0.16	0.52	0.20
delaunay_n20	0.07	0.08	0.08	1.25	OOT	0.07	0.00	0.00	1.25	0.20
inf-netherlands_osm	0.03	0.00	0.00	0.20	0.45	0.03 0.14	0.10	0.07	0.75	OOT
delaunay_n21	0.11 0.16	0.16	0.10 0.15	1.77	0.45 OOT	0.14 0.14	0.10 0.14	0.11 0.14	$\frac{0.73}{2.53}$	OOT
packing-500x100x100-b050		1396.72	45.17	71.86	OOT	0.14 0.41	0.14 0.44	0.14 0.45	55.11	OOT
venturiLevel3	0.12	0.18	0.18	1.60	OOT	0.41 0.20	0.44 0.18	$0.45 \\ 0.19$	0.39	OOT
hugetrace-00000	0.18 0.21	0.18 0.21	0.18 0.20	3.92	OOT	$0.20 \\ 0.22$	0.18 0.21	$0.19 \\ 0.22$	3.64	001 00T
9	0.21 0.32	0.21 0.32	0.20 0.31	6.14	OOT	0.22 0.35	0.21 0.29	0.22 0.28	5.04 5.07	1.04
delaunay_n22 hugetric-00000	0.32 0.29	0.32 0.26	$0.31 \\ 0.26$		OOT		0.29 0.28	0.28 0.29	5.86	00T
channel-500x100x100-b050				0.94	OOT	0.30				
	53.92	OOT	166.70	156.27		0.75	0.72	0.72	113.50	OOT
adaptive	0.30	0.31	0.30	0.75	OOT	0.33	0.29	0.31	5.31	1.09
inf-italy_osm	0.29	0.28	0.29	2.75	OOT	0.32	0.29	0.31	2.01	OOT
hugetric-00010	0.38	0.37	0.36	1.32	2.07	0.39	0.37	0.40	7.92	2.01
inf-great-britain_osm	0.36	0.36	0.37	3.91	143.46	0.47	0.36	0.39	4.21	OOT
hugetric-00020	0.40	0.40	0.38	8.60	OOT	0.48	0.41	0.43	1.41	OOT
inf-asia_osm	0.56	0.51	0.50	1.11	2.28	0.62	0.52	0.62	1.12	2.10
delaunay_n23	0.64	0.64	0.62	7.99	OOT	0.65	0.58	0.62	10.52	OOT
inf-germany_osm	0.59	0.56	0.56	1.49	TOO	0.65	0.56	0.59	1.46	ООТ
hugetrace-00010	0.69	0.66	0.74	2.38	TOO	0.81	0.65	0.71	12.48	3.92
\inf -road_central	0.86	0.89	0.86	2.93	OOT	1.05	0.86	1.00	3.03	OOT
hugetrace-00020	0.94	0.90	1.09	18.69	OOT	1.43	1.22	0.97	18.20	5.42
$delaunay_n24$	1.28	1.29	1.25	12.64	OOT	1.64	1.21	1.19	20.91	OOT
hugebubbles-00020	1.35	1.31	1.26	5.20	OOT	1.38	1.35	1.65	31.32	OOT
\inf -road_usa	1.31	1.61	1.27	12.37	OOT	1.42	1.30	1.44	3.48	OOT
$inf-europe_osm$	2.96	2.36	3.26	25.65	9.17	2.87	2.57	3.82	5.91	OOT
$kron_g500-logn16$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn17$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn18$	OOT	TOO	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
$kron_g500-logn19$	OOT	TOO	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn20$	OOT	TOO	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn21$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT

Table 6: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=20 on 10th-DIMACS graphs before filtering

			k=20		
Graph	Maple	$Maple_{com}$	$\mathrm{Maple_{hyb}}$	kPlexS	KpLeX
consph	273.93	880.98	268.40	TOO	OOT
connectus	OOT	OOT	OOT	OOT	OOT
$rgg_n_2_24s0$	3.78	3.28	4.08	67.92	OOT
$rgg_n_2_23_s0$	1.34	1.90	1.73	31.87	OOT
$rgg_n_2_22_s0$	0.53	0.62	0.69	14.64	OOT
$rgg_n_2_1s0$	0.24	0.29	0.26	1.39	1.71
soc-Epinions1	0.45	0.92	0.56	2.97	OOT
bio-pdb1HYS	1.75	5.38	OOT	1.70	OOT
co-papers-dblp	0.40	1.35	0.76	0.38	0.80
$rgg_n_2_20_s0$	0.12	0.14	0.16	0.52	0.79
citationCiteseer	0.04	0.04	0.03	0.47	2.00
co-papers-citeseer	0.12	0.13	0.13	0.11	0.36
tech-caida $RouterLevel$	0.05	0.13	0.05	0.07	0.22
$\operatorname{cnr-2000}$	0.11	0.19	0.11	0.12	0.14
${ m email}{ m -enron}$	0.09	0.09	0.09	0.07	2.46
$\operatorname{polblogs}$	0.00	0.00	0.00	0.00	0.00
c-62ghs	0.03	0.03	0.03	0.03	0.04
coAuthorsCiteseer	0.03	0.04	0.03	0.06	0.07
c-66b	0.02	0.02	0.02	0.02	0.04
che sape ake	0.00	0.00	0.00	0.00	0.00
$delaunay_n10$	0.00	0.00	0.00	0.00	0.02
bio-celegansneural	0.00	0.00	0.00	0.03	0.00
delaunay_n11	0.00	0.00	0.00	0.00	0.09
$delaunay_n12$	0.00	0.00	0.00	0.01	0.36
fe-4elt2	0.00	0.00	0.00	0.01	2.24
$delaunay_n13$	0.00	0.00	0.00	0.02	1.48
fe-sphere	0.00	0.00	0.00	0.01	4.10
cti	0.00	0.00	0.00	0.03	4.52
wing	0.00	0.00	0.00	0.05	52.04
$wing_nodal$	0.00	0.00	0.00	0.23	3.48
$delaunay_n14$	0.00	0.00	0.00	0.04	5.69
cs4	0.00	0.00	0.00	0.03	6.91
t60k	0.00	0.00	0.00	0.01	46.33
$delaunay_n15$	0.00	0.00	0.00	0.04	23.31
fe-body	0.00	0.00	0.00	0.05	16.09
$rgg_n_2_{15}s0$	0.00	0.00	0.00	0.05	0.02
$delaunay_n16$	0.00	0.01	0.00	0.08	90.74
\inf -luxembourg_osm	0.00	0.00	0.00	0.03	0.03
$rgg_n_2_{16}s0$	0.01	0.01	0.01	0.02	0.03
fe-ocean	0.01	0.01	0.01	0.14	322.83
fe-t10000.0h	0.01	0.01	TOO	0.34	135.09
$delaunay_n17$	0.01	0.01	0.01	0.15	363.76
598a	0.01	0.01	0.01	2.09	354.87
$rgg_n_2_{17}s0$	0.01	0.01	0.01	0.04	0.06
fe_rotor	0.01	0.01	0.01	1.40	286.05
144	0.02	0.02	0.02	2.53	632.33
$delaunay_n18$	0.02	0.02	0.02	0.32	1507.12
$rgg_n_2_{18}s0$	0.02	0.03	0.02	0.53	OOT
$cop20k_A$	0.02	0.03	0.02	23.36	364.85
m14b	0.03	0.03	0.03	4.30	1411.08
$delaunay_n19$	0.03	0.04	0.03	0.63	OOT
$rgg_n_2_{19}s0$	0.05	0.06	0.05	0.24	OOT
auto	0.06	0.08	0.05	8.35	555.30
\inf -belgium_osm	0.06	0.07	0.06	0.66	0.29
$delaunay_n20$	0.07	0.08	0.08	1.35	OOT

	0.44	0.40	0.44	0.01	0.40
inf-netherlands_osm	0.11	0.12	0.11	0.91	0.48
delaunay_n21	0.14	0.17	0.14	3.22	ТОО
packing- $500x100x100-b050$	0.15	0.18	0.15	63.55	TOO
venturiLevel3	0.25	0.20	0.19	3.05	OOT
hugetrace-00000	0.22	0.23	0.22	0.95	1.60
$delaunay_n22$	0.28	0.34	0.28	5.61	OOT
hugetric-00000	0.28	0.30	0.27	5.72	OOT
channel-500x100x100-b050	0.29	0.31	0.28	138.03	OOT
adaptive	0.32	0.33	0.30	1.02	1.51
$inf-italy_osm$	0.33	0.33	0.28	2.46	OOT
hugetric-00010	0.38	0.44	0.37	9.27	OOT
inf -great-britain_osm	0.38	0.41	0.37	1.07	1.76
hugetric-00020	0.41	0.45	0.40	9.32	OOT
\inf -asia_osm	0.54	0.58	0.53	1.51	OOT
$delaunay_n23$	0.57	0.69	0.55	11.21	OOT
\inf -germany_osm	0.61	0.64	0.61	2.30	OOT
hugetrace-00010	0.66	0.74	0.66	14.12	OOT
$inf-road_central$	0.88	0.95	1.10	11.24	OOT
hugetrace-00020	0.94	1.05	0.91	3.88	8.41
$delaunay_n24$	1.18	1.32	1.11	21.62	OOT
hugebubbles-00020	1.33	1.50	1.31	33.75	OOT
\inf -road_usa	1.51	1.54	1.29	13.19	OOT
$inf-europe_osm$	2.54	2.75	2.54	6.97	13.25
kron_g500-logn16	OOT	OOT	OOT	OOT	OOT
kron_g500-logn17	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn18$	OOT	OOT	OOT	OOT	OOT
kron_g500-logn19	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn20$	OOT	OOT	OOT	OOT	OOT
kron_g500-logn21	OOT	OOT	OOT	OOT	OOT

2.3 2nd-DIMACS Graphs

In this subsection, we show the running time comparison of five algorithms on 2nd-DIMACS graphs for k=2,5,10,15 and 20. Due to the limit of page size, we divide k values into 3 groups to demonstrate the results. The first group is k=2 and 5, the second group is k=10 and 15 and the last group is k=20.

Table 7: Running time of Maple, Maple $_{com}$, Maple $_{hyb}$, kPlexS and KpLeX for k=2 and 5 on 2nd-DIMACS graphs before filtering

			k=2						k=5		
Graph	Maple	$Maple_{com}$	Maple _{hyb}	kPlexS	KpLeX	Maj	ple	$Maple_{com}$	Maple _{hyb}	kPlexS	KpLeX
johnson8-4-4	3.25	4.83	3.85	17.05	2.58	OC	T	OOT	OOT	ООТ	407.74
C125.9	OOT	OOT	OOT	OOT	OOT	OO	T	OOT	OOT	OOT	OOT
$san200_0.7_2$	OOT	OOT	OOT	OOT	OOT	OC	T	OOT	OOT	OOT	OOT
hamming8-2	OOT	OOT	OOT	OOT	OOT	OO	T	OOT	OOT	OOT	OOT
$MANN_a27$	654.16	1332.14	685.81	OOT	OOT	0.8	80	0.45	1.03	0.94	0.21
$san200_0.7_1$	OOT	OOT	OOT	OOT	OOT	OC	T	OOT	OOT	OOT	1,309.52
hamming10-2	OOT	OOT	OOT	OOT	OOT	OO	$^{\mathrm{T}}$	OOT	OOT	OOT	OOT
$\sin 400 _{-}0.7 _{-}2$	OOT	OOT	OOT	OOT	OOT	OC	T	OOT	OOT	OOT	OOT
$\sin 400 _{-}0.7 _{-}1$	OOT	OOT	OOT	OOT	OOT	OC	T	OOT	OOT	OOT	OOT
$san 400 _ 0.7 _ 3$	OOT	OOT	OOT	OOT	OOT	OC	T	OOT	OOT	OOT	OOT
$MANN_a45$	OOT	OOT	OOT	OOT	OOT	22.0	02	9.49	28.44	21.70	1.58
hamming6-4	0.01	0.00	0.00	0.01	0.01	0.0)3	0.03	0.03	0.01	0.04
hamming6-2	0.75	0.99	0.83	17.84	18.21	87.	37	146.39	87.65	1,357.46	1,567.71
c-fat500-2	0.00	0.01	0.01	0.01	0.00	0.0)1	0.01	0.01	0.00	0.03
c-fat500-10	0.05	0.08	0.09	0.06	0.05	0.0	8	0.14	0.13	0.08	OOT
$MANN_a9$	0.01	0.03	0.02	0.92	0.29	0.0	00	0.00	0.00	0.00	0.00
c-fat 500 - 1	0.05	0.08	0.09	0.00	0.01	0.0	8	0.14	0.13	0.00	0.01
c-fat200-2	0.00	0.00	0.00	0.00	0.00	0.0	00	0.00	0.00	0.00	0.01
c-fat500-5	0.02	0.02	0.03	0.02	0.02	0.0)3	0.04	0.04	0.02	122.09

c-fat200-5	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.01	OOT
johnson8-2-4	0.00	0.01	0.01	0.01	0.00	0.06	0.06	0.05	0.03	0.02
c-fat200-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$brock200_{-}1$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$brock200_2$	17.12	41.79	38.89	208.96	4.63	OOT	OOT	OOT	OOT	OOT
$brock200_{-}3$	142.48	257.39	209.71	OOT	65.03	TOO	OOT	OOT	OOT	OOT
$brock200_{-4}$	457.09	677.29	617.40	OOT	261.46	TOO	OOT	OOT	OOT	OOT
$brock400_{-}1$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_{-2}$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_{-}3$	OOT	OOT	ООТ	ООТ	OOT	OOT	ООТ	ООТ	OOT	ООТ
brock400_4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
brock800_1	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
brock800_2	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
brock800_3	OOT	OOT	OOT	ООТ	OOT	OOT	OOT	OOT	OOT	OOT
brock800_4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C1000.9	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C2000.5	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C2000.9	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C250.9	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C4000.5	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
C500.9	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
DSJC1000 ₋₅	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
DSJC500_5	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
gen200_p0.9_44		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
gen200_p0.9_5		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_5		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_6		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_7		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
hamming10-4		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
hamming 10-4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
johnson16-2-4		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
johnson32-2-4		OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
keller4	194.35	259.61	240.55	1,713.13		OOT	OOT	OOT	OOT	OOT
keller5	194.33 OOT	OOT	OOT	1,713.13 OOT	OOT	OOT	OOT	OOT	OOT	OOT
keller6	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
MANN_a81	OOT	OOT	OOT	OOT	OOT			OOT		
		OOT				1650.64	1049.96		1,459.41	38.85
p_hat1000-1	TOO		TOO	OOT	184.60	TOO	OOT	TOO	TOO	TOO
p_hat1000-2	TOO	TOO	TOO	OOT	OOT	OOT	OOT	TOO	OOT	TOO
p_hat1000-3	TOO	TOO	TOO	OOT	OOT	OOT	OOT	TOO	OOT	TOO
p_hat1500-1	TOO	TOO	TOO	OOT	OOT	OOT	OOT	TOO	OOT	TOO
p_hat1500-2	TOO	TOO	TOO	OOT	TOO	OOT	OOT	TOO	OOT	TOO
p_hat1500-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT
p_hat300-1	4.78	0.10	0.10	18.06	0.67	1081.41	186.09	169.59	OOT	79.64
p_hat300-2	175.74	181.29	187.60	OOT	932.68	821.69	430.00	476.65	TOO	TOO
p_hat300-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	TOO
p_hat500-1	61.46	6.98	6.04	894.21	6.76	OOT	OOT	TOO	OOT	OOT
p_hat500-2	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT
p_hat500-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat700-1	349.34	291.74	287.16	OOT	31.13	OOT	OOT	TOO	OOT	OOT
p_hat700-2	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat700-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	ООТ
san1000	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT	OOT
san200_0.9_1	54.40	72.46	64.15	OOT	OOT	0.42	0.26	0.45	OOT	1.27
san200_0.9_2	TOO	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT
san200_0.9_3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
san400_0.5_1	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$san 400 _{-}0.9 _{-}1$	TOO	TOO	TOO	OOT	OOT	OOT	OOT	OOT	OOT	OOT
	4000	4-0	40-0 -:	~ ~ —	4 4 ~	~ ~ —	~ ~ ~	~ ~ -	~ ~ -	~ ~ -
$sanr200_0.7$	1386.36	1727.00	1679.80	OOT	1,194.07	OOT	OOT	OOT	OOT	OOT
	1386.36 OOT OOT	1727.00 OOT OOT	1679.80 OOT OOT	TOO TOO	1,194.07 OOT 554.79	TOO TOO TOO	TOO TOO TOO	TOO TOO	TOO TOO TOO	TOO TOO TOO

