Nonlinear Spacecraft Control

ASEN 5010

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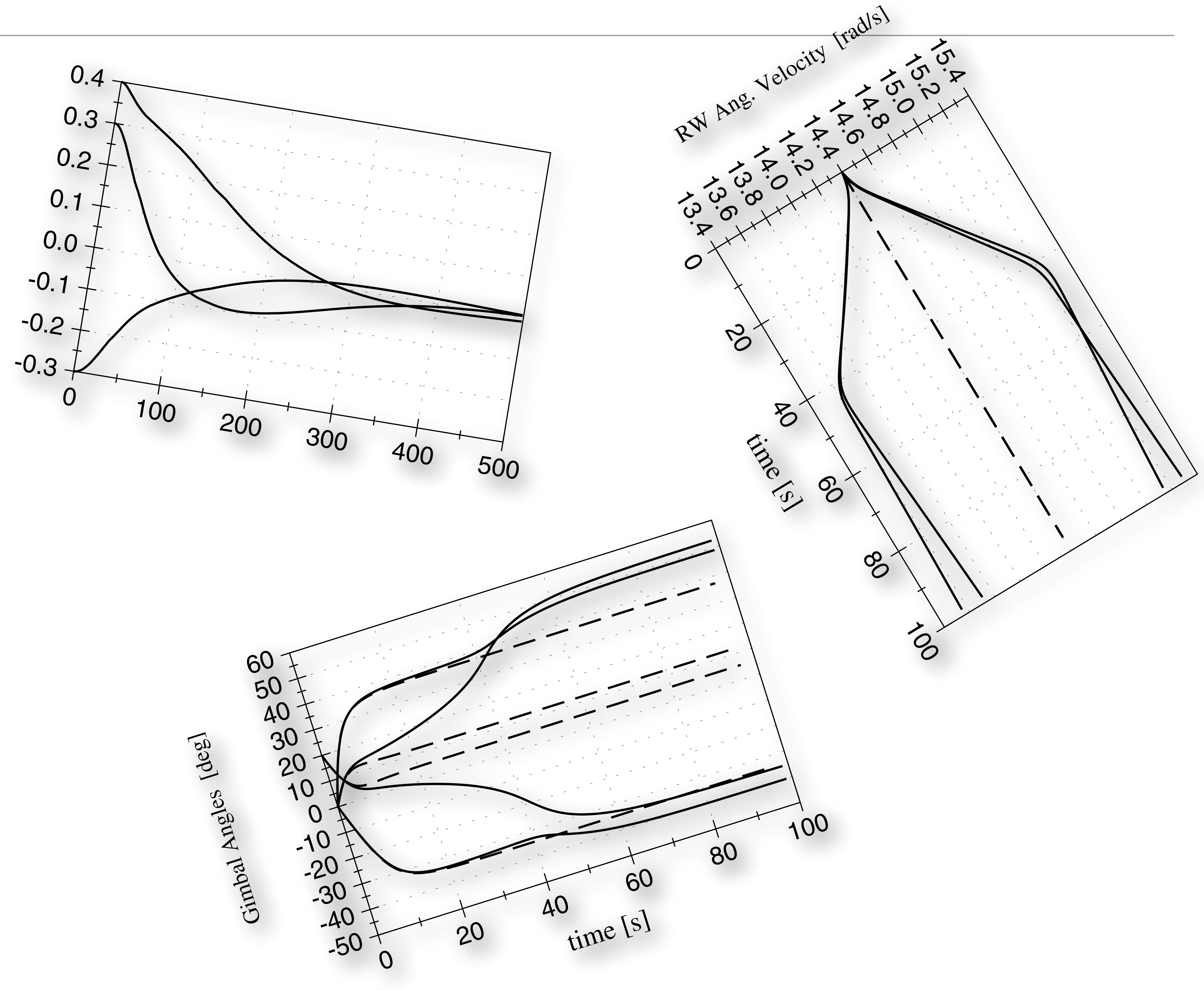
- Linear Closed-Loop Dynamics



Stability Definitions

Why isn’t stable just stable?

Definitions

State Vector: $$\boldsymbol{x} = (x_1 \cdots \; x_N)^T$$

EOM: $$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t)$$ — Non-Autonomous System

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$$ — Autonomous System

Control Vector: $$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$$

Closed-Loop System: $$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$$

Equilibrium State: A state vector point $\boldsymbol{x}_e$ is said to be an equilibrium state (or equilibrium point) of a dynamical system described by $\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x},t)$ at time t_0 if

$$\boldsymbol{f}(\boldsymbol{x}_e, t) = 0 \qquad \forall \; t > t_0$$

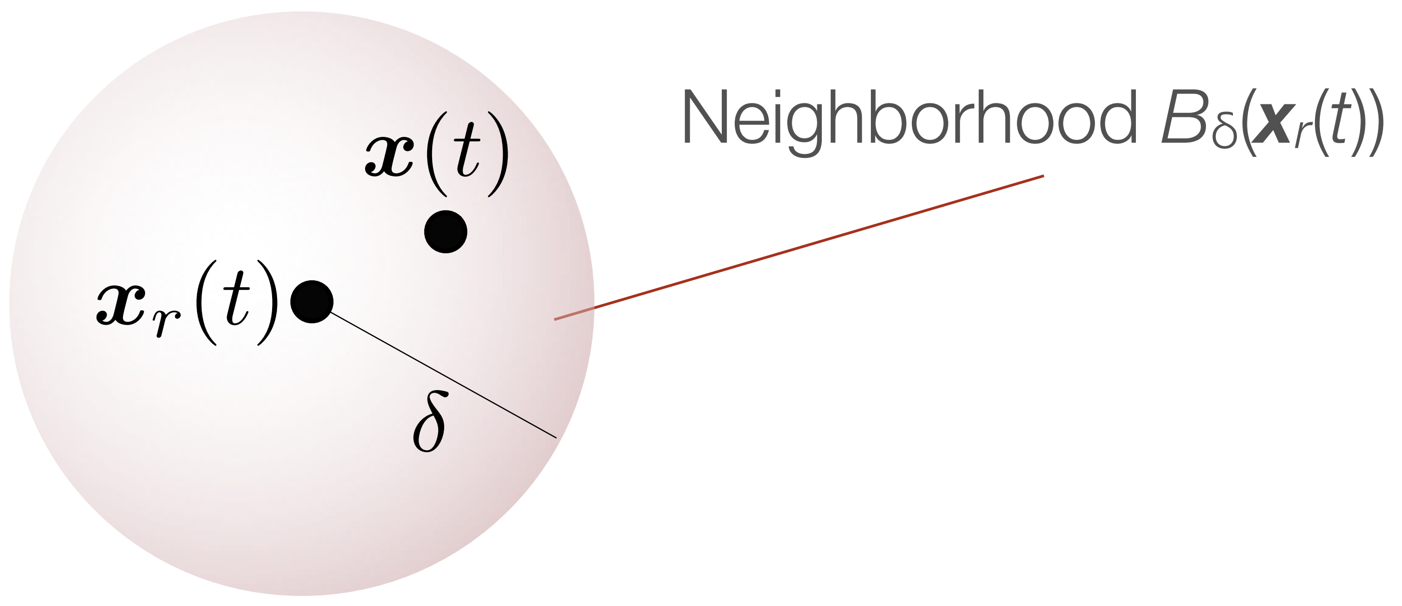
$$\dot{\boldsymbol{x}}_e = 0 \qquad \boldsymbol{x}_e = \text{constant}$$

Neighborhood: Given $\delta>0$, a state vector $\boldsymbol{x}(t)$ is said to be in the neighborhood $B_\delta(\boldsymbol{x}_r(t))$ of the state $\boldsymbol{x}_r(t)$ if

$$||\boldsymbol{x}(t) - \boldsymbol{x}_r(t)|| \; < \; \delta$$

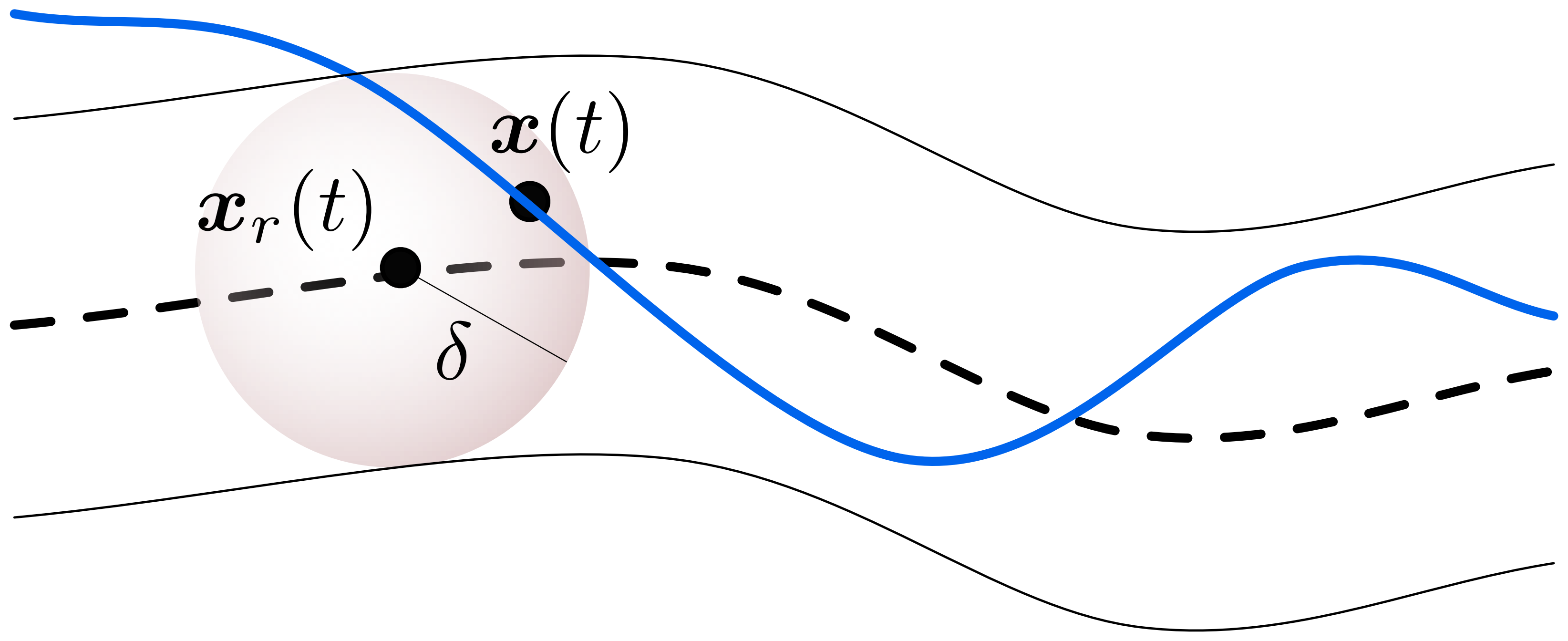
then

$$\boldsymbol{x}(t) \in B_\delta(\boldsymbol{x}_r(t))$$



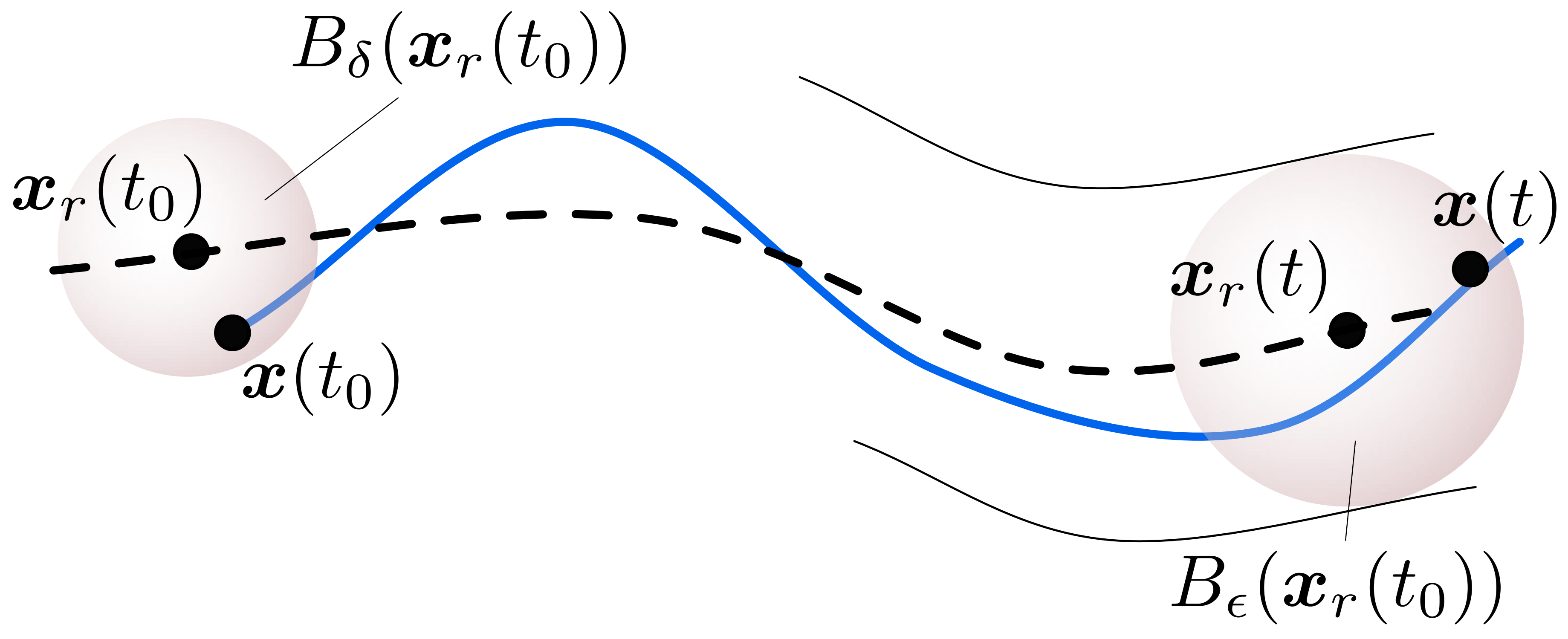
Lagrange Stability: The motion $\boldsymbol{x}(t)$ is said to be Lagrange stable (or bounded) relative to $\boldsymbol{x}_r(t)$ if there exists a $\delta>0$ such that

$$\boldsymbol{x}(t) \in B_\delta(\boldsymbol{x}_r(t)) \qquad \forall\ t > t_0$$



Lyapunov Stability: The motion $\boldsymbol{x}(t)$ is said to be Lyapunov stable (or stable) relative to $\boldsymbol{x}_r(t)$ if for each $\varepsilon>0$ there exists a $\delta(\varepsilon)>0$ such that

