eqs

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$$\text{Determinantes:} \tag{1}$$

$$sgn_n(\cdots, j_{p-1}, j_p, j_{p+1}, \cdots) := (-1)^{n-p} sgn_{n-1}(\cdots, j_{p-1}, j_{p+1}, \cdots); \tag{2}$$

$$\det(A) := \sum_{(j_1, j_2, \cdots, j_n)} sgn(j_1, j_2, \cdots, j_n) A_{1,j_1} A_{2,j_2} \cdots A_{n,j_n}; \det(AB) = \det(A)\det(B); \tag{3}$$

$$\det(A(k_1, k_2, \cdots, k_n)) := \sum_{(j_1, j_2, \cdots, j_n)} sgn(j_1, j_2, \cdots, j_n) A_{k_1,j_1} A_{k_2,j_2} \cdots A_{k_n,j_n}; \tag{4}$$

$$\det(A(\cdots, k_p, \cdots, k_q, \cdots)) = -\det(A(\cdots, k_q, \cdots, k_p, \cdots)); \tag{5}$$

$$\det(A(k_1, k_2, \cdots, k_n)) = sgn(k_1, k_2, \cdots, k_n)\det(A); \tag{6}$$

$$\det(A) = \frac{1}{n!} \sum_{(k_1, k_2, \cdots, k_n)} \sum_{(j_1, j_2, \cdots, j_n)} sgn(k_1, k_2, \cdots, k_n) sgn(j_1, j_2, \cdots, j_n) A_{k_1,j_1} A_{k_2,j_2} \cdots A_{k_n,j_n}; \tag{7}$$

$$\det(A^T) = \det(A); C_{k,j_k} = \alpha A_{k,j_k} + \beta B_{k,j_k} \Rightarrow \det(C) = \alpha\det(A) + \beta\det(B); \tag{8}$$

$$A = \begin{bmatrix} B & * \\ \mathbb{O} & b \end{bmatrix} \Rightarrow \det(A) = b\det(B); \det(A) = \sum_{k=1}^{n} A_{j,k} cof(A)_{j,k} = \sum_{j=1}^{n} A_{j,k} cof(A)_{j,k}; \tag{9}$$

$$cof(A) = \left(cof(A)_{j,k}\right) = \left((-1)^{j+k}\det(A^{(j,k)})\right); (A^{-1})_{j,k} = \frac{cof(A)_{k,j}}{\det(A)}; \tag{10}$$

$$A|x\rangle = |y\rangle, |y\rangle \neq |\oslash\rangle, |x\rangle_j = \frac{\det(K^{(j)})}{\det A}, K^{(j)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & A_{k-1,j-1} & |y\rangle_{k-1} & A_{k-1,j+1} & \cdots \\ \cdots & A_{k,j-1} & |y\rangle_k & A_{k,j+1} & \cdots \\ \cdots & A_{k+1,j-1} & |y\rangle_{k+1} & A_{k+1,j+1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}; \tag{11}$$

$$\langle L_j^A| \to \langle L_j^A| + c\langle L_l^A| \Rightarrow \det(A') = \det(A); |L_j^A\rangle \to |L_j^A\rangle - (A_{j,k}/A_{k,k})|L_k^A\rangle, j = k+1, \cdots, n; \tag{12}$$

$$\det \begin{bmatrix} T_{1,1} & T_{1,2} & T_{1,3} & \cdots & T_{1,n} \\ 0 & T_{2,2} & T_{2,3} & \cdots & T_{2,n} \\ 0 & 0 & T_{3,3} & \cdots & T_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & T_{n,n} \end{bmatrix} = \Pi_{j=1}^{n} T_{j,j}. \tag{13}$$

