Marking Sheet　　Assignment 4　　CS370 Winter 2018

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Section: 1 (8:30pm)　　2 (11:30pm)　　3 (1:30)

1.	(a) Google matrix	(5) __________	
	(b) PageRank algorithm	(10) __________	
	(c) verification	(5) __________	
			......../20
2.	positive Markov matrix		......../10
3.	quadratic solve algorithm		......../10
4.	PA = LU factorization	(5) __________	
	solution	(5) __________	......../10
5.	(a) recurrence equations	(10) __________	
	(b) cost	(5) __________	
	(c) cost	(5) __________	
			......../20
6.	(a) PageRank function	(10) __________	
	(b) small web	(5) __________	
	(b) math_uwaterloo.mat	(8) __________	
	(d) math_uwaterloo.mat	(7) __________	
			......../30

Total: (100) ______________

CS 370 A4

1. (a)

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

No dead $\Rightarrow Q = P$

$\because e = [1,1,1,1,1,1,1]^T$

$$\therefore M = aQ + \frac{(1-a)}{7} e e^T$$

$$= \begin{bmatrix} \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}+\frac{6}{7}a & \frac{1}{7}-\frac{a}{7} \\ \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} \\ \frac{5}{14}a+\frac{1}{7} & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} \\ \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} \\ \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} \\ \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}+\frac{6}{7}a \\ \frac{1}{7}-\frac{a}{7} & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}+\frac{6}{7}a & \frac{5}{14}a+\frac{1}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} & \frac{1}{7}-\frac{a}{7} \end{bmatrix}$$

(b) Using Matlab. $P^0 = [\frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}]^T$

After 15 iteration, $\vec{X} = [0.2147, 0.0505, 0.1326, 0.1108, 0.0680, 0.2223, 0.2011]^T$.

(c) I continue 10 more iterations.

The $\vec{x}$ stables at $[0.211, 0.0506, 0.133, 0.111, 0.0686, 0.223, 0.203]^T$.

Hence $M\vec{x} = \vec{x}$.

2.

$$\sum_i M_{ij} = a\left(\sum_i P_{ij} + \sum_i \frac{1}{R} ed^T\right) + (1-a)\sum_i \frac{1}{R} ee^T$$

$\because P_{ij}$ is probability vector $\Rightarrow \sum_i P_{ij} = 1$.

$$= a \cdot (1) + (1-a) \cdot (1)$$

$$= 1.$$

And $M = \alpha \underbrace{\left(P + \frac{1}{R} ed^T\right)}_{>0} + \underbrace{(1-\alpha)\frac{1}{R} ee^T}_{>0}$

Hence $M > 0$.

$\therefore$ M is a positive Markov matrix.

3. For $A = LU$, we can have $A^T = U^T \cdot L^T$.

~~We can~~ Then, $AA^T \cdot \vec{x} = \vec{b}$

$$\Rightarrow L \cdot U \cdot U^T \cdot L^T \cdot \vec{x} = \vec{b}.$$

~~Then~~ Hence, we can use following equation to solve $AA^T \cdot \vec{x} = \vec{b}$.

$$\begin{cases} L \cdot \vec{p} = \vec{b} \\ U \cdot \vec{q} = \vec{p} \\ U^T \cdot \vec{r} = \vec{q} \\ L^T \cdot \vec{x} = \vec{r}. \end{cases}$$

, which needs quadratic time procedure.

4. First stage:

$$\cancel{\begin{bmatrix} 2 & 2 & 2 \\ -4 & -2 & 6 \\ 2 & -1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 2 \\ -2 & 2 & 10 \\ 1 & -3 & 2 \end{bmatrix}}$$

$$P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 2 & 2 \\ -4 & -2 & 6 \\ 2 & -1 & 4 \end{bmatrix} \quad P_1 A = \begin{bmatrix} -4 & -2 & 6 \\ 2 & 2 & 2 \\ 1 & -3 & 2 \end{bmatrix}$$

so $\begin{bmatrix} -4 & -2 & 6 \\ 2 & 2 & 2 \\ 1 & -3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & -2 & 6 \\ -0.5 & 1 & 5 \\ -0.25 & -3.5 & 3.5 \end{bmatrix}$

Second stage:

$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

$$\begin{bmatrix} -4 & -2 & 6 \\ -0.25 & -3.5 & 3.5 \\ -0.5 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & -2 & 6 \\ -0.25 & -3.5 & 3.5 \\ -0.5 & -\frac{2}{7} & 6 \end{bmatrix} . \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ -0.5 & -\frac{2}{7} & 1 \end{bmatrix} \quad U = \begin{bmatrix} -4 & -2 & 6 \\ 0 & -3.5 & 3.5 \\ 0 & 0 & 6 \end{bmatrix}$$

$P_2 \cdot P_1 A = LU$. $LU\vec{x} = P_2 \cdot P_1 \vec{b}$

$\vec{t} = P_2 \cdot P_1 \cdot \vec{b} = P_2 \cdot \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix}$.

$$\begin{cases} L\vec{r} = \vec{t} \\ U\vec{x} = \vec{r} \end{cases} \Rightarrow \vec{r} = \begin{bmatrix} 4 \\ 8 \\ \frac{44}{7} \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} \frac{25}{21} \\ -\frac{26}{21} \\ \frac{22}{21} \end{bmatrix}$$

5. (a) $d_1 = 2$

$l_2 \cdot d_1 = 1$

$l_2 + d_2 = 4$

$l_3 \cdot d_2 = 1$

~~$d_i + l_n = 4$~~

if $i \neq n$: $d_i + l_i = 4$

else: $d_i + l_i = 2$.

$\Rightarrow d_i = 4 - l_i = 4 - \frac{1}{d_{i-1}}$ but $d_n = 2 - \frac{1}{d_{n-1}}$

$l_i = \frac{1}{d_{i-1}} = \frac{1}{4 - l_{i-1}}$ but $l_n = \frac{1}{2 - l_{i-1}}$.

(b) Following the recurrence equations in (a), we can find LU decomposition of A in $2 \times O(n)$ ~~iterat~~ recurrence. Then, it is $O(n)$.

(c) Using a $A\vec{x} = \vec{b}$ to solve a natural spline would be taking $O(n^2)$ time, but since all Δx_i are the same, we can compute the $A\vec{x} = \vec{b}$ in just $O(n)$. Hence, total is $O(n^2)$.

6. (a) Answer is on PageRank.m

(b) a4q6b.m & 6 1 7 3 4 5 2

(c) a4q6c.m

(d) $0.15 \to it = 9$

$0.35 \to it = 16$

$0.55 \to it = 27$

$0.75 \to it = 57$

$0.95 \to it = 313$

With the increment of alpha, iterations required dramatically increased. It is because the convergence speed decreases ~~wi~~ when the alpha is high.