COMPUTATION OF DYADIC GREEN'S FUNCTIONS FOR GENERALIZED CYLINDERS IN FREE SPACE

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Abstract

Dyadic Green's functions are powerful and efficient for solving the boundary problems in electromagnetic theory. The key problem involved is how to deal with the additional term in the electric dyadic Green's function at the source region. In this paper, the dyadic Green's functions for generalized cylinders in free space are derived with the technique in another paper of author (1984). The dyadic Green's functions for conducting wedges, half-plates and elliptic cylinders, whose integral of continuous spectrum h has been eliminated, are given particularly.

I. Introduction

The dyadic Green's functions for special geometries in cylindrical coordinate systems in free space are usually used to deal with the radiation of electric dipoles and slots in half-plates, the electromagnetic scattering and diffraction of infinite cylinders and half-plates. Differing from a bounded region (such as waveguides), the expansion of dyadic Green's functions in cylindrical coordinate systems in free space needs the vector wave functions whose eigenvalues h and λ are continuous. Thus, there are two possible expressions for which either the integral of continuous spectrum h or the integral of continuous spectrum λ has been eliminated. One form of the expressions may be more efficient than another in numerical calculation.

In this paper, the dyadic Green's functions for generalized cylinders in free space are derived by the spectral theory of dyadic operators established by the author in [1]. These expressions are fairly general and can be applied to obtain explicit expressions for special geometries in cylindrical coordinate systems in free space. As an example, these general expressions first reduce to those obtained previously for circular cylinders by Tai[2]. Then, the dyadic Green's functions for conducting wedges, half-plates and elliptic cylinders whose integral of continuous spectrum h has been eliminated, are given particularly. As far as we know, these results have not been reported in literature. Thus, the expressions derived in this paper are useful for dealing with electromagnetic boundary value problems.

II. Dyadic Green's Functions for Generalized Cylinders in Free Space

The dyadic version of wave equations can be written in the form

$$\nabla\times\nabla\times\bar{G}_e(R/R')-k^2\bar{G}_e(R/R')=\bar{I}\delta(R-R'), \tag{1a}$$

$$\nabla\times\nabla\times\bar{G}_m(R/R')-k^2\bar{G}_m(R/R')=\nabla'\times\bar{I}\delta(R-R'), \tag{1b}$$

where $\bar{G}_e(R/R')$ designates the electric dyadic Green's function and $\bar{G}_m(R/R')$ the magnetic dyadic Green's function. Eq. (1b) is a new expression derived strictly by the theory of distributions in [1] for the dyadic version of the magnetic wave equation. The operator ∇' in Eq. (1b) acts on source coordinates and latter vectors.

The coordinate system with respect to generalized cylinders is shown in Fig. 1.

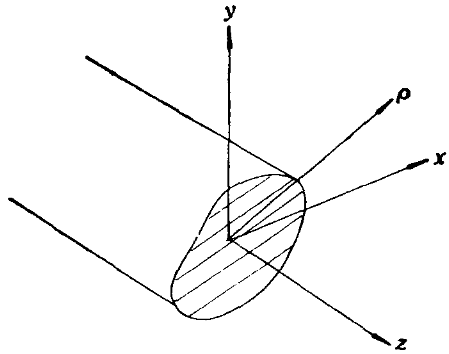

Fig. 1 Generalized cylinders in free space

The vector wave functions for generalized cylinders in free space are

$$M_{\substack{e\\o}V}(\lambda,h)=\nabla\times[\Phi_{\substack{e\\o}V}(\lambda,h)\hat{z}], \tag{2a}$$

$$N_{\substack{e\\o}V}(\lambda,h)=\frac{1}{K}\nabla\times\nabla\times[\Phi_{\substack{e\\o}V}(\lambda,h)\hat{z}], \tag{2b}$$

where $\Phi_{\substack{e\\o}V}(\lambda, h)$ is the solution of the scalar homogeneous Helmholtz equation

