# Continuous System

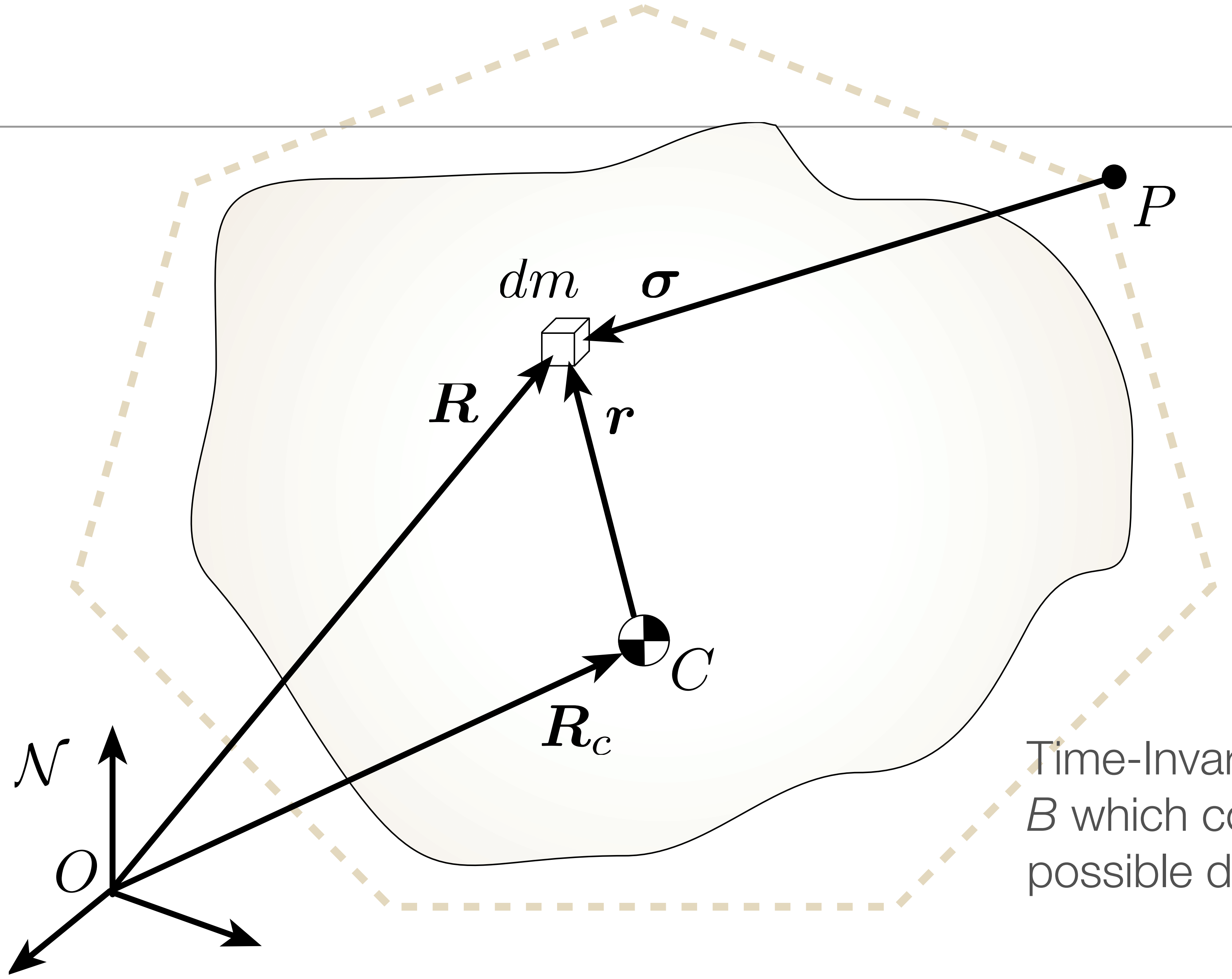
What does jello look like in space?

# Equations of Motion

Newton's Law: $\mathrm{d}\boldsymbol{F} = \ddot{\boldsymbol{R}}\mathrm{d}m$

Force Vector: $\mathrm{d}\boldsymbol{F} = \mathrm{d}\boldsymbol{F}_E + \mathrm{d}\boldsymbol{F}_I$

Total Force acting on System: $\boldsymbol{F} = \int_B \mathrm{d}\boldsymbol{F} = \int_B \mathrm{d}\boldsymbol{F}_E$




Center of Mass: $M\boldsymbol{R}_c = \int_B \boldsymbol{R}\mathrm{d}m = \int_{\mathcal{B}} (\boldsymbol{R}_c + \boldsymbol{r})\mathrm{d}m \Rightarrow \int_{\mathcal{B}} \boldsymbol{r}\mathrm{d}m = \boldsymbol{0}$

$$M\ddot{\boldsymbol{R}}_c = \int_B \ddot{\boldsymbol{R}}\mathrm{d}m = \int_B \mathrm{d}\boldsymbol{F} \Rightarrow \boxed{M\ddot{\boldsymbol{R}}_c = \boldsymbol{F}}$$

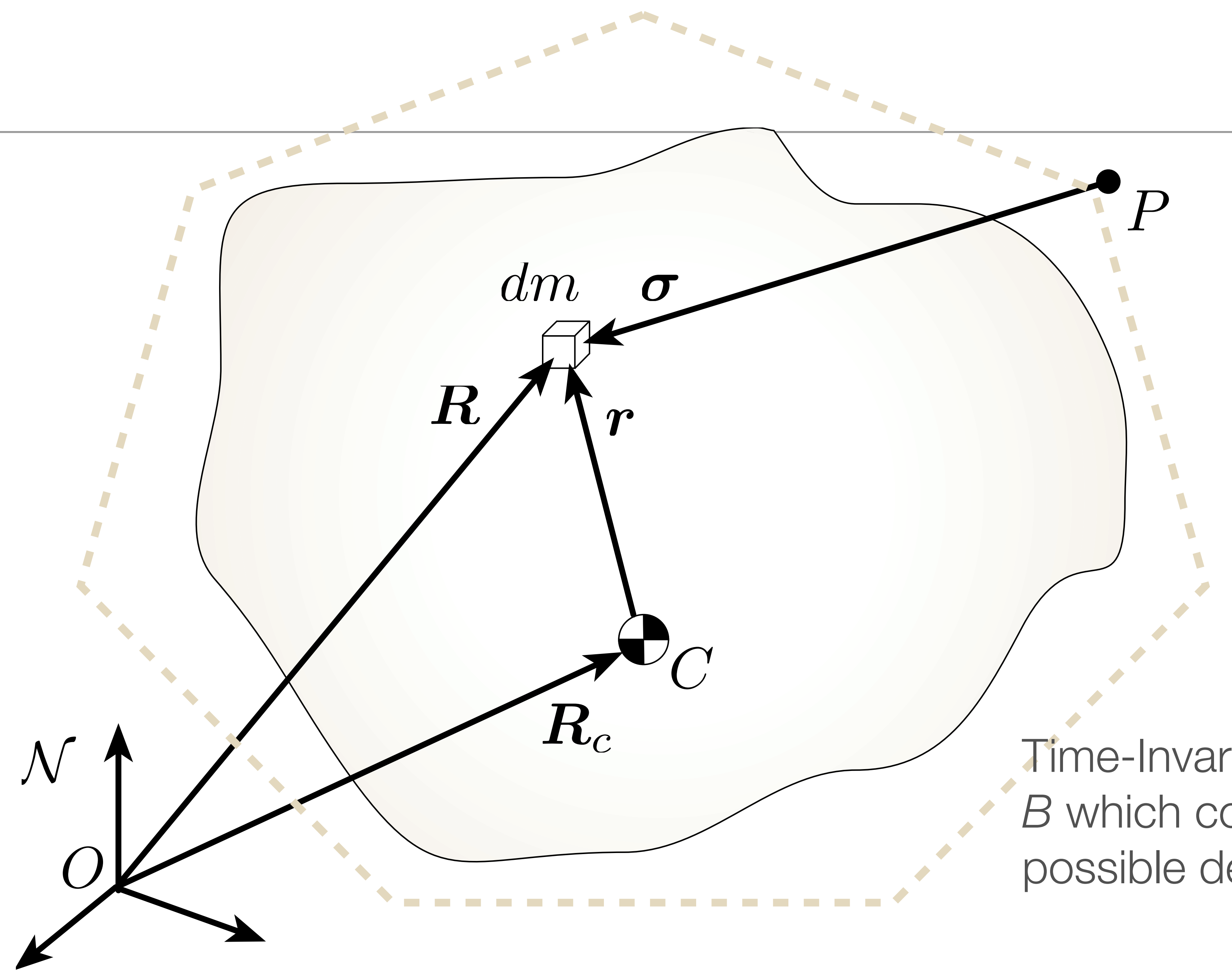
Super Particle Theorem

# Kinetic Energy

Definition: $$T = \frac{1}{2}\int_B \dot{\boldsymbol{R}} \cdot \dot{\boldsymbol{R}} \mathrm{d}m$$

$$\dot{\boldsymbol{R}} = \dot{\boldsymbol{R}}_c + \dot{\boldsymbol{r}}$$

$$T = \frac{1}{2}\left(\int_B \mathrm{d}m\right)\dot{\boldsymbol{R}}_c \cdot \dot{\boldsymbol{R}}_c + \dot{\boldsymbol{R}}_c \cdot \cancel{\int_B \dot{\boldsymbol{r}} \mathrm{d}m}$$
$$+\frac{1}{2}\int_B \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} \mathrm{d}m$$




Time-Invariant Boundary $B$ which contains all possible deformations.

$$\boxed{T = \frac{1}{2}M\dot{\boldsymbol{R}}_c \cdot \dot{\boldsymbol{R}}_c + \frac{1}{2}\int_B \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} \mathrm{d}m}$$

Energy of CM Energy about CM

# Work/Energy Principle

Differentiate Energy:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = M\ddot{\boldsymbol{R}}_c \cdot \dot{\boldsymbol{R}}_c + \int_B \dot{\boldsymbol{r}} \cdot \ddot{\boldsymbol{r}}\mathrm{d}m$$

$$M\ddot{\boldsymbol{R}}_c = \boldsymbol{F} \qquad \ddot{\boldsymbol{r}} = \ddot{\boldsymbol{R}} - \ddot{\boldsymbol{R}}_c$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \boldsymbol{F} \cdot \dot{\boldsymbol{R}}_c + \int_B \left(\ddot{\boldsymbol{R}}\mathrm{d}m\right) \cdot \dot{\boldsymbol{r}} - \ddot{\boldsymbol{R}}_c \cdot \underset{\text{C.M.}}{\cancel{\int_B \dot{\boldsymbol{r}}\mathrm{d}m}}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \boldsymbol{F} \cdot \dot{\boldsymbol{R}}_c + \int_B \mathrm{d}\boldsymbol{F} \cdot \dot{\boldsymbol{r}}$$

$$\boxed{T(t_2) - T(t_1) = \int_{\boldsymbol{R}(t_1)}^{\boldsymbol{R}(t_2)} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{R}_c + \int_B \int_{\boldsymbol{r}(t_1)}^{\boldsymbol{r}(t_2)} \mathrm{d}\boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}}$$

Work energy/principle for system of particles

# Linear Momentum

Definition: $$\mathrm{d}\boldsymbol{p} = \dot{\boldsymbol{R}}\mathrm{d}m$$

$$\boldsymbol{p} = \int_{\mathcal{B}} \mathrm{d}\boldsymbol{p} = \int_{\mathcal{B}} \dot{\boldsymbol{R}}\mathrm{d}m = \int_{\mathcal{B}} (\dot{\boldsymbol{R}}_c + \dot{\boldsymbol{r}})\mathrm{d}m = \left(\int_{\mathcal{B}} \mathrm{d}m\right) \dot{\boldsymbol{R}}_c + \underbrace{\cancel{\int_{\mathcal{B}} \dot{\boldsymbol{r}}\mathrm{d}m}}_{\text{C.M.}}$$

$$\boldsymbol{p} = M\dot{\boldsymbol{R}}_c$$

Linear Momentum Rate: $$\dot{\boldsymbol{p}} = \int_{\mathcal{B}} \ddot{\boldsymbol{R}}\mathrm{d}m = \int_{\mathcal{B}} \mathrm{d}\boldsymbol{F} = \boldsymbol{F}$$

$$\boldsymbol{F} = \frac{{}^{\mathcal{N}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{p})$$

