Marking Sheet Assignment 3 CS 370 Winter 2018

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Section: 1 (8:30pm) 2 (11:30pm) 3 (1:30)

1.	(a) Fourier	(5)	________	
	(b) Inverse Fourier	(5)	________	
				......../10
2.	(a) F_k	(7)	________	
	(b) F_{2k}	(8)	________	
				......../15
3.	(a) g, h	(5)	________	
	(b) G, H	(5)	________	
	(c) F	(5)	________	
	(d) F (butterfly)	(5)	________	
				......../20
4.	Edge Sharpening			
				......../15
5.	train / bird			
				......../20
6.	Image compression			
				......../20

Total: (100) ____________

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1a DFT of $f = (1, 2, 3, 2)$

$$F[0] = \frac{1}{4}\sum_{n=0}^{3} f_n W^{-0n} = \frac{1}{4}\cdot(1+2+3+2) = 2$$

$$F[1] = \frac{1}{4}\sum_{n=0}^{3} f_n W^{-n} = \frac{1}{4}\sum_{n=0}^{3} f_n e^{-\frac{i\cdot 2\pi}{4}\cdot n} = \frac{1}{4}\sum_{n=0}^{3} f_n(-i)^n = \frac{1}{4}\cdot(1 - 2i - 3 + 2i) = -\frac{1}{2}$$

$$F[2] = \frac{1}{4}\sum_{n=0}^{3} f_n(-i)^{2n} = \frac{1}{4}(1-2+3-2) = 0$$

$$F[3] = \frac{1}{4}\sum_{n=0}^{3} f_n(-i)^{3n} = \frac{1}{4}(1+2i-3-2i) = -\frac{1}{2}$$

1b. Inverse DFT of $F = (4, -1, 0, -1)$

$$f[0] = \sum_{k=0}^{3} F_k W^{nk} = \sum_{k=0}^{3} F_k e^{\frac{2\pi i n k}{4}} = \sum_{k=0}^{3} F_k (i)^{nk} = 4-1+0-1 = 2$$

$$f[1] = \sum_{k=0}^{3} F_k (i)^k = 4 - i - 0 + i = 4$$

$$f[2] = \sum_{k=0}^{3} F_k (i)^{2k} = 4+1+0+1 = 6$$

$$f[3] = \sum_{k=0}^{3} F_k (i)^{3k} = 4 + i - 0 - i = 4$$

2a. $$F_k = \frac{1}{N}\sum_{n=0}^{N-1}(-1)^n \cdot e^{\frac{2\pi i n k}{N}} = \frac{1}{N}\sum_{n=0}^{(N-1)/2} \cdot e^{\frac{2\pi i 2n k}{N}} - \frac{1}{N}\sum_{n=0}^{(N-1)/2} \cdot e^{\frac{2\pi i (2n+1)k}{N}}$$

$$F_k = \frac{1}{N}\sum_{n=0}^{(N-2)/2} e^{\frac{4\pi i n k}{N}} \cdot \left(1 - e^{\frac{2\pi i k}{N}}\right)$$ if N is even.

$$= \frac{1}{N}\left(1 - e^{\frac{2\pi i k}{N}}\right)\frac{e^{2\pi i k} - 1}{e^{\frac{4\pi i k}{N}} - 1} = 0$$

$$F_k = \frac{1}{N}\sum_{n=0}^{(N-3)/2}\left(e^{\frac{2\pi i 2nk}{N}} - e^{\frac{2\pi i (2n+1)k}{N}}\right) + \frac{1}{N} e^{\frac{2\pi i (N-1)k}{N}}$$ if N is odd

Similar to above, $= 0 + \frac{1}{N} e^{2\pi i \frac{(N-1)k}{N}} = \frac{1}{N}$

Hence, $F_k = \begin{cases} 0, & \text{if } N \text{ is even} \\ \frac{1}{N}, & \text{if } N \text{ is odd} \end{cases}$

2.(b) $F_{2k} = \frac{1}{N}\sum_{n=0}^{N-1} f_n W^{-2nk} = \frac{1}{N}\sum_{n=0}^{\frac{N}{2}-1} (-1)\cdot e^{\frac{2\cdot 2\pi i\cdot(-2nk)}{N}} + (1)\cdot e^{\frac{2\pi i(-2n+1)\cdot(2k)}{N}}$

$= \frac{1}{N}\sum_{n=0}^{\frac{N}{2}-1} (-1)\cdot e^{\frac{-8\pi i nk}{N}} + e^{\frac{-8\pi i nk+2\pi i 2k}{N}}$

$= \frac{1}{N}\sum_{n=0}^{\frac{N}{2}-1} e^{\frac{-8\pi i nk}{N}}\cdot\left(e^{\frac{4\pi i k}{N}}-1\right)$

~~$\because e^{\frac{-8\pi i nk}{N}} = 1$ for every n~~

$\because e^{\frac{4\pi i k}{N}} = 1$ for every integer N

$\therefore F_{2k} = 0$ because of $1-1=0$.