$$\text{Vetores:} \tag{14}$$
$$\sum_j a_j|w_j\rangle = |\emptyset\rangle \Rightarrow a_j = 0 \forall j \Rightarrow |w_j\rangle \text{ são LI;} \tag{15}$$
$$ext(\{|v_j\rangle\}_{j=1}^n) := \left\{\sum_{j=1}^n a_j|v_j\rangle \text{ para } a_j \in \mathbb{F}\right\}; \quad \text{base} = LI + ext(\{|v_j\rangle\}_{j=1}^n) = V; \tag{16}$$
$$\langle v| \left(\sum_j a_j|w_j\rangle\right) = \sum_j a_j\langle v|w_j\rangle; \langle v|w\rangle = (\langle w|v\rangle)^*; \langle v|v\rangle \geq 0 \text{ e } \langle v|v\rangle = 0 \Rightarrow |v\rangle = |\emptyset\rangle; \tag{17}$$
$$\langle v|w\rangle := |v\rangle^\dagger|w\rangle; \langle A|B\rangle_{hs} := Tr(A^\dagger B); ||v|| := \sqrt{\langle v|v\rangle}; |w_j\rangle := \frac{|v_j\rangle - \sum_{k=1}^{j-1}\langle w_k|v_j\rangle|w_k\rangle}{||(|v_j\rangle - \sum_{k=1}^{j-1}\langle w_k|v_j\rangle|w_k\rangle)||}; \tag{18}$$
$$|v\rangle = \sum_{j=1}^{\dim V}\langle b_j|v\rangle|b_j\rangle; \mathcal{H} = \left(\mathbb{C}^n, \langle\psi|\phi\rangle = |\psi\rangle^\dagger|\phi\rangle\right); \langle v|v\rangle\langle w|w\rangle \geq \langle v|w\rangle\langle w|v\rangle \tag{19}$$
$$\tag{20}$$

$$\text{Operadores:} \tag{21}$$
$$A(\sum_j c_j|v_j\rangle) = \sum_j c_j A(|v_j\rangle); (\alpha A + \beta B)(|v\rangle) = \alpha A(|v\rangle) + \beta B(|v\rangle); \tag{22}$$
$$A : V \to W, A(|v_j\rangle) = \sum_{k=1}^{\dim W} A_{k,j}|w_k\rangle, \text{ para } j = 1, \cdots, \dim V; \tag{23}$$
$$A|a\rangle \propto |a\rangle =: \alpha_a|a\rangle; \det(A - \alpha_a\mathbb{I}) = 0; c_n\alpha_a^n + c_{n-1}\alpha_a^{n-1} + \cdots + c_2\alpha_a^2 + c_1\alpha_a + c_0 = 0; \tag{24}$$
$$(|v\rangle, A|w\rangle) = (A^\dagger|v\rangle, |w\rangle), \forall|v\rangle, |w\rangle \in V \tag{25}$$
$$P_W(|v\rangle) := \sum_{j=1}^{\dim W}\langle w_j|v\rangle|w_j\rangle; P_W = \sum_{j=1}^{\dim W}|w_j\rangle\langle w_j|; P_W^\dagger = P_W; P_W + P_{W^\perp} = \mathbb{I}_V; \tag{26}$$
$$A \circ A^\dagger = A^\dagger \circ A; A = \sum_a \alpha_a P_a; \sum_a P_a = \mathbb{I}_{\mathcal{H}}; AP_a = \alpha_a P_a; \tag{27}$$
$$\langle A\rangle_\psi := \langle\psi|A|\psi\rangle; [A, B] := A \circ B - B \circ A; \{A, B\} := A \circ B + B \circ A; \tag{28}$$
$$A = \sum_j a_j P_j \text{ e } B = \sum_j b_j P_j \Leftrightarrow [A, B] = {}_{\mathcal{H}}; \tag{29}$$
$$A^\dagger \circ A = A \circ A^\dagger = \mathbb{I}_{\mathcal{H}} \Rightarrow (A|v\rangle, A|w\rangle) = (|v\rangle, |w\rangle); \tag{30}$$
$$Tr(AB) = Tr(BA); Tr(UAU^\dagger) = Tr(A); \sum_{j=1}^d\langle a_j|A|a_j\rangle = \sum_{k,l=1}^d \delta_{l,k}\langle b_k|A|b_l\rangle; Tr(|\eta\rangle\langle\xi|) = \langle\xi|\eta\rangle; \tag{31}$$
$$\langle\psi|A|\psi\rangle \geq 0, \forall|\psi\rangle \in \mathcal{H} \therefore A \geq {}_{\mathcal{H}} \Rightarrow A = A^\dagger; \alpha_a \geq 0; B^\dagger \circ B \geq {}_{\mathcal{H}}; X \geq Y \Rightarrow \langle X\rangle_\psi \geq \langle Y\rangle_\psi \tag{32}$$

