Datalog

Datalog

- A nonprocedural language based on Prolog
  - Describe what instead of how: specifying the information desired without giving a specific procedure of obtaining that information
  - Resemble the syntax of Prolog
- A purely declarative manner
  - Simplify writing simple queries
  - Make query optimization easier

Basic Example

- Define a view relation *v1* containing account numbers and balances for accounts at the Perryridge branch with a balance of over $700
  - *v1(A*, *B) :– account(A*, "Perryridge", *B), B* > 700
  - **for all** *A, B*

    if (*A*, "Perryridge", *B*) $\in$ *account* **and** *B* > 700

    then (*A, B*) $\in$ *v1*
- A Datalog program consists of a set of rules

Evaluation of a Datalog Program

- *v1(A, B) :– account(A*, "Perryridge", *B), B* > 700

account-number	*branch-name*	*balance*
A-101	Downtown	500
A-215	Mianus	700
A-102	Perryridge	400
A-305	Round Hill	350
A-201	Perryridge	900
A-222	Redwood	700
A-217	Perryridge	750

account-number	*balance*
A-201	900
A-217	750

Retrieving Tuples

- Retrieve the balance of account number “A-217” in the view relation *v1*

  $$? \; v1(\text{“A-217”}, B)$$

  – Answer: (A-217, 750)
- Find account number and balance of all accounts in *v1* that have a balance greater than 800

  $$? \; v1(A,B), B > 800$$

  – Answer: (A-201, 900)

A Program of Multiple Rules

- The interest rates for accounts

  interest-rate(A, 5*)* :– *account(A, N, B), B* < 10000

  interest-rate(A, 6*)* :– *account(A, N, B), B* >= 10000

- The set of tuples in a view relation is defined as the union of all the sets of tuples defined by the rules for the view relation

Negation

- Define a view relation *c* that contains the names of all customers who have a deposit but no loan at the bank

  c(N) :– depositor(N, A), **not** *is-borrower(N).*
  is-borrower(N) :–borrower (N,L)

- Using **not** *borrower (N, L)* in the first rule results in a different meaning, namely there is some loan L for which N is not a borrower
  - To prevent such confusion, we require all variables in negated "predicate" to also be present in non-negated predicates

Syntax of Datalog Rules

- Positive literal: $p(t_1, t_2 ..., t_n)$
  - p is the name of a relation with n attributes
  - Each t_i is either a constant or variable
  - Example: account(A, “Perryridge”, B)
- Negative literal: **not** $p(t_1, t_2 ..., t_n)$
- Comparison and arithmetic are treated as positive predicates
  - $X > Y$ is treated as a predicate $>(X, Y)$
  - A = B + C is treated as +(B, C, A)

Fact and Rules

- Fact $p(v_1, v_2, ..., v_n)$
  - Tuple $(v_1, v_2, ..., v_n)$ is in relation p
- Rules: $\underbrace{p(t_1, t_2, ..., t_n)}_{head}$:– $\underbrace{L_1, L_2, ..., L_m.}_{body}$
  - Each of the L_i's is a literal
  - Head – the literal $p(t_1, t_2, ..., t_n)$
  - Body – the rest of the literals
- A Datalog program is a set of rules

An Example Datalog Program

- Define interest on Perryridge accounts

```
interest(A, I) :- account(A, “Perryridge”, B),
                       interest-rate(A, R), I=B*R/100.
interest-rate(A, 5) :- account(A, N, B), B<10000.
interest-rate(A, 6) :- account(A, N, B), B>=10000.
```

Dependency of View Relations

- View relation v_1 depends directly on v_2 if v_2 is used in the expression defining v_1
  - Relation interest depends directly on relations interest-rate and account
- View relation v_1 depends indirectly on v_2 if there is a sequence of intermediate relations $v_1 = i_1, \ldots, i_n = v_2$ such that v_j depends directly on v_{j+1} for $1 \leq j < n$
  - Relation interest depends indirectly on relation account
- View relation v_1 depends on v_2 if v_1 depends directly or indirectly on v_2

Recursive Relation

- A view relation v is recursive if it depends on itself, otherwise, it is nonrecursive
- An example – defining the relation employment

  empl(X, Y) :- manager(X, Y).

  empl(X, Y) :- manager(X, Z), empl(Z, Y)

Semantics of Nonrecursive Datalog

- A ground instantiation of a rule (or simply instantiation) is the result of replacing each variable in the rule by some constant
  - Rule: *v1(A,B)* :– *account (A,*“Perryridge”, *B), B* > 700.
  - An instantiation:

    v1(“A-217”, 750) :– *account(*“A-217”, “Perryridge”, 750), 750 > 700.
- The body of rule instantiation *R’* is satisfied in a set of facts (database instance) *I* if
  - For each positive literal $q_i(v_{i,1}, ..., v_{i,ni})$ in the body of *R’, I* contains the fact $q_i(v_{i,1}, ..., v_{i,ni})$; and
  - For each negative literal **not** $q_j(v_{j,1}, ..., v_{j,nj})$ in the body of *R’, I* does not contain the fact $q_j(v_{j,1}, ..., v_{j,nj})$