$$(\nabla^2+K^2)\Phi_{\substack{e\\o}V}(\lambda,h)=0, \tag{3}$$

and subscripts e and o denote even and odd functions. The eigenvalue equation is

$$K^2=k^2_{V,\lambda}+h^2. \tag{4}$$

Since the magnetic wave equation describes zero-divergence fields, its complete solution can be expressed in terms of the solenoidal vector wave functions M and N. Hence, the complete relation for the generalized function $\bar{I}\delta(R-R')$ is

$$\bar{I}\delta(R-R')=\int_{-\infty}^{\infty}dh\int_0^{\infty}d\lambda\sum_V[C_{mV}M_{\substack{e\\o}V}(\lambda,h)M'_{\substack{e\\o}V}(\lambda,-h)$$
$$+C_{nV}N_{\substack{e\\o}V}(\lambda,h)N'_{\substack{e\\o}V}(\lambda,-h)], \tag{5}$$

where C_{mV} and C_{nV} are the normalized constants of the vector wave functions M and N. In an unbounded region, both eigenvalues h and λ are continuous spectrums. With the aid of the theory of Fourier transformation of generalized functions[3] and the vector identities

$$M_{\substack{e\\o}V}(\lambda,h)=\frac{1}{K}\nabla\times N_{\substack{e\\o}V}(\lambda,h), \tag{6a}$$

$$N_{\substack{e\\o}V}(\lambda,h)=\frac{1}{K}\nabla\times M_{\substack{e\\o}V}(\lambda,h), \tag{6b}$$

we obtain

$$\nabla' \times \bar{I}\delta(R-R') = \int_{-\infty}^{\infty} dh \int_{0}^{\infty} d\lambda \sum_{V} K[C_{mV} M_{^e_o V}(\lambda,h) N'_{^e_o V}(\lambda,-h) + C_{nV} N_{^e_o V}(\lambda,h) M'_{^e_o V}(\lambda,-h)]. \tag{7}$$

By means of the spectral theory of operators[1], the solution for dyadic wave equations can be written as

$$\bar{U}(R/R') = (\mathscr{L} - k^2)^{-1} \bar{F} = \sum \frac{U_V A_V}{\lambda_V - k^2}, \tag{8}$$

where $\sum$ means summation of the discrete spectrum and the integration of the continuous spectrum. In view of Eq. (8), we obtain

$$G_m(R/R') = \int_{-\infty}^{\infty} dh \int_{0}^{\infty} d\lambda \sum_{V} \frac{K}{K^2 - k^2} [C_{mV} M_{^e_o V}(\lambda,h) N'_{^e_o V}(\lambda,-h) + C_{nV} N_{^e_o V}(\lambda,h) M'_{^e_o V}(\lambda,-h)]. \tag{9}$$

Since the cross section of an infinite generalized cylinder does not change along z-coordinate, the scalar wave function can be rewritten as[4]

$$\Phi_{^e_o V}(\lambda, \pm h) = \varphi_{^e_o V}(\rho,\lambda) e^{\pm ihz}, \tag{10}$$

where ρ is the coordinate of the cross section perpendicular to z-coordinate and $\varphi_{^e_o V}(\rho,\lambda)$ is the solution of the two dimensional scalar wave equation. With the aid of the relation $\frac{\partial}{\partial z}[\varphi_{^e_o V}(\rho,\lambda)] = 0$ and the vector identities

$$\nabla \times [\Phi_{^e_o V}(\lambda,h)\hat{z}] = [\nabla_t \Phi_{^e_o V}(\lambda,h)] \times \hat{z}, \tag{11}$$

$$\nabla \times \nabla \times A = \nabla\nabla \cdot A - \nabla^2 A, \tag{12}$$

the vector wave functions can be written in the form

$$M_{^e_o V}(\lambda, \pm h) = m_{^e_o V}(\rho,\lambda) e^{\pm ihz}, \tag{13a}$$

$$N_{^e_o V}(\lambda, \pm h) = n^{\pm}_{^e_o V}(\rho,\lambda) e^{\pm ihz}, \tag{13b}$$

where

$$m_{^e_o V}(\rho,\lambda) = [\nabla_t \varphi_{^e_o V}(\rho,\lambda)] \times \hat{z}, \tag{14a}$$

$$n^{\pm}_{^e_o V}(\rho,\lambda) = \frac{1}{K}[\pm \tau ih \nabla_t + \hat{z} k^2_{c,\lambda}] \varphi_{^e_o V}(\rho,\lambda). \tag{14b}$$

Substituting Eq. (13) into Eq. (9) and separating the functions of z-coordinate from Eq. (9), the integral of continuous spectrum h in Eq. (9) can be evaluated by the residual theorem[5]. Thus

$$\begin{aligned}\bar{G}_m(R/R') = \int_{0}^{\infty} d\lambda \sum_{V} \frac{\pi ik}{h_1} \{ & C_{mV}[m_{^e_o V}(\rho,\lambda) n^{-}_{^e_o V}(\rho',\lambda) e^{ih_1(z-z')} \mu(z-z') \\ & + m_{^e_o V}(\rho,\lambda) n^{+}_{^e_o V}(\rho',\lambda) e^{ih_1(z'-z)} \mu(z'-z)] \\ & + [C_{nV}[n^{+}_{^e_o V}(\rho,\lambda) m_{^e_o V}(\rho',\lambda) e^{ih_1(z-z')} \mu(z-z') \\ & + n^{-}_{^e_o V}(\rho,\lambda) m_{^e_o V}(\rho',\lambda) e^{ih_1(z'-z)} \mu(z'-z)]\},\end{aligned} \tag{15}$$

where $\mu(z-z')$ is the Heaviside's symbol

$$\mu(z-z') = \begin{cases} 1, & z \geqslant z'; \\ 0, & z < z'. \end{cases} \tag{16}$$

Eq. (15) can be rewritten in the form.

$$\bar{G}_m(R/R')=\int_0^\infty d\lambda\sum_V \frac{\pi ik}{h_1}\{C_{mV}[M_{\substack{e\\o}V}(\lambda,h_1)N'_{\substack{e\\o}V}(\lambda,-h_1)\mu(z-z')$$
$$+M_{\substack{e\\o}V}(\lambda,-h_1)N_{\substack{e\\o}V}(\lambda,h_1)\mu(z'-z)]$$
$$+[C_{nV}[N_{\substack{e\\o}V}(\lambda,h_1)M'_{\substack{e\\o}V}(\lambda-h_1)\mu(z-z')$$
$$+N_{\substack{e\\o}V}(\lambda,-h_1)M'_{\substack{e\\o}V}(\lambda,h_1)\mu(z'-z)]\}. \tag{17}$$

The key problem involved in determining the electric dyadic Green's function is how to deal with the additional term. The additional term is usually determined by the method of $\bar{G}_m(R/R')$[2]. In the following, the additional term is derived with the method given in [1]. The incomplete solution $\bar{G}_{eo}(R/R')$ is expressed first in terms of the incomplete dyadic basis MM' and NN', and then the complete solution for the electric dyadic Green's function can be represented as

$$\bar{G}_e(R/R')=\bar{G}_{eo}(R/R')+\bar{G}_{eL}(R/R'), \tag{18}$$

where $\bar{G}_{eL}(R/R')$ is the additional term to be solved. $\bar{G}_{eL}(R/R')$ can be determined by the identities in the theory of distributions and the dyadic version of Maxwell's equation

$$\nabla'\times\bar{G}_e(R/R')=\bar{G}_m(R/R'). \tag{19}$$

The generalized function $\bar{I}\delta(R/R')$ is expanded in terms of incomplete dyadic basis MM' and NN'

$$\bar{I}\delta(R-R')=\int_{-\infty}^\infty dh\int_0^\infty d\lambda\sum_V[C_{mV}M_{\substack{e\\o}V}(\lambda,h)M'_{\substack{e\\o}V}(\lambda,-h)$$
$$+C_{nV}N_{\substack{e\\o}V}(\lambda,h)N'_{\substack{e\\o}V}(\lambda,-h)]. \tag{20}$$

