PROBLEM SET #2

For the problems below, you need to write programs into a single Jupyter notebook document. Use `Markdown` cells and the hash(`#`) symbol to indicate the problem numbers, explanation, and comments. Name your notebook document using your student `ID` number as `HW2_ID.ipynb`, and email it to your teaching assistant at `hangyeol@snu.ac.kr` before the deadline. No homework will be accepted after the deadline.

1. Write a (short) program to find the machine epsilon of your computer, using Python. Run your program to obtain the outputs.

2. This problem is to show that you need to be careful to avoid unstable algorithms in which roundoff errors can increase exponentially. The "golden mean", ϕ, is given by $\phi = (\sqrt{5} - 1)/2 \simeq 0.61803398875 \cdots$.

(a) Write a Python program to calculate the n-th power of ϕ, using successive multiplications

$$\phi^0 = 1, \quad \text{and} \quad \phi^n = \phi \cdot \phi^{n-1} \text{ for } n = 1, 2, 3, \cdots, \tag{1}$$

and plot ϕ^n as a function of n for $0 \leq n \leq 50$. (The ordinate should be in logarithmic scale.)

(b) Another (clever) way to calculate ϕ^n is to use following recursion relation

$$\phi^{n+1} = \phi^{n-1} - \phi^n \text{ for } n = 1, 2, 3, \cdots. \tag{2}$$

In a `Markdown` cell, show that Equation (2) is equivalent to Equation (1).

(c) Use Equation (2) to calculate ϕ^n for $0 \leq n \leq 50$. Compare the results with those in part (a) by overploting all the results in the same Figure.

(d) Why do you think are the results in parts (a) and (b) so different for high n? (Hint: there is another solution of Equation (2) whose magnitude is greater than unity.)

3. The Planck function (measured per unit wavelength) from a blackbody with temperature T is given by

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}, \tag{3}$$

where $h = 6.626 \times 10^{-34}$ J s, $k = 1.381 \times 10^{-23}$ J K^{-1}, and $c = 2.998 \times 10^8$ m s^{-1}. All of your answers should be accurate to at least four digits.

(a) Derive Wien's displacement law by solving $dB_\lambda/d\lambda = 0$.

(b) For a blackbody with $T = 10^4$ K, find two wavelengths corresponding to $B_\lambda = 10^{13}$ J s^{-1} m^{-3}.

4. In celestial mechanics, Kepler's equation is important. It reads $x = y - \epsilon \sin y$, in which x is planet's mean anomaly, y its eccentric anomaly, and ϵ the eccentricity of its orbit. Taking $\epsilon = 0.9$, construct a table of y for 30 equally spaced values of x in the interval $0 \leq x \leq \pi$. Use Newton-Raphson method to obtain each value of y. The y corresponding to an x can be used as the staring point for the interation when x is changed slightly.