Table 8: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=10 and 15 on 2nd-DIMACS graphs before filtering

			k=10					k=15		
Graph	Maple	$\mathrm{Maple_{com}}$	$\mathrm{Maple}_{\mathrm{hyb}}$	kPlexS	KpLeX	Maple	$Maple_{com}$	Maple _{hyb}	kPlexS	KpLeX
johnson8-4-4	OOT	OOT	OOT	OOT	OOT	89.08	155.33	97.55	676.19	OOT
C125.9	OOT	OOT	OOT	OOT	OOT	68.30	5.31	70.73	158.44	TOO
$san200_0.7_2$	OOT	OOT	OOT	2.54	0.62	0.00	0.00	0.00	0.00	OOT
hamming8-2	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	OOT
$MANN_a27$	270.05	254.04	278.36	OOT	2.62	0.01	0.01	0.01	0.01	OOT
$san200_0.7_1$	0.03	0.03	0.03	0.03	0.02	0.03	0.04	0.03	0.03	OOT
hamming10-2	OOT	OOT	OOT	OOT	OOT	0.07	0.07	0.07	0.06	OOT
$san 400_0.7_2$	OOT	OOT	OOT	OOT	OOT	0.19	0.25	0.25	0.19	OOT
$san 400_0.7_1$	0.15	2.41	0.25	0.17	0.16	0.18	0.27	0.27	0.19	OOT
$san 400_0.7_3$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$MANN_a45$	83.21	98.85	90.14	OOT	4.73	454.59	1196.16	548.36	OOT	OOT
hamming 6-4	0.00	0.01	0.01	0.01	0.54	0.01	0.00	0.01	0.00	0.00
hamming6-2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38
c-fat 500 - 2	0.01	0.02	0.01	0.01	0.03	0.00	0.01	0.01	0.01	59.86
c-fat 500 - 10	0.27	0.23	0.16	0.35	OOT	0.51	0.34	0.23	0.72	OOT
$MANN_a9$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	OOT
c-fat500-1	0.27	0.23	0.16	0.00	0.03	0.51	0.34	0.23	0.00	OOT
c-fat200-2	0.00	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.00	OOT
c-fat 500 - 5	0.05	0.07	0.04	0.06	170.85	0.08	0.37	0.08	0.09	OOT
c-fat200-5	0.03	0.03	0.03	0.03	OOT	0.04	0.37	0.05	0.05	0.02
johnson 8-2-4	0.02	0.02	0.01	0.02	0.05	0.00	0.00	0.00	0.00	OOT
c-fat200-1	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	219.08
$brock200_{-}1$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$brock200_{-2}$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	0.00
$brock200_3$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	46.47
$brock200_4$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	139.11
$brock400_{-}1$	OOT	TOO	OOT	TOO	OOT	OOT	OOT	OOT	OOT	0.00
$brock400_{-2}$	OOT	TOO	OOT	TOO	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_3$	OOT	OOT	TOO	TOO	OOT	OOT	OOT	OOT	OOT	TOO
$brock400_4$	OOT	OOT	TOO	TOO	OOT	OOT	OOT	OOT	OOT	TOO
$brock800_{-1}$	OOT	OOT	TOO	TOO	OOT	OOT	OOT	OOT	OOT	TOO
$brock800_{-2}$	OOT	OOT	TOO	TOO	OOT	OOT	OOT	OOT	OOT	OOT
$brock800_3$	OOT	OOT	TOO	TOO	OOT	OOT	OOT	OOT	OOT	OOT
brock800_4	OOT	TOO	OOT	TOO	ООТ	OOT	OOT	OOT	ООТ	358.95
C1000.9	OOT	OOT	TOO	TOO	ООТ	OOT	OOT	OOT	ООТ	ООТ
C2000.5	OOT	OOT	OOT	TOO	ООТ	OOT	OOT	OOT	ТОО	ООТ
C2000.9	TOO	OOT	TOO	TOO	ООТ	TOO	TOO	TOO	TOO	TOO
C250.9	TOO	OOT	TOO	TOO	ООТ	TOO	TOO	TOO	TOO	TOO
C4000.5	TOO	OOT	TOO	TOO	ООТ	TOO	TOO	TOO	TOO	ООТ
C500.9	OOT	OOT	OOT	TOO	ООТ	OOT	OOT	OOT	ООТ	0.00
DSJC1000_5	TOO	OOT	TOO	TOO	ООТ	OOT	TOO	TOO	TOO	TOO
DSJC500_5	TOO	OOT	TOO	TOO	ТОО	ООТ	TOO	TOO	TOO	TOO
$gen200_p0.9_44$		OOT	TOO	TOO	ООТ	OOT	TOO	TOO	TOO	TOO
gen200_p0.9_55		OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	TOO
gen400_p0.9_55		OOT	TOO	TOO	ТОО	ООТ	TOO	TOO	ООТ	OOT
gen400_p0.9_65		OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	0.00
gen400_p0.9_75		OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	TOO
hamming10-4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	0.00
hamming8-4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	TOO
johnson16-2-4	OOT	OOT	OOT	OOT	OOT	OOT	OOT	TOO	OOT	TOO
johnson32-2-4	ООТ	OOT	TOO	ООТ	ООТ	OOT	OOT	OOT	ООТ	0.00

keller4	ООТ	ООТ	ООТ	ООТ	ООТ	ООТ	ООТ	OOT	ООТ	ООТ
keller5	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
keller6	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$MANN_{-}a81$	OOT	1090.24	OOT	OOT	79.82	OOT	OOT	OOT	OOT	OOT
p_hat1000-1	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1000-2	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1000-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-1	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-2	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-3	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat300-1	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$p_hat300-2$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
p_hat300-3	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	OOT
$p_hat 500-1$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
$p_hat 500-2$	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	OOT
$p_hat 500-3$	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	OOT
p_hat700-1	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	OOT
$p_hat700-2$	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	0.03
$p_hat700-3$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	0.00
san1000	OOT	OOT	OOT	OOT	TOO	OOT	OOT	TOO	OOT	OOT
$san200_0.9_1$	OOT	OOT	OOT	OOT	TOO	TOO	OOT	TOO	OOT	OOT
$san200_0.9_2$	OOT	OOT	OOT	OOT	TOO	TOO	OOT	TOO	OOT	OOT
$san200_0.9_3$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	0.19
$san 400_0.5_1$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	0.10
$san 400 _ 0.9 _ 1$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
$\operatorname{sanr}200_0.7$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
$sanr200_0.9$	OOT	OOT	OOT	OOT	OOT	TOO	OOT	OOT	OOT	OOT
$\rm sanr 400_0.5$	OOT	TOO	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT
$sanr400_0.7$	OOT	OOT	OOT	OOT	OOT	OOT	OOT	OOT	ООТ	OOT

Table 9: Running time of Maple, Maple $_{\rm com},$ Maple $_{\rm hyb},$ kPlexS and KpLeX for k=20 on 2nd-DIMACS graphs before filtering

			k=20		
Graph	Maple	Maple _{com}	$Maple_{hyb}$	kPlexS	KpLeX
johnson8-4-4	0.00	0.00	0.00	0.00	0.00
C125.9	0.00	0.00	0.00	0.00	0.00
$san200_0.7_2$	0.00	0.00	0.00	0.00	0.00
hamming8-2	0.01	0.01	0.01	0.01	0.00
$MANN_a27$	0.01	0.01	0.01	0.01	0.00
$san200_0.7_1$	98.37	0.07	0.07	OOT	OOT
hamming10-2	0.07	0.07	0.07	0.06	0.00
$san 400 _ 0.7 _ 2$	0.17	0.22	0.22	0.18	0.14
$\sin 400 _{-}0.7 _{-}1$	0.18	0.27	0.25	0.19	0.20
$san 400 _ 0.7 _ 3$	0.18	0.23	0.23	0.18	0.11
$MANN_a45$	OOT	OOT	OOT	OOT	OOT
hamming6-4	0.01	0.00	0.01	OOT	0.22
hamming 6-2	0.00	0.00	0.00	0.00	0.00
c-fat 500 - 2	0.01	0.06	0.02	13.65	OOT
c-fat500-10	0.83	0.60	0.42	1.18	OOT
$MANN_a9$	0.00	0.00	0.00	0.00	0.00
c-fat 500 - 1	0.83	0.60	0.42	0.00	0.01
c-fat200-2	0.00	0.04	0.01	0.59	OOT
c-fat 500 - 5	0.08	0.54	0.09	0.16	OOT
c-fat200-5	0.05	0.32	0.05	0.10	OOT
johnson 8-2-4	0.00	0.00	0.00	0.00	0.00
c-fat200-1	0.00	0.00	0.00	0.00	0.00
$brock200_1$	OOT	OOT	OOT	OOT	OOT

$brock200_{-2}$	ООТ	OOT	OOT	ООТ	ООТ
$brock200_{-3}$	OOT	ООТ	OOT	ООТ	ООТ
brock200_4	OOT	OOT	OOT	OOT	OOT
brock400_1	OOT	OOT	OOT	OOT	OOT
brock400_2	TOO	OOT	OOT	OOT	OOT
brock400_3	OOT	OOT	OOT	OOT	OOT
brock400_4	TOO	OOT	OOT	OOT	OOT
brock800_1	TOO	OOT	OOT	OOT	OOT
brock800_2	TOO	OOT	OOT	OOT	OOT
brock800_3	OOT	OOT	OOT	OOT	OOT
brock800_4	OOT	OOT	OOT	OOT	OOT
C1000.9	TOO	OOT	OOT	OOT	OOT
C2000.5	OOT	OOT	OOT	OOT	OOT
C2000.9	OOT	OOT	OOT	OOT	OOT
C250.9	OOT	ООТ	OOT	ООТ	ООТ
C4000.5	OOT	OOT	OOT	OOT	OOT
C500.9	OOT	ООТ	OOT	ООТ	ООТ
DSJC1000_5	OOT	ООТ	OOT	ООТ	ООТ
$DSJC500_5$	OOT	ООТ	OOT	ООТ	ООТ
gen200_p0.9_44	OOT	ООТ	OOT	ООТ	ООТ
gen200_p0.9_55	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_55	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_65	OOT	ООТ	OOT	ООТ	ООТ
gen400_p0.9_75	OOT	OOT	OOT	OOT	OOT
hamming10-4	OOT	ООТ	OOT	ООТ	ООТ
hamming8-4	OOT	OOT	OOT	OOT	OOT
johnson16-2-4	OOT	OOT	OOT	OOT	OOT
johnson32-2-4	OOT	OOT	OOT	OOT	OOT
keller4	OOT	OOT	OOT	OOT	OOT
keller5	OOT	OOT	OOT	OOT	OOT
keller6	OOT	OOT	OOT	OOT	OOT
$MANN_a81$	OOT	OOT	OOT	OOT	367.50
p_hat1000-1	OOT	OOT	OOT	OOT	OOT
p_hat1000-2	OOT	OOT	OOT	OOT	OOT
p_hat1000-3	OOT	OOT	OOT	OOT	OOT
p_hat1500-1	OOT	OOT	OOT	OOT	OOT
p_hat1500-2	OOT	OOT	OOT	OOT	OOT
p_hat1500-3	OOT	OOT	OOT	OOT	OOT
p_hat300-1	OOT	OOT	OOT	OOT	OOT
p_hat300-2	OOT	OOT	OOT	OOT	OOT
p_hat300-3	OOT	OOT	OOT	OOT	OOT
$p_hat 500-1$	OOT	OOT	OOT	OOT	OOT
$p_hat 500-2$	OOT	OOT	OOT	OOT	OOT
$p_hat 500-3$	OOT	OOT	OOT	OOT	OOT
p_hat700-1	OOT	OOT	OOT	OOT	OOT
$p_hat700-2$	OOT	OOT	OOT	OOT	OOT
$p_hat700-3$	OOT	OOT	OOT	OOT	OOT
san 1000	TOO	OOT	OOT	OOT	OOT
$san200_0.9_1$	TOO	OOT	OOT	OOT	OOT
$san200_0.9_2$	TOO	OOT	OOT	OOT	OOT
$san200_0.9_3$	OOT	TOO	OOT	TOO	OOT
$san 400_0.5_1$	OOT	TOO	OOT	OOT	OOT
$san 400_0.9_1$	OOT	TOO	OOT	TOO	OOT
$\rm sanr 200_0.7$	OOT	TOO	OOT	OOT	OOT
$\rm sanr 200_0.9$	OOT	TOO	OOT	OOT	OOT
$\rm sanr 400_0.5$	OOT	TOO	OOT	OOT	OOT
sanr400_0.7	ООТ	TOO	TOO	ТОО	ООТ

3 Graph information

Tab. 10-18 show the information for each graph of different k values, including the graph name, the vertex number, the edge number, the degeneracy, the community-degeneracy, the maximum k-plex size, the degeneracy gap, the community-degeneracy gap. For the convenience of display, we also divide this section into 3 subsections by dataset Network-Repo graph, 10th-DIMACS graph and 2nd-DIMACS graph, which is the same as section 2 Unfiltered Running time.

3.1 Network-Repo Graphs

Note: If there is no non-trivial solution, $\omega_k(G)$ is set to 2k-2 and $g_k(G)/cg_k(G)$ is adjusted to 0, when 2k-2 is larger than the degeneracy bound.