In view of Eq. (8), we obtain

$$\bar{G}_{eo}(R/R')=\int_{-\infty}^\infty dh\int_0^\infty d\lambda\sum_V\frac{1}{K^2-k^2}[C_{mV}M_{\substack{e\\o}V}(\lambda,h)M'_{\substack{e\\o}V}(\lambda,-h)$$
$$+C_{nV}N_{\substack{e\\o}V}(\lambda,h)N'_{\substack{e\\o}V}(\lambda,-h)]. \tag{21}$$

Substituting Eq. (13) into Eq. (21) and eliminating the integral of continuous spectrum h in Eq. (21), we obtain

$$\bar{G}_{eo}(R/R')=\int_0^\infty d\lambda\sum_V\frac{\pi i}{h_1}\{C_{mV}[m_{\substack{e\\o}V}(\rho,\lambda)n^-_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z-z')}\mu(z-z')$$
$$+m_{\substack{e\\o}V}(\rho,\lambda)n^+_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z'-z)}\mu(z'-z)]$$
$$+[C_{nV}[n^+_{\substack{e\\o}V}(\rho,\lambda)m_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z-z')}\mu(z-z')$$
$$+n^-_{\substack{e\\o}V}(\rho,\lambda)m_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z'-z)}\mu(z'-z)]\}. \tag{22}$$

Taking the curl of Eq. (22) yields

$$\nabla'\times\bar{G}_{eo}(R/R')=\int_0^\infty d\lambda\sum_V\frac{\pi i}{h_1}\{kC_{mV}[m_{\substack{e\\o}V}(\rho,\lambda)n^-_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z-z')}\mu(z-z')$$
$$+m_{\substack{e\\o}V}(\rho,\lambda)n^+_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z'-z)}\mu(z'-z)]$$
$$+kC_{nV}[n^+_{\substack{e\\o}V}(\rho,\lambda)m_{\substack{e\\o}V}(\rho',\lambda)e^{ih_1(z-z')}\mu(z-z')$$

$$+\boldsymbol{n}^{-}_{\substack{e\\o}\nu}(\rho,\lambda)\boldsymbol{m}_{\substack{e\\o}\nu}(\rho',\lambda)e^{ih_1(z'-z)}\mu(z'-z)]$$
$$+C_{n\nu}(h_1^2/k^2)\nabla_t\varphi_{\substack{e\\o}\nu}(\rho,\lambda)\hat{z}\times\nabla_t\varphi_{\substack{e\\o}\nu}(\rho',\lambda)$$
$$\times[-e^{ih_1(z-z')}\delta(z-z')+e^{ih_1(z'-z)}\delta(z'-z)]$$
$$+C_{n\nu}(ih_1)\left(\frac{k^2_{\nu,\lambda}}{k^2}\right)\varphi_{\substack{e\\o}\nu}(\rho,\lambda)\hat{z}$$
$$\times\nabla_t\varphi_{\substack{e\\o}\nu}(\rho',\lambda)[e^{ih_1(z-z')}\delta(z-z')+e^{ih_1(z'-z)}\delta(z'-z)]$$
$$+C_{m\nu}\boldsymbol{m}_{\substack{e\\o}\nu}(\rho,\lambda)\hat{z}\times\boldsymbol{m}_{\substack{e\\o}\nu}(\rho',\lambda)[-e^{ih_1(z-z')}$$
$$\times\delta(z-z')+e^{ih_1(z'-z)}\delta(z'-z)]\}. \tag{23}$$

With the aid of the dyadic equation (Eq. (19)) and the identities in the sense of distributions

$$-e^{ih_1(z-z')}\delta(z-z')+e^{ih_1(z'-z)}\delta(z'-z)=0, \tag{24a}$$
$$e^{ih_1(z-z')}\delta(z-z')+e^{ih_1(z'-z)}\delta(z'-z)=2\delta(z-z'), \tag{24b}$$

we obtain the additional term to be solved.

$$\bar{G}_{eL}(R/R')=-\int_0^\infty d\lambda\sum_{\nu}C_{n\nu}\left(\frac{2\pi K^2_{\nu,\lambda}}{k^2}\right)\varphi_{\substack{e\\o}\nu}(\rho,\lambda)\varphi_{\substack{e\\o}\nu}(\rho',\lambda)\delta(z-z')\hat{z}\hat{z}. \tag{25}$$

The electric dyadic Green's function for generalized cylinders in free space is found readily by substituting Eqs. (22) and (25) into Eq. (18). These expressions are fairly general and can be applied to obtain explicit expressions for special geometries in cylindrical coordinate systems in free space. A procedure for evaluating $\bar{G}_e(R/R')$ and $\bar{G}_m(R/R')$ for some typical geometries is to be described in the next section.