Inferring Facts

- The set of facts that can be inferred from a given set of facts I using rule R as: $infer(R, I) = \{p(t_1, ..., t_n) \mid$ there is a ground instantiation R' of R where $p(t_1, ..., t_n)$ is the head of R', and the body of R' is satisfied in $I\}$
- Given a set of rules $\mathfrak{R} = \{R_1, R_2, ..., R_n\}$, define

$infer(\mathfrak{R}, I) = infer(R_1, I) \cup infer(R_2, I) \cup ... \cup infer(R_n, I)$

Example

- Rule: *v1(A,B)* :– *account (A,*“Perryridge”, *B), B* > 700

account-number	*branch-name*	*balance*
A-101	Downtown	500
A-215	Mianus	700
A-102	Perryridge	400
A-305	Round Hill	350
A-201	Perryridge	900
A-222	Redwood	700
A-217	Perryridge	750

A set of facts I

infer(R, I)

account-number	*balance*
A-201	900
A-217	750

Layer the View Relations

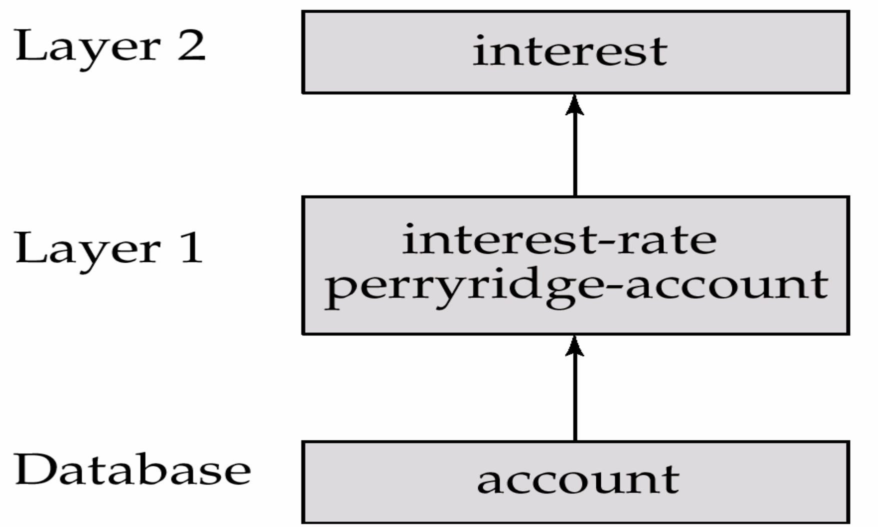
- Program

  interest(*A*, *I*) :– *perryridge-account*(*A*,*B*),
  interest-rate(*A*,*R*), *I* = *B* * *R*/100.

  perryridge-account(*A*,*B*) :–*account*(*A*, “Perryridge”, *B*).

  interest-rate(*A*,5) :–account(*N*, *A*, *B*), *B* < 10000.

  interest-rate(*A*,6) :–account(N, A, B), *B* >= 10000.



Layers

- A relation is in layer 1 if all relations used in the bodies of rules defining it are stored in the database
- A relation is in layer 2 if all relations used in the bodies of rules defining it are either stored in the database, or are in layer 1
- A relation p is in layer $i + 1$ if
  - It is not in layers 1, 2, ..., i
  - All relations used in the bodies of rules defining a p are either stored in the database, or are in layers 1, 2, ..., i

Semantics of a Program

- Let the layers in a given program be 1, 2, ..., n. Let $\Re_i$ denote the set of all rules defining view relations in layer i
- Define I_0 = the set of facts stored in the database
- Recursively define $I_{i+1} = I_i \cup infer(\Re_{i+1}, I_i)$
- The set of facts in the view relations defined by the program (also called the semantics of the program) is given by the set of facts I_n corresponding to the highest layer n

Example

- Program

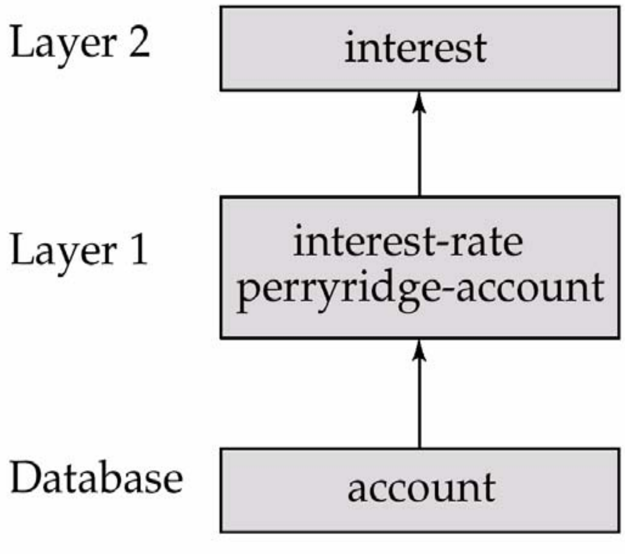
  interest(A, I) :– *perryridge-account(A,B),*
  *interest-rate(A,R), I = B * R/*100.

  perryridge-account(A,B) :–*account*(*A*, “Perryridge”, *B).*

  *interest-rate(A,*5) :–account(*N, A, B), B* < 10000.

  interest-rate(*A*,6) :–account(N, A, B), *B* >= 10000.