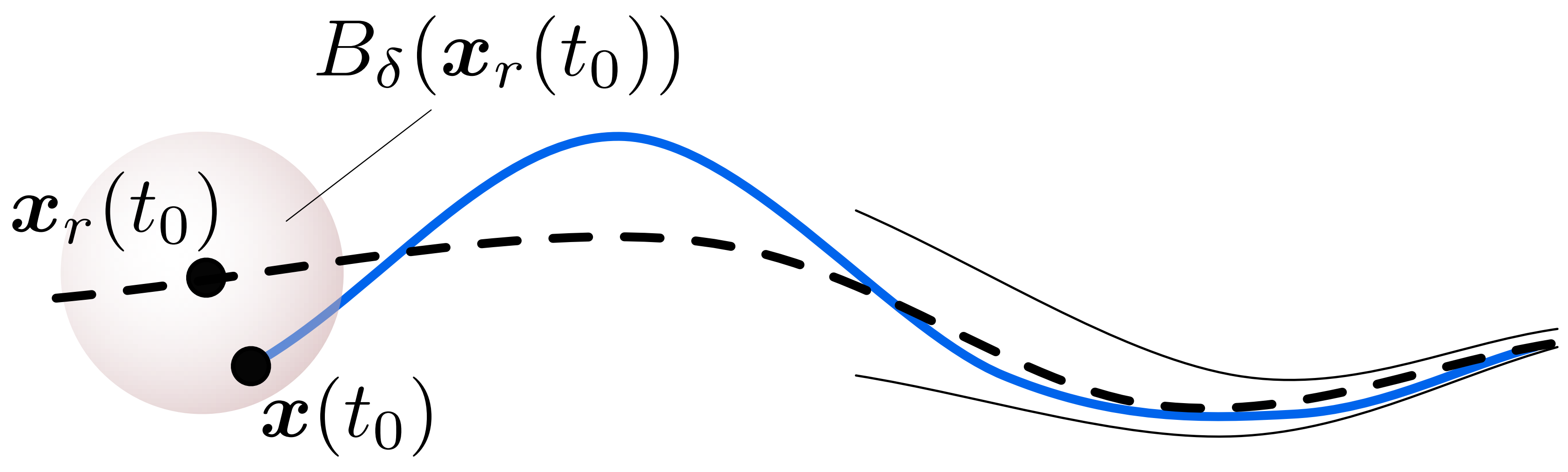
$$\boldsymbol{x}(t_0) \in B_\delta(\boldsymbol{x}_r(t_0)) \Longrightarrow \boldsymbol{x}(t) \in B_\epsilon(\boldsymbol{x}_r(t))$$
$$\forall\ t > t_0$$