3.(a) $g = (0,0,0,0)$ $h = (1,0,\cancel{0}\,{-2i},0)$

(b) $G_0 = \frac{1}{4}\sum_{n=0}^{3} g_0 W^{-2nk} = 0$, $G_1 = 0$, $G_2 = 0$, $G_3 = 0 \Rightarrow G = (0,0,0,0)$.

$H_0 = \frac{1}{4}\sum_{n=0}^{3} h_n W^{-2nk} = \frac{1}{4}\sum_{n=0}^{3} h_n (i)^{-2nk} = \frac{1}{4}(1-2i) = \frac{1}{4}-\frac{1}{2}i$

~~$\Rightarrow H = (1,0,0,0)$~~

~~$H_1 = 0$, $H_3 = 0$~~

~~$H_2 = \frac{1}{4}\sum_{n=0}^{3} h_2 (i)^{-2nk} = \frac{1}{4}(1+1-1+1) = 0$~~

$H_1 = \frac{1}{4}\sum_{n=0}^{3} h_n W^{-2n} = \frac{1}{4}(1+2i)$

$H_2 = \frac{1}{4}\sum_{n=0}^{3} h_n W^{-4n} = \frac{1}{4}(1-2i)$

$H_3 = \frac{1}{4}\sum_{n=0}^{3} h_n W^{-6n} = \frac{1}{4}(1+2i)$

(c) $F = (0, \frac{1}{4}-\frac{i}{2}, 0, \frac{1}{4}+\frac{i}{2}, 0, \frac{1}{4}-\frac{i}{2}, 0, \frac{1}{4}+\frac{i}{2})$

~~(c) $F = (0,1,0,0,0,1,0,0)$~~

(d) $(1\ \ 0\ \ 2\ \ 0\ \ -1\ \ 0\ \ -2\ \ 0)$

$(0\ \ 0\ \ 0\ \ 0)(1\ \ 0\ \ -2i\ \ 0)$

~~$(0\ \ 0)(0\ \ 0)(0\ \ 0)(1\ \ 0)$~~

~~$(0)(0)(0)(0)(0)(0)(\frac{1}{2})(\frac{1}{2})$~~

$(0\ \ 0)(0\ \ 0)(\frac{1-2i}{2}\ \ 0)(\frac{1+2i}{2}\ \ 0)$

~~$(0)(0)(0)(0)(\frac{1}{2}-i)(0.5+i)(0.5+i)$~~

~~$F = (0, \frac{1}{2}-i, 0, 0.5+i, 0, 0.5-i, 0, 0.5+i)$~~

$(0)(0)(0)(0)(\frac{1}{4}-\frac{i}{2})(\frac{1}{4}-\frac{i}{2})(\frac{1}{4}+\frac{i}{2})(\frac{1}{4}+\frac{i}{2})$

$F = (0, \frac{1}{4}-\frac{i}{2}, 0, \frac{1}{4}+\frac{i}{2}, 0, \frac{1}{4}-\frac{i}{2}, 0, \frac{1}{4}+\frac{i}{2})$

4.

$$f_x = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F_{k,l} \cos\left(\frac{2\pi l y}{N}\right) \cdot \left(-\sin\left(\frac{2\pi k x}{N}\right)\right) \cdot \frac{2\pi k}{N}$$

$$f_{xx} = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F_{k,l} \cos\left(\frac{2\pi l y}{N}\right) \cdot \left(-\cos\left(\frac{2\pi k x}{N}\right)\right) \cdot \left(\frac{2\pi k}{N}\right)^2$$

$$f_y = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F_{k,y} \cos\left(\frac{2\pi k x}{N}\right) \cdot \left(-\sin\left(\frac{2\pi l y}{N}\right)\right) \cdot \left(\frac{2\pi l}{N}\right)$$

$$f_{yy} = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F_{k,l} \cos\left(\frac{2\pi k x}{N}\right) \cdot \left(-\cos\left(\frac{2\pi l y}{N}\right)\right) \cdot \left(\frac{2\pi l}{N}\right)^2.$$

$$f^* = f - a \cdot (f_{xx} + f_{yy})$$

$$= \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F_{k,l} \cos\left(\frac{2\pi k x}{N}\right) \cos\left(\frac{2\pi l y}{N}\right) \cdot \underbrace{\left[1 + a\left(\frac{2\pi l}{N}\right)^2 + a \cdot \left(\frac{2\pi k}{N}\right)^2\right]}_{>0 \text{ since } a>0.}$$

The filter enhances the grid with higher $F_{k,l}$ value. It tends to sharpen edges because edges ~~are~~ usually have larger $F_{k,l}$, k and l value.