PROBLEM SET #3

For the problems below, you need to write programs into a single Jupyter notebook document. Use `Markdown` cells and the hash(`#`) symbol to indicate the problem numbers, explanation, and comments. Name your notebook document using your student `ID` number as `HW3_ID.ipynb`, and email it to your teaching assistant at `hangyeol@snu.ac.kr` before the deadline. No homework will be accepted after the deadline.

1. Integrate the following integral to 6 significant digits

$$\int_0^4 \sin(x^4)dx, \tag{1}$$

using (a) the composite trapezoidal rule, (b) the composite Simpson's rule, and (c) the Gaussian quadrature.

2. The exact limb darkening formula normalized at $\mu = 0$ is given by

$$H(\mu) = \frac{1}{1+\mu} \exp\left[\frac{1}{\pi}\int_0^{\pi/2} \frac{\phi \tan^{-1}(\mu \tan\phi)}{1-\phi\cot\phi} d\phi\right]. \tag{2}$$

Use the composite Simpson's rule to integrate the above equation and plot $H(\mu)$ as a function of μ for $0 \le \mu \le 1$.

3. Interpolate the data given in Table 1 using (a) piecewise linear functions, (b) 8-th order polynomials, (c) natural cubic splines, (d) clamped cubic splines, and (e) B splines, and plot the resulting curves together with the data in a single figure. Make a table for the expected values at $x = 1$, 2, and 3.5 from each of the interpolation methods.

x	y
0	−0.8
0.6	−0.34
1.5	0.59
1.7	0.23
2.2	0.1
2.3	0.28
2.8	1.03
3.1	1.44
4	0.74

Table 1: Data points for Problem 3

4. It has been well established that all galaxies contain supermassive black holes at their centers, and that the black hole mass $M_{\rm BH}$ is related to the stellar velocity dispersion σ_e in the bulge of its host galaxy such that $\log(M_{\rm BH}/1M_\odot) = a + b\log(\sigma_e/1{\rm km\ s^{-1}})$, with a and b being constants. The `BlackHall.txt` file in the class web page contains data for 67 galaxies: the first and second columns give $M_{\rm BH}$ and its measurement error $\Delta M_{\rm BH}$ in units of $M_\odot$, respectively; the third and fourth columns give σ_e and its error $\Delta\sigma_e$ in units of km s^{-1}, respectively.

(a) Ignore $\Delta\sigma_e$ and $\Delta M_{\rm BH}$, and fit the data to find a and b and their errors.

(b) Allowing for both $\Delta\sigma_e$ and $\Delta M_{\rm BH}$, fit the data to find a and b and their errors.

(c) Make a plot of the data with errorbars together with your fits in (a) and (b).

(d) Now fit the data as $\log(\sigma_e/1{\rm km\ s^{-1}}) = c + d\log(M_{\rm BH}/1M_\odot)$ allowing for $\Delta\sigma_e$ and $\Delta M_{\rm BH}$ to find the fitting coefficients c and d. Discuss the relation between the coefficients in (b) and (d).

5. The `hw3p5.dat` file in the class web page contains two-column data: x and y.

(a) Fit the data using a Gaussian function

$$y = p_0 + p_1 \exp\left[-\frac{(x-p_2)^2}{2p_3^2}\right], \tag{3}$$

and find the fitting coefficients and their errors, as well as χ^2.

(b) Fit the data using a Lorenzian function

$$y = q_0 + \frac{q_1}{q_2 + (x-q_3)^2}, \tag{4}$$

and find the fitting coefficients and their errors, as well as χ^2.

(c) Make a plot of the data together with your fits in (a) and (b). Which fit do you think is better between (a) and (b)?