III. Applications to Some Typical Geometries in Cylindrical Coordinate Systems in Free Space

The dyadic Green's functions for circular cylinders are given first by using the above results and reduced to those obtained in [2] with the technique in [1]. The dyadic Green's functions for conducting wedges, half-plates and elliptic cylinders, whose integral of continuous spectrum h has been eliminated, are derived next.

1. $\bar{G}_e(R/R')$ and $\bar{G}_m(R/R')$ for circular cylinders

The scalar wave functions for circular cylinders are

$$\Phi_{\substack{e\\o}\nu}(\lambda,h)=J_n(\lambda r)\begin{matrix}\cos\\ \sin\end{matrix}n\phi e^{ihz}. \tag{26}$$

Substituting the vector wave functions for circular cylinders obtained by Eq. (2) into Eqs. (17) and (22), the magnetic dyadic Green's function $\bar{G}_m(R/R')$ and the regular term $\bar{G}_{e0}(R/R')$ in the electric dyadic Green's functions, which are similar to those obtained by Tai[2], are given. The additional term obtained by Eq. (25) is

$$\bar{G}_{eL}(R/R')=-\int_0^\infty d\lambda\sum_n\left(\frac{2-\delta_0}{4\pi^2\lambda}\right)\left(\frac{2\pi\lambda^2}{k^2}\right)J_n(\lambda r)\begin{matrix}\cos\\ \sin\end{matrix}n\phi J_n(\lambda r')\begin{matrix}\cos\\ \sin\end{matrix}n\phi'\delta(z-z')\hat{z}\hat{z}, \tag{27}$$

which is the expansion of the eigenfunctions of δ function. With the aid of the identities in the sense of distributions

$$\frac{\delta(r-r')\delta(\phi-\phi')}{r}=\int_0^{\infty} J_n(\lambda r)J_n(\lambda r')\lambda d\lambda \sum_n\left(\frac{2-\delta_0}{2\pi}\right)\frac{\cos}{\sin}n\phi\frac{\cos}{\sin}n\phi', \tag{28}$$

we obtain the same expression as that given in [2], that is,

$$\bar{G}_{eL}(R/R') = -\frac{1}{k^2}\frac{\delta(r-r')\delta(\phi-\phi')\delta(z-z')}{r}\hat{z}\hat{z}. \tag{29}$$

2. $\bar{G}_e(R/R')$ and $\bar{G}_m(R/R')$ for conducting wedges and half-plates

The geometry of this problem is illustrated in Fig. 2.

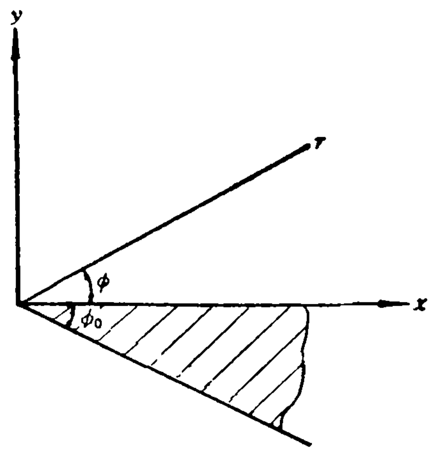

Fig. 2 The cross-section of wedges

The scalar wave functions for wedges are

$$\Phi_{\substack{e\\o}V}(\lambda,h)=J_V(\lambda r)\frac{\cos}{\sin}V\phi e^{ihz}, \tag{30}$$

where $V=n/(2-\varphi_0/\pi)$, $n=1, 2, \ldots$; φ_0 is the angle of wedges. When $\varphi_0=0$, a wedge reduces to a half-plate. The eigenvalue equation is