# Angular Momentum

Ang. Momentum about $P$: $$\boldsymbol{H}_P = \int_B \boldsymbol{\sigma} \times \dot{\boldsymbol{\sigma}} \mathrm{d}m$$

$$\boldsymbol{\sigma} = \boldsymbol{R} - \boldsymbol{R}_P$$

Inertial Time Derivative: $$\dot{\boldsymbol{H}}_P = \cancel{\int_B \dot{\boldsymbol{\sigma}} \times \dot{\boldsymbol{\sigma}} \mathrm{d}m} + \int_B \boldsymbol{\sigma} \times \boxed{\ddot{\boldsymbol{\sigma}}} \mathrm{d}m$$

$$\dot{\boldsymbol{H}}_P = \boxed{\int_B \boldsymbol{\sigma} \times \ddot{\boldsymbol{R}} \mathrm{d}m} - \boxed{\left(\int_B \boldsymbol{\sigma} \mathrm{d}m\right)} \boxed{\times \ddot{\boldsymbol{R}}_P}$$

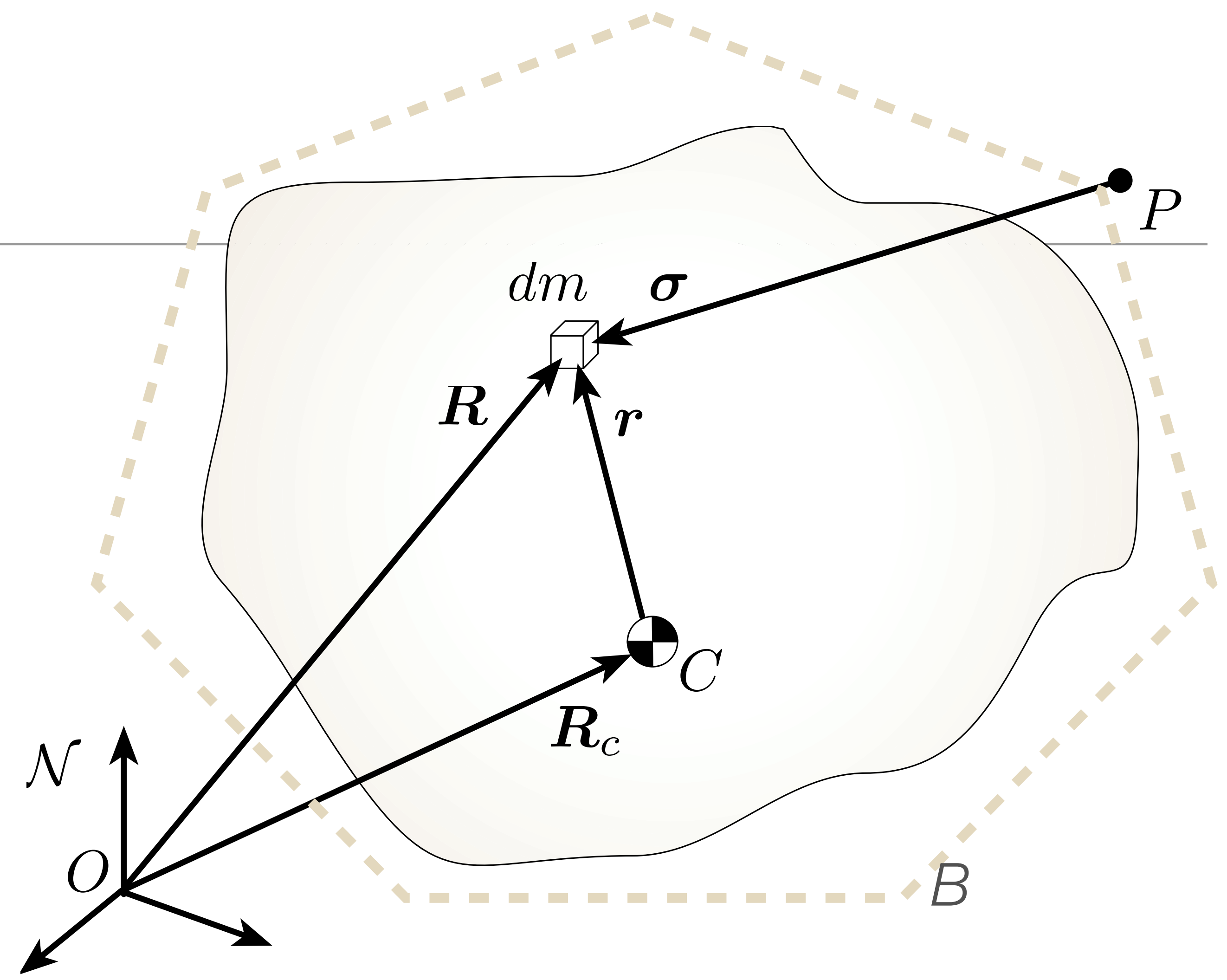
$$\int_B \boldsymbol{\sigma} \mathrm{d}m = \int_B \boldsymbol{R} \mathrm{d}m - \left(\int_B \mathrm{d}m\right) \boldsymbol{R}_P = \boxed{M(\boldsymbol{R}_c - \boldsymbol{R}_P)}$$

Torque about $P$: $$\boldsymbol{L}_P = \boxed{\int_B \boldsymbol{\sigma} \times \ddot{\boldsymbol{R}} \mathrm{d}m} = \int_B \boldsymbol{\sigma} \times \mathrm{d}\boldsymbol{F}$$

$$\dot{\boldsymbol{H}}_P = \boldsymbol{L}_P + M\ddot{\boldsymbol{R}}_P \times (\boldsymbol{R}_c - \boldsymbol{R}_P)$$

$$\boxed{\dot{\boldsymbol{H}}_P = \boldsymbol{L}_P}$$

If $P$ is CM or Inertial



# Rigid Body Dynamics

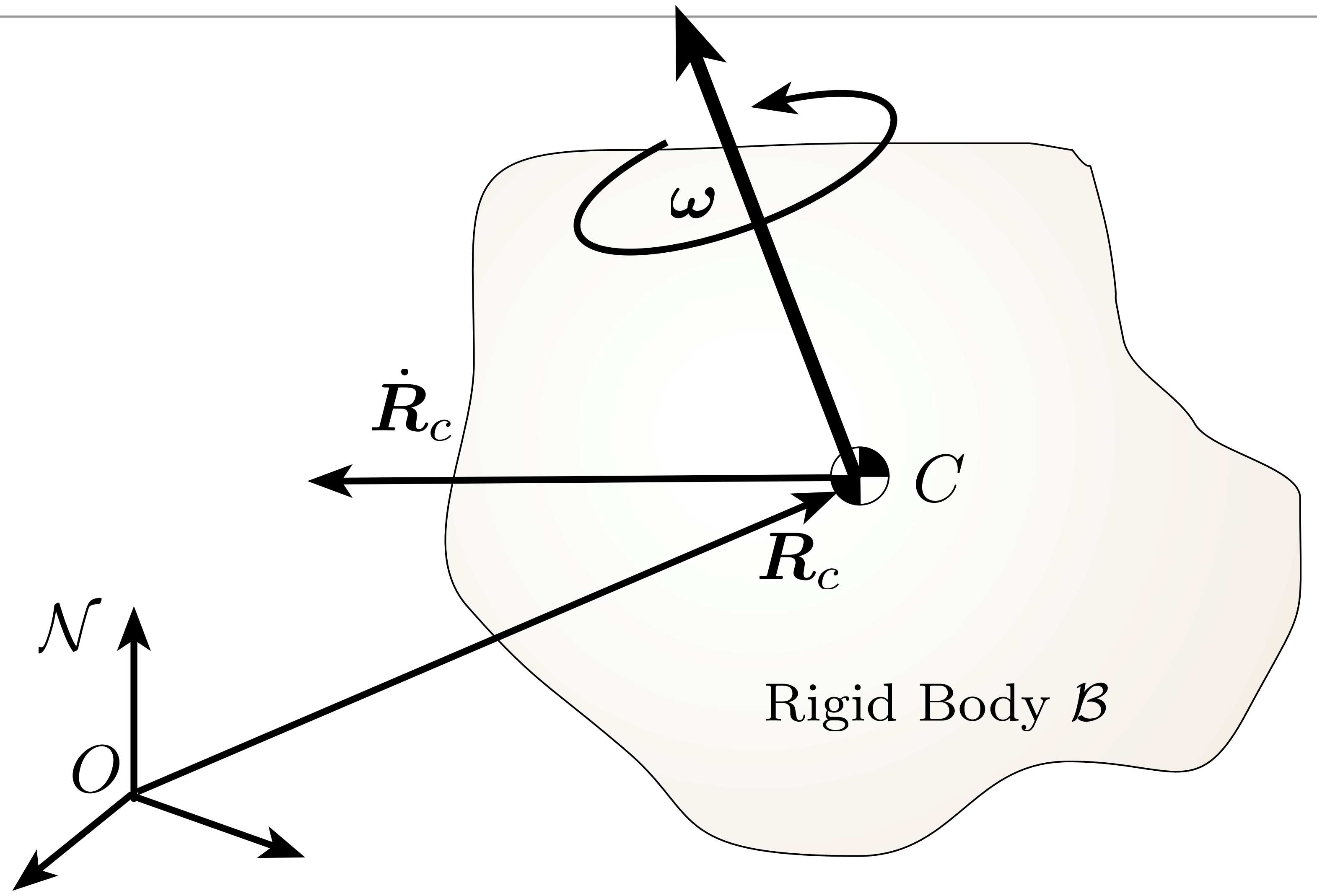
The 101 of spacecraft dynamics...

# General Angular Momentum

Definition: $$\boldsymbol{H}_O = \int_B \boldsymbol{R} \times \dot{\boldsymbol{R}} \mathrm{d}m$$

or

$$\boldsymbol{H}_O = \underbrace{\boldsymbol{R}_c \times M\dot{\boldsymbol{R}}_c}_{\text{CM}} + \int_B \boldsymbol{r} \times \dot{\boldsymbol{r}} \mathrm{d}m$$




Momentum about CM:

$$\boldsymbol{H}_c = \int_B \boldsymbol{r} \times \dot{\boldsymbol{r}} \mathrm{d}m$$

$$\dot{\boldsymbol{r}} = \frac{{}^{\mathcal{N}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \cancel{\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r})} + \boldsymbol{\omega} \times \boldsymbol{r} = \boldsymbol{\omega} \times \boldsymbol{r}$$

$$\boldsymbol{H}_c = \int_B \boldsymbol{r} \times (\boldsymbol{\omega} \times \boldsymbol{r})\, \mathrm{d}m = \left( \int_B -[\tilde{\boldsymbol{r}}][\tilde{\boldsymbol{r}}] \mathrm{d}m \right) \boldsymbol{\omega}$$

# Inertia Tensor Properties

Definition:

$$
{}^{\mathcal{B}}[I_c] = \int_B -[\tilde{\boldsymbol{r}}][\tilde{\boldsymbol{r}}]\mathrm{d}m = \int_B {}^{\mathcal{B}}\begin{bmatrix} r_2^2 + r_3^2 & -r_1 r_2 & -r_1 r_3 \\ -r_1 r_2 & r_1^2 + r_3^2 & -r_2 r_3 \\ -r_1 r_3 & -r_2 r_3 & r_1^2 + r_2^2 \end{bmatrix} \mathrm{d}m
$$

Angular Momentum Expression:

$$
\boldsymbol{H}_c = {}^{\mathcal{B}}\begin{pmatrix} H_{c_1} \\ H_{c_2} \\ H_{c_3} \end{pmatrix} = \int_B {}^{\mathcal{B}}\begin{bmatrix} r_2^2 + r_3^2 & -r_1 r_2 & -r_1 r_3 \\ -r_1 r_2 & r_1^2 + r_3^2 & -r_2 r_3 \\ -r_1 r_3 & -r_2 r_3 & r_1^2 + r_2^2 \end{bmatrix} {}^{\mathcal{B}}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \mathrm{d}m = [I_c]\boldsymbol{\omega}
$$

