

# Predator Prey Model Plastics Reduced

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## 1 Microplastic as Prey

Aleksandra Samotyja, Gordon Chambers, [John S Butler](#)  
TU Dublin

## 2 Introduction

This webpage will describe a mathematical model that simulates impact of the introduction of microplastics on fish and salmon population using Python. The webpage is divided into six parts: \* The first section outlines the normal predator prey interaction first observed in hare and lynx; \* The second section will mathematically model the predator-prey relationship of salmon and small fish; \* The third section describes and models the growth of the human population and increase in the production of microplastics; \* The fourth section describes the impact of microplastics on the small fish, salmon and human population; \* The fifth section shows the results of the simulations of the ecosystem for different levels of plastic production rates; \* The final section presents some questions that arise from this problem.

The webpage is written such that it can be read by some one with no programming experience but if you want to read the code you can click on the button below.

```
[1]: import numpy as np
import math

%matplotlib inline
import matplotlib.pyplot as plt # side-stepping mpl backend
import matplotlib.gridspec as gridspec # subplots
import warnings
warnings.filterwarnings("ignore")
from __future__ import print_function
from ipywidgets import interact, interactive, fixed, interact_manual
import ipywidgets as widgets
plastic_widget1 = widgets.FloatSlider(min=0.001, max=1, step=0.01, value=0.1)
plastic_widget = widgets.FloatSlider(min=0.001, max=1, step=0.01, value=0.1)
from IPython.display import HTML

HTML('''<script>
code_show=true;
function code_toggle() {
```

```

if (code_show){
  $('div.input').hide();
} else {
  $('div.input').show();
}
code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
<form action="javascript:code_toggle()"><input type="submit" value="Click here_
↳to toggle on/off the raw code."></form>'''

```

```

File "<ipython-input-1-6c5fc4bc23b4>", line 12
    plastic_widget1 = widgets.FloatSlider(min=0.001, max=1, step=0.
↳01,value=0.1)
    ^
SyntaxError: from __future__ imports must occur at the beginning of the file

```

### 3 Predator Prey

#### 3.1 Predators and prey

The cyclical relationship of Predators and Prey in the wild means that when there is an abundance of prey, the predators will eat more and the prey population will increase. On the other hand, if there is a shortage in the number of prey, the number of predators will decrease due to the scarcity of food source [1].

#### 3.2 Fish and Salmon

In a balanced eco system the relationship between small fish (prey) and salmon (predator) can be described mathematically using a Locketta-Volterra model [2-4]

$$\begin{aligned}\frac{dF}{dt} &= F - 0.02SF, \\ \frac{dS}{dt} &= -0.05S + 0.001FS,\end{aligned}\tag{1}$$

this set of equations models the interaction between small fish (F) and salmon (S).

The equations can be written in words as:

|   |             |  |
|---|-------------|--|
| The rate of change of population                  | is equal to | the current population with a control factor   |
| The increase in fish population $\frac{dF}{dt}$   | is equal to | the current fish population $F$ minus the amount of fish salmon eat $-0.02FS$        |
| The increase in salmon population $\frac{dS}{dt}$ | is equal to | the current salmon population $-0.05S$ plus the amount of fish salmon eat $+0.001FS$ |

Or they can be summed up as the salmon are the predators who eat the small fish to survive.

Figure 2 shows a simulation of the populations of salmon and small fish from 2000 to 2100, assuming that nothing is added to environment. The salmon population fluctuates between 45 and 55 billion while the small fish population fluctuates from 25 to 85 billion. The equation illustrates a relative equilibrium between the two populations ensuring neither population gets too big or too small.

```
[ ]: # INITIAL CONDITIONS
N=10000
time = np.linspace(2000, 2100, N )           # time
h=100/N
fish=50*np.ones(N) # 60 billions small fish
salmon=45*np.ones(N) # 30 billion salmon
SMALLFISH = np.array( [1,-0.02])
SALMON= np.array([-0.05,0.001])
for i in range (1,N):
    fish[i]=fish[i-1]+h*fish[i-1]*(SMALLFISH[0]+SMALLFISH[1]*salmon[i-1])
    salmon[i]=salmon[i-1]+h*salmon[i-1]*(SALMON[0]+SALMON[1]*fish[i-1])
## PLOTTING
fig = plt.figure(figsize=(8,4))
plt.plot(time, fish, 'r-', label='small fish')
plt.plot(time, salmon, 'b-', label='salmon')
plt.grid()
plt.xlabel('time (years)')
plt.ylabel('Population (billions)')
plt.title('Figure 2: Predator Prey relationship of Salmon and small fish')
plt.legend(loc='best')
plt.tight_layout()
plt.show()
```

