Universiteit van Amsterdam

Complex System Simulation

Emerging patterns of opinion formation in social structures

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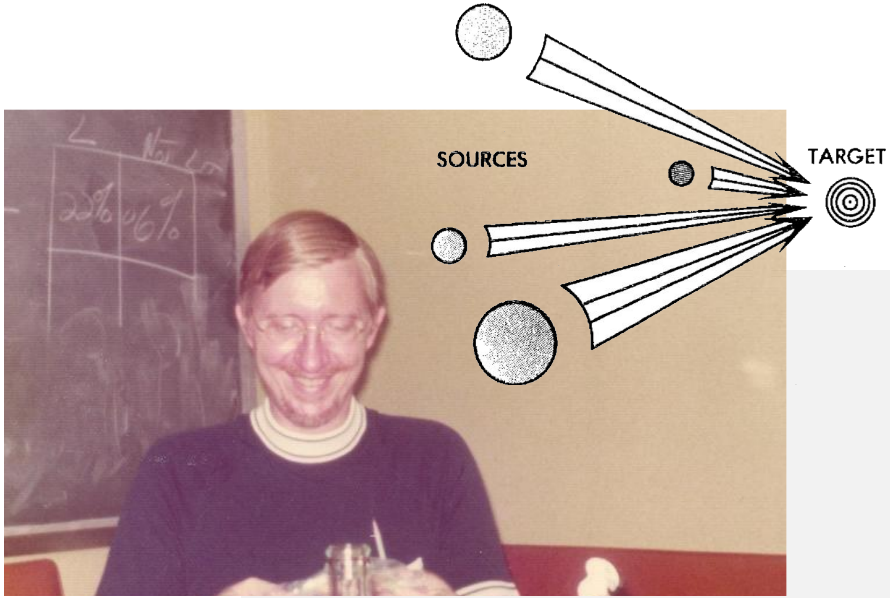
Methodology. 1

Social Impact Theory

Individuals change as a result of the presence and actions of other individuals, in accord to a multiplicative function of these three factors:

- Strength
- Immediacy
- Number of sources



Bibb Latané - Alchetron, The Free Social Encyclopedia

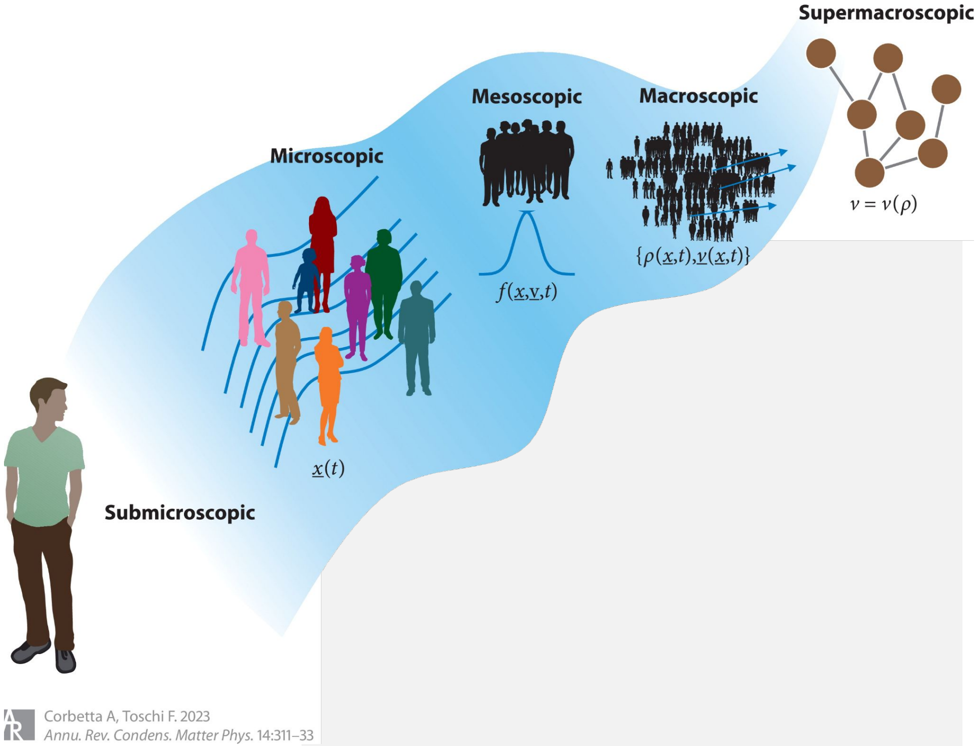
B. Latané - The psychology of social impact. *American Psychologist (1984)*

Methodology. 2

Studying Crowds

Humans can be observed at different length and timescales. At mesoscale humans are not described individually but at a statistical level.