Table 10: Network-Repo graph information for k=2 and 5

						k=	:2		k=5	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$	$g_k(G)$	$cg_k(G)$	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$
soc-LiveMocha	104,103	2,193,083	92	25	19	75	10	28	69	7
soc-youtube	$495,\!957$	1,936,748	49	17	20	31	1	26	28	1
$\operatorname{soc-youtube-snap}$	1,134,890	2,987,624	51	17	20	33	1	26	30	1
soc-lastfm	1,191,805	$4,\!519,\!330$	70	21	18	54	7	27	48	4
$\operatorname{soc-digg}$	770,799	5,907,132	236	71	57	181	18	72	169	9
ia-wiki-Talk	$92,\!117$	360,767	58	18	18	42	4	25	38	3
socfb-Duke 14	$9,\!885$	$506,\!437$	85	45	38	49	11	48	42	7
$\operatorname{soc-orkut}$	2,997,166	106,349,209	230	73	52	180	25	68	167	15
sc-ldoor	$909,\!537$	20,770,807	34	19	21	15	2	23	16	6
$\operatorname{sc-msdoor}$	404,785	9,378,650	34	19	21	15	2	23	16	6
sc-nasas rb	$54,\!870$	$1,\!311,\!227$	35	22	24	13	2	24	16	8
sc-pkustk11	87,804	$2,\!565,\!054$	47	34	36	13	2	36	16	8
soc-flixster	2,523,386	7,918,801	68	45	38	32	11	49	24	6
sc- $pwtk$	$217,\!891$	5,653,221	35	22	24	13	2	26	14	6
$scc_reality$	6,809	4,714,485	1235	1234	1236	1	2	1237	3	7
soc-gowalla	$196,\!591$	$950,\!327$	51	27	30	23	1	32	24	5
soc-epinions	$26,\!588$	100,120	32	16	18	16	2	25	12	1
socfb-B-anon	2,937,612	20,959,854	63	29	27	38	6	35	33	4
socfb-A-anon	3,097,165	23,667,394	74	28	28	48	4	37	42	1
soc-pokec	1,632,803	22,301,964	47	27	31	18	0	34	18	3
soc-twitter-follows	404,719	713,319	28	4	8	22	0	13	20	1
soc-FourSquare	639,014	3,214,986	63	36	35	30	5	44	24	2
socfb-Indiana	29,732	1,305,757	76	51	51	27	4	59	22	2
soc-slashdot	70,068	358,647	53	33	31	24	6	40	18	3
ia-enron-large	33,696	180,811	43	20	22	23	2	28	20	2
tech-WHOIS	7,476	56,943	88	69	64	26	9	76	17	3
ia-email-EU	32,430	54,397	22	11	15	9	0	20	7	1
socfb-Texas84	36,364	1,590,651	81	60	55	28	9	68	18	2
sc-pkustk13	94,893	3,260,967	41	34	36	7	2	36	10	8
socfb-UF	$35{,}111$	1,465,654	83	65	60	25	9	73	15	2
rt-retweet-crawl	1,112,702	2,278,852	18	11	14	6	1	17	6	4
tech-as-skitter	1,694,616	11,094,209	111	66	69	44	1	75	41	1
soc-livejournal	4,033,137	27,933,062	213	212	214	1	2	214	4	8
socfb-UIllinois	30,795	1,264,421	85	63	63	24	4	73	17	0
$scc_fb-messages$	1,303	531,893	706	705	708	0	1	709	2	6
scc_twitter-copen	2,623	473,614	582	579	581	3	2	582	5	7
socfb-Penn94	41,536	1,362,220	62	46	50	14	0	55	12	1
ca-hollywood-2009	1,069,126	56,306,653	2208	2207	2209	1	2	2209	4	8
web-uk-2005	129,632	11,744,049	499	498	500	1	2	500	4	8
socfb-Stanford3	11,586	568,309	91	58	59	34	3	67	29	1
socfb-MIT	6,402	251,230	72	39	37	37	6	48	29	1
soc-delicious	536,108	1,365,961	33	21	23	12	$\overline{2}$	30	8	1
sc-shipsec1	140,385	1,707,759	24	$\overline{22}$	$\frac{1}{24}$	2	$\overline{2}$	24	5	8
socfb-Berkeley13	22,900	852,419	64	45	$\frac{-1}{47}$	19	2	53	16	$\overset{\circ}{2}$
<i>y</i> -	,	,								

113	2,196	28	16	19	11	1	25	8	1
23,831	835,946	60	40	42	20	2	50	15	0
20,453	747,604	65	52	55	12	1	62	8	0
509,338	7,178,413	431	430	432	1	2	432	4	8
6,621		69	43	47	24	0	52	22	1
	,	29		24	7	2	26	8	6
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	,								8
	, ,								2
									$\frac{2}{4}$
,	,								1
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	,								
	,								1
									0
									1
	, ,								8
	,								1
	,								8
40,164	85,123			18	7			6	3
17,903	196,972	56	55	57	1	2	57	4	8
$226,\!413$	716,460	74	73	75	1	2	75	4	8
163,598	1,747,269	101	100	102	1	2	102	4	8
21,363	91,286	25	24	26	1	2	26	4	8
		17	14	17	2	1	19	3	5
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					1			1	5
3,031		29	28	30	1	2	30	4	8
2,113	6,632	15	14	17	0	1	20	0	4
16	13	2	1	4	0	1	8	0	3
10	5	1	0	2	1	2	8	0	2
7	5	2	1	3	1	2	8	0	3
13	7	1	0	3	0	1	8	0	2
5	6	2		4	0			0	3
22	12	1		3					2
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									6
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96	117	3	2	4	1	2	8	0	4
	23,831 20,453 509,338 6,621 179,104 63,392 540,486 488 1,206 17,206 332,689 190,914 14,917 4,767 317,080 26,475 227,320 40,164 17,903 226,413 163,598 21,363 410 121,422 146 56,739 11,204 10,972 11,358 17,151 16,062 4,158 273 113 12,305 127 127 135 3,031 2,113 16 10 7 13	23,831 835,946 20,453 747,604 509,338 7,178,413 6,621 249,959 179,104 2,200,076 63,392 816,886 540,486 15,245,729 488 71,011 1,206 65,990 17,206 604,867 332,689 820,644 190,914 607,610 14,917 482,215 4,767 37,375 317,080 1,049,866 26,475 53,381 227,320 814,134 40,164 85,123 17,903 196,972 226,413 716,460 163,598 1,747,269 21,363 91,286 410 2,765 121,422 334,419 146 9,828 56,739 212,945 11,204 117,619 10,972 175,573 11,358 47,606 17,151 24,015 16,062 25,593 4,150 13 </td <td>23,831 835,946 60 20,453 747,604 65 509,338 7,178,413 431 6,621 249,959 69 179,104 2,200,076 29 63,392 816,886 52 540,486 15,245,729 336 488 71,011 272 1,206 65,990 174 17,206 604,867 65 332,689 820,644 24 190,914 607,610 32 14,917 482,215 65 4,767 37,375 35 317,080 1,049,866 113 26,475 53,381 22 227,320 814,134 86 40,164 85,123 23 17,903 196,972 56 226,413 716,460 74 163,598 1,747,269 101 21,363 91,286 25 410 2,765 17</td> <td>23,831 835,946 60 40 20,453 747,604 65 52 509,338 7,178,413 431 430 6,621 249,959 69 43 179,104 2,200,076 29 22 63,392 816,886 52 34 540,486 15,245,729 336 335 488 71,011 272 264 1,206 65,990 174 164 17,206 604,867 65 51 332,689 820,644 24 23 190,914 607,610 32 17 14,917 482,215 65 58 4,767 37,375 35 21 317,080 1,049,866 113 112 26,475 53,381 22 14 227,320 814,134 86 85 40,164 85,123 23 15 17,903 196,972 56</td> <td>23,831 835,946 60 40 42 20,453 747,604 65 52 55 509,338 7,178,413 431 430 432 6,621 249,959 69 43 47 179,104 2,200,076 29 22 24 63,392 816,886 52 34 33 540,486 15,245,729 336 335 337 488 71,011 272 264 266 1,206 65,990 174 164 166 17,206 604,867 65 51 53 332,689 820,644 24 23 25 190,914 607,610 32 17 20 14,917 482,215 65 58 59 4,767 37,375 35 21 21 317,080 1,049,866 113 112 114 26,475 53,381 22 14</td> <td>23,831 835,946 60 40 42 20 20,453 747,604 65 52 55 12 509,338 7,178,413 431 430 432 1 6,621 249,959 69 43 47 24 179,104 2,200,076 29 22 24 7 63,392 816,886 52 34 33 21 540,486 15,245,729 336 335 337 1 488 71,011 272 264 266 8 1,206 65,990 174 164 166 10 17,206 604,867 65 51 53 14 332,689 820,644 24 23 25 1 190,914 607,610 32 17 20 14 4,767 37,375 35 21 21 16 317,080 1,049,866 113 112</td> <td>23,831 835,946 60 40 42 20 2 20,453 747,604 65 52 55 12 1 509,338 7,178,413 431 430 432 1 2 6,621 249,959 69 43 47 24 0 179,104 2,200,076 29 22 24 7 2 63,392 816,886 52 34 33 21 5 540,486 15,245,729 336 335 337 1 2 488 71,011 272 264 266 8 2 1,206 65,990 174 164 166 10 2 17,206 604,867 65 51 53 14 2 190,914 607,610 32 17 20 14 1 14,917 482,215 65 58 59 8 3 4,767<td>23,831 835,946 60 40 42 20 2 50 20,453 747,604 65 52 55 12 1 62 509,338 7,178,413 313 430 47 24 0 52 179,104 2,200,076 29 22 24 7 2 26 63,392 816,886 52 34 33 21 5 42 540,486 15,245,729 36 335 337 1 2 337 488 71,011 272 264 266 8 2 272 1,206 665,990 174 164 166 10 2 170 17,206 604,867 55 51 53 14 2 60 332,689 820,644 24 23 25 1 2 25 4,907 482,215 65 58 59 8 3</td><td>23.831 835,946 60 40 42 20 2 50 15 20,453 747,604 65 52 55 12 1 62 8 509,338 7,178,413 431 430 432 1 2 432 4 6,621 249,959 69 43 47 24 0 52 22 63,392 816,886 52 34 33 21 5 42 15 540,486 71,210 363 335 337 1 2 337 4 488 71,011 272 264 266 8 2 272 5 1,206 65,990 174 164 166 10 2 170 9 17,206 604,867 65 51 53 14 2 60 10 332,689 820,644 24 23 25 1 2 25</td></td>	23,831 835,946 60 20,453 747,604 65 509,338 7,178,413 431 6,621 249,959 69 179,104 2,200,076 29 63,392 816,886 52 540,486 15,245,729 336 488 71,011 272 1,206 65,990 174 17,206 604,867 65 332,689 820,644 24 190,914 607,610 32 14,917 482,215 65 4,767 37,375 35 317,080 1,049,866 113 26,475 53,381 22 227,320 814,134 86 40,164 85,123 23 17,903 196,972 56 226,413 716,460 74 163,598 1,747,269 101 21,363 91,286 25 410 2,765 17	23,831 835,946 60 40 20,453 747,604 65 52 509,338 7,178,413 431 430 6,621 249,959 69 43 179,104 2,200,076 29 22 63,392 816,886 52 34 540,486 15,245,729 336 335 488 71,011 272 264 1,206 65,990 174 164 17,206 604,867 65 51 332,689 820,644 24 23 190,914 607,610 32 17 14,917 482,215 65 58 4,767 37,375 35 21 317,080 1,049,866 113 112 26,475 53,381 22 14 227,320 814,134 86 85 40,164 85,123 23 15 17,903 196,972 56	23,831 835,946 60 40 42 20,453 747,604 65 52 55 509,338 7,178,413 431 430 432 6,621 249,959 69 43 47 179,104 2,200,076 29 22 24 63,392 816,886 52 34 33 540,486 15,245,729 336 335 337 488 71,011 272 264 266 1,206 65,990 174 164 166 17,206 604,867 65 51 53 332,689 820,644 24 23 25 190,914 607,610 32 17 20 14,917 482,215 65 58 59 4,767 37,375 35 21 21 317,080 1,049,866 113 112 114 26,475 53,381 22 14	23,831 835,946 60 40 42 20 20,453 747,604 65 52 55 12 509,338 7,178,413 431 430 432 1 6,621 249,959 69 43 47 24 179,104 2,200,076 29 22 24 7 63,392 816,886 52 34 33 21 540,486 15,245,729 336 335 337 1 488 71,011 272 264 266 8 1,206 65,990 174 164 166 10 17,206 604,867 65 51 53 14 332,689 820,644 24 23 25 1 190,914 607,610 32 17 20 14 4,767 37,375 35 21 21 16 317,080 1,049,866 113 112	23,831 835,946 60 40 42 20 2 20,453 747,604 65 52 55 12 1 509,338 7,178,413 431 430 432 1 2 6,621 249,959 69 43 47 24 0 179,104 2,200,076 29 22 24 7 2 63,392 816,886 52 34 33 21 5 540,486 15,245,729 336 335 337 1 2 488 71,011 272 264 266 8 2 1,206 65,990 174 164 166 10 2 17,206 604,867 65 51 53 14 2 190,914 607,610 32 17 20 14 1 14,917 482,215 65 58 59 8 3 4,767 <td>23,831 835,946 60 40 42 20 2 50 20,453 747,604 65 52 55 12 1 62 509,338 7,178,413 313 430 47 24 0 52 179,104 2,200,076 29 22 24 7 2 26 63,392 816,886 52 34 33 21 5 42 540,486 15,245,729 36 335 337 1 2 337 488 71,011 272 264 266 8 2 272 1,206 665,990 174 164 166 10 2 170 17,206 604,867 55 51 53 14 2 60 332,689 820,644 24 23 25 1 2 25 4,907 482,215 65 58 59 8 3</td> <td>23.831 835,946 60 40 42 20 2 50 15 20,453 747,604 65 52 55 12 1 62 8 509,338 7,178,413 431 430 432 1 2 432 4 6,621 249,959 69 43 47 24 0 52 22 63,392 816,886 52 34 33 21 5 42 15 540,486 71,210 363 335 337 1 2 337 4 488 71,011 272 264 266 8 2 272 5 1,206 65,990 174 164 166 10 2 170 9 17,206 604,867 65 51 53 14 2 60 10 332,689 820,644 24 23 25 1 2 25</td>	23,831 835,946 60 40 42 20 2 50 20,453 747,604 65 52 55 12 1 62 509,338 7,178,413 313 430 47 24 0 52 179,104 2,200,076 29 22 24 7 2 26 63,392 816,886 52 34 33 21 5 42 540,486 15,245,729 36 335 337 1 2 337 488 71,011 272 264 266 8 2 272 1,206 665,990 174 164 166 10 2 170 17,206 604,867 55 51 53 14 2 60 332,689 820,644 24 23 25 1 2 25 4,907 482,215 65 58 59 8 3	23.831 835,946 60 40 42 20 2 50 15 20,453 747,604 65 52 55 12 1 62 8 509,338 7,178,413 431 430 432 1 2 432 4 6,621 249,959 69 43 47 24 0 52 22 63,392 816,886 52 34 33 21 5 42 15 540,486 71,210 363 335 337 1 2 337 4 488 71,011 272 264 266 8 2 272 5 1,206 65,990 174 164 166 10 2 170 9 17,206 604,867 65 51 53 14 2 60 10 332,689 820,644 24 23 25 1 2 25

