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Name:

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Section: 1 (8:30am) 2 (11:30am) 3 (1:30pm)

1. (a) roots by quadratic formula (4) _____
(b) alternate quadratic formula (3) _____
(c) roots by alternate formula (3) _____
...../10
2. (a) Matlab function (4) _____
(b) stability analysis (6) _____
(c) number of steps (5) _____
...../15
3. (a) Conditions and values (6) _____
(b) Lagrange (4) _____
...../10
4. (a) Linear system (8) _____
(b) Spline coefficients (8) _____
(b) Graphic (4) _____
...../20
5. (a) points (4) _____
(b) lines (4) _____
(c) splines (12) _____
...../20

Total: (75) _____

CS 370 A1

$$1. (a) x_1 = \frac{-22.41 - \sqrt{22.41^2 - 4 \times 1.03 \times 0.1123}}{2 \times 1.03}$$

$$= \frac{-22.41 - \sqrt{502.21 - 0.46268}}{2.06}$$

$$= \frac{-22.41 - \sqrt{501.75}}{2.06}$$

$$= \frac{-22.41 - 22.400}{2.06}$$

$$= -21.752$$

$$x_2 = \frac{-22.41 + 22.4}{2.06} = -0.0048544$$

$$\text{Err}_{\text{rel} x_1} = \frac{|-21.752 + 21.752|}{|-21.752|} \approx 0$$

$$\text{Err}_{\text{rel} x_2} = \frac{|-0.0050123 + 0.0048544|}{|-0.0050123|} \approx 0.3 \times 10^{-1}$$

$$(b) x_1 \cdot x_2 = \frac{(-b - \sqrt{b^2 - 4ac}) \cdot (-b + \sqrt{b^2 - 4ac})}{2a \cdot 2a}$$

$$= \frac{(-b)^2 - (b^2 - 4ac)}{4a^2}$$

$$\text{Then } a \cdot x_1 \cdot x_2 = a \cdot \frac{4ac}{4a^2} = c$$

$$\text{Hence } x_2 = \frac{c}{ax_1}$$

(c) x_1 is the same as (a).

$$x_2 = \frac{0.1123}{1.03 \cdot (-21.752)} = \frac{0.1123}{-22.405} = -0.0050123$$

$$\text{Err}_{\text{rel} x_2} = \frac{|-0.0050123 + 0.0050123|}{|-0.0050123|} \approx 0$$

2. (a) The data in the 11th is larger than the 10th. And if I perform the recursion for the 21st times, the data get ~~error~~ messed up. And with the amount get larger, the answer get frustrating.

$$(b) x_1 = \frac{-a - \sqrt{a^2 - 4b}}{2} \quad x_2 = \frac{-a + \sqrt{a^2 - 4b}}{2}$$

$$q_n = C_1 \cdot \left(\frac{-a - \sqrt{a^2 - 4b}}{2} \right)^n + C_2 \cdot \left(\frac{-a + \sqrt{a^2 - 4b}}{2} \right)^n$$

$$\cancel{K(q_n)} = \frac{\ln(q_n)}{q_n} \quad K(p) = \frac{\ln(q'_n)}{q_n} \approx \frac{\ln \left| \ln \left(\frac{-a - \sqrt{a^2 - 4b}}{2} \right) \right| + \ln \left(\frac{-a + \sqrt{a^2 - 4b}}{2} \right)}{C_1 + C_2}$$

$$\approx \ln | \in O(\ln),$$

Because $n \rightarrow \infty$, ~~$K(p)$~~ P is unstable.

(c) Take $n = 25 \Rightarrow 2 \times 10^{-1}$. Then we can see there will be a O(1) error on the 8th number. In the experiment of (a), the 8th number got error, which match the prediction.

3. (a) By observation,

$$\cancel{a_1=19}, \cancel{a_2=27}, a_3=6, a_4=1$$

$$a_1=19, a_2=26, a_3=9, a_4=1$$

$$(b) \begin{aligned} S(-3) &= 28 + 19(-3) + 9(-3)^2 + (-3)^3 = 25 \\ S(-1) &= 26 + 19(-1) + 9(-1)^2 - (-1)^3 = 17 \\ S(0) &= 26 \\ S(3) &= 26 + 19 \times 3 + 3 \times 3^2 + 3^3 = 137. \end{aligned}$$

$$L_1(x) = \frac{(x+1)(x-0)(x-3)}{(-2)(-3)(-6)} = \frac{(x+1)x(x-3)}{-36}$$

$$L_2(x) = \frac{(x+3)(x-0)(x-3)}{(2)(-1)(-4)} = \frac{(x+3)x(x-3)}{8}$$

$$L_3(x) = \frac{(x+3)(x+1)(x-3)}{(3)(1)(-3)} = \frac{(x+3)(x+1)(x-3)}{-9}$$

$$L_4(x) = \frac{(x+3)(x+1)(x-0)}{(16)(4)(3)} = \frac{(x+3)(x+1)x}{72}$$

$$p(x) = 25 \cdot L_1(x) + 17 \cdot L_2(x) + 26 \cdot L_3(x) + 137 \cdot L_4(x).$$

4. (a) $s_1 = b_1 + 2c_1(-1+1) + 3d_1(-1+1)^2$

$$a_1 + b_1(1+1) + c_1(1+1)^2 + d_1(1+1)^3 = a_2 + b_2(1-1) + c_2(1)^2 + d_2(1-1)^3 = 1$$

$$s_2 = b_1 + 2c_1(1+1) + 3d_1(1+1)^2 = b_2 + 2c_2(1-1) + 3d_2(1)^2$$

$$a_2 + b_2(2-1) + c_2(2-1)^2 + d_2(2-1) = a_3 + b_3(2-2) + c_3(2-2)^2 + d_3(2-2)^3 = 4$$

..... twice-diff; $ss_1 = 2c_1 + 6d_1(-1+1)$ $ss_2 = 2c_1 + 6d_1(1+1) - 2c_2 + 6d_2(1)$

(b) $a_1 = 2$

$$a_2 = 1$$

$$a_3 = 4$$

$$a_4 = 3$$