- Cellular Automata
- Networks



[Physics of Human Crowds | Annual Review of Condensed Matter Physics]

Cellular Automata model

Social Impact

$$I_i = -s_i\beta - \sigma_i h - \sum_{j=1, j\neq i}^{N} \frac{s_j \sigma_i \sigma_j}{g(d_{ij})}$$

- ***N***: Individuals
- **σ_i**: Opposite opinions (±1)
- **s_i**: Influence strength (>0)
- **d_{ij}**: Distance
- ***β***: Self-support parameter
- **h**: External influence
- **g(x)**: Increasing function of social distance

The Dynamical Rule

$$\sigma_i(t+1) = \begin{cases} \sigma_i(t) & \text{with probability } \dfrac{\exp(-I_i/T)}{\exp(-I_i/T) + \exp(I_i/T)} \\ -\sigma_i(t) & \text{with probability } \dfrac{\exp(I_i/T)}{\exp(-I_i/T) + \exp(I_i/T)} \end{cases}.$$

- **I_i** : Social impact
- **T**: Social temperature (degree of randomness & average volatility)
- Deterministic limit: $\sigma_i(t+1) = -\mathrm{sign}(I_i \sigma_i)$

Influence: $s_L \gg s_i$

Quadratic grid

2D disc of radius R ≫ 1

J. Holyst, K. Kacperskia, F. Schweitzer - Phase transitions in social impact models of opinion formation

Cluster Radius $I = 0$

$$a \approx \frac{1}{16}\left[2\pi R - \sqrt{\pi} \pm \beta - h \pm \sqrt{(2\pi R - \sqrt{\pi} \pm \beta - h)^2 - 32 s_L}\right]$$

If cluster exist:

Limit for Stable State

$$(2\pi R - \sqrt{\pi} \pm \beta - h)^2 - 32 s_L \geqslant 0$$

$I_L = 0$

s_{Lmin} Against Majority Influence

$$s_{Lmin} = \frac{1}{\beta}(2\pi R - \sqrt{\pi} - h)$$

"Cluster or Unification?"

$$I_i = -s_i\beta - \sigma_i h - \sum_{j=1, j\neq i}^{N} \frac{s_j \sigma_i \sigma_j}{g(d_{ij})} \quad (1)$$

$\bar{s} = 1 \qquad g(r) = r$

- : stable solution

+: unstable solution

Methodology. 5

Research questions

1. Can we replicate the findings of the CA paper?
2. Which conditions lead to either SO and clustering or unification?
3. What is the relationship between the critical temperature and the leader influence?
4. How do our CA and network implementation compare to each other?

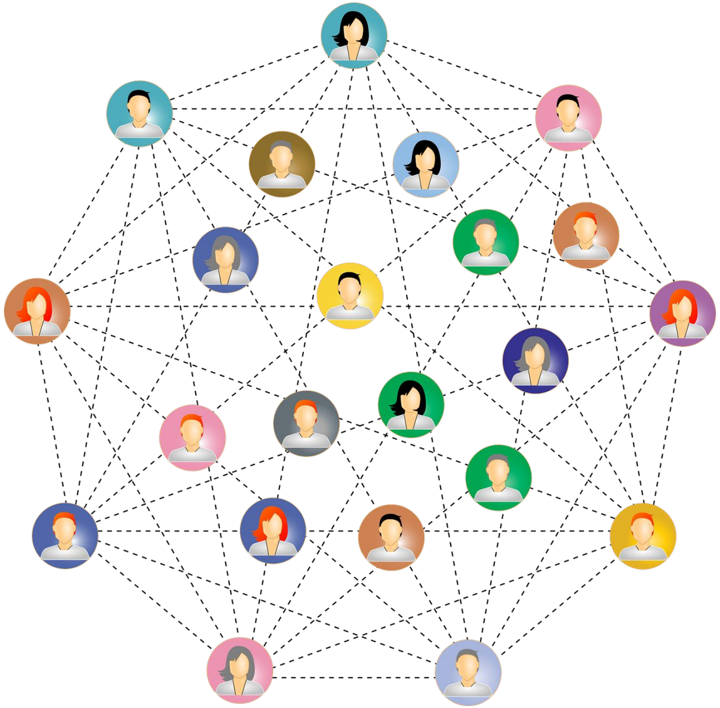


Photo by GDJ

Implementation details

```python
class CA(object):

    def __init__(self, gridsize_x, gridsize_y, p_occupation, p_opinion_1, temp, h, beta, beta_leader, s_mean
        # Parameters directly provided
        self.gridsize_x, self.gridsize_y = gridsize_x, gridsize_y
        self.p_occupation = p_occupation
        self.p_opinion_1 = p_opinion_1
        self.temp = temp
        self.h = h
        self.beta = beta
        self.beta_leader = beta_leader
        self.s_mean = s_mean
        self.s_leader = s_leader

        # Functions passed
        self.d = lambda x0, y0, x1, y1: distance_metric(
```

