Marking Sheet Assignment 2 CS370 Winter 2018

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Section: 1 (8:30am) 2 (11:30am) 3 (1:30pm)

1. (a)	first-order system	(6)	________
(b)	system dynamics function	(9)	________
			......../15
2. (a)	Solve (Euler)	(3)	________
(b)	Modified Euler function	(3)	________
(c)	Solve (Modified Euler)	(3)	________
(d)	Numerical evidence (Euler)	(3)	________
(e)	Numerical evidence (Mod Euler)	(3)	________
			......../15

3. local truncation error ..../10

4. stability analysis ..../10

5. Pursuit Problem code (10) & tests (15) ..../25

Total: (75) ____________

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1. (a) Let $z_1 = u$, $z_2 = v$, $z_3 = u'$, $z_4 = v'$

Then
$$z_1'(t) = z_3(t)$$
$$z_2'(t) = z_4(t)$$
$$z_3'(t) = \sin(t) - c_1(z_3(t) - z_4(t)) - k_1(z_1(t) - z_2(t)) - k_2 z_1(t)$$
$$z_4'(t) = c_1(z_3(t) - z_4(t)) - c_2(z_1(t) - z_2(t))$$

3. ~~$y(t_{n+1}) = y(t$~~

$$y_{n+1} = y_n + y'_n \cdot h + O(h^2)$$
$$= y_n + f(t_n, y_n)h + O(h^2)$$
$$= y_{n-1} + y'_{n-1} \cdot h + O(h^2) + f(t_n, y_n)h + O(h^2)$$
$$= y_{n-1} + h[f(t_{n-1}, y_{n-1}) + f(t_n, y_n)] + O(h^2).$$

$$\text{LocErr} = O(h^2) + [f(t_n, y_n) - f(t_{n-1}, y_{n-1})] \cdot h = O(h^2)$$

4. ~~$y'(t)$~~
$$y_{n+1} = y_n + \frac{h}{3}[-\lambda y_n + 2 \cdot (-\lambda)(y_n + 3hf(t_n + 3h/4, y_n)/4)]$$
$$= y_n + \frac{h}{3}(-\lambda y_n + -2\lambda(y_n + 3h(-\lambda)y(t_n + 3h/4)/4)]$$
$$= y_n[1 + \frac{h}{3} \cdot (-\lambda + -2\lambda(1 \cancel{+} - 3\lambda h/4)]$$
$$= y_n[1 + \frac{h}{3}(-\lambda + (-2\lambda + \frac{3}{2}\lambda h))]$$
$$= y_n \cdot [1 \cancel{+} - \lambda\frac{h}{3} + \frac{h}{3}(-2\lambda + \frac{3}{2}\lambda h)]$$
$$= y_n \cdot [1 - \lambda\frac{h}{3} \cancel{+ 9} - \frac{2}{3}h\lambda + \frac{3}{2}\lambda h]$$

$$e_{n+1} = [1 - \lambda\frac{h}{3} + \frac{5}{6}\lambda h]e_n = [1 - \frac{h}{3} + \frac{5}{6}\lambda h]^{n+1} e_0$$

We need
$$|1 - \lambda\frac{h}{3} + \frac{5}{6}\lambda h| < 1$$
$$|1 - \frac{1}{2}\lambda h| < 1$$

Then, $0 < \lambda h < 4$ to be stable.