## 4 Humans

### 4.1 Human population growth

Humans are adaptable omnivores who have fire and tools so we are able to eat most animals and plants [5]. This means humans violate this the cyclical relationship. When humans have eaten all of one prey, such as the dodo, we just move on to another prey with seemingly little impact on human population growth. From 1800 to 1900 the human population increased by 60% from 1 billion to 1.6 billion. From 1900 to 2000 the human population has increased by almost 400% from 1.6 to 6 billion. Figure 3 shows the predicted increase in human population (yellow) in billions from 6 billion in 2000 to around 16 billion in 2100. The population should level off as number of foods sources can only sustain so many huamns. This growth can be descibed as a the equation:

$$\frac{dH}{dt} = 0.05H - 0.003H^2, \quad (2)$$

where  $H$  is human population. The equations can be written in words as:

|  |             |  |
|--|-------------|--|
| Rate of change of Population                     | is equal to | the current population and a control factor  |
| The increase in human population $\frac{dH}{dt}$ | is equal to | the current human population $0.05H$ minus a factor to control for over population $-0.003H^2$ |

It can be summed up as the human population keeps increasing until there is nothing left to eat.

```
[ ]: humans=6*np.ones(N) # 6 billion humans

for i in range (1,N):
    humans[i]=humans[i-1]+h*humans[i-1]*(0.05-0.003*humans[i-1])
fig= plt.figure(figsize=(8,4))
plt.plot(time, humans, 'y-', label = 'Humans')
plt.xlabel('time (yrs)')
plt.ylabel('Population (billions)')
plt.legend(loc='best')
plt.tight_layout()
plt.ylim(0,20)
plt.title('Figure 3: Human Population')
plt.show()
```

## 4.2 Impact of Humans

Due to the increasing human population the demand on food sources is getting higher and higher. This demand could put popular food sources at risk of being eaten into extinction. In the recent decades, governments have tried to address this by introduction quotas and identifying at risk animals. While this can address the direct impact of humans on the ecosystem it does not address the indirect by products of modern life such as microplastics.

## 4.3 MicroPlastics and Humans

Microplastics is any plastic entity which is approximately less than 5mm in size [6] (<http://www.dit.ie/physics/research/microplastics/>). The increase in microplastics can be modelled as a relationship with the human population. This can be described by the equation:

$$\frac{dP}{dt} = plastic_{rate}H. \quad (3)$$

The parameter  $plastic_{rate}$  is used to describe the relationship between the human population (H) and the rate of increase in microplastics (P) production.

|   |             |   |
|---|-------------|---|
| The rate of change of population              | is equal to | the current population with a control factor  |
| The increase in microplastics $\frac{dP}{dt}$ | is equal to | the current human population $H$ times the plastic production rate $plastic_{rate}$ |

Positive values of production rate mean the more humans, the more microplastics. Even if the production of microplastics is stopped, the amount of microplastics will remain the same. So even

after humans are gone, our plastic foot print will remain.

```
[ ]: def plastic_model(plastic_rate):
    humans=6*np.ones(N) # 30 billion salmon

    plastics=1*np.ones(N) # 1 billion microplastics
    for i in range (1,N):
        plastics[i]=plastics[i-1]+h*humans[i-1]*(plastic_rate)
        humans[i]=humans[i-1]+h*humans[i-1]*(0.05-0.003*humans[i-1])

    fig= plt.figure(figsize=(8,3))
    plt.plot(time, plastics, 'g', label = 'Microplastics')
    plt.plot(time, humans, 'y-', label = 'Humans')
    plt.xlabel('time (yrs)')
    plt.ylabel('Population')
    plt.legend(loc='best')
    plt.tight_layout()
    plt.title('Figure 4: Relationship of Human Population and Microplastics_
    ↳with a plastic rate = %s'%(plastic_rate))
    plt.show()
```

In Figure 4 the rate of microplastic production can be manipulated to illustrate the result of a change in amount of microplastics. The figure plots modelled microplastics (green) and human population as a function of time from 2000 to 2100.

```
[2]: plastic_model(0.01);
```

```
↳
↳-----

NameError                                Traceback (most recent call↳
↳last)

<ipython-input-2-d7a351ca8cc0> in <module>
----> 1 plastic_model(0.01);

NameError: name 'plastic_model' is not defined
```

## 5 Microplastics and Fish

While fishing quotas account for the direct impact of man on fish it does not account for by-products of modern life like microplastics to the ecosystem. It has been documented that fish are now consuming microplastics which can impact the population of fish.

For these reasons we consider microplastics as a predator of small fish. To model the impact of microplastics on small fish (prey) and the knock on effect it has on salmon (prey).