Cellular Automata

- Our own implementation
- Class
- Functions in module
- Cluster size
- Analytical functions of paper

```python
class Network(object):

    def __init__(self, gridsize_x, gridsize_y, p_occupation, p_opinion_1, temp, h, beta, beta_leader,
                 s_mean, s_leader, dist_func='euclidean', dist_scaling_func='linear', dist_scaling_factor=1,
                 s_prob_dist_func='uniform', network_type='grid', ba_m=4, neighbor_dist=1, c_leader = 1):
        # Parameters directly provided
        self.gridsize_x, self.gridsize_y = gridsize_x, gridsize_y
        self.p_occupation = p_occupation
        self.p_opinion_1 = p_opinion_1
        self.temp = temp
        self.h = h
        self.beta = beta
        self.beta_leader = beta_leader
        self.s_mean = s_mean
        self.s_leader = s_leader
```

Networks

- Networkx
- Custom class
- Grid/Barabasi-Albert
- Nodes have edges with attribute distance
  - Manhattan
- Social impact formula

Results. 1

Phase Diagram for Circular Social Space

$$\sigma_i(t+1) = -\mathrm{sign}(I_i\sigma_i)$$

$$a \approx \frac{1}{16}\left[2\pi R - \sqrt{\pi} \pm \beta - h \pm \sqrt{(2\pi R - \sqrt{\pi} \pm \beta - h)^2 - 32 s_L}\right]$$

Cluster radius a vs. leader's strength s_L

Deterministic limit (T = 0)

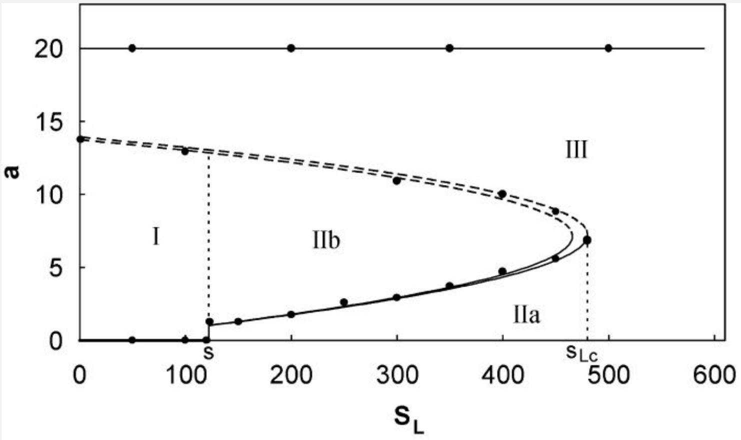
Solid lines: attractors -

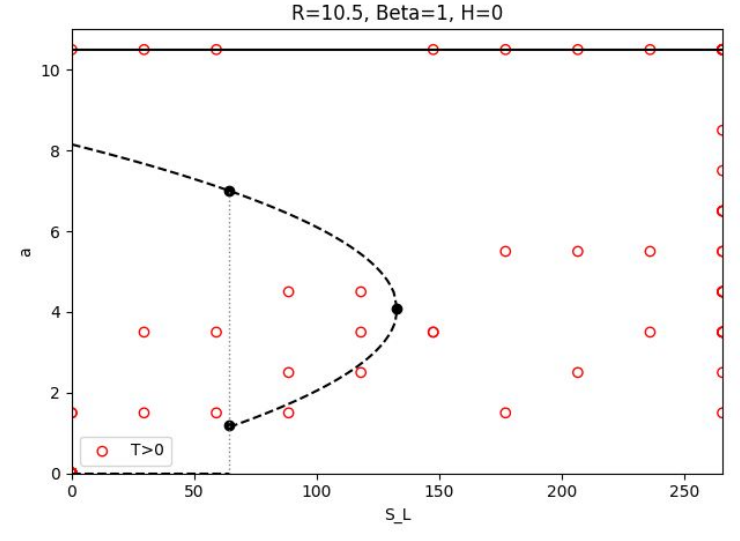
Dashed lines: unstable repellers +

Ⅰ: Evolves towards complete or no unification (a=20, a=0)

Ⅱ: The stable cluster attractor bifurcates the space into regions

Ⅲ: Leads to unification（a=20)