Funções matriciais: (33)

$$f(A) \neq (f(A_{j,k})); f(x) = \sum_{j=0}^{\infty}\left(\frac{1}{j!}\frac{d^j f(x)}{dx^j}\right)x^j; e^A = \mathbb{I}_{\mathcal{H}} + A + \frac{A^2}{2} + \frac{A^3}{3!} + \cdots; \tag{34}$$

$$e^{cB}Ae^{-cB} = A + c[B,A] + \frac{c^2}{2!}[B,[B,A]] + \frac{c^3}{3!}[B,[B,[B,A]]] + \cdots; \tag{35}$$

$$(\vec{n}\cdot\vec{\sigma})^2 = ||\vec{n}||^2\sigma_0; e^{i\theta\vec{n}\cdot\vec{\sigma}} = \sigma_0\cos\theta + i\vec{n}\cdot\vec{\sigma}\sin\theta; e^{c\sigma_z}\sigma_x e^{-c\sigma_z} = \sigma_x\cos(2ic) + \sigma_y\sin(2ic); \tag{36}$$

$$[A, A^\dagger] = \mathbb{O}_{\mathcal{H}} \Rightarrow A = \sum_a aP_a \Rightarrow f(A) := \sum_a f(a)P_a; \tag{37}$$

$$\rho = \sum_r rP_r \Rightarrow S(\rho) = -Tr(\rho\log_2(\rho)) = -\sum_r r\log_2(r); \tag{38}$$

$$H^\dagger = H, \theta\in\mathbb{R}, U := e^{i\theta H} \Rightarrow U^\dagger = U^{-1}. \tag{39}$$

Espaços compostos: (40)

$$\mathcal{H}_{ab} = \mathcal{H}_a\otimes\mathcal{H}_b \to |c_{jk}\rangle := |a_j\rangle\otimes|b_k\rangle, j = 1,\cdots,\dim\mathcal{H}_a, k = 1,\cdots,\dim\mathcal{H}_b; \tag{41}$$

$$A\otimes B = \begin{bmatrix} A_{1,1}B & \cdots & A_{1,d_a}B \\ \vdots & \cdots & \vdots \\ A_{d_a,1}B & \cdots & A_{d_a,d_a}B \end{bmatrix}; (A\otimes B)(C\otimes D) = AC\otimes BD; \tag{42}$$

$$A = \sum_{j,k=1}^{n}\langle\beta_j|A|\beta_k\rangle|\beta_j\rangle\langle\beta_k|; C = \sum_{j,p=1}^{n}\sum_{k,q=1}^{m}C_{jk,pq}(|\alpha_j\rangle\otimes|\beta_k\rangle)(\langle\alpha_p|\otimes\langle\beta_q|); \tag{43}$$

$$Tr_{\mathcal{H}_b}(C) := \sum_{l=1}^{d_b}(\mathbb{I}_a\otimes\langle\beta_l|)C(\mathbb{I}_a\otimes|\beta_l\rangle) = \sum_{j,p=1}^{n}\left(\sum_{l=1}^{m}C_{jl,pl}\right)|\alpha_j\rangle\langle\alpha_p| =: \sum_{j,p=1}^{n}A_{j,p}|\alpha_j\rangle\langle\alpha_p|; \tag{44}$$

$$A = UJ = KV, J = \sqrt{A^\dagger A}, K = \sqrt{AA^\dagger}, U^\dagger = U^{-1}, V^\dagger = V^{-1}; \tag{45}$$

$$A = UDW, U^\dagger = U^{-1}, W^\dagger = W^{-1}, D = diag(d_1,\cdots,d_n) \geq \mathbb{O}_{\mathbb{C}^n}; \tag{46}$$

$$|\Psi\rangle = \sum_{j=1}^{\dim\mathcal{H}_a}\sum_{k=1}^{\dim\mathcal{H}_b}c_{j,k}|\alpha_j\rangle\otimes| = \sum_{j=1}^{\min(\dim\mathcal{H}_a,\dim\mathcal{H}_b)}d_j|\tilde{\alpha}_j\rangle\otimes|\tilde{\beta}_j\rangle; \tag{47}$$

$$A := Tr_{\mathcal{H}_b}(P_\Psi) = \sum_j d_j^2|\tilde{\alpha}_j\rangle\langle\tilde{\alpha}_j|; B := Tr_{\mathcal{H}_a}(P_\Psi) = \sum_j d_j^2|\tilde{\beta}_j\rangle\langle\tilde{\beta}_j| \tag{48}$$