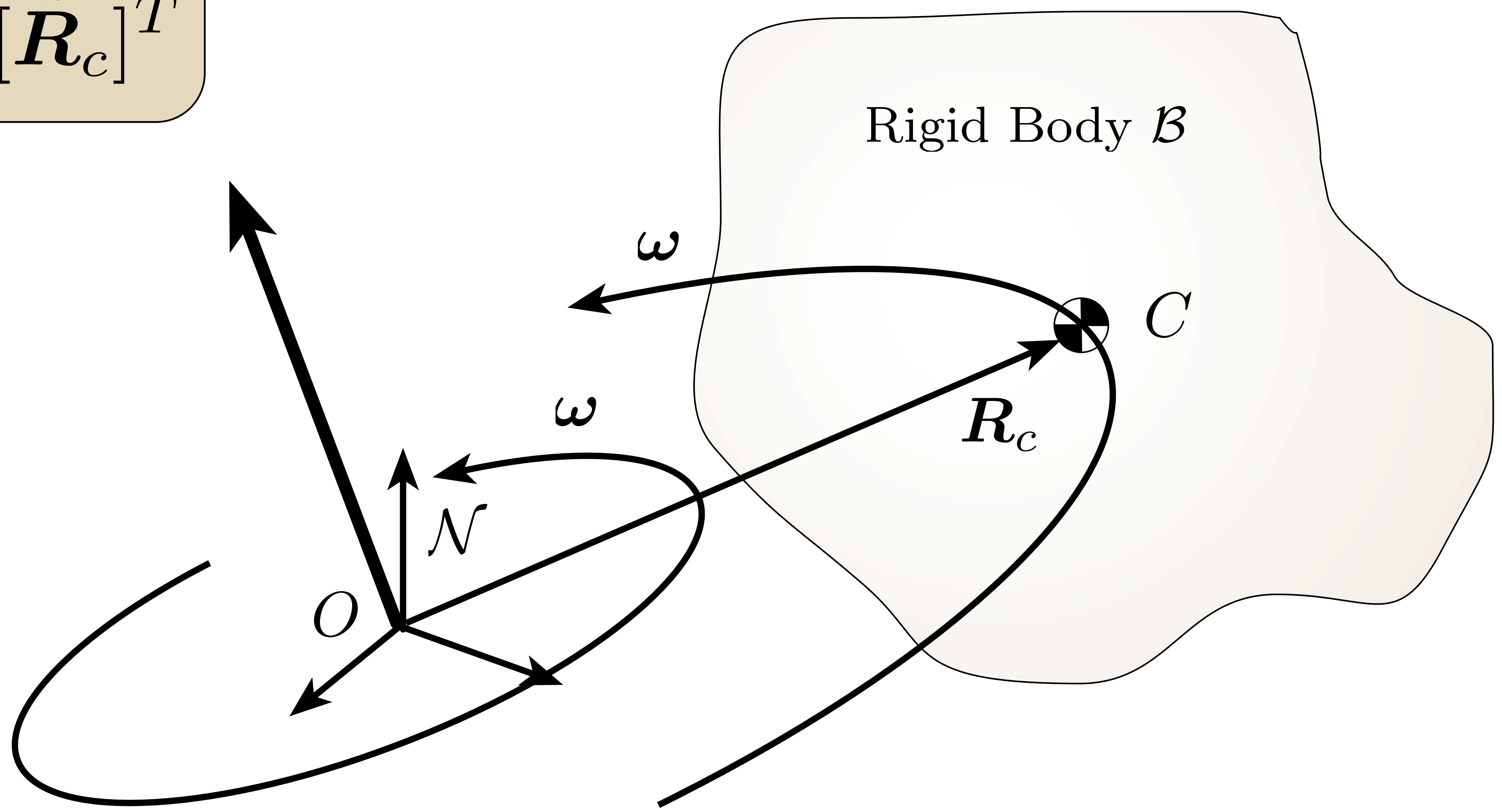
# Parallel Axis Theorem

Total Mass

CM Offset

$$[I_O] = [I_c] + M[\tilde{\boldsymbol{R}}_c][\tilde{\boldsymbol{R}}_c]^T$$

Inertia about CM



# Coordinate Transformation

$$^{\mathcal{F}}[I] = [FB]\,^{\mathcal{B}}[I]\,[FB]^T$$

*B* - Body Frame

*F* - 2nd Coordinate Frame

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} {}^{\mathcal{B}}\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} = {}^{\mathcal{F}}\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Rotation matrix [*C*] contains the eigenvectors of [*I*]

Principal inertia matrix whose diagonal entries are the eigenvalues of [*I*]

$$[C] = [V]^T = \begin{bmatrix} \boldsymbol{v}_1^T \\ \boldsymbol{v}_2^T \\ \boldsymbol{v}_3^T \end{bmatrix}$$

# Kinetic Energy

Total Energy:

$$T = \frac{1}{2} M \dot{\boldsymbol{R}}_c \cdot \dot{\boldsymbol{R}}_c + \frac{1}{2} \int_B \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} \mathrm{d}m = T_{\mathrm{trans}} + T_{\mathrm{rot}}$$

Rotational Energy:

$$T_{\mathrm{rot}} = \frac{1}{2} \int_B \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} \mathrm{d}m = \frac{1}{2} \int_B (\boldsymbol{\omega} \times \boldsymbol{r}) \cdot (\boldsymbol{\omega} \times \boldsymbol{r}) \, \mathrm{d}m$$

$$T_{\mathrm{rot}} = \frac{1}{2} \boldsymbol{\omega} \cdot \int_B \boldsymbol{r} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \, \mathrm{d}m = \frac{1}{2} \boldsymbol{\omega} \cdot \boldsymbol{H}_c = \frac{1}{2} \boldsymbol{\omega}^T [I] \boldsymbol{\omega}$$

Energy Rate:

$$\dot{T} = \boldsymbol{F} \cdot \dot{\boldsymbol{R}}_c + \boldsymbol{L}_c \cdot \boldsymbol{\omega}$$

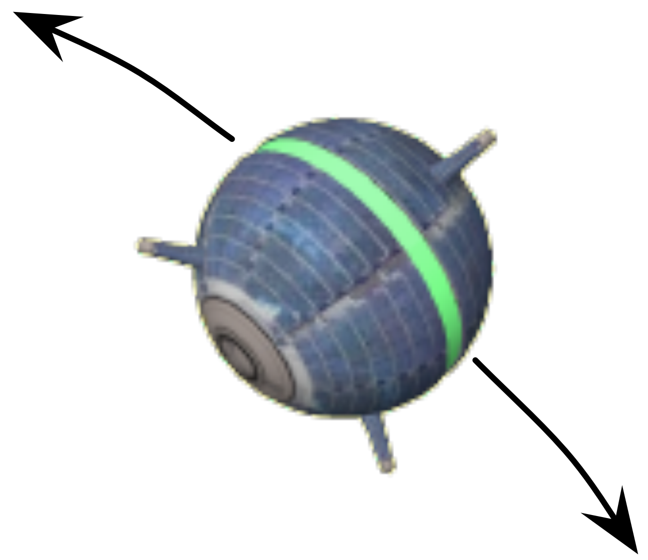
Work/Energy Principle:

$$W = T(t_2) - T(t_1) = \int_{t_1}^{t_2} \boldsymbol{F} \cdot \dot{\boldsymbol{R}}_c \mathrm{d}t + \int_{t_1}^{t_2} \boldsymbol{L}_c \cdot \boldsymbol{\omega} \mathrm{d}t$$

# Equations of Motion

Euler's equation: $$\dot{\boldsymbol{H}}_c = \boxed{\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{H}_c)} + \boldsymbol{\omega} \times \boldsymbol{H}_c = \boldsymbol{L}_c$$

$$\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{H}_c) = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}([I])\,\boldsymbol{\omega} + [I]\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\omega}) = [I]\dot{\boldsymbol{\omega}}$$

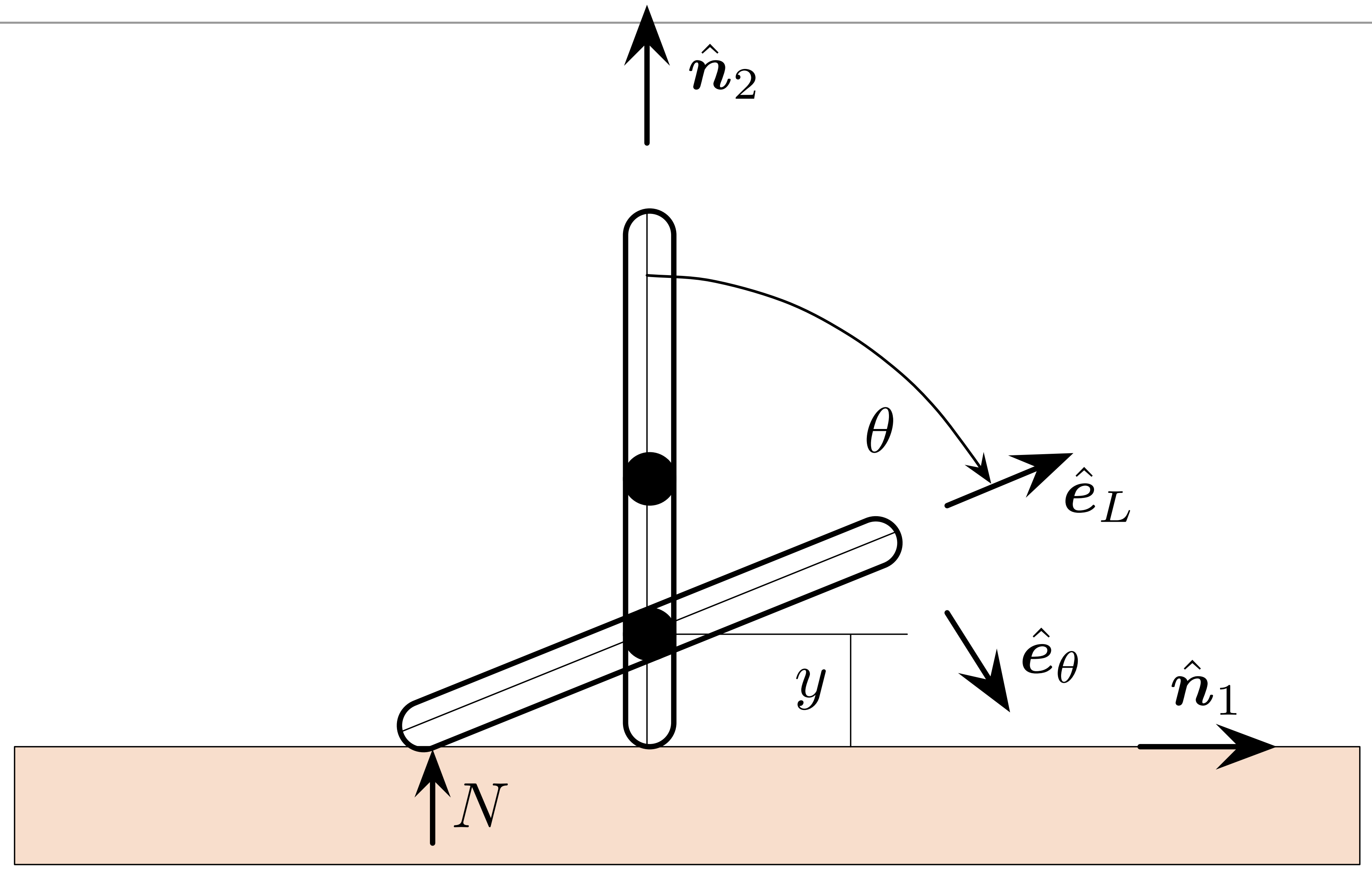


Euler's rotational equations of motion: $$[I]\dot{\boldsymbol{\omega}} = -[\tilde{\omega}][I]\boldsymbol{\omega} + \boldsymbol{L}_c$$

Principal axis version of rotational EOM:
$$I_{11}\dot{\omega}_1 = -(I_{33} - I_{22})\omega_2\omega_3 + L_1$$
$$I_{22}\dot{\omega}_2 = -(I_{11} - I_{33})\omega_3\omega_1 + L_2$$
$$I_{33}\dot{\omega}_3 = -(I_{22} - I_{11})\omega_1\omega_2 + L_3$$

Discuss how to integrate full EOM

# Example: Slender Rod Falling




Rod Inertia about CM: $I_c = \frac{m}{12}L^2$

Momentum about CM: $\boldsymbol{H}_c = I_c\dot{\theta}\hat{\boldsymbol{e}}_3$

Torque: $\boldsymbol{L}_c = \left(-\frac{L}{2}\hat{\boldsymbol{e}}_L\right) \times N\hat{\boldsymbol{n}}_2 = \frac{L}{2}N\sin\theta\hat{\boldsymbol{e}}_3$

Euler's Eqn: $\dot{\boldsymbol{H}}_c = \boldsymbol{L}_c$

$$\frac{m}{12}L^2\ddot{\theta} - \frac{L}{2}N\sin\theta = 0$$

Newton's Eqn: $m\ddot{y}\hat{\boldsymbol{n}}_2 = (N - mg)\hat{\boldsymbol{n}}_2$

$$y = \frac{L}{2}\cos\theta \Rightarrow \ddot{y} = -\frac{L}{2}\ddot{\theta}\sin\theta - \frac{L}{2}\dot{\theta}^2\cos\theta$$

$$\Rightarrow N = mg - m\frac{L}{2}\ddot{\theta}\sin\theta - m\frac{L}{2}\dot{\theta}^2\cos\theta$$

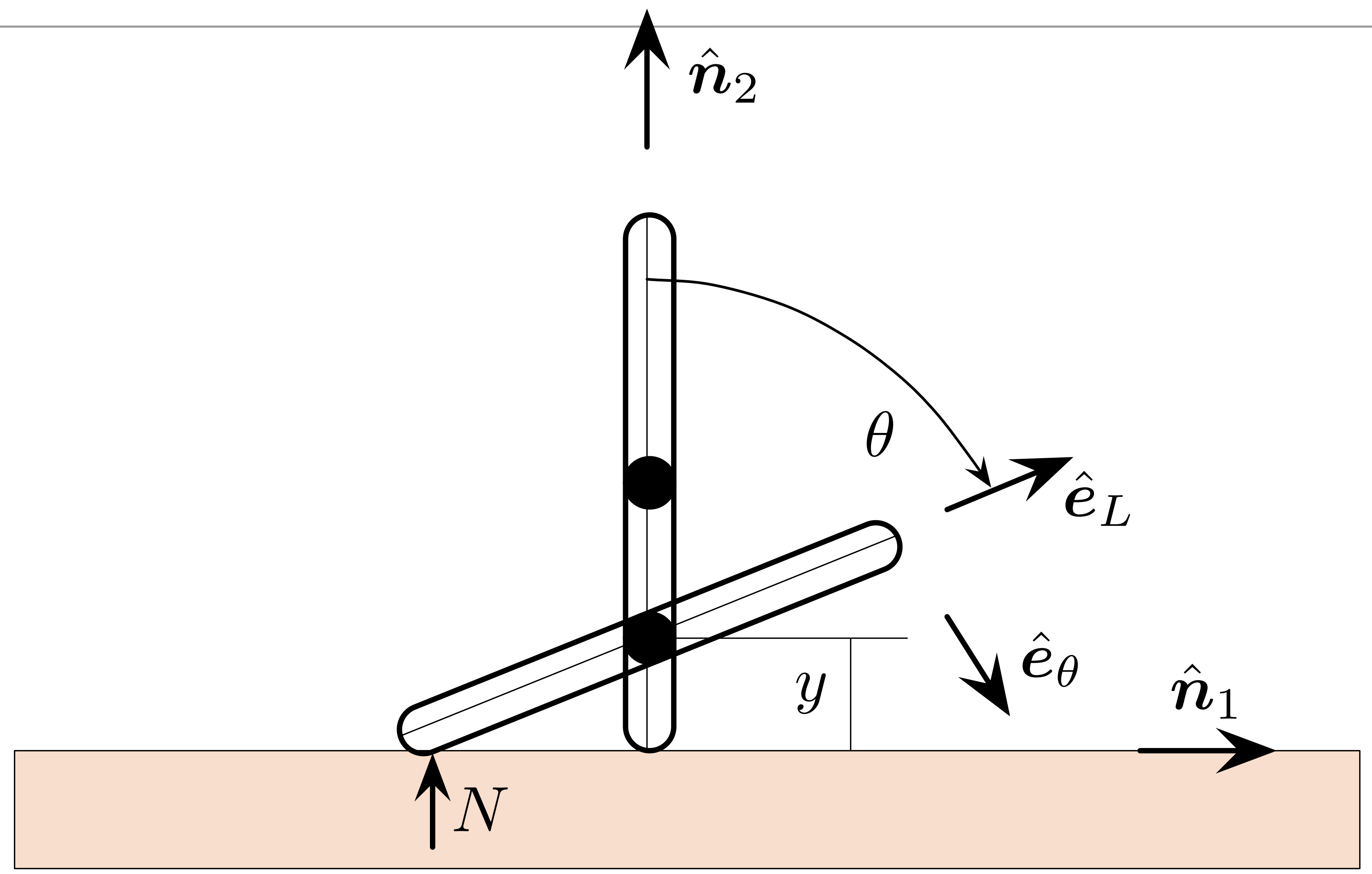
EOM: $\boxed{\frac{m}{12}L^2\ddot{\theta}(1 + 3\sin^2\theta) + \frac{m}{4}L^2\dot{\theta}^2\sin\theta\cos\theta - \frac{m}{2}Lg\sin\theta = 0}$

# Example: Slender Rod Falling

Energy functions:

$$V(\theta) = mgy = mg\frac{L}{2}\cos\theta$$

$$T(\theta,\dot\theta) = \frac{m}{2}\dot y^2 + \frac{I_c}{2}\dot\theta^2 = \frac{mL^2}{24}(1+3\sin^2\theta)\dot\theta^2$$




Initial energy level:

$$E(t_0) = T_0 + V_0 = mg\frac{L}{2}$$

$$T(\theta,\dot\theta) + V(\theta) = E(t_0)$$

$$\Rightarrow \quad \boxed{\dot\theta^2 = \frac{12g(1-\cos\theta)}{L(1+3\sin^2\theta)}}$$

Energy Conservation avoids
having to solve ODE.

# Momentum/Energy Surfaces

- Let's study a special class of rigid body motion where no external torque is acting on the body.
- In this case the kinetic energy and the angular momentum of the rigid body are conserved!
- In inertial frame vector components, we find

$$\boldsymbol{H} = {}^{\mathcal{N}}\boldsymbol{H} = [BN]^T \, {}^{\mathcal{B}}\boldsymbol{H}$$

$$\boldsymbol{H} = {}^{\mathcal{N}}\boldsymbol{H} = \begin{bmatrix} c\theta_2 c\theta_1 & c\theta_2 s\theta_1 & -s\theta_2 \\ s\theta_3 s\theta_2 c\theta_1 - c\theta_3 s\theta_1 & s\theta_3 s\theta_2 s\theta_1 + c\theta_3 c\theta_1 & s\theta_3 c\theta_2 \\ c\theta_3 s\theta_2 c\theta_1 + s\theta_3 s\theta_1 & c\theta_3 s\theta_2 s\theta_1 - s\theta_3 c\theta_1 & c\theta_3 c\theta_2 \end{bmatrix} \begin{pmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{pmatrix}$$

$$\dot{\boldsymbol{H}} = 0 = \boldsymbol{f}(\theta_1, \theta_2, \theta_3, \omega_1, \omega_2, \omega_3, \dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3)$$

- Notice that the constant angular momentum condition can become very complicated and difficult to study!

- Instead of writing ***H*** in inertial frame components, we chose to write it in the body frame where the inertia matrix is a constant for a rigid body.

- Assume the angular momentum vector ***H*** is written in body frame components, and that principal axes were chosen for the body frame *B*.

$$
\begin{aligned}
\boldsymbol{\omega} &= \omega_1 \hat{\boldsymbol{b}}_1 + \omega_2 \hat{\boldsymbol{b}}_2 + \omega_3 \hat{\boldsymbol{b}}_3 \\
\boldsymbol{H} = {}^{\mathcal{B}}\boldsymbol{H} &= H_1 \hat{\boldsymbol{b}}_1 + H_2 \hat{\boldsymbol{b}}_2 + H_3 \hat{\boldsymbol{b}}_3 \\
&= I_1 \omega_1 \hat{\boldsymbol{b}}_1 + I_2 \omega_2 \hat{\boldsymbol{b}}_2 + I_3 \omega_3 \hat{\boldsymbol{b}}_3
\end{aligned}
\qquad
[I] = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}
$$

- This allows us to write ***H*** as:

$$\boldsymbol{H} = {}^{\mathcal{B}}\boldsymbol{H} = {}^{\mathcal{B}}\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = {}^{\mathcal{B}}\begin{pmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{pmatrix}$$

- Because ***H*** is constant, all possible rigid body angular velocities must lie on the surface of the following momentum ellipsoid:

$$H^2 = \boldsymbol{H}^T\boldsymbol{H} = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2 = \text{constant}$$

- Similarly, since the kinetic energy is conserved, all possible rigid body angular velocities must also lie on the surface of the following energy ellipsoid:

$$T = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$$

- The final admissible angular velocities will be on the intersection of these two ellipsoids.

- Using the momenta coordinates

$$H_1 = I_1\omega_1 \qquad H_2 = I_2\omega_2 \qquad H_3 = I_3\omega_3$$

- we can write the momentum magnitude constraint as

$$H^2 = H_1^2 + H_2^2 + H_3^2 \implies \text{Sphere}$$

- and the kinetic energy constraint as

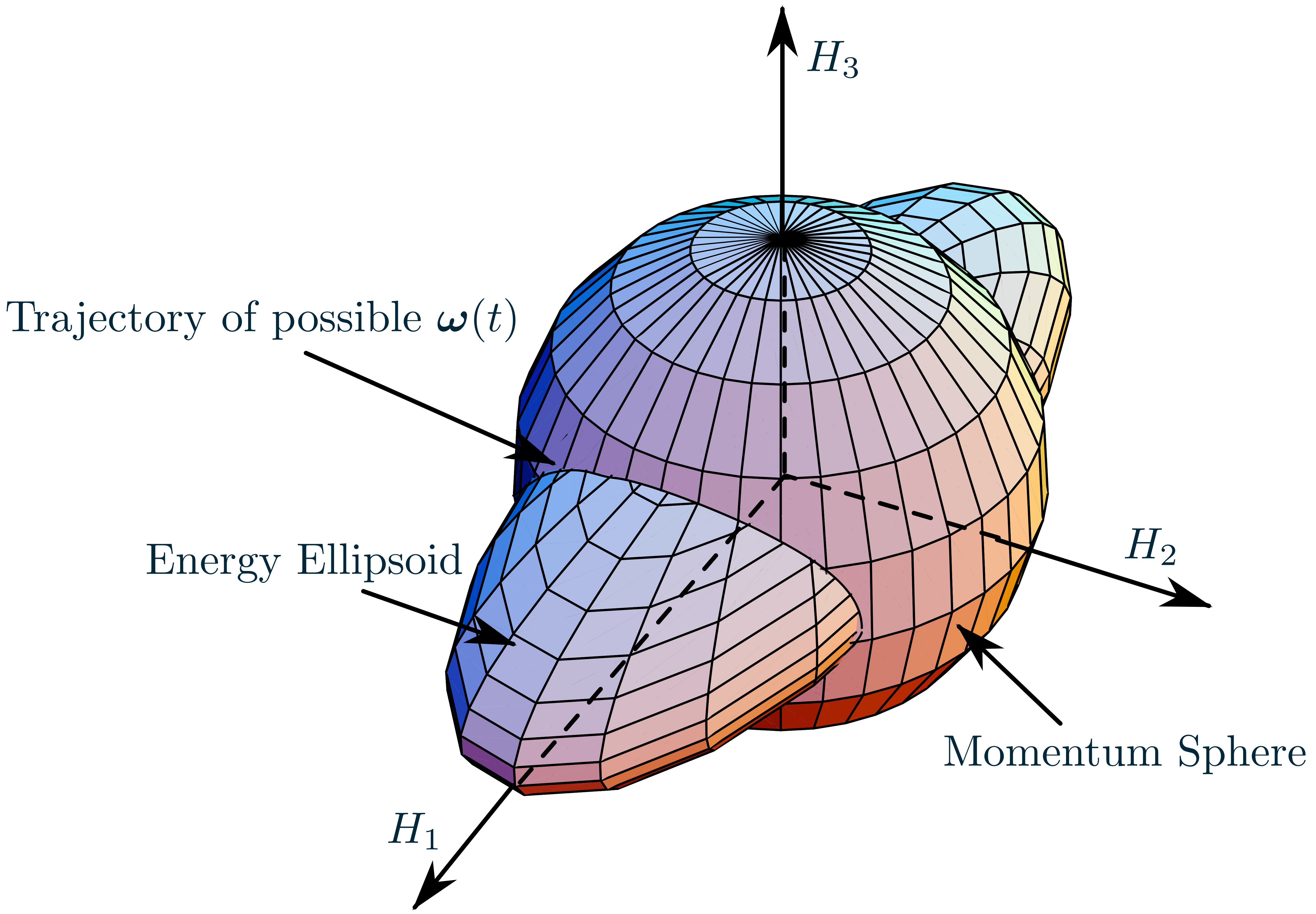
$$1 = \frac{H_1^2}{2I_1T} + \frac{H_2^2}{2I_2T} + \frac{H_3^2}{2I_3T}$$

Compare to ellipsoid equation:

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

$$\sqrt{2I_iT} \implies \text{semi-axes of ellipsoid}$$

- Clearly, for a given *H*, only a certain range of kinetic energies is possible.
- Let's assume the common notation: $I_1 \geq I_2 \geq I_3$



- Let's look at the Minimum Energy Case:

- The surfaces will only intersect at:

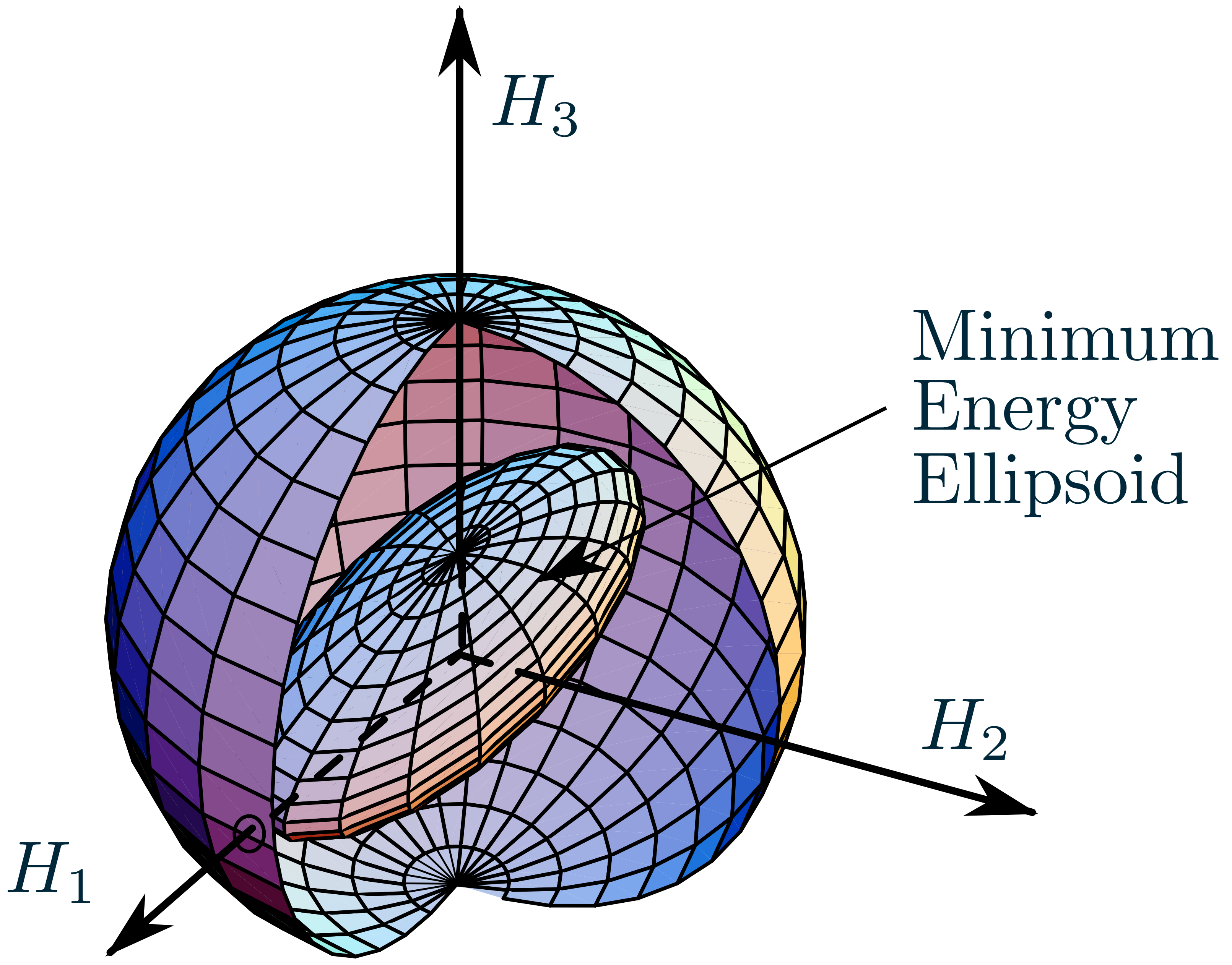
$$^{\mathcal{B}}\boldsymbol{H} = \pm H \hat{\boldsymbol{b}}_1$$

$$H_1 = H \qquad H_2 = H_3 = 0$$

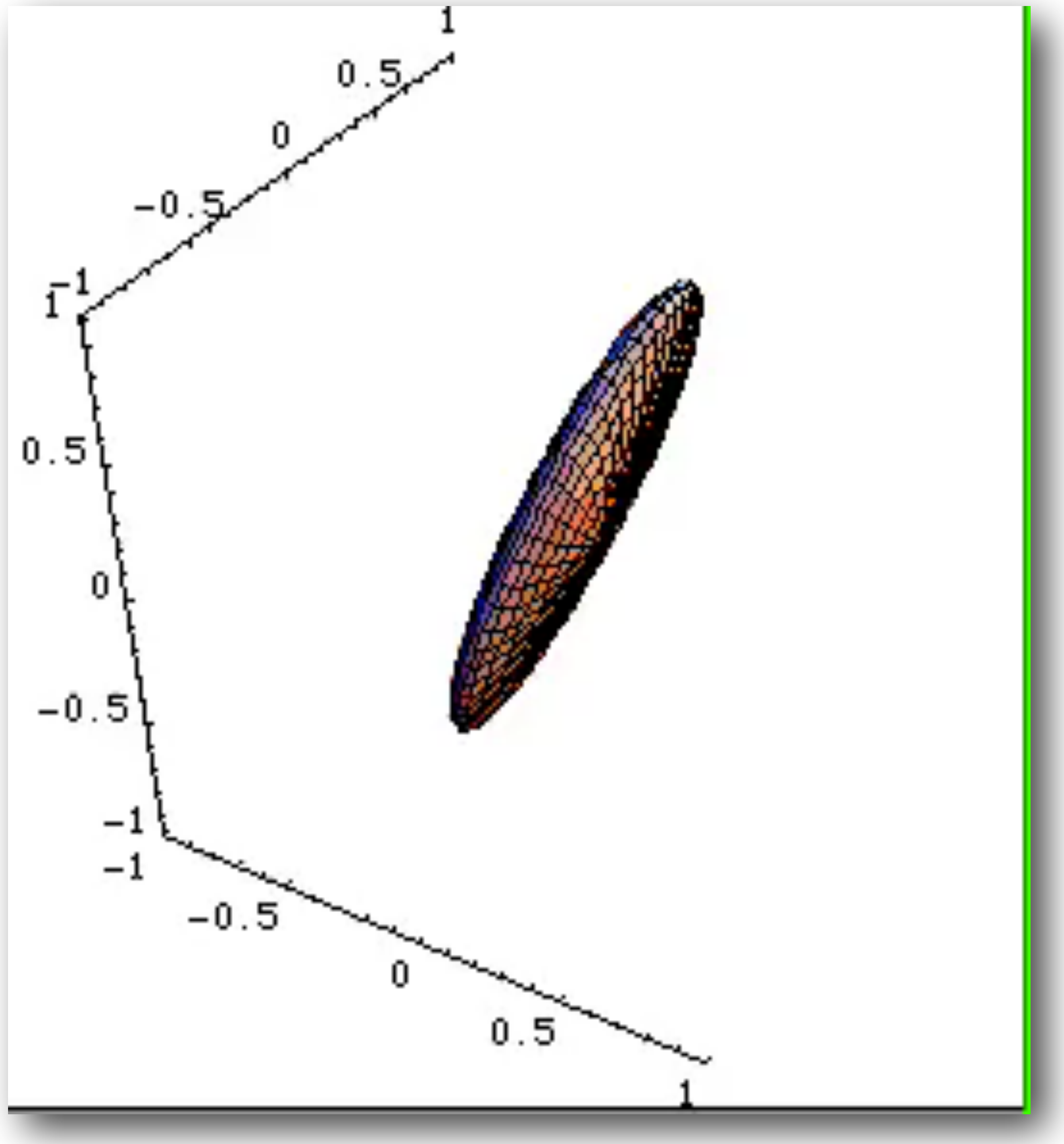
The kinetic energy is:

$$1 = \frac{H_1^2}{2I_1T} + \frac{H_2^2}{2I_2T} + \frac{H_3^2}{2I_3T}$$

$$\boxed{T_{\text{min}} = \frac{H^2}{2I_1}}$$

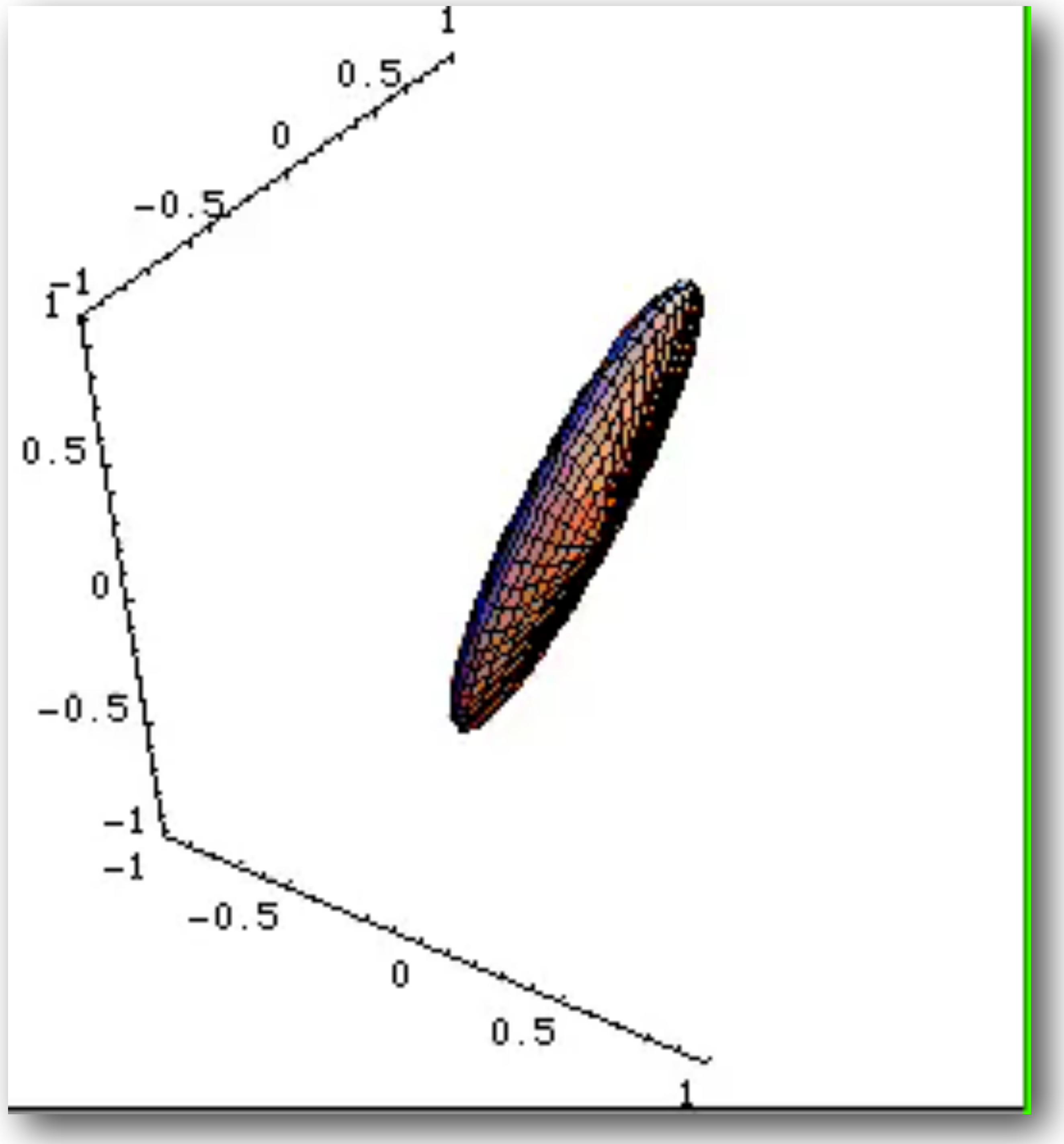


General Rotation of Rigid Body with $I_1 \geq I_2 \geq I_3$




Pure Spin

$\boldsymbol{\omega}_0 = (10^o, 0^o, 0^o)$ /s




Slightly Off Spin

$\boldsymbol{\omega}_0 = (10^o, 0.5^o, 0.5^o)$ /s

- Let’s look at the Intermediate Energy Case:

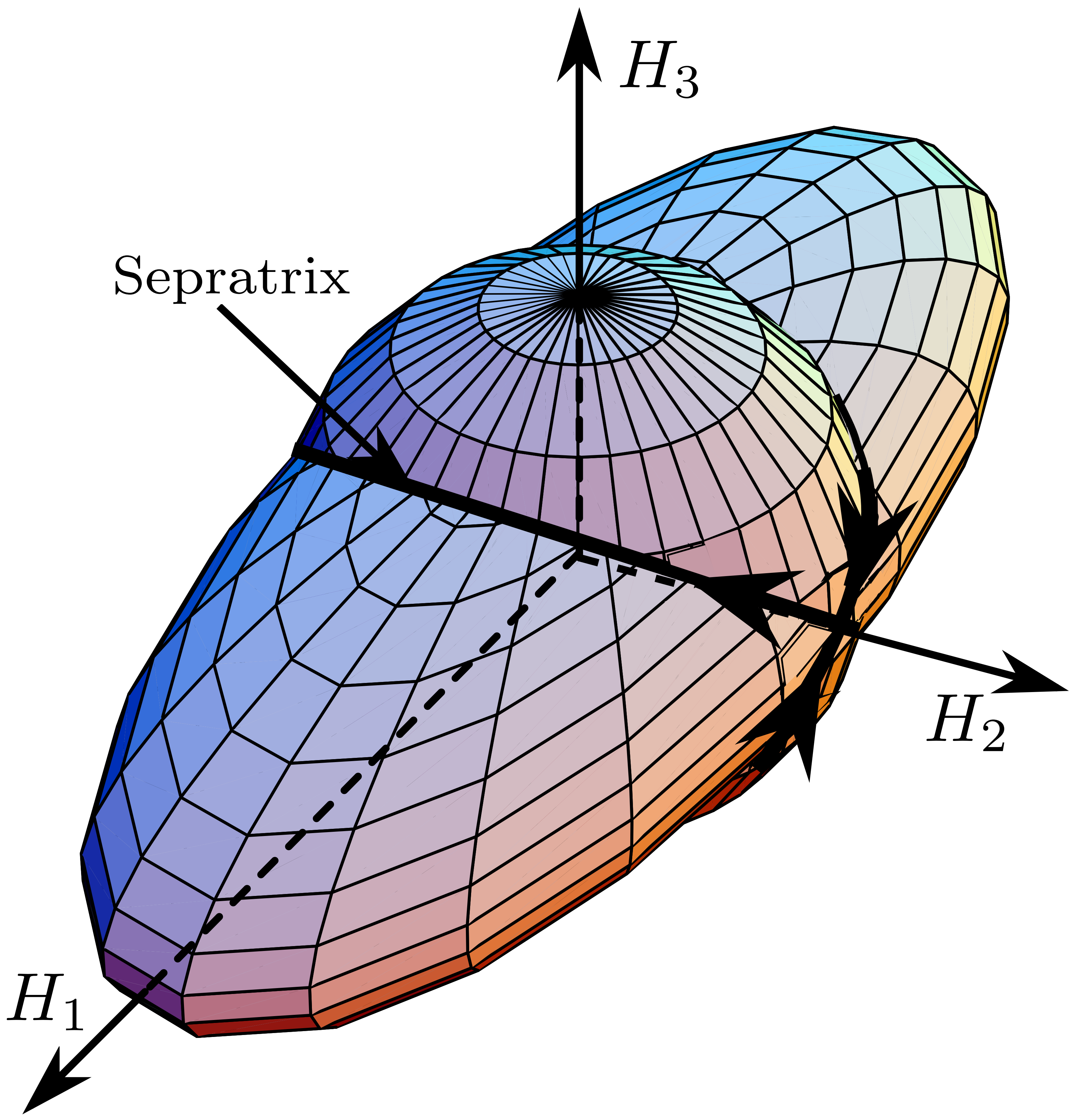
- The surfaces will only intersect at:

$$^{\mathcal{B}}\boldsymbol{H} = \pm H\hat{\boldsymbol{b}}_2$$

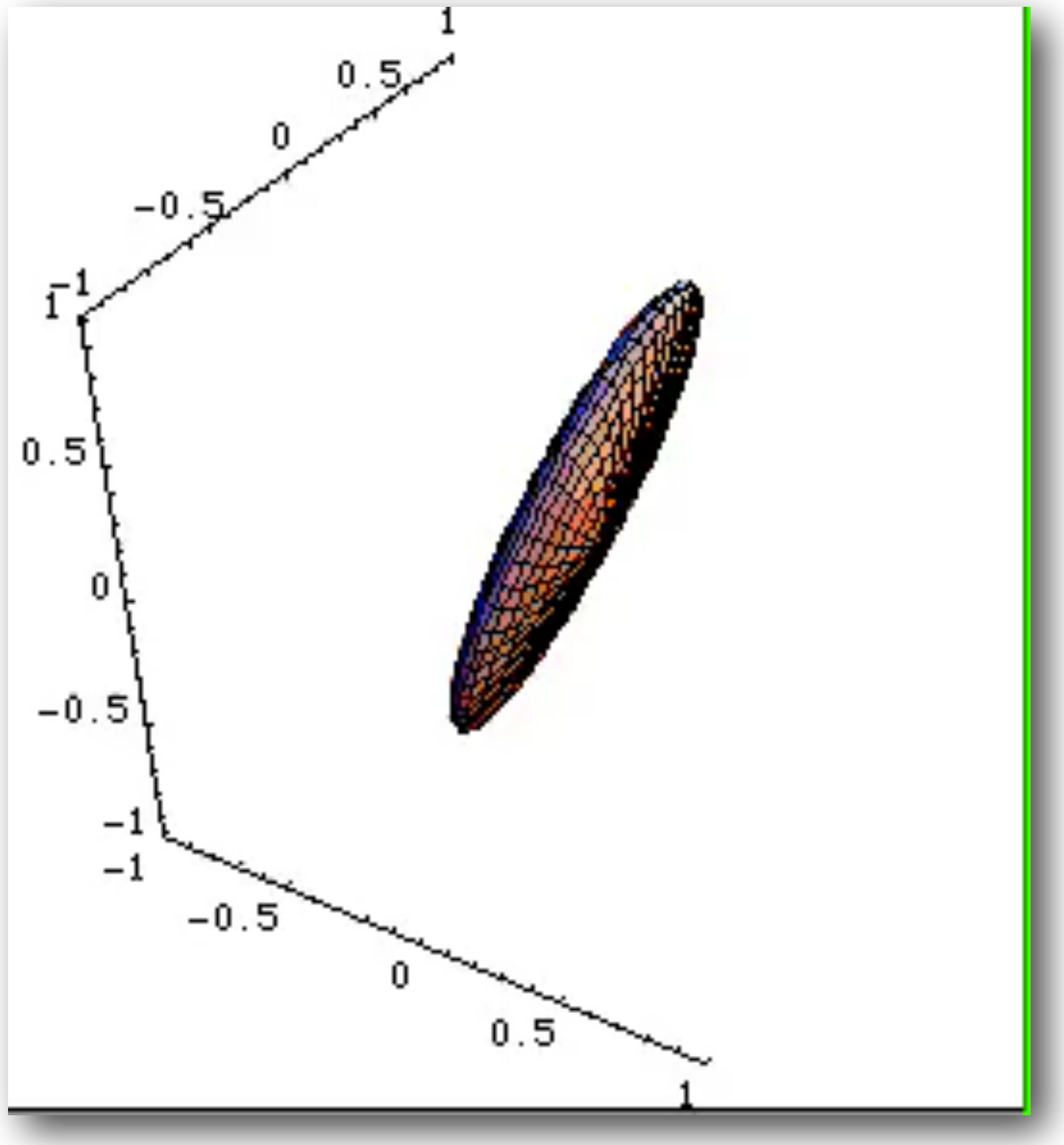
The kinetic energy is:

$$1 = \frac{H_1^2}{2I_1 T} + \frac{H_2^2}{2I_2 T} + \frac{H_3^2}{2I_3 T}$$

$$\boxed{T_{\text{int}} = \frac{H^2}{2I_2}}$$

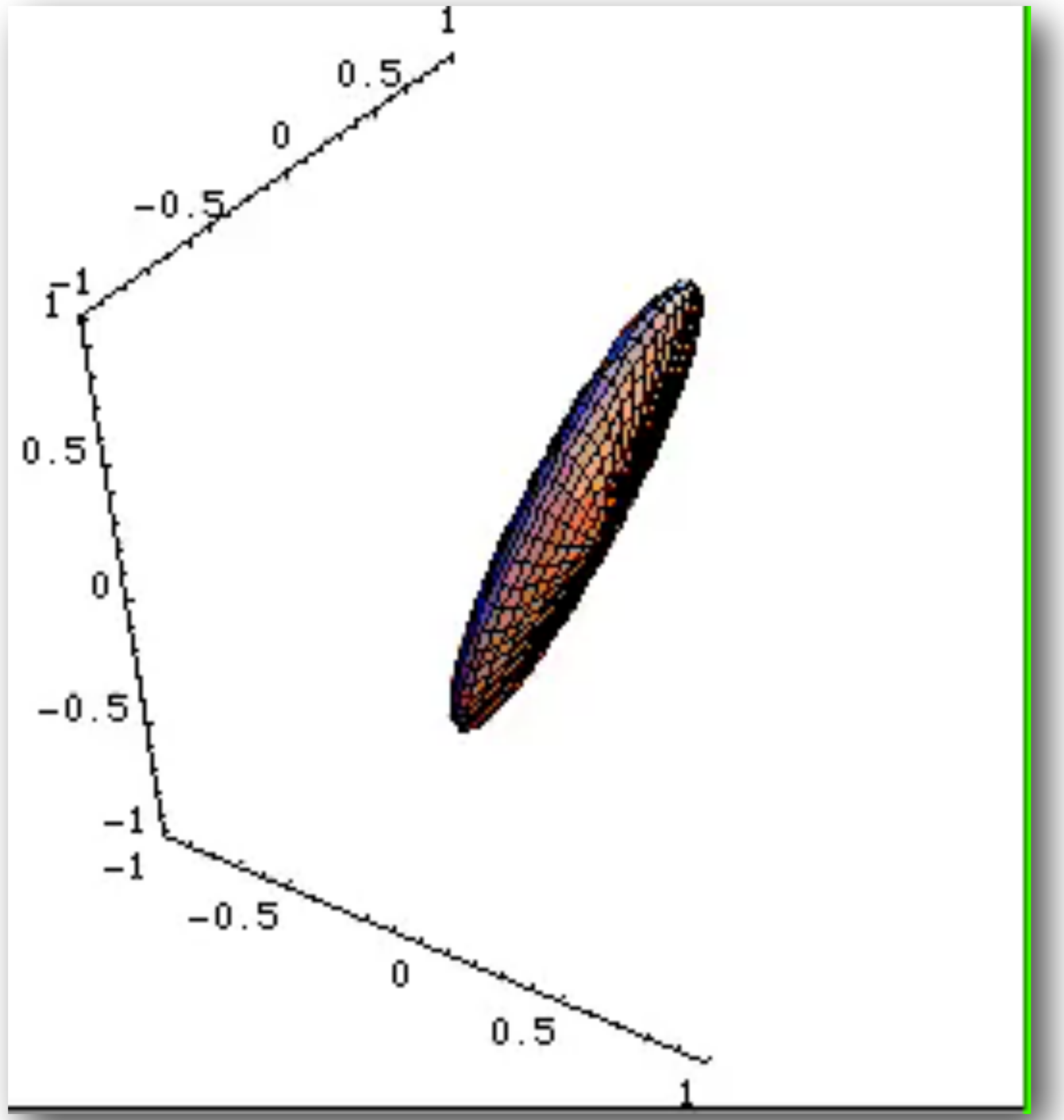


General Rotation of Rigid Body with $I_1 \geq I_2 \geq I_3$




Pure Spin

$\boldsymbol{\omega}_0 = (0^o, 10^o, 0^o)$ /s




Slightly Off Spin

$\boldsymbol{\omega}_0 = (0.5^o, 10^o, 0.5^o)$ /s

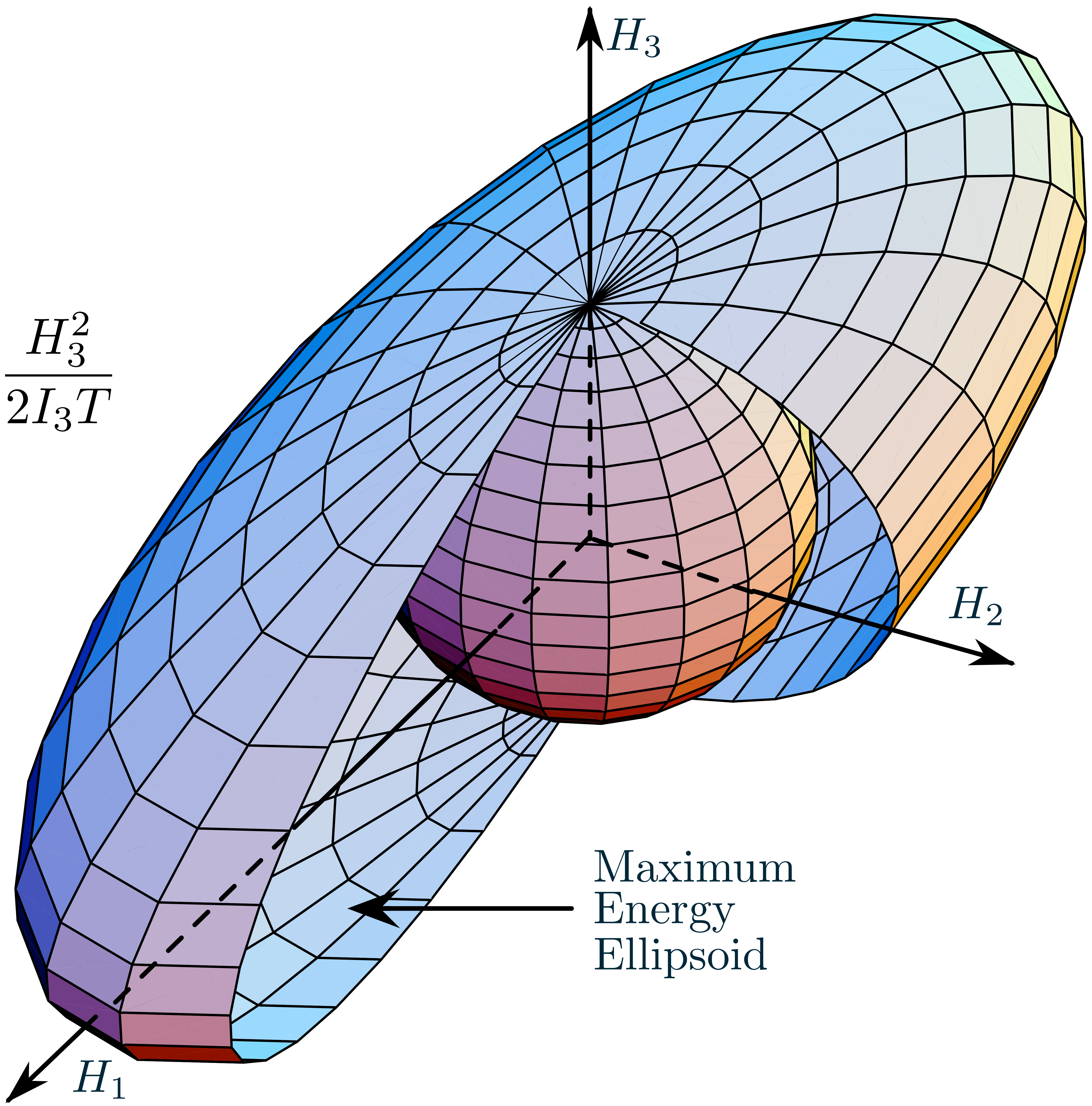
- Maximum Energy Case:

$$^{\mathcal{B}}\boldsymbol{H} = H\hat{\boldsymbol{b}}_3$$

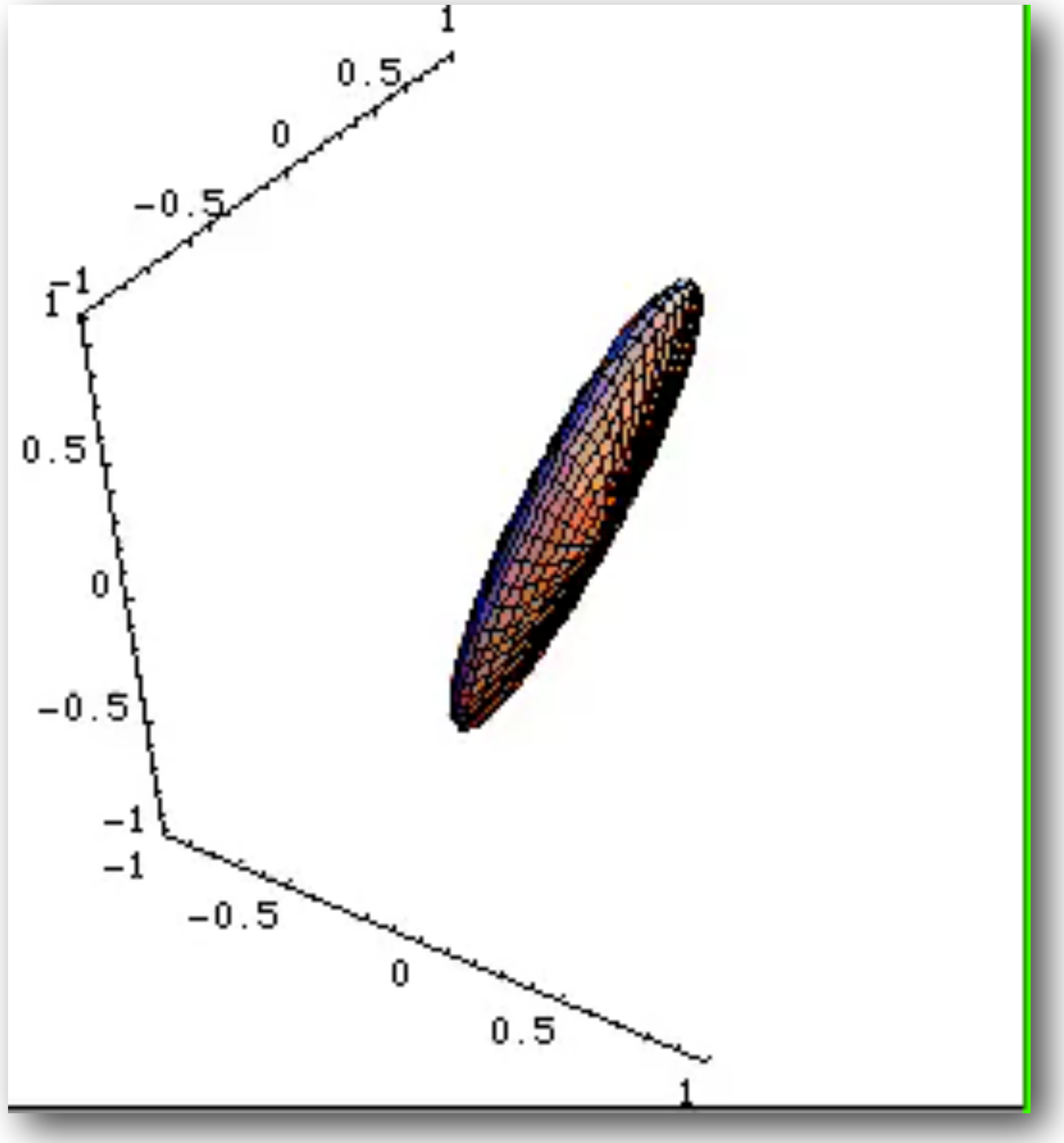
The kinetic energy is:

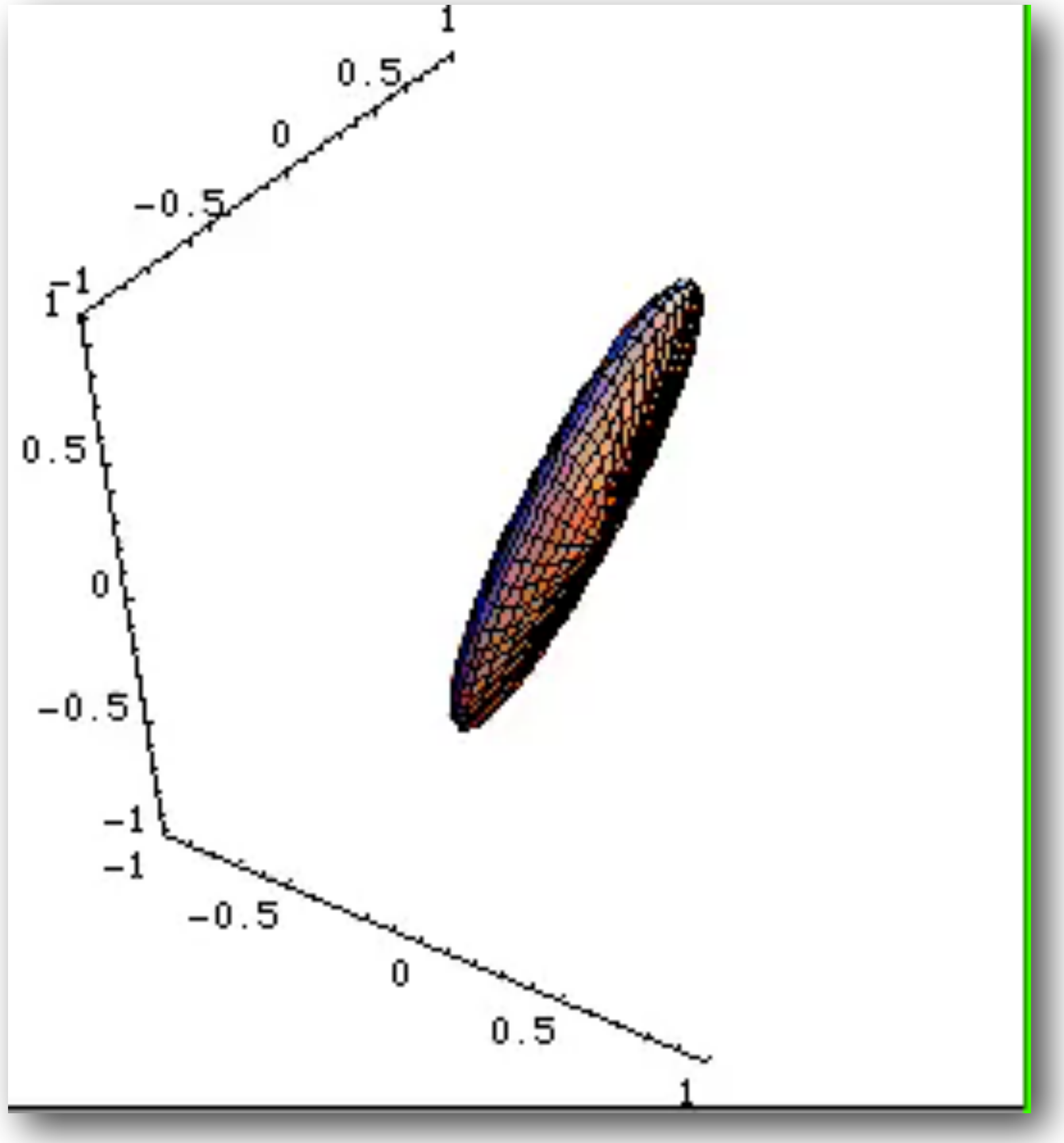
$$1 = \frac{H_1^2}{2I_1T} + \frac{H_2^2}{2I_2T} + \frac{H_3^2}{2I_3T}$$

$$\boxed{T_{\max} = \frac{H^2}{2I_3}}$$



General Rotation of Rigid Body with $I_1 \geq I_2 \geq I_3$



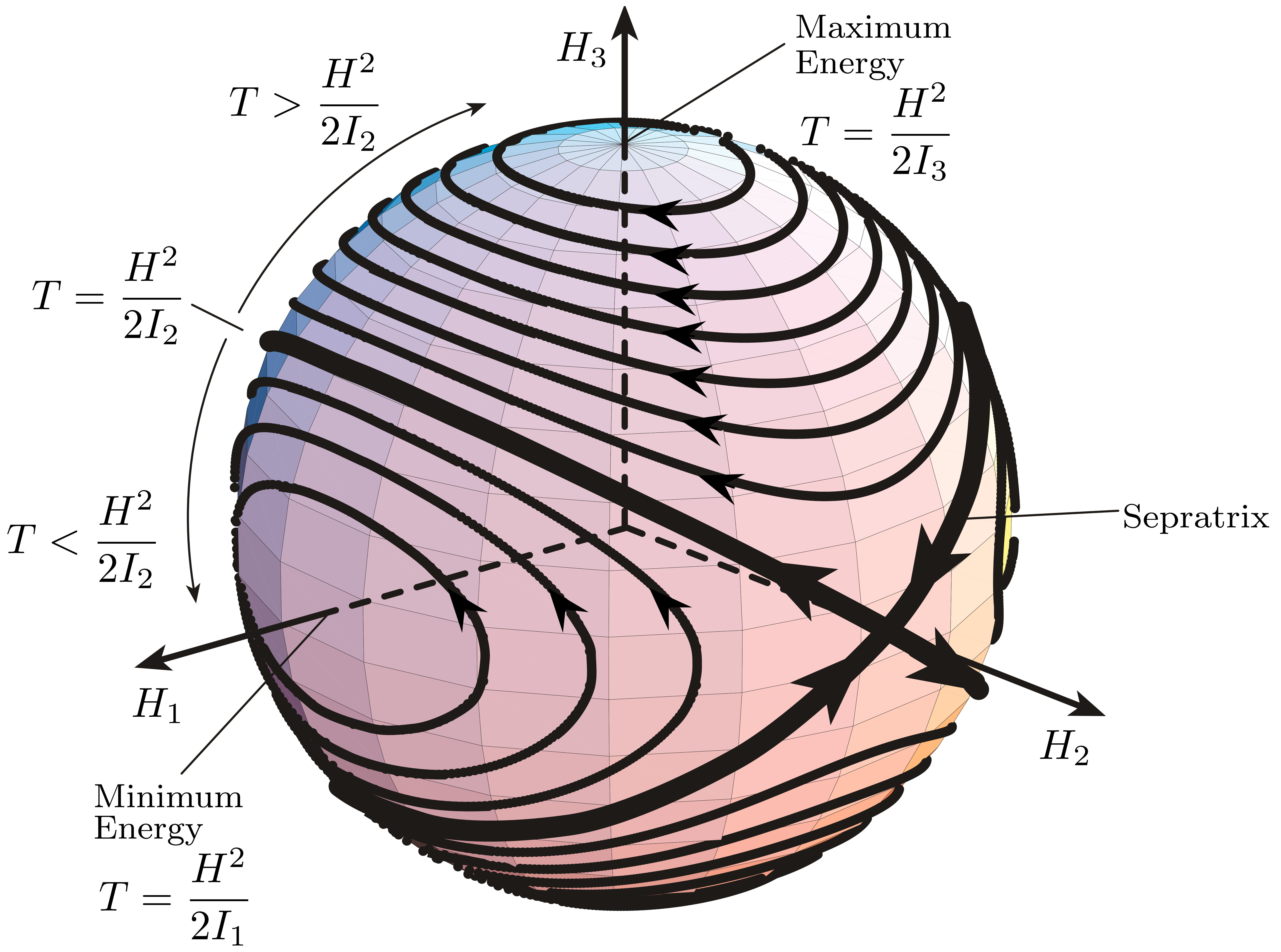




Pure Spin

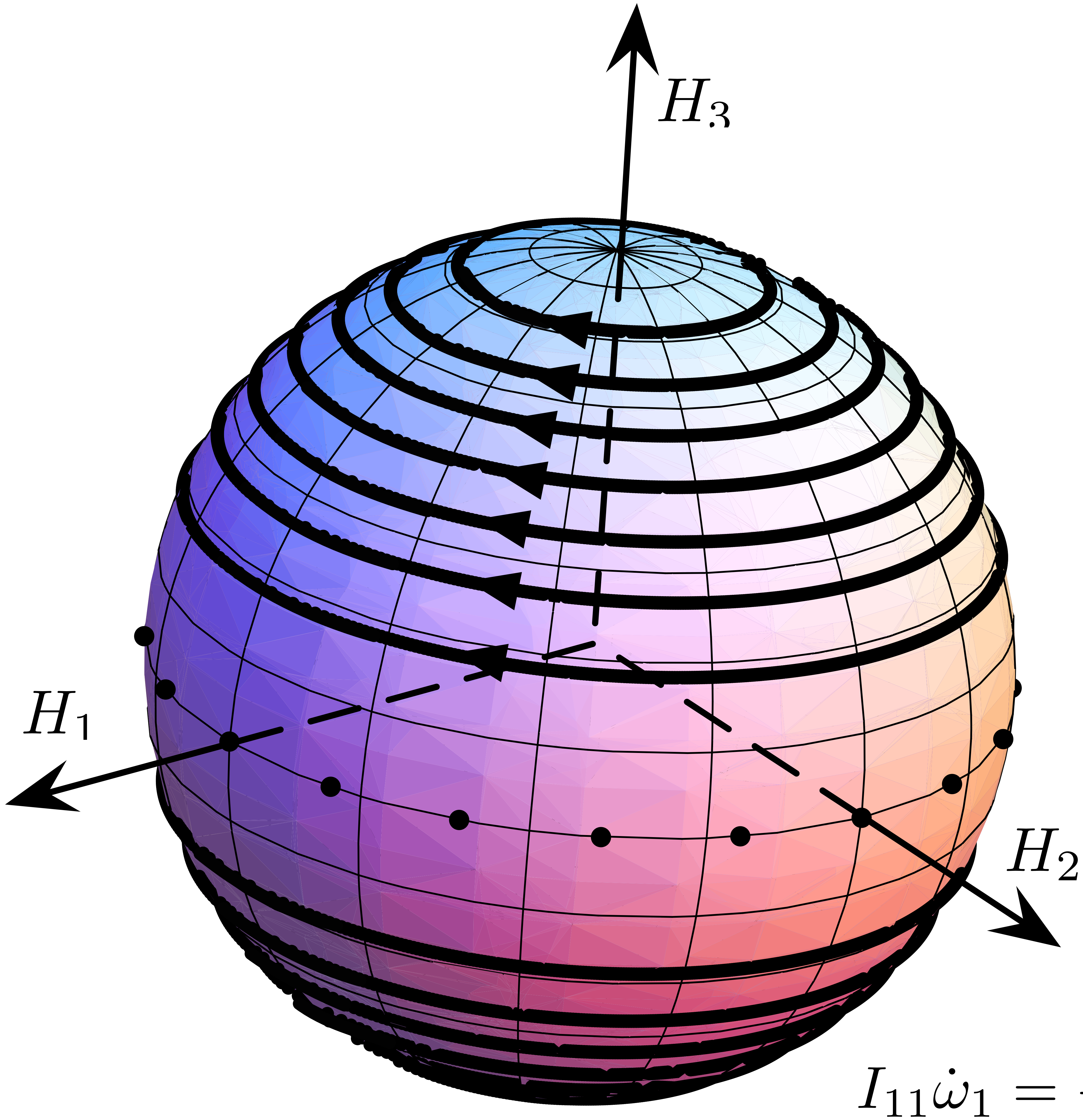
$\boldsymbol{\omega}_0 = (0^o, 0^o, 10^o)$ /s

Slightly Off Spin

$\boldsymbol{\omega}_0 = (0.5^o, 0.5^o, 10^o)$ /s

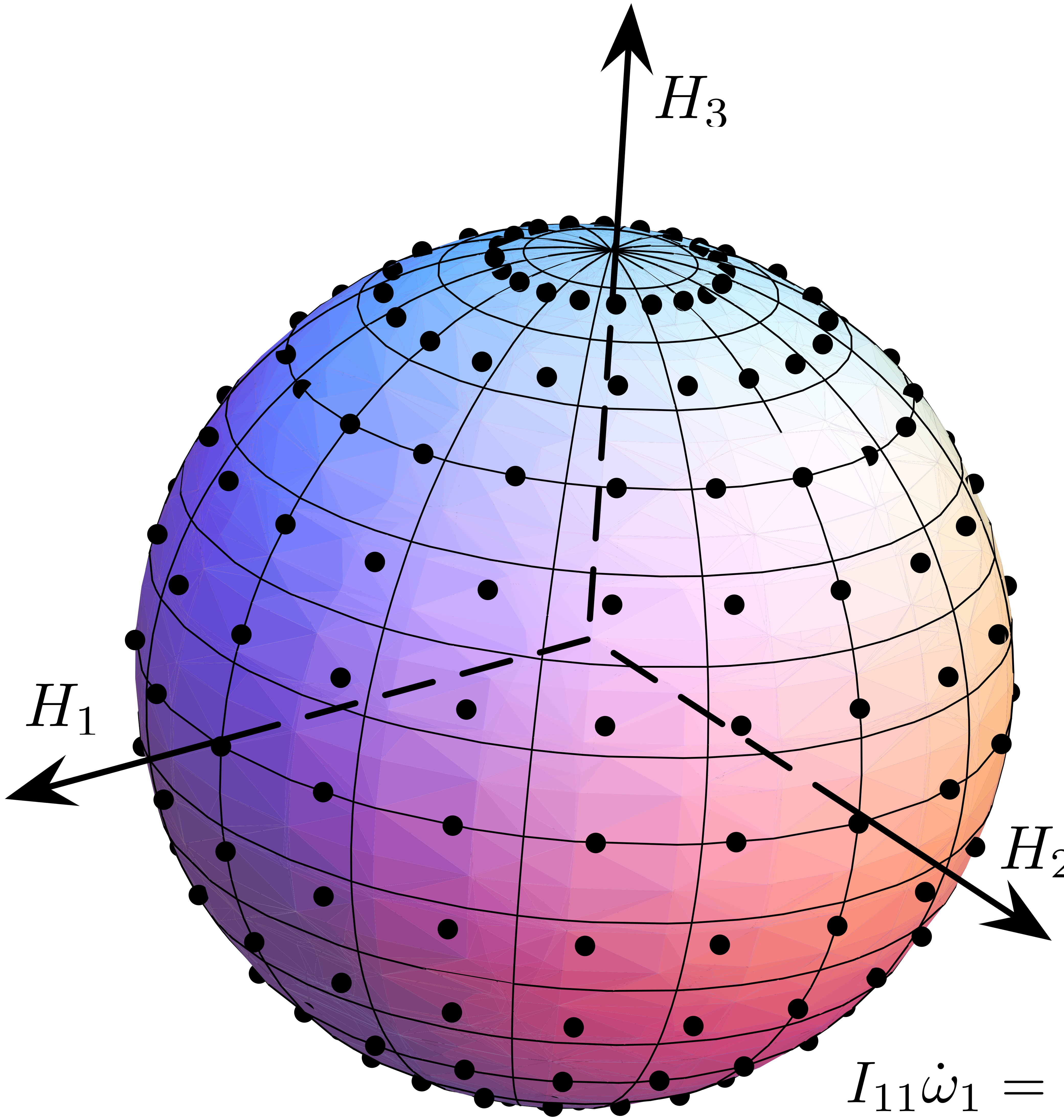



Family of energy ellipsoid and momentum sphere intersections.




What type of spacecraft body would yield this?

$$I_{11}\dot{\omega}_1 = -(I_{33} - I_{22})\omega_2\omega_3$$
$$I_{22}\dot{\omega}_2 = -(I_{11} - I_{33})\omega_3\omega_1$$
$$I_{33}\dot{\omega}_3 = -(I_{22} - I_{11})\omega_1\omega_2$$




What type of spacecraft body would yield this?

$$I_{11}\dot{\omega}_1 = -(I_{33} - I_{22})\omega_2\omega_3$$
$$I_{22}\dot{\omega}_2 = -(I_{11} - I_{33})\omega_3\omega_1$$
$$I_{33}\dot{\omega}_3 = -(I_{22} - I_{11})\omega_1\omega_2$$