$$k^2=h_1^2+\lambda^2.$$

The normalized constants are

$$C_{mV}=C_{nV}=\frac{2-\delta_0}{2\pi(2\pi-\varphi_0)\lambda}.$$

Substituting Eq. (30) into Eq. (2), we obtain the vector wave functions

$$M_{\substack{e\\o}V}(\lambda,h_1)=\left[\mp\frac{VJ_V(\lambda r)}{r}\frac{\sin}{\cos}V\phi\hat{r}-\frac{\partial J_V(\lambda r)}{\partial r}\frac{\cos}{\sin}V\phi\hat{\phi}\right]e^{ih_1z}, \tag{31a}$$

$$N_{\substack{e\\o}V}(\lambda,h_1)=\frac{1}{k}\left[ih_1\frac{\partial J_V(\lambda r)}{\partial r}\frac{\cos}{\sin}V\phi\hat{r}\mp\frac{ih_1V}{r}J_V(\lambda r)\frac{\sin}{\cos}V\phi\hat{\phi}\right.$$
$$\left.+\lambda^2J_V(\lambda r)\frac{\cos}{\sin}V\phi\hat{z}\right]e^{ih_1z}. \tag{31b}$$

Determining the subscript e or o for the vector wave functions by employing the boundary conditions and substituting these functions into Eqs. (15), (22) and (25), we obtain the magnetic dyadic Green's function

$$\bar{G}_m(R/R')=\int_0^\infty d\lambda\sum_V\left(\frac{\pi ik}{h_1}\right)\left(\frac{2-\delta_0}{2\pi(2\pi-\varphi_0)\lambda}\right)\{[M_{oV}(\lambda,h_1)N'_{oV}(\lambda,-h_1)\,\mu(z-z')$$
$$+M_{oV}(\lambda,-h_1)N'_{oV}(\lambda,h_1)\mu(z'+z)$$
$$+[N_{eV}(\lambda,h_1)M'_{oV}(\lambda,-h_1)\mu(z-z')]$$
$$+N_{eV}(\lambda,-h_1)M'_{eV}(\lambda,h_1)\mu(z'-z)]\}\tag{32}$$

and the electric dyadic Green's function conducting wedges and half-plates

$$\bar{G}_e(R/R')=\int_0^\infty d\lambda\sum_V\frac{\pi i}{h_1}\left(\frac{2-\delta_0}{2\pi(2\pi-\varphi_0)\lambda}\right)\{[M_{eV}(\lambda,h_1)M'_{eV}(\lambda,-h_1)\mu(z-z')$$
$$+M_{eV}(\lambda,-h_1)M'_{eV}(\lambda,h_1)\mu(z-z')]$$
$$+[N_{oV}(\lambda,h_1)N'_{oV}(\lambda,-h_1)\mu(z-z')$$
$$+N_{oV}(\lambda,-h_1)N'_{oV}(\lambda,h_1)\mu(z'-z)]\}+\bar{G}_{eL}(R/R'),\tag{33}$$

where

$$\bar{G}_{eL}(R/R')=-\int_0^\infty d\lambda\sum_V\left(\frac{2-\delta_0}{2\pi(2\pi-\varphi_0)\lambda}\right)\left(\frac{2\pi\lambda^2}{k^2}\right)J_V(\lambda r)\sin V\phi J_V(\lambda r')$$
$$\times\sin V\phi'\delta(z-z')\hat{z}\hat{z}.\tag{34}$$

By means of the identities in the sense of distributions

$$\frac{\delta(r-r')\delta(\phi-\phi')}{r}=\int_0^\infty J_V(\lambda r)J_V(\lambda r')\lambda d\lambda\sum_V\frac{2-\delta_0}{(2\pi-\varphi_0)}\sin V\phi\sin V\phi',\tag{35}$$

Eq. (33) can be written in the form

$$\bar{G}_{eL}(R/R')=-\frac{1}{k^2}\frac{\delta(r-r')\delta(\phi-\phi')\delta(z-z')}{r}\hat{z}\hat{z}.\tag{36}$$

3. $\bar{G}_e(R/R')$ and $\bar{G}_m(R/R')$ for elliptic cylinders

The geometry of this problem is illustrated in Fig. 3.