- I_0: account
- I_1: account, insterst-rate
- I_2: account, interst-rate, interest



Safety

- Unsafe rules – lead to infinite answers
  - *gt(X, Y)* :– *X* > *Y*
  - *not-in-loan(B, L)* :– **not** *loan(B, L)*
  - *P(A) :- q(B)*
- Safety conditions
  - Every variable that appears in the head of the rule also appears in a non-arithmetic positive literal in the body of the rule
  - Every variable appearing in a negative literal in the body of the rule also appears in some positive literal in the body of the rule
- If a nonrecursive Datalog program satisfies the safety conditions, then all the view relations defined in the program are finite

Relational Operations

- Project out attribute *account-name* from account.

$$query(A) :– account(A, N, B).$$

- Cartesian product of relations r_1 and r_2.

$$query(X_1, X_2, ..., X_n, Y_1, Y_1, Y_2, ..., Y_m) :–$$
$$r_1(X_1, X_2, ..., X_n), r_2(Y_1, Y_2, ..., Y_m).$$

- *Union of relations* r_1 and r_2.

$$query(X_1, X_2, ..., X_n) :– r_1(X_1, X_2, ..., X_n),$$
$$query(X_1, X_2, ..., X_n) :– r_2(X_1, X_2, ..., X_n),$$

- Set difference of r_1 and r_2.

$$query(X_1, X_2, ..., X_n) :– r_1(X_1, X_2, ..., X_n),$$
$$\textbf{not}\ r_2(X_1, X_2, ..., X_n)$$

Recursion

Relation schema manager(employee, manager)

empl-jones (X) :- manager (X, Jones).

empl-jones (X) :- manager (X, Y), empl-jones(Y).

employee-name	*manager-name*
Alon	Barinsky
Barinsky	Estovar
Corbin	Duarte
Duarte	Jones
Estovar	Jones
Jones	Klinger
Rensal	Klinger

Iteration number	Tuples in *empl-jones*
0	
1	(Duarte), (Estovar)
2	(Duarte), (Estovar), (Barinsky), (Corbin)
3	(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)
4	(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)

Datalog Fixpoint

- The view relations of a recursive program containing a set of rules $\mathfrak{R}$ are defined to contain exactly the set of facts *I* computed by the iterative procedure *Datalog-Fixpoint*

procedure Datalog-Fixpoint
I = set of facts in the database
repeat
$Old\_I = I$
$I = I \cup infer(\mathfrak{R}, I)$
until $I = Old\_I$

- At the end of the procedure, $infer(\mathfrak{R}, I) \subseteq I$
  - $infer(\mathfrak{R}, I) = I$ if we consider the database to be a set of facts that are part of the program
- *I* is called a fixed point of the program

Semantics of Recursion

- Fixpoint
  - Fixpoint is unique
- Transitive closure of a relation
  - *empl(X, Y) :–manager(X, Y).*
    empl(X, Y) :–manager(X, Z), empl(Z, Y)
- Another way
  - *empl(X, Y) :–manager(X, Y).*
    empl(X, Y) :–empl(X, Z), *manager(Z, Y).*
- Cannot use negation

The Power of Recursion

- Recursive views make it possible to write queries, such as transitive closure queries, that cannot be written without recursion or iteration
- Without recursion, a non-recursive non-iterative program can perform only a fixed number of joins
- Programs satisfy the safety condition will terminate
  - number(0). number(A) :- number(B), A=B+1.
  - Some programs not satisfying the safety condition do terminate

Monotonicity

- A view V is said to be monotonic if given any two sets of facts I_1 and I_2 such that $I_1 \subseteq I_2$, then $E_V(I_1) \subseteq E_V(I_2)$, where E_V is the expression used to define V
- A set of rules R is said to be monotonic if

  $$I_1 \subseteq I_2 \text{ implies } infer(\mathrm{R}, I_1) \subseteq infer(\mathrm{R}, I_2),$$
- Relational algebra views defined using only the operations: $\prod$, σ, $\times$, $\cup$, $\cap$, and ρ are monotonic
  - Relational algebra views defined using "–" may not be monotonic.
- Datalog programs without negation are monotonic, but Datalog programs with negation may not be monotonic
- Monotonic expressions can use the fixpoint technique

Summary

- Datalog: a prolog-like query language
- Using Datalog to write queries
- Semantics of Datalog programs

To-Do-List

- Examine the example queries in the relational algebra section, which ones can be rewritten in Datalog?