scc_rt_assad	34	96	7	6	9	0	1	12	0	4
$scc_rt_mittromney$	102	108	4	3	6	0	1	9	0	4
$scc_rt_one direction$	35	368	26	25	27	1	2	27	4	8
$scc_rt_bahrain$	72	129	7	6	9	0	1	11	1	5
$scc_rt_barackobama$	80	226	9	8	11	0	1	14	0	4
$scc_rt_alwefaq$	72	355	15	14	17	0	1	19	1	5
$scc_rt_justinbieber$	62	442	16	15	18	0	1	20	1	5
ia-enron-only	143	623	9	6	10	1	0	13	1	3
ca-netscience	379	914	8	7	9	1	2	10	3	7
ca-CSphd	1,882	1,740	2	1	4	0	1	8	0	3
rt-twitter-copen	761	1,029	4	2	5	1	1	9	0	3
bio-diseasome	516	1,188	10	9	11	1	2	11	4	8
web-polblogs	643	2,280	12	8	12	2	0	17	0	1
bio-yeast	1,458	1,948	5	4	6	1	2	8	2	6
bio-celegans	453	2,025	10	7	10	2	1	14	1	3
soc-wiki-Vote	889	2,914	9	5	8	3	1	12	2	3
web-google	$1,\!299$	2,773	17	16	19	0	1	19	3	7
ia-fb-messages	$1,\!266$	6,451	11	3	6	7	1	10	6	3
ia-email-univ	1,133	5,451	11	10	12	1	2	13	3	7
inf-power	4,941	$6,\!594$	5	4	6	1	2	9	1	5
ia-reality	6,809	7,680	5	3	6	1	1	9	1	4
ca-Erdos 992	5,094	7,515	7	6	8	1	2	11	1	5
bio-dmela	$7,\!393$	$25,\!569$	11	5	8	5	1	12	4	3
tech-p2p-gnutella	$62,\!561$	$147,\!878$	6	2	5	3	1	10	1	2
rec-amazon	$91,\!813$	125,704	4	3	6	0	1	8	1	5
soc-douban	154,908	327,162	15	9	12	5	1	17	3	2
\inf -roadNet-PA	1,087,562	$1,\!541,\!514$	3	2	5	0	1	8	0	4
\inf -roadNet-CA	1,957,027	2,760,388	3	2	5	0	1	8	0	4
inf-road-usa	23,947,347	28,854,312	3	2	5	0	1	8	0	4
socfb-uci-uni	58,790,782	92,208,195	16	5	9	9	0	13	8	2
soc-BlogCatalog	88,784	2,093,195	221	99	OOT	OOT	OOT	OOT	OOT	OOT
soc-buzznet	$101,\!163$	2,763,066	153	57	36	119	25	OOT	OOT	OOT
soc-flickr	513,969	$3,\!190,\!452$	309	151	OOT	OOT	OOT	OOT	OOT	OOT
web-wikipedia 2009	$1,\!864,\!433$	$4,\!507,\!315$	66	29	32	36	1	32	39	7

Table 11: Network-Repo graph information for k=10 and 15 $\,$

						k=1	10		k=15	i
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$	$g_k(G)$	$cg_k(G)$	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$
soc-LiveMocha	104,103	2,193,083	92	25	41	61	4	52	55	3
soc-youtube	495,957	1,936,748	49	17	35	24	2	43	21	4
soc-youtube-snap	1,134,890	2,987,624	51	17	35	26	2	43	23	4
soc-lastfm	1,191,805	$4,\!519,\!330$	70	21	38	42	3	47	38	4
$\operatorname{soc-digg}$	770,799	5,907,132	236	71	87	159	4	100	151	1
ia-wiki-Talk	$92,\!117$	360,767	58	18	35	33	3	44	29	4
socfb-Duke14	9,885	$506,\!437$	85	45	60	35	5	70	30	5
soc-orkut	2,997,166	106,349,209	230	73	89	151	4	101	144	2
sc-ldoor	$909,\!537$	20,770,807	34	19	35	9	4	42	7	7
$\operatorname{sc-msdoor}$	404,785	$9,\!378,\!650$	34	19	35	9	4	42	7	7
sc-nasasrb	$54,\!870$	1,311,227	35	22	31	14	11	36	14	16
sc-pkustk11	87,804	$2,\!565,\!054$	47	34	48	9	6	48	14	16
soc-flixster	2,523,386	7,918,801	68	45	62	16	3	72	11	3
$\operatorname{sc-pwtk}$	$217,\!891$	5,653,221	35	22	33	12	9	38	12	14
$scc_reality$	6,809	4,714,485	1235	1234	1239	6	15	1244	6	20
soc-gowalla	$196,\!591$	$950,\!327$	51	27	42	19	5	49	17	8
soc-epinions	$26,\!588$	$100,\!120$	32	16	33	9	3	40	7	6
socfb-B-anon	2,937,612	20,959,854	63	29	47	26	2	57	21	2
socfb-A-anon	3,097,165	23,667,394	74	28	47	37	1	54	35	4

soc-pokec	1,632,803	22,301,964	47	27	45	12	2	49	13	8
soc-twitter-follows	404,719	$713,\!319$	28	4	21	17	3	30	13	4
soc-FourSquare	639,014	3,214,986	63	36	53	20	3	59	19	7
socfb-Indiana	29,732	$1,\!305,\!757$	76	51	70	16	1	75	16	6
soc-slashdot	70,068	$358,\!647$	53	33	51	12	2	59	9	4
ia-enron-large	33,696	180,811	43	20	38	15	2	45	13	5
tech-WHOIS	7,476	56,943	88	69	87	11	2	96	7	3
ia-email-EU	32,430	54,397	22	11	26	6	5	33	4	8
socfb-Texas 84	36,364	1,590,651	81	60	79	12	1	87	9	3
sc-pkustk13	94,893	$3,\!260,\!967$	41	34	45	6	9	50	6	14
socfb-UF	35,111	1,465,654	83	65	83	10	2	91	7	4
rt-retweet-crawl	1,112,702	2,278,852	18	11	25	3	6	31	2	10
tech-as-skitter	1,694,616	11,094,209	111	66	84	37	2	95	31	1
soc-livejournal	4,033,137	27,933,062	213	212	217	6	15	221	7	21
socfb-UIllinois	30,795	1,264,421	85	63	82	13	1	89	11	4
$scc_fb-messages$	1,303	531,893	706	705	710	6	15	714	7	21
scc_twitter-copen	2,623	473,614	582	579	587	5	12	597	0	12
socfb-Penn94	41,536	1,362,220	62	46	60	12	6	64	13	12
ca-hollywood-2009	1,069,126	56,306,653	2208	2207	2209	9	18	2209	14	28
web-uk-2005	129,632	11,744,049	499	498	500	9	18	500	14	28
socfb-Stanford3	11,586	568,309	91	58	75	26	3	77	29	11
socfb-MIT	6,402	251,230	72	39	57	$\frac{25}{25}$	2	61	26	8
soc-delicious	536,108	1,365,961	33	21	37	6	$\frac{2}{4}$	43	5	8
sc-shipsec1	140,385	1,707,759	$\frac{33}{24}$	22	29	5	13	34	5	18
socfb-Berkeley13	22,900	852,419	64	45	62	12	3	69	10	6
ia-infect-hyper	113	2,196	28	16	33	5	3	39	4	7
socfb-Wisconsin87	23,831	835,946	60	40	59	11	1	67	8	3
socfb-UCLA	20,453	747,604	65	52	67	8	5	70	10	12
web-it-2004	509,338	7,178,413	431	430	432	9	18	432	14	28
socfb-CMU	6,621	249,959	69	43	57	22	6	62	22	11
sc-shipsec5	179,104	2,200,076	29	22	31	8	11	38	6	14
socfb-OR	63,392	816,886	$\frac{29}{52}$	$\frac{22}{34}$	$\frac{51}{53}$	9	1	59	8	5
ca-coauthors-dblp	540,486	15,245,729	$\frac{32}{336}$	$\frac{34}{335}$	337	9	18	337	0 14	28
scc_fb-forum	488	71,011	272	264	281	1	3	284	3	10
scc_retweet	1,206		174	$\frac{204}{164}$	176	8	8	184	5	10
socfb-UConn		65,990	65	51	67	8		73	3 7	8
ca-MathSciNet	17,206	604,867 820,644	$\frac{05}{24}$	23	$\frac{67}{25}$	9	4 18	29	10	$\frac{\circ}{24}$
	332,689	/				-				
tech-RL-caida	190,914	607,610	32	17	35 75	7	2	41	6	6
socfb-UCSB37	14,917	482,215	65	58	75 40	0	3	78	2	10
web-spam	4,767	37,375	35	21	40	5	1	47	3	4
ca-dblp-2012	317,080	1,049,866	113	112	114	9	18	114	14	28
tech-as-caida2007	26,475	53,381	22	14	29	3	5	36	1	8
ca-citeseer	227,320	814,134	86	85	87	9	18	87	14	28
tech-internet-as	40,164	85,123	23	15	29	4	6	37	1	8
ca-AstroPh	17,903	196,972	56	55	57	9	18	58	13	27
ca-dblp-2010	226,413	716,460	74	73	75	9	18	75	14	28
web-arabic-2005	163,598	1,747,269	101	100	102	9	18	102	14	28
ca-CondMat	21,363	$91,\!286$	25	24	28	7	16	32	8	22
ia-infect-dublin	410	2,765	17	14	27	0	7	32	0	12
web-sk- 2005	$121,\!422$	334,419	81	80	84	7	16	84	12	26
$scc_enron-only$	146	$9,\!828$	119	118	128	1	10	131	3	17
soc-brightkite	56,739	212,945	52	41	58	4	3	65	2	6
ca-HepPh	$11,\!204$	$117,\!619$	238	237	239	9	18	239	14	28
$scc_infect-dublin$	10,972	$175,\!573$	83	82	84	9	18	87	11	25
web-indochina-2004	11,358	47,606	49	48	50	9	18	50	14	28
$scc_retweet\text{-}crawl$	$17,\!151$	24,015	19	18	27	2	11	31	3	17
web-webbase- 2001	16,062	25,593	32	31	33	9	18	33	14	28
ca- $GrQc$	$4,\!158$	$13,\!422$	43	42	46	7	16	46	12	26
scc_rt_lolgop	273	4,510	41	40	49	2	11	53	3	17
$scc_infect-hyper$	113	$6,\!222$	105	104	107	8	17	110	10	24
web-Berk $Stan$	$12,\!305$	19,500	28	27	29	9	18	29	14	28

scc_rt_occupy	127	931	17	16	26	1	10	32	0	14
scc_rt_occupywallstnyc	127	931	17	16	26	1	10	32	0	14
$scc_rt_gmanews$	135	1,078	21	20	29	2	11	35	1	15
web-edu	3,031	6,474	29	28	30	9	18	30	14	28
tech-routers-rf	2,113	6,632	15	14	24	1	10	29	1	15
scc_rt_oman	16	13	2	1	18	0	3	28	0	3
scc_rt_lebanon	10	5	1	0	18	0	2	28	0	2
scc_rt_voteonedirection		5	2	1	18	0	3	28	0	3
scc_rt_gop	13	7	1	0	18	0	2	28	0	2
scc_rt_http	5	6	2	1	18	0	3	28	0	3
scc_rt_israel	22	12	1	0	18	0	2	28	0	2
scc_rt_qatif	14	11	2	1	18	0	3	28	0	3
scc_rt_tlot	13	8	1	0	18	0	2	28	0	2
scc_rt_obama	8	4	1	0	18	0	2	28	0	2
scc_rt_p2	26	15	1	0	18	0	2	28	0	2
scc_rt_uae	18	12	2	1	18	0	3	28	0	3
scc_rt_ksa	21	23	5	4	18	0	6	28	0	6
scc_rt_libya	27	26	2	1	18	0	3	28	0	3
scc_rt_tcot	26	18	2	1	18	0	3	28	0	3
scc_rt_dash	31	39	5	4	18	0	6	28	0	6
scc_rt_damascus	34	41	4	3	18	0	5	28	0	5
scc_rt_saudi	28	91	7	6	18	0	8	28	0	8
soc-karate	34	78 150	4	3	18	0	5	28	0	5
soc-dolphins	62	159	4	3	18	0	5	28	0	5
rt-retweet	96	117	3	2	18	0	4	28	0	4
scc_rt_assad	34	96	7	6	18	0	8	28	0	8
scc_rt_mittromney	102	108	4	3	18	0	5	28	0	5
scc_rt_onedirection	35	368	26	25	27	9	18	28	13	27
scc_rt_bahrain	72	129	7	6	18	0	8	28	0	8
scc_rt_barackobama	80	226	9	8	18	1	10	28	0	10
$scc_rt_alwefaq$	72	355	15	14	21	4	13	28	2	16
scc_rt_justinbieber	62	442	16	15	25	1	10	28	3	17
ia-enron-only	143	623	9	6	18	1	8	28	0	8
ca-netscience	379	914	8	7	18	0	9	28	0	9
ca-CSphd	1,882	1,740	2	1	18	0	3	28	0	3
rt-twitter-copen	761	1,029	4	2	18	0	4	28	0	4
bio-diseasome	516	1,188	10	9	18	2	11	28	0	11
web-polblogs	643	2,280	12	8	22	0	6	28	0	10
bio-yeast	1,458	1,948	5	4	18	0	6	28	0	6
bio-celegans	453	2,025	10	7	20	0	7	28	0	9
soc-wiki-Vote	889	2,914	9	5	18	1	7	28	0	7
web-google	1,299	2,773	17	16	19	8	17	28	4	18
ia-fb-messages	1,266	6,451	11	3	18	3	5	28	0	5
ia-email-univ	1,133	5,451	11	10	18	3	12	28	0	$\frac{12}{c}$
inf-power	4,941	6,594	5	4	18	0	6	28	0	6
ia-reality	6,809	7,680	5	3	18	0	5	28	0	5
ca-Erdos992	5,094	7,515	7	6	18	0	8	28	0	8
bio-dmela	7,393	25,569	11	5	19	2	6	28	0	7
tech-p2p-gnutella	62,561	147,878	6	2	18	0	4	28	0	4
rec-amazon	91,813	125,704	4	3	18	0	5	28	0	5
soc-douban	154,908	327,162	15	9	21	$\frac{4}{0}$	8	28	$\frac{2}{0}$	11
inf-roadNet-PA	1,087,562	1,541,514	3	2	18	-	4	28		4
inf-roadNet-CA	1,957,027	2,760,388	$\frac{3}{3}$	$\frac{2}{2}$	18	0	4	28	0	4
inf-road-usa	23,947,347	28,854,312			18	0	4	28	0	$\frac{4}{7}$
socfb-uci-uni	58,790,782		16	5	21 OOT	5 ООТ	4 OOT	28 OOT	3 OOT	7 OOT
soc-BlogCatalog	88,784	2,093,195	221 152	99 57	TOO	TOO	TOO	TOO		TOO
soc-buzznet	101,163	2,763,066	153	57	TOO	TOO	TOO	TOO		TOO
soc-flickr	513,969	3,190,452	309	151	OOT	OOT	OOT	TOO		TOO
web-wikipedia2009	1,864,433	4,507,315	66	29	33	43	16	001	ООТ	ООТ