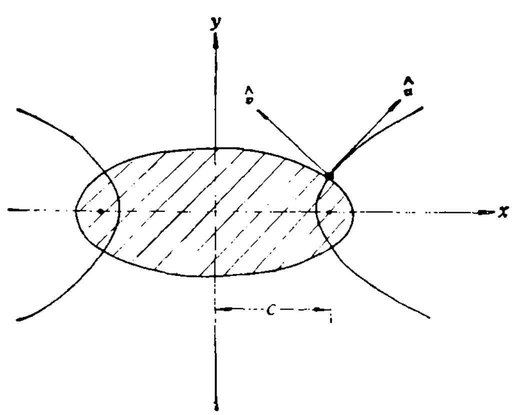

Fig. 3 The cross-section of elliptic cylinders

The scalar wave function for elliptic cylinders is

$$\Phi_{\substack{e\\o}m\lambda}(h)=S_{\substack{e\\o}m\lambda}(v)R_{\substack{e\\o}m\lambda}(u)e^{ihz}, \tag{37}$$

where $S_{\substack{e\\o}m\lambda}(v)$ and $R_{\substack{e\\o}m\lambda}(u)$ are the angular and radial functions of elliptic cylinders. The eigenvalue equation is

$$k^2=\lambda^2+h_1^2.$$

The normalized constants are

$$C_{mV}=C_{nV}=-\frac{1}{\pi^2\lambda I_{\substack{e\\o}m\lambda}},$$

where

$$I_{\substack{e\\o}m\lambda}=\int_0^{2\pi}S^2_{\substack{e\\o}m\lambda}(v)dv.$$

Substituting Eq. (37) into Eq. (2) yields the vector wave functions

$$M_{\substack{e\\o}m}(h_1)=\frac{1}{B}\left[R_{\substack{e\\o}m\lambda}(u)\frac{\partial S_{\substack{e\\o}m\lambda}(v)}{\partial v}\hat{u}-S_{\substack{e\\o}m\lambda}(v)\frac{\partial R_{\substack{e\\o}m\lambda}(u)}{\partial u}\hat{v}\right]e^{ih_1z}, \tag{38a}$$

$$N_{\substack{e\\o}m}(h_1)=\frac{1}{kB}\left[ih_1S_{\substack{e\\o}m\lambda}(v)\frac{\partial R_{\substack{e\\o}m\lambda}(u)}{\partial u}\hat{u}+ih_1R_{\substack{e\\o}m\lambda}(u)\frac{\partial S_{\substack{e\\o}m\lambda}(v)}{\partial v}\hat{v}\right.$$
$$\left.+B\lambda^2R_{\substack{e\\o}m\lambda}(u)S_{\substack{e\\o}m\lambda}(v)\hat{z}\right]e^{ih_1z}. \tag{38b}$$

Substituting Eq. (38) into Eqs. (15), (22) and (25), we obtain the magnetic dyadic Green's functions

$$\begin{aligned}\bar{G}_m(R/R')=\int_0^\infty d\lambda\sum_m\left(\frac{\pi ik}{h_1}\right)\left(\frac{1}{\pi^2\lambda I_{\substack{e\\o}m\lambda}}\right)\{&[M_{\substack{e\\o}m\lambda}(h_1)N'_{\substack{e\\o}m\lambda}(-h_1)\mu(z-z')\\&+M_{\substack{e\\o}m\lambda}(-h_1)N'_{\substack{e\\o}m\lambda}(h_1)\mu(z-z')]\\&+[N_{\substack{e\\o}m\lambda}(h_1)M'_{\substack{e\\o}m\lambda}(-h_1)\mu(z-z')\\&+N_{\substack{e\\o}m\lambda}(-h_1)M'_{\substack{e\\o}m}(\lambda,h_1)\mu(z'-z)]\}\end{aligned} \tag{39}$$

and the electric dyadic Green's function for the elliptic cylinders in free space