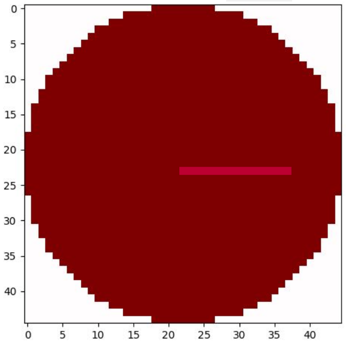


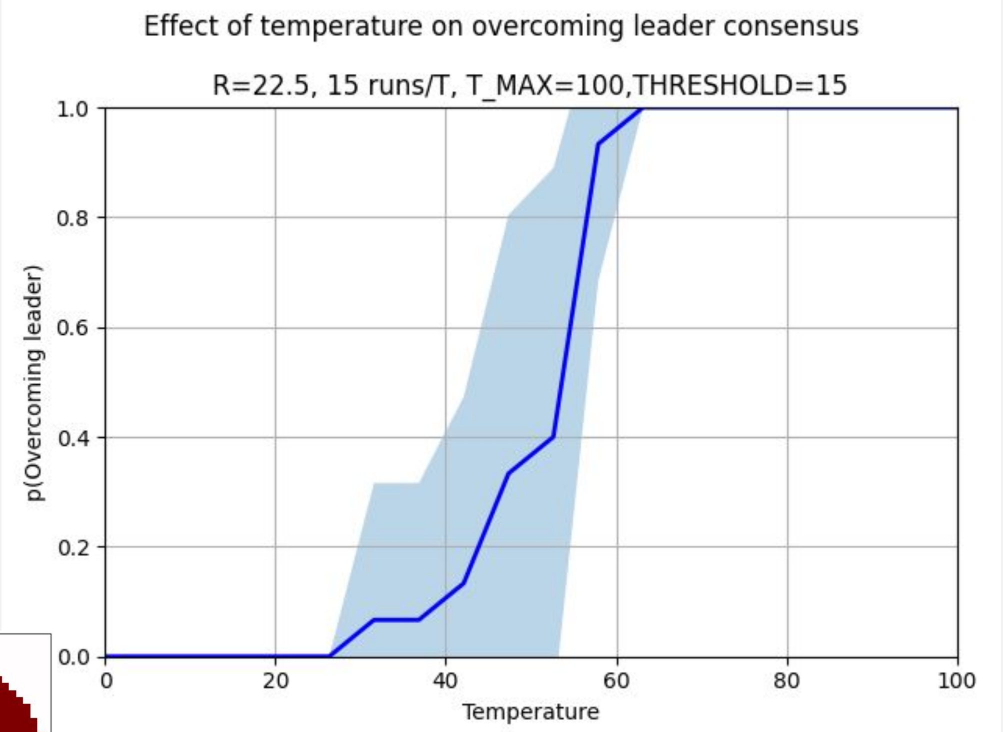
Results. 2,3

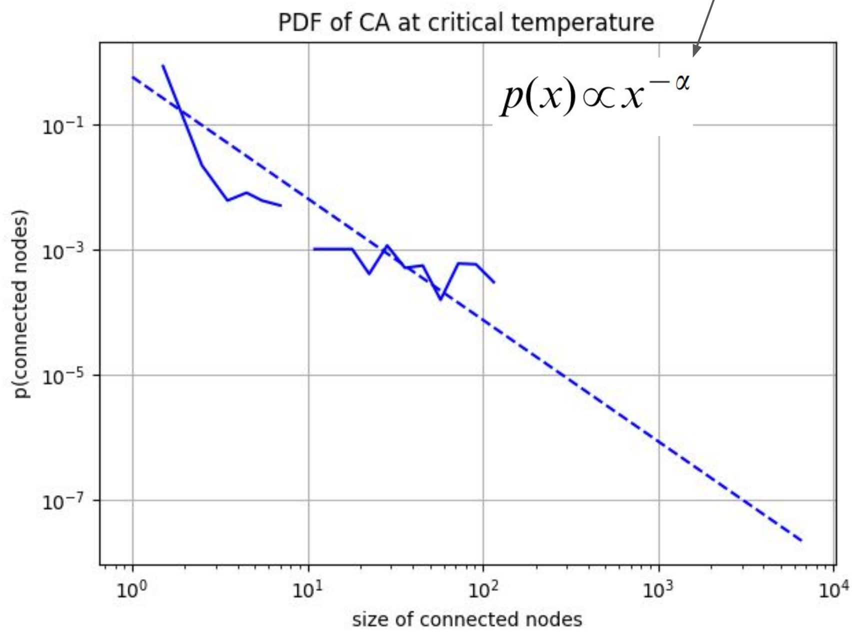
Overcoming leader

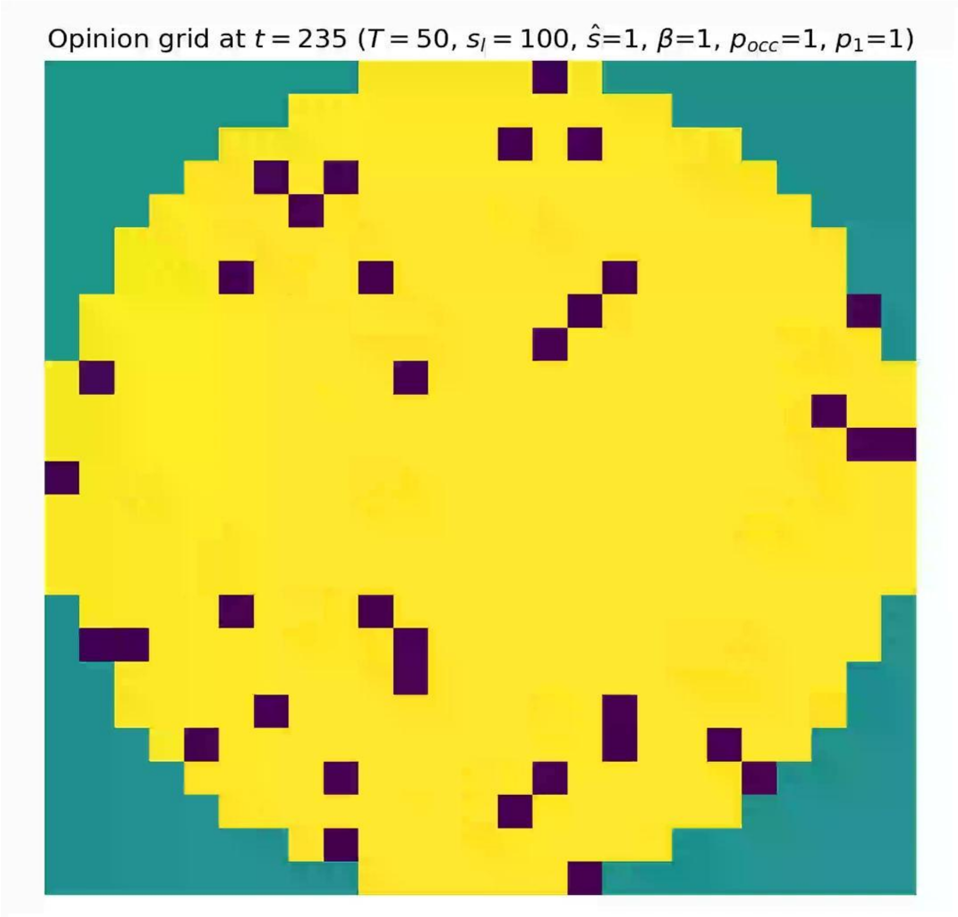
Begin with leader's opinion
Overcoming leader: end with <50% area proportion of the disc
Large simulations on each T
Count the times it exists

$$R_l = \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}$$









Log-Log plot: SOC
dashed lines: Power-law
Distribution of connected nodes with fixed T_c
Total number after N timesteps
Suddenly grow

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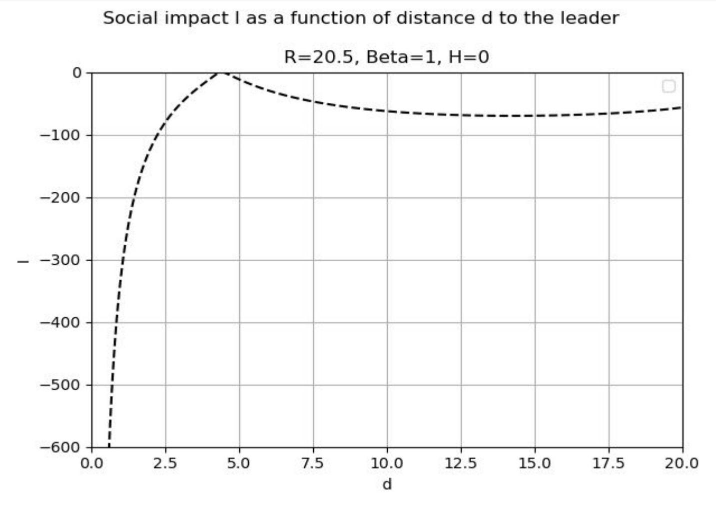