Table 12: Network-Repo graph information for k=20

						k=20	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_{\nu}(G)}$		$cg_k(G)$
T:M1	104 109	0.102.002	00	25			
soc-LiveMocha soc-youtube	$104,103 \\ 495,957$	2,193,083 $1,936,748$	92 49	$\frac{25}{17}$	60 50	$\frac{52}{19}$	5 7
soc-youtube-snap	1,134,890	2,987,624	49 51	17	50 51	20	6
soc-youtube-shap soc-lastfm	1,191,805	4,519,330	70	21	56	$\frac{20}{34}$	5
soc-digg	770,799	5,907,132	236	71	109	147	$\frac{3}{2}$
ia-wiki-Talk	92,117	360,767	58	18	OOT	ООТ	OOT
socfb-Duke14	9,885	506,437	85	45	81	24	4
soc-orkut	2,997,166	106,349,209	230	73	111	139	2
sc-ldoor	909,537	20,770,807	$\frac{250}{34}$	19	45	9	$\frac{2}{14}$
sc-msdoor	404,785	9,378,650	34	19	45	9	14
sc-nasasrb	54,870	1,311,227	35	22	42	13	20
sc-pkustk11	87,804	2,565,054	47	34	56	11	18
soc-flixster	2,523,386	7,918,801	68	45	81	7	4
sc-pwtk	217,891	5,653,221	35	22	46	9	16
scc_reality	6,809	4,714,485	1235	1234	1251	4	23
soc-gowalla	196,591	950,327	51	27	56	15	11
soc-epinions	26,588	100,120	32	16	46	6	10
socfb-B-anon	2,937,612	20,959,854	63	29	64	19	5
socfb-A-anon	3,097,165	23,667,394	74	28	61	33	7
soc-pokec	1,632,803	22,301,964	47	27	55	12	12
soc-twitter-follows	404,719	713,319	28	4	38	10	6
soc-FourSquare	639,014	3,214,986	63	36	65	18	11
socfb-Indiana	29,732	1,305,757	76	51	83	13	8
soc-slashdot	70,068	358,647	53	33	68	5	5
ia-enron-large	33,696	180,811	43	20	51	12	9
tech-WHOIS	7,476	56,943	88	69	104	4	5
ia-email-EU	$32,\!430$	$54,\!397$	22	11	39	3	12
socfb-Texas84	$36,\!364$	$1,\!590,\!651$	81	60	94	7	6
sc-pkustk13	$94,\!893$	$3,\!260,\!967$	41	34	55	6	19
$\operatorname{socfb-UF}$	$35{,}111$	$1,\!465,\!654$	83	65	99	4	6
rt-retweet-crawl	1,112,702	$2,\!278,\!852$	18	11	38	0	13
tech-as-skitter	1,694,616	11,094,209	111	66	104	27	2
soc-livejournal	4,033,137	27,933,062	213	212	222	11	30
socfb-UIllinois	30,795	1,264,421	85	63	96	9	7
scc_fb -messages	1,303	$531,\!893$	706	705	715	11	30
$scc_twitter-copen$	2,623	$473,\!614$	582	579	602	0	17
socfb-Penn94	41,536	$1,\!362,\!220$	62	46	69	13	17
ca-hollywood-2009	1,069,126	56,306,653	2208	2207	2209	19	38
web-uk-2005	129,632	11,744,049	499	498	500	19	38
socfb-Stanford3	11,586	568,309	91	58	77	34	21
socfb-MIT	6,402	251,230	72	39	64	28	15
soc-delicious	536,108	1,365,961	33	21	49	4	12
sc-shipsec1	140,385	1,707,759	24	22	41	3	21
socfb-Berkeley13	22,900	852,419	64	45	75	9	10
ia-infect-hyper	113	2,196	28	16	46	2	10
socfb-Wisconsin87	23,831	835,946	60	40	73 74	7	7
socfb-UCLA	20,453	747,604	65	52	74	11	18
web-it-2004	509,338	7,178,413	431	430	432	19	38
socfb-CMU	6,621	249,959	69	43	67	22	16
sc-shipsec5	179,104	2,200,076	29	22	44	5	18
socfb-OR ca-coauthors-dblp	63,392	816,886	$\frac{52}{336}$	$\frac{34}{335}$	$\frac{64}{337}$	8	$\frac{10}{38}$
scc_fb-forum	$540,\!486$ 488	15,245,729 71,011	$\frac{330}{272}$	333 264	337 289	19 3	38 15
scc_retweet	$\frac{488}{1,206}$	65,990	$\frac{272}{174}$	$\frac{204}{164}$	289 191	3	13
socfb-UConn	1,200 $17,206$	604,867	65	51	191 77	8	13 14
ca-MathSciNet	332,689	820,644	$\frac{00}{24}$	$\frac{31}{23}$	38	6	25
C. 11100112C111C0	552,005	020,011	<i>≟</i> 1	20	30	J	20

tech-RL-caida	190,914	$607,\!610$	32	17	48	4	9
socfb-UCSB37	14,917	482,215	65	58	83	2	15
web-spam	4,767	$37,\!375$	35	21	53	2	8
ca-dblp-2012	317,080	1,049,866	113	112	114	19	38
			$\frac{113}{22}$	14	42	0	12
tech-as-caida2007	26,475	53,381					
ca-citeseer	$227,\!320$	$814,\!134$	86	85	87	19	38
tech-internet-as	40,164	85,123	23	15	43	0	12
$\operatorname{ca-AstroPh}$	17,903	196,972	56	55	64	12	31
ca-dblp-2010	226,413	716,460	74	73	75	19	38
web-arabic-2005	$163,\!598$	1,747,269	101	100	102	19	38
ca-CondMat	21,363	91,286	25	24	38	7	26
ia-infect-dublin	410	2,765	$\frac{25}{17}$	$\frac{24}{14}$	$\frac{38}{38}$	0	16
		,					
web-sk-2005	121,422	334,419	81	80	84	17	36
$scc_enron-only$	146	9,828	119	118	132	7	26
soc-brightkite	56,739	212,945	52	41	70	2	11
ca-HepPh	11,204	117,619	238	237	239	19	38
scc_infect-dublin	10,972	175,573	83	82	91	12	31
web-indochina-2004	11,358	47,606	49	48	50	19	38
scc_retweet-crawl	17,151	24,015	19	18	38	1	20
web-webbase-2001	16,062	25,593	32	31	38	14	33
ca- $GrQc$	$4,\!158$	13,422	43	42	46	17	36
scc_rt_lolgop	273	4,510	41	40	57	4	23
$scc_infect-hyper$	113	6,222	105	104	112	13	32
web-BerkStan	12,305	19,500	28	27	38	10	29
scc_rt_occupy	127	931	17	16	38	0	18
scc_rt_occupywallstnyc	127	931	17	16	38	0	18
						3	
scc_rt_gmanews	135	1,078	21	20	38		22
web-edu	3,031	$6,\!474$	29	28	38	11	30
tech-routers-rf	2,113	6,632	15	14	38	0	16
scc_rt_oman	16	13	2	1	38	0	3
$scc_rt_lebanon$	10	5	1	0	38	0	2
$scc_rt_voteonedirection$	7	5	2	1	38	0	3
scc_rt_gop	13	7	1	0	38	0	$\overset{\circ}{2}$
scc_rt_http	5	6	2	1	38	0	3
-			1				
scc_rt_israel	22	12		0	38	0	2
scc_rt_qatif	14	11	2	1	38	0	3
scc_rt_tlot	13	8	1	0	38	0	2
scc_rt_obama	8	4	1	0	38	0	2
$\mathrm{scc_rt_p2}$	26	15	1	0	38	0	2
scc_rt_uae	18	12	2	1	38	0	3
$\operatorname{scc_rt_ksa}$	21	$\overline{23}$	5	$\overline{4}$	38	0	6
scc_rt_libya	$\frac{21}{27}$	26	$\frac{\circ}{2}$	1	38	0	3
scc_rt_tcot	26	18	2	1	38	0	3
scc_rt_dash	31	39	5	4	38	0	6
$scc_rt_damascus$	34	41	4	3	38	0	5
scc_rt_saudi	28	91	7	6	38	0	8
$\operatorname{soc-karate}$	34	78	4	3	38	0	5
soc-dolphins	62	159	4	3	38	0	5
rt-retweet	96	117	3	2	38	0	4
scc_rt_assad	34	96	7	6	38	0	8
	102	108	4	3	38		5
scc_rt_mittromney						0	
$scc_rt_onedirection$	35	368	26	25	38	8	27
$scc_rt_bahrain$	72	129	7	6	38	0	8
$scc_rt_barackobama$	80	226	9	8	38	0	10
$scc_rt_alwefaq$	72	355	15	14	38	0	16
scc_rt_justinbieber	62	442	16	15	38	0	17
ia-enron-only	143	623	9	6	38	0	8
ca-netscience	379	914	8	7	$\frac{38}{38}$	0	9
ca-CSphd	1,882	1,740	2	1	38	0	3
rt-twitter-copen	761	1,029	4	2	38	0	4
bio-diseasome	516	1,188	10	9	38	0	11

web-polblogs	643	2,280	12	8	38	0	10
bio-yeast	1,458	1,948	5	4	38	0	6
bio-celegans	453	2,025	10	7	38	0	9
soc-wiki-Vote	889	2,914	9	5	38	0	7
web-google	1,299	2,773	17	16	38	0	18
ia-fb-messages	1,266	6,451	11	3	38	0	5
ia-email-univ	1,133	5,451	11	10	38	0	12
inf-power	4,941	$6,\!594$	5	4	38	0	6
ia-reality	6,809	7,680	5	3	38	0	5
ca-Erdos 992	5,094	$7,\!515$	7	6	38	0	8
bio-dmela	7,393	25,569	11	5	38	0	7
tech-p2p-gnutella	$62,\!561$	147,878	6	2	38	0	4
rec-amazon	91,813	125,704	4	3	38	0	5
soc-douban	154,908	$327,\!162$	15	9	38	0	11
\inf -roadNet-PA	1,087,562	1,541,514	3	2	38	0	4
\inf -roadNet-CA	1,957,027	2,760,388	3	2	38	0	4
inf-road-usa	23,947,347	28,854,312	3	2	38	0	4
socfb-uci-uni	58,790,782	92,208,195	16	5	38	0	7
soc-BlogCatalog	88,784	2,093,195	221	99	OOT	OOT	OOT
soc-buzznet	101,163	2,763,066	153	57	OOT	OOT	OOT
soc-flickr	513,969	3,190,452	309	151	OOT	OOT	OOT
web-wikipedia2009	1,864,433	4,507,315	66	29	TOO	ООТ	ООТ

3.2 10th-DIMACS Graphs

Note: If there is no non-trivial solution, $\omega_k(G)$ is set to 2k-2 and $g_k(G)/cg_k(G)$ is adjusted to 0, when 2k-2 is larger than the degeneracy bound.

Table 13: 10th-DIMACS graph information for k=2 and 5 $\,$

				- (- 0)		k=	2		k=5	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$	$g_k(G)$	$cg_k(G)$	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$
consph	79,679	2,963,573	41	22	24	19	2	26	20	6
connectus	394,707	$1,\!127,\!491$	37	9	12	27	1	19	23	0
$rgg_n_2_24_s0$	16,777,215	$132,\!557,\!200$	20	19	22	0	1	25	0	4
$rgg_n_2_3s0$	8,388,607	63,501,393	20	19	22	0	1	24	1	5
$rgg_n_2_22s0$	4,194,301	30,359,198	19	18	20	1	2	23	1	5
$rgg_n_2_1s0$	2,097,148	14,487,995	18	17	19	1	2	22	1	5
soc-Epinions1	$75,\!879$	405,740	67	31	28	41	7	39	33	2
bio-pdb1HYS	$36,\!417$	$2,\!154,\!174$	74	58	60	16	2	63	16	5
co-papers-dblp	$540,\!486$	$15,\!245,\!729$	336	335	337	1	2	337	4	8
$rgg_n_2_20_s0$	1,048,575	6,891,620	17	15	18	1	1	20	2	5
citationCiteseer	$268,\!495$	$1,\!156,\!646$	15	11	13	4	2	18	2	3
co-papers-citeseer	$434,\!102$	16,036,720	844	843	845	1	2	845	4	8
tech-caida $RouterLevel$	192,244	609,066	32	17	20	14	1	26	11	1
cnr-2000	$325,\!557$	2,738,969	83	82	85	0	1	86	2	6
${ m email} ext{-}{ m enron}$	69,017	254,449	52	34	36	18	2	44	13	0
$\operatorname{polblogs}$	1,224	16,715	36	23	23	15	4	32	9	1
c- $62ghs$	41,731	$258,\!806$	60	0	4	58	0	10	55	0
coAuthorsCiteseer	$227,\!320$	814,134	86	85	87	1	2	87	4	8
c-66b	49,989	$224,\!509$	42	0	4	40	0	10	37	0
che sape ake	39	170	6	3	7	1	0	11	0	2
$delaunay_n10$	1,024	3,056	4	2	5	1	1	8	1	4
bio-celegansneural	297	2,148	10	6	10	2	0	13	2	3
delaunay_n11	2,048	$6,\!127$	4	2	5	1	1	8	1	4
$delaunay_n12$	4,096	12,264	4	2	5	1	1	8	1	4
fe-4elt2	$11,\!143$	$32,\!818$	4	2	5	1	1	8	1	4
$delaunay_n13$	8,192	$24,\!547$	4	2	5	1	1	8	1	4
fe-sphere	$16,\!386$	49,152	5	2	5	2	1	8	2	4
cti	16,840	48,232	4	1	4	2	1	8	1	3