$$\begin{aligned}\bar{G}_e(R/R')=\int_0^\infty d\lambda\sum_m\left(\frac{\pi i}{h_1}\right)\left(\frac{1}{\pi^2\lambda I_{\substack{e\\o}m\lambda}}\right)\{&[M_{\substack{e\\o}m\lambda}(h_1)M'_{\substack{e\\o}m\lambda}(-h_1)\mu(z-z')\\&+M_{\substack{e\\o}m\lambda}(-h_1)M'_{\substack{e\\o}m\lambda}(h_1)\mu(z-z')]\\&+[N_{\substack{e\\o}m\lambda}(h_1)N'_{\substack{e\\o}m\lambda}(-h_1)\mu(z-z')\\&+N_{\substack{e\\o}m\lambda}(-h_1)N'_{\substack{e\\o}m\lambda}(h_1)\mu(z'-z)]\}+\bar{G}_{eL}(R/R'),\end{aligned} \tag{40}$$

where

$$\begin{aligned}\bar{G}_{eL}(R/R')=-\int_0^\infty d\lambda\sum_m&\left(\frac{1}{\pi^2\lambda I_{\substack{e\\o}m\lambda}}\right)\left(\frac{2\pi\lambda^2}{k^2}\right)S_{\substack{e\\o}m\lambda}(v)R_{\substack{e\\o}m\lambda}(u)S_{\substack{e\\o}m\lambda}(v')\\&\times R_{\substack{e\\o}m\lambda}(u')\delta(z-z')\hat{z}\hat{z}.\end{aligned} \tag{41}$$

The angular and radial functions of elliptic cylinders are expanded in terms of the series for Bessel functions[6]

$$S_{em\lambda}(v)R_{em\lambda}(u)=\sqrt{\frac{\pi}{2}}\sum_n{}'(i)^{n-m}D_n^m\cos n\phi J_n(\lambda r), \tag{42a}$$

$$S_{om\lambda}(v)\,R_{om\lambda}(u)=\sqrt{\frac{\pi}{2}}\sum_n{}'(i)^{n-m}F_n^m\sin n\phi J_n(\lambda r). \tag{42b}$$

Substituting Eq. (42) into Eq. (41) and employing Eq. (28) yields

$$G_{eL}(R/R')=-\frac{1}{k^2}\,\frac{\delta(r-r')\delta(\phi-\phi')\delta(z-z')}{r}\hat{z}\hat{z}. \tag{43}$$

δ function in circular coordinates is transformed to that in elliptic ones[8]

$$\begin{aligned}\frac{\delta(r-r')\delta(\phi-\phi')\delta(z-z')}{r}&=\frac{1}{|J|}\delta(v-v')\delta(u-u')\delta(z-z')\\&=\frac{\delta(v-v')\delta(u-u')\delta(z-z')}{c^2(\cosh^2u-\cos^2v)},\end{aligned} \tag{44}$$

where $|J|$ is the Jocobian of the transformation of coordinates. Eq. (43) can be rewritten in the form

$$G_{eL}(R/R')=-\frac{1}{k^2}\,\frac{\delta(v-v')\delta(u-u')\delta(z-z')}{c^2(\cosh^2u-\cos^2v)}\hat{z}\hat{z}. \tag{45}$$

The another result obtained here is

$$\frac{\delta(v-v')\,\delta(u-u')}{c^2(\cosh^2u-\cos^2v)}=\int_0^\infty d\lambda\sum_m\left(\frac{2\lambda}{\pi I_{\substack{e\\o}m\lambda}}\right)S_{\substack{e\\o}m\lambda}(v)R_{\substack{e\\o}m\lambda}(u)S_{\substack{e\\o}m\lambda}(v')R_{\substack{e\\o}m\lambda}(u'). \tag{46}$$

IV. Conclusion

The dyadic Green's functions for generalized cylinders in free space are derived by the spectral theory of dyadc operators. The dyadic Green's functions for conducting wedges, half-plates and elliptic cylinders, whose integral of continuous spectrum h has been eliminated, are given particularly. As the complement of the expressions, whose integral of continuous spectrum λ has been eliminated, the results obtained in this paper are useful for dealing with electromagnetic boundary value problems.

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