Results. 4

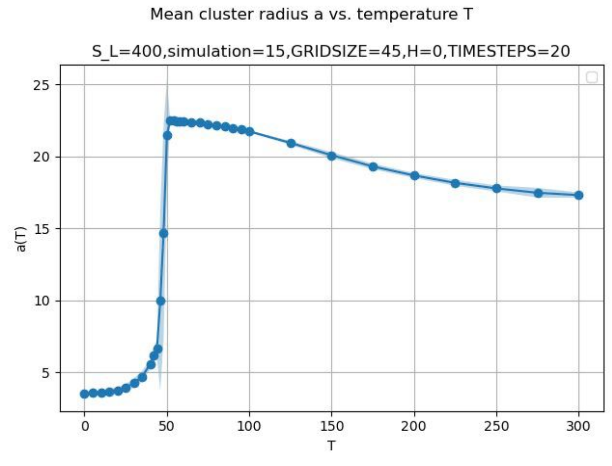
Low temperature mean-field approximation

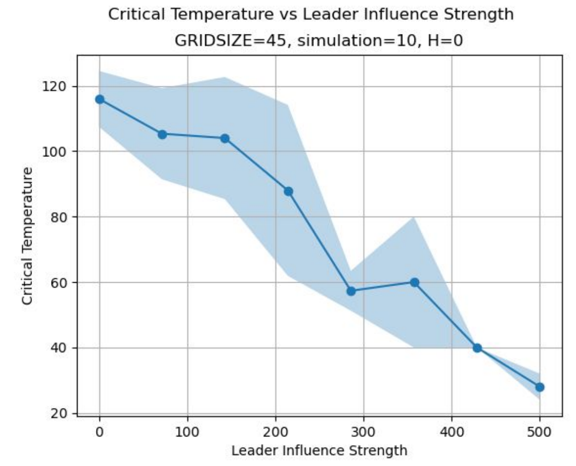
low T: the cluster is only slightly diluted
Asymmetry
0 at the border: sensitive to fluctuations
Inner: deeper confirmed, resistant against noise
suddenly increase
gradually going down

$$I_i(d) = -\frac{s_L}{d} - 8aE\left(\frac{d}{a}, \frac{\pi}{2}\right) + 4RE\left(\frac{d}{R}, \frac{\pi}{2}\right) + 2\sqrt{\pi} - \beta\,,$$

$$I_o(d) = \frac{s_L}{d} + 8aE\left(\frac{d}{a}, \arcsin\frac{a}{d}\right) - 4RE\left(\frac{d}{R}, \frac{\pi}{2}\right) + 2\sqrt{\pi} - \beta\,,$$