wing	10,937	75,488	8	4	7	3	1	10	3	4
wing_nodal	10,937 $10,937$	75,488	8	4	7	3	1	10	3	4
delaunay_n14	16,384	49,122	4	2	6	0	0	8	1	4
cs4	22,499	43,858	3	1	4	1	1	8	0	3
t60k	60,005	89,440	$\frac{3}{2}$	0	4	0	0	8	0	$\frac{3}{2}$
delaunay_n15	32,768	98,274	$\frac{2}{4}$	2	5	1	1	9	0	3
fe-body	44,775	163,734	6	$\frac{2}{4}$	8	0	0	10	1	$\frac{3}{4}$
rgg_n_2_15_s0	32,766	160,734 $160,240$	12	11	14	0	1	16	1	5
delaunay_n16	65,536	196,575	4	2	5	1	1	8	1	4
inf-luxembourg_osm	114,599	119,666	2	1	4	0	1	8	0	3
rgg_n_2_16_s0	65,532	342,127	13	12	15	0	1	18	0	$\frac{3}{4}$
fe-ocean	143,437	409,593	4	0	4	2	0	8	1	2
fe-t10000.0h	78,136	452,591	7	3	7	$\frac{2}{2}$	0	10	2	3
delaunay_n17	131,072	393,176	4	2	6	0	0	8	1	$\frac{3}{4}$
598a	110,971	741,934	8	5	7	3	2	11	2	4
rgg_n_2_17_s0	131,070	728,753	14	13	16	0	1	18	1	5
fe_rotor	99,617	662,431	8	3	7	3	0	11	2	$\frac{3}{2}$
144	144,649	1,074,393	9	5	8	3	1	12	$\frac{2}{2}$	3
delaunay_n18	262,144	786,396	$\frac{3}{4}$	$\frac{3}{2}$	6	0	0	8	1	$\frac{3}{4}$
rgg_n_2_18_s0	262,144	1,547,283	15	$\frac{2}{14}$	16	1	2	20	0	4
cop20k_A	99,843	1,347,263 $1,262,244$	18	12	14	6	$\frac{2}{2}$	17	6	5
m14b	214,765	1,679,018	9	5	8	3	1	12	2	3
delaunay_n19	524,288	1,572,823	$\frac{3}{4}$	$\frac{3}{2}$	5	1	1	9	0	3
rgg_n_2_19_s0	524,284	3,269,766	17	16	19	0	1	21	1	5
auto	448,695	3,314,611	9	5	8	3	1	12	2	3
inf-belgium_osm	1,441,295	1,549,970	3	1	5	0	0	8	0	3
delaunay_n20	1,048,576	3,145,686	4	2	5	1	1	9	0	3
inf-netherlands_osm	2,216,688	2,441,238	3	1	5	0	0	8	0	3
delaunay_n21	2,097,152	6,291,408	4	2	6	0	0	9	0	3
packing-500x100x100-b050		17,488,243	9	$\frac{2}{4}$	8	3	0	10	4	$\frac{3}{4}$
venturiLevel3	4,026,819	8,054,237	3	1	4	1	1	8	0	3
hugetrace-00000	4,588,484	6,879,133	$\frac{3}{2}$	0	4	0	0	8	0	$\frac{3}{2}$
delaunay_n22	4,194,304	12,582,869	$\frac{2}{4}$	$\frac{0}{2}$	6	0	0	9	0	3
hugetric-00000	5,824,554	8,733,523	2	0	4	0	0	8	0	$\frac{3}{2}$
channel-500x100x100-b050		42,681,372	9	4	8	3	0	10	4	$\frac{2}{4}$
adaptive	6,815,744	13,624,320	$\frac{3}{2}$	0	4	0	0	8	0	2
inf-italy_osm	6,686,493	7,013,978	3	1	5	0	0	8	0	3
hugetric-00010	6,592,765	9,885,854	$\frac{3}{2}$	0	$\frac{3}{4}$	0	0	8	0	$\frac{3}{2}$
inf-great-britain_osm	7,733,822	8,156,517	3	1	5	0	0	8	0	3
hugetric-00020	7,122,792	10,680,777	2	0	4	0	0	8	0	$\frac{3}{2}$
inf-asia_osm	11,950,757	12,711,603	3	$\frac{0}{2}$	5	0	1	8	0	$\frac{2}{4}$
delaunay_n23	8,388,608	25,165,784	4	2	6	0	0	9	0	3
inf-germany_osm	11,548,845	12,369,181	3	1	5	0	0	8	0	3
hugetrace-00010	12,057,441	18,082,179	2	0	4	0	0	8	0	2
inf-road_central	14,081,816	16,933,413	3	$\frac{0}{2}$	5	0	1	8	0	$\frac{2}{4}$
hugetrace-00020	16,002,413	23,998,813	2	0	4	0	0	8	0	2
delaunay_n24	16,777,216	50,331,601	$\frac{2}{4}$	$\frac{0}{2}$	6	0	0	9	0	3
hugebubbles-00020	21,198,119	31,790,179	2	0	4	0	0	8	0	$\frac{3}{2}$
inf-road_usa	23,947,347	28,854,312	3	$\frac{0}{2}$	5	0	1	8	0	$\frac{2}{4}$
inf-europe_osm	50,912,018	54,054,660	3	$\frac{2}{2}$	5	0	1	8	0	4
kron_g500-logn16	55,321	2,456,071	432	$\frac{2}{283}$	OOT	ООТ	OOT	ООТ	ООТ	TOO
kron_g500-logn17	107,909	5,113,985	551	$\frac{280}{380}$	TOO	OOT	OOT	OOT	TOO	OOT
kron_g500-logn18	210,155	10,582,686	695	502	TOO	OOT	OOT			OOT
kron_g500-logn19	409,175	21,780,787	935	655	TOO		OOT		OOT	OOT
kron_g500-logn20	1,048,576	44,619,837	OOT		TOO	OOT	OOT	OOT		OOT
kron_g500-logn21	2,097,152	91,041,471		OOT		OOT	OOT		OOT	OOT
M1011_5000-10g1121	2,001,102	01,011,111	001	001	501	001	001	501	001	001

Table 14: 10th-DIMACS graph information for k=10 and 15 $\,$

						k=1	.0		k=15	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$	$g_k(G)$	$cg_k(G)$	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$
consph	79,679	2,963,573	41	22	33	18	9	42	14	10
connectus	394,707	1,127,491	37	9	26	21	3	34	18	5
$rgg_n_2_24_s0$	16,777,215	132,557,200	20	19	29	1	10	33	2	16
$rgg_n_2_23_s0$	8,388,607	63,501,393	20	19	28	2	11	33	2	16
$rgg_n_2_2s0$	4,194,301	30,359,198	19	18	27	2	11	31	3	17
$rgg_n_2_1s0$	2,097,148	14,487,995	18	17	25	3	12	30	3	17
soc-Epinions1	75,879	405,740	67	31	49	28	2	58	24	3
bio-pdb1HYS	$36,\!417$	$2,\!154,\!174$	74	58	69	15	9	74	15	14
co-papers-dblp	$540,\!486$	$15,\!245,\!729$	336	335	337	9	18	337	14	28
$rgg_n_2_20_s0$	$1,\!048,\!575$	6,891,620	17	15	26	1	9	32	0	13
citationCiteseer	$268,\!495$	1,156,646	15	11	23	2	8	29	1	12
co-papers-citeseer	$434,\!102$	16,036,720	844	843	845	9	18	845	14	28
tech-caidaRouterLevel	$192,\!244$	609,066	32	17	35	7	2	41	6	6
cnr-2000	$325,\!557$	2,738,969	83	82	89	4	13	89	9	23
email-enron	69,017	254,449	52	34	51	11	3	59	8	5
$\operatorname{polblogs}$	1,224	16,715	36	23	41	5	2	50	1	3
c-62ghs	41,731	$258,\!806$	60	0	20	50	0	30	45	0
coAuthorsCiteseer	$227,\!320$	814,134	86	85	87	9	18	87	14	28
c-66b	49,989	$224,\!509$	42	0	20	32	0	30	27	0
che sapeake	39	170	6	3	18	0	5	28	0	5
$delaunay_n10$	1,024	3,056	4	2	18	0	4	28	0	4
bio-celegansneural	297	2,148	10	6	19	1	7	28	0	8
$delaunay_n11$	2,048	$6,\!127$	4	2	18	0	4	28	0	4
$delaunay_n12$	4,096	12,264	4	2	18	0	4	28	0	4
fe-4elt2	11,143	32,818	4	2	18	0	4	28	0	4
$delaunay_n13$	8,192	$24,\!547$	4	2	18	0	4	28	0	4
fe-sphere	$16,\!386$	49,152	5	2	18	0	4	28	0	4
cti	16,840	48,232	4	1	18	0	3	28	0	3
wing	10,937	$75,\!488$	8	4	18	0	6	28	0	6
${ m wing_nodal}$	10,937	$75,\!488$	8	4	18	0	6	28	0	6
$delaunay_n14$	$16,\!384$	49,122	4	2	18	0	4	28	0	4
cs4	22,499	$43,\!858$	3	1	18	0	3	28	0	3
t60k	$60,\!005$	89,440	2	0	18	0	2	28	0	2
$delaunay_n15$	32,768	$98,\!274$	4	2	18	0	4	28	0	4
fe-body	44,775	163,734	6	4	18	0	6	28	0	6
$rgg_n_2_{15}s0$	32,766	$160,\!240$	12	11	20	2	11	28	0	13
$delaunay_n16$	$65,\!536$	$196,\!575$	4	2	18	0	4	28	0	4
\inf -luxembourg_osm	$114,\!599$	119,666	2	1	18	0	3	28	0	3
$rgg_n_2_{16}$ s0	$65,\!532$	342,127	13	12	22	1	10	28	0	14
fe-ocean	143,437	$409,\!593$	4	0	18	0	2	28	0	2
fe-t10000.0h	$78,\!136$	$452,\!591$	7	3	18	0	5	28	0	5
$delaunay_n17$	131,072	$393,\!176$	4	2	18	0	4	28	0	4
598a	110,971	741,934	8	5	18	0	7	28	0	7
$rgg_n_2_17_s0$	131,070	728,753	14	13	22	2	11	28	1	15
fe_rotor	$99,\!617$	$662,\!431$	8	3	18	0	5	28	0	5
144	$144,\!649$	1,074,393	9	5	18	1	7	28	0	7
$delaunay_n18$	262,144	$786,\!396$	4	2	18	0	4	28	0	4
$rgg_n_2_{18}s0$	262,141	$1,\!547,\!283$	15	14	25	0	9	28	2	16
$ m cop20k_A$	99,843	$1,\!262,\!244$	18	12	23	5	9	28	5	14
m14b	214,765	1,679,018	9	5	18	1	7	28	0	7
$delaunay_n19$	$524,\!288$	$1,\!572,\!823$	4	2	18	0	4	28	0	4
$rgg_n_2_{19}s0$	$524,\!284$	3,269,766	17	16	23	4	13	28	4	18
auto	448,695	3,314,611	9	5	18	1	7	28	0	7
\inf -belgium_osm	$1,\!441,\!295$	$1,\!549,\!970$	3	1	18	0	3	28	0	3
$delaunay_n20$	1,048,576	$3,\!145,\!686$	4	2	18	0	4	28	0	4
$inf-netherlands_osm$	2,216,688	2,441,238	3	1	18	0	3	28	0	3
$delaunay_n21$	2,097,152	6,291,408	4	2	18	0	4	28	0	4
packing- $500x100x100-b050$	$2,\!145,\!839$	$17,\!488,\!243$	9	4	18	1	6	28	0	6

venturiLevel3	4,026,819	8,054,237	3	1	18	0	3	28	0	3
hugetrace-00000	$4,\!588,\!484$	$6,\!879,\!133$	2	0	18	0	2	28	0	2
$\rm delaunay_n22$	4,194,304	12,582,869	4	2	18	0	4	28	0	4
hugetric-00000	$5,\!824,\!554$	8,733,523	2	0	18	0	2	28	0	2
channel- $500x100x100$ -b050	4,802,000	$42,\!681,\!372$	9	4	18	1	6	28	0	6
adaptive	6,815,744	13,624,320	2	0	18	0	2	28	0	2
$\inf - italy _osm$	$6,\!686,\!493$	7,013,978	3	1	18	0	3	28	0	3
hugetric-00010	$6,\!592,\!765$	$9,\!885,\!854$	2	0	18	0	2	28	0	2
inf -great-britain_osm	7,733,822	8,156,517	3	1	18	0	3	28	0	3
hugetric-00020	7,122,792	10,680,777	2	0	18	0	2	28	0	2
\inf -asia_osm	11,950,757	12,711,603	3	2	18	0	4	28	0	4
$\rm delaunay_n23$	8,388,608	$25,\!165,\!784$	4	2	18	0	4	28	0	4
\inf -germany_osm	11,548,845	12,369,181	3	1	18	0	3	28	0	3
hugetrace-00010	12,057,441	18,082,179	2	0	18	0	2	28	0	2
$inf-road_central$	14,081,816	16,933,413	3	2	18	0	4	28	0	4
hugetrace-00020	16,002,413	23,998,813	2	0	18	0	2	28	0	2
$delaunay_n24$	16,777,216	50,331,601	4	2	18	0	4	28	0	4
hugebubbles-00020	$21,\!198,\!119$	31,790,179	2	0	18	0	2	28	0	2
$inf-road_usa$	23,947,347	28,854,312	3	2	18	0	4	28	0	4
$inf-europe_osm$	50,912,018	54,054,660	3	2	18	0	4	28	0	4
$kron_g500-logn16$	$55,\!321$	$2,\!456,\!071$	432	283	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn17$	107,909	$5,\!113,\!985$	551	380	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn18$	$210,\!155$	10,582,686	695	502	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500-logn19$	$409,\!175$	21,780,787	935	655	OOT	OOT	OOT	OOT	OOT	OOT
$kron_g500$ - $logn20$	$1,\!048,\!576$	$44,\!619,\!837$	OOT							
$kron_g500-logn21$	$2,\!097,\!152$	91,041,471	OOT							