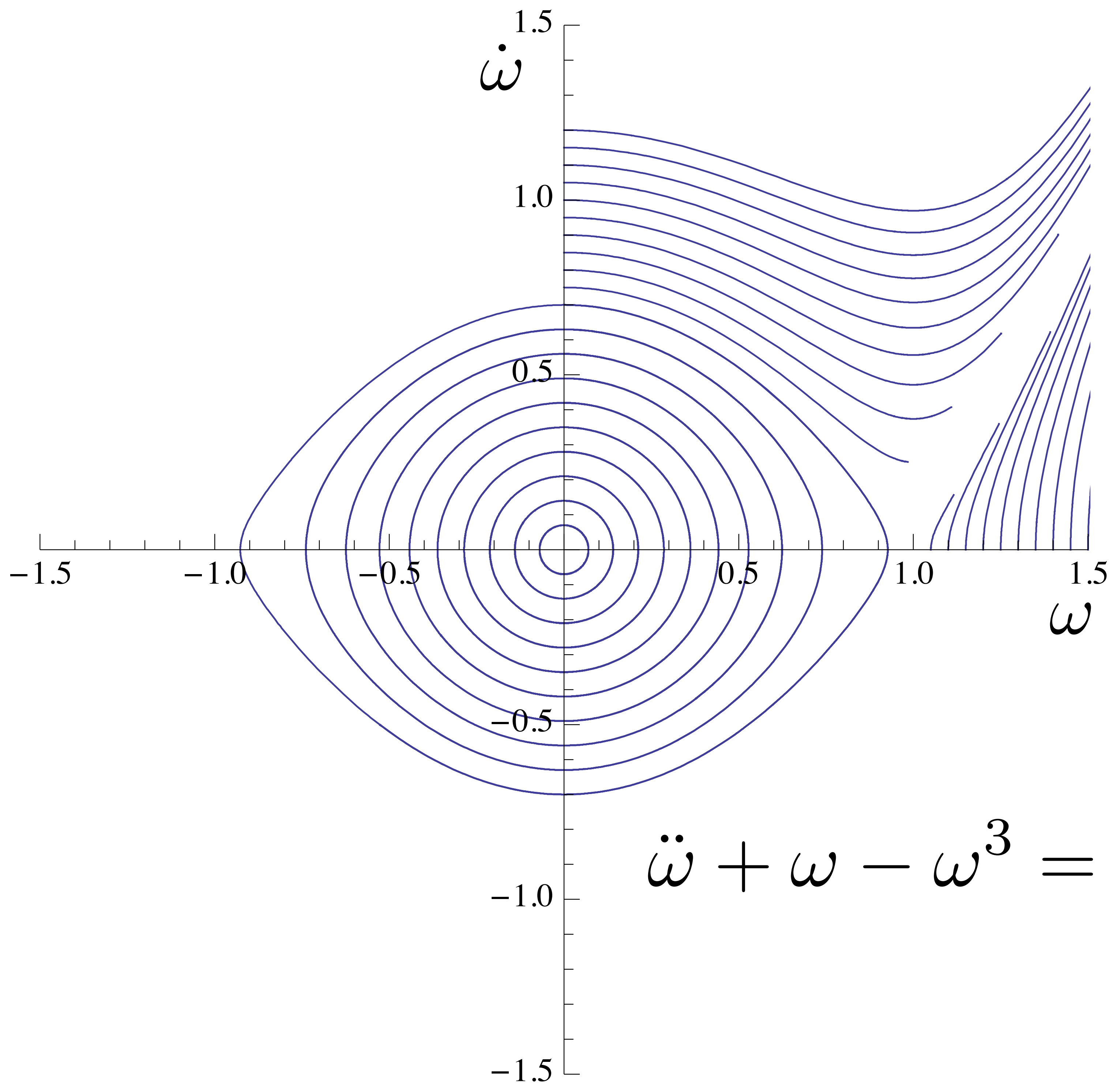


Asymptotic Stability: The motion $\boldsymbol{x}(t)$ is asymptotically stable relative to $\boldsymbol{x}_r(t)$ if $\boldsymbol{x}(t)$ is Lyapunov stable and there exists a δ>0 such that

$$\boldsymbol{x}(t_0) \in B_\delta(\boldsymbol{x}_r(t_0)) \Longrightarrow \lim_{t\to\infty} \boldsymbol{x}(t) = \boldsymbol{x}_r(t)$$

Global Stability: The motion $\boldsymbol{x}(t)$ is globally stable relative to $\boldsymbol{x}_r(t)$ if $\boldsymbol{x}(t)$ is stable for any initial state vector $\boldsymbol{x}(t_0)$.





$$\ddot{\omega} + \omega - \omega^3 = 0$$

(Show Mathematica Example)

Linearization of Dynamical System

Feedforward control

Reference motion given by: $\dot{\boldsymbol{x}}_r = \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)$

Nonlinear EOM: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$

Feedback control: $\delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}_r$

Departure motion: $\delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_r$

Performing a Taylor Series expansion of ***x*** about ($\boldsymbol{x}_r$,$\boldsymbol{u}_r$) we obtain

$$\delta\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r) + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}}\delta\boldsymbol{x} + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}}\delta\boldsymbol{u} + H.O.T - \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)$$

$$\delta\dot{\boldsymbol{x}} \simeq \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}}\delta\boldsymbol{x} + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}}\delta\boldsymbol{u}$$

Let us define:

$$[A] = \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}}$$

$$[B] = \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}}$$

The linearized system is then written in standard form as

$$\boxed{\delta\dot{\boldsymbol{x}} \simeq [A]\delta\boldsymbol{x} + [B]\delta\boldsymbol{u}}$$

If the nominal reference motion is an equilibrium state $\boldsymbol{x}_e$, then the linearized EOM simplify to:

$$\dot{\boldsymbol{x}} \simeq [A]\boldsymbol{x} + [B]\boldsymbol{u}$$