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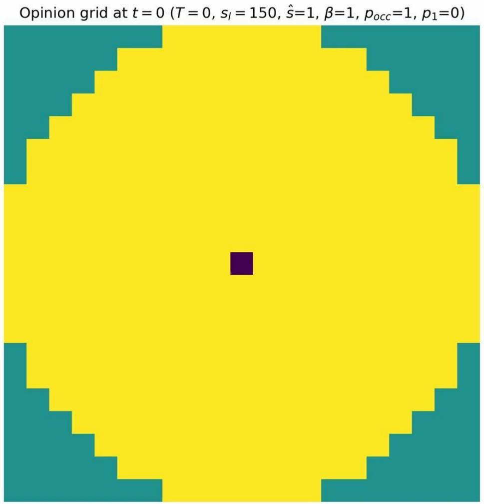
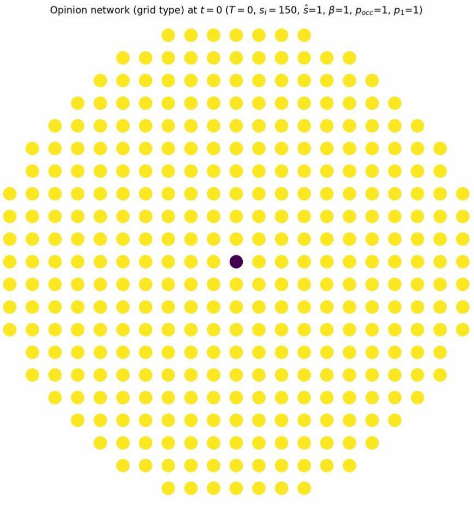
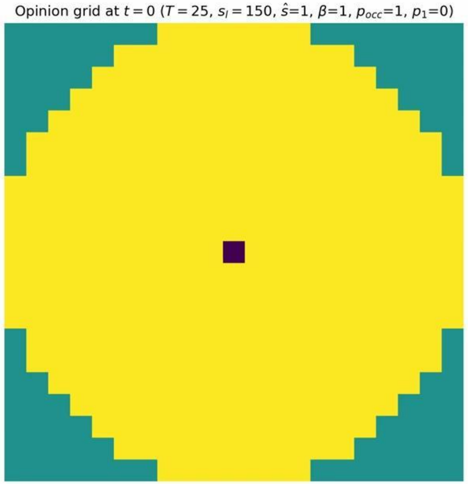
Results. 5

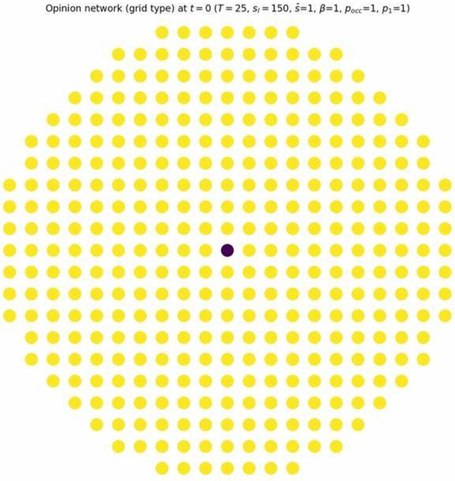
Network / CA equivalency

- Similar behaviour
- Network slightly slower







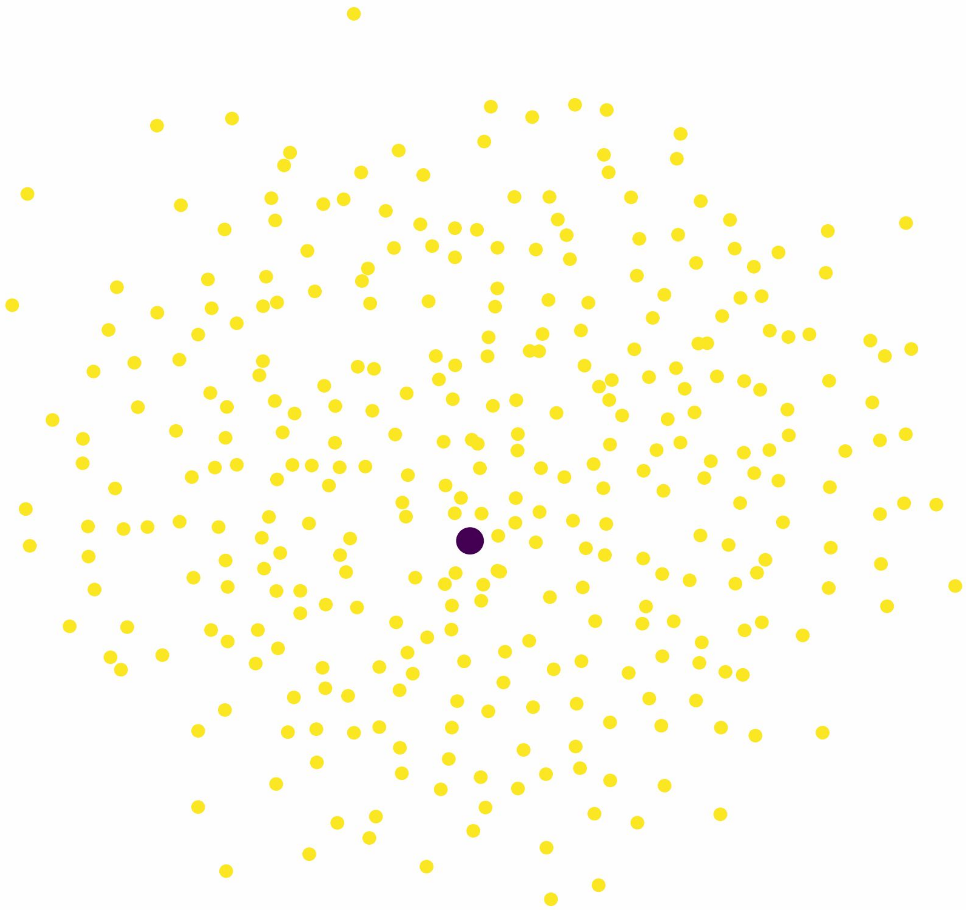


Results. 6

Next step: Barabasi-Albert Network

- More realistic
- Scale free, preferential attachment
- Same social impact formula
- Factor C_L
    - Ratio degree leader node : mean node degree
- Longest path length

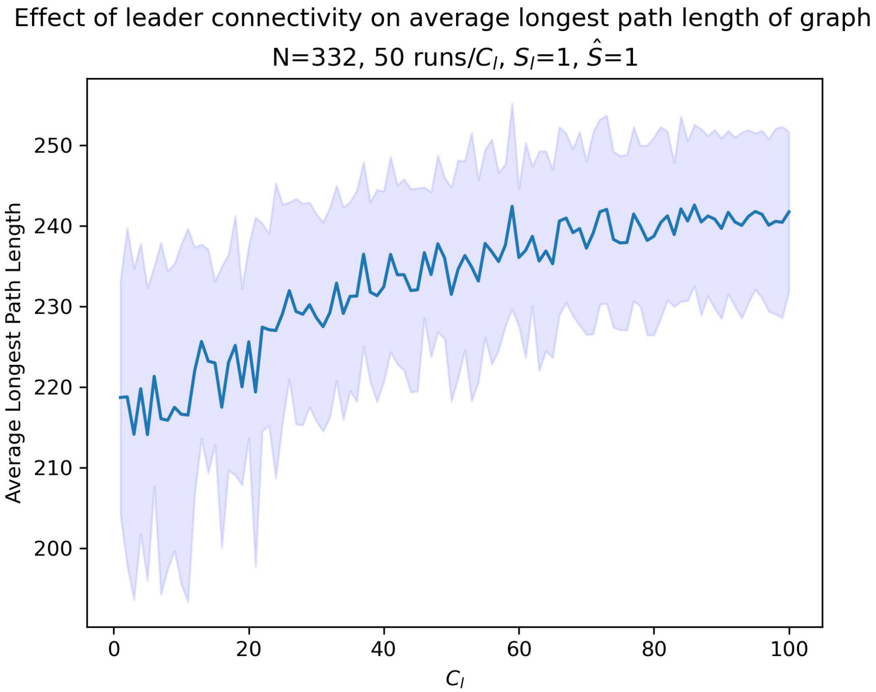
Opinion network (barabasi-albert type) at $t = 0$ ($T = 0$, $c_l = 10$, $<k> = 7$, $\beta = 1$, $p_{occ} = 1$, $p_1 = 1$)

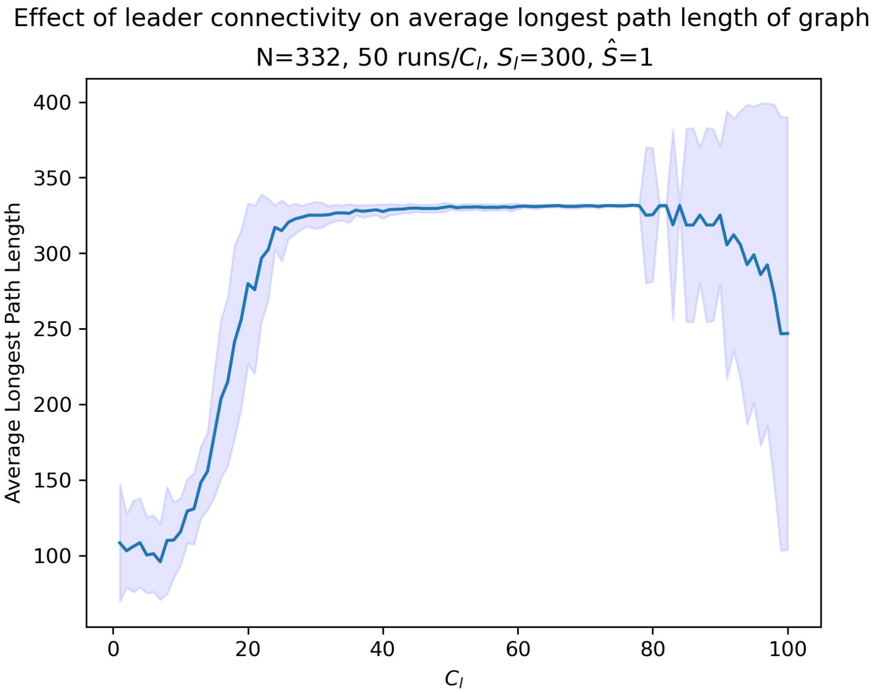


Longest path length= 0

Results. 7

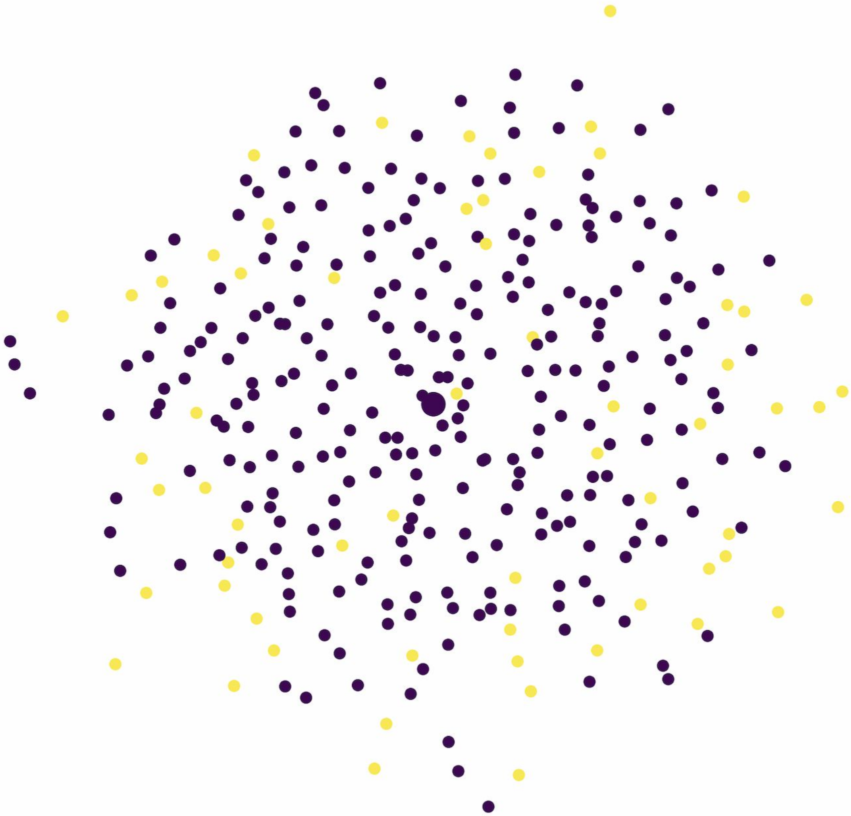
Leader strength



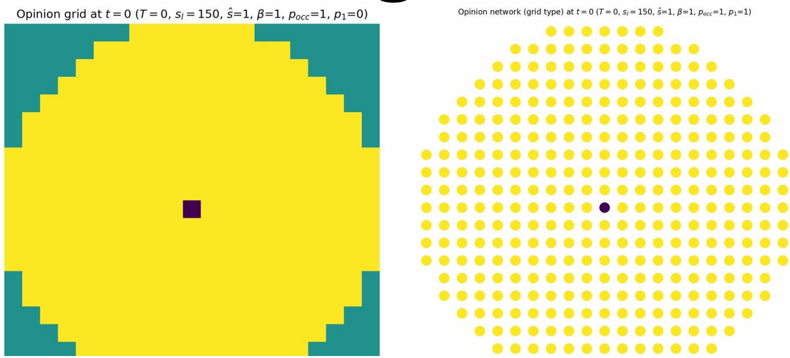


Network findings

- Similar patterns of spreading
- Leader influence
- Supercritical regime
    - Isolated nodes



Main findings



CAs can model opinion change

Findings replicated
Leader influence and temperature contribute to clustering and SO
Critical temperature threshold
SOC

CAs and networks can be bridged

Networks allow a more complete representation

Similarities and differences

Thank You

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