To model the negative impact of microplastics on the small fish population an extra term is added to the small fish equation:

$$\frac{dF}{dt} = F - 0.02SF - 0.01FP. \quad (4)$$

|  |               |  |
|--|---------------|--|
| The rate of change of population         | is equal to = | the current population with a control factor   |
| The change in small fish $\frac{dF}{dt}$ | is equal to = | the current fish population $F$ minus the amount salmon eat $-0.02SF$ minus the amount that die due to microplastics $-0.01FP$ |

## 5.1 The ecosystem model

Bringing all these equations together results in a system of four first order differential equations which describe a simple ecosystem of,  
the population of Small Fish (F),

$$\frac{dF}{dt} = F - 0.02SF - 0.01FP; \quad (5)$$

the population of salmon (S),

$$\frac{dS}{dt} = -0.05S + 0.001FS; \quad (6)$$

plastic production (P),

$$\frac{dP}{dt} = plastic_{rate}H; \quad (7)$$

and human population (H),

$$\frac{dH}{dt} = 0.0001H - 0.002H^2 + 0.0006SH + 0.001HF. \quad (8)$$

The system was modeled from 2000 to the year 2100 are shown in the Figures bellow for different plastic rates. The figures show the small fish population in red, the salmon population in blue, the plastics in green and human population in yellow.

```
[ ]: def plastic_fish_model(plastic_rate):
    # INITIAL CONDITIONS
    fish=50*np.ones(N) # 60 billions small fish
    salmon=45*np.ones(N) # 30 billion salmon
    plastics=0.001*np.ones(N) # 1 billion microplastics
    humans=6*np.ones(N) # 7 billion humans

    SMALLFISH = np.array( [1,-0.02,-0.01])
    SALMON= np.array([-0.05,0.001,0])
    for i in range (1,N):
        ↵
        ↪fish[i]=fish[i-1]+h*fish[i-1]*(SMALLFISH[0]+SMALLFISH[1]*salmon[i-1]+SMALLFISH[2]*(plastics
        ↪[i-1]))
```

```

    ↪salmon[i]=salmon[i-1]+h*salmon[i-1]*(SALMON[0]+SALMON[1]*fish[i-1]+SALMON[2]*plastics
    ↪[i-1])
        plastics[i]=plastics[i-1]+h*humans[i-1]*(plastic_rate)
        humans[i]=humans[i-1]+h*humans[i-1]*(0.05-0.003*humans[i-1])
    fig = plt.figure(figsize=(8,6))
    plt.title('Figure: Impact of plastic rate = %s on the
    ↪ecosystem'%(plastic_rate))
    plt.plot(time, fish, 'r-', label='small fish')
    plt.plot(time, salmon, 'b-', label='salmon')
    plt.plot(time, plastics, 'g', label='Microplastics')
    plt.plot(time, humans, 'y', label='Humans')
    plt.grid()
    plt.title('The impact of microplastics')
    plt.xlabel('time (years)')
    plt.ylabel('Population in Billions')
    plt.legend(loc='best')
    plt.tight_layout()
    plt.show()

```

## 6 Results

The figure below shows the output of the model for a plastic rate of 0.05. The introduction of microplastics results in a decrease in the small fish population which then impacts the salmon population as small fish are their primary food source.

```
[ ]: plastic_fish_model(0.05)
```

If the production rate of plastics is doubled to 0.1. The introduction of microplastics in to the ecosystem will lead to the extinction of small fish (red) by 2080 and salmon (blue) will be more or less extinct by 2100.

```
[ ]: plastic_fish_model(0.1)
```

## 7 Open Questions

The model shows the destructive impact of microplastics on a simple ecosystem model of small fish, salmon, microplastics and humans. Mathematical models of this type can be used to discuss

The immediate questions that arise from this simulation are: 1. What is our current rate of plastic production? 2. How will the extinction of small fish impact the human population? 3. How can we do to prevent this from happening?

## 8 References

[1] Lynx-hare fur trade.<https://theglyptodon.wordpress.com/2011/05/02/the-fur-trades-records/>. Accessed:2018-03-20.

[2] Wikipedia contributors. (2018, July 27). Lotka–Volterra equations. In Wikipedia, The Free Encyclopedia. Retrieved 09:33, September 7, 2018, from [https://en.wikipedia.org/w/index.php?title=Lotka%E2%80%93Volterra\\_equations&oldid=852191432](https://en.wikipedia.org/w/index.php?title=Lotka%E2%80%93Volterra_equations&oldid=852191432)

[3] Alfred J Lotka. Analytical note on certain rhythmic relations inorganic systems. Proceedings of the National Academy of Sciences, 6(7):410–415, 1920.

[4] Vito Volterra. Fluctuations in the abundance of a species considered mathematically. Nature, 118:558–560, 1926.

[5] Harari, Yuval N. (2015). Sapiens : a brief history of humankind. New York :Harper.

[6] Gordon Chambers, (2017) Microplastics, Retrieved 09:33, September 7, 2018, <http://www.dit.ie/physics/research/microplastics/>

[ ]: