Hardware & Software Verification

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Lecture 7: Foundations of Hardware Verification
18 November 2025

This lecture

- Temporal logic
- Binary decision diagrams

Temporal logic

- SymbiYosys can verify *immediate* assertions like:

```
assert (count > 0);
```

- And *concurrent* assertions like:

```
assert (wen > 0 |=> waddr > 0);
```

- And *coverage* statements like:

```
cover (count == 24);
```

- These can be generalised to formulas in *temporal logic*.

Linear Temporal Logic

ϕ ::= true
| false
| p // atom
| $\neg\,\phi$ // negation
| $\phi \wedge \phi$ // conjunction
| $\phi \vee \phi$ // disjunction
| $\phi \rightarrow \phi$ // implication
| **X** ϕ // ne**X**t state
| **F** ϕ // some **F**uture state
| **G** ϕ // all future states (**G**lobally)
| ϕ **U** ϕ // **U**ntil

count > 0

count > 0 $\rightarrow$ waddr $\neq$ raddr

en $\rightarrow$ **X** (count > 0)

go $\rightarrow$ **F** (heat)

dead $\rightarrow$ **G** (dead)

G (req $\rightarrow$ **F** (ack))

liftmoving **U** (floor = 5)

Amir Pnueli
(1941–2009)

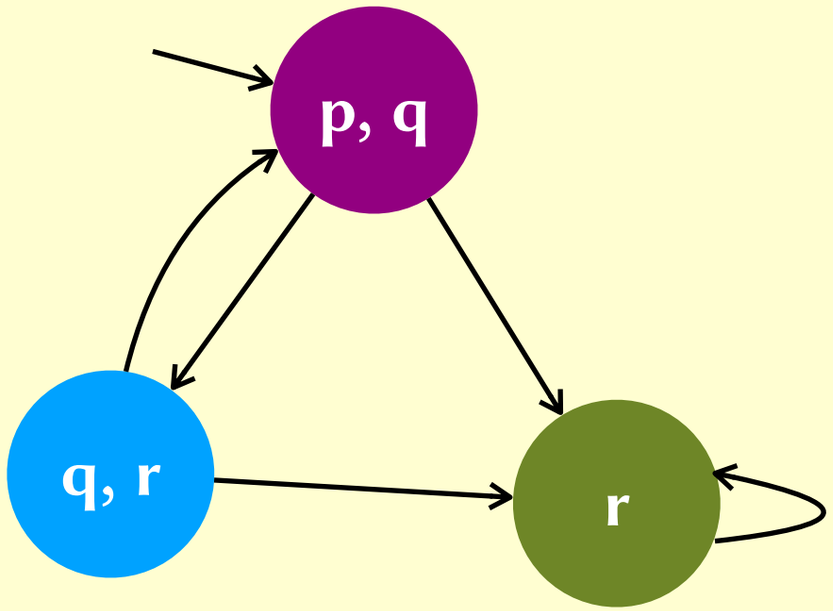
Paths

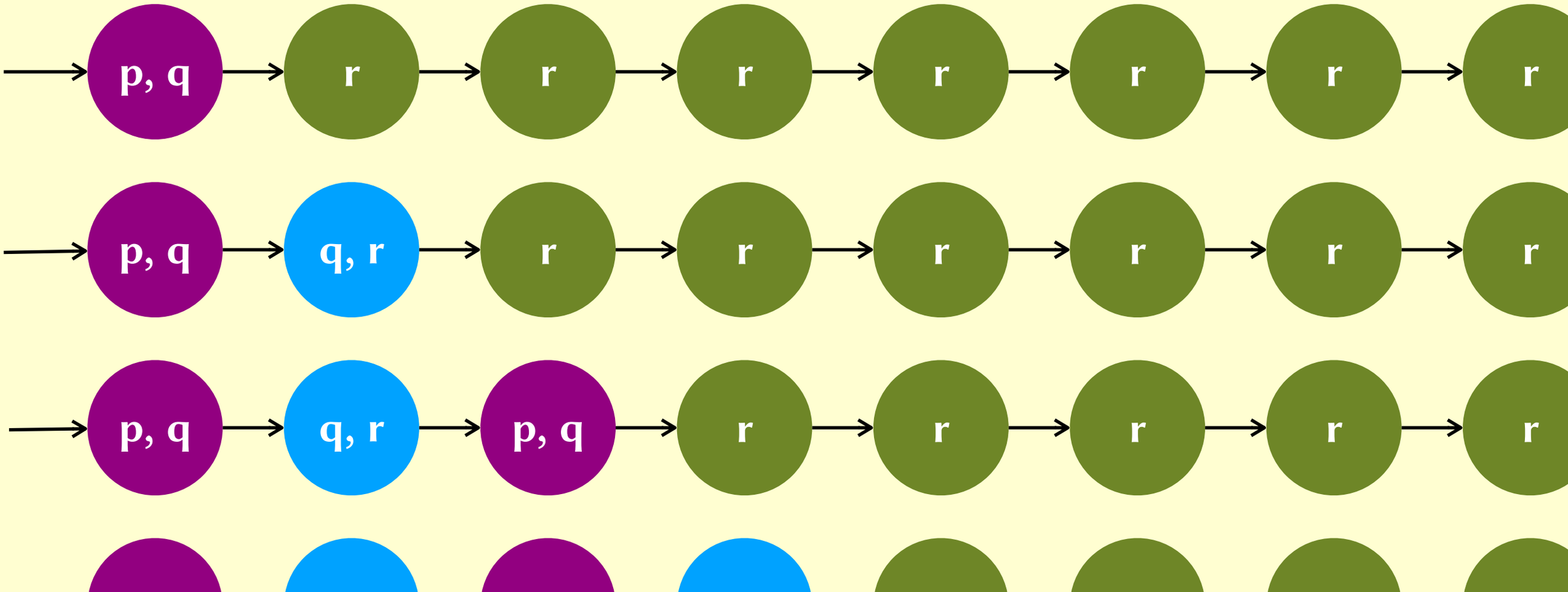
$p \rightarrow q$ ✓

$\mathbf{G}\,(p \rightarrow q)$ ✓

$\mathbf{X}\,r$ ✓ $\mathbf{F}\,r$ ✓ $\mathbf{G}\,r$ ✗ $\mathbf{G}\,\mathbf{F}\,r$ ✓ $\mathbf{F}\,\mathbf{G}\,r$ ✗

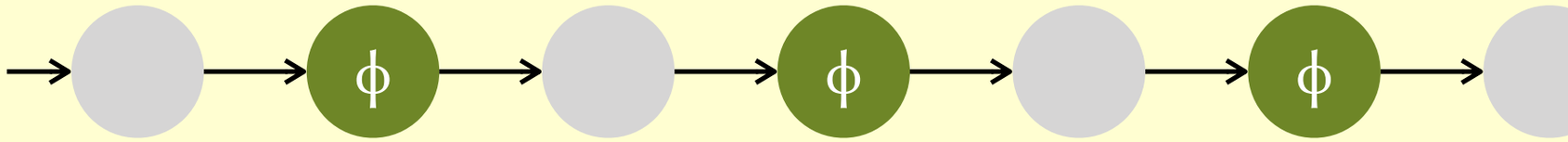
$\mathbf{F}\,(\neg q)$ ✗ $q\ \mathbf{U}\ r$ ✓ $q\ \mathbf{U}\ \neg q$ ✗





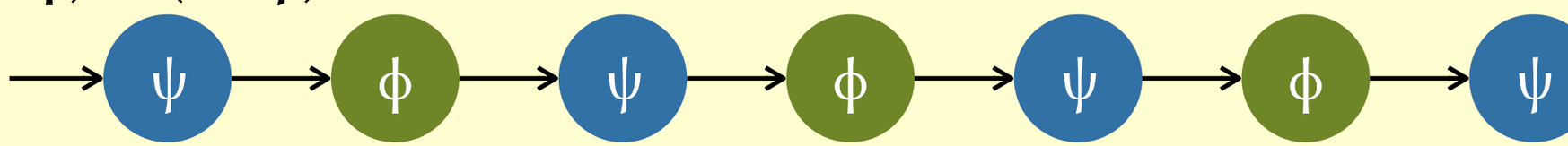
Some identities

- $\mathbf{G}\,\phi \overset{\checkmark}{\equiv} \neg\,\mathbf{F}\,(\neg\,\phi)$
- $\mathbf{F}\,\mathbf{F}\,\phi \overset{\checkmark}{\equiv} \mathbf{F}\,\phi$
- $\mathbf{G}\,\mathbf{G}\,\phi \overset{\checkmark}{\equiv} \mathbf{G}\,\phi$
- $\mathbf{G}\,\mathbf{F}\,\phi \overset{\times}{\equiv} \mathbf{F}\,\mathbf{G}\,\phi$

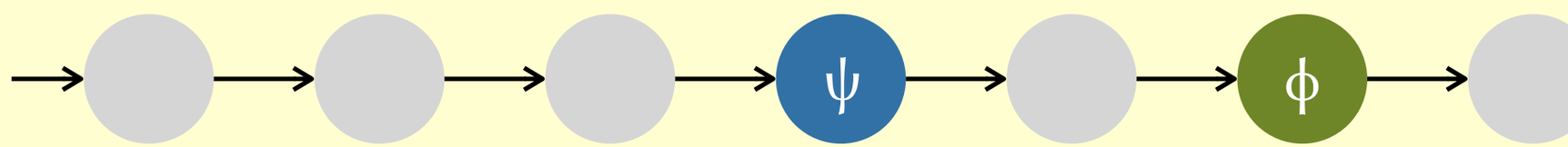


Some more identities

- $\mathbf{F}\,(\phi \vee \psi) \overset{\checkmark}{\equiv} (\mathbf{F}\,\phi) \vee (\mathbf{F}\,\psi)$
- $\mathbf{G}\,(\phi \vee \psi) \overset{\times}{\equiv} (\mathbf{G}\,\phi) \vee (\mathbf{G}\,\psi)$



- $\mathbf{F}\,(\phi \wedge \psi) \overset{\times}{\equiv} (\mathbf{F}\,\phi) \wedge (\mathbf{F}\,\psi)$



- $\mathbf{G}\,(\phi \wedge \psi) \overset{\checkmark}{\equiv} (\mathbf{G}\,\phi) \wedge (\mathbf{G}\,\psi)$

Temporal logic

- SymbiYosys can verify *immediate* assertions like:

```
assert (count > 0);
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$\mathbf{G}\ (\text{count} > 0)$ ✔

- And *concurrent* assertions like:

```
assert (wen > 0 |=> waddr > 0);
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$\mathbf{G}\ (\text{wen} > 0 \longrightarrow \mathbf{X}\ (\text{waddr} > 0))$ ✔

- And *coverage* statements like:

```
cover (count == 24);
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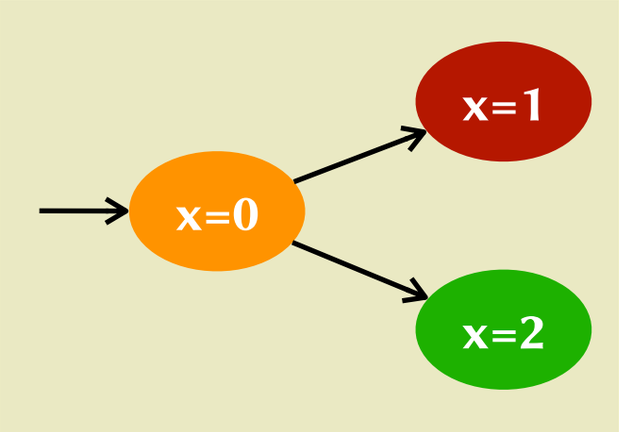
$\mathbf{F}\ (\text{count} = 24)$ ✘

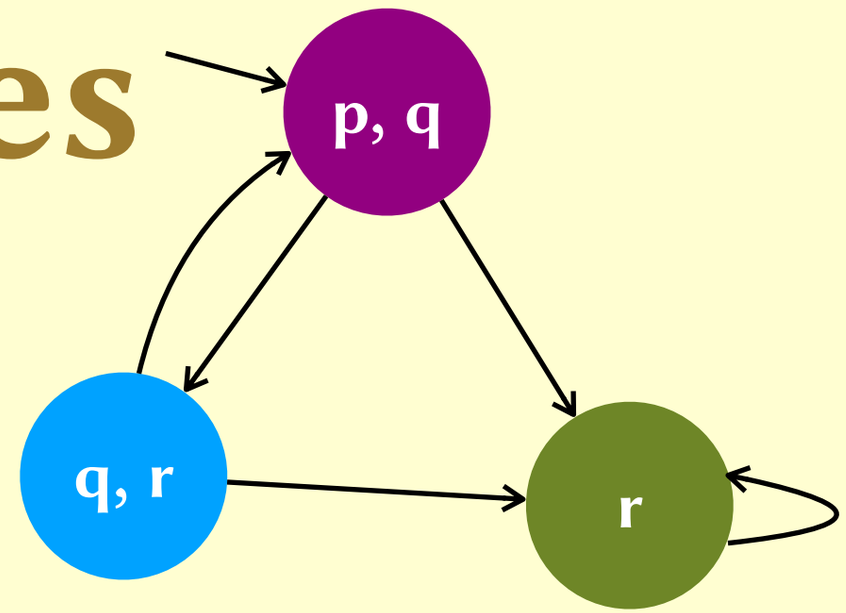
- These can be generalised to formulas in *temporal logic*.

LTL's downfall

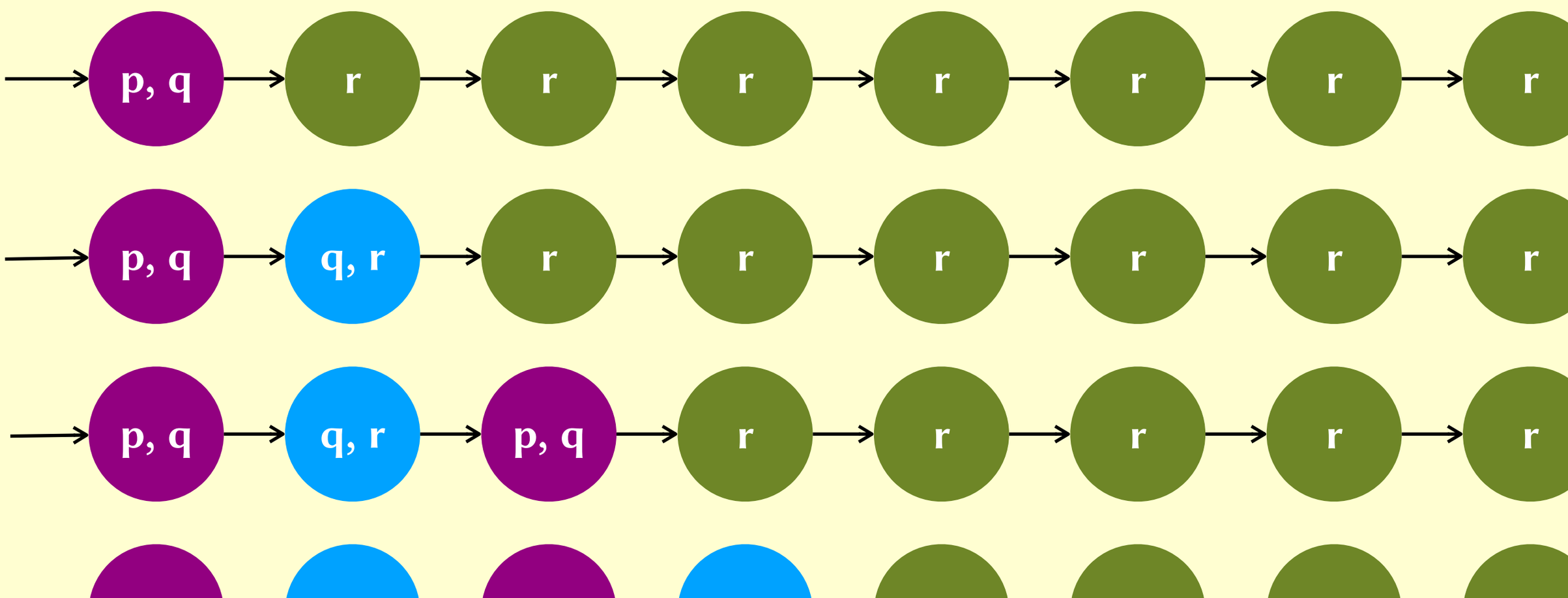


- `cover(x == 1)` should be true.
- But **F**(x=1) is not true because it doesn't hold on *all paths*.
- Conclusion: there are some interesting temporal properties that cannot be expressed in LTL.
- The problem with LTL is that formulas have to hold on *all paths*.

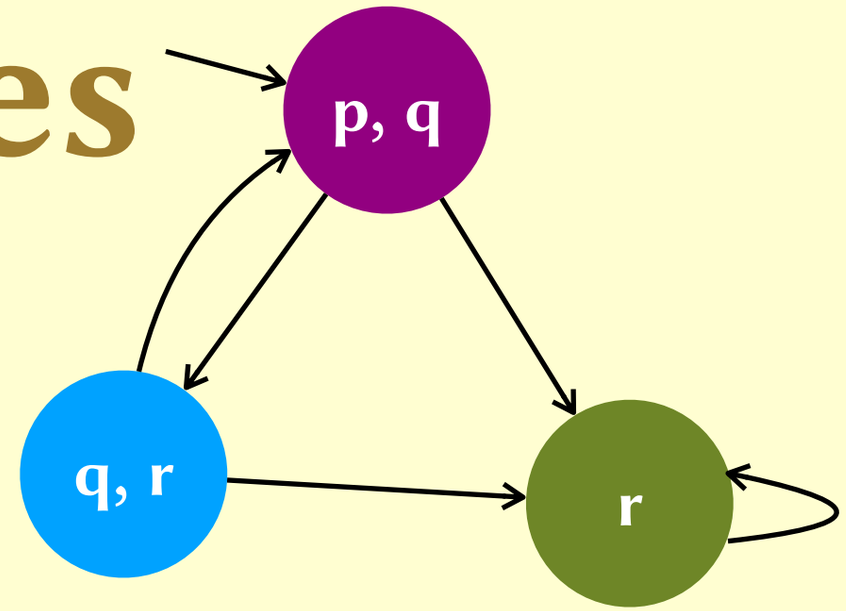
From paths to trees



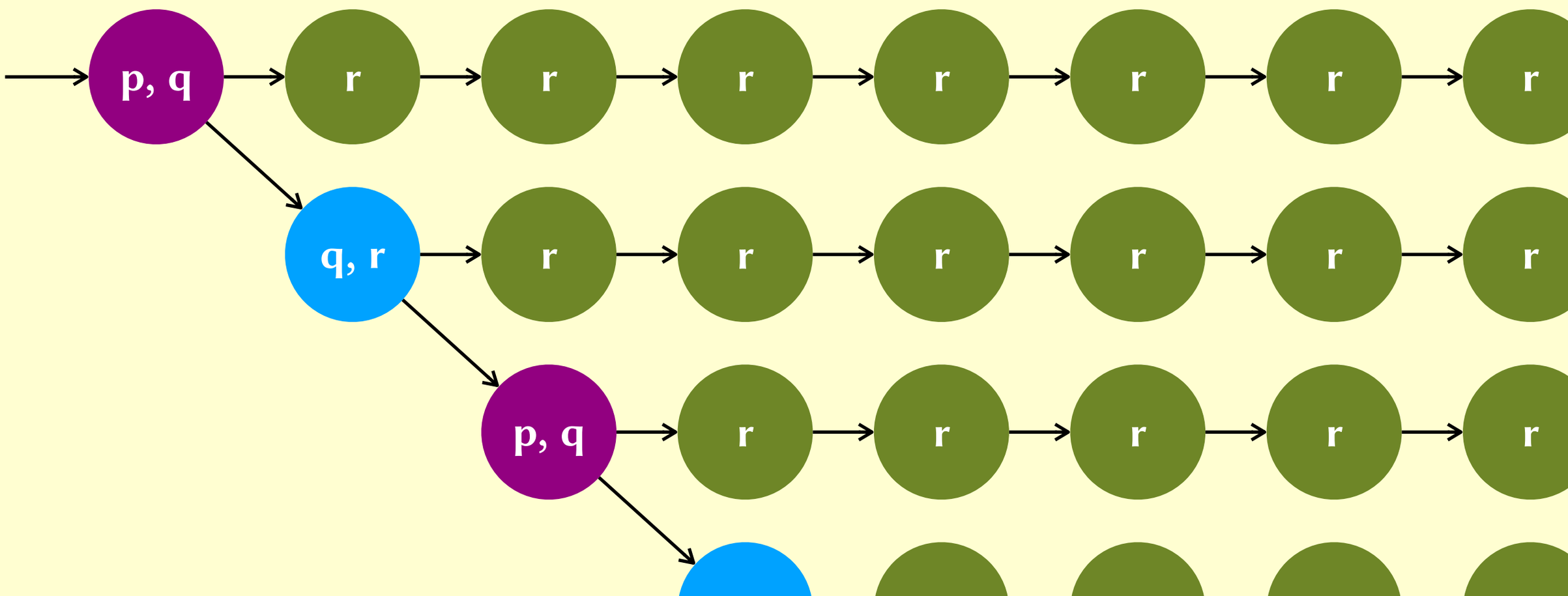
- Rather than unfolding the transition system into a set of paths...



From paths to trees



- ...unfold it into a single tree.
- Now formulas talk about this tree, rather than individual paths.



Computational Tree Logic

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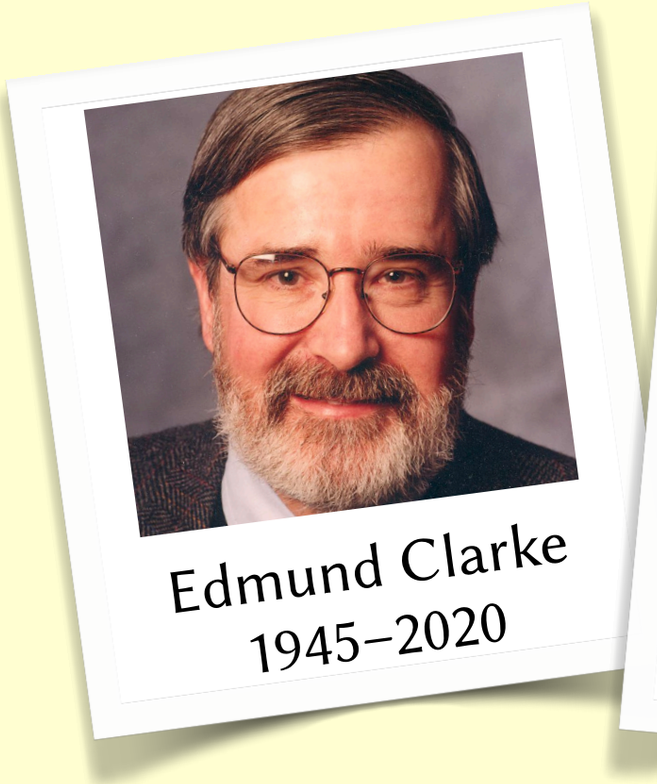
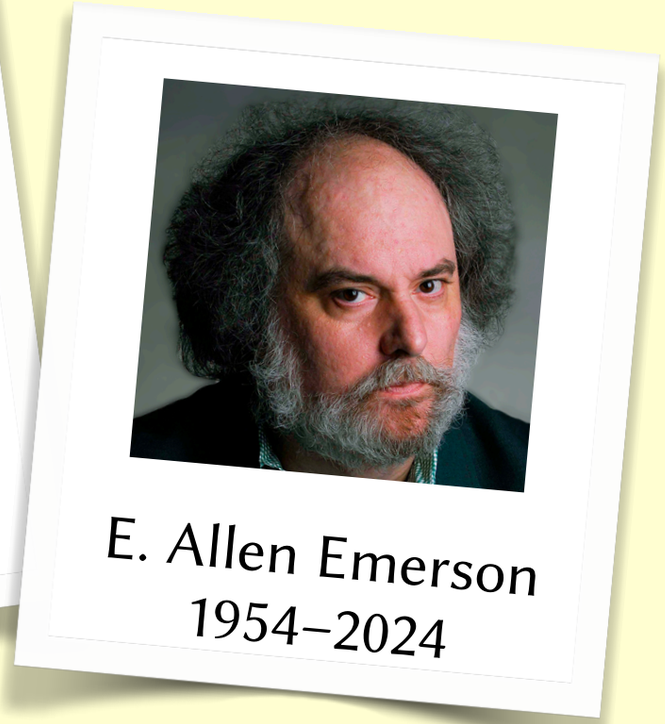
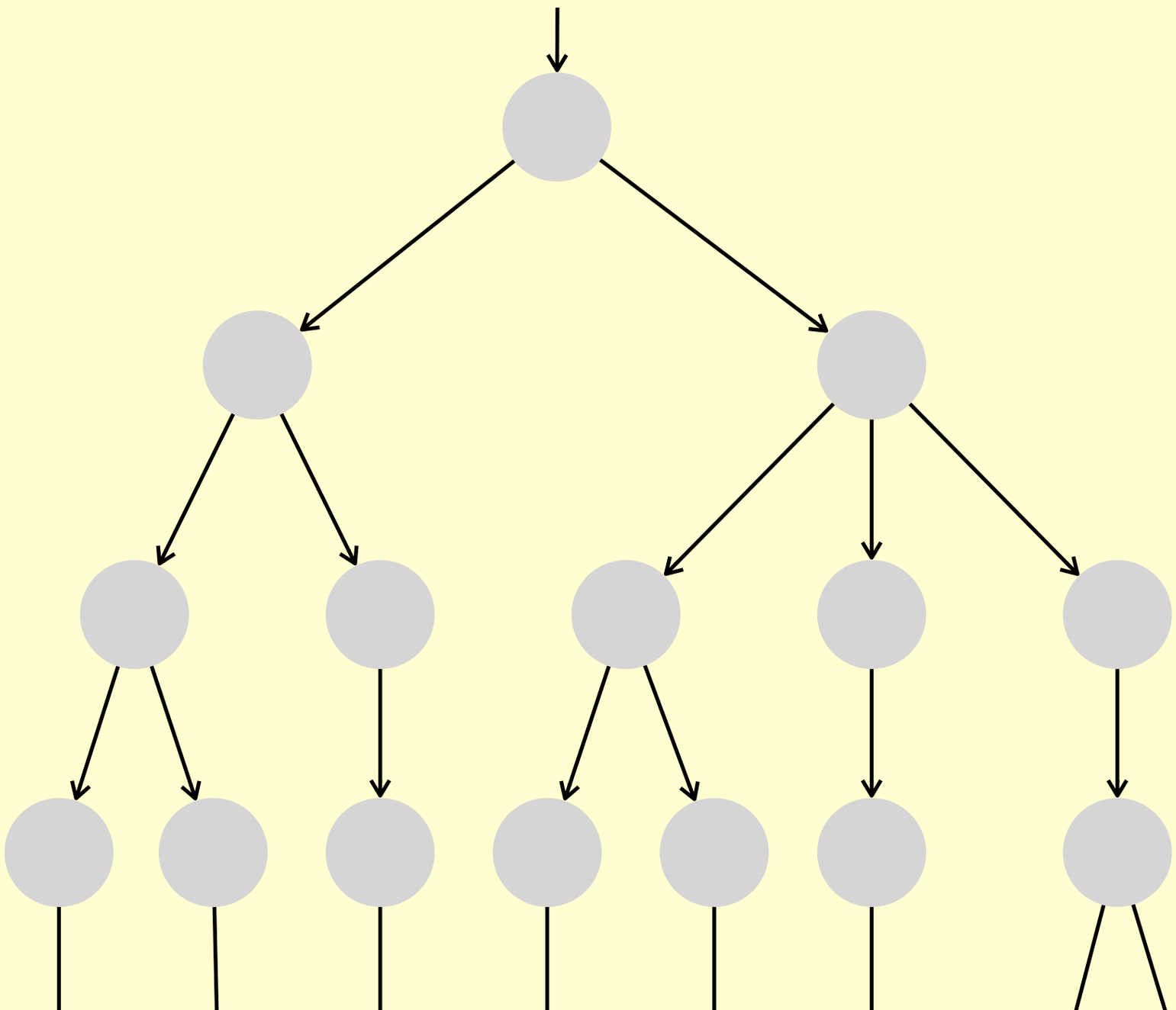
| **AX** ϕ | **EX** ϕ // ne**X**t state (**A**ll / there **E**xists)
| **AF** ϕ | **EF** ϕ // some **F**uture state
| **AG** ϕ | **EG** ϕ // all future states (**G**lobally)
| **A**[ϕ **U** ϕ] | **E**[ϕ **U** ϕ] // **U**ntil


Edmund Clarke
1945–2020

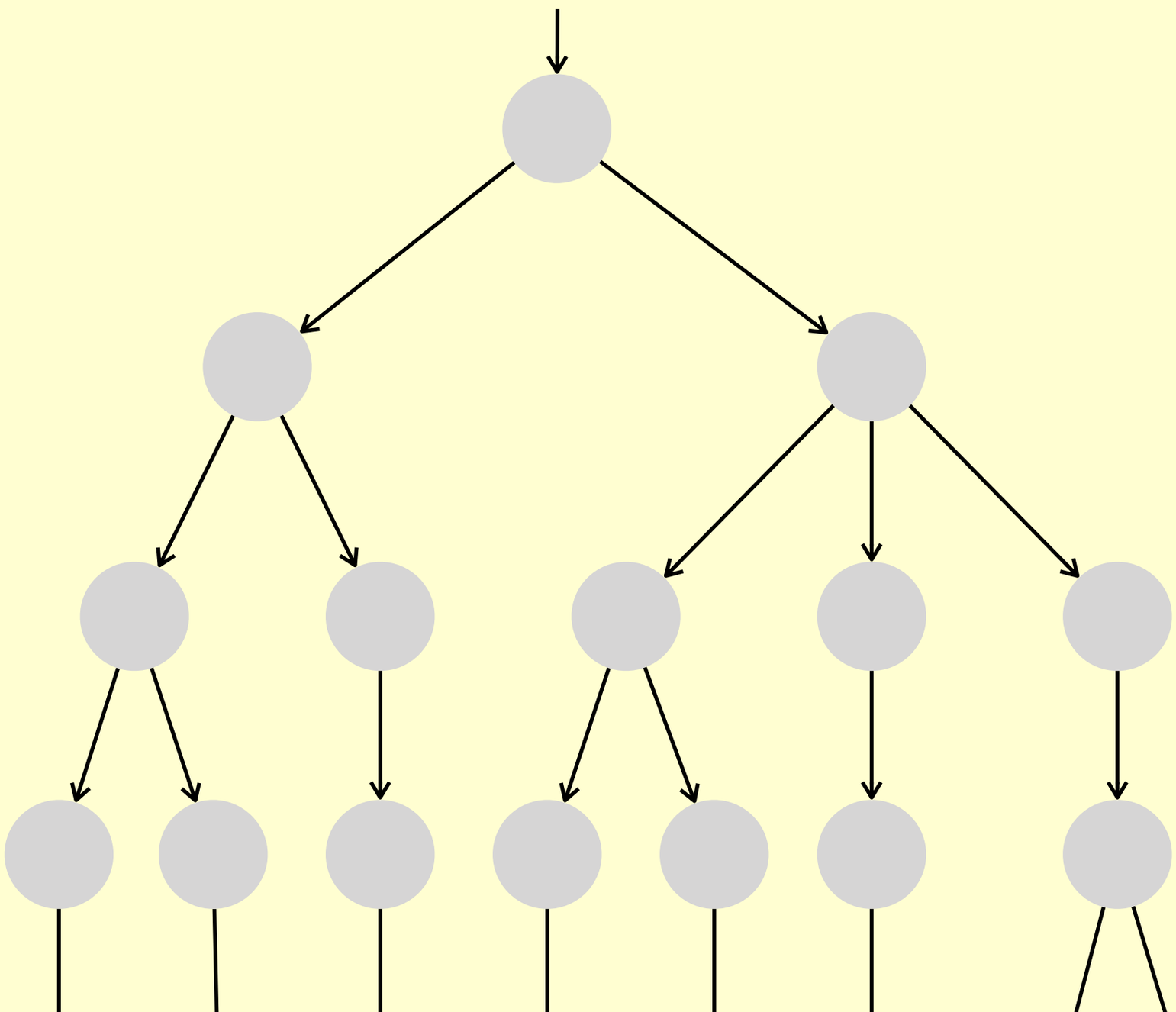

E. Allen Emerson
1954–2024

Computational Tree Logic



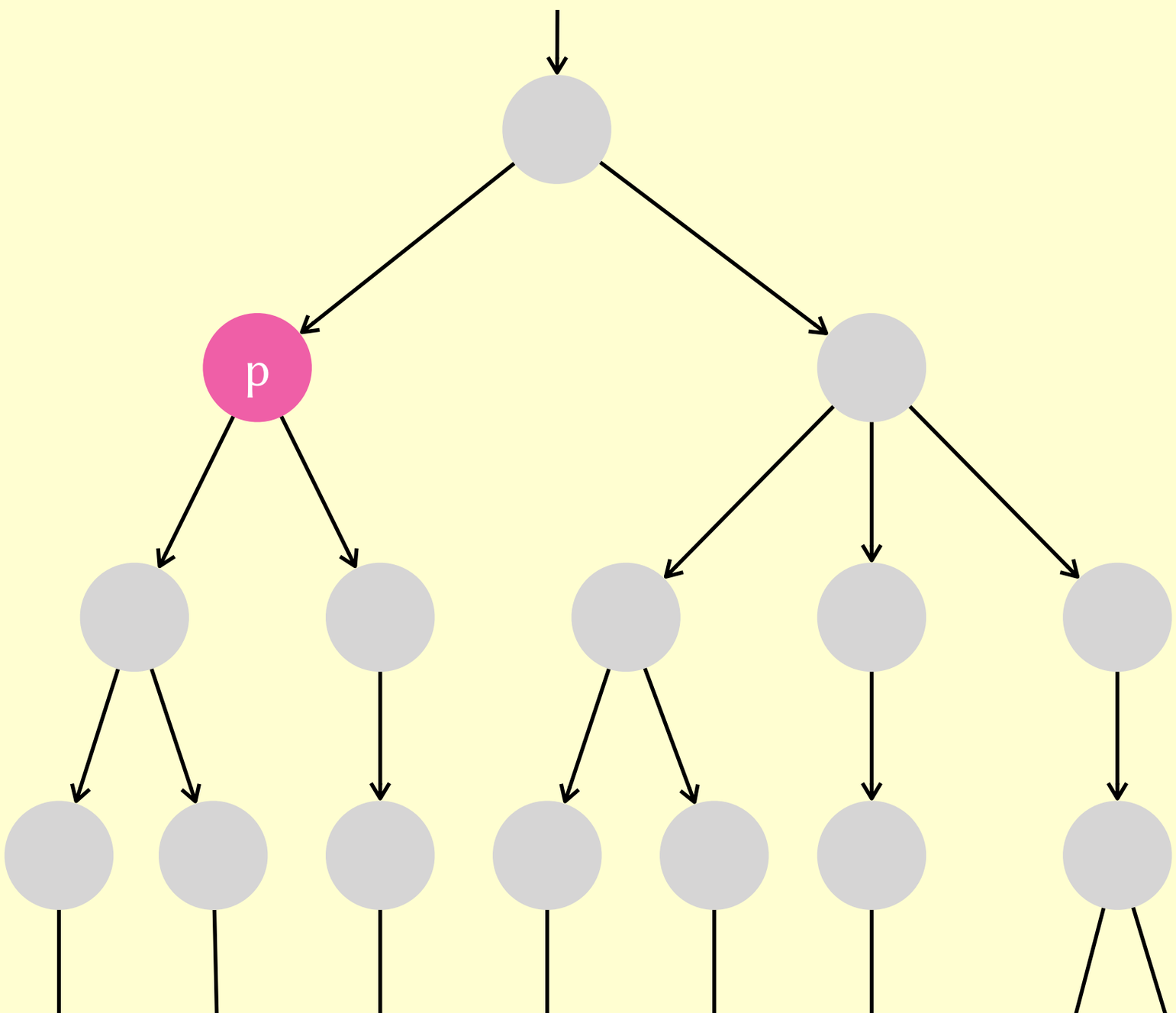
Computational Tree Logic

EX p



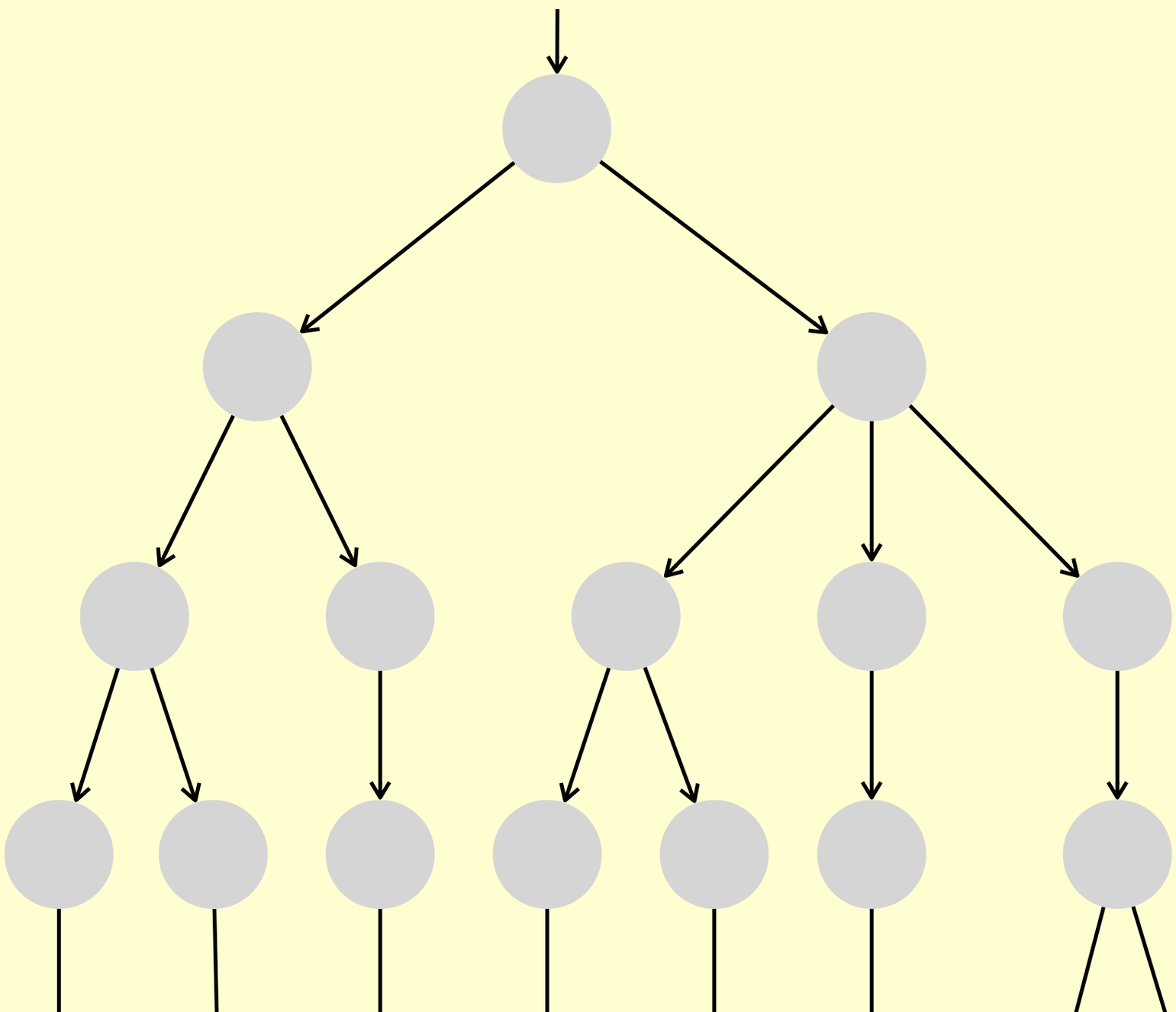
Computational Tree Logic

EX p



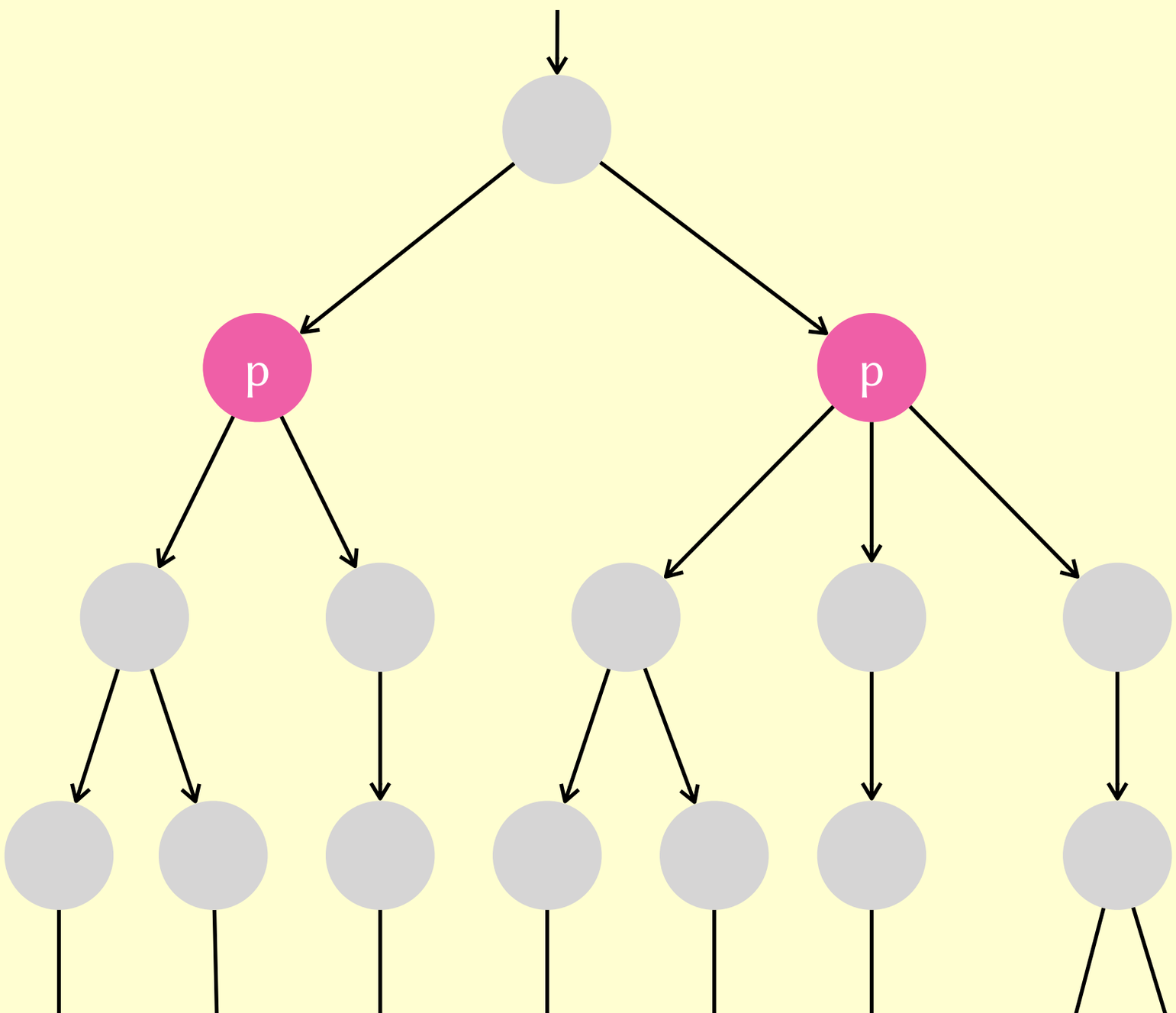
Computational Tree Logic

AX p



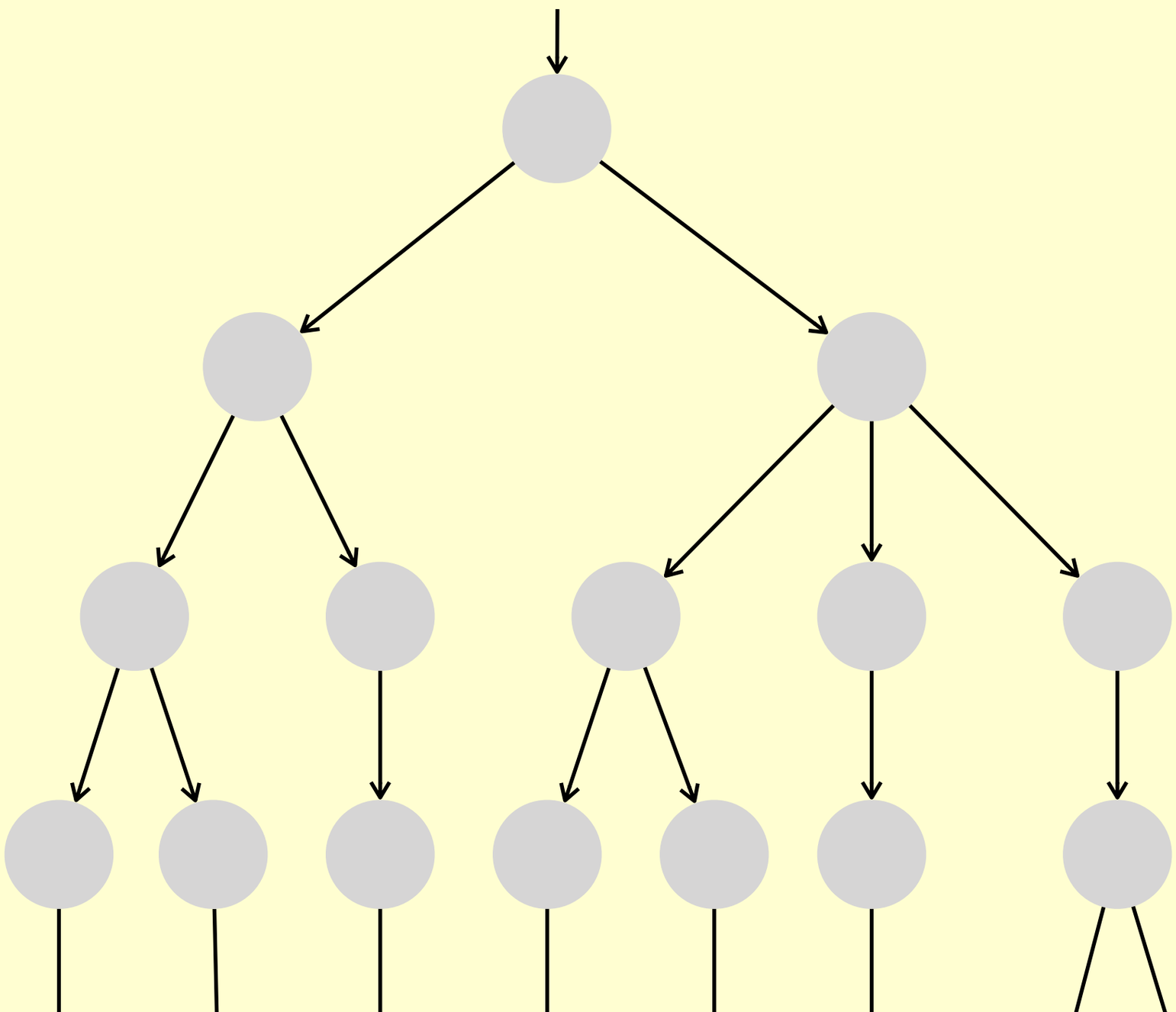
Computational Tree Logic

AX p

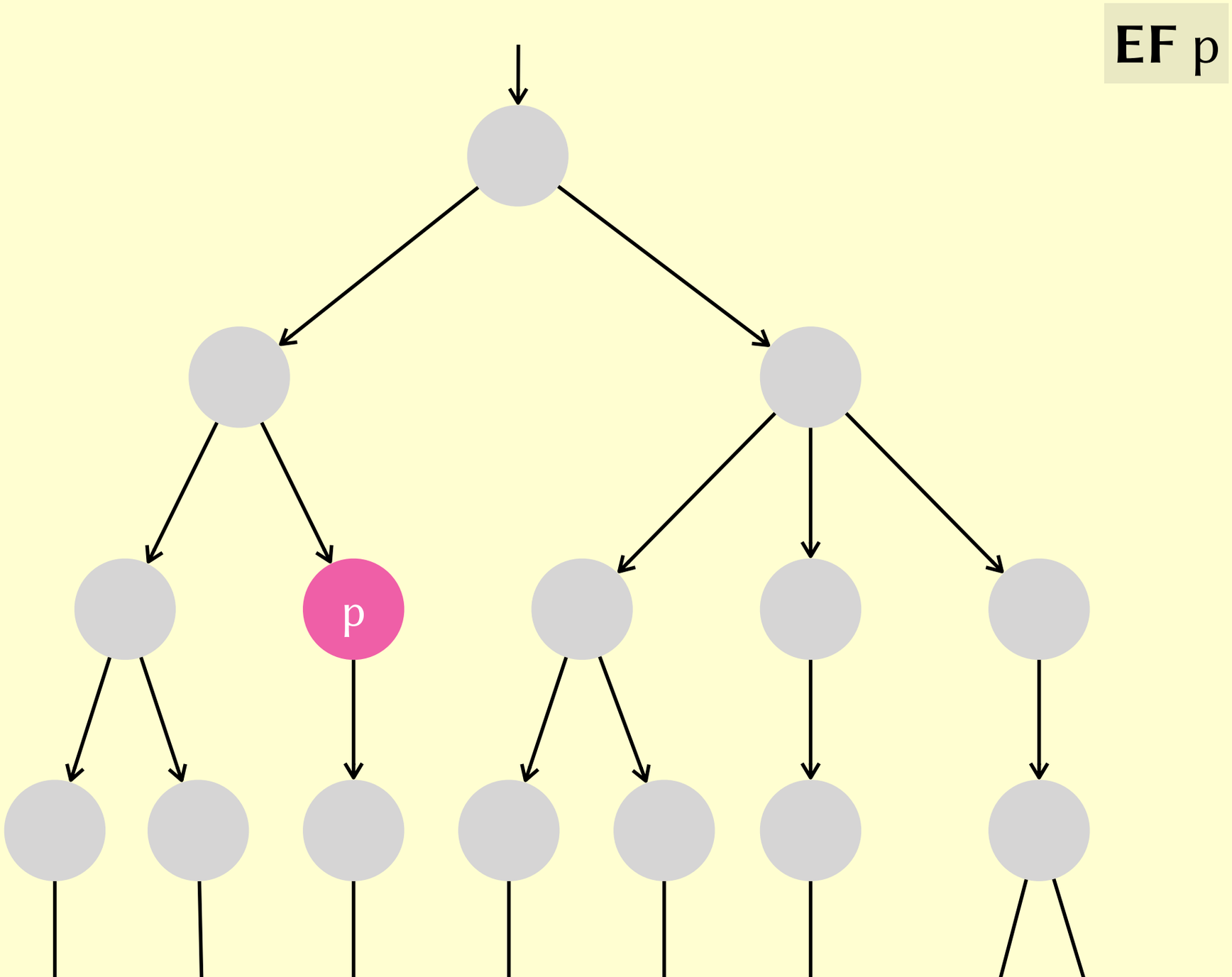


Computational Tree Logic

EF p

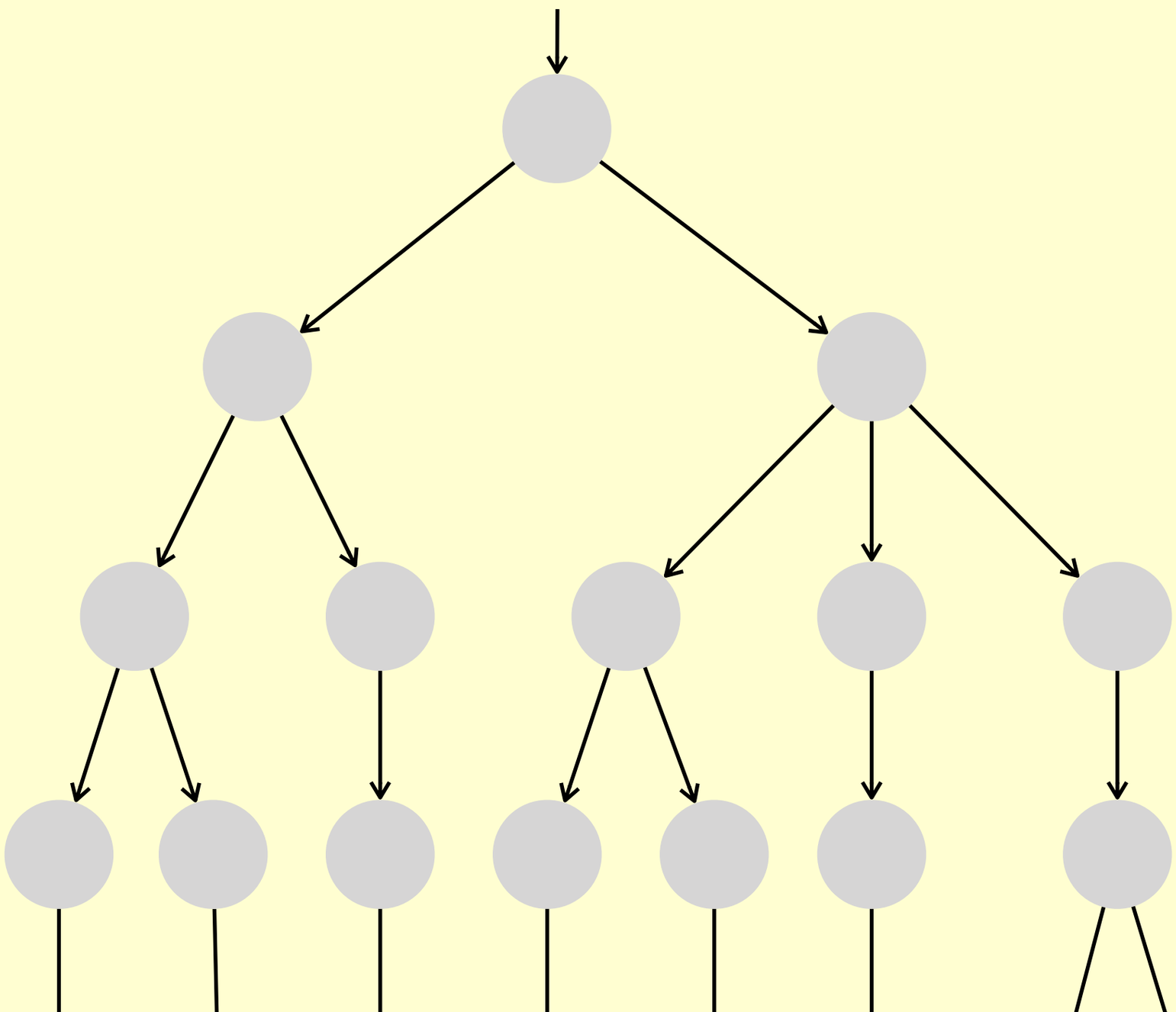


Computational Tree Logic



Computational Tree Logic

AF p



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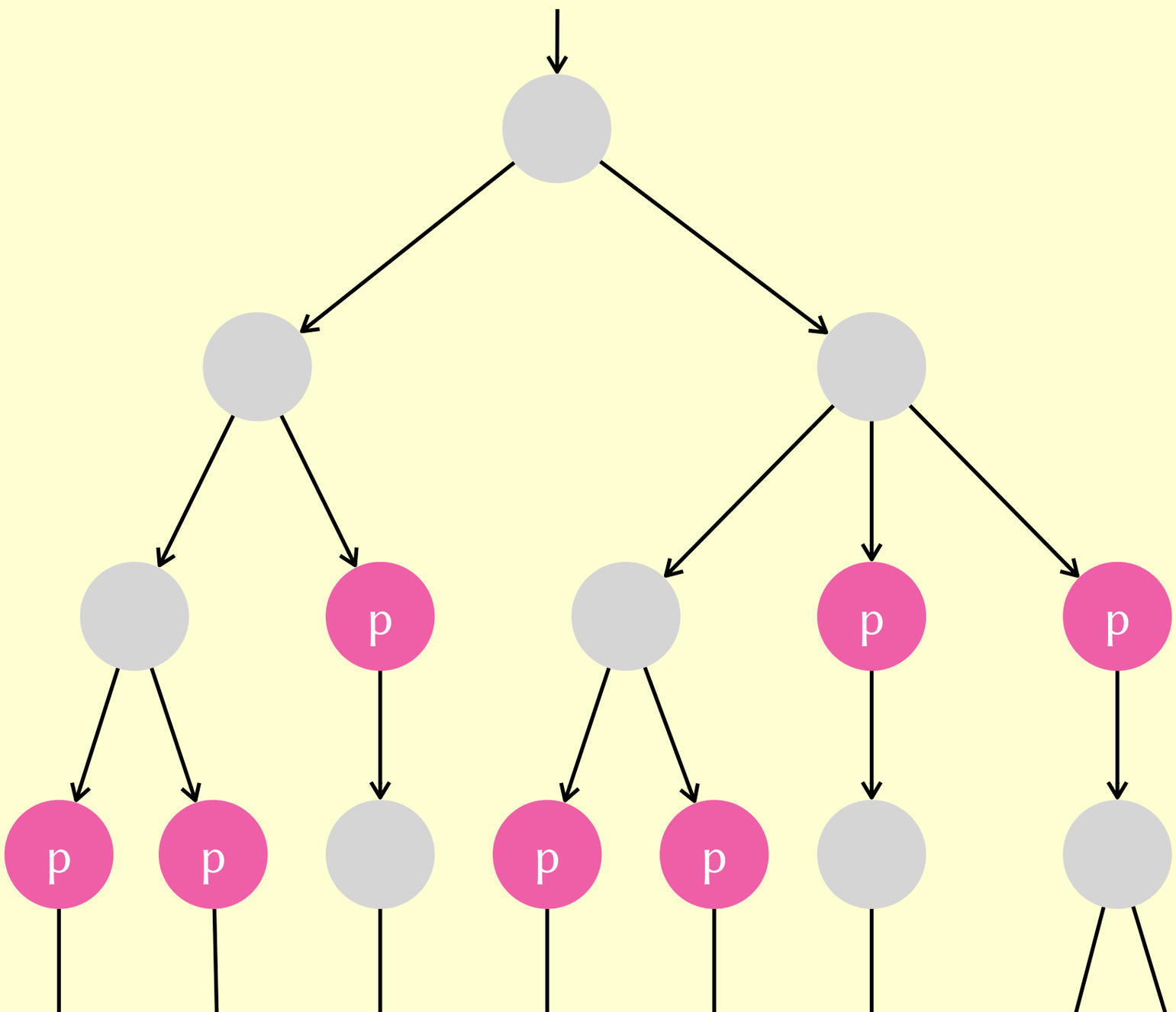
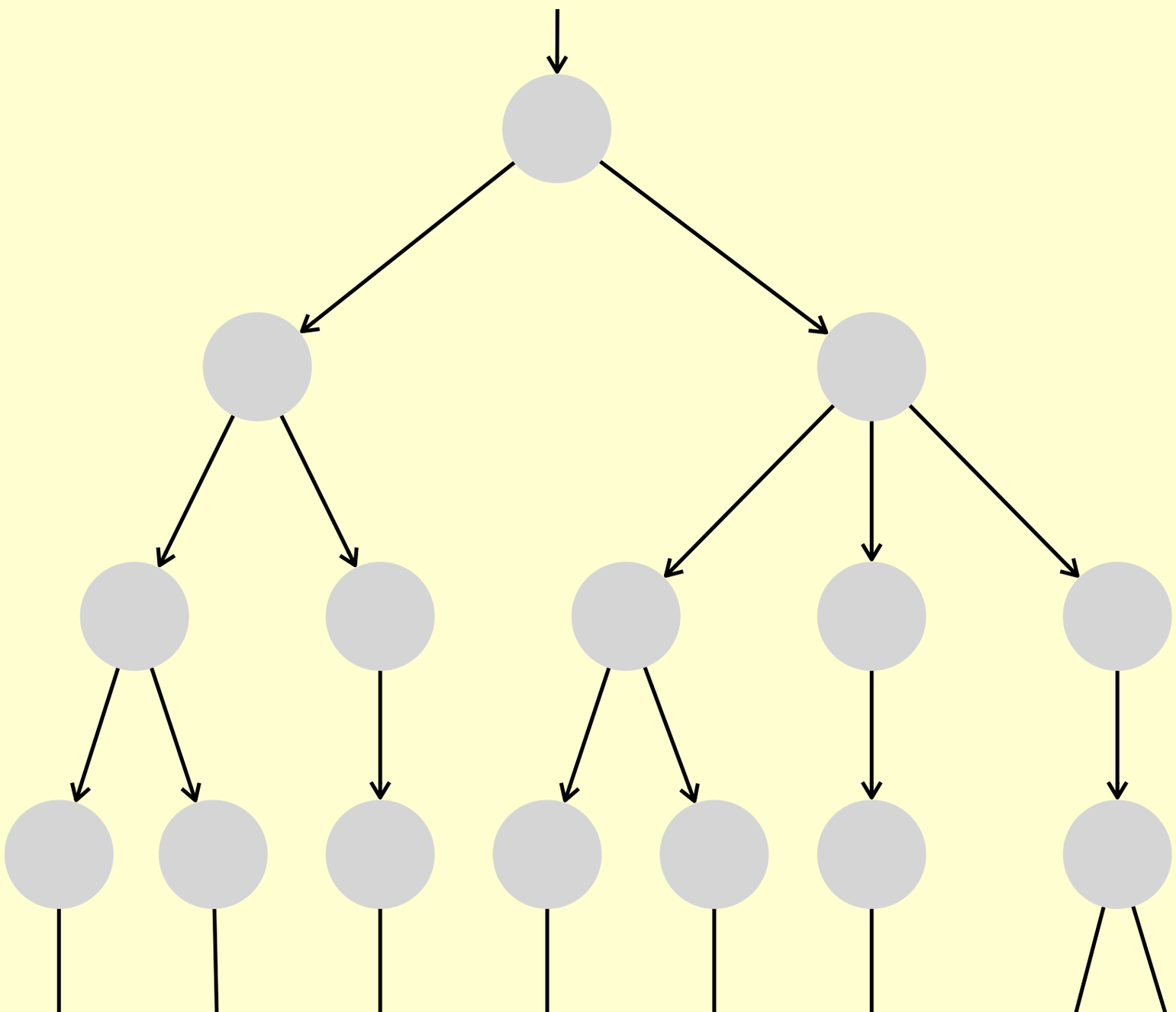


Computational Tree Logic

EG p



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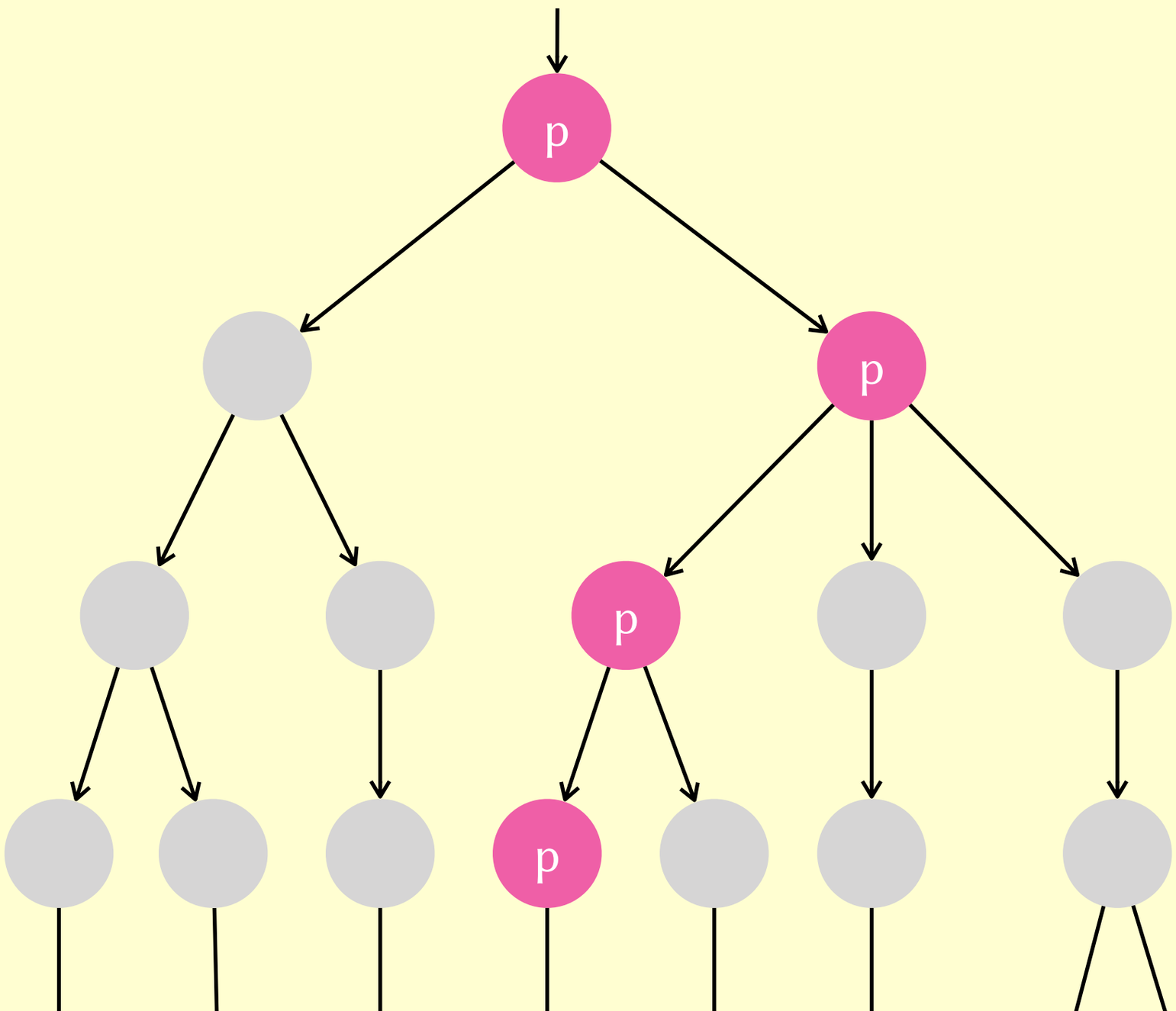
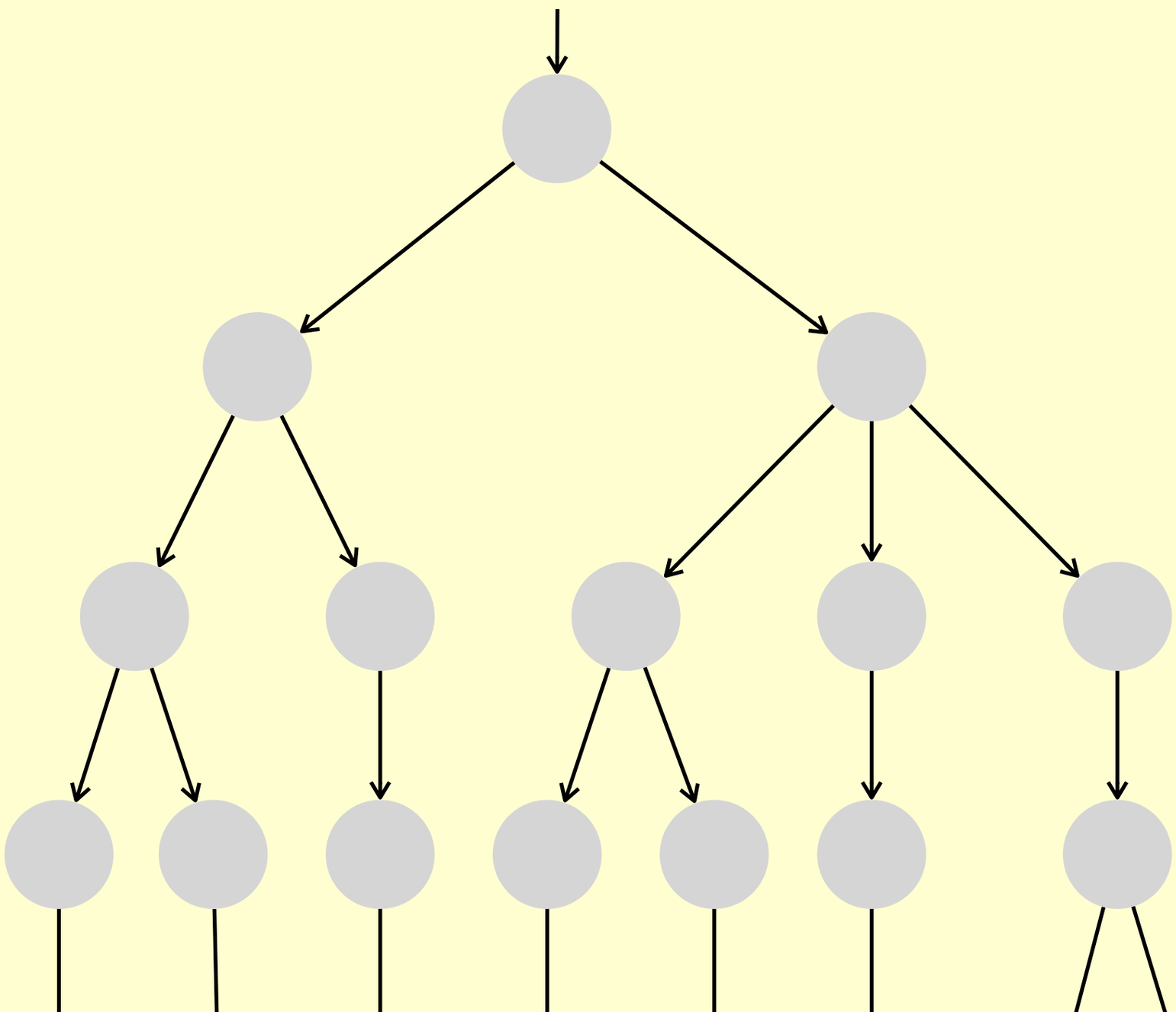


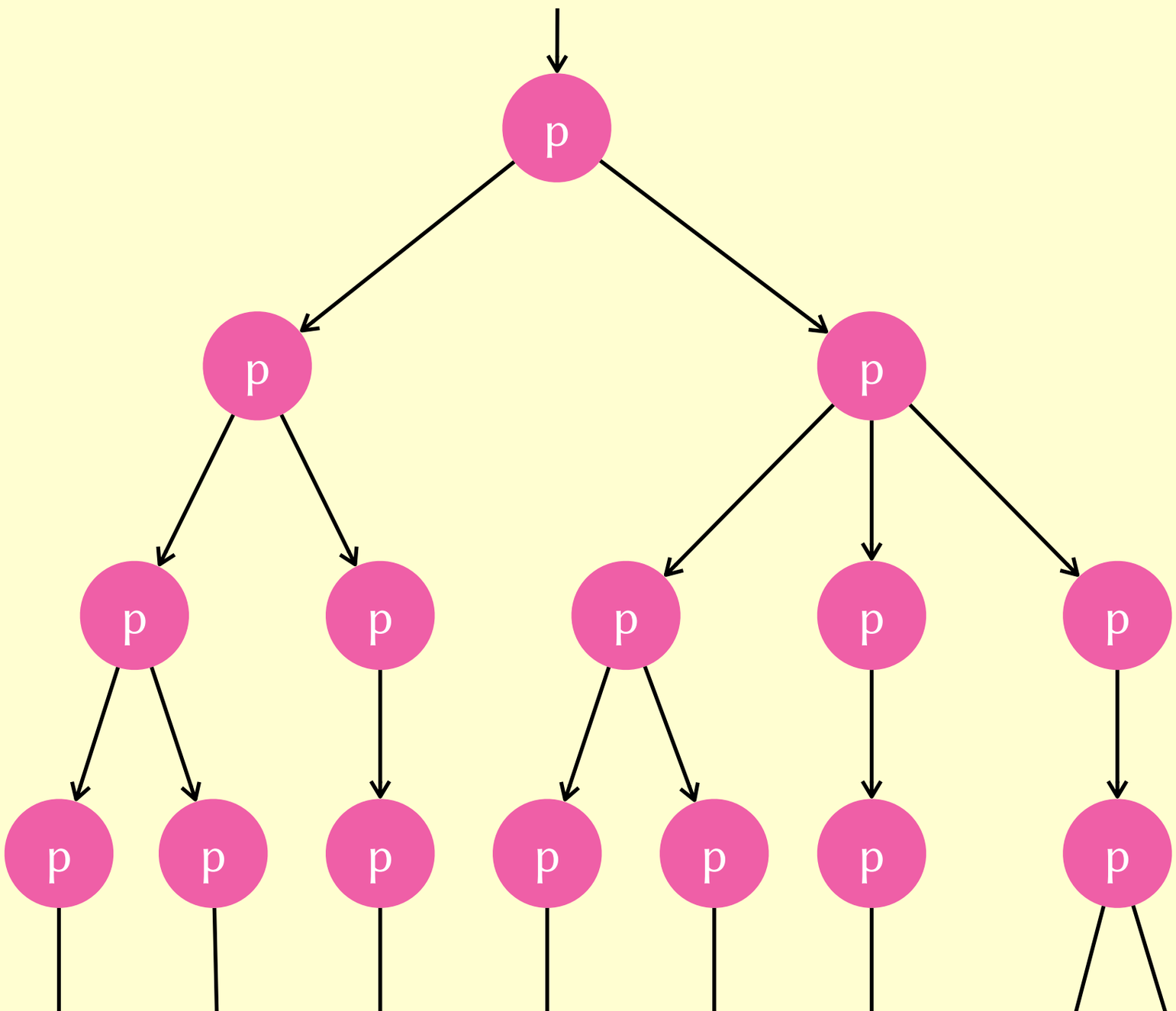
Computational Tree Logic

AG p



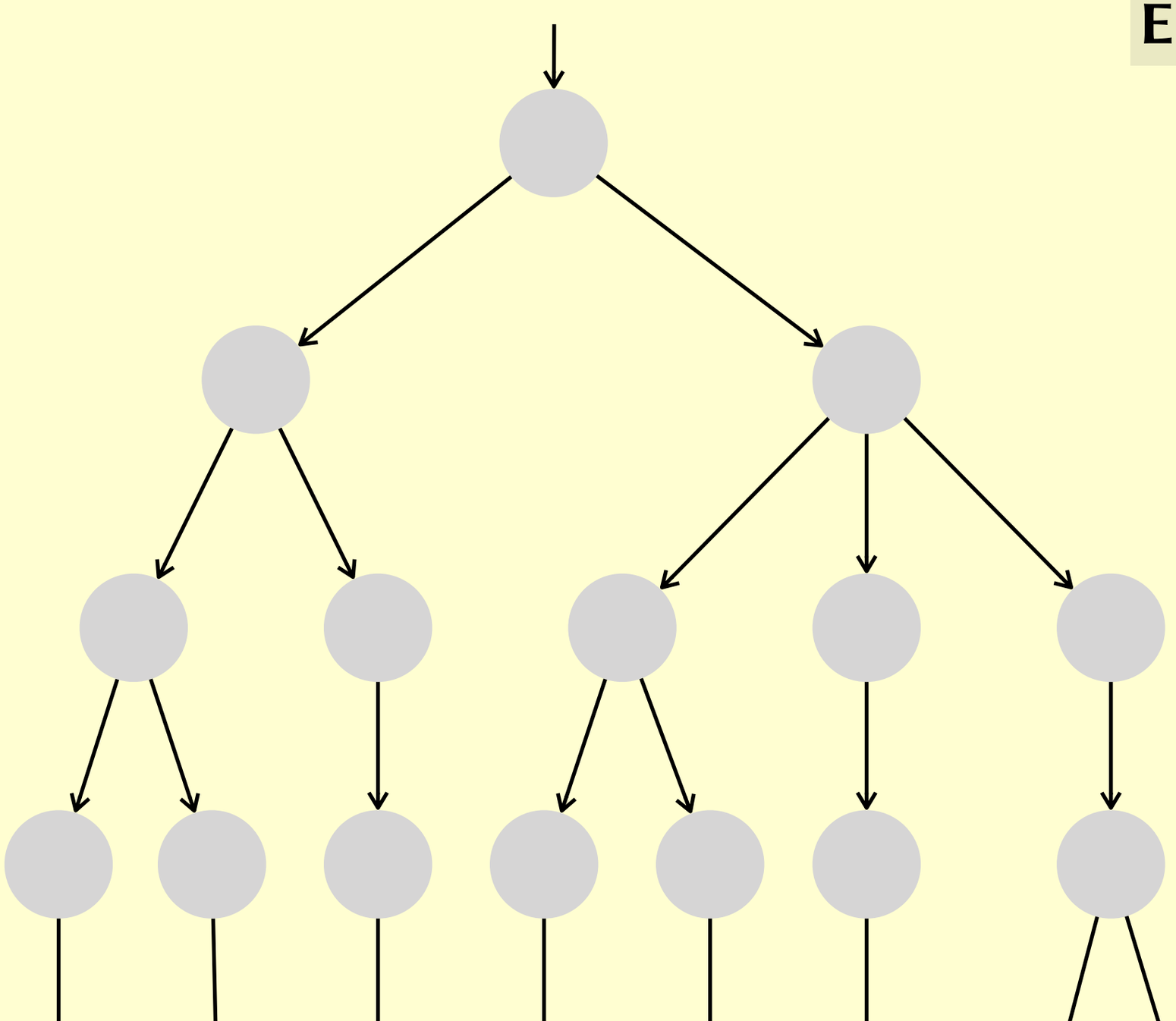
Computational Tree Logic

AG p



Computational Tree Logic

E[q **U** p]



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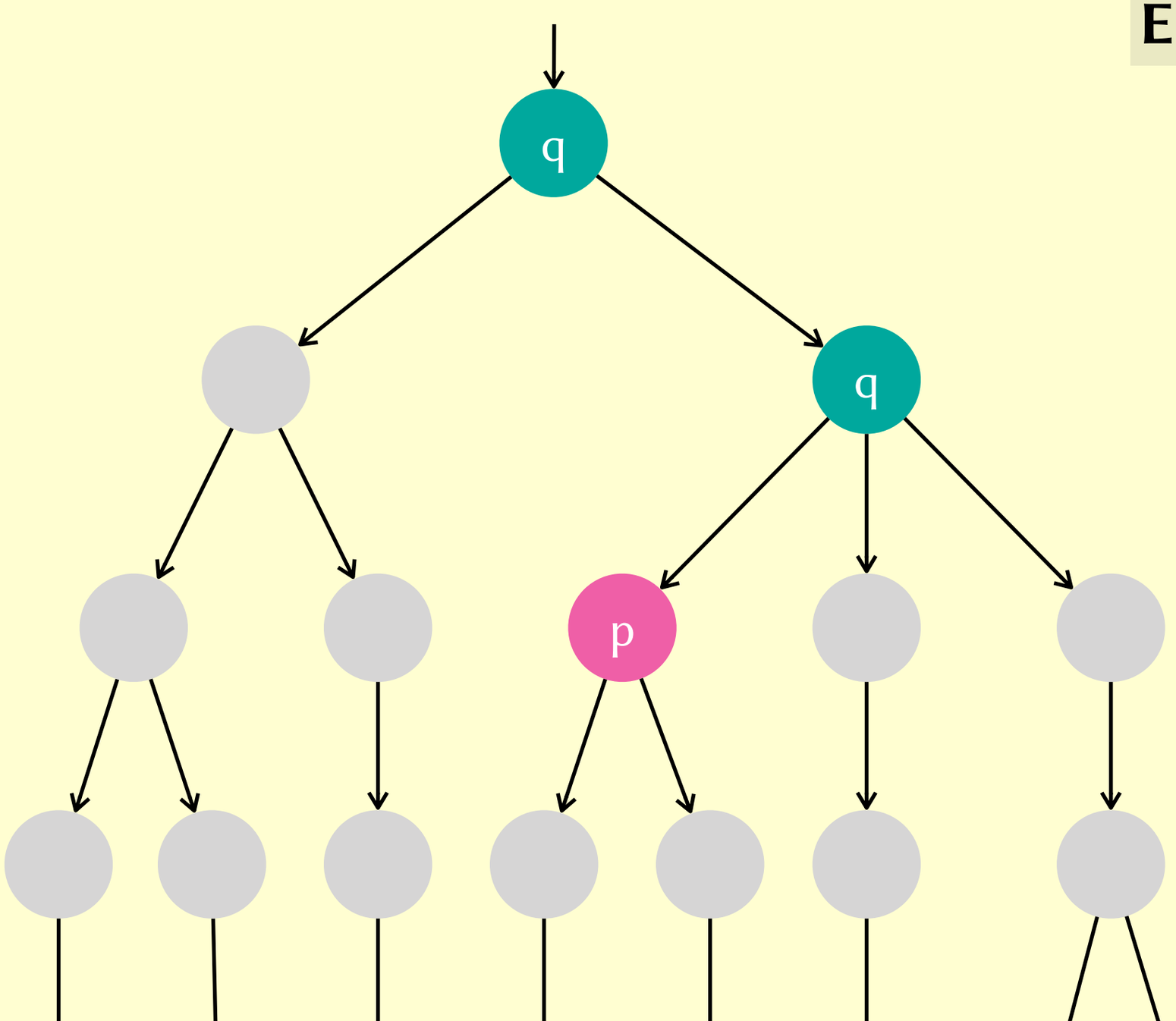

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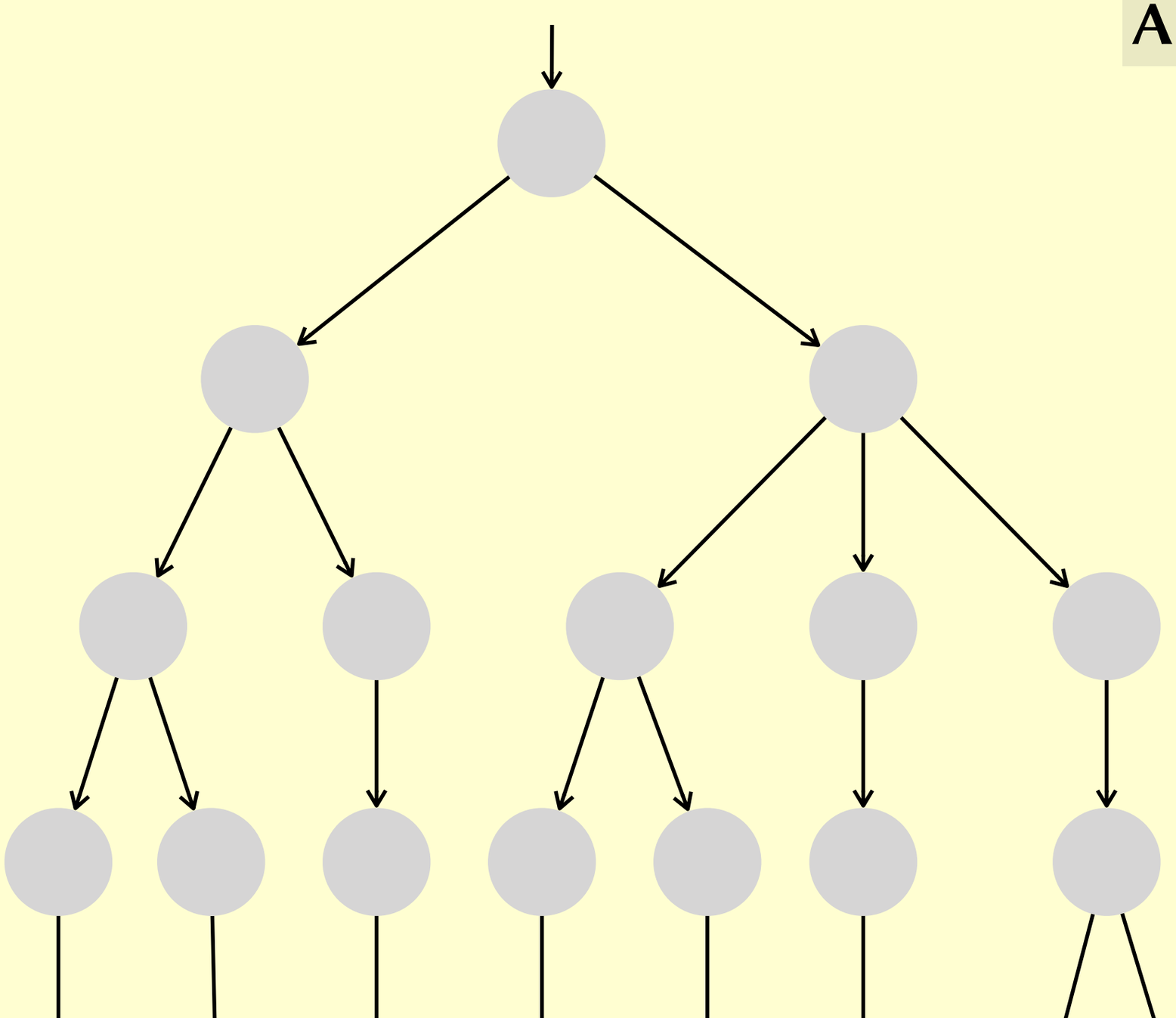

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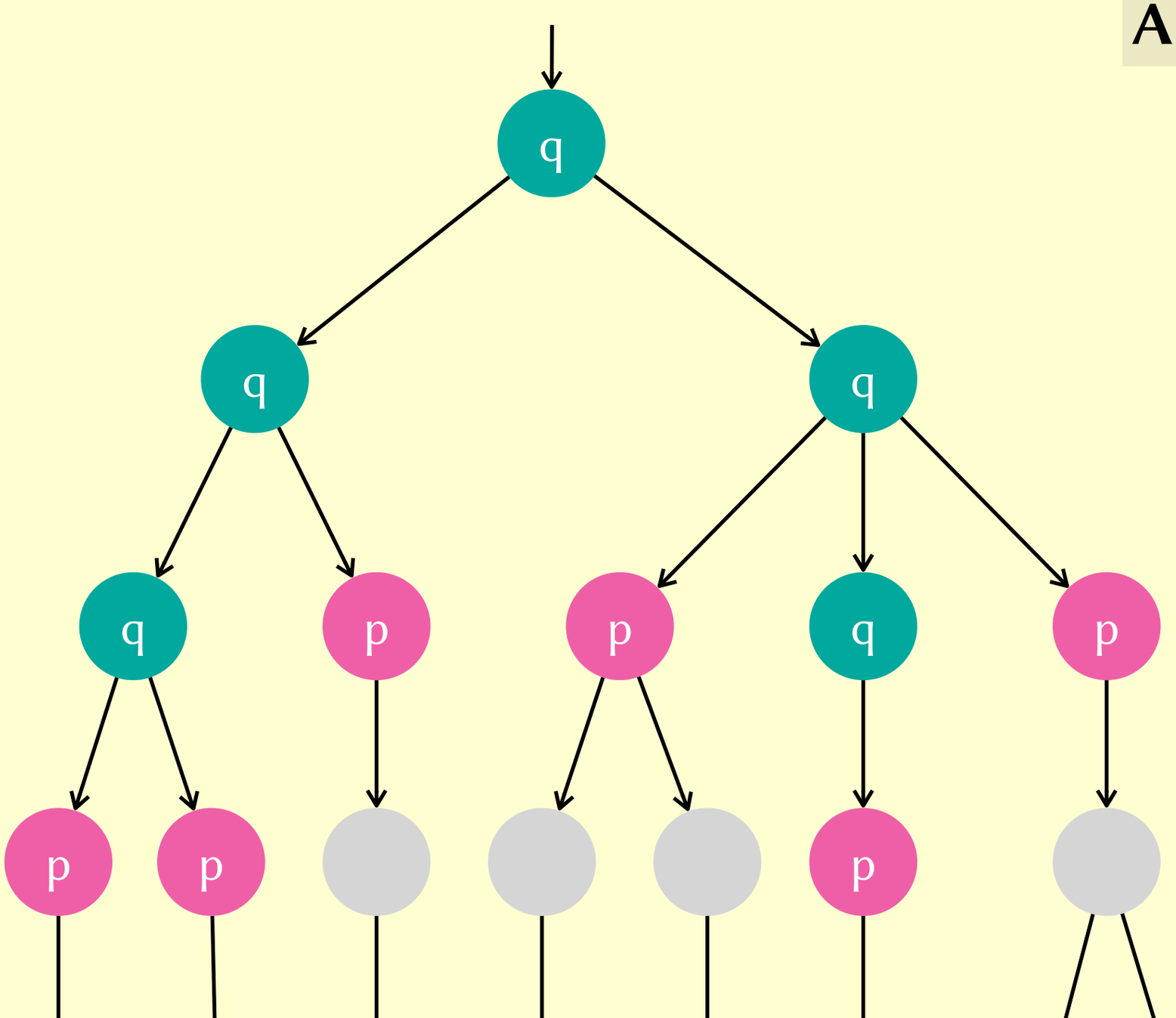

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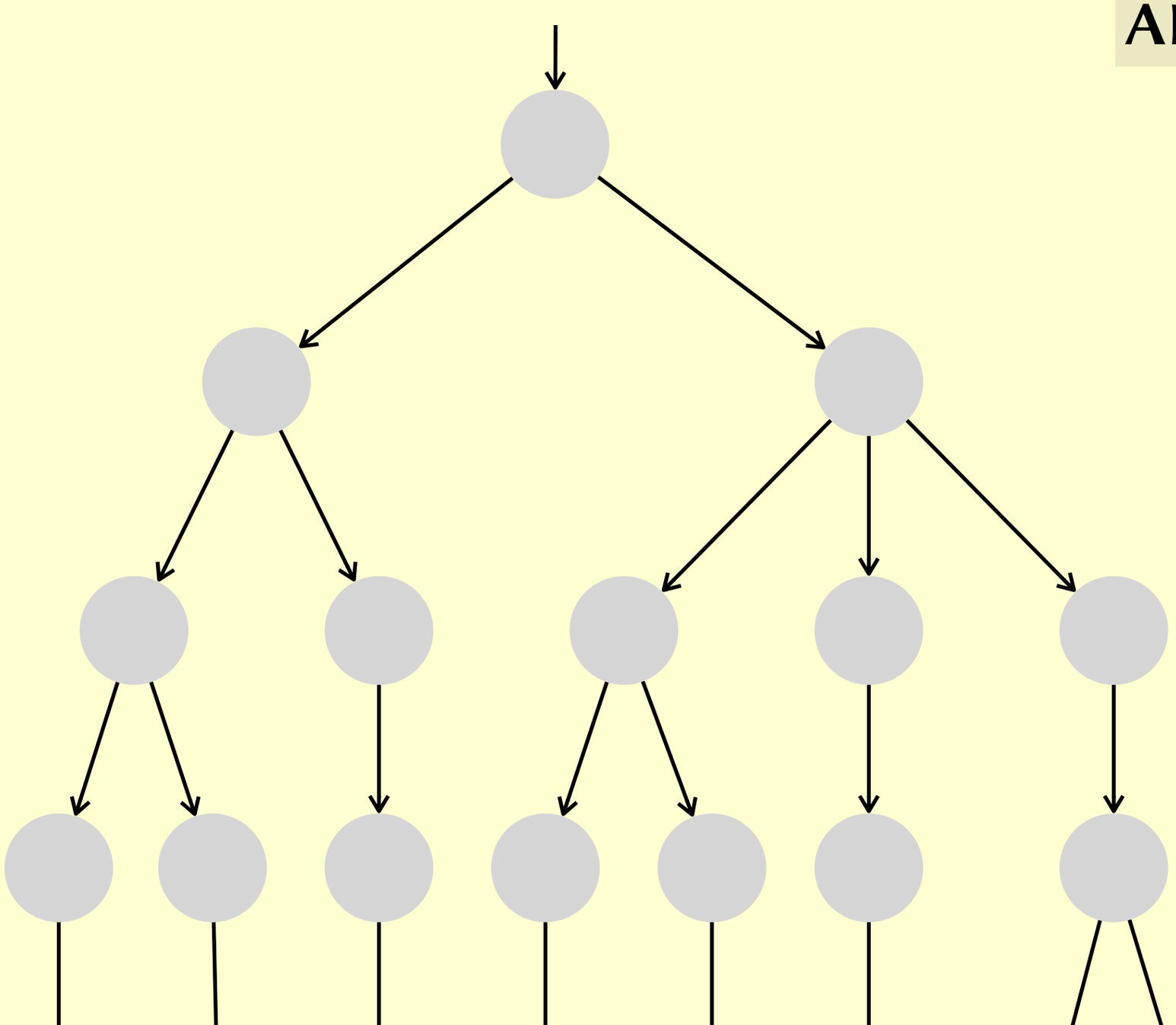

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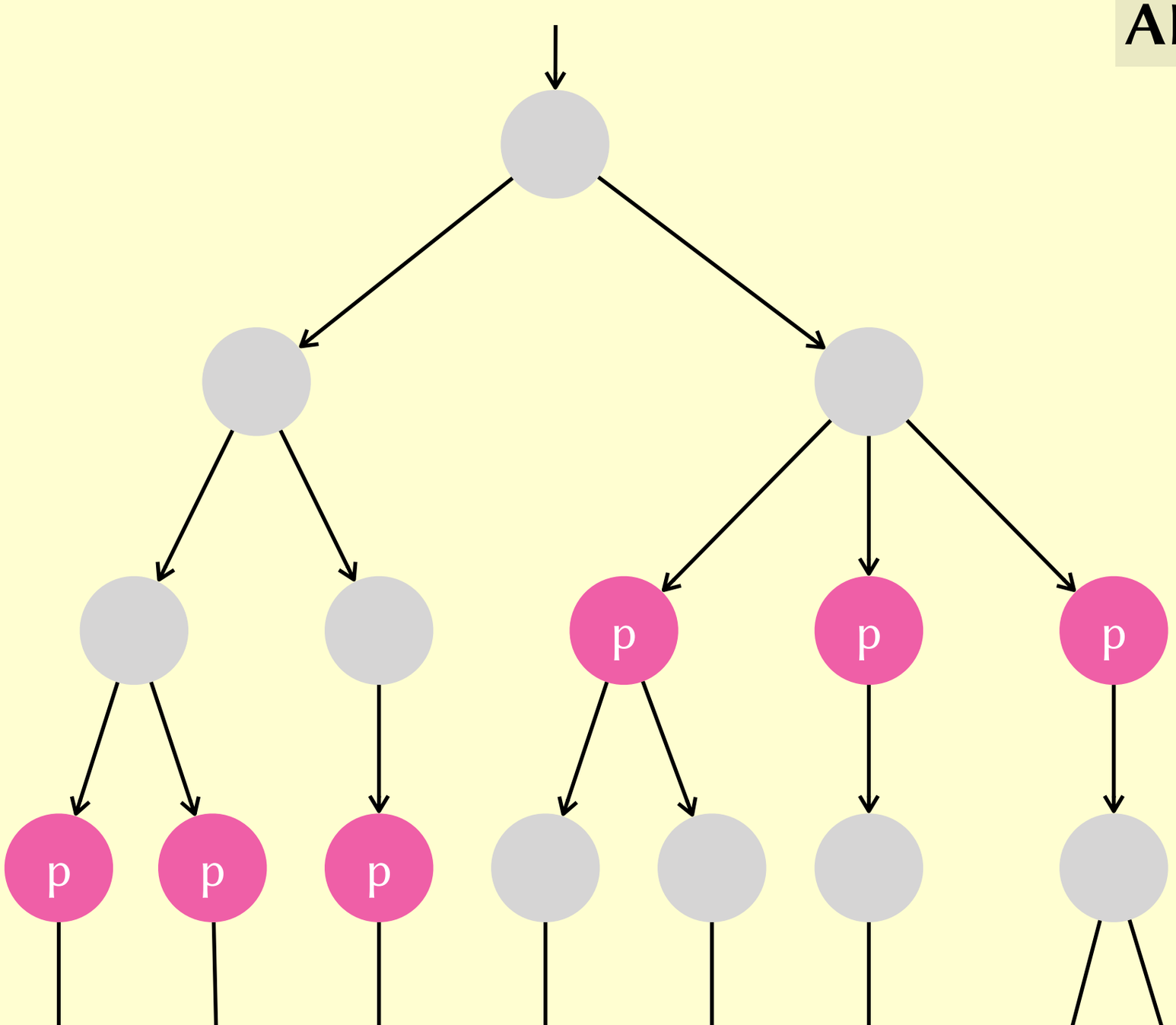

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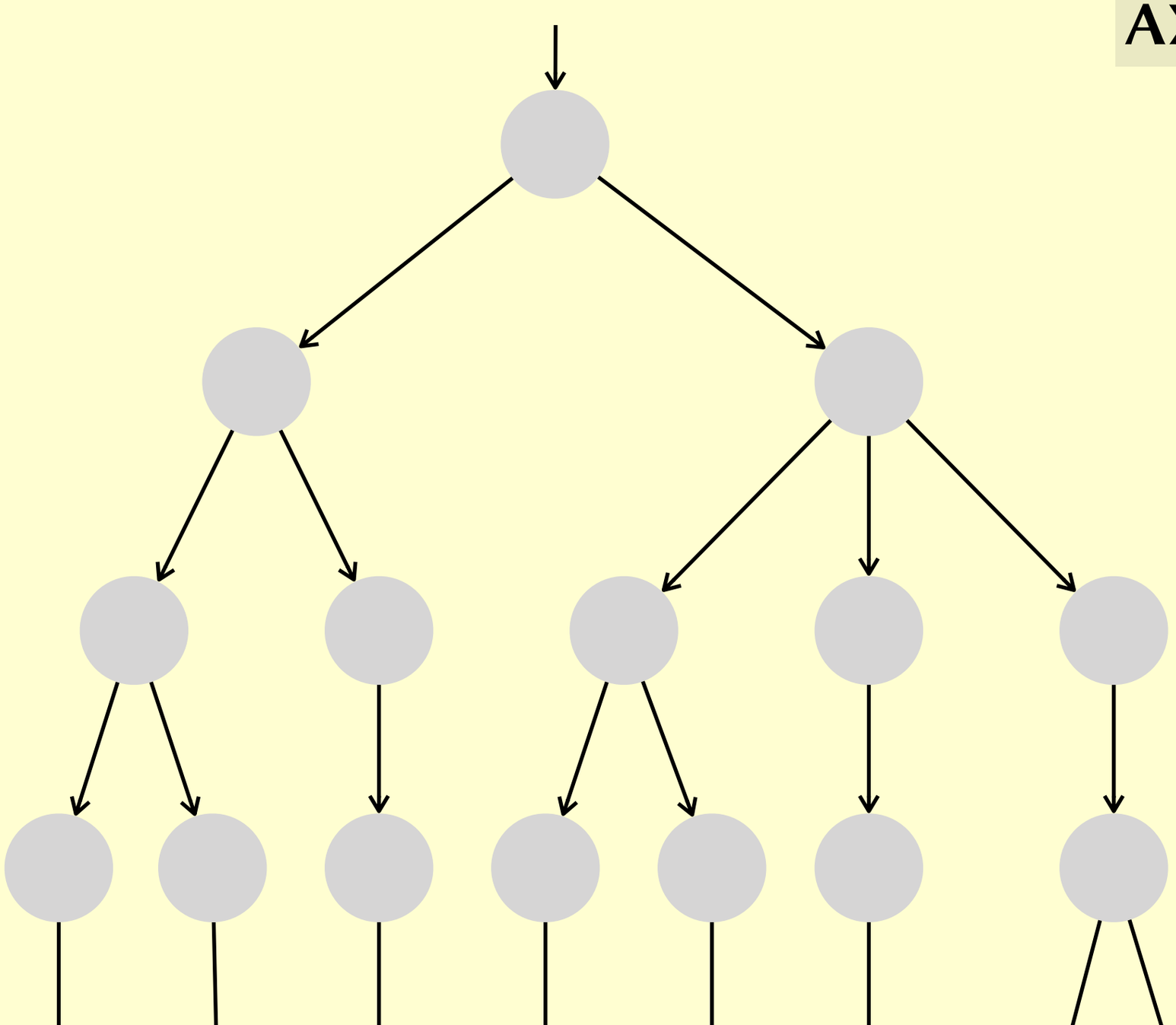

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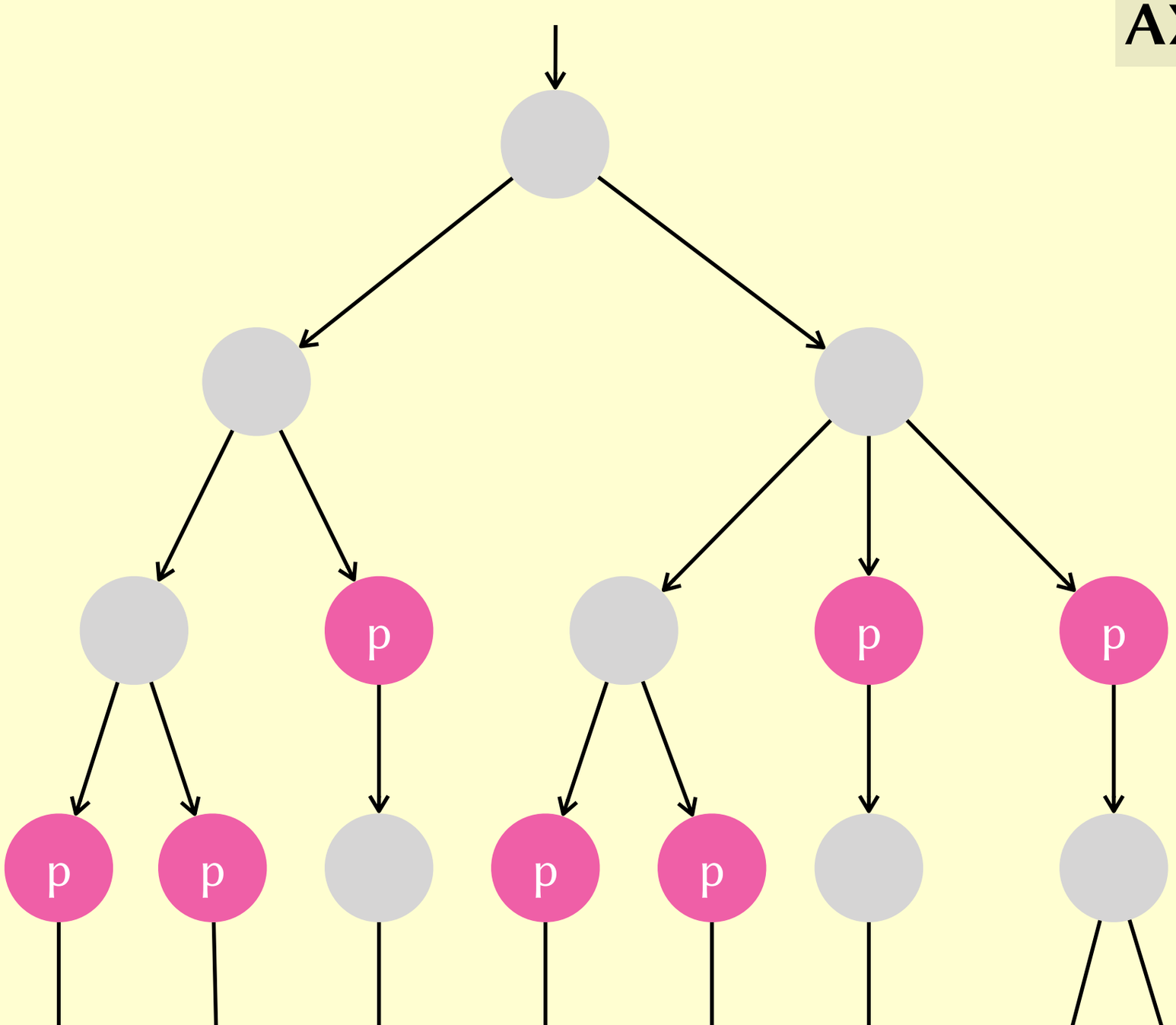

CTL to the rescue...

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assert (count > 0);
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$\mathbf{G}\ (\text{count} > 0)$ ✔ $\mathbf{AG}\ (\text{count} > 0)$ ✔

- And *concurrent* assertions like:

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assert (wen > 0 |=> waddr > 0);
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$\mathbf{G}\ (\text{wen} > 0 \longrightarrow \mathbf{X}\ (\text{waddr} > 0))$ ✔ $\mathbf{AG}\ (\text{wen} > 0 \longrightarrow \mathbf{AX}\ (\text{waddr} > 0))$ ✔

- And *coverage* statements like:

```
cover (count == 24);
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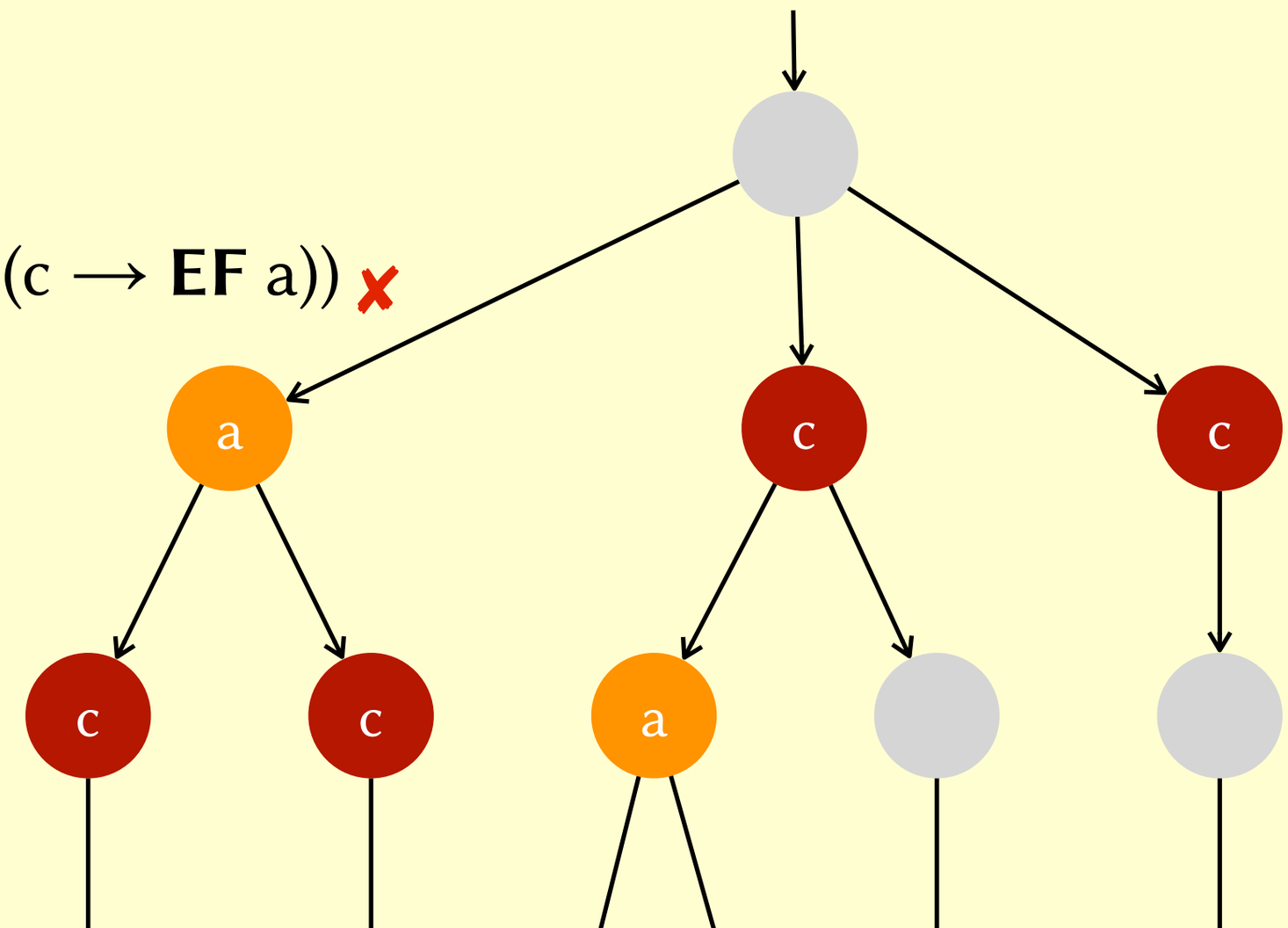
$\mathbf{F}\ (\text{count} = 24)$ ✘ $\mathbf{EF}\ (\text{count} = 24)$ ✔

- These can be generalised to formulas in *temporal logic*.

CTL's downfall...

- "All paths that contain an a also contain a c."
- (**AF** a) ⟶ (**AF** c) ✗
- **AF** (a ⟶ **EF** c) ✗
- **AF** ((a ⟶ **EF** c) ∧ (c ⟶ **EF** a)) ✗
- **F** a ⟶ **F** c ✓



LTL vs CTL

- Some properties can be expressed in CTL but not LTL, e.g. "it is possible to reach a state satisfying foo".
- Some properties can be expressed in LTL but not CTL, e.g. "every path that has a foo also has a baz".
- LTL formulas may be simpler to understand.
- CTL formulas may be more efficient to model-check.

CTL*

"state formulas"

$$\phi ::= \text{true} \mid \text{false} \mid p \mid \neg\,\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \longrightarrow \phi \mid \mathbf{A}\,\alpha \mid \mathbf{E}\,\alpha$$

"path formulas"

$$\alpha ::= \phi \mid \neg\,\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \longrightarrow \alpha \mid \mathbf{X}\,\alpha \mid \mathbf{F}\,\alpha \mid \mathbf{G}\,\alpha \mid \alpha\,\mathbf{U}\,\alpha$$

$\mathbf{EGF}\ p$

"There exists a path with infinitely many ps."

$\mathbf{A}(\mathbf{F}\ a \longrightarrow \mathbf{F}\ c)$

"All paths that contain an a also contain a c."

- Not actually used; just nice to know that LTL and CTL are both special cases of a more general temporal logic.

This lecture

- Temporal logic
- Binary decision diagrams

Boolean functions

- We often need to compare two Boolean functions for equivalence.
- Could represent Boolean functions via their truth tables.
- The time taken to check f≡g is exponential in the number of variables.

A	B	C	f
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**0**

A	B	C	g
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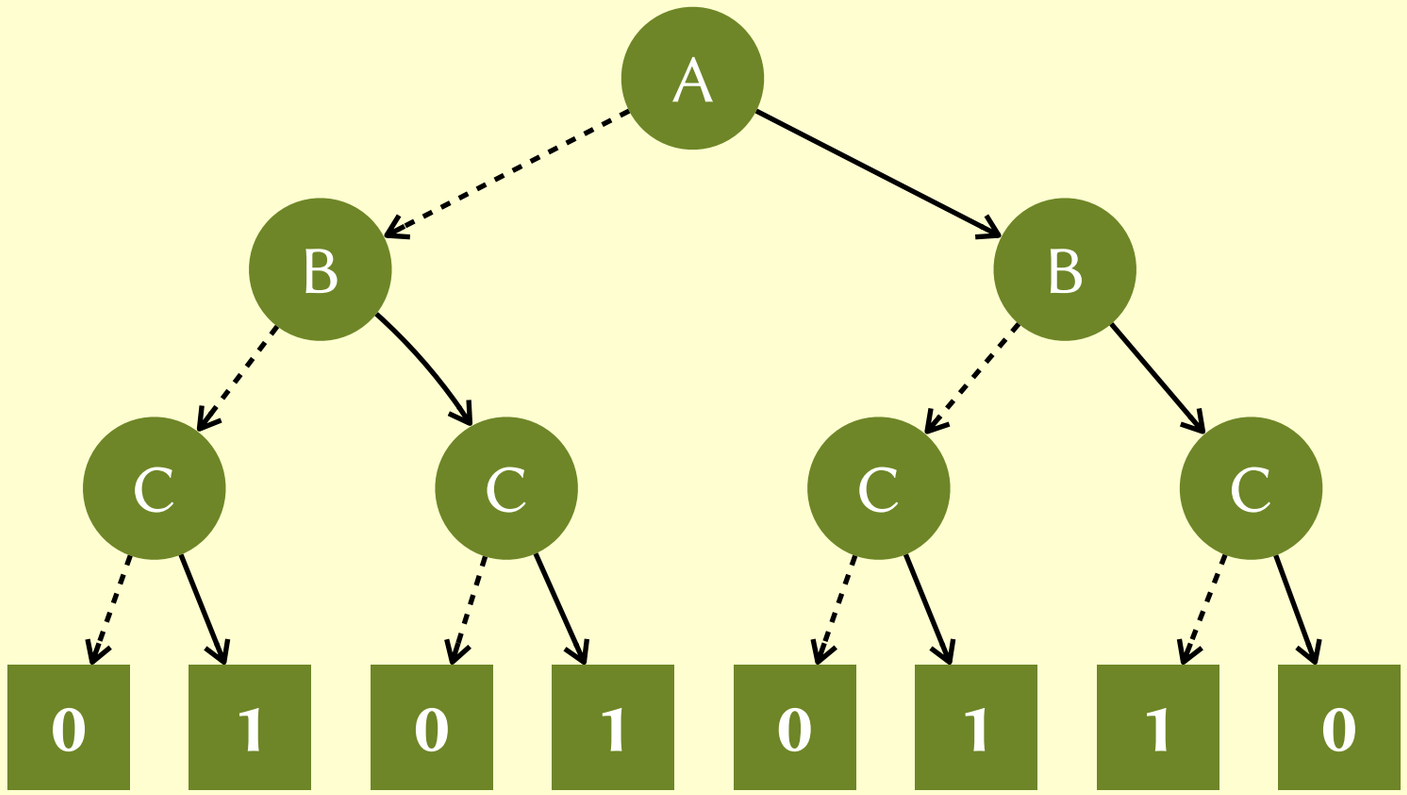
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**0**

Boolean functions

- Could represent Boolean functions as syntactic formulas.
- E.g. $f = (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge (B \oplus C))$
  $g = (\neg A \wedge C) \vee (A \wedge (B \oplus C))$
- The formulas may be compact, but the time taken to check $f \equiv g$ is still exponential in the number of variables.

Boolean functions

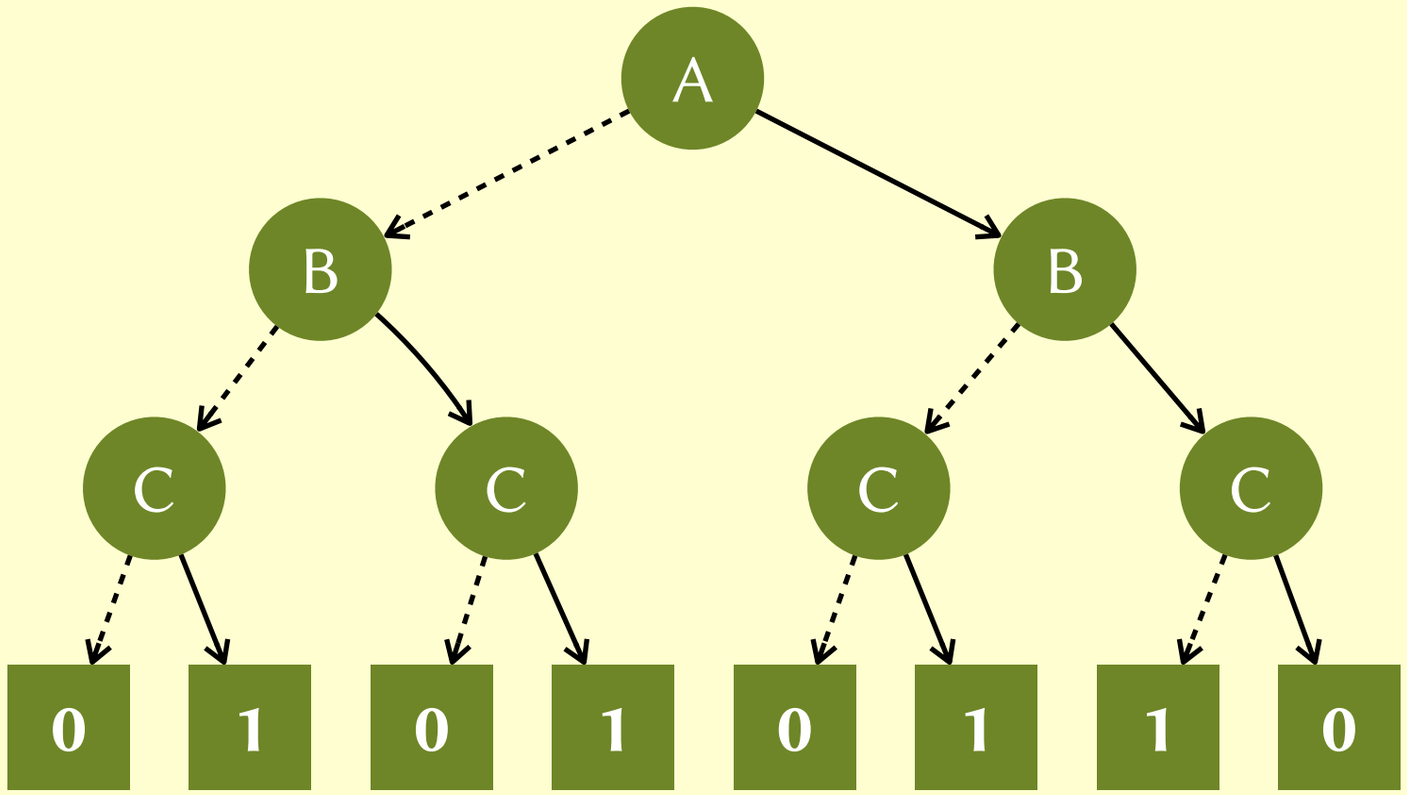
- Could represent Boolean functions as a binary decision tree.



- This is no more space efficient than the truth table, but…

Boolean functions

1. Share all the 0s and 1s.



Boolean functions

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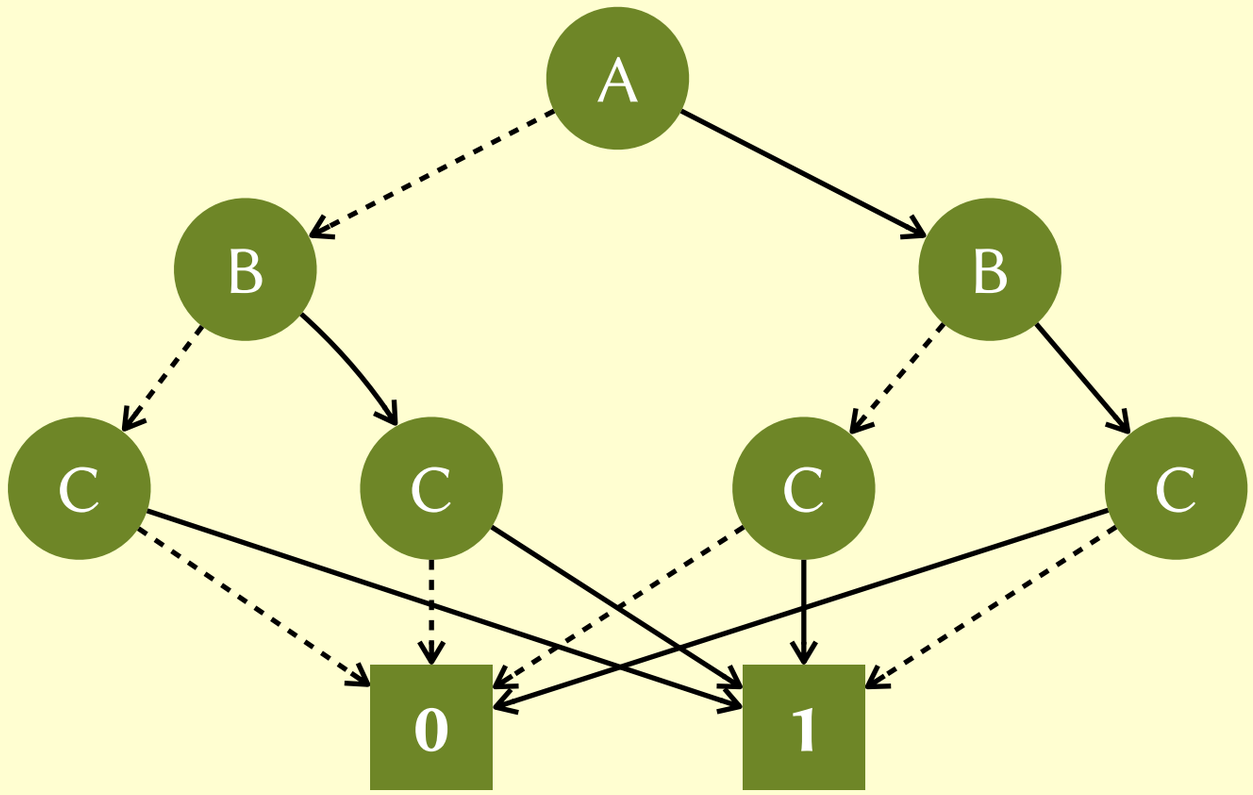
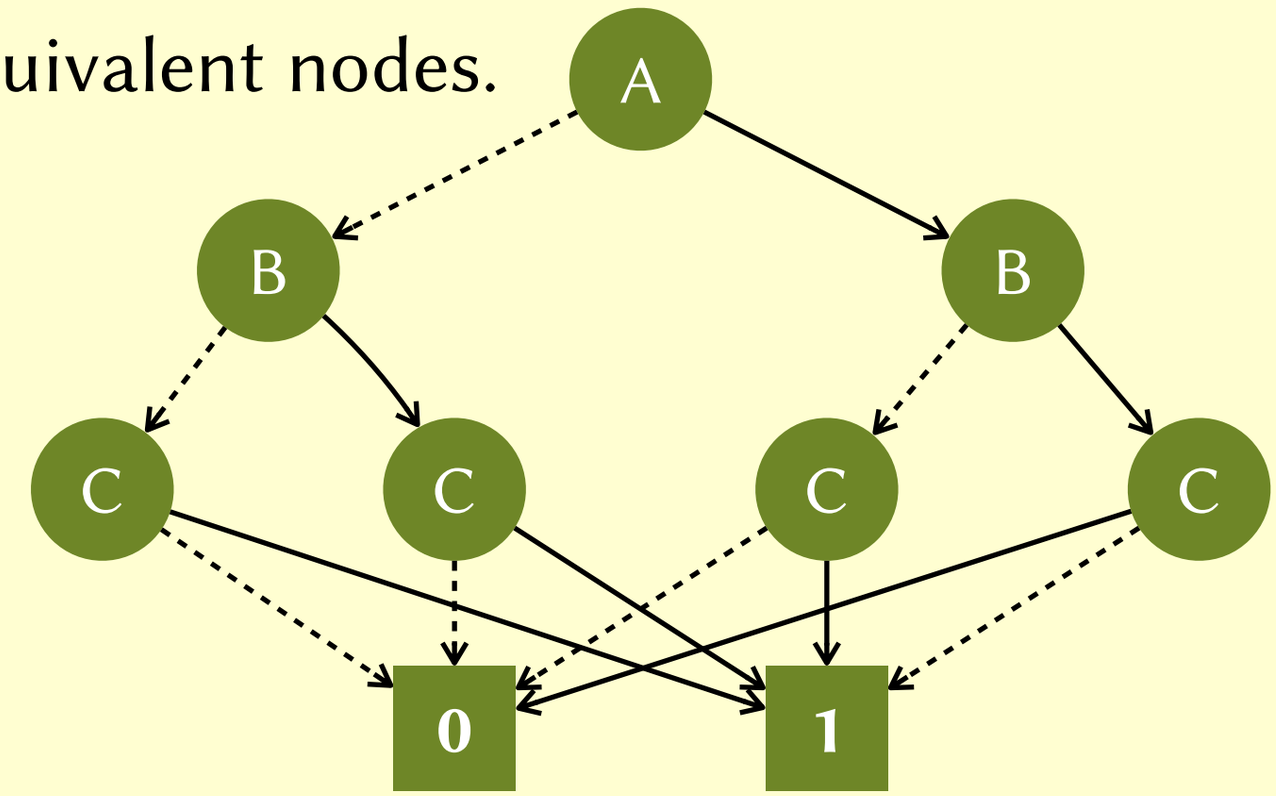


Boolean functions

1. Share all the 0s and 1s.

2. Merge equivalent nodes.



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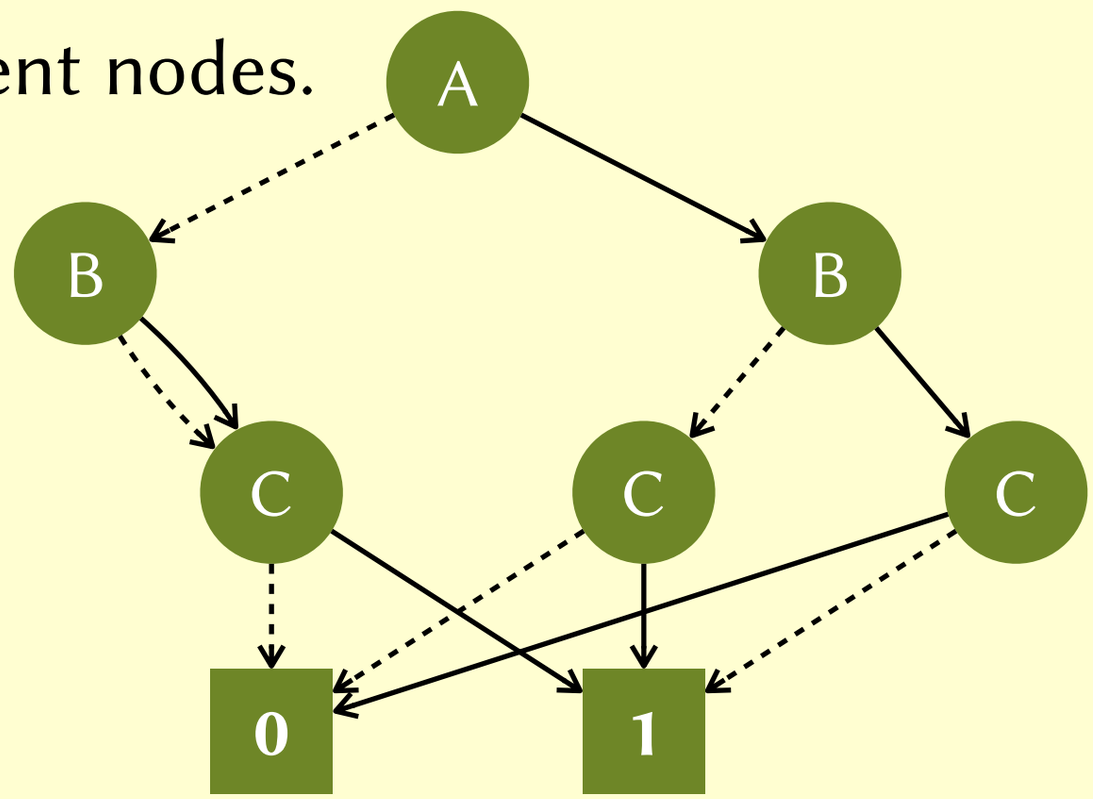
2. Merge equivalent nodes.



Boolean functions

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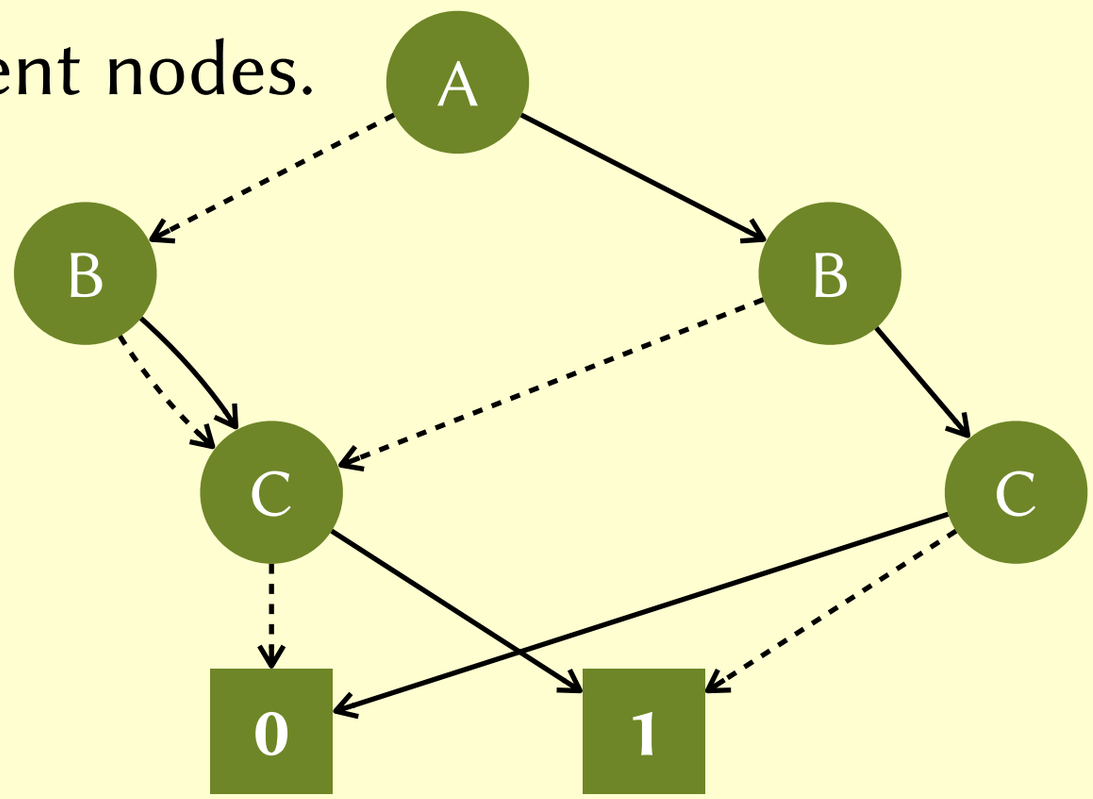
2. Merge equivalent nodes.



Boolean functions

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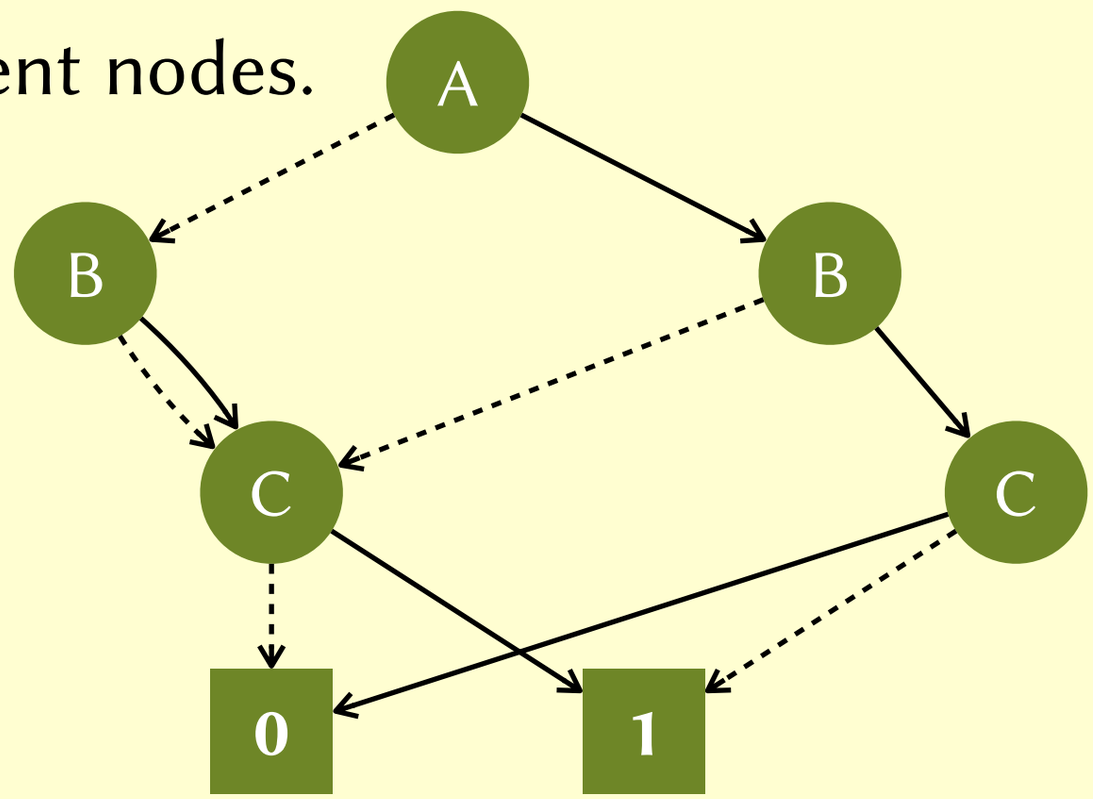
3. If both arrows from a node point to the same place, remove the node.



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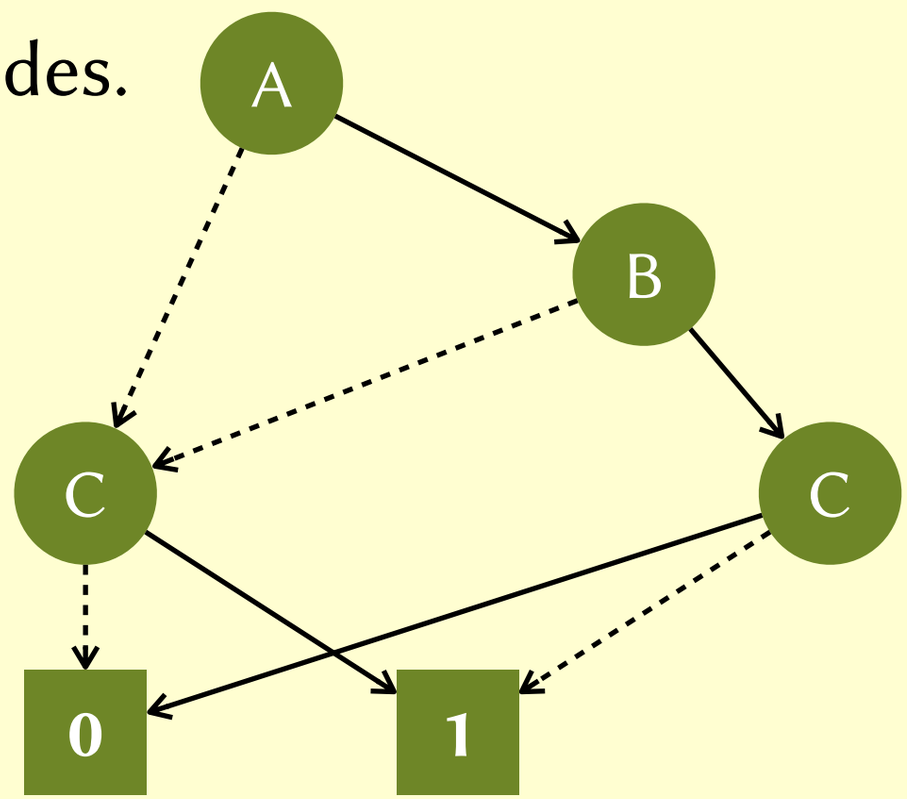


- We have produced a **binary decision diagram** (BDD).

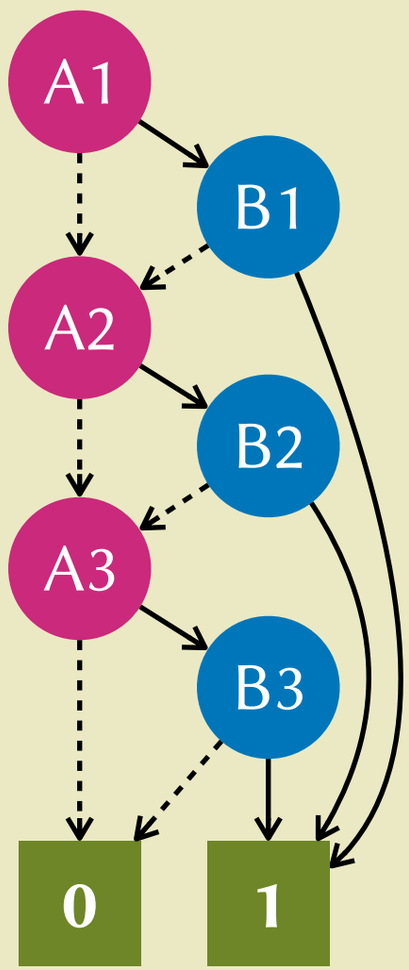
BDDs

- BDDs can be *compact*.
- BDDs are a *canonical representation*: if two formulas are equivalent, their BDDs will be the same. (Assuming BDDs are fully reduced and the order of variables is fixed.)
- Choosing a good order of variables is critical!

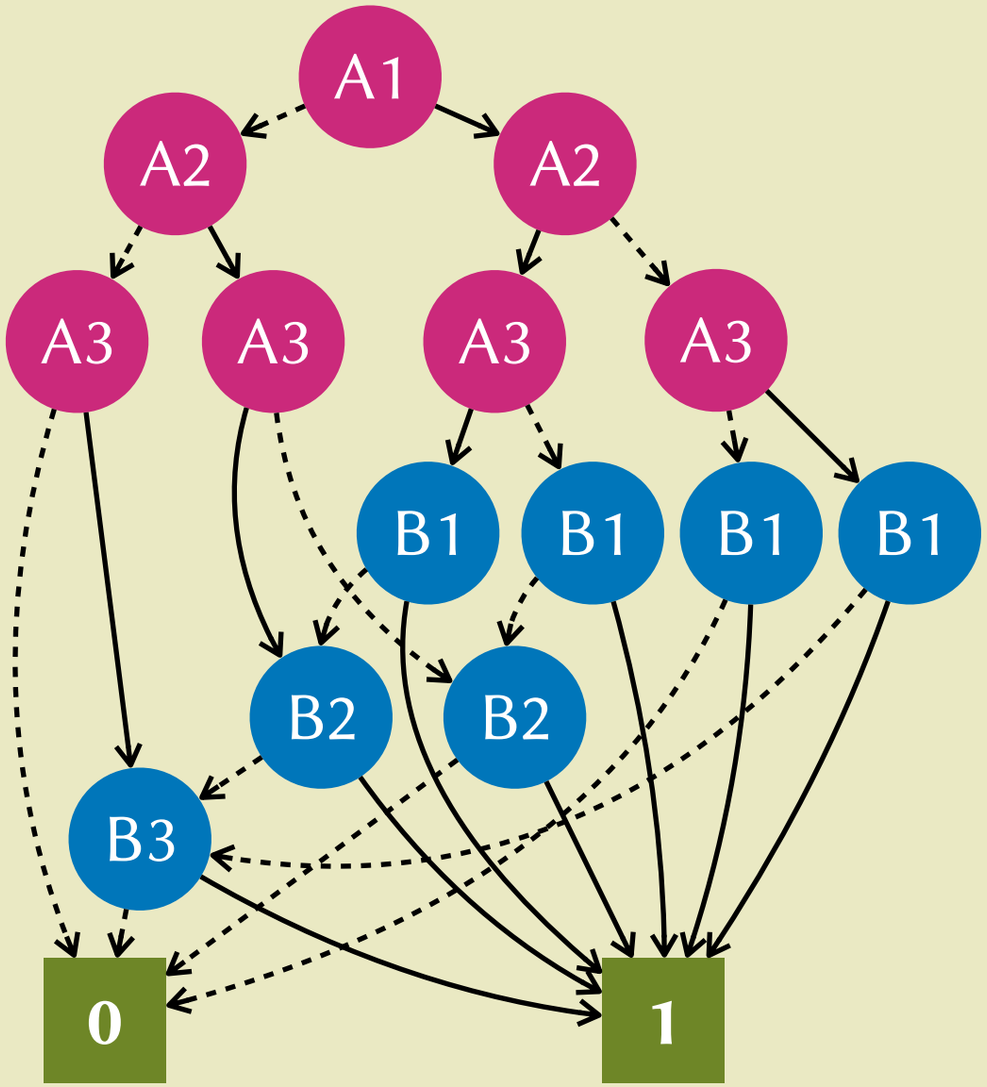
BDDs

- Suppose our formula is $(A1 \wedge B1) \vee (A2 \wedge B2) \vee (A3 \wedge B3)$.

Order A1, B1, A2, B2, A3, B3:



Order A1, A2, A3, B1, B2, B3:



BDDs

- BDDs can be *compact*.
- BDDs are a *canonical representation*: if two formulas are equivalent, their BDDs will be the same. (Assuming BDDs are fully reduced and the order of variables is fixed.)
- Choosing a good order of variables is critical!
- BDDs aren't a silver bullet, but are often quite efficient.

This lecture

- Temporal logic
- Binary decision diagrams