Table 15: 10th-DIMACS graph information for k=20 $\,$

a ,	le el	LE		1(0)		k=20	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$	$g_k(G)$	$cg_k(G)$
consph	79,679	2,963,573	41	22	45	16	17
connectus	394,707	$1,\!127,\!491$	37	9	OOT	OOT	OOT
$rgg_n_2_24s0$	16,777,215	$132,\!557,\!200$	20	19	38	2	21
$rgg_n_2_23_s0$	8,388,607	63,501,393	20	19	38	2	21
$rgg_n_2_22s0$	4,194,301	30,359,198	19	18	38	1	20
$rgg_n_2_21_s0$	2,097,148	14,487,995	18	17	38	0	19
soc-Epinions1	$75,\!879$	405,740	67	31	65	22	6
bio-pdb1HYS	$36,\!417$	$2,\!154,\!174$	74	58	79	15	19
co-papers-dblp	$540,\!486$	15,245,729	336	335	337	19	38
$rgg_n_2_20_s0$	1,048,575	6,891,620	17	15	38	0	17
citationCiteseer	$268,\!495$	1,156,646	15	11	38	0	13
co-papers-citeseer	434,102	16,036,720	844	843	845	19	38
tech-caida $RouterLevel$	192,244	609,066	32	17	48	4	9
cnr-2000	$325,\!557$	2,738,969	83	82	93	10	29
email-enron	69,017	254,449	52	34	66	6	8
polblogs	1,224	16,715	36	23	55	1	8
c-62 ghs	41,731	258,806	60	0	40	40	0
coAuthorsCiteseer	227,320	814,134	86	85	87	19	38
c-66b	49,989	224,509	42	0	40	22	0
che sape ake	39	170	6	3	38	0	5
$delaunay_n10$	1,024	3,056	4	2	38	0	4
bio-celegansneural	297	2,148	10	6	38	0	8
delaunay_n11	2,048	$6,\!127$	4	2	38	0	4
delaunay_n12	4,096	12,264	4	2	38	0	4
fe-4elt2	11,143	32,818	4	2	38	0	4
$delaunay_n13$	8,192	24,547	4	2	38	0	4
fe-sphere	16,386	$49,\!152$	5	2	38	0	4

wing 10,937 75,488 8 4 38 0 6 6 wing.nodal 10,937 75,488 8 4 38 0 6 6 delaumay.n14 16,384 49,122 4 2 38 0 4 6 64,000 66 60,005 89,440 2 0 38 0 2 6 6 60,005 89,440 2 0 38 0 2 6 6 6 60,005 89,440 2 0 38 0 2 6 6 6 60,005 89,440 2 0 38 0 2 6 6 6 6 60,005 89,440 2 0 38 0 2 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	cti	16,840	48,232	4	1	38	0	3
wing_nodal 10,937 75,488 8 4 38 0 4 delaunay_n14 16,384 49,122 4 2 38 0 3 t60k 60,005 89,440 2 0 38 0 2 delaunay_n15 32,768 98,274 4 2 38 0 2 fe-body 44,775 163,734 6 4 38 0 6 reg_n.2_15_s0 32,766 160,240 12 11 38 0 3 delaumay_n16 65,553 196,575 4 2 38 0 4 fe-coccan 143,437 409,593 4 0 38 0 2 fe-t10000.0h 78,136 452,591 7 3 38 0 5 fe-t10000.0h 78,136 452,591 7 3 38 0 7 rgg_n_2_17.50 131,070 728,753 14								
Celaumay.n14	_							
cs4 22,499 43,858 3 1 38 0 3 delaunay.n15 32,768 89,274 4 2 38 0 2 delaunay.n16 65,536 160,240 12 11 38 0 13 delaunay.n16 65,536 196,575 4 2 38 0 4 inf-luxembourg.osm 114,599 119,666 2 1 38 0 3 rgg.n.2.16.s0 65,532 342,127 13 12 38 0 3 fe-cocan 143,437 409,593 4 0 38 0 5 delaunay.n17 131,072 393,176 4 2 38 0 5 delaunay.n17 131,072 393,176 4 2 38 0 7 rgg.n.2.17.80 131,070 728,753 14 13 38 0 7 delaunay.n18 262,144 786,396 <t< td=""><td>_</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	_							
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rgg.n.2.15.s0 32,766 160,240 12 11 38 0 13 delaumay.n16 65,536 196,575 4 2 38 0 4 in-luxembourg.osm 114,599 119,666 2 1 38 0 1 fe-crocan 143,437 409,593 4 0 38 0 2 fe-t10000.0h 78,136 452,591 7 3 38 0 2 fe-t10000.0h 78,136 452,591 7 3 38 0 4 598a 110,971 741,934 8 5 38 0 7 rgg.n.2.17-so 131,070 728,753 14 13 38 0 5 fe.rotor 99,617 662,431 8 3 38 0 7 delaunay.n18 262,141 786,396 4 2 38 0 7 delaunay.n20 214,581 1,572,823 <th< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></th<>								
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$\begin{array}{c} \text{hugetric-00000} & 5,824,554 & 8,733,523 & 2 & 0 & 38 & 0 & 2 \\ \text{channel-500x100x100-b050} & 4,802,000 & 42,681,372 & 9 & 4 & 38 & 0 & 6 \\ \text{adaptive} & 6,815,744 & 13,624,320 & 2 & 0 & 38 & 0 & 2 \\ \text{inf-italy_osm} & 6,686,493 & 7,013,978 & 3 & 1 & 38 & 0 & 3 \\ \text{hugetric-00010} & 6,592,765 & 9,885,854 & 2 & 0 & 38 & 0 & 2 \\ \text{inf-great-britain_osm} & 7,733,822 & 8,156,517 & 3 & 1 & 38 & 0 & 3 \\ \text{hugetric-00020} & 7,122,792 & 10,680,777 & 2 & 0 & 38 & 0 & 2 \\ \text{inf-asia_osm} & 11,950,757 & 12,711,603 & 3 & 2 & 38 & 0 & 4 \\ \text{delaunay_n23} & 8,388,608 & 25,165,784 & 4 & 2 & 38 & 0 & 4 \\ \text{inf-germany_osm} & 11,548,845 & 12,369,181 & 3 & 1 & 38 & 0 & 3 \\ \text{hugetrace-00010} & 12,057,441 & 18,082,179 & 2 & 0 & 38 & 0 & 2 \\ \text{inf-road_central} & 14,081,816 & 16,933,413 & 3 & 2 & 38 & 0 & 4 \\ \text{hugetrace-00020} & 16,002,413 & 23,998,813 & 2 & 0 & 38 & 0 & 2 \\ \text{delaunay_n24} & 16,777,216 & 50,331,601 & 4 & 2 & 38 & 0 & 4 \\ \text{hugebubbles-00020} & 21,198,119 & 31,790,179 & 2 & 0 & 38 & 0 & 2 \\ \text{inf-road_usa} & 23,947,347 & 28,854,312 & 3 & 2 & 38 & 0 & 4 \\ \text{kron_g500-logn16} & 55,321 & 2,456,071 & 432 & 283 & OOT & OOT & OOT \\ \text{kron_g500-logn17} & 107,909 & 5,113,985 & 551 & 380 & OOT & OOT & OOT \\ \text{kron_g500-logn18} & 210,155 & 10,582,686 & 695 & 502 & OOT & OOT & OOT \\ \text{kron_g500-logn19} & 409,175 & 21,780,787 & 935 & 655 & OOT & OOT & OOT \\ \text{kron_g500-logn19} & 409,175 & 21,780,787 & 935 & 655 & OOT & OOT & OOT \\ \text{kron_g500-logn19} & 409,175 & 21,780,787 & 935 & 655 & OOT & OOT & OOT \\ \text{kron_g500-logn19} & 409,175 & 21,780,787 & 935 & 655 & OOT & OOT & OOT \\ \text{kron_g500-logn20} & 1,048,576 & 44,619,837 & OOT & OOT & OOT & OOT & OOT \\ \text{kron_g500-logn20} & 1,048,576 & 44,619,837 & OOT & OOT & OOT & OOT & OOT \\ \text{kron_g500-logn20} & 1,048,576 & 44,619,837 & OOT & OOT & OOT & OOT & OOT \\ \text{OOT} & OOT \\ \text{Non_g500-logn20} & 1,048,576 & 44,619,837 & OOT & OOT & OOT & OOT & OOT \\ \text{Non_g500-logn20} & 1,048,576 & 44,619,837 & OOT & OOT & OOT & OOT & OOT \\ Non_g$	<u> </u>				2			
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	hugetrace-00020	16,002,413	23,998,813		0	38	0	2
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	\inf -road_usa	23,947,347	28,854,312		2	38	0	4
kron_g500-logn17 107,909 5,113,985 551 380 OOT OOT OOT OOT OOT Kron_g500-logn18 210,155 10,582,686 695 502 OOT OOT OOT OOT Kron_g500-logn19 409,175 21,780,787 935 655 OOT OOT OOT OOT OOT OOT OOT OOT kron_g500-logn20 1,048,576 44,619,837 OOT OOT OOT OOT OOT OOT OOT	$inf-europe_osm$						0	4
kron_g500-logn17 107,909 5,113,985 551 380 OOT OOT OOT OOT OOT Kron_g500-logn18 210,155 10,582,686 695 502 OOT OOT OOT OOT Kron_g500-logn19 409,175 21,780,787 935 655 OOT OOT OOT OOT OOT OOT OOT OOT kron_g500-logn20 1,048,576 44,619,837 OOT OOT OOT OOT OOT OOT OOT								
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kron_g500-logn19 409,175 21,780,787 935 655 OOT OOT OOT kron_g500-logn20 1,048,576 44,619,837 OOT OOT OOT OOT						OOT	OOT	OOT
kron_g500-logn20 1,048,576 44,619,837 OOT OOT OOT OOT						OOT	OOT	OOT
						OOT	OOT	OOT
	$kron_g500-logn21$	$2,\!097,\!152$	$91,\!041,\!471$	OOT	OOT	OOT	OOT	OOT

3.3 2nd-DIMACS Graphs

Note: If there is no non-trivial solution, $\omega_k(G)$ is set to 2k-2 and $g_k(G)/cg_k(G)$ is adjusted to 0, when 2k-2 is larger than the degeneracy bound.

Table 16: 2nd-DIMACS graph information for k=2 and 5 $\,$

						k=	2		k=5	
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)}$		$cg_k(G)$	$\omega_k(G)$		$cg_k(G)$
johnson8-4-4	70	1855	53	36	14	41	26	28	30	18
C125.9	125	6963	102	84	OOT		OOT	OOT	OOT	ООТ
$ san 200 _ 0.7 _ 2 $	200	13930	122	110	OOT		OOT	OOT	OOT	OOT
hamming8-2	256	31616	247	238	OOT	ООТ	OOT	OOT	OOT	OOT
MANN_a27	378	70551	364	350	236	130	118	351	18	9
$\sin 200 _ 0.7 _ 1$	200	13930	125	91	OOT		OOT	ООТ	OOT	OOT
hamming10-2	1024	518656	1013	1002	OOT		ООТ	ООТ	ООТ	ООТ
$ san 400_0.7_2 $	400	55860	259	177	OOT	OOT	OOT	OOT	OOT	OOT
$san 400_0.7_1$	400	55860	261	180	OOT	OOT	OOT	OOT	OOT	OOT
$san 400_0.7_3$	400	55860	253	180	OOT	OOT	OOT	OOT	OOT	OOT
$MANN_a45$	1035	533115	1012	989	OOT	OOT	OOT	990	27	9
hamming6-4	64	704	22	6	6	18	4	12	15	4
hamming6-2	64	1824	57	50	32	27	22	48	14	12
c-fat500-2	500	9139	35	24	26	11	2	26	14	8
c-fat500-10	500	46627	185	124	126	61	2	126	64	8
$MANN_{-}a9$	45	918	40	35	26	16	13	45	0	0
c-fat 500 - 1	500	46627	185	124	126	61	2	126	64	8
c-fat200-2	200	3235	32	22	24	10	2	24	13	8
c-fat 500 - 5	500	23191	92	62	64	30	2	64	33	8
c-fat200-5	200	8473	83	56	58	27	2	58	30	8
johnson 8-2-4	28	210	15	6	5	12	5	12	8	4
c-fat200-1	200	1534	14	10	12	4	2	14	5	6
$brock200_{-}1$	200	14834	134	89	OOT	OOT	OOT	OOT	OOT	OOT
$brock200_2$	200	9876	84	33	13	73	24	OOT	OOT	OOT
$brock200_3$	200	12048	105	54	17	90	41	OOT	OOT	OOT
$brock200_4$	200	13089	117	66	20	99	50	OOT	OOT	OOT
$brock400_{-}1$	400	59723	277	190	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_2$	400	59786	278	191	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_3$	400	59681	278	190	OOT	OOT	OOT	OOT	OOT	OOT
$brock400_4$	400	59765	277	191	OOT	ООТ	OOT	OOT	OOT	OOT
$brock800_{-}1$	800	207505	487	290	OOT	ООТ	OOT	OOT	OOT	OOT
$brock800_2$	800	208166	486	290	OOT	ООТ	TOO	OOT	OOT	TOO
$brock800_3$	800	207333	483	287	OOT	ООТ	OOT	OOT	OOT	TOO
$brock800_4$	800	207643	485	289	OOT		TOO	TOO	OOT	OOT
C1000.9		450079	874	762	TOO		ООТ		ООТ	TOO
C2000.5	2000		940	433	TOO	OOT	TOO	TOO	TOO	TOO
C2000.9		1799532		1547	TOO		TOO	TOO	TOO	TOO
C250.9	250	27984	210	179	TOO		TOO	ООТ	OOT	TOO
C4000.5		4000268		897	TOO		TOO	TOO	OOT	TOO
C500.9	500	112332	432	371	OOT		OOT	OOT	OOT	OOT
DSJC1000_5	1000	249826	459	205	TOO		TOO	TOO	OOT	TOO
DSJC500_5	500	62624	225	97	TOO		TOO	TOO	OOT	TOO
gen200_p0.9_44		17910	167	139	TOO	OOT	TOO	OOT	OOT	TOO
gen200_p0.9_55		17910	166	140	OOT		OOT	OOT	OOT	OOT
gen400_p0.9_55		71820	336	285	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_65		71820	336	284	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_75		71820	336	285	OOT	OOT	OOT	OOT	OOT	OOT
hamming10-4	1024	434176	848	672	TOO	OOT	OOT	TOO	TOO	TOO
hamming8-4	256	20864	163	80	TOO	OOT	OOT	TOO	TOO	TOO
johnson16-2-4	120	5460	91	66	OOT	OOT	OOT	TOO	OOT	TOO
johnson32-2-4	496	107880	435	378	OOT	ООТ	OOT	OOT	ООТ	ООТ

keller4	171	9435	102	52	15	89	41	ООТ	ООТ	ООТ
keller5	776	225990	560	377	OOT	OOT	OOT	OOT	OOT	OOT
keller6	3361	4619898	2690	2082	OOT	OOT	OOT	OOT	OOT	OOT
$MANN_{-}a81$	3321	5506380	3280	3239	OOT	OOT	OOT	3240	45	9
p_hat1000-1	1000	122253	163	45	13	152	36	OOT	OOT	OOT
p_hat1000-2	1000	244799	327	194	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1000-3	1000	371746	609	386	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-1	1500	284923	252	73	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-2	1500	568960	504	312	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1500-3	1500	847244	929	595	OOT	OOT	OOT	OOT	OOT	OOT
p_hat300-1	300	10933	49	13	10	41	7	16	38	7
p_hat300-2	300	21928	98	57	30	70	31	46	57	21
p_hat300-3	300	33390	180	116	TOO	OOT	TOO	OOT	TOO	OOT
$p_hat 500-1$	500	31569	86	23	12	76	15	OOT	OOT	OOT
$p_hat 500-2$	500	62946	170	100	TOO	OOT	TOO	OOT	TOO	OOT
$p_hat500-3$	500	93800	303	195	TOO	OOT	TOO	OOT	TOO	OOT
p_hat700-1	700	60999	117	32	13	106	23	OOT	TOO	OOT
$p_hat700-2$	700	121728	235	141	OOT	TOO	TOO	OOT	OOT	TOO
$p_hat700-3$	700	183010	426	271	OOT	TOO	TOO	OOT	OOT	TOO
san1000	1000	250500	464	397	TOO	OOT	TOO	OOT	TOO	TOO
$\sin 200 - 0.9 - 1$	200	17910	162	132	90	74	46	125	42	17
$san200_0.9_2$	200	17910	169	141	TOO	OOT	TOO	OOT	TOO	TOO
$\sin 200 - 0.9 - 3$	200	17910	169	143	OOT	TOO	TOO	OOT	OOT	OOT
$\sin 400 _{-}0.5 _{-}1$	400	39900	183	152	OOT	TOO	TOO	OOT	OOT	OOT
$\sin 400_{-}0.9_{-}1$	400	71820	344	292	OOT	TOO	TOO	OOT	OOT	OOT
$sanr200_0.7$	200	13868	124	76	22	104	58	OOT	OOT	OOT
$sanr200_0.9$	200	17863	166	139	TOO	OOT	TOO	OOT	TOO	OOT
$sanr 400 _ 0.5$	400	39984	177	75	15	164	64	OOT	OOT	OOT
$sanr400_0.7$	400	55869	258	162	OOT	OOT	OOT	OOT	OOT	OOT

Table 17: 2nd-DIMACS graph information for k=10 and 15 $\,$

			- 4	- 4		k=1	10		k=15	
Graph	V	E	d(G)	cd(G)	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$	$\omega_k(G)$	$g_k(G)$	$cg_k(G)$
johnson8-4-4	70	1855	53	36	TOO	TOO	OOT	60	8	6
C125.9	125	6963	102	84	FOO	TOO	OOT	112	5	2
$san200_0.7_2$	200	13930	122	110	FOO	TOO	OOT	134	3	6
hamming8-2	256	31616	247	238	256	1	2	256	6	12
$MANN_a27$	378	70551	364	350	351	23	19	378	1	2
$san200_0.7_1$	200	13930	125	91	105	30	6	105	35	16
hamming10-2	1024	518656	1013	1002	FOO	TOO	OOT	1024	4	8
$san 400 _ 0.7 _ 2$	400	55860	259	177	Γ OO	TOO	TOO	205	69	2
$san 400_0.7_1$	400	55860	261	180	200	71	0	200	76	10
$san 400 _ 0.7 _ 3$	400	55860	253	180	Γ OO	TOO	TOO	OOT	OOT	OOT
$MANN_a45$	1035	533115	1012	989	990	32	19	990	37	29
hamming6-4	64	704	22	6	20	12	6	30	7	6
hamming6-2	64	1824	57	50	64	3	6	64	8	16
c-fat 500 - 2	500	9139	35	24	31	14	13	39	11	15
c-fat500-10	500	46627	185	124	126	69	18	126	74	28
$MANN_a9$	45	918	40	35	45	5	10	45	10	20
c-fat 500 - 1	500	46627	185	124	126	69	18	126	74	28
c-fat200-2	200	3235	32	22	30	12	12	35	12	17
c-fat 500 - 5	500	23191	92	62	64	38	18	64	43	28
c-fat200-5	200	8473	83	56	58	35	18	58	40	28
johnson 8-2-4	28	210	15	6	21	4	5	28	2	8
c-fat200-1	200	1534	14	10	18	6	12	28	1	12
$brock200_1$	200	14834	134	89	TOO	TOO	OOT	OOT	OOT	OOT
$brock200_2$	200	9876	84	33	FOO	TOO	OOT	OOT	OOT	OOT

11-200 2	200	10040	105	F 4	ООТ	ООТ	ООТ	ООТ	ООТ	ООТ
brock200_3	200	12048	105	54 66	TOO	TOO	TOO	TOO	TOO	TOO
brock200_4	200	13089	117	66	TOO	TOO	TOO	TOO	TOO	TOO
brock400_1	400	59723	277	190	TOO	TOO	ООТ	TOO	OOT	OOT
brock400_2	400	59786	278	191	OOT	OOT	ООТ	TOO	TOO	TOO
brock400_3	400	59681	278	190	OOT	OOT	ООТ	TOO	TOO	OOT
brock400_4	400	59765	277	191	TOO	TOO	OOT	TOO	TOO	TOO
brock800_1	800	207505	487	290	TOO	TOO	TOO	TOO	TOO	TOO
brock800_2	800	208166	486	290	OOT	ООТ	ООТ	TOO	OOT	ООТ
$brock800_3$	800	207333	483	287	OOT	OOT	OOT	OOT	ООТ	OOT
$brock800_4$	800	207643	485	289	OOT	OOT	OOT	OOT	TOO	OOT
C1000.9	1000	450079	874	762	OOT	OOT	OOT	OOT	OOT	OOT
C2000.5	2000	999836	940	433	OOT	OOT	OOT	OOT	OOT	OOT
C2000.9	2000		1758	1547	OOT	OOT	OOT	OOT	OOT	OOT
C250.9	250	27984	210	179	OOT	OOT	OOT	OOT	OOT	OOT
C4000.5	4000	4000268	1909	897	OOT	OOT	OOT	OOT	OOT	OOT
C500.9	500	112332	432	371	OOT	OOT	OOT	OOT	OOT	OOT
$DSJC1000_5$	1000	249826	459	205	OOT	OOT	OOT	OOT	OOT	OOT
$DSJC500_5$	500	62624	225	97	OOT	OOT	OOT	OOT	OOT	OOT
$gen200_p0.9_44$	200	17910	167	139	OOT	OOT	OOT	OOT	OOT	OOT
$gen200_p0.9_55$	200	17910	166	140	OOT	OOT	OOT	OOT	OOT	OOT
$gen 400_p 0.9_5 5$	400	71820	336	285	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_65	400	71820	336	284	OOT	OOT	OOT	OOT	OOT	OOT
gen400_p0.9_75	400	71820	336	285	OOT	OOT	OOT	OOT	OOT	OOT
hamming10-4	1024	434176	848	672	OOT	OOT	OOT	OOT	OOT	OOT
hamming8-4	256	20864	163	80	OOT	OOT	OOT	OOT	OOT	OOT
johnson16-2-4	120	5460	91	66	OOT	OOT	OOT	OOT	OOT	OOT
johnson32-2-4	496	107880	435	378	OOT	OOT	OOT	OOT	OOT	OOT
keller4	171	9435	102	52	OOT	ООТ	OOT	ООТ	ООТ	ООТ
keller5	776	225990	560	377	OOT	ООТ	OOT	ООТ	ООТ	ООТ
keller6		4619898		2082	OOT	OOT	OOT	OOT	OOT	OOT
MANN_a81		5506380		3239	3240	50	19	OOT	ООТ	OOT
p_hat1000-1	1000	122253	163	45	OOT	OOT	ООТ	TOO	OOT	OOT
p_hat1000-2	1000	244799	327	194	OOT	OOT	OOT	OOT	OOT	OOT
p_hat1000-3	1000	371746	609	386	OOT	OOT	OOT	TOO	OOT	OOT
p_hat1500-1	1500	284923	252	73	OOT	OOT	OOT	TOO	OOT	OOT
p_hat1500-2	1500	568960	504	312	OOT	OOT	OOT	TOO	OOT	OOT
p_hat1500-3	1500	847244	929	595	OOT	OOT	OOT	TOO	OOT	OOT
p_hat300-1	300	10933	$\frac{323}{49}$	13		OOT	OOT		OOT	OOT
p_hat300-2	300	21928	98	57	OOT	OOT	OOT	TOO	OOT	OOT
p_hat300-3	300	33390	180	116	OOT	OOT	OOT	TOO	OOT	OOT
p_hat500-1	500	31569	86	23	OOT		OOT	TOO		OOT
p_hat500-1 p_hat500-2	500	62946	170	100	OOT		OOT	TOO	OOT	OOT
p_hat500-3	500	93800	303	195	OOT		OOT	TOO	OOT	OOT
p_hat700-1	700	60999	117	$\frac{195}{32}$	OOT		OOT	TOO	OOT	OOT
p_hat700-1 p_hat700-2	700	121728	$\frac{117}{235}$	$\frac{32}{141}$	OOT		OOT	TOO	OOT	OOT
p_hat700-2 p_hat700-3					001 00T			TOO	OOT	
•	700	183010	426	271			TOO			TOO
	1000	250500	464	397	TOO		TOO	TOO	TOO	TOO
san200_0.9_1	200	17910 17010	162	132	TOO		TOO	TOO	TOO	TOO
san200_0.9_2	200	17910	169	141	TOO		TOO	TOO	TOO	TOO
san200_0.9_3	200	17910	169	143	TOO		TOO	TOO	TOO	TOO
san400_0.5_1	400	39900	183	152	TOO		TOO	TOO	TOO	TOO
san400_0.9_1	400	71820	344	292	OOT		ООТ	TOO	OOT	OOT
$sanr200_{-}0.7$	200	13868	124	76	OOT		ООТ	TOO	OOT	OOT
sanr200_0.9	200	17863	166	139	OOT		ООТ	TOO	OOT	TOO
sanr400_0.5	400	39984	177	75	TOO		ООТ	OOT	OOT	OOT
$sanr400_0.7$	400	55869	258	162	OOT	OOT	ООТ	TOO	OOT	ООТ

Table 18: 2nd-DIMACS graph information for k=20

					k=20)
Graph	V	E	d(G)	cd(G)	$\overline{\omega_k(G)} \ g_k(G)$	$cg_k(G)$
johnson 8-4-4	70	1855	53	36	70 3	6
C125.9	125	6963	102	84	122 0	2
$\sin 200 _{-}0.7_{-}2$	200	13930	122	110	134 8	16
hamming8-2	256	31616	247	238	256 11	22
MANN_a27	378	70551	364	350	378 6	12
san200_0.7_1	200	13930	125	91	105 40	26
hamming10-2	1024	518656	1013	1002	1024 9	18
san400_0.7_2	400	55860	259	177	205 74	12
san400_0.7_1 san400_0.7_3	400 400	55860	$\frac{261}{253}$	180 180	200 81 216 57	$\begin{array}{c} 20 \\ 4 \end{array}$
MANN_a45	1035	55860 533115	$\frac{255}{1012}$	989	OOT OOT	
hamming6-4	64	704	22	6	38 4	8
hamming6-2	64	1824	57	50	64 13	$\frac{3}{26}$
c-fat500-2	500	9139	35	$\frac{30}{24}$	39 16	$\frac{25}{25}$
c-fat500-10	500	46627	185	124	126 79	38
MANN_a9	45	918	40	35	45 15	30
c-fat500-1	500	46627	185	124	126 79	38
c-fat200-2	200	3235	32	22	38 14	24
c-fat 500 - 5	500	23191	92	62	70 42	32
c-fat200-5	200	8473	83	56	67 36	29
johnson 8-2-4	28	210	15	6	38 0	8
c-fat200-1	200	1534	14	10	$38 \qquad 0$	12
$brock200_{-}1$	200	14834	134	89	OOT OOT	OOT
$brock200_2$	200	9876	84	33	OOT OOT	OOT
$brock200_3$	200	12048	105	54	OOT OOT	OOT
$brock200_4$	200	13089	117	66	OOT OOT	OOT
$brock400_{-}1$	400	59723	277	190	OOT OOT	
$brock 400_2$	400	59786	278	191	OOT OOT	
$brock400_3$	400	59681	278	190	OOT OOT	
$brock400_{-}4$	400	59765	277	191	OOT OOT	
brock800_1	800	207505	487	290	OOT OOT	
brock800_2	800	208166	486	290	OOT OOT	
brock800_3	800	207333	483	287	OOT OOT	
brock800_4	800	207643	485	289	OOT OOT	
C1000.9	1000	450079	874	762	OOT OOT	
C2000.5	2000	999836	940	433	OOT OOT	
C2000.9		1799532 27984	1758	1547	OOT OOT	
$C250.9 \\ C4000.5$	250	4000268	210 1909	$179 \\ 897$	TOO TOO TOO TOO	
C4000.5 C500.9	500	112332	432	371	OOT OOT	
DSJC1000 ₋₅	1000	249826	$452 \\ 459$	$\frac{371}{205}$	OOT OOT	
DSJC500_5	500	62624	$\frac{455}{225}$	97	OOT OOT	
gen200_p0.9_44		17910	167	139	OOT OOT	
gen200_p0.9_55		17910	166	140	OOT OOT	
gen400_p0.9_55		71820	336	285	OOT OOT	
gen400_p0.9_65		71820	336	284	OOT OOT	
gen400_p0.9_75		71820	336	285	OOT OOT	
hamming10-4	1024	434176	848	672	OOT OOT	OOT
hamming8-4	256	20864	163	80	OOT OOT	OOT
johnson16-2-4	120	5460	91	66	OOT OOT	OOT
johnson32-2-4	496	107880	435	378	OOT OOT	OOT
keller4	171	9435	102	52	OOT OOT	OOT
keller5	776	225990	560	377	OOT OOT	
keller6		4619898	2690	2082	OOT OOT	
$MANN_a81$		5506380		3239	OOT OOT	
p_hat1000-1	1000	122253	163	45	OOT OOT	
p_hat1000-2	1000	244799	327	194	OOT OOT	
p_hat1000-3	1000	371746	609	386	OOT OOT	ООТ

p_hat1500-1	1500	284923	252	73	TOO TOO TOO
p_hat1500-2	1500	568960	504	312	TOO TOO TOO
p_hat1500-3	1500	847244	929	595	TOO TOO TOO
p_hat300-1	300	10933	49	13	TOO TOO TOO
$p_hat300-2$	300	21928	98	57	TOO TOO TOO
$p_hat300-3$	300	33390	180	116	TOO TOO TOO
$p_hat 500-1$	500	31569	86	23	TOO TOO TOO
$p_hat 500-2$	500	62946	170	100	TOO TOO TOO
$p_hat 500-3$	500	93800	303	195	TOO TOO TOO
$p_hat700-1$	700	60999	117	32	TOO TOO TOO
$p_hat700-2$	700	121728	235	141	TOO TOO TOO
$p_hat700-3$	700	183010	426	271	TOO TOO TOO
san1000	1000	250500	464	397	TOO TOO TOO
$\sin 200 _ 0.9 _ 1$	200	17910	162	132	TOO TOO TOO
$\sin 200 _{-}0.9 _{-}2$	200	17910	169	141	TOO TOO TOO
$\sin 200 _{-}0.9 _{-}3$	200	17910	169	143	TOO TOO TOO
$\sin 400 _0.5 _1$	400	39900	183	152	TOO TOO TOO
$\sin 400 _ 0.9 _ 1$	400	71820	344	292	TOO TOO TOO
$sanr200_0.7$	200	13868	124	76	TOO TOO TOO
$sanr200_0.9$	200	17863	166	139	TOO TOO TOO
$\rm sanr 400_0.5$	400	39984	177	75	TOO TOO TOO
$sanr 400 _ 0.7$	400	55869	258	162	TOO TOO TOO